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# Optimized Full-Duplex Multi-Antenna Relay in Single-Input Single-Output Link

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This thesis studies the performance evaluation and optimization of full-duplex multiple-input multiple-output (MIMO) relaying systems in single-input single-output (SISO) link, based on signal-to-interference-plus-noise ratio (SINR). Relays are transceivers which can improve the throughput of a system by coverage extension in a power-efficient manner, whereas full-duplex (FD) systems are point-to-point communication systems, in which transmission and reception occurs simultaneously on a single frequency band. Deploying relaying systems in the full-duplex mode, however, causes self-interference, because the signal transmitted from the transmitter side of the relay couples at its receiver side. This interference causes performance degradation in these systems. In this thesis, a one-way SISO communication link with a MIMO relay connecting the source and the destination nodes is studied. The relay is considered to be implementing either amplify-and-forward (AF) or decode-and-forward (DF) protocol. First, the end-to-end SINR of the system is derived. With the knowledge of SINR, numerical evaluation is made via computer simulations. The numerical results are reached by introducing different assumptions to the general system, as well as by keeping the system intact. Although the numerical solutions provide high performance, they require much time and computational power. Hence, this thesis offers some computationally efficient analytical solutions to the problem. For example, after setting the transmit filter of the relay, minimum mean square error (MMSE) method is applied on the first hop to optimize the system; or by assuming the relay self-interference channel is a rank-one matrix, a closed-form solution for the transmitter and receiver relay filters eliminating the self-interference is derived. Then, the performance of these methods are compared and discussed in different aspects; such as high SINR and computational requirement. The results indicate that each scheme has certain benefits over the others depending on the system design requirements.

Keywords: full-duplex, relay, MIMO, SINR, self-interference, MMSE, nonlinear optimization.

# Preface

This master thesis is a small contribution to the mathematical understanding of wireless relays and their efficient implementation within a system in a theoretical sense.

This thesis is primarily aimed at people that have knowledge in the basics of communication systems, and/or holding, at least, a bachelor's degree or equivalent in the field. It contains the work done from May to November, 2012. This thesis has been written by its author; some of the creative content, however, is based on the research of others. I have done my best to provide the references to these sources.

Several people have contributed to this thesis. I would therefore like to thank my supervisor Professor Risto Wichman for his time and support during the thesis period. I am grateful to him for giving me the opportunity to work in this topic for the thesis. Furthermore, I would like to thank M.Sc. (Tech) Taneli Riihonen and D.Sc. Stefan Werner for their help and constructive comments. This thesis, truly, would not exist without any of those helps. I am, especially, thankful to my thesis instructor Taneli Riihonen for his support, guidance throughout the entire work, and our discussions, which helped me always keep an open mind to the possibilities. He has reviewed and refined every draft of this thesis, yet any errors that might be left are entirely my responsibility.

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# Abbreviations

AWGN	Additive white Gaussian noise
AF	Amplify-and-forward
CF	Compress-and-forward
DF	Decode-and-forward
EF	Estimate-and-forward
FD	Full-duplex
FDD	Frequency-division duplexing
HD	Half-duplex
MIMO	Multiple-input multiple-output
MMSE	Minimum mean square error
MSE	Mean square error
OF	Observe-and-Forward
QF	Quantize-and-Forward
SF	Store-and-forward
SINR	Signal-to-interference-plus-noise ratio
SISO	Single-input single-output
SNR	Signal-to-noise ratio
SVD	Singular value decomposition
TDD	Time-division duplexing
WiMAX	Worldwide Interoperability for Microwave Access

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# Chapter 1

## Introduction

### 1.1 Background

Communication has always been a fundamental necessity through the course of human history. Because we are living in a society, contributing to society, sharing within the society, communication is a natural part of who we are. The efforts to make communication a better experience have always been with us. The limitations of communication have been continuously reduced by the technology. For instance, one of the early improvements to make the communication a better experience was the invention of smoke signal. It extended the maximum distance to communicate. It ameliorated the life of mankind.

Different requirements of communication systems have come into play as we progressed. The extent of security, speed, accuracy, distance has put the challenge into different fields, compelling us to invent different forms of communication. Yet, there has always been one goal which is common for all forms of communication systems: achieving higher data rate. Because we are living in a world in which the frequency spectrum is scarce, and in which the population has been increasing faster than ever, the challenge that of the next-generation communication systems being able to provide improved throughput and coverage to increasing number of users is becoming more and more difficult.

Relays are transceivers, which receive the signal on one end, and transmit it from the other end. They may involve either a power amplification or decoding/re-encoding process, depending on the implemented protocol, amplify-and-forward (AF) or decode-and-forward (DF), respectively. The use of relays in wireless communication systems brings some benefits. For instance, relays improve the system coverage by repeating the signal towards farther distances. Relays lead to higher system throughput, and more efficient power consumption.

Full-duplex (FD) systems are point-to-point communication systems, which allow simultaneous two-way communication on the same frequency band. FD communication systems provide higher capacity compared to half-duplex (HD) systems due



to the fact that FD systems utilize spectrum reuse, which is an efficient way to combat the problem of spectrum scarcity. For these reasons, utilizing the concepts of relaying and full-duplex communication within the same system, in theory, results in better performance in various aspects.

However, full-duplex relay systems suffer from self-interference signal, which decreases the overall system signal-to-interference-plus-noise ratio (SINR). The self-interference occurs as a result of the coupling of the transmitted signal to the received signal at the relay. Even though full-duplex relay, theoretically, is a promising topic in terms of capacity, in practice, extreme amounts of self-interference sometimes make it not so much feasible to implement. Therefore, most of the academic research on wireless relay systems has been based on half-duplex implementation, with the exception of some recent works.

The full-duplex relaying systems can provide effectively higher SINR performance than the half-duplex systems if the problem of self-interference is solved and the rate loss factor is taken into account. For instance, having multiple-input multiple-output (MIMO) relay enables the system to make self-interference suppression in spatial domain. By selecting an appropriate set of beamforming vectors both for the transmitter and receiver sides of the relay, the effects of the self-interference signal can be mitigated, leading to higher end-to-end SINR performance so as to optimize the system.

## 1.2 Research Problem

As described above, full-duplex MIMO relay systems have great theoretical potential to offer higher performance than conventional half-duplex relays, if the adverse effect of the self-interference is, somehow, mitigated. This thesis aims to optimize the full-duplex relay systems in terms of overall SINR performance by developing new schemes using beamforming filters and transmit power allocation to combat the self-interference at the relay. The proposed schemes are covering all of the common relay protocols, which are amplify-and-forward and decode-and-forward. The main scope is on the systems where the direct source-to-destination link is weak and the relay extends the coverage area of the main source transmitter.

## 1.3 Contributions of the Thesis

The main contributions of this thesis can be summarized as follows.

- The search for the optimal implementation of full-duplex MIMO relay systems is formalized for both amplify-and-forward and decode-and-forward protocols in a pure mathematical sense as two exclusive optimization problems; one by aiming for the maximization of overall system SINR, and another by adopting complete self-interference cancellation approach in spatial domain.

- The literature survey reviews key recent works offering several theoretical methods to cope with the interference so as to improve the system performance. It is noted that they all fail to reach the global maximum system capacity, because of the constraints and assumptions defined along the way. Instead, this thesis takes the problem as a whole and presents the global optimization result as an upper limit via numerical analysis.
- An analytical solution reducing the optimization problem down to a single open parameter is proposed. First, the optimal receiver beamforming filter is derived by fixing the transmitter beamforming filter and the relay transmit power. Then, with the knowledge of the optimal receiver beamforming filter, the optimal relay transmit power given any transmitter beamforming filter is derived for both amplify-and-forward and decode-and-forward protocols. This proposed solution not only produces promising performance itself, but also takes us one step closer to the global analytical solution.
- An iterative scheme which is available in literature is modified to propose a new solution in a closed form without much sacrificing the resulting SINR performance. With the assumption of rank-one matrix for the self-interference channel, the optimal beamforming filters maximizing the overall system performance are derived with the constraint of complete interference suppression.
- Newly proposed schemes are studied based on different system conditions and their performance is compared to numerical evaluation results, as well as to some conventional reference schemes. It is shown that a proper combination of those schemes can promise even more effective solution depending on the system parameters.

## 1.4 Outline of the Thesis

This thesis is organized as follows. In Chapter 2, literature survey conducts full-duplex relay systems, mainly focusing on the mitigation of self-interference. Performance analysis is presented on various mitigation schemes. Chapter 3 presents a full-duplex relay system model in a SISO link with two hops assuming either AF or DF protocols at the relay. The end-to-end system SINR is derived for both of these protocols. In Chapter 4, the system SINR definition is transformed to an optimization problem. It is shown that given the knowledge of channel responses, a certain set of directions for the relay receiver and transmitter beams maximizes the system SINR. Numerical evaluation results supporting this claim are also presented. In Chapter 5, some analytical schemes are derived and proposed as a substitute for the numerical solutions due to their comparable performance but yet simpler implementation. Chapter 6 presents simulation results for the analytical schemes offered in Chapter 5, as well as the numerical evaluation results. Their performance is compared and discussed in terms of different aspects, e.g., high SINR, computational requirements, for different test cases. Finally, Chapter 7 concludes the outcomes of the thesis.

# Chapter 2

## Literature Survey

This chapter summarizes the work that has been done in earlier literature, mostly about full-duplex transmission and relaying. In the following sections, firstly, the definitions of full-duplex and relaying systems are given. Their benefits and drawbacks are pointed out. Later, the systems combining full-duplex and relay systems are discussed. The previous efforts and various proposed solutions on those are presented. The last but not least, their performance in various aspects is discussed.

### 2.1 Relay Systems

A relay, by its simplest definition, is a wireless transceiver. It has the ability to receive a wireless signal at its receiver, and transmit it from the other end. Ironically, as opposed to their such simple functionalities, relay-based communication systems have crucial roles and widespread uses in various applications. Relays can be employed to divert traffic from congested areas of a cellular system to cells with lower traffic load [1]. In ad-hoc networks, using greater number of relays leads to higher network capacity proportional to the logarithm of the number of relays [2, 3]. In addition, relays provide higher mobile cell coverage and indoor coverage [4]. They extend the edge of the cell by forwarding the data signal to the areas where the signal coming directly from the source cannot reach. Relays can also increase cell coverage by filling uncovered territories, particularly in urban areas. Since, by deploying relays, the shadowing effect which is a result of the presence of high buildings can be eliminated [4, 5]. All in all, relay systems are efficient in power consumption, and they lead to higher throughput.

The first relay systems were introduced in 1971 by van der Meulen [6]. A single relay channel between a receiver and a transmitter node is studied for the first time. Most general strategies for relay networks were developed in 1979 by Cover and El Gamal later in [7]. Maximum relay channel capacities were extensively studied. Since then, relay channels have been an interesting subject in an information theoretic perspective for a long time. In cellular systems, however, the first practical applications were studied in [5], [9] and [10]. The authors in [5] give practical examples, e.g., for deploying cooperative relays to increase capacity through an antenna

array, deploying fixed relay stations to increase the coverage in mobile broadband systems through clusters of relays. In [9], transmit precoding schemes are suggested to be deployed in MIMO relay systems since they provide a lower capacity bound. In [10], the authors discuss the practical advantages and disadvantages of a multi hop relay system for Worldwide Interoperability for Microwave Access (WiMAX) in terms of network planning and cost analysis.

Figure 2.1 shows a basic two-hop relay communication model. After the information is transmitted from the source node,  $S$ , it goes through the channel,  $h_{SR}$ , between the source,  $S$ , and the relay,  $R$ . Then, the received signal is processed in the relay before it is transmitted towards the destination node,  $D$ . Finally, the signal goes through the channel,  $h_{RD}$ , between the relay,  $R$ , and the destination,  $D$ , and it is received at the destination,  $D$ .

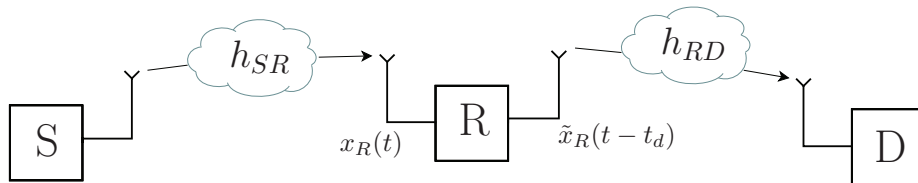


Figure 2.1: A two-hop SISO relaying (R) system enabling the communication between the source (S) and the destination (D) nodes, where  $t_d$  is the relay processing delay

There is a time delay ( $t_d$ ), called *processing delay*, between the reception and re-transmission of the signal at the relay, occurring as a result of the process the received signal goes through before being transmitted towards the destination node. This process, depending on the protocol implemented on the relay, may entail the process of decoding/re-encoding the signal, or compressing the signal, or merely a power amplification on the signal.

## 2.2 Relaying Protocols

There are several relaying protocols. Each of them have advantages and disadvantages over the others. Relays with different protocols are utilized in different applications depending on what is needed. Some of the common relaying protocols are described below.

**Amplify-and-Forward (AF)** Relays with AF protocol amplify the received signal from the source and transmit it towards the destination. The signal does not go through any decoding/re-encoding process [10]. Therefore, relays implementing AF protocol are known as *non-regenerative relays* [11]. However, the

received signal at the relay is generally transmitted with a different gain. As a result, if the gain is greater than one, the power of the noise signal within the received signal, as well as the power of the actual useful signal, is increased, which causes noise amplification [5]. Among all the relay protocols described below, AF gives the smallest delay. Because, it requires the least computational power of all. In summary, it is fast and simple. Hence, AF protocol is widely used in practical systems [12, 13].

**Decode-and-Forward (DF)** Relays with DF protocol decode the received signal block by block, and then transmit the re-encoded signal [10]. These relays are known as *regenerative relays*. They are also referred as digital repeaters, bridges, or routers [5]. DF gives good signal-to-noise ratio (SNR) performance. Yet, it requires high computational power. DF protocol does not work as fast as AF does [14].

**Compress-and-Forward (CF)** Relays with CF protocol are also called Estimate-and-Forward (EF) [7, 10], Observe-and-Forward (OF) [12], or Quantize-and-Forward (QF) [15]. CF is similar to DF protocol. But, unlike DF, CF goes beyond the quantization process by applying source coding techniques on the received signal [16, 17]. Thus, CF protocol can be regarded as a hybrid solution of DF and AF protocols. The received signal is not decoded at the relay. But, the signal is quantized and source-coded before it is transmitted. So, the transmitted signal contains estimation errors. At the destination, the relay estimation can be used as side information when coding the signal coming through the direct source-to-destination link [10].

**Store-and-Forward (SF)** Relays with SF protocol store the received signal, and transmit it at a later time to the destination. Unlike AF, DF or CF protocols, which are implemented at the physical layer, SF is a network layer protocol. Relays with SF protocol are mostly used in multiple hop network systems with high error rates, or in the systems which require long delays [18].

Among these protocols, AF and DF are the most commonly implemented protocols. Therefore, only these protocols are considered in the following chapters throughout the rest of this thesis.

## 2.3 Full-Duplex Relaying

In this section, first of all, half-duplex and full-duplex systems in general are explained. They are compared both in general communication system and in relaying system aspects. Advantages and disadvantages coming with half-duplex and full-duplex schemes are contrasted. Then, the self-interference<sup>1</sup> concept occurring due

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<sup>1</sup>When a signal is transmitted from the relay towards a different node, it is also unintentionally received at the relay itself, thus interfering with the actual received signal. Therefore, it is called *self-interference*. In some literature, self-interference is referred as *loop interference*, such as [19, 20, 21, 22].

to the implementation of full-duplex protocol on the relaying systems is explained. Possible consequences of the presence of self-interference to the system performance in general are discussed.

### 2.3.1 Half-Duplex vs. Full-Duplex Systems

Point-to-point communication systems can be separated into two groups based on their ability to allow simultaneous transmission on different directions.

**Half-Duplex Systems (HD)** Half-duplex systems are point-to-point communication systems, in which simultaneous communication is not possible. If different signals are transmitted from different ends at the same time, the signals interfere with each other. Each node in the system should wait for its turn to transmit [10]. One way to achieve that is to allocate short time intervals for each node. By doing that, the communication on each direction looks practically uninterrupted. This is called *time-division duplexing (TDD)* [10].

**Full-Duplex Systems (FD)** Full-duplex systems are point-to-point communication systems, in which simultaneous communication works. This can be possible if the channel for the relay operation consists of two ideally orthogonal subchannels [10]. It can be achieved by allocating different spectrums for each node to transmit on. Since the transmitted signals are carried over different frequency bands, they do not interfere with each other. This is called *frequency-division duplexing (FDD)* [10].

The word *duplex* refers to two-way point-to-point communication systems. However, the relaying systems with only one-way communication can also be considered to be HD or FD systems. We can say that the reason for that lies on the fact that at the relay, both reception and transmission occur<sup>2</sup>. And the choice of handling these two processes at the same time or letting them wait for each other draws the line between HD and FD characteristics of the relay systems<sup>3</sup>.

In HD relaying systems, the same frequency band is used on both of the hops from the source to the relay and from the relay to the destination. However, different time slots are occupied by each hop [23]. Thus, the received and the transmitted signals at the relay do not interfere with each other. However, the time to send a symbol or to receive a symbol becomes two-fold compared to FD. Due to this spectral efficiency loss [24], half of the time spent on the communication process is wasted in HD systems. Therefore, FD systems are more efficient than HD systems in terms of system capacity. Theoretically, FD systems provide twice as much capacity as HD systems do [25, 26]. The HD mode may only be preferred over the FD mode for the

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<sup>2</sup>In other words, relay is a transceiver.

<sup>3</sup>However, this thesis understands full-duplex arrays as transmission on a single band without TDD or FDD.

systems of which the overall capacity is not too important, due to their much easier implementation [27, 28, 29].

Relay nodes operating in the FD mode can receive and transmit simultaneously on the same frequency [4]. Thus, relaying systems working on the FD mode are spectrally more efficient [30]. Yet, the FD mode causes a disadvantage. The transmitted signal from the relay to the destination is also received at the receiver side of the relay itself. In other words, self-interference occurs at the relay. Because of such drawback, FD relaying systems conventionally are not deployed as widely as HD relays are, as discussed in [4].

### 2.3.2 Self-Interference in Full-Duplex Relays

Full-duplex relays allow simultaneous reception and transmission. The signal transmitted towards the relay is received by the receiver antennas of the MIMO relay, while the signal received at the relay in some previous time slot is being retransmitted from the transmitter antennas after being processed. In the next time slot, the signal transmitted from the transmitter antennas of the relay unintentionally interferes with the signal received by the receiver antennas of the relay. Assuming zero or low correlation between the signals received in different time slots, this self-interference signal coming from the transmitter side of the relay degrades the system SINR performance.

## 2.4 Self-Interference Mitigation

This section introduces some terms and concepts about self-interference mitigation schemes, which are used or referred throughout this thesis.

The main motivation behind using the FD mode instead of the HD mode in relaying systems is to achieve higher capacity. Because reception and transmission happens at the same time on the same channel, the FD mode brings higher data rate to the system. Yet, the FD mode comes with self-interference, which should be cancelled or, at least, mitigated. Needless to say, the simplest way to avoid self-interference is to use the HD mode, which is widely adopted in several articles [16, 31, 32]. Having said these, if, however, FD relaying is to be used, there are several techniques to combat self-interference. With these mitigation techniques, the FD mode can supersede the HD mode in terms of data rate.

It may, possibly, be confusing for some to use the terms *interference mitigation* and *throughput maximization* interchangeably. Because, the ultimate goal is supposed to be maximizing the system capacity, not minimizing the interference, which is only a step towards that goal. It cannot be denied that, theoretically speaking, there is not always a monotonic relation between the system capacity and the self-

interference level. However, in practical systems, since the self-interference is much stronger than the desired signal, the system capacity converges to its upper limit, as the self-interference is fully cancelled out at the relay [33].

An important parameter for the efficiency of those signal processing methods in improving the system capacity is the *degrees of freedom* of the system. Degrees of freedom refers to the dimension of the system. For a relay communication system (i.e. that shown in Fig. 2.1), the degrees of freedom increase as the number of antennas at the receiver and transmitter sides of the relay, together with the number of antennas at the source,  $S$ , or at the destination,  $D$ , increases [34, 35]. However, throughout this thesis, only SISO links will be taken into consideration. In other words, there is only one degree of freedom at the source and the destination nodes.

Having more degrees of freedom leads to better optimization<sup>4</sup>. More specifically, increasing the number of antennas at the receiver and transmitter sides of the relay in the system given in Fig. 2.1 will possibly result in higher upper limit for the end-to-end SINR. Following the same logic, it can be considered that having only a single antenna both at the receiver side and at the transmitter side of the relay would mean having only one degree of freedom in the system.

After having compared the HD and FD modes, it can be argued that HD relaying system is a fair choice for a practical system implementation, since HD relaying systems are easy to deploy and do not suffer from self-interference [16, 31, 32]. However, the optimal choice between the HD and FD modes depends on the system parameters changing incessantly in mobile systems [37]. Hence, an adaptive system switching on real time between the HD and FD modes depending on the channel conditions can result in higher spectral efficiency [36].

There are different methods to reduce the self-interference present in FD relaying systems. The most straightforward and intuitive way to mitigate the self-interference might be to estimate it at the relay, possibly by using a training signal<sup>5</sup>, and then subtract the estimation from the received signal. But, the self-interference is easy to estimate only in fixed relay systems, of which the transmitter and the receiver are not mobile, and possibly, there is line-of-sight between them. This simple way of removing the self-interference does not work well in mobile systems, due to the difficulties in channel estimation. Because, in mobile systems, the distance between the receiver and the transmitter varies, and an obstacle may temporarily block the line-of-sight.

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<sup>4</sup> "Better optimization" implies whatever the optimization problem is aiming at. For our case, it is higher system capacity/SINR.

<sup>5</sup>Prior to the actual data transmission, if the source node is kept silent, and if an array of training bits is transmitted from the transmitter side of the relay, it is fair to expect only the self-interference signal at the receiver side of the relay, which is nothing but the weighted training signal with time delay.



In the following, some interference mitigation techniques are covered. For the ease of understanding, they are presented in three groups; physical isolation, time-domain cancellation, spatial suppression; as they are grouped in [30].

### 2.4.1 Physical Isolation

Physical isolation, or antenna isolation, refers to any means of interference mitigation due to physical deployment characteristics of any on-frequency repeaters in general. On-frequency repeaters are the repeaters which receive and transmit signals on identical frequencies. Their implementations are cost-effective. Yet, they are effective only with sufficient physical isolation between the receive and transmit antennas [38]. In case of insufficient physical isolation, excessive feedback from the transmitter side to the receiver side reduces system SINR. It may even cause the repeater to go into oscillation. Therefore, the transmit power of the repeater should be less than the physical isolation by a certain factor known as *gain margin* for reliable transmission [39]. For the relays with separated antennas, greater distance between the receive and the transmit antennas results in less interaction between the receiver and transmitter sides of the relay, which produces less self-interference. Using directional antennas on each side of the relay and placing them towards opposite directions reduces the interference [39]. The environment in which the relay is deployed is also significant in determining the physical isolation, e.g., having a physical obstacle between the receive and the transmit antennas may substantially reduce the self-interference [39].

### 2.4.2 Time-Domain Cancellation

The self-interference signal at the receiver side of a relay can be considered the response of the self-interference channel<sup>6</sup> to the signal transmitted from the relay in some previous time slot. In other words, all it takes to determine the self-interference signal is to have the knowledge of the transmit signal of the relay, and the self-interference channel response. Once the self-interference signal is determined, it can be cancelled out at the instant the signal is received at the receiver side, since the knowledge of the transmit signal is already available at the relay [40]. However, the self-interference channel should be estimated, which is a critical task. Since cancellation is done at the receiver by subtraction, in case of large channel estimation errors, time-domain cancellation may cause even more interference instead of mitigating it[30].

### 2.4.3 Spatial Suppression

Self-interference at the relay can be also mitigated by using beamforming filters on the relay. The signal transmitted from the relay, firstly, goes through the transmitter filter, then after traveling through the self-interference channel, the receiver filter of

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<sup>6</sup>In some literature, self-interference channel is called *leakage channel*, i.e. [38, 39].

the relay is applied on the signal. The idea behind interference mitigation lies on the possibility that these beamforming filters can be adaptively modified in such a way that the information stored in the useful signal is preserved as much as possible, while the adverse effect of the interfering signal to the overall system performance is reduced [30].

Spatial suppression can be applied by utilizing different methods. Some of them are introduced below.

By selecting only one of the antennas, instead of using both of them at the receiver and transmitter sides, the self-interference mitigation can be obtained [30]. With antenna selection method, the best combination of the number of antennas for each side producing the lowest interference is selected.

The self-interference channel transfer function of a MIMO relay can be regarded as a matrix which can be written in singular value decomposition (SVD) form. Following the steps of (20), (21), (22) in [30], the set of singular values resulting in the lowest interference can be selected. This method is called beam selection [30].

Unlike the methods mentioned above, null-space projection aims to eliminate the interference completely, instead of mitigating it to some extent. Similar to the time-domain cancellation method, the self-interference signal goes through the receiver and transmitter filters of the relay, as well as the self-interference channel. But, instead of estimating the self-interference and subtracting it from the received signal at the receiver side, the self-interference can be directly set to be zero by choosing a proper pair of relay filters [20, 33]. Obviously, there is no control over the self-interference channel. Yet, the receiver and transmitter filters of the relay are system design parameters, which can be set to adaptively fit to the system in case of self-interference channel variations. For any given transmitter (or receiver) filter, the set of receiver (or transmitter) filters which cancels the self-interference can be obtained due to vector orthogonality. Then, among all those filters, the one producing the highest end-to-end performance can be chosen. Even without the knowledge of any of the relay filters, an iterative approach can be adopted [33, 41].

Instead of trying to mitigate the self-interference at the relay, the system can be considered as a single optimization problem. Using minimum mean square error (MMSE) method, the interference-plus-noise of the system can be mitigated [30]. This method is elaborated more in Section 5.2.

## 2.5 Survey on Performance Analysis

In this section, current literature on the performance optimization of FD relaying systems is summarized. Various techniques based on different assumptions to maximize SINR or capacity are explained. For most of the cases, the key to obtain

higher SINR can be regarded as reducing the interference as much as possible. Some of the literature concentrates on self-interference mitigation, whereas others try to maximize the end-to-end SINR<sup>7</sup> to achieve the optimal system. The techniques are reviewed below in a rather chronological order.

Due to the simultaneous occurrence of reception and transmission, FD relays suffer from self-interference, which causes a decrease in system capacity. In the ideal interference-free case, full-duplex systems promise twice the capacity of half-duplex systems. However, the presence of self-interference within the relay degrades the capacity. Therefore, capacity improvement of full-duplex relays over the half-duplex relays depends upon the level of the self-interference. In [37], the authors discuss the feasibility of full-duplex SISO relay systems by comparing them to conventional half-duplex relays. After considering several power allocation schemes, they conclude that if the self-interference is below a certain level, the full-duplex mode can be preferable over the conventional half-duplex mode. Similarly, the authors in [42] study the feasibility of the full-duplex mode MIMO relay systems with amplify-and-forward protocol. They compare the average capacity for the full-duplex and half-duplex modes. And they derive a condition for the full-duplex mode to outperform the half-duplex mode in terms of capacity.

However, in practical systems, it is not always the case to make an estimation about the self-interference level prior to the system design. Therefore, making the choice in advance between the full-duplex and half-duplex modes for system design may not be possible. Instead, an adaptive system switching the full-duplex mode on/off can achieve higher capacity. In [36], the authors propose a hybrid technique that switches between the full-duplex and half-duplex modes, and that uses transmit power adaptation. The authors show that their proposed technique works better in optimizing both instantaneous and long-term spectral efficiency.

In practice, self-interference mitigation techniques cannot completely eliminate the self-interference due to imperfect channel estimation. Therefore, the authors in [43] propose relay transmit power control schemes, which can adaptively set the relay transmit power to combat the residual self-interference. High self-interference degrades the first hop channel SINR. Since the end-to-end SINR is limited by the lower hop SINR, the relay transmit power can be reduced to boost the first hop SINR at the expense of second hop SNR. Or inversely, low self-interference results in high first hop channel SINR. So, the relay transmit power is increased to favor the lower second hop SNR while sacrificing the first hop SINR. The authors in [44] present a transmit power control scheme for MIMO full-duplex decode-and-forward relay systems minimizing the residual self-interference. They compare their scheme to the half-duplex mode, and conclude that it is not only more power efficient, but also results in higher capacity.

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<sup>7</sup>In the following chapters of this thesis, focusing on reducing the self-interference is referred as *null-space projection*, a method forcing the self-interference to be zero, and maximizing the end-to-end SINR is referred as *global optimization problem*.

Self-interference mitigation in full-duplex MIMO relays is crucial for higher capacity. The suppression of self-interference can be achieved via time-domain subtraction, as well as via multi-antenna techniques in spatial domain. In [45], the authors propose null-space projection and MMSE filtering schemes to mitigate self-interference in full-duplex relaying systems in MIMO link. With null-space projection, self-interference is forced to be zero. With MMSE filtering scheme, optimal relay receiver filter is derived with the knowledge of relay transmitter filter. In [33], the authors propose an algorithm which searches for the optimal beamforming vectors in a full-duplex relay canceling the self-interference via null-space projection. The proposed algorithm uses an iterative approach to optimize the system. The authors in [41] propose a new spatial-domain method to eliminate self-interference both in amplify-and-forward and decode-and-forward full-duplex relays. Their proposed method uses an iterative algorithm. The authors conclude that their method results in better performance than the conventional SVD-based method. In [30], the authors propose schemes for spatial-domain interference suppression in full-duplex MIMO relays. They conclude that the proposed schemes succeed in mitigating the self-interference to a tolerable level.

Conventional full-duplex relays use antennas separately at the receiver and at the transmitter. The authors in [25] suggest that the efficiency of antenna resources can be improved if each antenna at the relay transmits and receives simultaneously on the same frequency band to obtain higher capacity. They propose a method for full-duplex relaying system which relies on antenna sharing between the receiver and transmitter sides of the relay. They claim that their proposed method outperforms the conventional full-duplex relaying systems. However, it remains questionable whether single-array full-duplex transceiver can be implemented in practice.

One of the main motives of deploying relay systems is to extend the indoor coverage by mirroring the outdoor signal to indoor. For this application, full-duplex relays can clearly promise higher capacity than half-duplex relays. However, one of the fundamental challenges of indoor full-duplex relays, as opposed to those of outdoor, is that using directional antennas and increasing the physical distance between the antennas is not a viable solution to suppress the self-interference. In [46], the authors conduct experiments and present self-interference measurement results for outdoor-to-indoor full-duplex relays. The authors claim that indoor full-duplex relaying systems are feasible in practice.

The system performance can be improved more by maximizing the end-to-end SINR instead of eliminating the self-interference at the relay. The authors in [47] propose an algorithm to mitigate the self-interference in full-duplex amplify-and-forward relaying systems by considering the system as a whole, instead of using zero forcing schemes on the self-interference signal. In the proposed algorithm, the receiver and transmitter relay filters optimizing the system are searched. The authors conclude that their algorithm outperforms any null-space projection scheme. However, they

neglect the processing delay at the relay, which tends to give more optimistic system performance in the simulations.

More literature related to wireless relaying systems can be found in [9, 12, 16, 31, 32, 35, 48, 49, 50, 51, 52, 53, 54].

Accurate channel estimation is crucial to the validity of any optimization technique, even though it is not a focal point in this thesis. Therefore, throughout the thesis, the channels are always assumed to be perfectly estimated without any justification. For the readers in interest, some literature related to channel estimation on relay systems can be found in [40, 55, 56, 57, 58, 59, 60].

# Chapter 3

## System Model

In this chapter, firstly, a system model is defined for a two-hop SISO communication link with a single MIMO relay connecting those two hops. Then, the system parameters are explained, and the symbols used for those parameters are introduced. Later, some approximations and constraints to the system are given. Finally, the derivation of system SINR is presented for both AF and DF protocols.

### 3.1 MIMO Relay in SISO Link

We are considering a two-hop communication system with a full-duplex relay,  $R$ , between a source,  $S$ , and a destination,  $D$ . The source,  $S$ , and the destination,  $D$ , are single-antenna nodes. The relay may have multiple receiver and transmitter antennas, with the number of antennas represented by  $N_{rx}$  for the receiver side, and  $N_{tx}$  for the transmitter side. The illustration of this model is given in Fig. 3.1.

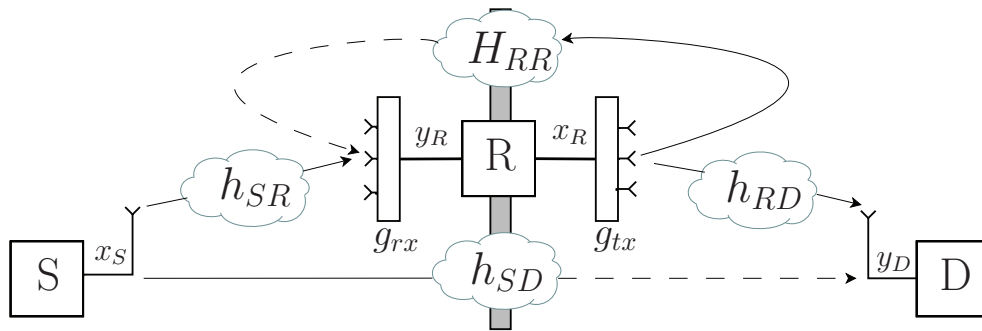


Figure 3.1: System model of a two-hop communication system with a full-duplex MIMO relay

Having multiple antennas at the relay results in multiple subchannels. There are, apparently,  $N_{rx}$  subchannels between the source,  $S$ , and the relay,  $R$ , nodes. Similarly, there are  $N_{tx}$  subchannels between the relay,  $R$ , and the destination,  $D$ , nodes. And there are  $N_{rx} \times N_{tx}$  subchannels within the relay as the self-interference channel.

Hence, the channel transfer functions are considered as matrix and vectors. The channel responses presented in Fig. 3.1 represent the corresponding channels given below.

$$\begin{aligned}
 h_{SR} &\in \mathbb{C}^{N_{rx} \times 1} && \text{the channel between the source, } S, \text{ and the relay, } R \\
 h_{RD} &\in \mathbb{C}^{1 \times N_{tx}} && \text{the channel between the relay, } R, \text{ and the destination, } D \\
 H_{RR} &\in \mathbb{C}^{N_{rx} \times N_{tx}} && \text{the self-interference channel within the relay, } R \\
 h_{SD} &\in \mathbb{C} && \text{the channel between the source, } S, \text{ and the destination, } D
 \end{aligned}$$

For any channel in the system model, additive white Gaussian noise (AWGN) is assumed to be present. The additive noise terms at the relay and at the destination are represented as

$$\begin{aligned}
 n_R &\in \mathbb{C}^{N_{rx} \times 1} && \text{vector of independent AWGN terms at the relay, } R \\
 n_D &\in \mathbb{C} && \text{AWGN term at the destination, } D
 \end{aligned}$$

The relay has multiple antennas both at the receiver and at the transmitter. This is useful for combating self-interference, although spatial multiplexing is not possible with SISO relays [36]. The beamforming vectors at the transmitter and at the receiver are represented in Fig. 3.1 as given below.

$$\begin{aligned}
 g_{rx} &\in \mathbb{C}^{1 \times N_{rx}} && \text{the beamforming filter at the receiver side} \\
 g_{tx} &\in \mathbb{C}^{N_{tx} \times 1} && \text{the beamforming filter at the transmitter side}
 \end{aligned}$$

For a communication system which has multiple antennas at the receiver, the same signal is received multiple times, each copy being slightly different, as a result of multipath scattering, having different angles and different distances to different antennas at the receiver. In other words, the same copies of the signal that is sent from the transmitter go through slightly different channels before they reach the receiver. The similar logic can be carried on for a communication system which has multiple antennas at the transmitter. The same copies of the transmitted signal go through slightly different channels before they are received at the receiver.

Hence, by applying beamforming filters, useful information over the multiple copies can be combined. This way, the maximum useful signal power at the receiver is attained, while keeping the noise and interference signal powers low. As a result, higher SINR performance is achieved. As more antennas are used at the transmitter or at the receiver side of the relay, degrees of freedom of the system increases, thus leading to higher SINR [34, 35].

Since  $g_{rx}n_R \sim \mathcal{CN}(0, \sigma_R^2)$  and  $n_D \sim \mathcal{CN}(0, \sigma_D^2)$ , the noise powers can be calculated as given below. Note that noise signal at the relay,  $n_R$ , is affected by the receiver beamforming filter,  $g_{rx}$ , for which we assume  $\|g_{rx}\|_2 = 1$ .

$$\sigma_R^2 = E\{|g_{rx}n_R|^2\} \quad (3.1)$$

$$\sigma_D^2 = E\{|n_D|^2\} \quad (3.2)$$

The average power of the input signal transmitted from the source,  $p_S$ , and the average power of the signal transmitted from the relay,  $p_R$ , are defined as

$$p_S = E\{|x_S|^2\} \quad (3.3)$$

$$p_R = E\{|x_R|^2\} \quad (3.4)$$

In this system model, two of the relaying protocols introduced in Section 2.2 are considered separately. These are AF and DF protocols.

- With amplify-and-forward (AF) protocol, the relay amplifies the received signal,  $y_R$ , in every time slot,  $k$ , to obtain the relay transmitted signal,  $x_R$ . The relationship between the relay input,  $y_R$ , and the relay output,  $x_R$ , can be given [12] as

$$x_R[k] = \beta y_R[k - k_D^{\text{AF}}] \quad (3.5)$$

where  $\beta$  is the relay processing gain, and  $k_D^{\text{AF}}$  is the processing delay. Because the relay processing gain,  $\beta$ , is simply the gain of the signal in amplitude, it can be calculated by taking the square root of the power gain, which is the ratio of the output signal power to the input signal power of the relay.

- With decode-and-forward (DF) protocol, the relay decodes a block of the received signal,  $y_R$ , and re-encodes it before transmission. In this decoding/re-encoding process, the same coding schemes are used. Therefore, the signal is regenerated as it was at the source,  $S$ . For the ideal case in which there is no decoding error, the signal transmitted from the relay is exactly the equivalent of the signal transmitted from the source,  $S$ , but with a delay and power normalization to relay transmit power,  $p_R$ . Therefore, in the ideal case, the signal at the relay transmitter is written [12] as

$$x_R[k] = \sqrt{\frac{p_R}{p_S}} x_S[k - k_D^{\text{DF}}] \quad (3.6)$$

where  $k_D^{\text{DF}}$  is the relay processing delay.

Note that DF requires more computational power, and causes longer delay than AF does. Yet, DF results in higher SINR performance [14, 49].



## 3.2 Signal Transmission

The input signal,  $x_S$ , is transmitted from the source,  $S$ , to the destination,  $D$ , through two independent paths. It is received both after going through the relaying system and directly through the source-to-destination channel,  $h_{SD}$ . For the direct source-to-destination transmission path, the input signal,  $x_S$ , travels through the channel,  $h_{SD}$ , before it is directly received at the destination,  $D$ , node.

For the relay transmission path, the input signal,  $x_S$ , is transmitted through the *relay link* channel,  $h_{SR}$  [10]. Multiple copies of this information are received at the receiver side of the relay together with the interference signal coming from the transmitter side of the relay. This combined signal,  $y_R$ , is processed at the relay according to either AF or DF protocol. For AF protocol, the signal goes through a power amplification process. For DF protocol, the signal goes through a decoding/re-encoding process. The output signal,  $x_R$ , of either of these relay processes is transmitted via multiple antennas at the transmitter side of the relay. This signal is received both at the receiver side of the relay and at the destination,  $D$ , node of the system. It goes through the self-interference channel,  $H_{RR}$ , before being received at the receiver side of the relay as the self-interference signal [26, 37]. And it also goes through the access link channel,  $h_{RD}$ , before it is received at the destination,  $D$  [10].

The signal coming through the relay link and the signal coming directly from the source,  $S$ , node have different delays, mostly because of the relay processing and the different physical distances of the channels. So, when they are received at the destination,  $D$ , they interfere with each other. However, the system model is considered for the cases in which the direct transmission from the source,  $S$ , to the destination,  $D$ , does not perform satisfactorily. Mainly for that reason, a relaying system, enabling a better indirect connection is in use. Based on this motivation, it is a valid and acceptable assumption that the direct link channel between the source and the destination,  $h_{SD}$ , is too weak to be considered an interference signal for the relay link. Thus, the channel,  $h_{SD}$ , will be assumed to be non-existent throughout this thesis due to acute path loss as discussed in [61, 62, 63], i.e.,

$$h_{SD} \approx 0 \tag{3.7}$$

## 3.3 System Constraints

The beamforming filters at the receiver of the relay,  $g_{rx}$ , and at the transmitter of the relay,  $g_{tx}$ , are assumed to have unit Euclidean norms.

$$\|g_{rx}\|_2 = 1 \qquad \|g_{tx}\|_2 = 1 \tag{3.8}$$

Therefore, the power of the signal going through either of these filters is preserved

on average<sup>1</sup>. With this assumption, the relay transmit power can be extracted from beamforming vectors, and considered as a separate system parameter.

In practice, the transmit power of a signal is set based on several criteria, e.g., channel conditions, the availability of a power source around the transmitter, etc. Since these practical conditions are not easy to be estimated in theory, it is better to define a theoretical bound for the transmit power, and take those practical conditions into account by setting the channel SNRs based on real-life measurements. Due to the simplicity it brings to the calculations, the transmit powers at the source,  $p_S$ , and at the relay,  $p_R$ , are normalized to be upper bounded by 1. Since the power of a signal is always non-negative, the constraints on the transmit powers,  $p_S$  and  $p_R$ , can be written as

$$0 \leq p_S \leq 1 \quad (3.9)$$

$$0 \leq p_R \leq 1 \quad (3.10)$$

The effect of  $p_S$  can only be observed at the first hop of the system (source-to-relay). Since the direct link is assumed to be too weak, as given in (3.7), higher  $p_S$  results in higher end-to-end SINR performance. Therefore, it is straightforward to conclude that the end-to-end system optimization is achieved when

$$p_S = 1 \quad (3.11)$$

However, if the direct source-to-destination link were not weak enough to be neglected, then, the optimal  $p_S$  would not necessarily be one, due to the fact that the signals arriving at the destination,  $D$ , through different links would interfere with each other.

Changing the relay transmit power,  $p_R$ , however, affects both the relay-to-destination link and the amount of self-interference simultaneously. Thus, there is a trade-off between obtaining low self-interference and achieving high relay-to-destination channel SNR in terms of  $p_R$ . The optimal  $p_R$  maximizing the end-to-end SINR depends upon the set of channel realizations.

### 3.4 Signal to Interference plus Noise Ratio (SINR)

The received signal at the relay,  $y_R$ , and the received signal at the destination,  $y_D$ , can be written as

$$\begin{aligned} y_R &= g_{rx} (h_{SR}x_S + H_{RR}g_{tx}x_R) + g_{rx}n_R \\ &= g_{rx}h_{SR}x_S + g_{rx}H_{RR}g_{tx}x_R + g_{rx}n_R \end{aligned} \quad (3.12)$$

---

<sup>1</sup>It is impossible to preserve the signal power for each realization of the received signal with unit filter norm. However, average power of the signal at the relay over many independent realizations should converge to input signal power.

$$y_D = h_{RD}g_{tx}x_R + h_{SD}x_S + n_D \quad (3.13)$$

The average power of the received signal for each hop of the system is

$$E\{|y_R|^2\} = E\{|g_{rx}h_{SR}x_S + g_{rx}H_{RR}g_{tx}x_R + g_{rx}n_R|^2\} \quad (3.14)$$

$$E\{|y_D|^2\} = E\{|h_{RD}g_{tx}x_R + h_{SD}x_S + n_D|^2\} \quad (3.15)$$

We assume that the samples of input signal,  $x_S$ , transmitted from the source,  $S$ , over time are independent of each other. Thus, due to the processing delay, the signals  $x_S$  and  $x_R$  are also considered to be uncorrelated, i.e.,  $E\{x_Sx_R\} \approx 0$ . Then, the power terms can be simplified as

$$E\{|y_R|^2\} = E\{|g_{rx}h_{SR}x_S|^2\} + E\{|g_{rx}H_{RR}g_{tx}x_R|^2\} + E\{|g_{rx}n_R|^2\} \quad (3.16)$$

$$E\{|y_D|^2\} = E\{|h_{RD}g_{tx}x_R|^2\} + E\{|h_{SD}x_S|^2\} + E\{|n_D|^2\} \quad (3.17)$$

On the right-hand side of (3.16) and (3.17), the first terms are the useful signal powers, the second terms are the self-interference signal powers, whereas the third terms are the additive noise powers. Thus, SINR values for the first hop,  $\gamma_R$ , and for the second hop,  $\gamma_D$ , can be written as

$$\gamma_R = \frac{E\{|g_{rx}h_{SR}x_S|^2\}}{E\{|g_{rx}H_{RR}g_{tx}x_R|^2\} + E\{|g_{rx}n_R|^2\}} \quad (3.18)$$

$$\gamma_D = \frac{E\{|h_{RD}g_{tx}x_R|^2\}}{E\{|h_{SD}x_S|^2\} + E\{|n_D|^2\}} \quad (3.19)$$

After the constant terms are taken out of the expectations, SINR values,  $\gamma_R$  and  $\gamma_D$ , become

$$\gamma_R = \frac{|g_{rx}h_{SR}|^2 E\{|x_S|^2\}}{|g_{rx}H_{RR}g_{tx}|^2 E\{|x_R|^2\} + E\{|g_{rx}n_R|^2\}} \quad (3.20)$$

$$\gamma_D = \frac{|h_{RD}g_{tx}|^2 E\{|x_R|^2\}}{E\{|h_{SD}x_S|^2\} + E\{|n_D|^2\}} \quad (3.21)$$

Because the direct source-to-destination link is assumed to be weak, its channel

response,  $h_{SR}$ , is approximated to be zero as given in (3.7). Then, the interference term in the denominator of (3.21) is cancelled out as

$$E\{|h_{SD}x_S|^2\} \approx 0 \quad (3.22)$$

Then,  $\gamma_R$  and  $\gamma_D$  are simplified as

$$\gamma_R = \frac{|g_{rx}h_{SR}|^2 p_S}{|g_{rx}H_{RR}g_{tx}|^2 p_R + \sigma_R^2} \quad (3.23)$$

$$\gamma_D = \frac{|h_{RD}g_{tx}|^2 p_R}{\sigma_D^2} \quad (3.24)$$

since

$$p_S = E\{|x_S|^2\} \quad (3.25)$$

$$p_R = E\{|x_R|^2\} \quad (3.26)$$

$$\sigma_R^2 = E\{|g_{rx}n_R|^2\} \quad (3.27)$$

$$\sigma_D^2 = E\{|n_D|^2\} \quad (3.28)$$

Let us denote the channel SNRs of the system as

$$\gamma_{SR} = \frac{|g_{rx}h_{SR}|^2}{\sigma_R^2} \quad (3.29)$$

$$\gamma_{RR} = \frac{|g_{rx}H_{RR}g_{tx}|^2}{\sigma_R^2} \quad (3.30)$$

$$\gamma_{RD} = \frac{|h_{RD}g_{tx}|^2}{\sigma_D^2} \quad (3.31)$$

By substituting (3.29), (3.30) and (3.31) into (3.23) and (3.24), SINR of each hop of the system can be represented in terms of channel SNRs as

$$\boxed{\gamma_R = \frac{p_S \gamma_{SR}}{p_R \gamma_{RR} + 1}} \quad (3.32)$$

$$\boxed{\gamma_D = p_R \gamma_{RD}} \quad (3.33)$$

### 3.4.1 SINR with Amplify-and-Forward Protocol

An important parameter for the system performance optimization is the end-to-end SINR. For the end-to-end system with AF protocol, the received signal at the destination node,  $D$ , is derived using (3.5), (3.12) and (3.13). After (3.5) and (3.12) are substituted into (3.5), the signal at the destination,  $D$ , is found as

$$y_{D,e2e} = h_{RD}g_{tx}\beta(g_{rx}h_{SR}x_S + g_{rx}H_{RR}g_{tx}x_R + g_{rx}n_R) + h_{SD}x_S + n_D \quad (3.34)$$

After substituting (3.7) and (3.28) into the mean square of (3.34), we obtain

$$\begin{aligned} E[|y_{D,e2e}|^2] &= |h_{RD}g_{tx}|^2\beta^2|g_{rx}h_{SR}|^2E[|x_S|^2] + |h_{RD}g_{tx}|^2\beta^2|g_{rx}H_{RR}g_{tx}|^2E[|x_R|^2] \\ &\quad + |h_{RD}g_{tx}|^2\beta^2E[|g_{rx}n_R|^2] + \sigma_D^2 \end{aligned} \quad (3.35)$$

The first term on the right-hand side of (3.35) is the useful signal power, whereas the second and the third terms are the interference signal and noise powers, respectively.

Since SINR is defined as the ratio of the useful signal power to the interference signal and noise powers, the end-to-end SINR value can be written as

$$\gamma_{SRD} = \frac{|h_{RD}g_{tx}|^2\beta^2|g_{rx}h_{SR}|^2E[|x_S|^2]}{|h_{RD}g_{tx}|^2\beta^2|g_{rx}H_{RR}g_{tx}|^2E[|x_R|^2] + |h_{RD}g_{tx}|^2\beta^2E[|g_{rx}n_R|^2] + \sigma_D^2} \quad (3.36)$$

After substituting (3.25), (3.26) and (3.27), the SINR of the end-to-end system is calculated as

$$\gamma_{SRD} = \frac{|h_{RD}g_{tx}|^2\beta^2|g_{rx}h_{SR}|^2p_S}{|h_{RD}g_{tx}|^2\beta^2|g_{rx}H_{RR}g_{tx}|^2p_R + |h_{RD}g_{tx}|^2\beta^2\sigma_R^2 + \sigma_D^2} \quad (3.37)$$

For the AF protocol, the signal is not regenerated at the relay. Instead, the process is a power amplification. The gain factor,  $\beta$ , normalizes the relay transmit power to  $p_R$ . Therefore, it can be represented as a function of the power ratio between the relay output signal and the relay input signals. Hence, by using (3.5), (3.16), (3.27), (3.25) and (3.26), the processing gain,  $\beta$ , can be defined as

$$\beta = \sqrt{\frac{p_R}{|g_{rx}h_{SR}|^2p_S + |g_{rx}H_{RR}g_{tx}|^2p_R + \sigma_R^2}} \quad (3.38)$$

After substituting the processing gain,  $\beta$ , in (3.38) into the end-to-end SINR (3.37),  $\gamma_{SRD}$  is obtained as

$$\gamma_{SRD} = \frac{|h_{RD}g_{tx}|^2p_R|g_{rx}h_{SR}|^2p_S}{|h_{RD}g_{tx}|^2|g_{rx}H_{RR}g_{tx}|^2p_R^2 + |h_{RD}g_{tx}|^2p_R\sigma_R^2 + \dots} \quad (3.39)$$

$$\cdots \sigma_D^2 |g_{rx} h_{SR}|^2 p_S + \sigma_D^2 |g_{rx} H_{RR} g_{tx}|^2 p_R + \sigma_D^2 \sigma_R^2$$

The channel SNRs in (3.29), (3.30) and (3.31) are substituted into the end-to-end SINR (3.39). Then,  $\gamma_{SRD}$  becomes

$$\gamma_{SRD} = \frac{p_R \gamma_{RD} \sigma_D^2 \gamma_{SR} \sigma_R^2 p_S}{\gamma_{RD} \sigma_D^2 \gamma_{RR} \sigma_R^2 p_R^2 + \gamma_{RD} \sigma_D^2 p_R \sigma_R^2 + \sigma_D^2 \gamma_{SR} \sigma_R^2 p_S + \cdots} \quad (3.40)$$

$$\cdots \sigma_D^2 \gamma_{RR} \sigma_R^2 p_R + \sigma_D^2 \sigma_R^2$$

After dividing both the nominator and the denominator of (3.40) by the term  $(\sigma_R^2 \sigma_D^2) [\gamma_{RR} p_R + 1]$ , we obtain

$$\gamma_{SRD} = \frac{p_R \gamma_{RD} p_S \gamma_{SR} / [\gamma_{RR} p_R + 1]}{p_R \gamma_{RD} + \gamma_{SR} p_S / [\gamma_{RR} p_R + 1] + 1} \quad (3.41)$$

The first hop SINR in (3.32) and the second hop SINR in (3.33) are substituted into the end-to-end SINR in (3.41). Then, the overall system SINR,  $\gamma_{SRD}$ , for amplify-and-forward (AF) protocol is found as

$$\boxed{\gamma_{SRD} = \frac{\gamma_R \gamma_D}{\gamma_R + \gamma_D + 1}} \quad (3.42)$$

### 3.4.2 SINR with Decode-and-Forward Protocol

For the end-to-end system with DF protocol, the received signal at the destination node,  $D$ , depends upon the particular coding and modulation choices. The maximum average mutual information for repetition-coded DF full-duplex relay can be found in [12] as

$$I_{SRD} = \min\{\log_2(1 + \gamma_R), \log_2(1 + \gamma_D)\} \quad (3.43)$$

Because the system capacity is equal to the maximum mutual information and there is no direct link, capacity for DF is defined as

$$C_{SRD} = \min\{\log_2(1 + \gamma_R), \log_2(1 + \gamma_D)\} \quad (3.44)$$

We know that SINR values are always non-negative by definition.

$$\gamma_R \geq 0, \quad \gamma_D \geq 0 \quad (3.45)$$

We also know that logarithm functions are strictly monotonically increasing functions for non-negative real numbers. Thus, the system capacity in (3.44) can be written as

$$C_{SRD} = \log_2(1 + \min\{\gamma_R, \gamma_D\}) \quad (3.46)$$

According to Shannon limit [64], the relation between the capacity of a communication system and its SINR value is defined for a single-hop system as

$$C_{SRD} = \log_2\{1 + \gamma_{SRD}\} \quad (3.47)$$

Therefore, comparing (3.46) and (3.47), the end-to-end SINR for decode-and-forward (DF) protocol can be defined as

$$\boxed{\gamma_{SRD} = \min\{\gamma_R, \gamma_D\}} \quad (3.48)$$

Note that the SINR expression derived for DF in (3.48) is an effective SINR value. The nonlinear decoding/re-encoding process makes it difficult to measure or even define the real end-to-end SINR.

# Chapter 4

## Optimization Problems

In this chapter, the system model stated in the previous chapter is transformed to a global optimization problem, and various constraints are introduced to obtain subproblems which will enable us to derive closed-form solutions. At the end of the chapter, global numerical solutions to these problems are given.

### 4.1 Suboptimal Solutions

The idea behind optimizing the system is to maximize the performance in the source-to-relay and relay-to-destination links, while minimizing the negative effect of the self-interference channel,  $H_{RR}$ . Apparently, it is impossible<sup>1</sup> to perfectly achieve these three criteria at the same time.

Therefore, the fundamental trade-off in this optimization problem can be argued as finding a set of beamforming vectors,  $\{g_{rx}, g_{tx}\}$ , which are keeping the first and second hop SINRs as high as possible, as well as keeping the self-interference as low as possible to achieve the highest possible end-to-end system SINR. It can be easily observed that there are some suboptimal solutions to the problem, such as neglecting the self-interference by maximizing the channel SNR of each hop. In that case, the beamforming vectors become

$$g_{rx} = \arg \max_{g_{rx}} \{\|g_{rx} h_{SR}\|^2\} \quad (4.1)$$

$$g_{tx} = \arg \max_{g_{tx}} \{\|h_{RD} g_{tx}\|^2\} \quad (4.2)$$

Due to Cauchy-Schwarz inequality, we know that

$$\|v_1 v_2\|^2 \leq \|v_1\|^2 \|v_2\|^2 \quad \text{for any vectors } v_1, v_2 \quad (4.3)$$

---

<sup>1</sup>Theoretically, however, it is possible to have a set of channel responses such that a single pair of  $\{g_{rx}, g_{tx}\}$  may optimize each of those three criteria mentioned above, just by chance. Yet, it is safe to say impossible in practice.



Because of the unit norm constraint, the optimal beamforming vectors,  $g_{rx}$ ,  $g_{tx}$ , maximizing (4.1) and (4.2) by satisfying equality in (4.3) are the normalized hermitian transposes of the channel responses corresponding to them as given below. They are called the *matched filter* solutions.

$$g_{rx} = \frac{h_{SR}^H}{\|h_{SR}\|} \quad (4.4)$$

$$g_{tx} = \frac{h_{RD}^H}{\|h_{RD}\|} \quad (4.5)$$

Another possible suboptimal solution is to take the self-interference mitigation as the primary design criterion. Based on the assumption that the self-interference is always zero, an iterative approach to find an optimal set of beamforming vectors to maximize SINR can be adopted [33, 41].

The overall relaying system now is considered to be applied in the cases where the direct source-to-destination link cannot provide high performance due to poor channel conditions. As this claim suggests, the subsolution to the general problem with the zero self-interference assumption can be thought to be a fairly close estimate to the global solution.

## 4.2 Global Optimization

The global optimization problem can be stated as maximizing the end-to-end SINR,  $\gamma_{SRD}$  given in (3.42) and (3.48), while preserving the transmit power limits, given in (3.9) and (3.10), and unit beamforming vector norm constraints, given in (3.8), with the design parameters  $g_{rx}$ ,  $g_{tx}$ , and  $p_R$ . So, it is defined as

$$\begin{aligned} \text{maximizing } \gamma_{SRD}, \text{ (3.42 \& 3.48), with the constraints} \quad & 0 \leq p_R \leq 1, \\ & \|g_{rx}\|_2 = 1, \\ & \|g_{tx}\|_2 = 1 \end{aligned}$$

However, the vector norm operations in (3.42) and (3.48) introduce nonlinearity to the problem. That makes the analytical analysis cumbersome. Hence, the global optimization is only considered as a numerical problem, which is solved on a computer simulation. The performance results of this numerical analysis are used as an upper limit for comparison to the analytical solutions of the subproblems defined with some special assumptions and/or constraints.

### 4.3 Optimization with Null-Space Projection

The SINR of the first hop of the system is given in (3.23). The denominator of this equation consists of additive noise and self-interference. A set of beamforming vectors,  $\{g_{rx}, g_{tx}\}$ , can be chosen in such a way that the self-interference becomes zero [20, 33] as

$$g_{rx}H_{RR}g_{tx} = 0 \quad (4.6)$$

By following the same idea, but in opposite direction, (4.6) can be set as a design criterion. Null-space projection does not consume all degrees freedom. Thus, the signal of interest can be also improved. With this zero self-interference constraint at the relaying system, a subsolution to the global optimization problem can be obtained.

Because the self-interference is completely eliminated, SINR for the first hop becomes

$$\gamma_R = \frac{|g_{rx}h_{SR}|^2 p_S}{\sigma_R^2} \quad (4.7)$$

Note that the optimal relay transmit power,  $p_R$ , becomes one, independent of the channel responses, due to the fact that increasing  $p_R$  does not increase the self-interference term in the first hop SINR, simply because there is no self-interference at all.

Then, the optimization problem is simplified to be

$$\begin{aligned} \text{maximizing } \gamma_{SRD}, \text{ (3.42 \& 3.48), with constraints} \quad & \|g_{rx}\|_2 = 1, \\ & \|g_{tx}\|_2 = 1, \\ & g_{rx}H_{RR}g_{tx} = 0 \end{aligned}$$

### 4.4 Numerical Evaluations

Numerical evaluation results for various cases regarding the general unconstrained problem and the null-space projection problem are presented in this section. These results are obtained using the function *fmincon* of the nonlinear optimization toolbox on MATLAB<sup>®</sup> platform.

The system setup can be described as follows. The receiver and transmitter antennas of the relay are varied from 1 to 5, while always being kept equal to each other. Source-to-relay, relay-to-destination and self-interference channels are assumed to experience Rayleigh fading. The channels are also assumed to have independent identically distributed elements. The self-interference channel is modeled as a full-rank matrix. The noise variances at the relay and at the destination are assumed to be 1. The average power gain per each subchannel for the source-to-relay,  $\phi_{SR}$ , and relay-to-destination,  $\phi_{RD}$ , links are set to be 12 dB, whereas for the self-interference

subchannels,  $\phi_{RR}$  is set to be 0 dB as given below.

$$\phi_{SR} = \frac{E\{\|h_{SR}\|_2^2\}}{N_{rx}} = 12 \text{ dB} \quad (4.8)$$

$$\phi_{RD} = \frac{E\{\|h_{RD}\|_2^2\}}{N_{tx}} = 12 \text{ dB} \quad (4.9)$$

$$\phi_{RR} = \frac{E\{\|H_{RR}\|_F^2\}}{N_{rx}N_{tx}} = 0 \text{ dB} \quad (4.10)$$

The unconstrained global problem is defined as

$$\gamma_{SRD} = \frac{\gamma_R \gamma_D}{\gamma_R + \gamma_D + 1} \quad \text{for amplify-and-forward} \quad (4.11)$$

$$\gamma_{SRD} = \min\{\gamma_R, \gamma_D\} \quad \text{for decode-and-forward} \quad (4.12)$$

Null-space projection constraint, though, introduces an additional constraint (4.6) to the global problem. The beamforming filters are generated to produce zero self-interference.

In Fig. 4.1, the results for the general optimization problem are given. In the evaluation, four different cases are considered for DF and AF protocols each. First, the general problem is taken as it is. The beamforming vectors,  $g_{rx}$  and  $g_{tx}$ , and the relay transmit power,  $p_R$ , are assumed to be the optimization parameters. Second, receiver beamforming filter,  $g_{rx}$ , is assumed to be matched to the channel between the source and the relay as in (4.4), while transmitter beamforming vector,  $g_{tx}$ , and the relay transmit power,  $p_R$ , are taken as the optimization parameters. Third, transmitter beamforming vector,  $g_{tx}$ , is assumed to be matched to the channel between the relay and the destination as in (4.5), while receiver beamforming vector,  $g_{rx}$ , and the relay transmit power,  $p_R$ , are taken as the optimization parameters. Fourth, both the receiver beamforming vector,  $g_{rx}$ , and the transmitter beamforming vector,  $g_{tx}$ , are assumed to be matched to the channel between the source and the relay, and to the channel between the relay and the destination, respectively as in (4.4), and (4.5), while the relay transmit power,  $p_R$ , is taken as the only optimization parameter. For each case above, the evaluation is run over a sufficient number of channel realizations for different number of antennas cases, and the resulting SINR values are averaged over those realizations. The number of antennas are considered to be equal on receiver and transmitter sides of the relay<sup>2</sup>. Then, the final SINR values for each case are shown in Fig. 4.1.

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<sup>2</sup>The effect of number of antenna variations on the SINR performance is investigated more deeply in Section 6.4

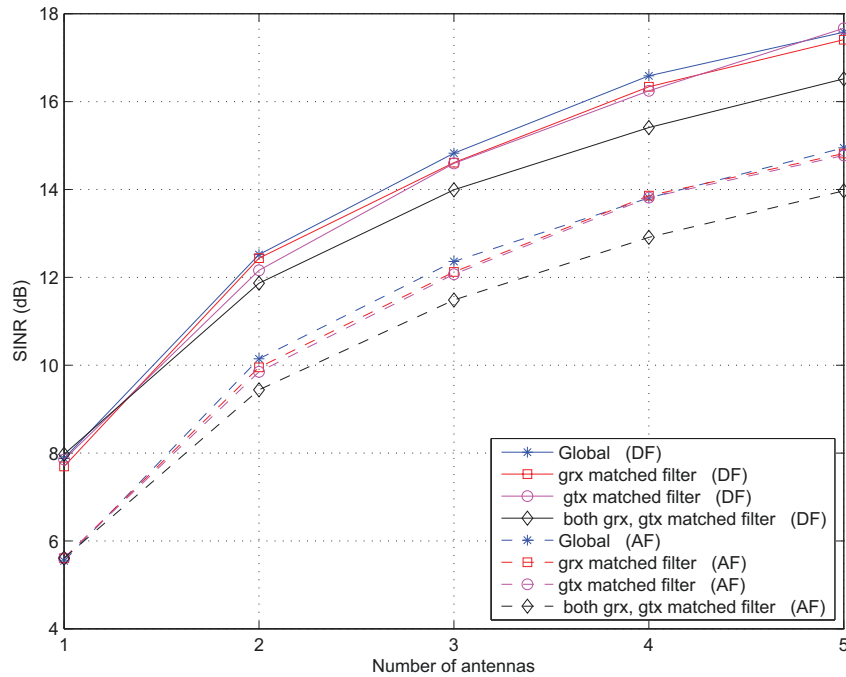


Figure 4.1: Numerical evaluation results of the global optimization problem for the general and the matched filter solutions

The second, third, and fourth cases, which utilize the matched filter concept, result in suboptimal solutions to the global problem. Yet, they are computationally faster than the general problem, since they reduce the uncertainty within the non-linear optimization algorithm by introducing some constraints. The idea behind these constraints is, simply, to maximize channel SNR of one or both of the hops of the system by matching the beamforming filters to the channel by neglecting the self-interference. Then, the other system parameters are obtained through numerical analysis.

As the number of antennas increases, SINR performance for each case improves. However, the additional SINR gain for each antenna decreases as the number of antennas increases. Global solution gives the highest SINR values, which are, in fact, the theoretical upper limits. The case in which the receiver beamforming vector,  $g_{rx}$ , is matched and the case in which the transmitter beamforming vector,  $g_{tx}$ , is matched produce almost identical SINR performance, whereas the case which sets both of the beamforming vectors,  $g_{rx}$  and  $g_{tx}$ , as matched filters results in slightly lower SINR values. An important observation that can be made out of Fig. 4.1 is that setting only one of the beamforming vectors as matched filter does not cause much SINR loss in practice, especially as compared to the case in which both of the beamforming vectors are set to be matched filters. In the next chapter, this observation is taken into account to find an effective analytical solution by focusing on

deriving the closed-form for one of the beamforming vectors after fixing the other.

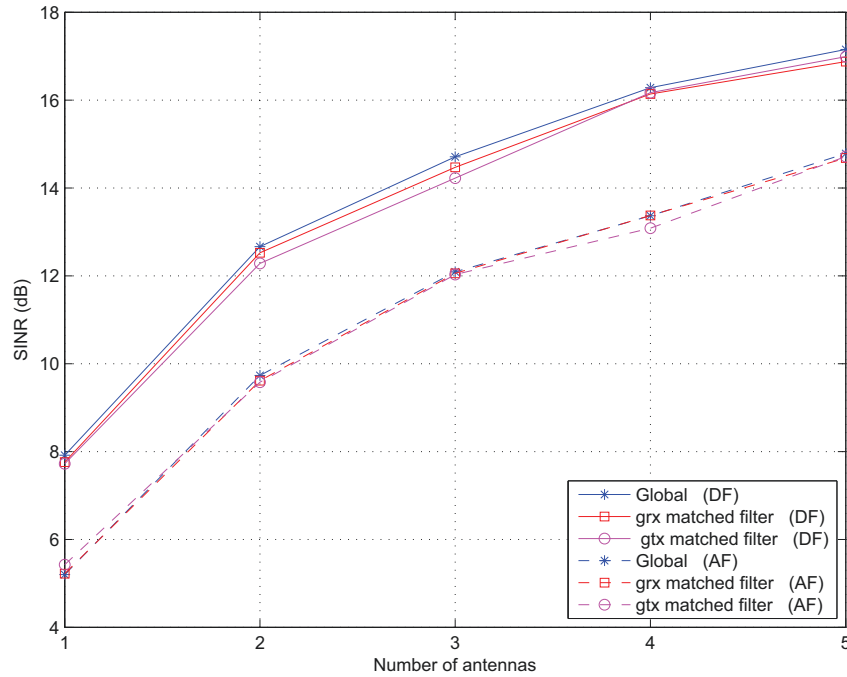


Figure 4.2: Numerical evaluation results of the optimization problem with the null-space projection constraint for the general and the matched filter solutions

In Fig. 4.2, the same system conditions for those in Fig. 4.1 are repeated. For this setup, however, null-space projection constraint (4.6) is applied to the evaluation. Since the interference is eliminated, the relay transmit power,  $p_R$ , is set to its upper limit. Thus, the case in which both of the beamforming vectors are set to be matched filters is excluded from the evaluation. The results for the other three cases are given for both DF and AF protocols using different number of antennas in Fig. 4.2.

The SINR performance results of numerical evaluation given in Fig. 4.2 and Fig. 4.1 do not substantially differ from each other. When the number of antennas is small, there is a slight difference. Yet, especially, when the number of antennas is higher, the performance gap between the general case and the null-space projection case disappears. This is because greater number of antennas provides more degrees of freedom, which enable the system to be optimized more accurately. This observation can be used to support the conjecture that the null-space projection assumption is a good subsolution for the general problem.

# Chapter 5

## Analytical Schemes

In this chapter, some assumptions will be introduced to simplify the general optimization problem<sup>1</sup>, of which the numerical evaluation is presented in the previous chapter, so that a suboptimal analytical solution, hopefully close to the global optimum point, can be derived. In the following sections, three different analytical schemes are presented. Firstly, by assuming that both of the beamforming vectors,  $g_{rx}$ ,  $g_{tx}$ , are known, an equation giving the optimal relay transmit power,  $p_R$ , for DF and AF protocols is presented. Secondly, if the second hop of the system is fixed, the optimal receiver beamforming vector,  $g_{rx}$ , optimizing the first hop SINR is derived in terms of relay transmit power,  $p_R$ , and transmitter beamforming vector,  $g_{tx}$ , using minimum mean square error (MMSE) method. Then, using that optimal receiver beamforming vector,  $g_{rx}$ , the relay transmit power,  $p_R$ , is optimized for both AF and DF protocols. Thirdly, the null-space projection method is forced on the system. Then, if one of the beamforming vectors is known, the other beamforming vector eliminating the self-interference is calculated. When the self-interference channel is assumed to be a rank-one matrix, an analytical solution giving the optimal pair of beamforming vectors,  $\{g_{rx}, g_{tx}\}$ , under the null-space projection constraint is derived.

### 5.1 Optimal Relay Transmit Power for Fixed Beamforming Filters

For any given pair of beamforming vectors,  $g_{rx}$  and  $g_{tx}$ , the optimal  $p_R$  value maximizing the end-to-end SINR can be found using (3.42) or (3.48) as shown in [36, 44].

For AF protocol, the optimal  $p_R$  can be derived by straightforwardly taking the derivative of the end-to-end SINR expression (3.42) with respect to  $p_R$  and setting it to zero. After the calculations, the optimal relay transmit power,  $p_R$ , for AF can

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<sup>1</sup>Finding an analytical solution to the general global problem seems too difficult, if not impossible.

be obtained as

$$p_R = \sqrt{\frac{\gamma_{SR} + 1}{\gamma_{RD}\gamma_{RR}}} \quad (5.1)$$

For DF protocol, the end-to-end SINR expression (3.48) is maximized when its arguments, which are the first hop and the second hop channel SINRs,  $\gamma_R$  and  $\gamma_D$ , are equal to each other. The optimal relay transmit power,  $p_R$ , for DF can be derived as

$$p_R = \frac{1}{2\gamma_{RR}} \left( \sqrt{\frac{4\gamma_{SR}\gamma_{RR}}{\gamma_{RD}} + 1} - 1 \right) \quad (5.2)$$

With the relay transmit power constraint,  $0 \leq p_R \leq 1$ , the optimal  $p_R$  can be stated as

$$p_R = \begin{cases} \min \left[ 1, \sqrt{\frac{\gamma_{SR} + 1}{\gamma_{RD}\gamma_{RR}}} \right], & \text{for AF} \\ \min \left[ 1, \frac{1}{2\gamma_{RR}} \left( \sqrt{\frac{4\gamma_{SR}\gamma_{RR}}{\gamma_{RD}} + 1} - 1 \right) \right], & \text{for DF} \end{cases} \quad (5.3)$$

## 5.2 Optimal Receive Beamforming Filter and Relay Transmit Power for Fixed Transmit Beamforming Filter

If we assume  $p_R$  and  $g_{tx}$  are known, the SNR of the second hop can be directly calculated. Fixing  $p_R$  and  $g_{tx}$  also fixes the direction from where the interference is arriving at the first hop. As it is pointed out in Section 4.1, maximizing the end-to-end SINR is the same as maximizing SINR of source-to-relay channel and SNR of relay-to-destination channel while minimizing the self-interference at the relay. Since the relay-to-destination channel SNR is fixed, the optimization problem can be interpreted as maximizing the source-to-relay channel SINR alone while keeping the self-interference as low as possible. In other words, with a fixed relay-to-destination channel SNR, maximizing first hop SINR maximizes also the overall end-to-end SINR. To achieve this goal, minimum mean square error (MMSE) method is used on the received signal at the relay node (3.12). After the optimal receiver beamforming vector,  $g_{rx}$ , is obtained in terms of the relay transmit power,  $p_R$ , the same logic explained in the previous section is applied to find the optimal relay transmit power,  $p_R$ , given only a fixed transmitter beamforming vector,  $g_{tx}$ , both for AF and DF separately.

Apparently, we do not expect to reach the performance of global solution, which is the theoretical upper limit. But, since the idea is to get close to it as much as possible, we prefer to choose the value of  $g_{tx}$  wisely, since the whole optimization problem is based on the trade-off between maximizing the source-to-relay, relay-to-destination SINRs and minimizing the self-interference power.

The derivation of optimal  $g_{rx}$  with the MMSE method is given step by step below.

Without the unit norm constraint on  $g_{rx}$  as given in (3.8), the ideal signal received at the relay after beamforming should be

$$y_{R,\text{ideal}} = x_S \quad (5.4)$$

The error signal for MMSE is the difference between what we have and what we expect to have at the receiver. It can be defined as

$$e = y_R - y_{R,\text{ideal}} \quad (5.5)$$

After substituting the ideal signal to be received (5.4) into the error signal (5.5), the error signal becomes

$$e = y_R - x_S \quad (5.6)$$

When  $y_R$ , given in (3.12), is substituted, the error signal,  $e$ , becomes

$$e = (g_{rx}h_{SR} - 1)x_S + g_{rx}H_{RR}g_{tx}x_R + g_{rx}n_R \quad (5.7)$$

The absolute square value of the error signal to be minimized is calculated as

$$|e|^2 = |(g_{rx}h_{SR} - 1)x_S + g_{rx}H_{RR}g_{tx}x_R + g_{rx}n_R|^2 \quad (5.8)$$

The expectation of (5.8) is calculated, i.e., the mean square error (MSE) is written as

$$E[|e|^2] = E[|(g_{rx}h_{SR} - 1)x_S + g_{rx}H_{RR}g_{tx}x_R + g_{rx}n_R|^2] \quad (5.9)$$

After some calculations, the mean square error is found as

$$\begin{aligned} E[|e|^2] &= E\left[|(g_{rx}h_{SR}-1)x_S|^2\right] + E\left[|g_{rx}H_{RR}g_{tx}x_R|^2\right] + E\left[|g_{rx}n_R|^2\right] \\ &\quad + E\left[(g_{rx}h_{SR} - 1)x_S (g_{rx}H_{RR}g_{tx}x_R)^H + (g_{rx}H_{RR}g_{tx}x_R)((g_{rx}h_{SR} - 1)x_S)^H\right. \\ &\quad \left. + (g_{rx}h_{SR} - 1)x_S (g_{rx}n_R)^H + (g_{rx}n_R)((g_{rx}h_{SR} - 1)x_S)^H\right. \\ &\quad \left. + (g_{rx}H_{RR}g_{tx}x_R)(g_{rx}n_R)^H + (g_{rx}n_R)(g_{rx}H_{RR}g_{tx}x_R)^H\right] \quad (5.10) \end{aligned}$$

Since the signals transmitted from the source,  $x_S$ , and the relay,  $x_R$ , have zero



means, and are uncorrelated due to processing delay,

$$E[x_R x_S] = E[x_R]E[x_S] = 0 \quad (5.11)$$

Since  $g_{rx}$  and  $n_R$  are uncorrelated, and  $n_R$  is zero-mean noise,

$$E[g_{rx} n_R] = g_{rx} E[n_R] = 0 \quad (5.12)$$

Hence, the mean square error is simplified by setting the cross terms to zero as

$$\begin{aligned} E[|e|^2] &= E[|g_{rx} h_{SR} - 1|^2 |x_S|^2] + E[|g_{rx} H_{RR} g_{tx}|^2 |x_R|^2] + E[|g_{rx} n_R|^2] \\ &= (1 - g_{rx} h_{SR} - (g_{rx} h_{SR})^H + g_{rx} h_{SR} (g_{rx} h_{SR})^H) E[|x_S|^2] \\ &\quad + (g_{rx} H_{RR} g_{tx})(g_{rx} H_{RR} g_{tx})^H E[|x_R|^2] + E[|g_{rx} n_R|^2] \end{aligned} \quad (5.13)$$

The autocorrelations of the signals,  $x_S$  and  $x_R$ , are equal to their powers. Then, we can substitute them in (5.13) with the power terms as defined in (3.3) and (3.4).

$$E[|x_S|^2] = p_S, \quad \text{and} \quad E[|x_R|^2] = p_R \quad (5.14)$$

Since the unit-norm constraint on  $g_{rx}$  is not momentarily preserved, its norm can be taken out of the expectation as

$$E[|g_{rx} n_R|^2] = \|g_{rx}\|^2 E\left[\left|\frac{g_{rx}}{\|g_{rx}\|} n_R\right|^2\right] \quad (5.15)$$

Using the definition given in (3.1), we can write

$$E[|g_{rx} n_R|^2] = \|g_{rx}\|^2 \sigma_R^2 \quad (5.16)$$

Then, mean square error in (5.13) becomes

$$\begin{aligned} E[|e|^2] &= g_{rx} g_{rx}^H \sigma_R^2 + p_S - g_{rx} h_{SR} p_S - h_{SR}^H g_{rx}^H p_S + g_{rx} h_{SR} h_{SR}^H g_{rx}^H p_S \\ &\quad + g_{rx} H_{RR} g_{tx} g_{tx}^H H_{RR}^H g_{rx}^H p_R \end{aligned} \quad (5.17)$$

Identity matrix,  $I_{N_{rx}}$ , is inserted into the first term on the right-hand side of (5.17). After taking some of the terms in (5.17) into common multipliers, mean square error can be written as

$$E[|e|^2] = p_S - g_{rx} h_{SR} p_S - h_{SR}^H g_{rx}^H p_S$$

$$+ g_{rx} \left( h_{SR} h_{SR}^H p_S + H_{RR} g_{tx} g_{tx}^H H_{RR}^H p_R + \sigma_R^2 I_{N_{rx}} \right) g_{rx}^H \quad (5.18)$$

Let  $R$  be defined as the covariance matrix of the relay input before the receiver beamforming filter.

$$R = h_{SR} h_{SR}^H p_S + H_{RR} g_{tx} g_{tx}^H H_{RR}^H p_R + \sigma_R^2 I_{N_{rx}} \quad (5.19)$$

Note that the covariance matrix,  $R$ , is a hermitian matrix. Then,  $(h_{SR}^H R^{-1} h_{SR} p_S^2)$  is added to and subtracted from the mean square error in (5.18).

$$\begin{aligned} E[|e|^2] = p_S - g_{rx} h_{SR} p_S - h_{SR}^H g_{rx}^H p_S + g_{rx} R g_{rx}^H \\ + (h_{SR}^H R^{-1} h_{SR} p_S^2 - h_{SR}^H R^{-1} h_{SR} p_S^2) \end{aligned} \quad (5.20)$$

After reorganizing some of the terms in (5.20), the mean square error becomes

$$\begin{aligned} E[|e|^2] = p_S - h_{SR}^H R^{-1} h_{SR} p_S^2 + g_{rx} R g_{rx}^H - g_{rx} h_{SR} p_S \\ - h_{SR}^H g_{rx}^H p_S + h_{SR}^H R^{-1} h_{SR} p_S^2 \end{aligned} \quad (5.21)$$

Identity matrix,  $I_{N_{rx}} = R R^{-1}$ , can be inserted into some terms in (5.21), just for the ease of calculation. After these modifications,

$$\begin{aligned} E[|e|^2] = p_S - h_{SR}^H R^{-1} h_{SR} p_S^2 + g_{rx} R g_{rx}^H - g_{rx} R R^{-1} h_{SR} p_S \\ - h_{SR}^H R^{-1} R g_{rx}^H p_S + h_{SR}^H R^{-1} R R^{-1} h_{SR} p_S^2 \end{aligned} \quad (5.22)$$

The last four terms on the right-hand side of (5.22) can be written as one term after being taken into the parenthesis of common multipliers and common multiplicands. Then, the mean square error becomes

$$E[|e|^2] = p_S - h_{SR}^H R^{-1} h_{SR} p_S^2 + (g_{rx} - h_{SR}^H R^{-1} p_S) R (g_{rx} - h_{SR}^H R^{-1} p_S)^H \quad (5.23)$$

The first two terms on the right-hand side of (5.23) are independent of  $g_{rx}$ . Thus, the mean square error is minimized when the last term is minimized.

Because the matrix  $R$  is positive semi-definite, all its eigenvalues are non-negative. In particular, let the eigenvalues of  $R$  be  $\lambda_0, \lambda_1, \dots, \lambda_{N_{rx}-1}$ . Then,  $R$  can be defined as

$$R = U_{N_{rx} \times N_{rx}} \Sigma_{N_{rx} \times N_{rx}} U_{N_{rx} \times N_{rx}}^H \quad (5.24)$$

where  $U$  is a unitary matrix consisting of its eigenvectors, and  $\Sigma$  is a diagonal matrix with the eigenvalues  $\{\lambda_0, \lambda_1, \dots, \lambda_{N_{rx}-1}\}$  as the diagonal terms.

Let the vector  $v_{1 \times N_{rx}}$  be defined as

$$v = (g_{rx} - h_{SR}^H R^{-1} p_S) \quad (5.25)$$

Then, the last term on the right-hand side of (5.23) can be written as

$$(g_{rx} - h_{SR}^H R^{-1} p_S) R (g_{rx} - h_{SR}^H R^{-1} p_S)^H = (vU) \Sigma (vU)^H \quad (5.26)$$

Because  $\Sigma$  is a diagonal matrix, the right-hand side of (5.26) can be written as

$$(vU) \Sigma (vU)^H = \sum_{j=0}^{N_{rx}-1} \lambda_j |v u_j|^2 \quad (5.27)$$

where  $u_j$  is the  $(j+1)$ th column vector of matrix  $U$ .

By combining (5.25), (5.26), and (5.27), it can be seen that the last term on the right-hand side of (5.23) is non-negative for any  $g_{rx}$  as shown below.

$$(g_{rx} - h_{SR}^H R^{-1} p_S) R (g_{rx} - h_{SR}^H R^{-1} p_S)^H \geq 0 \quad \text{for } \forall g_{rx} \in \mathbb{C}^{1 \times N_{rx}} \quad (5.28)$$

Therefore, the global minimum of the term in (5.28) is reached when

$$(g_{rx} - h_{SR}^H R^{-1} p_S) = 0 \quad (5.29)$$

Hence,  $g_{rx}$  minimizing MSE is found as

$$g_{rx} = h_{SR}^H R^{-1} p_S \quad (5.30)$$

Now, we need to see that the beamforming vector,  $g_{rx}$ , in (5.30), which minimizes the mean square error, is also the optimal solution maximizing SINR. The justification is given below.

The first hop SINR (3.23) can be written as

$$\gamma_R = \frac{(g_{rx} h_{SR})(g_{rx} h_{SR})^H p_S}{(g_{rx} H_{RR} g_{tx})(g_{rx} H_{RR} g_{tx})^H p_R + E[|(g_{rx} n_R)(g_{rx} n_R)^H|^2]} \quad (5.31)$$

After multiplications, the SINR expression becomes

$$\gamma_R = \frac{g_{rx} (h_{SR} h_{SR}^H) g_{rx}^H p_S}{g_{rx} (H_{RR} g_{tx} g_{tx}^H H_{RR}^H) g_{rx}^H p_R + g_{rx} (\sigma_R^2 I_{N_{rx}}) g_{rx}^H} \quad (5.32)$$

The denominator of (5.32) is taken into common multipliers. Then, the first hop SINR,  $\gamma_R$ , becomes

$$\gamma_R = \frac{g_{rx}(h_{SR}h_{SR}^H)g_{rx}^H p_S}{g_{rx}[H_{RR}g_{tx}g_{tx}^H H_{RR}^H p_R + \sigma_R^2 I_{N_{rx}}]g_{rx}^H} \quad (5.33)$$

After adding and subtracting 1, (5.33) becomes

$$\begin{aligned} \gamma_R &= \frac{g_{rx}(h_{SR}h_{SR}^H)g_{rx}^H p_S}{g_{rx}[H_{RR}g_{tx}g_{tx}^H H_{RR}^H p_R + \sigma_R^2 I_{N_{rx}}]g_{rx}^H} + 1 - 1 \\ &= \frac{g_{rx}[h_{SR}h_{SR}^H p_S + H_{RR}g_{tx}g_{tx}^H H_{RR}^H p_R + \sigma_R^2 I_{N_{rx}}]g_{rx}^H}{g_{rx}[H_{RR}g_{tx}g_{tx}^H H_{RR}^H p_R + \sigma_R^2 I_{N_{rx}}]g_{rx}^H} - 1 \end{aligned} \quad (5.34)$$

By substituting (5.19) into (5.34), we obtain  $\gamma_R$  as

$$\begin{aligned} \gamma_R &= \frac{g_{rx} R g_{rx}^H}{g_{rx}[R - h_{SR}h_{SR}^H p_S]g_{rx}^H} - 1 \\ &= \frac{g_{rx} R g_{rx}^H}{g_{rx} R g_{rx}^H - g_{rx} h_{SR} h_{SR}^H g_{rx}^H p_S} - 1 \end{aligned} \quad (5.35)$$

The optimal  $g_{rx}$  minimizing MSE given in (5.30) is then substituted and  $\gamma_R$  becomes

$$\gamma_R = \frac{(h_{SR}^H R^{-1} p_S) R (R^{-1} h_{SR} p_S)}{(h_{SR}^H R^{-1} p_S) R (R^{-1} h_{SR} p_S) - (h_{SR}^H R^{-1} p_S) h_{SR} h_{SR}^H (R^{-1} h_{SR} p_S) p_S} - 1 \quad (5.36)$$

After some multiplications, some of the terms are cancelled out. Then, we obtain

$$\gamma_R = \frac{h_{SR}^H R^{-1} h_{SR} p_S^2}{h_{SR}^H R^{-1} h_{SR} p_S^2 - h_{SR}^H R^{-1} h_{SR} h_{SR}^H R^{-1} h_{SR} p_S^3} - 1 \quad (5.37)$$

After dividing both the nominator and denominator of (5.37) by  $h_{SR}^H R^{-1} h_{SR} p_S^2$ , the first hop SINR,  $\gamma_R$ , becomes

$$\gamma_R = \frac{1}{1 - h_{SR}^H R^{-1} h_{SR} p_S} - 1 \quad (5.38)$$

Minimum mean square error can be found by inserting the optimal receiver beamforming vector,  $g_{rx}$ , in (5.30) into the mean square error term in (5.23).

$$\begin{aligned} \min\{E[|e|^2]\} &= p_S - h_{SR}^H R^{-1} h_{SR} p_S^2 + 0 \\ &= p_S [h_{SR}^H R^{-1} h_{SR} p_S] \end{aligned} \quad (5.39)$$

Using the first hop SINR obtained in (5.38) and the minimum mean square error in (5.39), it can be shown that

$$\frac{p_S}{\min\{E[|e|^2]\}} = 1 + \gamma_R \quad (5.40)$$

Equation (5.40) shows that there is an inverse relation between MMSE and SINR. As MMSE decreases, SINR increases. When MMSE becomes infinitesimal, SINR goes to infinity. In other words, maximum SINR is achieved when minimum possible MMSE is attained. Therefore, we can conclude that since the beamforming vector,  $g_{rx}$ , in (5.30) is minimizing MMSE, it is also maximizing SINR.

The solution in (5.30), however, does not take the unit norm constraint (3.8) into account. The beamforming vector,  $g_{rx}$ , in (5.30) will not necessarily have unit norm. Thus, the result can be normalized as

$$g_{rx} = \frac{h_{SR}^H R^{-1} p_S}{\|h_{SR}^H R^{-1} p_S\|} \quad (5.41)$$

Normalized  $g_{rx}$  may not always give the MMSE solution, even though it still maximizes SINR. Because for a receiver beamforming vector,  $\tilde{g}_{rx} = K g_{rx}$ , where  $K$  is any nonzero constant, the first-hop SINR,  $\tilde{\gamma}_R$ , is obtained to be equal to the first-hop SINR,  $\gamma_R$ , in (3.23) for  $g_{rx}$ .

$$\tilde{\gamma}_R = \frac{K^2 |g_{rx} h_{SR}|^2 p_S}{K^2 |g_{rx} H_{RR} g_{tx}|^2 p_R + K^2 \sigma_R^2} = \gamma_R \quad (5.42)$$

since the constant  $K^2$  can be cancelled out.

In other words, the first hop SINR is independent of the receiver beamforming vector norm by definition. This fact can be utilized to simplify the result in (5.30) a step further. The beamforming vector,  $g_{rx}$ , is found as

$$g_{rx} = h_{SR}^H [h_{SR} h_{SR}^H p_S + H_{RR} g_{tx} g_{tx}^H H_{RR}^H p_R + \sigma_R^2 I_{N_{rx}}]^{-1} p_S \quad (5.43)$$

Woodburry matrix identity [65] given below can be used to simplify the matrix sum inversion in (5.43).

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1} \quad (5.44)$$

Let  $R_Z$  be defined as the covariance matrix of the interference-plus-noise signal before the relay beamforming filter.

$$\boxed{R_Z = H_{RR} g_{tx} g_{tx}^H H_{RR}^H p_R + \sigma_R^2 I_{N_{rx}}} \quad (5.45)$$

Then, the parameters  $A$ ,  $B$ ,  $C$ ,  $D$  in (5.44) are defined as given below.

$$A = R_Z \quad B = h_{SR} p_S \quad C = 1 \quad D = h_{SR}^H \quad (5.46)$$

Due to (5.44),  $g_{rx}$  can be written as

$$\begin{aligned} g_{rx} &= h_{SR}^H \left[ R_Z + h_{SR} p_S h_{SR}^H \right]^{-1} p_S \\ &= h_{SR}^H \left[ R_Z^{-1} - R_Z^{-1} h_{SR} p_S (1 + h_{SR}^H R_Z^{-1} h_{SR} p_S)^{-1} h_{SR}^H R_Z^{-1} \right] p_S \end{aligned} \quad (5.47)$$

The term  $(1 + h_{SR}^H R_Z^{-1} h_{SR} p_S)^{-1} h_{SR}^H R_Z^{-1}$  is factored out on the right-hand side of (5.47). Then, we obtain

$$\begin{aligned} g_{rx} &= \left[ (1 + h_{SR}^H R_Z^{-1} h_{SR} p_S) - h_{SR}^H R_Z^{-1} h_{SR} p_S \right] \\ &\quad \times (1 + h_{SR}^H R_Z^{-1} h_{SR} p_S)^{-1} h_{SR}^H R_Z^{-1} p_S \end{aligned} \quad (5.48)$$

The terms in square brackets in (5.48) is equal to 1. So,  $g_{rx}$  becomes

$$g_{rx} = (1 + h_{SR}^H R_Z^{-1} h_{SR} p_S)^{-1} h_{SR}^H R_Z^{-1} p_S \quad (5.49)$$

where  $(1 + h_{SR}^H R_Z^{-1} h_{SR} p_S)^{-1}$  is a constant scalar.

As shown in (5.42), the norm of the receiver beamforming vector,  $g_{rx}$ , does not affect SINR. Therefore, the optimal  $g_{rx}$  maximizing SINR can be restated as

$$g_{rx} = K h_{SR}^H R_Z^{-1} p_S \quad (5.50)$$

where  $R_Z = H_{RR} g_{tx} g_{tx}^H H_{RR}^H p_R + \sigma_R^2 I_{N_{rx}}$  and  $K$  is any nonzero constant.

Due to the unit-norm constraint of system setup over the beamforming vectors given in (3.8), constant  $K$  in (5.50) is chosen as

$$K = \frac{1}{\|h_{SR}^H R_Z^{-1} p_S\|} \quad (5.51)$$

Finally, the resulting beamforming vector,  $g_{rx}$ , becomes

$$\boxed{g_{rx} = \frac{h_{SR}^H R_Z^{-1} p_S}{\|h_{SR}^H R_Z^{-1} p_S\|}} \quad (5.52)$$

### 5.2.1 Finding Relay Transmit Power

The optimal direction of the relay receiver beamforming vector,  $g_{rx}$ , maximizing the system SINR, given any relay transmit power,  $p_R$ , and given any relay transmitter beamforming vector,  $g_{tx}$ , is obtained in (5.50). However, that result can be taken a step further. Preserving the assumption of fixed transmitter beamforming vector,  $g_{tx}$ , the optimal  $g_{rx}$  result can be used to maximize the end-to-end SINR of the system. Then, the relay transmit power,  $p_R$ , resulting in the maximum SINR can be concluded to be the optimal transmit power.

In the following, first of all, the first hop SINR given in (3.23) is modified into a different form for the ease of calculation throughout the derivation. Then, DF and AF protocols are considered separately to find the optimal relay transmit power,  $p_R$ , due to the end-to-end SINR definitions given in (3.42) and (3.48) for AF and for DF, respectively.

The SINR of the first hop,  $\gamma_R$ , (3.23) is calculated using the optimal beamforming vector,  $g_{rx}$ , in (5.50) as

$$\gamma_R = \frac{(g_{rx}h_{SR})(g_{rx}h_{SR})^H p_S}{(g_{rx}H_{RR}g_{tx})(g_{rx}H_{RR}g_{tx})^H p_R + E[(g_{rx}n_R)(g_{rx}n_R)^H]} \quad (5.53)$$

After the multiplications and factoring out  $g_{rx}$ , first hop SINR becomes

$$\gamma_R = \frac{g_{rx}(h_{SR}h_{SR}^H)g_{rx}^H p_S}{g_{rx}(H_{RR}g_{tx}g_{tx}^H H_{RR}^H)g_{rx}^H p_R + g_{rx}(\sigma_R^2 I_{N_{rx}})g_{rx}^H} \quad (5.54)$$

The denominator of (5.54) is taken into common multipliers. Then, we obtain

$$\gamma_R = \frac{g_{rx}(h_{SR}h_{SR}^H)g_{rx}^H p_S}{g_{rx}[H_{RR}g_{tx}g_{tx}^H H_{RR}^H p_R + \sigma_R^2 I_{N_{rx}}]g_{rx}^H} \quad (5.55)$$

After substituting  $R_Z$  and  $g_{rx}$  as given in (5.45) and (5.50), respectively, the first hop SINR can be written as

$$\gamma_R = \frac{(h_{SR}^H R_Z^{-1} p_S)h_{SR}h_{SR}^H(h_{SR}^H R_Z^{-1} p_S)^H p_S}{(h_{SR}^H R_Z^{-1} p_S)R_Z(h_{SR}^H R_Z^{-1} p_S)^H} \quad (5.56)$$

The hermitian transpose of each term in  $(h_{SR}^H R_Z^{-1} p_S)^H$  of (5.56) is written. Then,  $\gamma_R$  becomes

$$\gamma_R = \frac{h_{SR}^H R_Z^{-1} h_{SR} h_{SR}^H R_Z^{-1} h_{SR} p_S^3}{h_{SR}^H R_Z^{-1} R_Z R_Z^{-1} h_{SR} p_S^2} \quad (5.57)$$

After substituting the identity matrix  $I_{N_{rx}} = R_Z R_Z^{-1}$  at the denominator of (5.57),

the constant terms  $h_{SR}^H R_Z^{-1} h_{SR} p_S^2$  at the numerator and at the denominator are cancelled out. Then,  $\gamma_R$  is simplified as

$$\gamma_R = h_{SR}^H R_Z^{-1} h_{SR} p_S \quad (5.58)$$

Woodbury identity (5.44) is applied on (5.58) to extract the transmit power,  $p_R$ , within  $R_Z^{-1}$ . The parameters for the identity are defined as

$$A = \sigma_R^2 I_{N_{rx}} \quad B = H_{RR} g_{tx} p_R \quad C = 1 \quad D = g_{tx}^H H_{RR}^H \quad (5.59)$$

Then,  $\gamma_R$  in (5.58) is written as

$$\gamma_R = h_{SR}^H \left[ \frac{1}{\sigma_R^2} I - \frac{1}{\sigma_R^2} H_{RR} g_{tx} p_R \left( 1 + g_{tx}^H H_{RR}^H H_{RR} g_{tx} \frac{p_R}{\sigma_R^2} \right)^{-1} g_{tx}^H H_{RR}^H \frac{1}{\sigma_R^2} \right] h_{SR} p_S \quad (5.60)$$

After the multiplications,  $\gamma_R$  is simplified as

$$\gamma_R = \frac{1}{\sigma_R^2} \|h_{SR}\|^2 p_S - \frac{1}{\sigma_R^2} |h_{SR}^H H_{RR} g_{tx}|^2 p_S \frac{p_R}{\sigma_R^2 + \|H_{RR} g_{tx}\|^2 p_R} \quad (5.61)$$

First,  $\gamma_R$  is written as a single ratio, and then, the relay transmit power,  $p_R$ , is factored out. Then, the first hop SINR,  $\gamma_R$ , is concluded as

$$\gamma_R = \frac{\|h_{SR}\|^2 p_S + \frac{1}{\sigma_R^2} p_S \left[ \|h_{SR}\|^2 \|H_{RR} g_{tx}\|^2 - |h_{SR}^H H_{RR} g_{tx}|^2 \right] p_R}{\sigma_R^2 + \|H_{RR} g_{tx}\|^2 p_R} \quad (5.62)$$

### For amplify-and-forward protocol:

If a point is a local extremum, minimum or maximum, of an expression, the first derivative of that expression at that particular point is equal to zero. Hence, the end-to-end SINR for AF (3.42) is maximized when

$$\frac{d}{dp_R} \left( \frac{\gamma_R \gamma_D}{\gamma_R + \gamma_D + 1} \right) = 0 \quad (5.63)$$

After substituting the relay-to-destination SINR (3.24) and source-to-relay SINR (5.62), the equation in (5.63) becomes

$$\frac{d}{dp_R} \left[ \frac{\frac{\|h_{SR}\|^2 p_S + \frac{1}{\sigma_R^2} p_S \left[ \|h_{SR}\|^2 \|H_{RR} g_{tx}\|^2 - |h_{SR}^H H_{RR} g_{tx}|^2 \right] p_R}{\sigma_R^2 + \|H_{RR} g_{tx}\|^2 p_R} \frac{|h_{RD} g_{tx}|^2 p_R}{\sigma_D^2}}{\frac{\|h_{SR}\|^2 p_S + \frac{1}{\sigma_R^2} p_S \left[ \|h_{SR}\|^2 \|H_{RR} g_{tx}\|^2 - |h_{SR}^H H_{RR} g_{tx}|^2 \right] p_R}{\sigma_R^2 + \|H_{RR} g_{tx}\|^2 p_R} + \frac{|h_{RD} g_{tx}|^2 p_R}{\sigma_D^2} + 1} \right] = 0 \quad (5.64)$$



Let non-negative constants<sup>2</sup>  $C_1, C_2, C_3, C_4$  be defined as

$$C_1 = \|H_{RR}g_{tx}\|^2 |h_{RD}g_{tx}|^2 \quad (5.65)$$

$$C_2 = \sigma_R^2 |h_{RD}g_{tx}|^2 \quad (5.66)$$

$$C_3 = \frac{\sigma_D^2}{\sigma_R^2} p_S \left( \|h_{SR}\|^2 \|H_{RR}g_{tx}\|^2 - |h_{SR}^H H_{RR}g_{tx}|^2 \right) \quad (5.67)$$

$$C_4 = \sigma_D^2 \|h_{SR}\|^2 p_S \quad (5.68)$$

Then, (5.64) can be written in terms of  $C_1, C_2, C_3, C_4$  as

$$\frac{d}{dp_R} \left[ \frac{p_R^2 \frac{1}{\sigma_R^2 \sigma_D^2} C_2 C_3 + p_R \frac{1}{\sigma_R^2 \sigma_D^2} C_2 C_4}{p_R^2 C_1 + p_R [C_2 + \sigma_R^2 \sigma_D^2 \frac{C_1}{C_2} + C_3] + C_4 + \sigma_R^2 \sigma_D^2} \right] = 0 \quad (5.69)$$

After the derivative with respect to  $p_R$  is calculated, the numerator of the result is set to zero to find the global maximum point. Then, we obtain

$$\begin{aligned} & \left[ p_R \frac{2}{\sigma_R^2 \sigma_D^2} C_2 C_3 + \frac{1}{\sigma_R^2 \sigma_D^2} C_2 C_4 \right] \times \left[ p_R^2 C_1 + p_R [C_2 + \sigma_R^2 \sigma_D^2 \frac{C_1}{C_2} + C_3] + C_4 + \sigma_R^2 \sigma_D^2 \right] \\ & - \left[ p_R 2 C_1 + [C_2 + \sigma_R^2 \sigma_D^2 \frac{C_1}{C_2} + C_3] \right] \times \left[ p_R^2 \frac{1}{\sigma_R^2 \sigma_D^2} C_2 C_3 + p_R \frac{1}{\sigma_R^2 \sigma_D^2} C_2 C_4 \right] = 0 \quad (5.70) \end{aligned}$$

After the multiplications, the equation is simplified as

$$\begin{aligned} & p_R^2 \left[ \frac{1}{\sigma_R^2 \sigma_D^2} C_2^2 C_3 - \frac{1}{\sigma_R^2 \sigma_D^2} C_1 C_2 C_4 + C_1 C_3 + \frac{1}{\sigma_R^2 \sigma_D^2} C_2 C_3^2 \right] \\ & + p_R \left[ \frac{2}{\sigma_R^2 \sigma_D^2} C_2 C_3 C_4 + 2 C_2 C_3 \right] + \frac{1}{\sigma_R^2 \sigma_D^2} C_2 C_4^2 + C_2 C_4 = 0 \quad (5.71) \end{aligned}$$

Let the non-negative constant  $C_5$  be defined as

$$C_5 = p_S |h_{SR}^H H_{RR}g_{tx}|^2 \quad (5.72)$$

which can be written in terms of  $C_1, C_2, C_3, C_4$  as

$$C_5 = \frac{\sigma_R^2}{\sigma_D^2} \left( \frac{C_1 C_4}{C_2} - C_3 \right) \quad (5.73)$$

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<sup>2</sup>One can easily see  $C_3$  is also always non-negative due to Cauchy-Schwarz inequality,  $\|uv\|^2 \leq \|u\|^2 \|v\|^2$  for any vectors  $u$  and  $v$ .

After some terms are cancelled out, we obtain

$$p_R^2 \left[ -\frac{1}{\sigma_R^4} C_2^2 C_5 + C_1 C_3 + \frac{1}{\sigma_R^2 \sigma_D^2} C_2 C_3^2 \right] + p_R \left[ \frac{2}{\sigma_R^2 \sigma_D^2} C_2 C_3 C_4 + 2C_2 C_3 \right] + \left[ \frac{1}{\sigma_R^2 \sigma_D^2} C_2 C_4^2 + C_2 C_4 \right] = 0 \quad (5.74)$$

The roots of the second order polynomial in (5.74) are found as

$$p_{R,1,2} = \frac{\mp \sqrt{\left( \frac{1}{\sigma_R^2 \sigma_D^2} C_4 + 1 \right) \left[ C_2^2 C_3^2 + \frac{1}{\sigma_D^4} C_2^3 C_4 C_5 - C_1 C_2 C_3 C_4 \right]} - \frac{1}{\sigma_R^2 \sigma_D^2} C_2 C_3 C_4 - C_2 C_3}{-\frac{1}{\sigma_R^4} C_2^2 C_5 + C_1 C_3 + \frac{1}{\sigma_R^2 \sigma_D^2} C_2 C_3^2} \quad (5.75)$$

Let the constant  $\psi$  be defined as

$$\psi = -\frac{1}{\sigma_R^4} C_2^2 C_5 + C_1 C_3 + \frac{1}{\sigma_R^2 \sigma_D^2} C_2 C_3^2 \quad (5.76)$$

Assuming  $\psi \neq 0$ , since  $C_1, C_2, C_3, C_4$ , and  $C_5$  are all non-negative constants, there are two possible cases for the signs of the roots of the polynomial in (5.74).

### Case 1:

If  $\psi > 0$ , both of the roots are negative. That means, for any non-negative relay transmit power,  $p_R$ , it is clear that

$$\frac{d}{dp_R} \left( \frac{\gamma_R \gamma_D}{\gamma_R + \gamma_D + 1} \right) > 0 \quad , \text{ for } \forall p_R \geq 0 \quad (5.77)$$

Therefore, due to maximum transmit power constraint in (3.10), the optimal relay transmit power,  $p_R$ , for AF protocol is chosen as

$$p_{R,\text{case1}} = 1 \quad (5.78)$$

### Case 2:

If  $\psi < 0$ , the roots  $p_{R,1}$  and  $p_{R,2}$  have different signs. Then, the positive root is chosen as the optimal transmit power. Due to the maximum transmit power constraint (3.10), the optimal  $p_R$  is concluded as

$$p_{R,\text{case2}} = \min \left\{ 1, \xi \right\} \quad (5.79)$$

where

$$\xi = \frac{-\sqrt{\left(\frac{1}{\sigma_R^2\sigma_D^2}C_4 + 1\right)\left[C_2^2C_3^2 + \frac{1}{\sigma_R^4}C_2^3C_4C_5 - C_1C_2C_3C_4\right]} - \frac{1}{\sigma_R^2\sigma_D^2}C_2C_3C_4 - C_2C_3}{-\frac{1}{\sigma_R^4}C_2^2C_5 + C_1C_3 + \frac{1}{\sigma_R^2\sigma_D^2}C_2C_3^2} \quad (5.80)$$

After combining these two cases above, the optimal transmit power,  $p_R$ , for AF protocol is found as

$$p_R^{\text{AF}} = \begin{cases} 1, & \psi \geq 0 \\ \min\{1, \xi\}, & \psi < 0 \end{cases} \quad (5.81)$$

where  $\psi = -\frac{1}{\sigma_R^4}C_2^2C_5 + C_1C_3 + \frac{1}{\sigma_R^2\sigma_D^2}C_2C_3^2$

✓ **Remark:** After substituting the constants  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_5$  into (5.76),  $\psi$  is found in terms of the system parameters as

$$\begin{aligned} \psi = & -p_S |h_{RD}g_{tx}|^4 |h_{SR}^H H_{RR}g_{tx}|^2 \\ & + p_S \frac{\sigma_D^2}{\sigma_R^2} |h_{RD}g_{tx}|^2 \|H_{RR}g_{tx}\|^2 (\|h_{SR}\|^2 \|H_{RR}g_{tx}\|^2 - |h_{SR}^H H_{RR}g_{tx}|^2) \\ & + p_S^2 \frac{\sigma_D^2}{\sigma_R^4} |h_{RD}g_{tx}|^2 (\|h_{SR}\|^2 \|H_{RR}g_{tx}\|^2 - |h_{SR}^H H_{RR}g_{tx}|^2) \end{aligned} \quad (5.82)$$

The first term on the right-hand side of (5.82) is always negative, whereas the second and the third terms are always positive. The term  $|h_{SR}^H H_{RR}g_{tx}|^2$  can be regarded as the self-interference term, due to the direction of optimal  $g_{rx}$  in (5.50) being similar to that of  $h_{SR}^H$ . By inspection, one can say that when  $|h_{SR}^H H_{RR}g_{tx}|^2$  is maximized to its upper limit, which is  $\|h_{SR}\|^2 \|H_{RR}g_{tx}\|^2$  due to Cauchy-Schwartz inequality,  $\psi$  is minimized. And minimum  $\psi$ , which is negative, minimizes  $p_R^{\text{AF}}$ . Since the channels  $h_{RD}$ ,  $h_{SR}$ , and  $H_{RR}$  are independent of each other, it is likely that when transmitter beamforming filter,  $g_{tx}$ , is matched to  $h_{RD}$ , the self-interference term,  $|g_{rx}^H H_{RR}g_{tx}|^2$ , becomes large. At this point, due to the direction of optimal receiver beamforming filter,  $g_{rx}$ , in (5.50), the value of  $\psi$  is obtained to be quite small, resulting in a low value of the optimal relay transmit power. If the lower limit of  $p_R$  is 1 (when  $\xi = 1$ ), we can conclude that the global optimal solution for  $g_{tx}$  converges to the matched filter solution. Yet, the lower limit of  $p_R$  is most likely to be smaller than 1, because of the independent random channels. As the direction of the transmitter beamforming filter,  $g_{tx}$ , is carried over to the vicinity of the orthogonal space of the vector  $h_{SR}^H H_{RR}$ , the self-interference is mitigated, resulting in greater  $\psi$ , and thus higher relay transmit power,  $p_R$ . In other words, first hop SINR,  $\gamma_R$ , is increased

at the expense of second hop SNR,  $\gamma_D$ . Later, at one point, however, either  $\psi$  becomes zero or  $\xi$  becomes 1. After that point, mitigating the self-interference more, obviously, does not improve the end-to-end SINR. Therefore, the global solution for  $g_{tx}$  can be approached by solving the equation below.

$$g_{tx,\text{optimal}} = \arg \max_{g_{tx}} \left\{ \gamma_{SRD} \Big|_{g_{tx}, \text{matched}}, \gamma_{SRD} \Big|_{\psi=0}, \gamma_{SRD} \Big|_{\xi=1} \right\} \quad (5.83)$$

Despite of not being a closed-form solution, the significance of (5.83) lies upon the fact that it only involves equalities. It does not consist of min\max problems, which make a computational search somewhat unreliable in differentiating the local and global converging minima\maxima<sup>3</sup>. It, presumably, works faster in a computational search.

### For decode-and-forward protocol:

The end-to-end SINR expression for DF (3.48) is a min function with the arguments of the first-hop and the second-hop SINRs. We know that the first-hop SINR can be increased only at the expense of the second-hop SNR, or vice versa. Therefore, the end-to-end SINR for DF is maximized when the first-hop SINR is equal to the second-hop SNR.

$$\gamma_R = \gamma_D \quad (5.84)$$

First, the relay-to-destination SINR in (3.24) and the source-to-relay SINR in (5.62) are substituted into (5.84). Then, the equality becomes

$$\frac{\|h_{SR}\|^2 p_S + \frac{1}{\sigma_R^2} p_S \left[ \|h_{SR}\|^2 \|H_{RR}g_{tx}\|^2 - |h_{SR}^H H_{RR}g_{tx}|^2 \right] p_R}{\sigma_R^2 + \|H_{RR}g_{tx}\|^2 p_R} = \frac{|h_{RD}g_{tx}|^2 p_R}{\sigma_D^2} \quad (5.85)$$

After the cross multiplication, we obtain

$$\begin{aligned} \|H_{RR}g_{tx}\|^2 |h_{RD}g_{tx}|^2 p_R^2 + \left[ \sigma_R^2 |h_{RD}g_{tx}|^2 - \frac{\sigma_D^2}{\sigma_R^2} p_S \left( \|h_{SR}\|^2 \|H_{RR}g_{tx}\|^2 - |h_{SR}^H H_{RR}g_{tx}|^2 \right) \right] p_R \\ - \sigma_D^2 \|h_{SR}\|^2 p_S = 0 \end{aligned} \quad (5.86)$$

After substituting non-negative constants  $C_1, C_2, C_3, C_4$  defined in (5.65), (5.66), (5.67), (5.68), we obtain

$$C_1 p_R^2 + (C_2 - C_3) p_R - C_4 = 0 \quad (5.87)$$

---

<sup>3</sup>A numerical search algorithm for min\max problems starts the iterations with a random initial guess, and if the iterations converge, a local minimum\maximum is obtained. There is no systematic way of finding the global minimum\maximum point. The only way is to run sufficient number of independent trials for local points with different initial guesses.

Equation (5.87) is a second-order polynomial with one positive and one negative root. The positive root is the optimal  $p_R$  maximizing the end-to-end SINR for DF protocol. It is found as

$$p_R = \frac{-C_2 + C_3 + \sqrt{(C_2 - C_3)^2 + 4C_1C_4}}{2C_1} \quad (5.88)$$

Due to the maximum transmit power constraint (3.10), the optimal relay transmit power,  $p_R$ , is concluded as

$$p_R^{\text{DF}} = \min \left\{ 1, \frac{-C_2 + C_3 + \sqrt{(C_2 - C_3)^2 + 4C_1C_4}}{2C_1} \right\} \quad (5.89)$$

## 5.3 Optimal Beamforming Filters with Null-Space Projection

The beamforming vectors,  $g_{rx}$  and  $g_{tx}$ , can be chosen in such a way that the self-interference power can be suppressed to zero. For that case, increasing the relay transmit power,  $p_R$ , does not affect the self-interference. So, due to (3.24), the optimal relay transmit power,  $p_R$ , maximizing the SINR becomes one, for any channel realization. Then, the general optimization problem is reduced to finding the beamforming vectors,  $g_{rx}$  and  $g_{tx}$ .

### 5.3.1 Full-Rank Interference Channel

Given the constraint of zero self-interference, an iterative approach can be adopted to find the pair of beamforming vectors achieving the highest possible end-to-end SINR [20, 33, 41].

For DF protocol, the optimization problem can be defined as in the following.

$$\begin{aligned} & \text{Finding the pair of vectors, } \{g_{rx}, g_{tx}\} \text{ maximizing } \min\{\|g_{rx}h_{SR}\|^2, \|g_{tx}h_{RD}\|^2\} \\ & \text{subject to } \|g_{rx}H_{RR}g_{tx}\| = 0 \text{ and } \|g_{rx}\| = \|g_{tx}\| = 1 \end{aligned}$$

The self-interference constraint implies that for any given  $g_{tx}$ , the vector  $g_{rx}$  is orthogonal to the vector  $H_{RR}g_{tx}$ . The orthogonal projection matrix,  $P_T$ , of  $H_{RR}g_{tx}$  is

$$P_T = I_{N_{rx}} - \frac{H_{RR}g_{tx}g_{tx}^H H_{RR}^H}{\|H_{RR}g_{tx}\|^2} \quad (5.90)$$

Similarly, the orthogonal projection matrix,  $P_R$ , of  $g_{rx}H_{RR}$  is

$$P_R = I_{N_{tx}} - \frac{g_{rx}H_{RR}H_{RR}^H g_{rx}^H}{\|g_{rx}H_{RR}\|^2} \quad (5.91)$$

So,  $g_{rx}$  satisfying the zero self-interference constraint can be expressed as

$$g_{rx} = \tilde{g}_{rx}P_T \quad (5.92)$$

where  $\tilde{g}_{rx}$  is any vector with unit norm.

Similarly, for any given  $g_{rx}$ , the vector  $g_{tx}$  satisfying the zero self-interference constraint is

$$g_{tx} = P_R\tilde{g}_{tx} \quad (5.93)$$

where  $\tilde{g}_{tx}$  is any vector with unit norm.

Now, since the zero self-interference constraint is satisfied, the first and second hop SINRs can be maximized with the optimal pair of vectors  $\{\tilde{g}_{rx}, \tilde{g}_{tx}\}$  using the matched filter solutions in (4.4), and (4.5) for the effective channels. So, for any given  $g_{tx}$ , the vector  $\tilde{g}_{rx}$  that maximizes SINR is written as

$$\tilde{g}_{rx} = \frac{g_{tx}^H H_{RR}^H P_T}{\|g_{tx}^H H_{RR}^H P_T\|} \quad (5.94)$$

Similarly, for any given  $g_{rx}$ , the vector  $\tilde{g}_{tx}$  that maximizes the SINR is written as

$$\tilde{g}_{tx} = \frac{P_R H_{RR}^H g_{rx}}{\|P_R H_{RR}^H g_{rx}\|} \quad (5.95)$$

Apparently, the optimal  $\tilde{g}_{rx}$  and  $\tilde{g}_{tx}$  depend on each other. Any pair of the vectors,  $\tilde{g}_{rx}$  and  $\tilde{g}_{tx}$ , optimizes both the first and second hop channel SNRs to some extent. Two extreme cases for the optimization point is either the case in which  $g_{rx}$  is set to be matched as  $g_{rx} = \frac{h_{SR}}{\|h_{SR}\|}$ , while  $g_{tx}$  is set only to satisfy the zero self-interference constraint, or vice-versa. Different pairs of vectors,  $\{\tilde{g}_{rx}, \tilde{g}_{tx}\}$ , can be generated heuristically by making the convex combinations of these two cases. Therefore, the vector,  $\tilde{g}_{rx}$ , of any of those different pairs can be written as

$$g_{rx} = \frac{\alpha g_{rx,1} + (1 - \alpha)g_{rx,2}}{\|\alpha g_{rx,1} + (1 - \alpha)g_{rx,2}\|} \quad (5.96)$$

where  $0 \leq \alpha \leq 1$ ,  $g_{rx,1} = \frac{h_{SR}}{\|h_{SR}\|}$  and  $g_{rx,2} = \frac{h_{SR}P_T}{h_{SR}P_T}$  with  $P_T = I_{N_{rx}} - \frac{H_{RR}h_{RD}^H h_{RD}H_{RR}^H}{\|H_{RR}h_{RD}^H\|^2}$

Finally, the value of  $\alpha$ , producing the pair of  $\{g_{rx}, g_{tx}\}$  which gives the highest end-to-end SINR of all can be found numerically. However, this does not still give exactly global solution.

### 5.3.2 Rank-One Interference Channel

The method for full-rank interference channel above can be taken a step further to derive a closed form solution for the beamforming filters by making an additional assumption. If we assume the environment around the relay is unlikely to be interacting much with the subchannels between the receiver and transmitter side antennas, the multipath effects, i.e. scattering, reflections, etc., of wireless signals are not expected to be observed much within the relay. Therefore, it is reasonable to think that there is only a line-of-sight channel. In other words, the line-of-sight channel is much stronger than the other multipath components. So, it is acceptable to think of the self-interference channel,  $H_{RR}$ , as a rank-one matrix.

The derivation for the null-space projection method is given below.

The self-interference channel SNR is

$$\gamma_{RR} = \frac{|g_{rx} H_{RR} g_{tx}|^2}{\sigma_R^2} \quad (5.97)$$

The singular value decomposition (SVD) of the matrix  $H_{RR}$  is

$$H_{RR} = U_{N_{rx} \times N_{rx}} \Sigma_{N_{rx} \times N_{tx}} V_{N_{tx} \times N_{tx}}^H \quad (5.98)$$

If  $H_{RR}$  is a rank-one matrix, it can be written as the multiplication of its corresponding left-singular and right-singular vectors to its only nonzero singular scaled by its Frobenius norm as given below.

$$H_{RR} = u \|H_{RR}\|_F v^H \quad (5.99)$$

where  $u_{N_{rx} \times 1}$  and  $v_{N_{tx} \times 1}$  are the left and right-singular vectors, respectively.

Then, the self-interference channel SNR is written as

$$\gamma_{RR} = \frac{|g_{rx} u \|H_{RR}\|_F v^H g_{tx}|^2}{\sigma_R^2} \quad (5.100)$$

Then, after taking the constant term out and separating the vector multiplications,  $\gamma_{RR}$  becomes

$$\gamma_{RR} = \frac{\|H_{RR}\|_F^2 |g_{rx} u|^2 |v^H g_{tx}|^2}{\sigma_R^2} \quad (5.101)$$

The self-interference can only be eliminated when  $\gamma_{RR}$ , as given in (5.101), becomes zero. Since the beamforming vectors,  $g_{rx}$ ,  $g_{tx}$ , are written as the multiplication of separate terms in (5.101), either of them, independent of the other, can be defined to make the whole equation zero. Note that after cancelling out the self-interference by setting one of the beamforming vectors, the other beamforming vector can be used to maximize the first-hop or second-hop channel SNR. In other words, it is futile

to use both of the beamforming vectors to remove the self-interference. Therefore, there are two independent cases that can satisfy the zero self-interference criterion. Each case is investigated separately below.

**Case 1:**

Assuming that the second multiplier in the nominator of (5.101) is zero given as

$$|g_{rx}u|^2 = 0 \quad (5.102)$$

The receiver beamforming vector,  $g_{rx}$ , can be calculated as

$$g_{rx} = w_R P_u \quad (5.103)$$

where  $P_u = I_{N_{rx}} - \frac{u u^H}{\|u\|^2}$  is a projection matrix, and  $w_R$  is a row vector.

Among all possible  $w_R$  row vectors, the one that makes the vector  $g_{rx}$  maximize the SNR is chosen.

$$\max |g_{rx}h_{SR}|^2 = \max |w_R P_u h_{SR}|^2 \quad (5.104)$$

Then, the row vector  $w_R$  is found as

$$\begin{aligned} w_R &= \arg \max_{w_R} |w_R P_u h_{SR}|^2 \\ &= h_{SR}^H P_u \end{aligned} \quad (5.105)$$

So, after the normalization due to unit-norm constraint (3.8),  $g_{rx}$  is concluded as

$$g_{rx, \text{case1}} = \frac{h_{SR}^H P_u}{\|h_{SR}^H P_u\|} \quad (5.106)$$

Since, the zero interference criterion is already satisfied,  $g_{tx}$  can be chosen independently to maximize the second hop SNR.

$$g_{tx} = \arg \max_{g_{tx}} |h_{RD}g_{tx}|^2 \quad (5.107)$$

Then, the transmitter beamforming vector,  $g_{tx}$ , is concluded as the matched filter solution in (4.5).

$$g_{tx, \text{case1}} = \frac{h_{RD}^H}{\|h_{RD}^H\|} \quad (5.108)$$



**Case 2:**

Assuming that the multiplicand in the nominator of (5.101) is zero given as

$$|v^H g_{tx}|^2 = 0 \quad (5.109)$$

The transmitter beamforming vector,  $g_{tx}$ , can be calculated as

$$g_{tx} = P_{v^H} w_C \quad (5.110)$$

where  $P_{v^H} = I_{N_{tx}} - \frac{v v^H}{\|v^H\|^2}$  is a projection matrix and  $w_C$  is a column vector.

Among all possible  $w_C$  column vectors, the one that makes the vector  $g_{tx}$  maximize the SNR is chosen.

$$\max |h_{RD} g_{tx}|^2 = \max |h_{RD} P_{v^H} w_C|^2 \quad (5.111)$$

Then, the column vector  $w_C$  is found as

$$\begin{aligned} w_C &= \arg \max_{w_C} |h_{RD} P_{v^H} w_C|^2 \\ &= P_{v^H} h_{RD}^H \end{aligned} \quad (5.112)$$

So, after the normalization due to unit-norm constraint (3.8),  $g_{tx}$  is concluded as

$$g_{tx, \text{case2}} = \frac{P_{v^H} h_{RD}^H}{\|P_{v^H} h_{RD}^H\|} \quad (5.113)$$

Since, the zero interference criterion is already satisfied,  $g_{rx}$  can be chosen independently to maximize the first hop SNR.

$$g_{rx} = \arg \max |g_{rx} h_{SR}|^2 \quad (5.114)$$

Then, the receiver beamforming vector,  $g_{rx}$ , is concluded as the matched filter solution in (4.4).

$$g_{rx, \text{case2}} = \frac{h_{SR}^H}{\|h_{SR}^H\|} \quad (5.115)$$

Each case above gives a pair of beamforming vectors,  $g_{rx}$ ,  $g_{tx}$ . The resulting end-to-end SNRs in (3.42) and (3.48) are compared, and the pair providing the higher

SNR is chosen as shown below.

$$\boxed{(g_{rx}, g_{tx}) = \begin{cases} (g_{rx,case1}, g_{tx,case1}), & \gamma_{SRD,case1} > \gamma_{SRD,case2} \\ (g_{rx,case2}, g_{tx,case2}), & \text{otherwise} \end{cases}} \quad (5.116)$$

Even though the self-interference channel is not exactly a rank-one matrix, the solution above can give an approximated result. If the strongest path within the self-interference channel is assumed to be the only path, by taking the eigenvector corresponding to the highest eigenvalue, the same derivation above can be followed. Since the self-interference channel matrix, in practice, is not expected to be of full rank, this approximation will still presumably give rather satisfactory results.

# Chapter 6

## Numerical Results

In this chapter, simulation results for different numerical and analytical methods are discussed. First, contour plots for AF and DF protocols are presented to observe the effect of channel SNRs in (3.29) and (3.31). Then, the performance of the analytical schemes presented in the previous chapter are compared. Those comparison results are also considered for the presence of higher self-interference. In Section 6.3, the computational requirements for numerical solutions are compared, and some hybrid solutions combining the higher SINR performance of numerical solutions with the faster implementation of analytical solutions are proposed. Then, the number of antennas on each side of the relay is varied with respect to another, and their SINR performance is compared.

### 6.1 SINR Contours

The end-to-end SINR obviously depends upon channel SNRs. The higher channel SNRs for the first and the second hops of the system lead to higher system performance, as clearly seen in (3.32) and (3.33). The higher channel SNR for the self-interference channel of the system leads to lower system performance<sup>1</sup>, as seen in (3.32). However, it may not be straightforward to see the performance contribution of each hop to the end-to-end system performance, independent of the other hop. For this reason, it is worth investigating SINR contour plots of global SINR expressions. They allow us to study the upper limits for all channel conditions.

In Fig. 6.1, SINR contour plots for AF and DF protocols are given. The channel SNR of the first hop is given on the horizontal axis, whereas the channel SNR of the second hop is given on the vertical axis. Each point on any curve represents a different pair of  $\{\gamma_{SR}, \gamma_{RD}\}$ . And every point on each curve results in the same end-to-end SINR value in decibels. The relay is assumed to have two antennas both on the receiver and transmitter sides. The source-to-relay, relay-to-source and self-

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<sup>1</sup>To be literally correct, it is safer to say non-decreasing end-to-end performance for higher system hop SNRs, and non-increasing end-to-end performance for higher self-interference channel SNR.

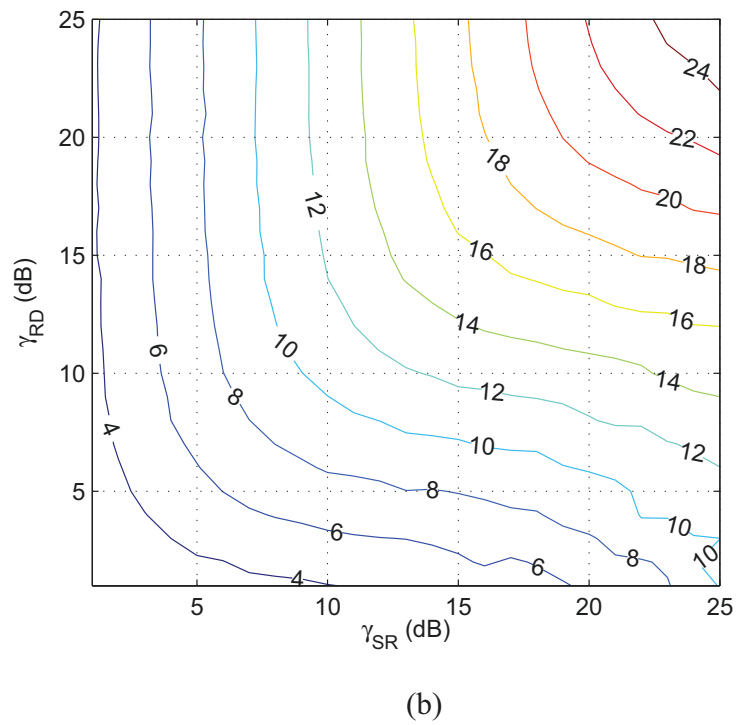
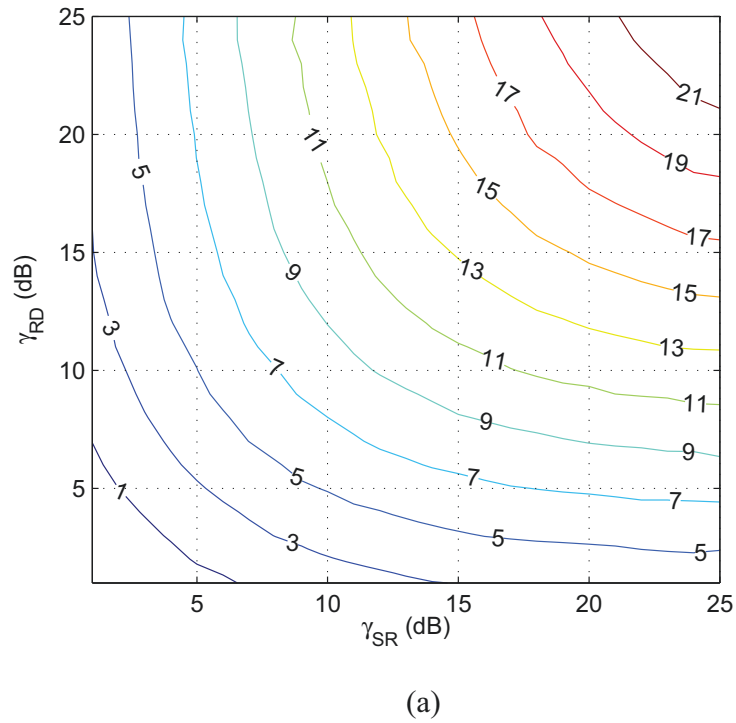


Figure 6.1: SINR contour plots in dB for global optimization problem (a) with AF protocol and (b) with DF protocol when the number of antennas at both sides of the relay is two.

interference channels are assumed to experience Rayleigh fading. The noise powers are assumed to be 1. The average power gain per each subchannel for the self-interference channel,  $\phi_{RR}$ , is set to be 0 dB.

$$\phi_{RR} = \frac{E\{\|H_{RR}\|_F^2\}}{N_{rx}N_{tx}} = 0 \text{ dB} \quad (6.1)$$

According to Fig. 6.1a, as both of the channel SNRs,  $\gamma_{SR}$  and  $\gamma_{RD}$ , increase, the end-to-end SINR increases. If only one of the channel SNRs increases, system performance still increases. However, the amount of increase depends on the level of the fixed channel SNR. If the fixed channel is the one with higher SNR, the proportional increase in the end-to-end SINR turns out to be greater compared to the case in which the fixed channel is lower than the other. In other words, lower channel SNR acts as a limiting factor in the system performance.

In Fig. 6.1b, increasing both  $\gamma_{SR}$  and  $\gamma_{RD}$  increases the end-to-end SINR. For unequal channel SNR cases, the smaller channel SNR acts as a limiting factor. However, unlike that for AF, this limit is much evident, due to min function in the end-to-end SINR definition (3.48). It is also worth noting that DF protocol is 2.5 - 3 dB superior to AF in terms of end-to-end SINR performance under any channel condition.

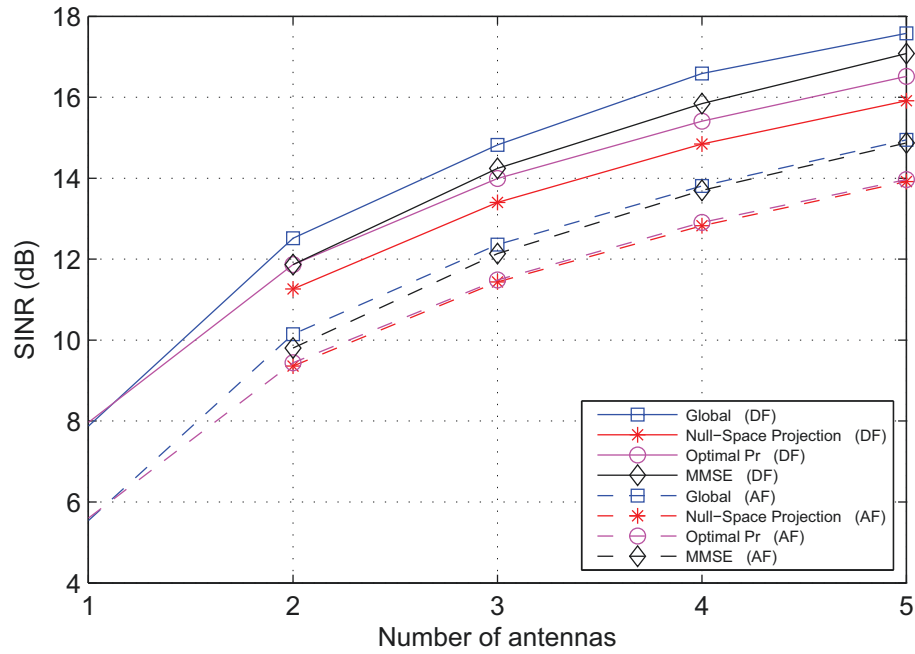
The lower right-hand side of the contour plot in Fig. 6.1b looks rather distorted, due to the result of averaging over insufficient number of channel realizations in the simulation. As the simulation is extended to calculate over more channel realizations, the corners of the contour lines, which are the points where the channel SNRs,  $\gamma_{SR}$  and  $\gamma_{RD}$ , are equal, are expected to look closer to being perpendicular.

## 6.2 Results for Analytical Schemes

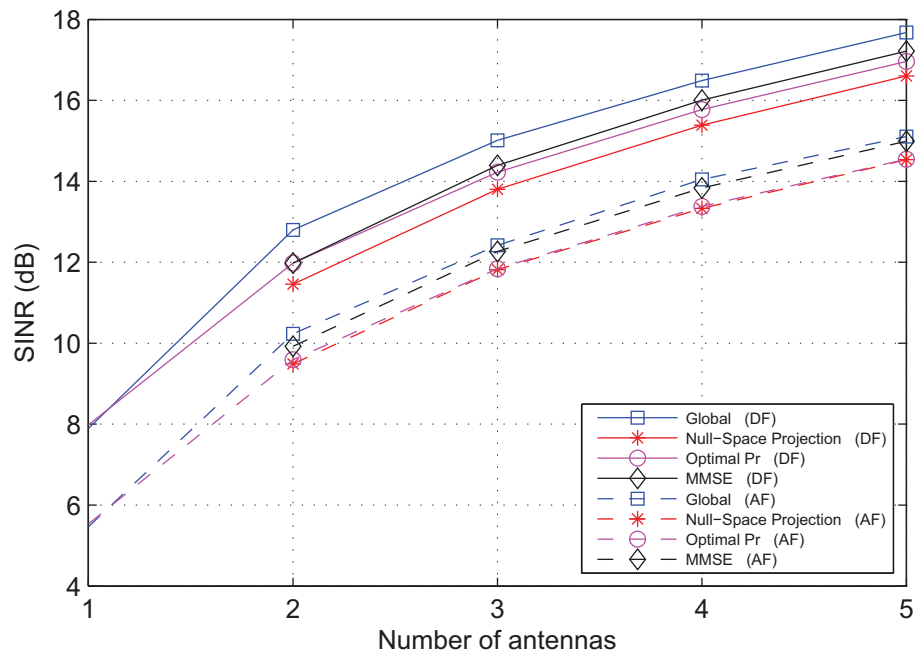
Simulations for the analytical schemes defined in the previous chapter are implemented on MATLAB<sup>®</sup>. More specifically, these are the schemes derived in Sections 5.1, 5.2, and 5.3.

In Fig. 6.2a and Fig. 6.2b, the simulation results of these methods versus different number of antennas are presented for both AF and DF protocols. Therefore, it is essential to keep the common parameters equal for a fair performance comparison. For each scheme, the same number of antennas are used for both of the beamforming vectors,  $g_{rx}$  and  $g_{tx}$ , which has been varied from  $N_{rx} = 1$  (and  $N_{tx} = 1$ ) to  $N_{rx} = 5$  (and  $N_{tx} = 5$ ). The numerical simulation results of the global optimization problem for AF and DF protocols are also added to the graph for comparison, since they represent the theoretical upper limits for SINRs.

Source-to-relay, relay-to-destination, and self-interference channels are assumed to



(a)



(b)

Figure 6.2: Simulation results for analytical schemes (a) with full-rank self-interference channel matrix, and (b) with rank-one self-interference channel matrix, when  $\phi_{RR} = 0$  dB

experience Rayleigh fading with independent identically distributed elements. The self-interference channel is modeled as a full-rank matrix in Fig. 6.2a, whereas in Fig. 6.2b, it is modeled as a rank-one matrix. The noise variances at the relay and at the destination nodes are assumed to be 1. The average power gain per each subchannel for the source-to-relay,  $\phi_{SR}$ , and relay-to-destination,  $\phi_{RD}$ , links are set to be 12 dB, whereas for the self-interference channel,  $\phi_{RR}$ , it is set to be 0 dB.

$$\phi_{SR} = \frac{E\{\|h_{SR}\|_2^2\}}{N_{rx}} = 12 \text{ dB} \quad (6.2)$$

$$\phi_{RD} = \frac{E\{\|h_{RD}\|_2^2\}}{N_{tx}} = 12 \text{ dB} \quad (6.3)$$

$$\phi_{RR} = \frac{E\{\|H_{RR}\|_F^2\}}{N_{rx}N_{tx}} = 0 \text{ dB} \quad (6.4)$$

The channels,  $h_{SR}$ ,  $h_{RD}$ ,  $H_{RR}$ , are generated over 2000 times independently. The results are averaged over those realizations for each number of antenna case. The scheme with optimal  $p_R$  uses the matched filter solutions, (4.4) and (4.5), for both of the beamforming vectors,  $g_{rx}$ ,  $g_{tx}$ , whereas MMSE scheme uses the matched filter solution (4.5) for the transmitter beamforming vector,  $g_{tx}$ .

In Fig. 6.2a, MMSE scheme gives the highest SINR performance among all the analytical schemes, especially as the number of antennas is increased. Null-space projection and the scheme optimizing  $p_R$ , also give quite promising results, not too far from the global solution. Note that when the number of antennas is one, null-space projection method fails to give sensible result. Apparently, it relies on the cancellation of the self-interference term. When the number of antennas is one, there are not enough degrees of freedom to achieve the cancellation in (5.102) or in (5.109). Therefore, it fails to produce sensible results when there is a single antenna at each side of the relay. For that reason, null-space projection simulation results for one antenna case are omitted. Similarly, MMSE method also does not produce sensible results when the number of relay antennas is one, because it reaches the optimal receiver beamforming vector by searching for the optimal direction by neglecting its norm. When there is one antenna on either side of the relay, there is, simply, no optimal direction for the receiver beamforming vector.

The MMSE scheme, unlike optimal  $p_R$  scheme, optimizes multiple parameters; the receive beamforming vector,  $g_{rx}$ , and the relay transmit power,  $p_R$ . Therefore, the observation that MMSE method is outperforming the optimal  $p_R$  scheme is not unexpected. However, null-space projection also optimizes multiple parameters; both the receiver and transmitter beamforming vectors,  $g_{rx}$  and  $g_{tx}$ . Yet, it underperforms optimal  $p_R$  scheme. The reason for that lies upon the suboptimality of the null-space projection itself. Without the zero self-interference constraint, the global problem seeks for the optimal set of parameters, which will produce the highest

SINR. Moreover, the null-space projection method also makes the assumption of rank-one self-interference channel response, whereas preserving the maximum relay transmit power. These are the underlying reasons for the lower SINR performance of the null-space projection scheme in Fig. 6.2a.

In Fig. 6.2b, the simulation results are presented for the case in which the self-interference channel is modeled as a rank-one matrix. Since the rank-one self-interference, compared to the full-rank, can be considered as causing less self-interference to the system, the global optimization point gets closer to the matched filter cases. For this reason, MMSE method, which uses the matched filter solution for the transmitter beamforming vector, and the optimal  $p_R$  scheme, which uses the matched filter solutions for both of the beamforming vectors, give better SINR performance. The SINR difference to global solution gets smaller. Another evident point in Fig. 6.2b is the relative performance improvement in the null-space projection method, because its self-interference cancellation scheme works accurately with the presence of actual rank-one self-interference channel response. Yet, it still underperforms other schemes, due to the inexistence of high self-interference term making interference elimination at the expense of much lower first hop and second hop channel SNRs unnecessary.

### 6.2.1 High Self-Interference Case

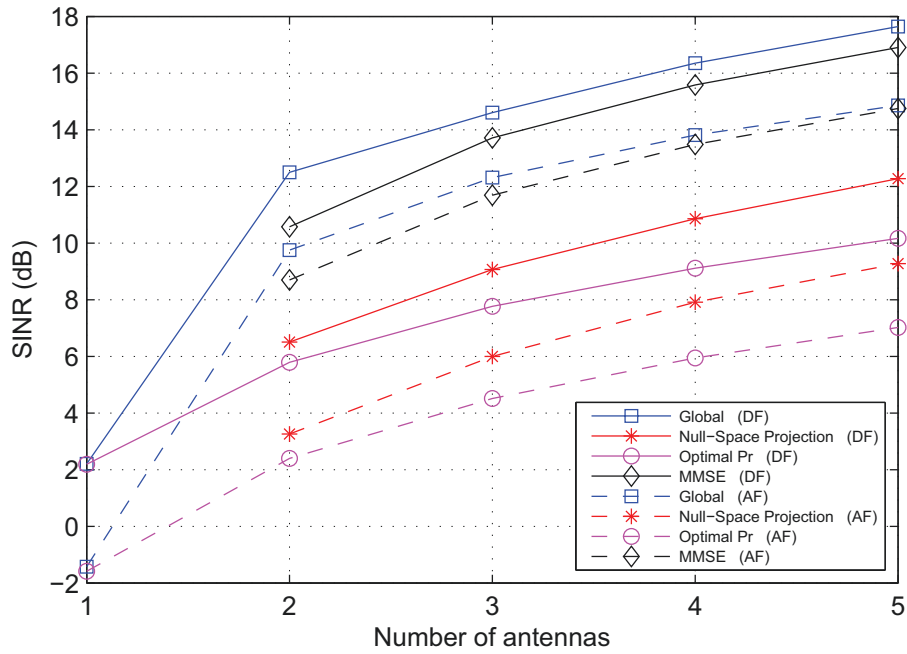
In the simulation results given in Fig. 6.2, the self-interference channels are generated to have zero mean, unit variance. However, if the self-interference channel is considered to have more adverse effects on the system than it has with the unit variance assumption, the relative performance of different schemes to each other will alter. Therefore, it is important to see the effect of higher self-interference on the schemes.

The simulations, of which the results are given in Fig. 6.2a and Fig. 6.2b, for the aforementioned analytical schemes are repeated next when the self-interference is higher, namely when

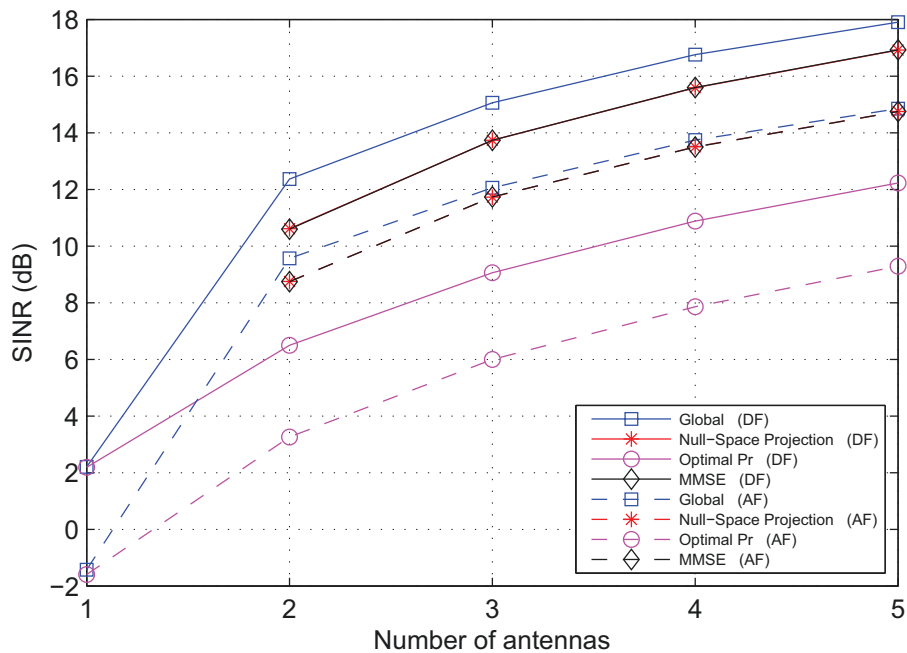
$$\phi_{RR} = \frac{E\{\|H_{RR}\|_F^2\}}{N_{rx}N_{tx}} = 10 \text{ dB} \quad (6.5)$$

In Fig. 6.3a, when higher self-interference is introduced to the system, some schemes give rather different performance results. The most conspicuous change is the relative performance decrease in null-space projection method and the optimal  $p_R$  scheme. Another evident point is that for AF protocol, MMSE method is hardly affected by the higher self-interference. Higher self-interference takes the global optimization solution closer to the zero self-interference point. In other words, the global optimization resides closer onto the point of mitigating the self-interference and further away from the point of increasing the first hop and second hop channel





(a)



(b)

Figure 6.3: Simulation results for analytical schemes (a) with full-rank self-interference channel matrix, and (b) with rank-one self-interference channel matrix, when  $\phi_{RR} = 10$  dB

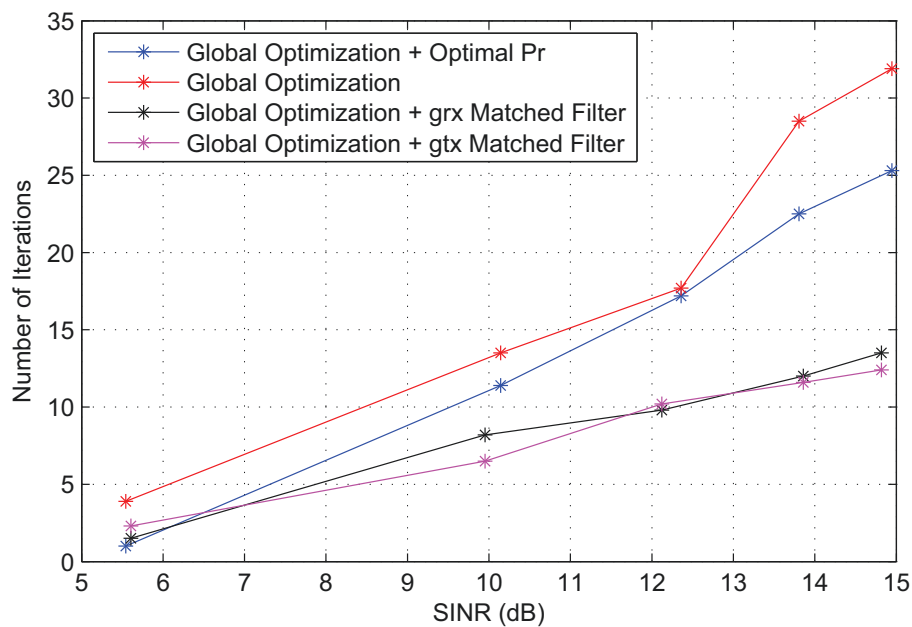
SNRs. Thus, the idea of focusing on the cancellation of self-interference gives higher SINR. For these reasons, the optimal  $p_R$  scheme maximizing the channel SNRs by neglecting the self-interference results in low performance. The null-space projection method, unlike those in Fig. 6.2a and Fig. 6.2b, outperforms the optimal  $p_R$  scheme. However, it performs worse than MMSE method. The reason for this lower performance is that the null-space projection scheme makes the assumption of rank-one self-interference channel response for the mitigation, even though the self-interference is of full-rank. Since the self-interference is not completely cancelled out, using maximum transmit power worsens the performance.

In Fig. 6.3b, when the self-interference channel is modeled as a rank-one matrix, the null-space projection method performs much better, similar to MMSE scheme. The optimal  $p_R$  scheme gives a bit better SINR performance, although it still suffers from the high self-interference.

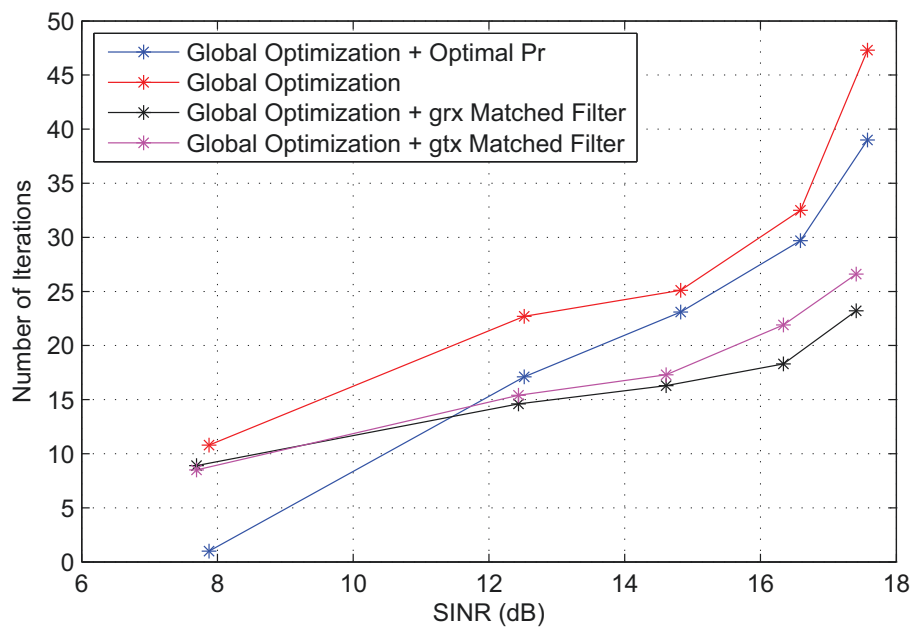
### 6.3 Computational Efficiency

Global numerical optimization, as being the theoretical upper limit, always outperforms other numerical methods, as well as analytical schemes, in terms of SINR. However, it is not quite efficient in terms of time and computational requirements. Introducing some constraints may reduce these inefficiencies while still keeping the SINR performance high. As seen in Fig. 4.1, introducing matched filter solutions to only one of the beamforming filters of the relay still keeps the SINR performance quite close to that of the global solution. However, the efficiency of the computing time and power also needs to be considered for a fair comparison. Thus, it is worth investigating the computational gains obtained with the subsolutions. For this purpose, the average number of iterations is calculated for each simulation, and regarded as the comparison criterion to measure the computing time, or the computing power. In reality, however, the relation between the number of iterations and the computing power, or the computing time, is not linear and depends heavily on the implementation of the optimization code. Based on the observations on the simulations, it can be still argued that higher number of iterations requires more and more computational power.

In Fig. 6.4a and Fig. 6.4b, the required number of iterations to achieve a certain SINR value is given for AF and DF protocols, respectively. Each marker in the plots is the result of a simulation with different number of relay antennas. For all the schemes, higher number of antennas requires more iterations. The global optimization requires higher number of iterations than the other methods. Introducing the optimal  $p_R$  constraint does not cause any SINR decrease, whereas the required number of iterations decreases, particularly as the number of antennas are increased. The schemes with matched beamforming vectors result in slightly lower SINR values (Fig. 4.1) compared to the global optimization. Yet, their implementation is computationally faster.



(a)



(b)

Figure 6.4: Number of iterations vs. SINR for global optimization (a) with AF protocol, and (b) with DF protocol

## 6.4 Effect of Antenna Variation at Relay

Up to this point, all the simulations have been run with different number of beamforming antennas. However, the number of antennas for the receiver beamforming vector,  $g_{rx}$ , and the transmitter beamforming vector,  $g_{tx}$ , have been kept equal. Yet, it is interesting to study the effect of the number of antenna variations for each beamforming vector separately.

In order to fairly compare the results of different number of antenna ratios between  $g_{rx}$  and  $g_{tx}$ , it is important to keep the total number of antennas constant, since increase in the number of antennas, obviously, results in higher SINR performance. Therefore, in this simulation setup, the total number of antennas is assumed to be six<sup>2</sup>.

$$N_{rx} + N_{tx} = 6 \quad (6.6)$$

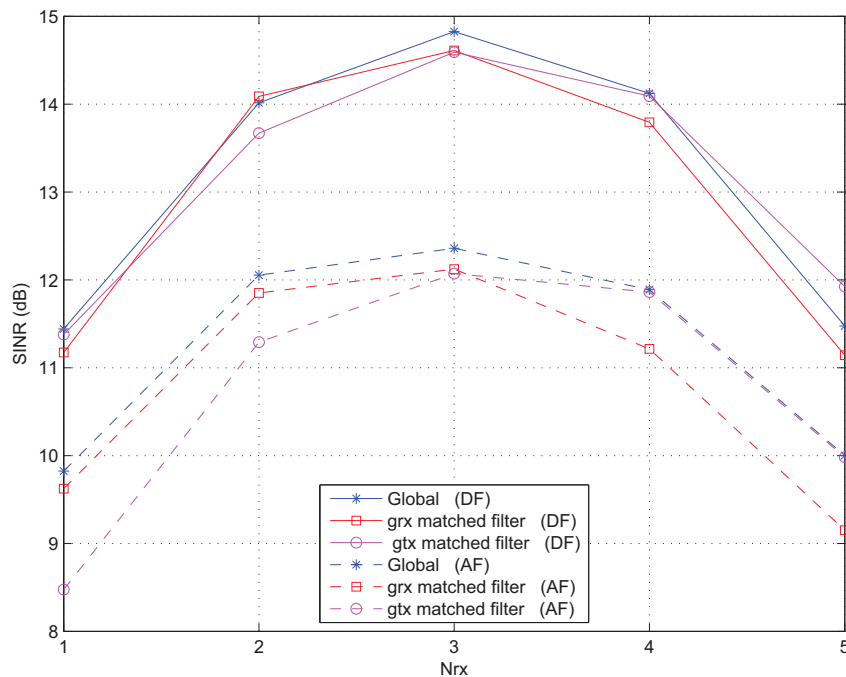


Figure 6.5: Simulation results of numerical optimization of global and matched filter solutions with AF and DF for different  $N_{rx}$  and  $N_{tx}$ , given  $N_{rx} + N_{tx} = 6$

In Fig. 6.5, for all the schemes, SINR performance is maximized when the number of antennas for beamforming vectors are kept equal. In horizontal axis,  $N_{rx}$  varies

<sup>2</sup>For our case, 6 is a convenient choice for analyzing various ratios between  $N_{rx}$  and  $N_{tx}$ , and still completing the simulations rather fast.

from 1 to 5, while  $N_{tx}$  varies from 5 to 1, respectively. In other words, on the left-hand side of the plot,

$$N_{tx} > N_{rx} \quad (6.7)$$

And on the right-hand side of the plot,

$$N_{rx} > N_{tx} \quad (6.8)$$

The simulation plots for matched filter constraints are not vertically symmetrical. A simulation with the matched filter constraint results in higher SINR when the number of antennas for the matched beamforming vector is smaller, compared to the case of setting the other beamforming vector as the matched filter, as expected.

Hence, if the matched filter solution is to be used on one side of the relay, the first step for better end-to-end system performance optimization is to use the same -or as close as possible- number of antennas for each of the beamforming vectors,  $g_{rx}$  and  $g_{tx}$ . Then, with a given set of number of antennas for the beamforming vectors on each side of the relay, assigning the matched filter solution to the beamforming vector with the smaller number of antennas, and optimizing the other beamforming vector independently has a higher end-to-end SINR upper limit.

# Chapter 7

## Conclusions

This thesis considers a one-way two-hop SISO communication link with a full-duplex relay. It is shown that the end-to-end system SINR depends upon both source-to-relay and relay-to-destination SINRs, and that the smaller of those acts as a limiting factor -as a soft or hard limit depending on the relay protocol, AF or DF, respectively- for the end-to-end SINR. The system is aimed to be optimized by maximizing the end-to-end SINR through beamforming at the receiver and transmitter sides of the relay. For that purpose, several analytical solutions are derived and their performance results are compared to each other, as well as to the theoretical upper limits on simulations.

The analytical solutions and the simulation results indicate that the optimization point relies on the set of channel realizations and the noise power at the relay and the destination nodes. For rather low self-interference levels comparative to the channel gains of the first hop and the second hop links, the optimization point gets closer to the point which optimizes the gain of the useful signal. In other words, the optimal solution for the beamforming filters converges to the matched filter solutions. In contrast, in the presence of rather high self-interference, the optimization point becomes closer to the null-space solution, which aims to eliminate the self-interference independent of the channel gains of the useful signal.

The numerical analysis of the system shows that setting only one of the beamforming filters as the matched filter, and using the optimal solution for the other beamforming filter does not degrade the SINR performance much, particularly compared to the case in which both of them are matched. This simple, yet important, observation encouraged us to aim for the optimization of one of the beamforming vectors using minimum mean square error (MMSE) method, instead of trying to find the suboptimal solutions for both of the filters by introducing a constraint to the system, which may not necessarily work well under any channel conditions.

Two of the proposed new schemes are central to this thesis. First, by fixing the transmitter beamforming filter, the mean square error of the first hop SINR is constructed. The optimal receiver beamforming vector minimizing the mean square

error is derived, and using this solution, the optimal relay transmit power, in terms of the transmitter beamforming filter, is obtained. Second, by assuming the self-interference channel is a rank-one matrix, null-space projection is considered, and an analytical solution for the set of beamforming filters is derived. In the simulations, this scheme is proven to give promising results, especially when the self-interference level of the system is high.

## 7.1 Future Work

This thesis offers several solutions for optimizing the full-duplex relay system performance, one working better than the other depending on the channel conditions. However, a single unified closed-form solution producing the optimal beamforming filter directions and the relay transmit power under any condition still stays undiscovered, even though we have found that solution via numerical search. What is more, the direct link between the source and the destination nodes, which is neglected in this work, can be taken into consideration. Instead of regarding the signal coming through the direct link as self-interference, its correlation with the received signal through the relay link can be combined to obtain higher SNR. Furthermore, in this thesis, the system model is constrained to be a SISO link. In a later work, the results of this thesis can be transported into a more generalized MIMO system setup with spatial multiplexing.

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