Aalto University
School of Science
Degree Programme of Engineering Physics and Mathematics

Markus Karppinen

Steering collective motion

Master's Thesis
Espoo, January 18, 2013

Supervisor: Professor Jouko Lampinen
Instructor: Professor Jari Saramäki

Aalto University
School of Science
Degree Programme of Engineering Physics and Mathematics

ABSTRACT OF
MASTER'S THESIS

Author:	Markus Karppinen		
Title:			
Steering collective motion			
Date:	January 18, 2013	**Pages:**	54
Professorship:	Computational Science	**Code:**	BECS-114
Supervisor:	Professor Jouko Lampinen		
Instructor:	Professor Jari Saramäki		

Collective motion is an umbrella term for both biological and non-organic coherent motion, in which tens to tens of millions units take part in. The huge fish schools that fishing ships pursue and the nightmarish legions of locusts which destroy entire countries' worth of crops are just a few examples of collective motion in nature that have a direct effect on us humans.

This thesis focuses on the complex behavior of collective motion and studies how such movement can be steered. As a tool, the original Vicsek model for simulating collective behavior is used. An agent-based model, the Vicsek model was introduced in 1995 and has been extensively utilized and studied since. The Vicsek model consists of units that move independently but prefer to take the common movement direction of their neighbors. Although it is a simplified model, the Vicsek model exhibits flocking behavior that is similar to what is observed in nature.

The results of this thesis show that in this context, collective motion of hundreds of units is greatly affected by just a small percentage of special units, called leaders. The leaders don't adhere to the common rules of the other units, but move in a constant direction. It is observed that the relative amount of leaders needed to steer the entire flock actually decreases as the flock size grows or if we wait sufficiently long. This leads to the conclusion that in the limit of an infinite system size, a finite amount of leaders would suffice to control the flock.

Keywords: collective motion, flocking, swarming, locusts, agent-based modeling, Vicsek Model, leader units

Language: English

Aalto-yliopisto
Perustieteiden korkeakoulu
Teknillisen fysiikan ja matematiikan tutkinto-ohjelma

DIPLOMITYÖN TIIVISTELMÄ

Tekijä: Markus Karppinen

Työn nimi:
Kollektiivisen liikkeen ohjaaminen

Päiväys: 8.1.2013 **Sivumäärä:** 54

Professuuri: Laskennallinen tiede **Koodi:** BECS-114

Valvoja: Professori Jouko Lampinen

Ohjaaja: Professori Jari Saramäki

Kollektiivinen liike on kattotermi sekä biologiselle että ei-orgaaniselle koherentille liikkeelle, johon osallistuu kymmenistä kymmeniin miljooniin yksikköä. Luonnossa esiintyvä kollektiivinen liike vaikuttaa suoraan meihin ihmisiinkin, kuten esimerkiksi kalastuslaivastojen metsästämät kalaparvet tai valtavat kulkusirkkaparvet jotka tuhoavat kokonaisten valtioiden viljasatoja osoittavat.

Tämä diplomityö keskittyy kollektiivisen liikkeen kompleksiseen käytökseen sekä tutkii kuinka tällaista liikettä voidaan ohjata. Työkaluna käytetään kollektiivisen käytöksen simuloimiseen tarkoitettua Vicsekin alkuperäismallia. Vicsekin malli on agenttipohjainen malli joka esiteltiin vuonna 1995, ja jota on siitä lähtien käytetty ja tutkittu laajasti. Vicsekin malli koostuu yksiköistä jotka liikkuvat itsenäisesti, mutta suosivat läheisten yksiköiden keskimääräistä liikesuuntaa. Vaikkakin Vicsekin malli on yksinkertaistettu, sen tuottama parvikäytös vastaa luonnossa havaittavaa käytöstä.

Tämän diplomityön tulokset osoittavat satojen yksikköjen kollektiivisen liikkeen käytöksen olevan riippuvainen vain pienen prosenttiosuuden muodostavien erityisten johtoyksiköiden käytöksestä. Johtoyksiköt eivät noudata samoja sääntöjä kuin muut yksiköt, vaan liikkuvat vakiosuuntaan. Kun parven koko kasvaa tai odotettaessa riittävän kauan, koko parvea ohjaamaan tarvittavien johtoyksiköiden suhteellinen lukumäärää vähenee. Tästä voidaan päätellä että äärettömän kokoisessa systeemissä äärellinen määrä johtoyksiköitä riittää kontrolloimaan koko parvea.

Asiasanat: kollektiivinen liike, parveilu, kulkusirkka, agenttipohjainen mallintaminen, Vicsekin malli, johtoyksikkö

Kieli: Englanti

Preface

This work was carried out at Aalto University School of Science, the Department of Biomedical Engineering and Computational Science.

This is as good a time as any to officially thank those who have especially helped in making this thesis a reality, like Paavo Niskala who let me use his code as a foundation for my own; Jelena Luketina who read and commented on the thesis in a crucial phase; Joonas Piili and Ville Backlund, the guys in my room, for advice and company; Lauri Kovanen and Mikko Kivelä for being older and wiser than me but still willing to offer invaluable advice; and of course the rest of the research group.

The biggest thank you, however, goes to a supervisor who has gone above and beyond any duty he might have had and instead mentored, helped and guided me for years. Jari, without your input I would never have made it this far!

Otaniemi, Espoo, January 18, 2013

Markus Karppinen

Symbols

N	total number of units
N_L	number of leaders
ϕ	average normalized velocity
η	noise
$\overrightarrow{v}_i$	the velocity of particle i
$\overrightarrow{x}_i$	the position of particle i
$\vartheta_i(t)$	the angle of the direction of motion at time t
R	interaction range
θ_i	the angle of motion of the ith particle
η_A	the amplitude of noise
ξ	uniformly distributed δ-correlated white noise
A	adjacency matrix
a_{ij}	element in ith row and jth column in the adjacency matrix
β	tuning parameter affecting the strength of influence
L	side length of simulation area
ρ	unit density
p_L	probability that the flock is following the leader

Abbreviations

SPP	self-propelled particle
VM	Vicsek Model
ABM	agent-based model/modeling
AN	angular noise
VN	vectorial noise
BU	backward update
FU	forward update
CSM	Cucker-Smale model

Contents

List of Tables

List of Figures

Chapter 1

Introduction

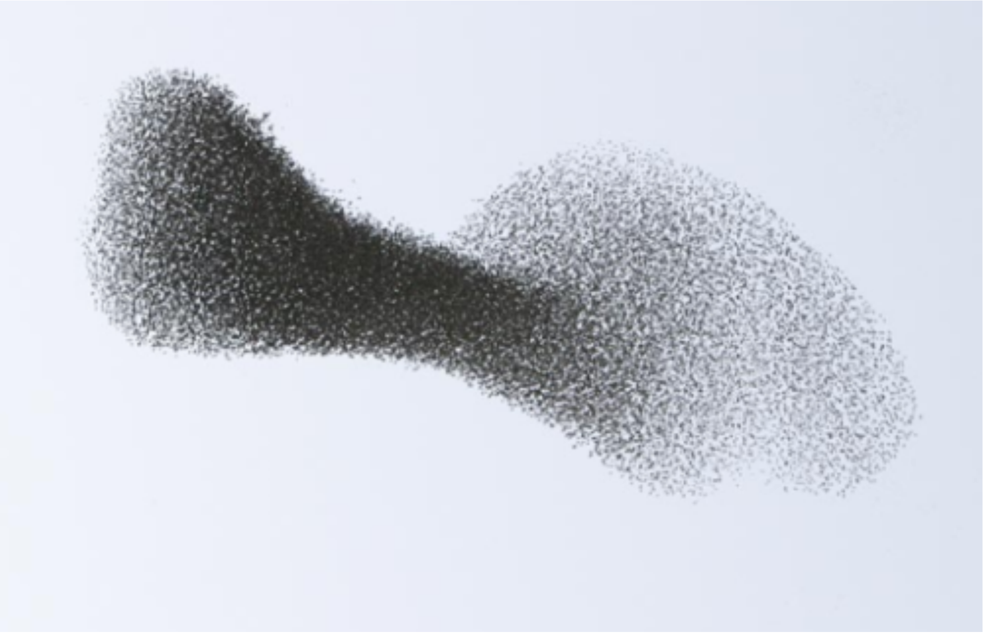


Figure 1.1: **A flock of starlings demonstrating the shape-shifting capabilities of flocks.** Source: `www.flowingdata.com`

Living in a world with crowded cities, coral reefs full of fish schools and skies filled with both insect swarms and bird flocks, synchronized movement surrounds us in all directions. A huge number of units moving in unison, ducking and rolling across skies or ocean floors, is one of nature's magnificent displays of physics in action. Collective motion can be at the simplest defined as just synchronized movement of many units of the same type. A sub-field of collective behavior, collective motion is, however, a lot more diverse than can be expected at first glance. The science of collective motion builds on experimental results in the wild, but also on results from physics and computer simulations. Uniting biology, physics and computer science, it is a try to create an understanding of what makes flocks tick.

What makes flocks, schools and swarms so interesting is the great speed and agility of the group as a whole, i.e. the ability of the group to behave as a single entity while consisting of a great number of simple biological units. Without bumping into each other or into obstacles, the units can achieve great coordination and a flock behaves almost like a fluid. This behavior is characteristic of swarms which are so large that information can't possibly travel from one side of the flock to the other fast enough so that all units could be aware of the actions of all other units. This has led some experts in the past to cite even telepathy as a means of communication in a swarm. However, it has turned out that such behavior can arise from fairly simple rules of interaction between the members of a swarm. Such rules of interaction are readily investigated with computer simulations of swarming behavior.

The forerunner in studying flocking behavior using simulation, C.W. Reynolds, pretty much initiated the field in 1987 with his seminal paper [1]. The next leap forward was the Vicsek model [2] in 1995, where statistical physics was fused with algorithmic treatment of flocks. Since our focus is in steering collective motion, a crucial paper that came out in 2000 is [3], where an actual experiment shows that a flock consisting of golden shiners, a type of small fish, can be hijacked and steered to a location of the experimenter's choosing using trained individuals. There have been some books on collective motion, e.g. [4] which focuses on designing and optimizing complex systems based on models of social insect behavior. I also reference throughout this thesis Vicsek's detailed review paper on collective motion [5], which came out in early 2012, and is pretty much the latest word on this rich field of study.

This thesis focuses on mathematical modeling of collective motion, and computer simulations of swarming behavior. In particular, I aim to study how one can affect collective motion, and how difficult it is to steer a herd or a school in the desired direction by manipulating the behavior of a small number of its units. This setting is inspired by recent empirical results, where actual fish schools were manipulated by inserting robot fish, who could then guide the rest of the school to a desired position in the fish tank [6]. Figure 1.2 is taken from the same paper, and it shows a robotic predator in a tank with a real fish school.

As a tool I use a simple toy model, the Vicsek Model (VM), which has been studied comprehensively [5] since its inception in 1995. VM is an agent-based model, where many units depicting fish or birds interact with all other units within a certain interaction range. They correct their flight direction to match the neighboring units, and thus form flocks that move coherently.

In addition to a better understanding of insect swarms and fish schools in nature, I am motivated by the possibility of man-made, robotic swarms and their control. The robot revolution has been coming for a long time, and it might not be long before simple helper robots are as common-place as cell phones today. Before this can happen though, we will need to be able to program the robots to abide by the subconscious social norms which govern our movement through the thick that is rush hour pedestrian traffic. Additionally, in settings such as disaster relief swarms of robots may prove to be very useful e.g. in locating earthquake victims or inspecting damaged installations deemed too dangerous for humans. Unless technological advances enable such swarms to operate autonomously, they need external guidance and direction, which of course is slow, cumbersome and expensive.

What I intend to do is to study how one can control collective movement. Is it possible to take a toy model of swarming behavior and modify it so that the flock flies in the direction one wants, if only a small fraction of the swarm members obey some directional preference? How big a percentage has to be controlled before we can steer the swarm? Would it be possible to further extend this into the real world and e.g. take control of huge locust swarms and direct their movement away from inhabited areas and so spare the crops of many a poor farmer or to help fish without causing widespread environmental damage in the process? These are questions with large stakes, and I'll try to answer the first two, while leaving the last ones for robot engineers and biologists.

Figure 1.2: **Overhead snapshot of robotic koi predator with live golden shiner school in a tank** [6].

Chapter 2

Theory

2.1 Collective motion - basics

Collective motion is a part of a greater class of phenomena called collective behavior, of which there is a huge variety of systems stretching over many orders of magnitude. Collective behavior can be defined as action where the system's units interact with each other locally, which gives rise to emergent behavior on a system-wide scale. We assume that a system exhibiting collective motion is made out of units that

(a) are quite similar,

(b) can change their direction of movement,

(c) have a specific interaction range and

(d) are subject to varying amounts of noise, i.e. random perturbations in their movement direction [5].

Starlings, for example, form big flocks[1] that can consist of hundreds of birds. They can change their direction of flight quickly, but still they stick close to each other so that the flock looks almost as a blob of liquid when viewed from afar (see Figure 1.1). Another example is human rush hour pedestrian traffic. Everyone is moving at pretty much the same pace and no matter how crowded the street, people still manage to not bump into each other too hard (see Figure 2.1). Thus it's very rare that anyone walks into someone so hard that they would fall down or get injured.

---

[1] Flocks of starlings are actually called murmurations, as a part of the long tradition of many species having their own collective nouns in English.

Collective behavior is not limited only to macrobiological systems, however. Although one of the ways to define life is to ask the question: 'does it move on its own accord?', it has recently been found out that also several physical and chemical systems possess interacting, "self-propelled" units, or particles, known shortly as SPPs. Collective motion has been observed in non-living systems such as shaken metallic rods, simple robots, boats, etc. in addition to living systems such as macromolecules, bacterial colonies, amoeba, cells, insects, fish, birds and mammals including humans.

Figure 2.1: **People crossing the street in the crowded Shibuya, Tokyo, Japan.** Source: `http://blogs.usyd.edu.au/theoryandpractice/2006/10/japan_travelgoue.html`

The mechanisms or "rules" of interaction between many similar units, such as molecules, bacteria, flocks of birds or schools of fish, can be very simple as we'll see, or a complex combination of many simple interactions. These interactions can take place between neighbors in 2D or 3D space, or be mediated through an underlying lattice or network. Also the medium itself can affect the interactions: e.g. in a dissipative medium the interactions may have shorter range, and external fields such as wind or magnetic fields can affect the movement of the particles. As with many complex systems, the complexity itself emerges from unit-to-unit interactions.

With collective motion defined, the choice of methodology must be made carefully. Comparing diverse motion patterns, we have to ask ourselves, are these system-specific or general? In biological sciences it is common that all cases differ, with every new case being mostly unique. For example all bird species have their own nesting and mating behavior, which have developed with time. In statistical physics, however, many seemingly unique phenomena can be grouped under just a few universality classes. For example, the behavior near a second order phase transition coincides in very different phenomena, such as magnetic systems, superfluid transition and alloy physics. As a physicist I opt for the second approach, but in reality the truth probably lies somewhere in between.

There are a lot of good reasons why swarming and flocking is so widely observed in nature, i.e. the benefits greatly outweigh the potential drawbacks of staying closely together. When the units stick together, they are a lot safer when a predator appears than what they would be if they were on their own. Not only are big fish schools, for example, harder to attack with the flock outsizing any marine predator, but if need be the flock can disperse and hence significantly lower the probability of a single unit being caught. Flocking is also beneficial for exploration for resources or hunting, as many units can cover a bigger area and communicate any finds to the rest of the flock. Decision-making is also improved in larger groups, as in many cases practically any unit can initiate a group movement and hence reach the intended goal without scattering or conflicts taking place.

In the previous few decades a lot of information has amassed due to the myriad of experiments and observations done in the field [5]. We now know that movement and a tendency to adopt the direction of motion of the neighbors is the main reason for collective ordered motion. It's been noted that very similar behaviors occur in systems of very different origin and both 2D (land-based) and 3D (aquatic or marine) systems contain many more similarities than one would expect. This suggests the possibility of the existence of universal classes of collective motion patterns. Also, boundary conditions significantly affect the essential features of flocking. Collective decision making is usually made in a globally highly disordered, locally moderately ordered state.

The simplest forms of collective motion can be found in systems where all members can be considered identical. Many insect, fish and bird species live in big groups, where they may be unable to recognize each other at the level of individuals. In these cases leadership is usually only temporary (e.g. for as long as an informed individual leads the flock to food), or non-existent, i.e. the movement is dictated by a general consensus that is formed by some units moving in a random direction and other units imitating the movement of them. Mammals, on the other hand, have the capacity to recognize individuals, and hence can form hierarchical groups, where certain individuals permanently act as leaders.

It is thus an important question how flocks of animals form the common decision to move in a coherent way. Two different mechanisms have been suggested in the literature [7]: "Consensus decision" is a process in which the members of a group choose between two or more mutually exclusive actions with the aim of reaching a consensus. "Leadership" on the other hand is the initiation of new directions of locomotion by one or more individuals, which are then followed by other group members. As mentioned before, such leadership can be either permanent or temporary, depending on the species and situation.

Generally, collective decision-making can be divided into two rules: 'individual-based' and 'self-organized' [5]. Individual-based consists of differences in social status, physiology, inner state, etc. Self-organization refers to passive interactions and simple, automated responses among individuals.

2.2 The physics of collective motion

In various models of flocking, coherent motion emerges through a transition from an unordered state to an ordered as a function of the parameters of the models. Thus to understand the theory of collective motion we need to know the physics behind such transitions. For example, here has been quite a lot of discussion about whether one of the simplest of models depicting collective motion, the Vicsek model, introduced in [2], has a continuous or discontinuous phase transition where the disordered motion of the flock is replaced by unidirectionality. But what exactly is a phase transition and what do such transitions have to do with SPPs?

Generally, a phase transition is a process during which a system, consisting of a huge number of interacting particles, undergoes a transition from one phase to another, typically from a disordered into an ordered phase, as a function of one or more external parameters [5]. A typical example of this process is the freezing of water, when a liquid becomes a solid. The degree of order and symmetry of a phase is characterized by the order parameter. Mathematically, this value is usually zero in the disordered phase and non-zero in the ordered phase.

When it comes to collective motion, the parameter usually chosen is the average normalized velocity ϕ,

$$\phi = \frac{1}{Nv_0} \left| \sum_{i=1}^{N} \overrightarrow{v}_i \right| \tag{2.1}$$

where N is the total number of the units, v_0 is the average absolute velocity of the units in the system and $\overrightarrow{v}_i$ is the vector of velocity of particle i.

If the motion is disordered, the velocities of the individual units point in random directions and average out resulting in a velocity vector of small magnitude, whereas for ordered motion the velocities all add up to a vector of absolute velocity close to Nv_0. Thus the order parameter for large N can vary from about 0 to about 1.

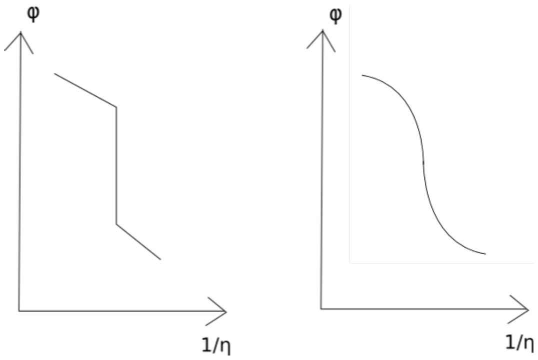


Figure 2.2: **The behavior of the order parameter $\phi \in [0, 1]$ as a function of system noise η.** The left side represents a first-order phase transition, defined by the non-continuous behavior exhibited. The right side represents a second-order phase transition, and is smooth.

There are two kinds of phase transition, first and second order, named so due to the behavior of the derivatives of the parameter. If the order param-

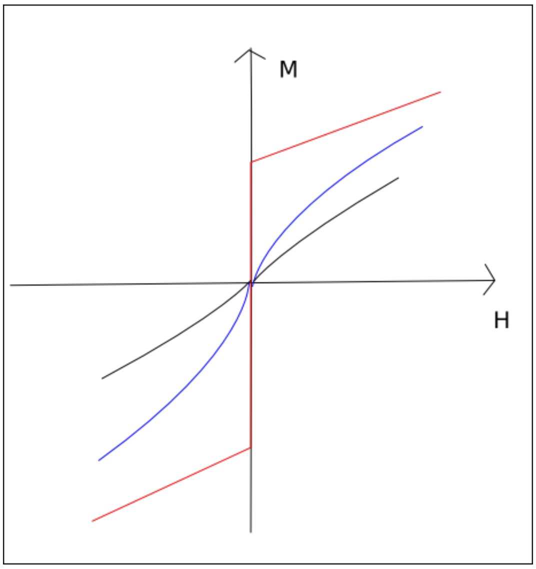


Figure 2.3: **The behavior of magnetization M in an external field H, an example of system size affecting the behavior of the system.** The gray line is a small-sized system, the blue one is a large-sized system, and the red one is an infinite-sized system. The bigger the system, the more ideal the transition.

eter changes discontinuously during the phase transition, the transition is defined as a first-order transition, and it contains a latent amount of energy, e.g. latent heat. Latent heat is the heat released or absorbed by a body or a thermodynamic system during a process that occurs without a change in temperature. Sticking to the example of water, when melting ice the temperature does not rise above zero until all the ice has been converted into water. Only then does the energy start going to heating the water.

In second-order transitions, on the other hand, the order parameter changes continuously, while its derivative is discontinuous. Second-order transitions are always accompanied by large fluctuations of some relevant quantities at the transition point. See Fig. 2.2 for a visual example. Phenomena associated with a continuous phase transition are called critical phenomena. This is because the transition takes place at an exact critical point. Near the critical point, the behavior of the quantities describing the system are very sensitive to small perturbations, and are characterized by the so-called critical exponents.

For example, if we consider a spin system not unlike one described by the

Ising model [8], we have particles with either spin up or down, with their collective behavior accounting for macroscopic magnetization of a physical body. Depending on the temperature the system has two separate phases; a disordered one where the particles have random spin, and an ordered one where the particles have the same spin, also known as the magnetized phase. At low temperatures the spins of the particles align themselves to imitate the spins of nearby particles, but as temperature rises the couplings of the spins are broken by thermal motion. If we have an external field, the spins align themselves according to the field. System size is a key factor in how the magnetization behaves: the bigger the system, the steeper the transition from one phase to another, as can be seen in Figure 2.3. For finite-size systems, the transition is always somewhat smeared-out, and becomes sharp at the thermodynamic limit of infinite system size.

A key physical element of collective motion is noise (η), the added randomness to the direction of motion, caused by the unideal nature of interactions. When it comes to systems of SPPs, noise plays the order-destructive role of temperature. The flock behaves less coherently if the units have bigger uncertainty of the locations and velocities of their neighbors, and will break down if the value of the noise becomes too large. Interestingly, various different physical systems follow similar laws and even their different critical exponents are related to each other. It is worth noting that the results of statistical physics are only exact in the thermodynamic limit, i.e. when the number of particles of the system tends to infinity. Often the number of units participating in collective motion is far from the huge quantities of particles that statistical physics usually deal with. Most real-life observations and experiments involve this mesoscopic scale of a few dozen to a few thousand SPPs.

2.3 Observations and experiments

The main difficulty in observations and experiments concerning collective motion is to keep track of all the trajectories and in some cases velocities of all the particles. This is because there are many particles and they are both almost indistinguishable and moving very fast in unpredictable ways. We have only recently started to understand the rich variety of phenomena that are connected to collective motion and the fact that also non-living particles can participate. Still, there have been many ingenious experiments devised to observe this fascinating phenomenon.

2.3.1 Inanimate objects

This thesis does not focus on collective motion at the cellular level or in inanimate objects, but it is worth noting that there have been some relevant results. In a recent experiment [9], researchers have studied inanimate objects using commercial radio-controlled boats moving in a circular pool, interacting through inelastic collisions only. Using varying amounts of noise in the communication between boats, various kinds of patters were recorded, such as clustering, jamming, disordered and ordered motion, depending on the noise level. It was also found that a few steerable boats, which acted as leaders, were able to "hijack" the group and steer its movements. To do this, it was enough to manipulate just 5 to 10 % of the boats.

When it comes to inanimate objects, such as metallic rods shaken in a container where they form swarm-like clusters, the assumption that only a few parameters and factors dictate the emergence of collective motion is increasingly supported [5]. It has turned out that one of these parameters is the particle density, or the density of the objects, inanimate or living, that exhibit collective motion.

2.3.2 Insects

One of the prime examples that comes to mind when mentioning collective motion are ants. They use pheromone trails to create tracks between the nest and food sources and move efficiently between the two. For example, New World army ants stage huge swarm raids with up to 200 000 individuals forming trail systems that are up to 100 meters long and 20 meters wide [5].

However, the chemical signaling used by ants is somewhat a special case, and coordinated movement in insects may be driven by mechanisms as simple as physical collisions. As an example, it has been shown that coordinated marching behavior in marching locusts strongly depends on the animal-density, and is mostly caused by the locusts' cannibalistic tendencies. The locusts are actually just trying to eat each other, but the emergent phenomenon is a huge swarm moving in a coordinated way. The transition between disordered and ordered states has been shown to exhibit hysteresis and a behavioral first order phase transition [10].

Figure 2.4: **A farmer caught in a huge locust swarm.** Source: `http://knowingthese.blogspot.fi/2010/06/insect-swarms-problem.html`

2.3.3 Fish

Two terms, "shoal" and "school" are commonly used to describe collective groups of fish. These terms carry different meanings. In a shoal fish relate to each other in a looser manner than in a school, and they might include fish of various species. Shoals are more vulnerable to predator attack, whereas in a school fish swim in a more tightly organized way considering their speed and direction [5].

Fish schools are typically leaderless aggregations of individuals following selfish survival strategies, with only a limited range of observation. Only the few outer layers of the school can actually access external stimuli from outside the school. The majority of fish relay on social cues from their neighbors for information about the school's environment. For example, an individual fish that does not perceive a predator directly can react to a neighbor turning fast and compensate its movement to match that of its neighbors, thus staying with the school and avoiding the predator [6].

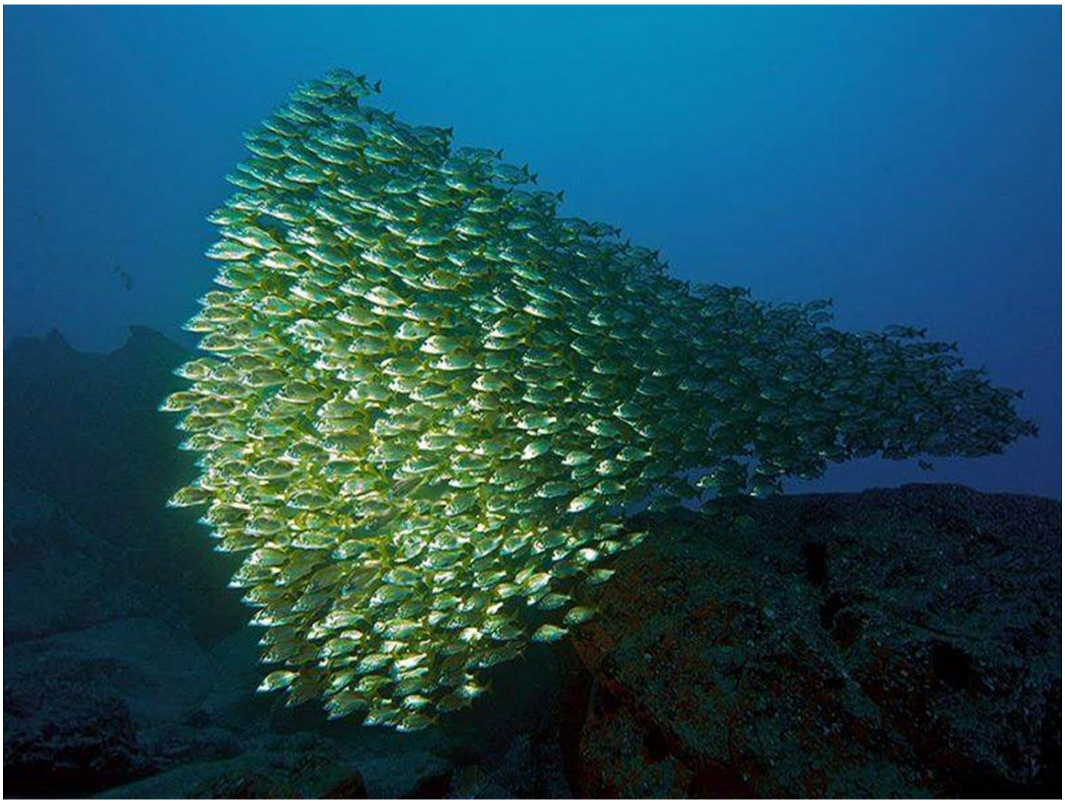


Figure 2.5: **A close-swimming fish school can manage lightning-quick movements without the fish bumping into either other fish or obstacles.** Source: `http://caesarom.com/`

The trajectories of young fish in a school were studied in [11]. Both individual and collective behavior were studied as a function of animal density and a transition from disordered to ordered motion was noted. It was experimentally shown that fish behave like attracting entities, selecting the mean orientation of their neighbors. The interactions between schooling golden shiners were studied in [12]. It was found that changes in speed are the main form of interaction between the fish, and alignment only modulates the strength of speed regulation, instead of being a force in itself. The forces that do play a role are attraction and repulsion, as could be expected from other results.

Herring populations spanning over tens of thousands of square kilometers were observed during spawning using a technique called 'Ocean Acoustic

Waveguide Remote Sensing" (OAWRS) [13]. A rapid transition from disordered to highly synchronized behavior was observed at a critical population density. It was found that a small set of leaders can significantly influence the actions of a much larger group. Pretty much the same conclusion was reached when Reebs et al. [3] trained twelve golden shiners to expect food around midday in one of the brightly lit corners of their tanks. Then the informed individuals were placed in uninformed shoals. It was examined whether the trained individuals could lead their uninformed shoal-mates to the food-site. Surprisingly, as few as a single informed individual was enough to guide the entire shoal to the food. Interestingly, the shoals never split up into smaller parts and they were always led by the same fish.

2.3.4 Birds

When bird movement was studied through a frame-by-frame analysis of high-speed film of sandpiper flocks in [14], it was argued that any individual can initiate a flock movement. When initiated, the movement propagates through the flock in a wave-like form radiating in all directions from the initiation site. Very high propagation speeds are achieved by individuals observing approaching maneuver waves and timing their own execution to coincide with its arrival, just as sports fans doing "the wave" demonstrate.

In the EU FP6 NEST project Starflag [5], the team measured the 3D positions of individual European starlings in flocks containing up to 2600 birds using stereometric and computer vision techniques. They studied the angular orientation of each bird and its nearest neighbors and found that starlings only interact with their 6-7 closest neighbors. This is known as a "topological approach", in contrast to "metrical approach" where the birds interact with others within a given distance. There have been opposite views expressed since, and the interaction mechanism is still unclear, but it is possible that the approach differs from species to species.

Studies on homing birds have shown that when a pair of birds is flying together the actions depend on the amount of difference in preferred direction. With small disagreements most pairs studied averaged out their routes, but upon reaching a critical threshold in disagreement, the pairs either split up or one of the birds became the leader [15]. It was also observed that almost all pairs navigated more efficiently than the individuals they were composed of, even if there was no leadership present.

2.3.5 Mammals

Leadership in zebras has been divided into effects of identity, i.e. dominance and kinship relations, and inner state, i.e. whether the individual is in its lactation period. As investigated in [16], lactating females initiated group movement more often than non-lactating ones, which points towards inner states having an important role in leadership. Put simply, those individuals with the greatest motivation to move will most vigorously try to initiate group movements. Beef cows, however, seem to initiate short-distance travel and foraging movements in a graded manner, i.e. the higher an individual is in the group hierarchy, the bigger influence it can have on the movements of the herd [17].

2.3.6 Robots

The first steps towards robots interacting amongst human populace have been taken by building and studying simple robots following a few basic rules: avoidance, aggregation, dispersion and homing. With these rules the scientists were able to achieve flocking behavior. Turgut et al. [18] examined the swarming of units which were equipped with a digital compass, an infrared-based short range sensing system, and other tools for sensing the direction of other units. They found that the main factor for defining the size of the swarm is the communication range between units, and that the size is robust against the amount and nature of the noise disturbing the sensing systems and the number of neighbors a unit has.

2.4 Models and simulations

2.4.1 A quick look at agent-based modeling

There are many different models used for studying and simulating collective motion in a variety of dimensions, and the most use an agent-based approach [5]. Agent-based modeling (ABM) is a common term for computer simulation models where the system consists of several autonomous units, agents, that follow simple rules, which lead to system wide complex behavior by way of emergence from the micro level to the macro level. As an example of an ABM, Figure 2.6 (taken from a special assignment done by the author) shows a model called the Sugarscape, where units eat, breed and evolve.

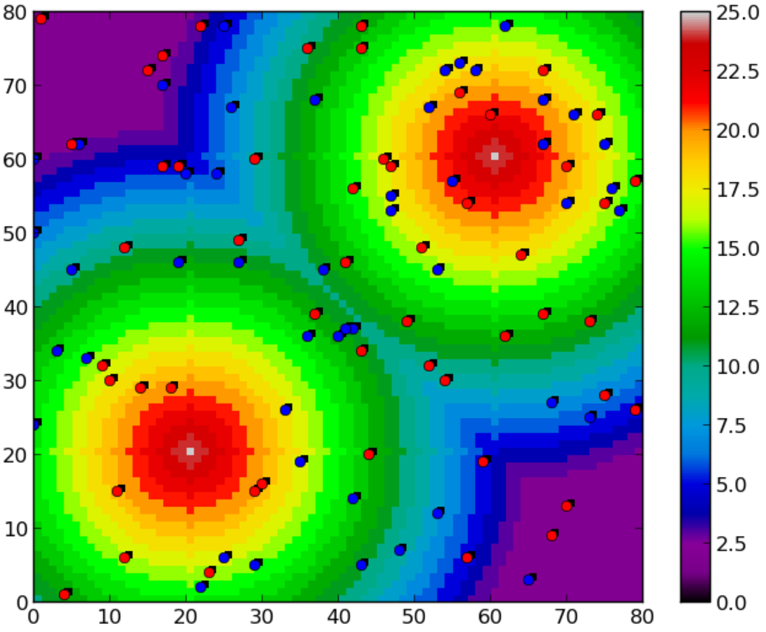


Figure 2.6: **A picture of the Sugarscape, as an example of an ABM.** The red and blue dots are units that scavenge for food (indicated by the background color of the lattice), and try to survive. [19]

Using ABM to study collective motion is useful, as it links the simple rules followed by the agents and the emergent behavior of the swarm. Here's a list of common rules applied in many agent-based models used in studying collective motion [5]:

(i) a long-range force avoiding being alone, e.g. moving towards the center of the swarm's mass

(ii) short-range repulsive force aiming to avoid collision with flock-mates and obstacles

(iii) adjusting the velocity vector according to the rest of the flock, e.g. taking the direction of the neighboring units

(iv) noise, i.e. random added perturbations to the unit's movement

(v) some kind of optional drag force caused by the medium in which the individuals move

There are some limitations in ABMs; different combinations of rules and parameters may provide similar patterns and collective behaviors. Hence, it's not enough to provide a rule and a parameter set and demonstrate that they reproduce observed behavior. This will not prove that a certain biological system obeys some given principles [20]. On the other hand, it has been demonstrated in [21] that the same rule and parameter set may result in different collective behavior, even in the same system, depending on the history of the system. It was however, noted in the same paper that their model led to the first evidence for collective memory in animal groups, i.e. the previous history of group structure influencing the collective behavior exhibited. Using ABMs might be a double-edged sword, but they still remain a useful tool for testing emergent behavior.

2.4.2 Precursors: The Reynolds and Aoki models

The model of Reynolds [1] is one of the first widely-known flocking simulations. He studied bird-like particles, called "boids", moving along trajectories defined by differential equations. He took into account only three types of interactions:

(a) Separation: trying to maintain a safe distance in order to avoid crowding or colliding with flock-mates, using the metrical approach.

(b) Alignment: objects steer towards the average heading direction of their local flock-mates and mimic their movements.

(c) Cohesion: objects move towards the average position of their neighboring flock-mates.

The boids of Reynold's model work independently and try both to stick together and avoid collisions with one another and with objects in their vicinity. Boids released near another begin to flock together forming small groups with its members heading approximately in the same direction and they change direction in synchronization. Smaller groups join to become bigger flocks and when they encounter external obstacles the flocks can split into smaller groups. The original simulations also corresponded visually to how flocks look like in nature and gave further confirmation that the model was onto something.

Even earlier, Aoki used pretty similar rules to simulate the collective motion of fish [22]. The units adhered to rules of avoidance, parallel orientation movements and approach.

The speed and direction of the units were stochastic in this model, but the direction of the units was related to the location and heading of the neighbors. It was also stated that collective motion can occur without a leader and that the individuals don't need information regarding the movement of the entire flock.

2.4.3 The Vicsek model

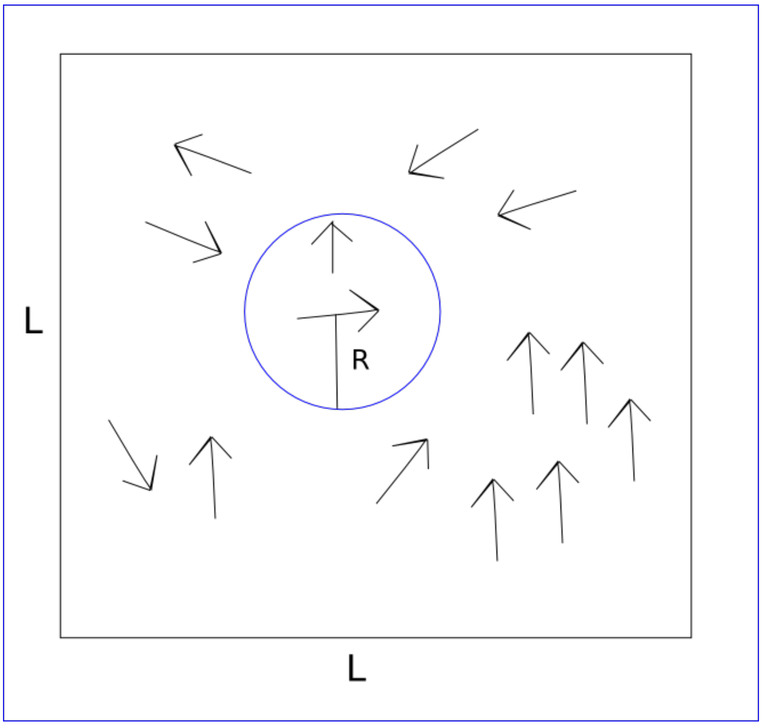


Figure 2.7: **Illustration of the Vicsek model.** The arrows are "birds", units that fly in the direction of the arrow with a constant velocity. The blue circle shows the interaction range R, i.e. the bird takes the average direction of the birds within a circle around it. The area where the system is simulated is a square with side length L with periodic boundary conditions.

A versatile and well-studied toy model, the Vicsek model (VM) [2] takes its approach from statistical physics. This makes it possible to study more quantitatively the behavior of huge flocks in the presence of noise. Another benefit is the model's simplicity as it only has one rule: at each timestep a given particle driven with a constant absolute velocity v_0 assumes the average direction of motion of the particles in its neighborhood of radius R with some noise/perturbation added. For a concept sketch of the model, see Figure 2.7.

The simulations of VM are done in two dimensions where pointlike particles move continuously and without a lattice in an area with a finite side-length L with periodic boundary conditions. Each particle (or unit) is labeled with an integer index (i) and hence its position and velocity are denoted by $\overrightarrow{x}_i$ and $\overrightarrow{v}_i$. As stated above, the absolute value of velocity of all units is constant, i.e. $v_i = v_0, \forall i$.

There are several different versions of the model around nowadays, with different ways of adding the noise and with different update rules. Using slightly different variations of the original model has been thought to be irrelevant to the results, but it has actually led to many conflicting results and a confusion about what type of a phase transition the model has. This confusion was clarified in [23], where the most common versions were clearly defined and compared. The following definitions and notations are taken from that paper, and introduce the two different ways to add noise and the two different update rules, as demonstrated in Table 2.1.

Table 2.1: The different variants of the Vicsek model considered

name	**abbreviation**	**rule**
Angular noise	AN	$\theta_i^{t+1} = Arg\left[\sum e^{i\theta_j^t}\right] + \eta_A \xi_i$
Vectorial noise	VN	$\theta_i^{t+1} = Arg\left[\sum e^{i\theta_j^t} + \eta_A n_i e^{i\xi_i}\right]$
Backward update	BU	$\overrightarrow{x}_i(t+\Delta t) = \overrightarrow{x}_i + \overrightarrow{v}_i(t)\Delta t$
Forward update	FU	$\overrightarrow{x}_i(t+\Delta t) = \overrightarrow{x}_i + \overrightarrow{v}_i(t+\Delta t)\Delta t$

The original Vicsek paper [2] used *angular noise* (AN). Using AN consists of determining the angle of motion θ of the ith particle as the average angle of motion of the neighboring j particles, also including i itself. This is then affected by the noise term, which consists of the amplitude of noise η_A and the uniformly distributed δ-correlated white noise ξ:

$$\theta_i^{t+1} = Arg\left[\sum e^{i\theta_j^t}\right] + \eta_A \xi_i. \tag{2.2}$$

The AN term can be thought to arise from the error committed by the the unit as it tries to adjust to its neighbors' average flight direction.

In [24] it was argued that the noise could also arise from each interaction between the ith particle and one of its neighbors. This translates into *vectorial noise* (VN):

$$\theta_i^{t+1} = Arg\left[\sum e^{i\theta_j^t} + \eta_A n_i e^{i\xi_i}\right]. \tag{2.3}$$

The difference is that the magnitude of AN is independent of the degree of local order, while VN becomes weaker when the local order is increased.

In the original Vicsek paper *backward update* (BU) was used. In it one first evaluates the direction of motion and then proceeds to update the position of the particle according to

$$\overrightarrow{x}_i(t+\Delta t) = \overrightarrow{x}_i + \overrightarrow{v}_i(t)\Delta t. \tag{2.4}$$

In some newer papers [23] another update rule has been used expecting it to give same results. This method, *forward update* (FU) is defined as

$$\overrightarrow{x}_i(t+\Delta t) = \overrightarrow{x}_i + \overrightarrow{v}_i(t+\Delta t)\Delta t. \tag{2.5}$$

It was shown in [23] that the combination of [AN+BU] yields a second-order phase transition, but the combination of [AN+FU] gives a clear first-order transition. This shows that the update procedure (FU vs BU) may actually influence the results dramatically. Also, the occurrence of first-order transitions are linked to VN and second-order to AN. Clearly, the choice of both update rule and noise affects the results.

2.4.4 Models without an explicit alignment rule

There are also models which do not contain a rule for alignment, but rather portray some collisions between particles in the presence of some interaction potential. One of the simplest models concerning SPPs is a model where the particles are trying to maintain a given absolute velocity and they only interact pairwise through a short-distanced repulsive linear force. This kind of system exhibits collective motion because each of these inelastic collisions between isotropic particles induces alignment, which in turn results in an increased overall velocity correlation.

An SPP model where the particles interact only through attraction was considered by Strömbom [27]. He found a variety of patterns, such as swarms (a set of particles with low and varying alignment), undirected mills (a group in which the particles move in a circular path around a common center) and moving aligned groups (in which the units move in a highly aligned manner). Importantly, these structures were stable only in the presence of noise.

2.4.5 The Cucker-Smale model

Exact formulation is a term used for results obtained analytically with a minimum amount of assumptions or approximations concerning the moving particles, except the rules they abide by. Cucker and Smale studied an exact formulation of the convergence to consensus in a population of autonomous agents in [28] and [29]. In their model (CSM) birds, denoted by $i = 1, ..., k$, are moving in 3 dimensional (Euclidean) space, $\mathbb{R}_3$, trying to reach a common direction or consensus. The position of the ith bird is given by $x_i(t)(\in \mathbb{R}_3)$. Every bird adjusts its velocity by adding to it a weighted average of the differences of its velocity with those of other birds. That is, at time t and for bird i

$$v_i(t+1) - v_i(t) = \sum_{j=1}^{k} a_{ij}(v_j(t) - v_i(t)) \tag{2.6}$$

The weights a_{ij} quantify the way the birds influence each other, and are a function of the distance between birds.

Let us define the adjacency matrix $A = (a_{ij})$, where the before-mentioned element a_{ij} measures the ability of birds i and j to communicate with each other, or their influence on each other. The elements of A should take values from the interval (0...1], and the closer unit i is to unit j, the bigger a_{ij} should be (since they exert a stronger influence on each other).

β is a tuning parameter affecting the strength of the influence,

$$a_{ij} = \frac{1}{(1 + ||x_i - x_j||^2)^\beta}, \tag{2.7}$$

where $\beta \geq 0$. The main benefit of this formulation is that it is a smooth function allowing analytical treatment. The adjacency matrix changes with time, since the positions of the birds keep changing with time.

The two main differences between CSM and VM are: 1) the range of interaction is continuous/discontinuous in CSM/VM, and 2) there is noise in VM, but in CSM there is not. In CSM the interaction range is a long-range action decaying with β, while in the VM it has the same intensity for all the neighboring units around a given particle, but only within a well-defined range. Although the two models are related, there are differences in behavior that stand out. The adjacency matrix that can be made based on the VM corresponds to a simple graph, whereas the matrix associated with CSM corresponds to a complete weighted graph. In VM convergence to a common direction depends on how the birds are connected to each other, whereas in CSM the weights decrease to zero as birds separate. In CSM the decay is polynomial and converges only with some initial values.

2.4.6 Network and control theoretical models

Networks have been used in recent years to depict the intricate underlying interactions of many a complex system. It has been shown that in many complex systems the number of connections is described by a power law, not a Poissonian distribution as previously thought. [30]

A flock of collectively moving units can be associated with a temporal network [5]. In such a network two units are connected if they interact, and only at the time of the interaction. The complex time evolution of the temporal network can reproduce the effect of moving units and changing environment. This kind of topology is referred to in control theory as a switching topology.

Using control theory, we can reformulate the problem of consensus in collective motion as follows: given a set of agents, who want to reach an agreement, regarding a certain quantity (direction of movement) that depends on the state of the agents. The interaction rule that defines the information exchange between a unit and its neighbors is called the consensus algorithm (or "protocol"). This system can be represented by a graph $G = (V, E)$, in which the agents are the nodes $V = 1, 2, ..., n$. Two nodes are connected with an edge $e \in E$ if, and only if, they communicate with each other. In this case they are neighbors. Accordingly, the neighbors of node i are $N_i = j \in V : (i, j) \in E$.

Within this framework, reaching a consensus means to converge asymptotically to an agreement via local communication. Let $A = (a_{ij})$ denote the adjacency matrix, which defines the communication pattern among the agents: if i and j interact with each other, then $a_{ij} > 0$, otherwise $a_{ij} = 0$. Notably, in the case of flocks both A and G vary with time, i.e. $A = A(t)$ and $G = G(t)$ [5].

Assuming a simple protocol, the state of agent i can change according to

$$\frac{d}{dt}x_i(t) = \sum_{j \in N_i} a_{ij}(x_j(t) - x_i(t)) \tag{2.8}$$

and hence defines a distributed consensus algorithm, i.e. it asymptotically solves an average-consensus problem for all initial states. In the case of undirected graphs the equation simplifies to the collective decision being the average of the initial state of the nodes. Once the protocol problem has been solved, we know how the system behaves in trying to reach consensus. [31, 32]

2.4.7 Models based on insect behavior

As an example of a 2D model, in [10] it was shown that Mormon cricket and Desert locust individuals with escape and pursuit behavior exhibit collective motion. The escape reaction is triggered in an individual if it is approached from behind by another individual. This causes the individual to increase its speed to avoid being attacked. If the individual notices one of its mates moving away, it increases its velocity in the direction of the fleeing individual, participating in pursuit behavior. Other cases do not trigger any response. According to the simulations, at moderate noise intensity and high particle density, these interactions (pursuit and escape) lead to global collective motion, irrespective of the detailed model parameters.

Another insect collective motion phenomena is the tendency of locusts to suddenly and coherently switch their direction of movement. It has been suggested in [33] that these ergodic directional switches might be the result of small errors made by the insects when trying to mimic the motion of their neighbors. These errors usually cancel each other out, but on a very large time scale the errors might accumulate and produce such a switch in motion. In Figure 2.8 can be seen a swarm of locusts in a lab setting.

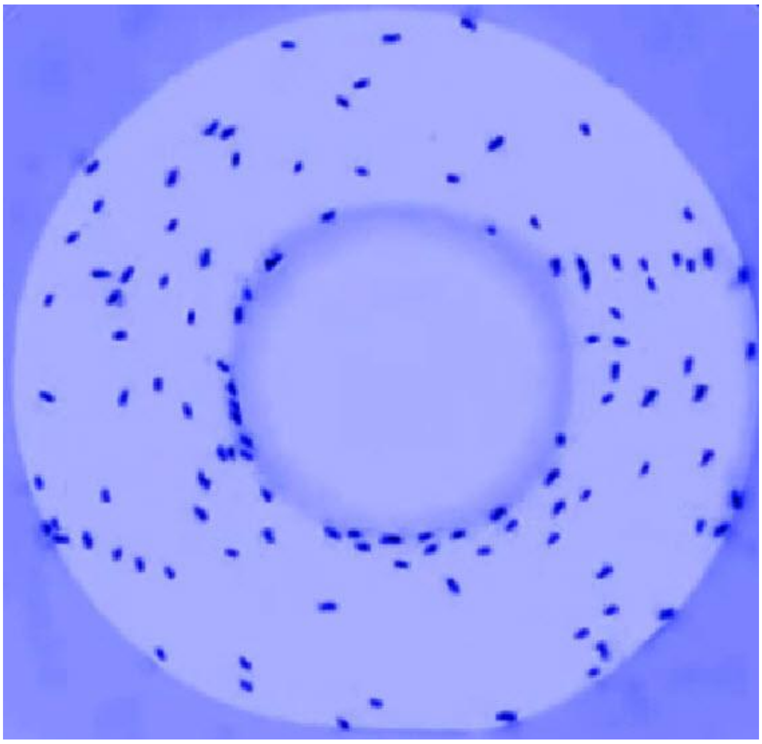


Figure 2.8: **A picture from a study of marching locusts.** The locusts circle around the container, until suddenly they simultaneously change directions. This has been explained by the accumulation of small errors made by the locusts as they try to mimic the motion of their neighbors. Source: `http://www.kurzweilai.net/social-networking-for-locusts`

Chapter 3

Methods

3.1 Vicsek model with leaders

In this thesis, I have studied the Vicsek model using angular noise (AN) and backwards update (BU), as was done in the original Vicsek paper [2]. AN consists of calculating the average of all near enough neighbors' flight directions and adding noise to the result. BU refers to calculating the direction of motion and then updating the position of the unit according to (2.4).

My focus is on controlling the swarm's direction of movement. To do this I add an additional component: leaders. Leaders are units that don't obey the normal update rules, but instead keep a constant direction. I study if these leaders can 'hijack' the flock and make them fly in the same direction as the leaders are flying.This setting has been motivated by the experimental results of [3], as discussed in Section 2.3.3, where it was shown that as few as one trained fish individual could direct the entire fish school to a feeding site without the school breaking up.

In brief, there are N units, of which N_L leaders are chosen, that have a constant flight direction ($\phi = 0$), don't follow other units, and are unaffected by noise.

As a control parameter, the average flight direction in radians,

$$v_{avg} = \frac{1}{N v_0}(v_1 + v_2 + ... + v_N) \tag{3.1}$$

is recorded at the end of each cycle (i.e. once all units have been updated). v_0 is a normalization factor, it's the VM unit speed. I used $v_0 = 0.03$, as was done in the original Vicsek paper for the same reasons; using this speed the particles always interact with their actual neighbors and move fast enough to change the configuration after a few direction updates.

Noise η is a key factor in VM. Using AN, we can calculate the noise according to (2.2). The density

$$\rho = \frac{N}{L^2} \tag{3.2}$$

of the units also affects the behavior of the system. As was stated in the original paper, noise and density decide if flocking behavior is even possible in the system: with high densities and noise the movement is random, with high density and low noise the motion is ordered. With small densities and high noise the particles tend to form groups moving coherently in random directions, and this is manifested in the order parameter ϕ (eq. (2.1)) attaining non-zero values. When most units are moving in the same direction, the values of ϕ approach 1.

3.2 Simulations

I wrote my code in Python. The code was based on the original Vicsek algorithm, implemented by Paavo Niskala. Initially all the units are distributed randomly, with a random initial flight direction. The amount of leaders was varied and the order parameter ρ, i.e. the average flight direction of all the units was recorded after each cycle. A cycle is defined to be when each unit has been updated once.

To find out the right amount of noise with a certain density used, I repeated the measures made in the original paper as follows; I take different values of noise and compute the ensemble average of the order parameter corresponding to that value of noise. With small enough values of noise, the order parameter is non-zero and we have flocking behavior.

I then look into the effect of N_L on the behavior of the flock. I increase the amount of leaders and see how much they affect the group's movements. Without leaders and with a few leaders the movement of the units should be random, and the order parameter should be practically zero. When the amount of leaders increases, the movement of the units as a whole should start indicating ordered movement.

Both the systems being investigated and the time frames used are finite. Hence, it must be chosen when the system state is measured and what is interpreted as uni-directional movement. When the different variables and settings are varied, we can better determine what happens in systems of different or infinite size. By choosing a specific ending time for the runs, I can define a variable which will tell about the behavior of the units. This binary variable, p_L, describes whether the run ended with the flock adopting the leaders' flight direction or not. In the time window $[t_1, t_2]$ the order parameter is calculated at the end of each cycle. If ρ is between $[-x, x]$ at every cycle with $t \in [t_1, t_2]$, then $p_L = 1$, otherwise $p_L = 0$. This in itself does not tell a lot, but by using an ensemble approach, we can calculate how many runs m of total M runs end with "hijacking".

After I have shown that leaders do have a great effect on the flock's movements even when their percentage is small, I go on to see what the critical leader densities

$$f_{L,C} = \frac{N_{L,C}}{N} \tag{3.3}$$

are so that above this limit the flock follows the leaders with a probability of $\frac{1}{2}$. So the critical value is defined as the value above which it is more probable to follow the flock than not.

After calculating the critical values, the effect of system size can be studied. By keeping ρ fixed and varying L and N, the critical leader amount can be plotted against system size. In accordance to Figure 2.3, one would expect the transition to become more and more abrupt with growing system size.

Due to computational restrictions, the normal simulations use $t_1 = 150$, $t_2 = 200$. The effect of waiting time, i.e. the point at which the simulations are terminated, can be studied, however. This is done by having the simulations run for the maximum time needed (in this case 1250 cycles) and then using the same data to examine the earlier time windows. That is, once we have the data for 1250 cycles, we can use different (t_1, t_2) pairs and see how the results change if we terminate the run at different points. This way we can make observations about what might have happened if we'd just let the runs used for earlier experiments continue. To keep the results comparable, the difference $t_2 - t_1$ is fixed to the value of 50.

Chapter 4

Results

4.1 Determining the level of noise η

As was stated in [2], noise and density decide if flocking behavior is possible in a system. To have particles that form groups that move in a flock-like manner, high densities and low levels of noise are needed.

Values of N and L were chosen based on a hunch, and then tested whether they led to the desired effect, after which they were fixed. Using $N = 500$ and $L = 25$, we get from equation (3.2): $\rho = 0.8$. Using this density I plot the average velocity as a function of η to find out when the order parameter gets a non-zero value. When the order parameter, depicting the average direction of movement of the whole system, is non-zero, we know there is flocking behavior to be detected. The plot can be seen in Figure 4.1. I choose the value $\eta = 0.5$ as it is clearly in the non-zero area. This noise level is used in all the following simulations, and hence ρ is kept constant.

Table 4.1: The parameters used for the flock simulations.

Parameter	value used
N (amount of units)	500
N_L (amount of leaders)	0-99
L (length of the side of the simulation area)	25
v_0 (the speed of the units)	0.03
η (noise)	0.5

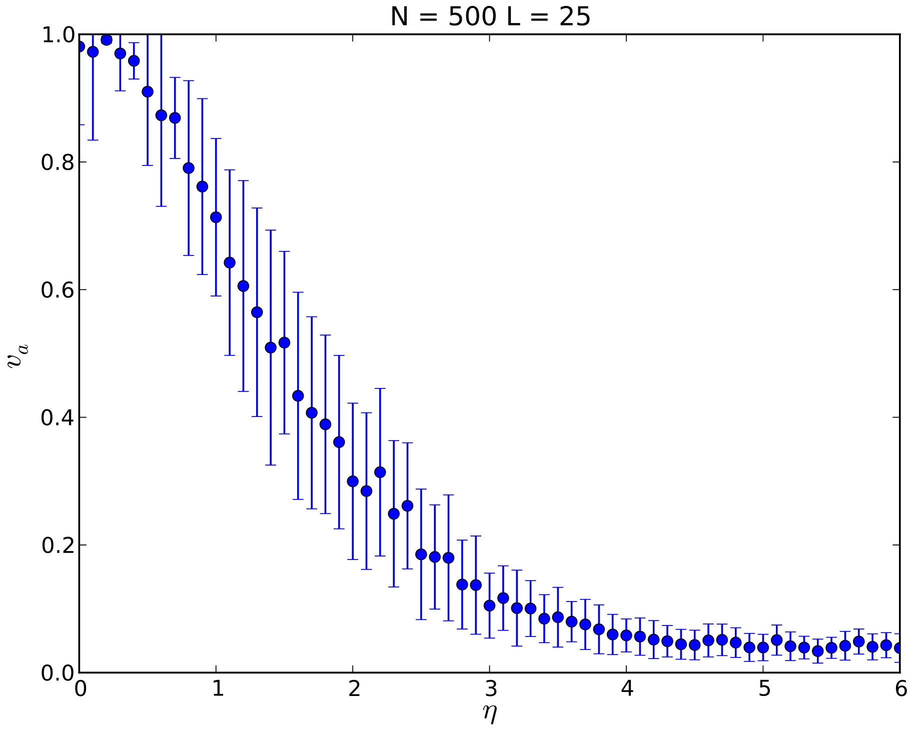
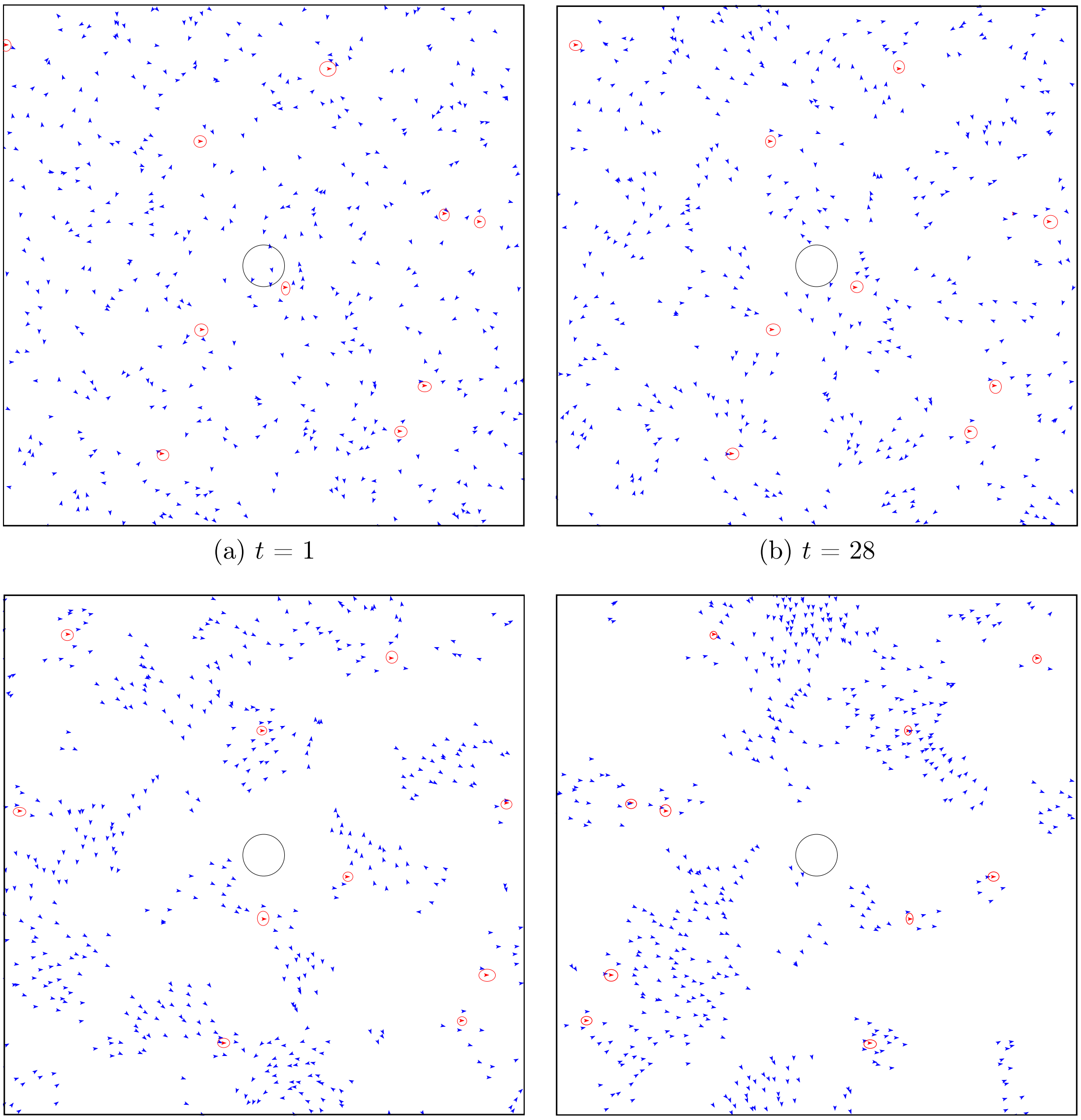


Figure 4.1: **The order parameter as a function of system noise.** Each data point is an ensemble average of 50 runs made. The value $\eta = 0.5$ is chosen as it is clearly in the non-zero area.

4.2 The effect of the number of leaders N_L

A series of example runs with $N_L = 10$ can be seen in Fig. 4.2. As time passes, the units form flocks that mostly move in the direction of the leaders, with some deviance accounted for by the effect of noise. Running simulations with the parameters indicated in Table 4.1, we see clear indication of the flock being hijacked by the leaders already with just a couple of leaders. To illustrate this, we can compare the average flight direction as a function of time to the constant flight direction of the leaders. As can be seen in Fig. 4.3, the average flight direction (the blue curve) converges to oscillations around the leader's flight direction (marked with red). This happens the faster the more leaders there are. The closer the blue curve is to the constant red line, the more the leaders dominate the flight direction. Even with a value of $N_L = 8$, the average flight direction reaches the constant line $\phi = 0$ before the half-way mark. That's just 1.6% of the flock needed to steer the group



(a) $t = 1$

(b) $t = 28$

(c) $t = 100$

(d) $t = 249$

Figure 4.2: **An example of how leaders affect the behavior of the flock, with $N_L = 10$.** The leaders are marked and circled in red. As time passes, the birds form flocks which merge into bigger flocks. The leaders take control of the flocks and most birds fly in the direction $\phi = 0$ (with some perturbations due to noise). The black circle shows the range of interaction.

into a constant direction.

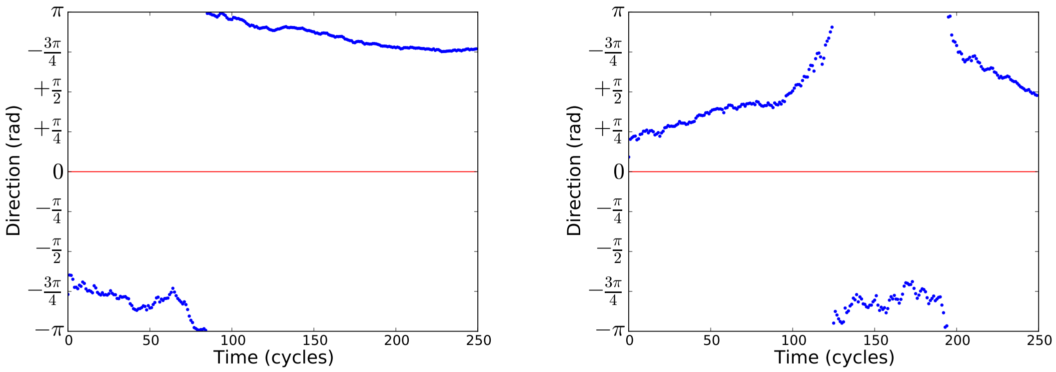
To study the leaders effect more rigorously, the criteria for the complete "hijacking" must be decided. I define this as follows: The flock follows the leaders at the end of the run, if every value of the order parameter at time $t \in [150, 200]$ is between $[\frac{-\pi}{8}, \frac{\pi}{8}]$. I then make runs for different values of N_L and record whether the run ended with the flock following the leader or not, i.e. a binary variable. For each value of N_L, I performed 100 runs of the simulation, and calculated p_L, the fraction of runs at N_L where the flock was seen to follow the leaders between $t \in [150, 200]$. Thus we get Figure 4.4, which shows that as the amount of leaders increases, the fraction p_L grows monotonically towards 1.

4.3 Varying the system size N and the waiting time t

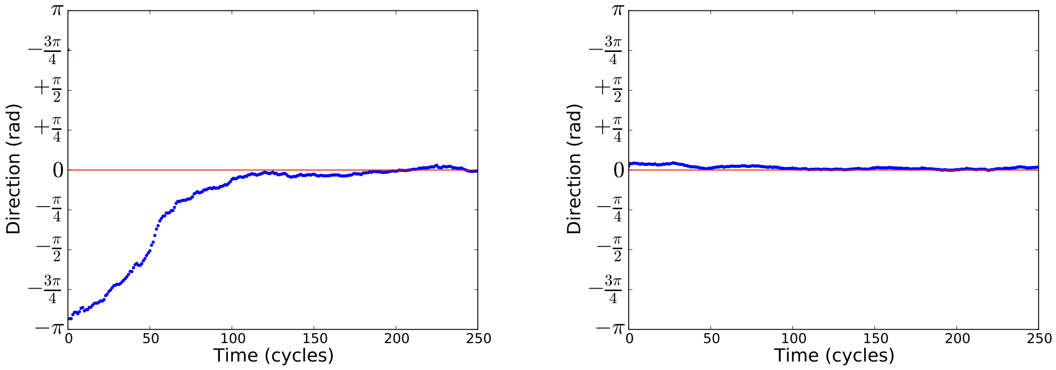
I study the effect of both system size and waiting time on flock hijacking, with the intention to use these results to extrapolate to a infinite sized system. We want to get a general result, by studying the system as it grows towards infinite size but we can also study a finite system with a infinite waiting time. If we get finite results for the amount of leaders in an infinite system, it tells us that we can control huge swarms without the need for immense amounts of leaders.

By keeping ρ practically constant, the effects of system size can be studied. The values used for this part are gathered in Table 4.2. By taking the six system sizes and comparing the absolute and relative amounts of leaders we get Figure 4.5. We can see that as the system size grows, the amount of leaders needed to control the flock decreases. For example, where nearly 10% leaders are needed for $N = 500$, only round 5% are needed when $N = 1000$.

Considering that the percentage of leaders needed goes down as a function of system size, it seems feasible that we can get similar results with a finite system by just waiting long enough. That's why I also study the effect of the time window chosen, i.e. when the ending check is performed. The base case is the time window $t = [150, 200]$ used for all the other simulations. What we want to see is whether waiting longer affects the amount of runs finishing in a leader-controlled state. This was done using a run lasting 1250 cycles,



(a) $N_L = 0$ (b) $N_L = 4$



(c) $N_L = 8$ (d) $N_L = 74$

Figure 4.3: **Four examples of the flock's average flight direction as a function of time, with flock size N being 500.** The red line indicates the leaders' flight direction. The closer the dots are to the red line, the more the units are flying in the same direction as the leaders.

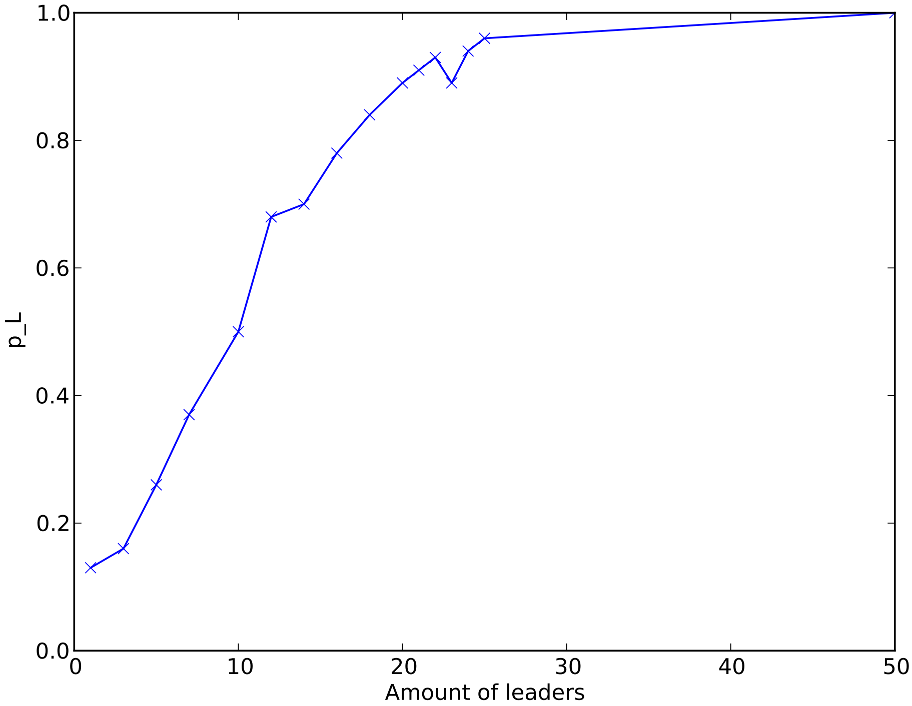


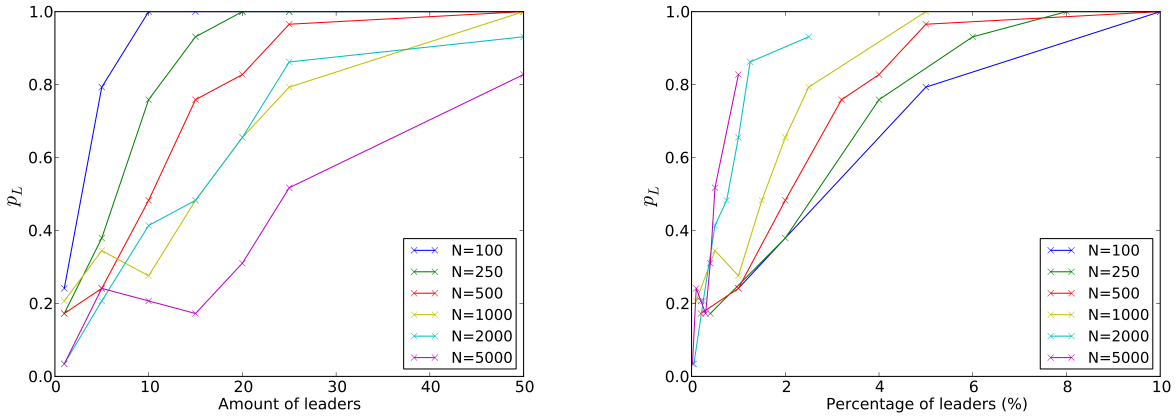
Figure 4.4: **The probability p_L that the flock is following the leader at the end of 200 cycles as a function of the amount of leaders.** $N = 500$, $L = 25$ and $\rho = 0.8$. Each data point is an average of 100 runs.

and then checking whether the run had ended using different time windows. Thus we can see how the results of the other tests might have behaved if we had been able to wait longer.

From the size and time plots I determine the critical values of leaders needed for group "hijacking". This choice is rather arbitrary, since the single runs are stochastic in nature and taking the value $p_L = 1$ is not really sensible. I do this by taking the percentage corresponding to (3.3), $p_L = 0.5$ and say that this is the corresponding $p_{L,c,N}/p_{L,c,t}$. These values can be seen in Table 4.3 and plotted in Figure 4.7. In the same figure we have the inverses of both system size and waiting time plotted against $p_{L,c}$. This is done so that we can extrapolate these values to infinity and see whether the percentage of leaders goes to zero. By doing an approximate fit (the red line) to the first values of the curves and reading the point where it intersects the y-axis, we get an estimate for the percentage of leaders needed in an infinite system. As

Table 4.2: The parameters used for critical simulations.

N	L	ρ
100	11	0.826
250	18	0.772
500	25	0.800
1000	35	0.816
2000	50	0.800
5000	79	0.801



(a) Absolute values

(b) Relative values

Figure 4.5: **The effect of system size on the leader's and their flock "hijacking".**

can be seen from the figure, we get the value of about 0.2% for both varied inverse time and size, which is reasonably within error estimates and can be thought to be zero. This value of 0.2% is, evidently, very close to zero, and it would be interesting to investigate whether, in fact, any non-zero fraction of leaders would be enough to hijack the flock. However, this would require simulations on much larger systems, and is thus beyond the scope of this work.

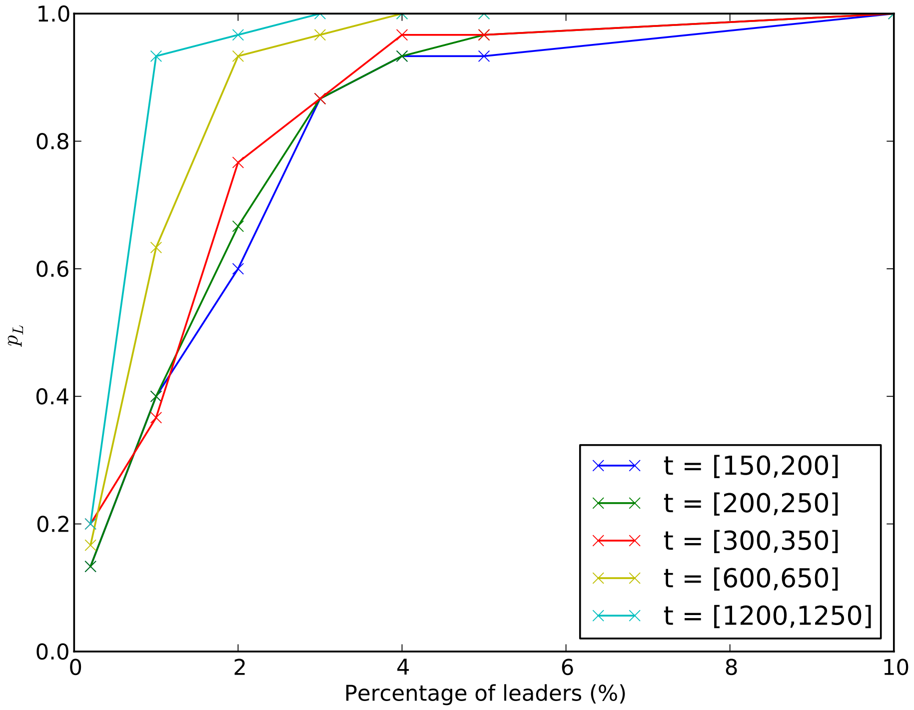
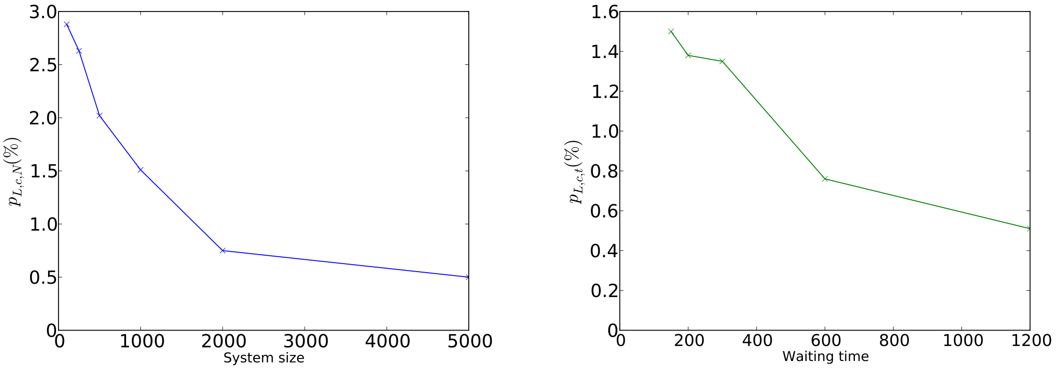


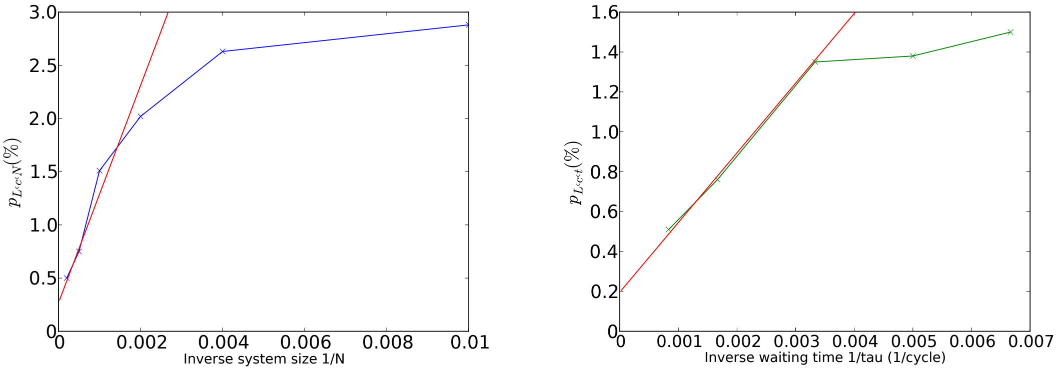
Figure 4.6: **The effect of waiting time window choice on the "hijacking" of the flock.** If every value of the order parameter in the time window is between $[\frac{-\pi}{8}, \frac{\pi}{8}]$, then the flock has adopted the leader's flight direction.

Table 4.3: The critical values from the size and time simulations.

N	$p_{L,c,N}$ (%)	t	$p_{L,c,t}$ (%)
100	2.88	$t = [150, 200]$	1.50
250	2.63	$t = [200, 250]$	1.38
500	2.02	$t = [300, 350]$	1.35
1000	1.51	$t = [600, 650]$	0.76
2000	0.75	$t = [1200, 1250]$	0.51
5000	0.50		



(a) System size (b) Waiting time



(c) Inverse system size (d) Inverse waiting time

Figure 4.7: **The effect of system size and waiting time on the "hijacking" of the flock.** Clearly the bigger the system and the longer we wait, the less percentage of leaders is needed to steer the entire flock into the direction wanted. The lower pictures show the inverse of both system size and waiting time. By making an approximate fit to the part closest to zero, we can extrapolate to find the probable point where the curve intersects the y-axis. This corresponds to the ideal limit $N(\tau) \to \infty, \frac{1}{N}(\frac{1}{\tau}) \to 0$. In reality the value for both is about 0.2%.

Chapter 5

Discussion

5.1 Summary of results

In this thesis I have looked at collective motion by both reviewing related literature and by simulating flocking behavior using a well-established model: the Vicsek model. My emphasis has been on manipulating and controlling collective motion using special individuals, dubbed leaders, who do not change their movement based on the other units, but rather keep a constant flight direction. Thus they are not influenced by the motion of others, but they exert an effect on the normal units, increasing the probability that the entirety of the flock is flying in the direction of the leaders.

In the Vicsek Model, the behavior of the system is determined by the level of noise, η. Flocking takes place only in the low-noise region, so the amount of added noise had to be determined. In Section 4.1 the value of $\eta = 0.5$ was chosen because the corresponding value of order parameter v_a is non-zero, as can be seen in Figure 4.1.

In Section 4.2, I found that with the constant unit-density chosen ($\rho = 0.8$), the amount of added leaders to the Vicsek model needed to divert the flock to the preferred constant direction was very low. As can be seen in Figure 4.3, the more leaders, the faster the flock matches the flight direction of the leaders. By defining p_L, the probability that the flock is flying in the same direction as the leaders at the end of the run, we can clearly see the effect of the amount of leaders present, as can be seen in Figure 4.4. With additional leaders the probability that the flock will fly in the leaders' direction grows steadily.

In Section 4.3, ρ was kept constant and the effect of the system's size was studied. As can be seen in Figure 4.5, the relative amount of leaders needed for leader control decreases with bigger systems, approaching the ideal case depicted in Figure 2.3. The same effect can be noticed with having the runs continue for longer. Figure 4.6 shows how longer run times results in a steeper curve.

Depending on how long the simulations were allowed to run and on system size, the amount of leaders needed was between $0.5\% - 3\%$, with the clear tendency for the percentage needed to be inversely monotonically correlated with both system size and wait time, as can be seen in Figure 4.7. This supports the conclusion that with infinite time and an infinite system the percentage goes to zero, meaning that a single leader can control the entire infinite flock.

5.2 Conclusions

As I have stated throughout this thesis, collective motion control is not just abstract theory. It can have applied uses in the animal kingdom. As I have mentioned before, locust swarming and migrating is a serious threat to the welfare of millions of Africans. Being able to even slightly divert their path so that they would avoid the most heavily farmed and populated areas would already make a significant difference. As my research gives reason to believe that the amount of controlled units will remain reasonable no matter how huge the swarms, it seems that we now only need robotic locusts that fool the others. There has already been studies of remote controlled individuals in animals as complex as rats [34], so it seems like only a matter of time.

Other practical applications for a real-world version of leader control that come to mind can have more dire consequences. If applied to fishing, entire shoals might be diverted towards a waiting fishing fleet, thus worsening the overfishing of our oceans. On the other hand it might be a tool for counting the populations of various potentially endangered species. A final example is the military potential of remote controlled ants, since one would not need immense amounts of controlled units if a minority can lead entire hills into attack. Ants are already masters of cooperation, as can be seen in Figure 5.1, and such could be a real tool for demoralizing of enemy troops, as they can circumnavigate most common hindrances.

While mostly speculation at this point, I still find that there's real potential in further developing theoretical tools for collective motion control and the implementation of much more detailed and realistic simulations and models. If a few can unnoticeably influence the behavior of the many, then subtle manipulation might be the way forward in many a field.

Figure 5.1: **An army of ants demonstrating the power of cooperation.** Source: `http://fofoa.blogspot.fi/2010_04_01_archive.html`

Bibliography

[1] C.W. Reynolds *Flocks, herds, and schools: A distributed behavioral model* Computer Graphics 25-34 (1987)

[2] T. Vicsek, A. Czirok, E. Ben-Jacob, I: Cohen, O. Shochet *Novel type of phase transition in a system of self-driven particles* Physical Review Letters Vol.75, No.6 (1995)

[3] S.G. Reebs *Can a minority of informed leaders determine the foraging movements of a fish shoal?* Animal Behaviour, 59:4033-409 (2000)

[4] E. Bonabeau, M. Dorigo, G. Theraulaz *Swarm Intelligence, from natural to artificial systems* Oxford University Press (1999)

[5] T. Vicsek, A. Zafeiris *"Collective motion"* arXiv:1010.5017v2 (2012)

[6] D.T. Swain, I.D. Couzin, N.E. Leonard *Real-Time Feedback-Controlled Robotic Fish for Behavioral Experiments With Fish Schools* Proceedings of the IEEE, Vol. 100, No.1 (2012)

[7] L. Conradt, T.J. Roper *Deciding group movements: Where and when to go* Behavioral Processes, 84:675-677 (2010)

[8] L.E. Reichl *A Modern Course in Statistical Physics* University of Texas Press (1980)

[9] N. Tarcai, C. Viragh, D. Abel, N. Nagy, P.L. Varkonyi, G. Vasarhelyi, T. Vicsek *Patterns, transitions and the role of leaders in the collective dynamics of a simple robotic flock* Journal of Statistical Mechanics: Theory and Experiment (2011)

[10] P. Romanczuk, I.D. Couzin, L. Schimansky-Geier *Collective motion due to individual escape and pursuit response* Physical Review Letters, 102:010602 (200)

[11] C. Becco, N. Vandewalle, J. Delcourt, P.Poncin *Experimental evidences of a structural and dynamical transition in fish school* Physica A, 367:487-493 (2006)

[12] Y. Katz, K. Tunstrom, C.C. Ioannou, C.Huepe, I.D. Couzin *Inferring the structure and dynamics of interactions in schooling fish* Proceeding of the national academy of sciences of the United States of America (2011)

[13] N.C. Makris, P. Ratilal, S.Jagannathan, Z.Gong, M. Andrews, I.Bertsatos, O.R.Godo, R.W. Nero, J.M. Jech *Critical population density triggers rapid formation of vast oceanic fish shoals* Science, 323:1734-1737 (2009)

[14] W.K. Potts *The chorus-line hypothesis of coordination in avian flocks* Nature, 24:344-345 (1984)

[15] D. Biro, D.J.T. Sumpter, J. Meade, T. Guilford *From compromise to leadership in pigeons homing* Current Biology, 16:2123-2128 (2006)

[16] I.R. Fischhoff, S.R. Sundaresan, J.Cordingley, H.M. Larkin, M.-J. Sellier, D.I. Rubenstein *Social relationships and reproductive state influence leadership roles in movements of plains zebra, Equus burchellii* Animal Behaviour, 73:825-831 (2007)

[17] R. Sarova, M. Spinka, J.L. Arias Panama, P. Simecek *Graded leadership by dominant animals in a herd of female beef cattle on pasture* Animal Behaviour 79:1037-1045 (2010)

[18] A.E. Turgut, H. Celikkanat, F. Gokce, E. Sahin *Self-organized flocking in mobile robot swarms* Swarm Intelligence 2:97-120 (2008)

[19] M. Karppinen *Kinship and Happiness on the Sugarscape* Aalto University Special Assignment (2012)

[20] A. Eriksson, M.N. Jacobi, J. Nystrom, K. Tunstrom *Determining interaction rules in animal swarms* Behavioral Ecology, 21:1106-1111 (2010)

[21] I.D. Couzin, J. Krause, R. James, G.D. Ruxton, N.R. Franks *Collective memory and spatial sorting in animal groups* Journal of Theoretical Biology 218:1-11 (2002)

[22] I. Aoki *A simulation study on the schooling mechanism in fish* Bulletin of the Japanese Society of Scientific Fisheries, 48(8):1081-1088 (1982)

[23] G. Baglietto, E.V. Albano *Nature of order-disorder transition in the Vicsek model for the collective motion of self-propelled particles* Physical Review E 80, 050103 (2009)

[24] G. Gregoire, H. Chate *Onset of collective and cohesive motion* Physical Review Letters 92, 025702 (2004)

[25] C. Huepe, M. Aldana *New tools for characterizing swarming systems: A comparison of minimal models* Physica A 387:2809-2822 (2008)

[26] H. Chate, F. Ginelli *Collective motion of self-propelled particles interacting without cohesion* Physical Review E 77, 046113 (2008)

[27] D. Strombom *Collective motion from local attraction* Journal of Theoretical Biology, 283:145-151 (2011)

[28] F. Cucker, S. Smale *Emergent behavior in flocks* IEEE Transactions on Automatic Control 52:852-862 (2007)

[29] F. Cucker, S. Smale *On the mathematics of emergence* Japanese Journal of Mathematics, 2:197-227 (2007)

[30] M.E.J. Newman *Networks - An Introduction* Ofxord University Press (2010)

[31] R. Olfati-Saber *Consensus Problems in Networks of Agents With Switching Topology and Time-Delays* IEEE Transactions on Automatic Control, Vol. 49, No. 9 (2004)

[32] R. Olfati-Saber *Flocking for Multi-Agent Dynamic Systems: Algorithms and Theory* IEEE Transactions on Automatic Control, Vol. 51, No. 3 (2006)

[33] C. Escudero, C.A. Yates, J.Buhl, I.D. Couzin, R. Erban, I.G. Kevreidis, P.K. Maini *Ergodic directional switching in mobile insect groups* Physical Review E, 82:011926 (2010)

[34] S.K. Talwar, S. Xu, E.S. Hawley, S.A. Weiss, K.A. Moxon, J.K. Chapin *Behavioural neuroscience: Rat navigation guided by remote control* Nature, Vol. 417, No. 6884 (2002)