Limited feedback MIMO techniques for temporally correlated channels and linear receivers

Eduardo Zacarías B.

Aalto University

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Limited feedback MIMO techniques for temporally correlated channels and linear receivers

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Abstract

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Abstract

Advanced mobile wireless networks will make extensive use of multiantenna (MIMO) transceivers to comply with high requirements of data rates and reliability. The use of feedback channels is of paramount importance to achieve this goal in systems employing frequency division duplexing (FDD). The design of the feedback mechanism is challenging due to the severe constraints imposed by computational complexity and feedback bandwidth restrictions.

This thesis addresses the design of transmission strategies in both single-user and multi-user MIMO systems, based on compact feedback messages. First, recursive feedback mechanisms for single-user transmission scenarios are proposed, including stochastic gradient techniques, deterministic updates based on Givens rotations and low computational complexity schemes based on partial update filtering concepts. Thereafter, channel feedback algorithms are proposed, and a convergence analysis for static channels is presented. These algorithms can be used to provide channel side information to any multi-user MIMO solution. A limited-feedback decentralized multi-user MIMO solution is proposed, which avoids the need for explicit channel feedback. A feed-forward technique is proposed, which allows our methods to operate in presence of feedback errors.

The performance of all the proposed algorithms is illustrated via link-level simulations, where the effect of different parameter values is assessed. Our results show that the proposed methods outperform existing limited-feedback counterparts over a range of low to medium mobile speeds, for moderate antenna array sizes that are deemed practical for commercial deployment. The computational complexity reduction of some of the proposed algorithms is also shown to be considerable, when compared to existing techniques.

Keywords Multiantenna systems, MIMO, limited feedback, wireless communications

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Preface

The research work presented in this thesis was carried out under supervision of Prof. Risto Wichman from the Department of Signal Processing and Acoustics, Aalto University of Technology (former Signal Processing Laboratory, Helsinki University of Technology / TKK).

First, I would like to warmly thank my supervisor Prof. Risto Wichman, for all his guidance and support throughout the long period that the work has spanned. I wish to express my thanks also to my instructor and co-author D. Sc. Stefan Werner. His constructive criticism and support improved both the work and the quality of the publications. It has been a pleasure to work with and learn from them, both being remarkably capable persons of great integrity.

Then, I acknowledge gratefully the work of the pre-examiners of the thesis, Dr.-Ing. Patrick Marsch and Dr Joakim Jaldén, for their valuable reviews and suggestions.

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The work was carried out in two stages. I worked full-time during the years 2003-2006 in the Laboratory at TKK, with funding from the Finnish Academy,

the Finnish Cultural Foundation, the University itself and Nokia Research Center. From 2007 onwards I have been working for Nokia-Siemens Networks with the exception of four months in 2009, during which I was back in the Laboratory full-time. This second stage has seen the doctoral research relegated to a background plane, and also my relocation to Germany, where I presently live. I am extremely thankful to Mikko Säily and Jari Hulkkonen in NSN for all their efforts in organizing my subcontracting with TKK throughout 2007-2009, their support in times when the doctoral research required efforts beyond my spare time, and the friendliness that has characterized our relationship. Finally, I thank my former boss Norbert Schulz for his efforts in my relocation to Germany, and my current boss Andreas Klug for his understanding and support in this last phase of the doctoral studies.

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Finally, I want to thank my beloved wife Silvia, for all her tender care, unwavering support and understanding.

Augsburg, January 2012

Eduardo A. Zacarías Brach

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List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

I E. Zacarías B., R. Wichman and S. Werner. Filtered gradient algorithm for closed loop MIMO systems. *Proc. IEEE VTC-Spring*, Stockholm, May 2005.

II E. Zacarías B., S. Werner and R. Wichman. Adaptive transmit eigenbeamforming with stochastic unitary plane rotations in MIMO systems with linear receivers. *Proc. IEEE IZS 2006*, Zurich, February 2006.

III E. Zacarías B., S. Werner and R. Wichman. Partial update adaptive transmit beamforming with limited feedback. *Proc. IEEE ICASSP 2006*, Tolousse, May 2006.

IV E. Zacarías B., S. Werner and R. Wichman. Enhanced partial update beamforming for closed loop MIMO systems. *Proc. IEEE PIMRC*, Helsinki, September 2006.

V E. Zacarías B., S. Werner and R. Wichman. Distributed Jacobi eigenbeamforming for closed-loop MIMO systems. *IEEE Commun. Lett.*, vol. 10, no. 12, December 2006.

VI E. Zacarías B., S. Werner and R. Wichman. Link adaptation with distributed Jacobi eigenbeamforming for MIMO systems. *Proc. IEEE ISWCS 2007*, Trondheim, October 2007.

VII E. Zacarías B., S. Werner, R. Wichman and T. Riihonen. Single-bit closed-loop quasi-orthogonal space-time codes for MIMO systems. *Proc. IEEE SPAWC*, Perugia, June 2009.

VIII E. Zacarías B., S. Werner and R. Wichman. Limited feedback multiuser MIMO techniques for time-correlated channels. *EURASIP Journal on Advances in Signal Processing*, doi:10.1155/2009/104950, October 2009.

IX E. Zacarías B., S. Werner and R. Wichman. Decentralized limited-feedback multiuser MIMO for temporally correlated channels. *Journal of Electrical and Computer Engineering*, doi:10.1155/2010/915653, July 2010.

List of abbreviations

3GPP	Third Generation Partnership Project
ALE-PUB	Alignment-enhanced partial update beamforming
BD	Block diagonalization
BEP	Bit error probability
BER	Bit error rate
BZ	Banister & Zeidler
CBBF	Codebook beamforming
CDF	Cumulative density function
CORDIC	Coordinate rotation dIgital computer
CQI	Channel quality indicator
CSI	Channel state information
CSIR	Channel state information at the receiver
CSIT	Channel state information at the transmitter
DAO	Distributed alternating optimization
DFT	Discrete Fourier transform
D-JAC	Distributed Jacobi eigenbeamforming algorithm
DL	Downlink
DPC	Dirty paper coding
DVB	Digital video broadcast
EBF	Eigenbeamforming
EGT	Equal gain combining
EVD	Eigenvalue decomposition
EXPM	Matrix exponential
FDD	Frequency-division duplexing
FFT	Fast Fourier transform
GPRS	General packet radio service
GSL	GNU scientific library
i.i.d.	Independent and identically distributed

IEEE	Institute of Electrical and Electronics Engineers
IGREB	Incremental Givens rotations eigenbeamforming
IIR	Infinite impulse response
IMT-2000	International Mobile Telecommunications-2000
IRC	Interference rejection combining
ISI	Intersymbol interference
LBG	Linde-Buzo-Gray
LMMSE	Linear minimum mean square error
LMS	Least mean squares
LPC	Linear predictive coding
LTE	Long term evolution
MCS	Modulation and coding scheme
MIMO	Multiple-input, multiple-output
MISO	Multiple-input, single-output
MMSE	Minimum mean square error
MRC	Maximum ratio combining
MSE	Mean square error
MSIP	Mean square inner product
MI	Mutual information
MUI	Multi-user interference
MU	Multi-user
OFDM	Orthogonal frequency-division multiplexing
OL	Open loop
OSTBC	Orthogonal space-time block codes
PDF	Probability density function
PSK	Phase shift keying
PVQ	Predictive vector quantization
QAM	Quadrature amplitude modulation
QOSTBC	Quasi-orthogonal space-time block code
QPSK	Quadrature phase shift keying
RBD	Regularized block diagonalization
RPUCF	Ranked partial-update channel feedback
RVQ	Random vector quantization
SBRVT	Single-bit real-valued tracker
SCGAS	Stochastic gradient search over an angle space
SGE	Stochastic geodesic
SINR	Signal to interference-plus-noise ratio
SISO	Single-input, single-output

SMF	Set membership filtering
SNR	Signal to noise ratio
SPUCF	Sequential partial-update channel feedback
STBC	Space-time block code
STC	Space-time code
STTC	Space-time trellis code
SU	Single-user
SVD	Singular-value decomposition
SVQ	Sequential vector quantization
TCM	Trellis-coded modulation
TDD	Time-division duplexing
THP	Tomlinson-Harashima precoding
TPMI	Transmit precoding matrix indicator
VQ	Vector quantization
WCDMA	Wideband code-division multiple access
WLAN	Wireless local area network
ZF	Zero-forcing

List of symbols

a, A	complex-valued scalars
$\lvert a \rvert$	absolute value of scalar a
a^*	complex-conjugate of scalar a
$\mathbf{a}$	complex-valued vector
$\mathbf{a}^*$	complex-conjugate (element-wise) of vector $\mathbf{a}$
$\lvert\lvert\mathbf{a}\rvert\rvert$	Euclidean norm of vector $\mathbf{a}$
$\mathbf{A}$	complex-valued matrix
$\lvert\lvert\mathbf{A}\rvert\rvert_F$	Frobenius norm of matrix $\mathbf{A}$
$\lvert\mathbf{A}\rvert$	determinant of matrix $\mathbf{A}$
$\mathbf{A}^\dagger$	Hermitian transpose of matrix $\mathbf{A}$
$\mathbf{A}^{-1}$	inverse of matrix $\mathbf{A}$
$\mathbf{A}^*$	complex-conjugate (element-wise) of matrix $\mathbf{A}$
min	minimum value
max	maximum value
$\arg\min$	minimizing argument
$\arg\max$	maximizing argument
$\mathrm{tr}\{\mathbf{A}\}$	trace of matrix $\mathbf{A}$
sign	sign operator
$\mathrm{E}\{\cdot\}$	Expectation of random variables
k	time (symbol period) index
l	slot index
exp	exponential function
expm	matrix exponential function
$\mathbf{J}^{mn}(\alpha, \beta)$	complex-valued Givens or Jacobi transformation on coordinate plane mn, with angles α, β
$\mathbf{Q}$	covariance matrix of the interference-plus-noise signals
$\mathbf{H}_i$	channel matrix for the i-th user

1. Introduction

1.1 Motivation

Commercial wireless communications systems evolve in a continuous quest for enhanced efficiencies. This fact derives mostly from the highly competitive characteristic of the market, where a strong pressure to deliver novel services, increase the user basis and reduce costs results in a very active research community.

Multiantenna techniques are one of the most important ways to improve spectral efficiency or throughput. The use of antenna arrays in wireless communications has evolved from traditional receive diversity and interference cancellation concepts [138], to a comprehensive set of techniques enabling higher spectral efficiencies, lower deployment costs and more sophisticated services including applications demanding high bit rates and low latencies.

Several benefits arise from the use of multiple-input, multiple-output (MIMO) techniques. In the single user (SU) case, for example, exploiting transmit and receive diversity simultaneously through beamforming results in received signal to noise ratio (SNR) improvements, which allow to extend coverage ranges or decrease the transmit power, or enable higher data rates through the use of higher order symbol constellations. On the other hand, the use of spatial multiplexing (SM) allows to send parallel data streams over the wireless channel, effectively increasing the link data rate. Further spectral efficiency enhancements to SU or scheduled systems comes from transmitting data to several users over the same time and frequency resources. This MIMO multi-user (MU) multiplexing can be implemented using low computational complexity receivers.

Current commercial wireless mobile systems have adopted orthogonal frequency division multiplexing (OFDM) techniques, because of their inherent

robustness against frequency selective fading of the wireless channels. OFDM techniques allow simple channel equalization after processing the received signal with the Fast Fourier Transform (FFT). This enables the deployment of MIMO techniques designed for frequency flat fading channels, as the transmission can be treated separately for groups of adjacent subcarriers, which experience approximately narrowband fading. Examples of MIMO-OFDM systems being currently deployed include the 3GPP Long Term Evolution (LTE) system [3], IEEE 802.16e (mobile WiMAX) [55] and IEEE 802.11n (WLAN) [56]. Furthermore, MIMO techniques will continue to play a crucial role in upcoming mobile wireless networks. For example, the IMT-Advanced requirements [57] will be fulfilled by LTE release 10 ("LTE-Advanced") by employing four-by-four MIMO configurations and cooperative MIMO schemes are being studied, where one terminal is served simultaneously by more than one access point (see, e.g., [92] for an overview of LTE Advanced and the IMT-Advanced requirements).

Multiantenna transceiver techniques for mobile wireless communications have been the focus of a large body of literature [37, 49]. A substantial amount of research has been devoted to understand the performance limits of single user or multi user systems employing antenna arrays, and to develop link level algorithms that can perform near those limits, operating under a variety of restrictions. These restrictions arise from implementation cost considerations, regulatory laws, etc. They take the form of constraints to the computational complexity of the algorithms, transmission power and bandwidth, and also the dynamic range of the transmitted signals (due to power amplifier efficiency considerations).

The concept of channel side information at the transmitter (CSIT) is well-known in mobile wireless communications. It can be described as information that the transmitter has about the wireless channel conditions, which enables it to adapt the transmitted signal to optimize a given metric. Simple examples of CSIT for adaptation of the modulation and coding scheme (MCS) are the bit error probability (BEP) statistics in General packet radio service (GPRS) [6] and the channel quality indicator (CQI) reports in 3GPP LTE [4]. More elaborated types of CSIT include statistical properties of the wireless channel (e.g., mean and covariance matrices) used for long-term beamforming and instantaneous CSIT for the purposes of a) short-term beamforming, b) data multiplexing based on linear precoding, e.g., the precoding matrix indicator in 3GPP LTE [4], c) multiuser multiplexing based on linear or non-linear precoding, d) transmit diversity-assisted interference cancellation at the

receiver.

In some cases, the CSIT can be acquired directly by the access point transmitter. This is the case of systems employing time division multiplex (TDD) to separate the access point to mobile link (downlink) and the mobile to access point link (uplink), where the wireless channel is essentially the same for both links since they use the same frequency range. This is not the case for frequency division multiplex (FDD) systems, where uplink and downlink employ different frequency bands. On such systems, the use of a feedback channel is required to provide CSIT. The feedback or closed-loop mechanism is designed to enable performance close to that of perfect CSIT, given the feedback restrictions.

One of the limiting factors for the performance of closed-loop systems is the variability of the wireless channel, compared to the capacity of the feedback link. The performance penalty of the outdated feedback will depend on the purpose of the feedback. For example, closed-loop linear precoding for single user transmission is less sensitive to outdated CSIT than closed-loop multiuser multiplexing.

Another limiting factor for the design of the feedback mechanism is the available processing power available at the mobile receiver. This limits the allowed computational complexity of the feedback calculation. For example, codebook-based feedback mechanisms incur search costs proportional to the codebook size. Depending on the allowed complexity, the codebook size can be severely limited, thus restricting the feedback accuracy that can be obtained. This can be partially alleviated by using small nested codebooks. Alternatively, partial update schemes can be designed with lower computational complexity and allow to efficiently exploit the available channel temporal correlation.

Yet another issue in closed-loop systems is the handling of feedback errors. A mismatch between the CSI fed back by the mobile receiver and the actual CSIT will in general degrade the performance, depending on the purpose and type of the feedback. The effects can range from mild to severe. For example, an error in an index belonging to a fine nested codebook may not be severe if it occurs within the deepest codebook layer. An error in a recursive feedback scheme will propagate and can compromise the tracking stability if uncorrected. One possibility is that the transmitter will forward the received feedback message to the mobile receivers, so that the feedback error can be detected and corrected when generating the next feedback message. A form of this strategy currently specified as the “transmit precoding indicator” (TPMI) in 3GPP LTE [4]. Additionally, a restarting policy can be used in the case of recursive feedback

schemes.

1.2 Scope of the thesis

The goal of this thesis is to develop transmission strategies for MIMO systems employing limited-rate feedback links, for use in MIMO channels exhibiting time correlation. These channels are experienced, for example, by mobile receivers traveling at pedestrian speeds and using frequency bands around 2 GHz. Feedback methods are developed and analyzed in conjunction with transmission strategies, for the purposes of increasing the wireless link reliability in the single user case, and to increase the spectral efficiency of the system through multiuser multiplexing. The techniques developed here can be used in transceiver design and may be applicable in the development of MIMO-OFDM based systems such as WiMAX and 3GPP LTE. The designs emphasize low computational complexity.

1.3 Contributions and organization of the thesis

This thesis contributes to the field of partial CSI multi-antenna transmission in wireless networks. The contributions fall into two broad categories, as described below. The first pertains closed-loop single-user MIMO systems. Five novel limited-feedback eigenbeamforming methods are proposed. One of them is further applied into adaptive transmission under heavy interference conditions and can also be combined with basic link adaptation to further boost the link reliability. The single feedback bit proposals are further applied in combination with quasi-orthogonal space-time block codes (QOSTBC), and obtain bit error rate (BER) performance advantages over existing methods. All the proposed methods outperform existing alternatives based on static codebooks in terms of uncoded bit error rate, over a range of low to medium mobile speeds.

The second category relates to multi-user (MU) MIMO systems with limited-rate feedback channels. A large body of literature has been devoted to full-CSI MU-MIMO, which is a realistic assumption in TDD systems, but not in FDD systems. Existing solutions based on static or adaptive codebooks normally incur in a large computational complexity, or require extensive simulation-based designs to tune different parameters as functions of the mobile speed and feedback rate. The proposed channel feedback methods combine partial

update concepts with single-bit encoding techniques from early audio compression methods. This results in low complexity methods for feeding back the complete channel matrix to the transmitter, which have good performance in low mobility conditions. Additionally, a novel multiuser multiplexing technique that avoids explicit channel feedback is proposed. The performance is analyzed both as a function of the mobile speed and the probability of feedback bit errors. A compensation mechanism is introduced, which allows the algorithm to operate under moderate amounts of feedback errors. The same mechanism can be applied to the other contributions of this dissertation.

This thesis is organized as follows. Chapter two presents system models for adaptive transmission based on feedback channels. The formulation includes as a general case multiple users and external interfering signals. A general description of the feedback channel is given. This channel will be used to transport different types of information, as will be detailed in Chapters 3 to 5. Formulas for the basic linear receiver structures are given.

Chapter 3 introduces closed-loop eigenbeamforming methods for single user MIMO systems. First a survey of the existing methods is presented. Thereafter a summary of the contributions of the thesis is given. These methods will be applied also in the context of Chapters 4 and 5.

Chapter 4 describes channel feedback methods for unstructured matrices, as opposed to the feedback of orthogonal matrices presented in Chapter 3. A review of existing methods is given, followed by a summary of the thesis contributions.

Chapter 5 deals with multiuser MIMO schemes. First a brief review of full-CSI MU-MIMO solutions is presented, including minimum mean square error (MMSE), decomposition and non-linear approaches. Thereafter the thesis contributions on limited-feedback MU-MIMO are described.

The introductory part of this thesis concludes in Chapter 6 with a summary of its results and contributions.

1.4 Publication summary

This thesis encompasses nine original publications, which are listed on page 7, and appended starting on page 115. Six papers are related to subspace tracking through the feedback channel and its application to single user MIMO communications. One publication deals with both interference-tolerant MIMO communications and channel feedback techniques for unstructured matrices. One article deals with closed-loop enhancements to quasi-orthogonal space-

time codes. Finally, one publication presents a decentralized multiuser MIMO solution based on limited feedback. All the contributions are based on recursive formulas, where the feedback channel provides information about incremental changes. Therefore, the performance will depend on the fading rate (equivalently the mobile receiver speed) and the allowed number of bits per use of the feedback channel. Simulations are used to analyze the link performance as a function of the mobile speed. A comparison to the performance of non-recursive techniques (e.g., feedback schemes based on static codebooks) is presented with each contribution.

In [P. I], a single feedback bit scheme is considered in a single user, single stream scenario. A performance enhancement is introduced by means of filtering the limited-feedback stochastic gradient estimate. This provides an improvement in convergence speed and tracking capabilities, and results in better BER performance under moderate mobile speeds, compared to the unfiltered version.

In [P. II], the case of a single user with multiple stream transmission and linear receivers is considered. Two single-bit recursions are proposed to enable tracking of the eigenbeams at the transmitter. The algorithms are shown to outperform existing single-bit proposals. Both schemes are based on complex-valued Givens rotations, which can be implemented efficiently in hardware by using the COordinate Rotation DIgital Computer (CORDIC) algorithm and its variations [40].

Publication [P. III] departs from the single-bit feedback messages scenarios treated in the previous work, but remains within the scope of single user, single stream systems. Partial-update concepts from adaptive filtering theory are applied to design a low-complexity eigenbeamformer for MISO systems. The resulting algorithm features a strong complexity reduction, when compared to codebook beamforming alternatives, and is shown to outperform them at low mobilities. Two variants are developed: a sequential and a data-dependent selective update.

The article [P. IV] extends the work in partial-update eigenbeamforming, and solves the partial-update problem in a general MIMO system, thus lifting the single antenna receiver restriction. The resulting algorithm also has both sequential and selective update variants. Simulations results show that near-optimal BER performance can be reached at pedestrian speeds, outperforming existing codebook beamforming alternatives over a low-to medium range of speeds. Furthermore, the computational complexity is still significantly lower than that of the codebook-based alternatives.

Publication [P. V] introduces the distributed Jacobi (D-JAC) eigenbeamforming algorithm, a low computational complexity subspace tracking algorithm based on complex-valued Givens rotors and the Jacobi algorithm for Hermitian matrices. This method allows tracking of orthogonal bases of a given dimension. In particular, the dimension is taken equal to the number of data streams transmitted to the user. The algorithm reaches near-optimal BER performance in low mobility conditions and does not need to enforce orthogonality conditions. Furthermore, Section 3.4 and Chapters 4 and 5 show how D-JAC is applied to the interference-tolerant transmission, channel feedback and limited-feedback MU-MIMO problems.

Publication [P. VI] shows that the feedback message structure of the D-JAC algorithm of [P. V] can be further exploited to allow some basic link adaptation mechanisms. An uncoded BEP criterion is applied to switch between the use of one or two parallel streams, while keeping the data rate constant. It is shown that the link adaptation information can be sent along with the feedback messages by alternately decreasing the quantization resolution associated to the D-JAC algorithm, without drastically affecting the performance and providing an overall performance gain.

Publication [P. VII] proposes a single-bit feedback-assisted transmit weight enhancements to quasi-orthogonal STBCs. In this work the signed stochastic gradient is applied to a link performance metric such as the BER, which enables to track an optimal beamformer under low mobility conditions. The algorithm is exemplified with the well-known ABBA QOSTBC, and it is shown to outperform existing single-bit proposals in low mobility scenarios.

Publication [P. VIII] addresses both single-user interference tolerant MIMO communications and channel feedback methods for unstructured matrices. In the first part, the feedback channel is used to adapt the transmission precoding in order to optimize the performance under interfering signals (e.g., inter-cell interference). An algorithm tailored for the single-stream case is introduced, which is based on the D-JAC algorithm of [P. V]. Thereafter, the more general case of weight adaptation for multiple-streams is considered, based on stochastic perturbations through unitary matrices and matrix exponentials. In the second part, channel feedback methods are proposed and analyzed. These partial-update methods can operate on unstructured matrices, thus avoiding approximations based on orthonormal matrices. Furthermore, they have reduced complexity, when compared to vector predictive quantization schemes. Simulation results show that the output of the channel feedback methods can be used with full-CSI MU-MIMO solutions, producing only a small BER per-

formance degradation.

Publication [P. IX] deals with limited-feedback MU-MIMO solutions. The performance degradation of full-CSI solutions employing quantized channel matrices is shown. Enhanced MU-MIMO performance is obtained by means of a decentralized solution which avoids the explicit feedback of the channel matrices. Simulation results analyze the performance of the proposed algorithm as a function of the speed of the mobile receivers.

The simulation software for all the contributions of this thesis was written solely by the author. The simulations were implemented in C++ and Matlab®. The C++ code was built on top of the Open Source software libraries IT++ [22], ATLAS [126] and GNU GSL [33]. The author was responsible for the development of the original ideas and writing all the papers. The role of the co-authors was to supervise and steer the work.

2. System model

This chapter describes the basic system model used throughout this thesis. It consists of a fixed transmitter equipped with N_t antennas and N_u mobile receivers, each with N_r antennas. The transmitter delivers signals simultaneously to the receivers, multiplexing them by means of linear precoding. The receivers employ linear combiners as means of demultiplexing and adaptation to the channel conditions. First, the notation and key assumptions are introduced. Thereafter, the feedback model and the basic receiver structures are presented. A brief discussion of link performance measures and transmission strategies is also given.

2.1 Introduction

Wireless channels have been traditionally considered "harsh" point-to-point communication media, requiring complicated design methodologies to enable reliable communications [95]. On the other hand, the random variations of the channels allow to use the transmission medium more effectively when multiple users can be scheduled according to their channel conditions, see for example [134]. In addition, the combination of random variability and a rich scattering environment facilitate the transmission of multiple parallel streams through multiple antenna transmission and reception [93].

The use of multiple antenna transceivers has been extensively covered in the literature. In particular, the use of feedback channels for multi-antenna transmission has produced a large body of literature (see for example the extensive review in [79]). It is by now well-known that the use of feedback channels to provide partial CSI at the transmitter can enhance the performance of single users employing linear receivers, and enable multiuser multiplexing. The algorithms presented in this thesis propose feedback methods providing partial CSIT for both purposes.

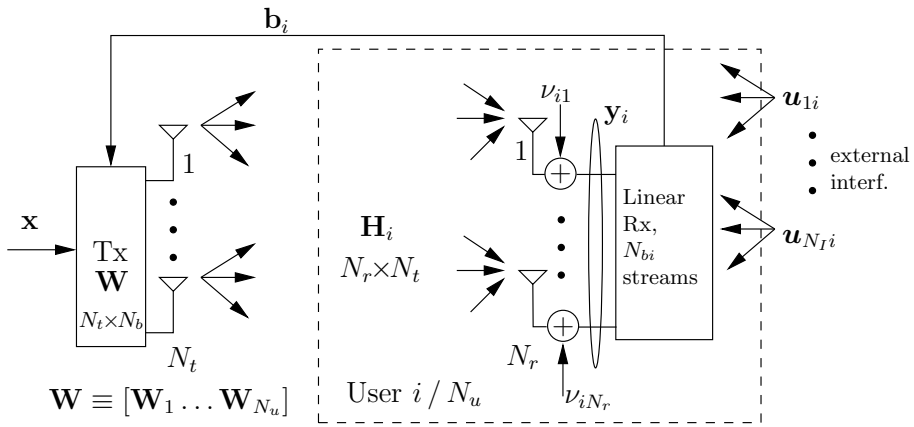

Figure 2.1. System model overview for closed-loop MU-MIMO. The transmitter launches a total of N_b parallel streams through its N_t antennas. Each user $i \in \{1, \ldots, N_u\}$ is equipped with N_r antennas, a linear receiver, subject to a flat-fading matrix $\mathbf{H}_i$, and possibly to external interfering signals.

We assume that the wireless channel has an approximately flat frequency response. This is the case when the transmitted signal bandwidth is smaller than the coherence bandwidth of the channel. This condition is satisfied, for example, for a subcarrier group in OFDM. This simplified model is also justified by the fact that the emphasis of this thesis is in the development and analysis of the feedback schemes, and not on channel modeling issues. Indeed, we assume that the receivers acquire perfect CSI from training sequences embedded within the data, e.g., "midambles in WCDMA" [5] and demodulation reference signals in LTE [2].

The system model overview is shown in Fig. 2.1, where one out of the N_u users of the MU-MIMO is depicted. The complete explanation for the symbols is given in Sections 2.2.2, 2.3 and 2.4.

2.2 Discrete time model

In this thesis, we consider a discrete time model representing a sampled version of the underlying continuous time signals and wireless channels. Linear modulations are considered with no intersymbol interference and ideal power amplifiers. This simplified system model allows the study to focus on the design of the feedback schemes themselves.

The following section briefly describes a basic baseband model for SISO links. Thereafter the discrete-time MIMO model used throughout this thesis is presented. A more detailed overview can be found in [17, 21, 69].

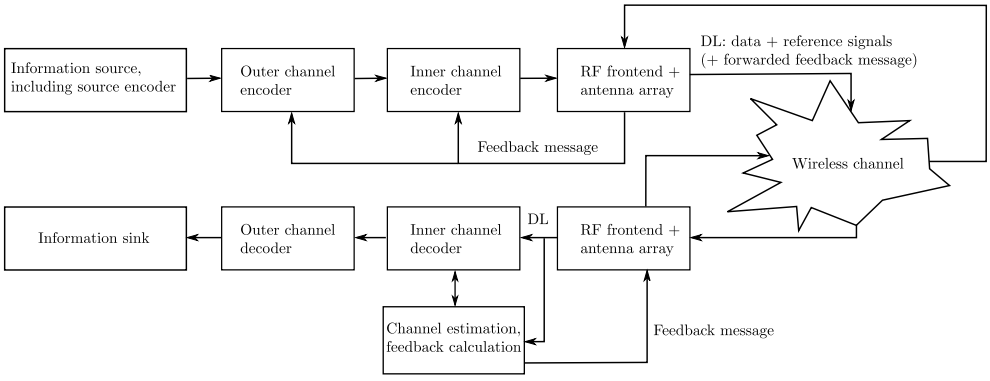

Figure 2.2. Information model and schematic view of a point-to-point communications system.

2.2.1 Basic baseband model

The traditional model for a digital communications system can be split in five blocks, as described in 1948 by Shannon [113]. It consists of an information source, a transmitter device, the communications channel including a noise source, a receiver device and the information destination. In many wireless systems, the transmitter device can be further split into a source encoder, an outer channel encoder, a symbol mapper, an inner channel encoder possibly including space-time modulation, the RF transmitter frontend and one or more transmission antennas.

The classical description of the system describes the physical signals involved as continuous-time functions, and the baseband signals as discrete-time series, where a sampling mechanism is employed. The following is a summary of each block. A high level overview is shown in Fig. 2.2.

The information source provides what is intended to be conveyed to the receiver. This can take many forms, such as continuous time signals representing, e.g., sounds or voice, video and audio signals, or blocks of bits representing parts of a computer file to be transferred. The information source can be modeled as an alphabet of messages, followed by a bit mapper, which produces a bit-stream from the information-bearing alphabet-elements. This bitstream is typically assumed to have independent and equally probable bits, with most of the information redundancy removed.

The outer channel encoder converts the source bit-stream to another bit-stream, possibly introducing redundancy for error detection and correction purposes. The symbol mapper takes this bit-stream and produces a stream of complex-valued symbols from a given symbol constellation. Alternatively, trellis-coded modulation (TCM) can map directly the source bit-stream to a symbol stream with added redundancy. TCM in SISO systems was proposed

by Ungerböck in [130, 131], and the extension to MIMO systems appeared later in Tarokh's space-time trellis codes (STTC) [123, 124].

The inner channel encoder adapts the transmission of one or more antennas, and may include space-time modulation, possibly employing information fed back by the receiver. The RF frontend for the transmitter applies a pulse shaping filter and translates the baseband signals in frequency, subsequently filtering the power-amplified signal to reduce interference to systems using adjacent frequency bands. The RF frontend can introduce perturbations to the information-bearing signal, specially timing jitter from the oscillators and amplitude distortions from non-linearities in the power amplifiers. The wireless channel is typically modeled as a finite impulse response filter, either time-varying or time-invarying. The receiver has an RF frontend, which filters the received signal and brings it back to baseband, possibly oversampling the signal and outputs a discrete time series. In case of TCM, sequence estimation is needed and typically the Viterbi algorithm is used. In case of split inner and outer encoders, the samples from the frontend are provided to the inner channel decoder. This can perform soft or hard detection, before feeding the outer decoder, which ultimately provides the information destination with an estimate of the transmitted bits.

2.2.2 MIMO model

At its simplest form, a MIMO system is a linear system where the complex-valued output vector at sampling instant k, $\mathbf{y}(k)$, can be written in terms an input vector $\mathbf{s}(k) \in \mathbb{C}^{N_t \times 1}$, D symbol-spaced channel matrices $\mathbf{H}(d,k) \in \mathbb{C}^{N_r \times N_t}$ and a noise vector $\mathbf{n}(k)$ as

$$\mathbf{y}(k) = \sum_{d=0}^{D-1} \mathbf{H}(d,k)\mathbf{s}(k-d) + \mathbf{n}(k) \tag{2.1}$$

In this thesis we are mostly concerned for the frequency-flat fading model, that is with $D = 1$, and will subsequently write $\mathbf{H}(0,k) \equiv \mathbf{H}(k)$. For a comprehensive summary of channel models, please consult [100, 108]. The transmitted vector $\mathbf{s}(k)$ will be restricted to be a linear function of an i.i.d. vector of complex-valued symbols from a fixed set. This so-called "linear precoding" is described in detail in Section 2.3 and covers both single/multiple-user MIMO with one or more data streams per user. Non-linear approaches based on Tomlinson-Harashima precoding [41, 128] are not considered.

2.3 Linear precoding

The discrete time index k represents the basic period of the system, referred to as the symbol period. During this time lapse, the transmitter can launch one or more information-bearing symbols to the wireless channel. These multiple symbols can be intended for one or more users, and the formation of the composite transmit signal by a matrix-vector multiplication is referred to as linear precoding, as opposed to non-linear strategies such as those applying Tomlinson-Harashima precoding [119].

Let the transmitter have N_t transmit antennas, and each user have N_r antennas and a feedback channel. The information bearing symbols belonging to all the users for the symbol period k are stacked in the vector $\mathbf{x}(k) = [\mathbf{x}_1^T(k) \dots \mathbf{x}_{N_u}^T(k)]^T \in \mathbb{C}^{N_b \times 1}$. This vector is multiplexed into a single signal by the use of the precoding or beamforming matrix $\mathbf{W}(k) \in \mathbb{C}^{N_t \times N_b}$, so that the transmitted signal during the symbol period k is $\mathbf{s}(k) = \mathbf{W}(k)\mathbf{x}(k)$. The precoding matrix $\mathbf{W}(k)$ may or may not be changed, depending on the CSIT assumptions. This is discussed in the feedback model description given in Section 2.4.

The transmission is arranged into slots, which comprise an integer number $L >> 1$ of symbol periods, indexed by $l \geq 0$. The system allows one feedback message per user.

Each user is allocated N_{bi} out of the N_b streams, with the corresponding beamforming weights and symbols represented by $\mathbf{W}_i(k) \in \mathbb{C}^{N_t \times N_{bi}}$ and $\mathbf{x}_i(k) \in \mathbb{C}^{N_{bi} \times 1}$, respectively. All the per-user precoding matrices are assembled into the overall beamforming matrix $\mathbf{W}(k) = [\mathbf{W}_1(k) \dots \mathbf{W}_{N_u}(k)] \in \mathbb{C}^{N_t \times N_b}$, where $N_b = \sum_i N_{bi}$. Each data stream corresponds to an entry of $\mathbf{x}(k)$ and we restrict our attention to uniform power allocation per stream, that is, $\mathrm{E}\left\{|x_m|^2\right\} = \mathcal{P}\, \forall m \in \{1, \dots, N_b\}$.

Let each user i have a frequency-flat fading matrix $\mathbf{H}_i$. The output of the antenna array of user i at symbol period k is

$$\begin{aligned} \mathbf{y}_i(k) &= \mathbf{H}_i(k)\mathbf{W}_i(l)\mathbf{x}_i(k) + \mathbf{n}_i(k) \\ \mathbf{n}_i(k) &= \mathbf{H}_i(k)\left(\sum_{m \neq i} \mathbf{W}_m(l)\mathbf{x}_m(k)\right) + \boldsymbol{\nu}_i(k) + \sum_{m=1}^{N_I} \boldsymbol{u}_{mi}(k) \\ k &= (l-1)L + s\,,\ s = 0, 1, \dots, L-1\,,\ l = 1, 2, \dots \end{aligned} \tag{2.2}$$

where $\mathbf{n}_i(k)$ includes Gaussian thermal noise $\boldsymbol{\nu}_i(k)$ with $\mathrm{E}\left\{\boldsymbol{\nu}_i\boldsymbol{\nu}_i^\dagger\right\} = \sigma^2\mathbf{I}$, N_I inter-cell interfering signals $\boldsymbol{u}_{mi}(k)$ received by user i and the intra-cell interference from the beams belonging to the other $N_u - 1$ users. We will subsequently use $\mathbf{W}(l)$ rather than $\mathbf{W}(k)$ to emphasize the slot-based update of $\mathbf{W}$, which

is common to all the contributions of this thesis. The index notation (k, l) will be used to remark the fact that some quantities have a component that is updated on a per-slot basis. Each receiver is assumed to know perfectly its own channel matrix on every symbol period k, i.e., we assume no error in $\mathbf{H}_i(k)$ at receiver i.

The conditional noise-plus-interference covariance matrices $\mathbf{Q}_i(k, l)$ are defined upon the channel matrices, the transmit weights and the SNRs as

$$\begin{aligned}\mathbf{Q}_i(k,l) &\equiv \mathrm{E}\left\{\mathbf{n}_i(k)\mathbf{n}_i^\dagger(k) \mid \mathbf{H}_1(k), \ldots, \mathbf{H}_{N_u}(k), \mathbf{W}_1(l), \ldots \mathbf{W}_{N_u}(l)\right\} \\ &= \mathcal{P}\sum_{m\neq i}\mathbf{H}_i(k)\mathbf{W}_m(l)\mathbf{W}_m^\dagger(l)\mathbf{H}_i^\dagger(k) + \sum_{q=1}^{N_I}\mathrm{E}\left\{\boldsymbol{u}_{qi}(k)\boldsymbol{u}_{qi}^\dagger(k)\right\} + \sigma_i^2\mathbf{I}\end{aligned} \tag{2.3}$$

2.4 Feedback model

As already mentioned, the system allows the transmission of one feedback message per user and slot. The feedback message of user i in slot l is denoted $\mathbf{b}_i(l) \in \{0, 1\}^{n_b}$, where n_b is the feedback capacity per channel use in bits. The feedback frequency is related to the symbol period through $f_{\mathbf{b}} = f_{\mathbf{x}}/L$, with L the number of transmitter channel uses per slot. Under full-CSIT assumptions, the transmitter could update $\mathbf{W}$ every symbol period k. However, as this thesis deals with limited-feedback systems, the update of $\mathbf{W}$ occurs only at the beginning of every slot. We assume that the receivers can compute the required feedback at the end of the slot, and neglect the feedback transmission and processing delay. Therefore, the transmitter can act upon the received feedback at the beginning of the next slot. These delays are application-dependent and have not been considered. However, the effect of outdated feedback is still included in the system, because the feedback frequency and capacity may be too low to maintain good CSIT at different fading rates (equivalently, mobile speeds).

The feedback messages can belong to recursive or non-recursive feedback strategies. In the former, the CSIT is updated from its value in the previous slot, given the incremental information contained in $\mathbf{b}_i$. In contrast, in non-recursive strategies the message $\mathbf{b}_i$ carries the complete CSIT of user i (assuming it is not corrupted by feedback errors). One can therefore expect that recursive methods will provide tracking advantages if the fading rate, feedback frequency and capacity are such that the non-recursive message $\mathbf{b}_i$ repeats itself for one or more slots. For a given feedback channel and transmis-

sion strategy, there exists a crossing point in terms of receiver speed, below which the recursive methods can offer significant advantages over the non-recursive counterparts. This will be studied for each of the contributions of this thesis.

2.4.1 Compensation of feedback errors

In order to cope with eventual feedback errors, the transmitter may forward the received feedback message to the receiver. This would be protected through strong channel coding in the downlink, so that chances of errors in the forwarded message should be low. The receiver can then determine when a feedback error has occurred, and take appropriate action. In the case of recursive methods such as the contribution of this thesis, the receiver can reproduce the wrong update made by the transmitter, and attempt to recover in the next update. This introduces perturbations in the convergence of the tracking algorithms, but allows them to operate under moderate amounts of feedback bit errors. Current commercial systems such as 3GPP LTE have provisions for this feedback verification. For example, the transmit precoding matrix indicator (TPMI) [1] which indicates which precoding codebook entry has been received by the transmitter.

2.5 Linear receivers

This thesis focuses on the particular class of linear receivers, which are of practical interest due to their limited computational complexity, compared to the non-linear counterparts. Transmission strategies devised for usage with non-linear receivers such as the successive interference cancellation scheme or decision-feedback equalizers are not considered here. Examples of such strategies are the uniform channel decomposition and the tunable channel decomposition. Please refer to [60] for details and additional information.

The linear receiver can be considered as a bank of combiners which process the output of the receive antenna array. Each user computes a matrix $\mathbf{\Omega}_i(k,l) \in \mathbb{C}^{N_{bi} \times N_r}$, which is used to determine the signals employed in the detection process

$$\begin{aligned}
\mathbf{z}_i(k,l) &= \mathbf{\Omega}_i^{\dagger}(k,l)\mathbf{y}_i(k) \\
&= \mathbf{G}_i(k,l)\mathbf{x}_i(k) + \mathbf{n}_i'(k,l) \\
\mathbf{G}_i(k,l) &= \mathbf{\Omega}_i^{\dagger}(k,l)\mathbf{H}_i(k)\mathbf{W}_i(l) \\
\mathbf{n}_i'(k,l) &= \mathbf{\Omega}_i^{\dagger}(k,l)\mathbf{n}_i(k)
\end{aligned} \tag{2.4}$$

where user i receives N_{b_i} independent data streams, $\mathbf{G}_i(k,l)$ and $\mathbf{n}'_i(k,l)$ define an equivalent linear system.

Examples of linear receivers are the minimum mean square error (MMSE), zero forcing (ZF) and the MIMO interference rejection combining (IRC) receiver, which in the single user case are given by

$$\mathbf{\Omega}^{\dagger}_{\mathrm{MMSE}}(k,l) = \left[[\mathbf{H}(k)\mathbf{W}(l)]^{\dagger}[\mathbf{H}(k)\mathbf{W}(l)] + \sigma^2\mathbf{I}\right]^{-1}[\mathbf{H}(k)\mathbf{W}(l)]^{\dagger} \quad (2.5)$$

$$\mathbf{\Omega}^{\dagger}_{\mathrm{ZF}}(k,l) = \left[[\mathbf{H}(k)\mathbf{W}(l)]^{\dagger}[\mathbf{H}(k)\mathbf{W}(l)]\right]^{-1}[\mathbf{H}(k)\mathbf{W}(l)]^{\dagger} \quad (2.6)$$

$$\mathbf{\Omega}^{\dagger}_{\mathrm{IRC}}(k,l) = [\mathbf{H}(k)\mathbf{W}(l)]^{\dagger}\mathbf{Q}^{-1}(k,l) \quad (2.7)$$

where $\mathbf{Q}(k,l)$ is the covariance matrix of the total noise-plus-interference, as defined in (2.3).

Throughout this thesis, the double index (k,l) is used in quantities that have a component which is updated on a slot basis. For example, the combiner matrix $\mathbf{\Omega}(k,l)$ depends explicitly on the beamforming weights $\mathbf{W}(l)$. In case of the disturbance signal $\mathbf{n}(k)$, we omit the l index because the dependency is not always present (e.g., in case of SU-MIMO). In contrast, we use the double index for the covariance matrix $\mathbf{Q}(k,l)$ because this quantity is only of interest when the slot-dependency is explicit, namely in SU-MIMO with external interference (c.f. Section 3.4) and in MU-MIMO (c.f. Section 5.2). Full CSIR is assumed, and therefore each channel matrix $\mathbf{H}(k)$ is known without error by the associated receiver on each symbol period k. We drop the indices for readability when refering to variables assumed static and when there is no possibility of confusion.

2.6 Performance measures

Given a fixed channel, transmit and receive matrices, the system can be considered as an additive noise system or noisy system, possibly with cross-talk between users/streams. In [125], Telatar computed the capacity of an SU-MIMO system, for a fixed channel $\mathbf{H}$ and i.i.d. circularly symmetric Gaussian noise. The results involve diagonalizing the MIMO channel (eigenbeamforming) and waterfilling to set the number of streams and adjust the power on each stream. The capacity for this channel under a total power constraint is

$$C = \sum_i [\log_2(\mu\lambda_i)]^+ \quad (2.8)$$

where $(\cdot)^+ = \max(\cdot, 0)$, μ is computed to satisfy the transmit power constraint and λ_i are the squared singular values of $\mathbf{H}$.

Therefore, capacity-wise ideal feedback requires both the beamforming matrix that diagonalizes the channel (the right singular vectors or "eigenbeams"), the power gains for each subchannel (the "eigengains") and the signal to noise ratio.

If the transmitter has no CSI, the optimal power allocation is arguably uniform over all transmit antennas, in which case the mutual information (MI) under Gaussian signalling becomes [91]

$$I_{\text{unif}} = \log_2 \left| \mathbf{I}_{N_r} + \frac{1}{N_t} \frac{P_{tx}}{\sigma^2} \mathbf{H}\mathbf{H}^\dagger \right| \tag{2.9}$$

where P_{tx} is the total transmit power.

When the channel matrix is random, two well-known measures are the ergodic capacity and the capacity with outage. The ergodic capacity is equal to the expected value of the capacity over the channel distribution. The capacity with outage, on the other hand, is the probability that the capacity will be less than a given data rate.

The ergodic capacity without CSIT for i.i.d. Gaussian channels was computed in [125] as the expectation $\mathrm{E}\{MI_{\text{unif}}\}$. This is accomplished by integrating over the joint distribution of the eigenvalues of the Wishart matrix $\mathbf{H}^\dagger\mathbf{H}$, which can be found, e.g., in [28].

For the full-CSIT case, on the other hand, the ergodic capacity is more difficult to compute due to the waterfilling step. A simplified case, however, consists of only diagonalizing the channel, and keeping the number of streams and power allocation constant. In this case, the bit loading for each stream can be optimized given the statistics of the squared singular values. For example, for $N_t = 4$, $N_r = 2$ the probability density function (PDF) of the ordered eigenvalues λ_1, λ_2 can be computed from the joint density given in [28]. These densities are illustrated in Fig. 2.3 and given by

$$\begin{aligned} f_{\lambda_1}(x) =& \frac{1}{6} e^{-2x} x^2 [e^x (x(x-6)+12) - 12 - x(x+6)] \\ f_{\lambda_2}(x) =& \frac{1}{6} e^{-2x} x^2 [12 + x(6+x)] \end{aligned} \tag{2.10}$$

The corresponding BER performance can be computed by integrating the conditional BER expression of each symbol constellation over the corresponding PDF from (2.10). Useful approximations for conditional BER in the case of M-QAM constellations can be found in [61]. Moreover, precise approximations to the Gaussian Q function can be found in [68]. The resulting BER curves are illustrated in Fig. 2.4 for different data rates and bit allocations. Alternatively, one can use the PDFs of (2.10) to obtain the capacity of each parallel channel, which is the upper limit for reliable transmission [113].

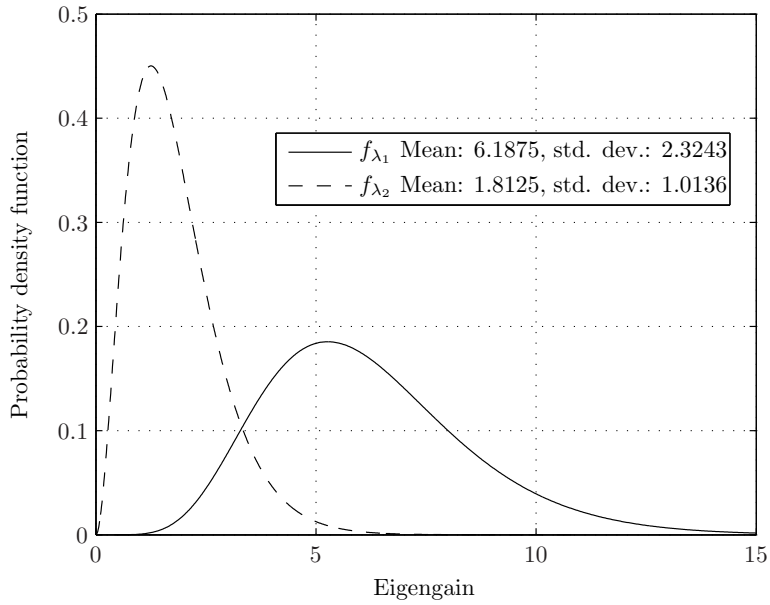

Figure 2.3. Distribution of parallel MIMO channel gains with perfect CSIT in the case of uncorrelated $N_t = 4, N_r = 2$ Gaussian channels. λ_1, λ_2 are largest and smallest eigenvalues of $\mathbf{H}$, with PDF according to (2.10).

For the particular case of $N_t = 4$, $N_r = 2$, it is seen that transmitting eight bits per channel use performs about four dB worse (at uncoded BER level of 1%) than using six bits per channel use, and even some crude link adaptation strategies can provide a performance enhancement. For example, if the bit rate is to be kept constant, one bit feedback can be used to signal to the transmitter whether all the bits should be loaded into the first stream, or whether both streams should be used according to a fixed allocation.

More generally, the mutual information given the channel matrix $\mathbf{H}$ and the interference-plus-noise covariance matrix $\mathbf{Q}$ can be written in terms of the transmit weights. Assuming even and fixed power allocation to all the streams, the mutual information is

$$I_{unif}(\mathbf{W}|\mathbf{H}) = \log_2 \left(\left| \mathbf{Q} + \mathcal{P}\mathbf{H}\mathbf{W}\mathbf{W}^{\dagger}\mathbf{H}^{\dagger} \right| \left| \mathbf{Q}^{-1} \right| \right) \tag{2.11}$$

2.7 Simulation assumptions

Most of the simulation software was written in the C++ language [122]. Three main simulation programs were used: single user transmission with closed-loop assisted space-time codes, single user closed-loop eigenbeamforming and feedback-assisted interference cancellation, and multiple-user transmission. The programs utilize polymorphism to facilitate reusing the same simulation chain with different algorithms. For example, an interface for channel feedback is defined, which provides functions to execute the chan-

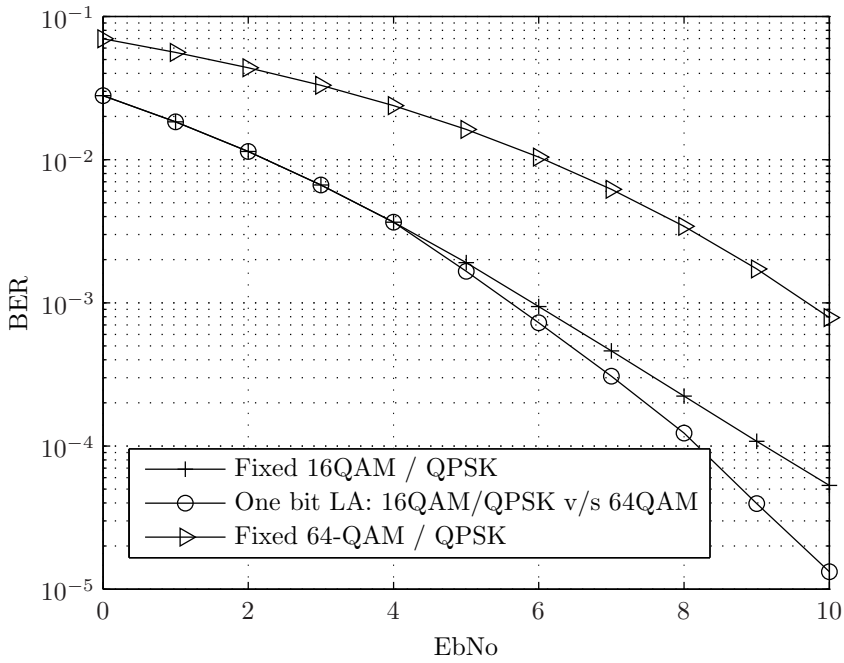

Figure 2.4. BER performance for double stream SU-MIMO for the $N_t = 4, N_r = 2$ eigengain densities from (2.10). The benefit of a simple link adaptation (LA) scheme is shown for the case of six bits per channel use: the transmitter switches between a single stream with 64QAM symbols and two streams with 16QAM and QPSK symbols, respectively. There is no benefit from such strategy for the case of eight bits per channel use.

nel feedback update given the current channel matrix, the number of bits per feedback channel use and possibly the previous channel feedback state and message. Another function is provided to return the current value of the channel matrix, as tracked by the transmitter. This interface is implemented by different derived classes, such as the sequential and ranked partial update feedback channel contributions described in Chapter 4.

Basic linear algebra data types like complex-valued matrices and vectors are provided by the IT++ library [22]. This library provides efficient vector matrix multiplication and matrix decompositions (SVD, EVD) through the linear algebra subsystems of ATLAS [126] and LAPACK [89]. Additionally, bit to symbol mappers for PSK and QAM modulations from IT++ are also used. Finally, the random number generators and histogram facilities are provided by the GSL [33] library.

The C++ code is compiled in Linux with the GNU "g++" compiler, part of the GNU compiler collection [32]. All the calculations are carried on using floating point arithmetic with type "double" for each complex number part. The number of bits allocated to a floating point number of type double depends on the particular compiler [122]. This, in turn, depends on the application binary interface (ABI) that the compiler adheres to. In the case of g++, this is a vendor-neutral specification intended to allow different compilers to operate

consistently on the same platform (see, e.g., [32] under "Binary Compatibility" and "Architecture"). Since simulations have been performed on Linux platforms operating in both 32 and 64 bits, the size of the fundamental types should be the same. The C++ function "sizeof()" allows to check the number of bytes used to store a given type [122]. It has thus been verified that in both 32 and 64-bit platforms, a "double" floating point type is stored using 8 bytes (64 bits), and therefore the numerical accuracy is the same.

2.7.1 Channel modeling

Throughout this thesis, a Rayleigh channel model is assumed. When correlation is considered, the correlation matrices defined by the I-METRA project [112] are used. Temporal correlation is induced by filtering temporally-white circularly Gaussian complex-valued variables with a low-pass filter matched to the maximum Doppler frequency. An IIR digital Butterworth filter is used, in order to provide a sharp cut-off with low computational complexity. This is somewhat simpler to implement, compared to sampling the classical Clarke's spectrum and using the inverse FFT, or using Jakes model [58].

The Butterworth filter is designed using the bilinear transform technique (see, e.g., [96]). The transformation is adjusted so that digital and analog cutoff frequencies correspond through the mapping. Figure 2.5 shows the autocorrelation function computed from channel samples saved from the C++ simulation chain. This is compared to the autocorrelation of Clarke's "bowl" spectrum, which is [58].

$$J_0\left(2\pi\frac{v}{\lambda}f_{\mathbf{b}}n\right)$$

where λ is the carrier wavelength in meters, v is the speed in meters/second, $nf_{\mathbf{b}}$ is the time duration of n slots and $J_0(\cdot)$ is the Bessel function of the first kind, order zero [8].

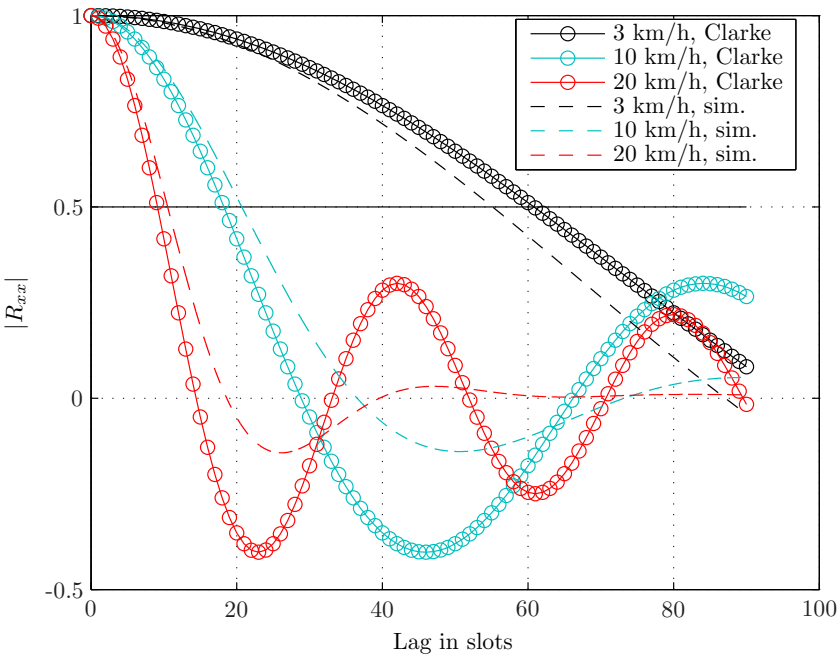

Figure 2.5. Autocorrelation function for SISO channels with speed simulated through a digital Butterworth filter of order 4, for speeds of 3, 10, 20 km/h with carrier frequency of 2.1 GHz and sampling frequency of 240 kHz. "sim" refers to the samples as saved from the C++ simulation chain, which is used for most simulations in this thesis. "Clarke" refers to the autocorrelation of Clarke's model [58].

3. Limited-feedback closed-loop techniques in MIMO systems

This chapter studies closed-loop MIMO techniques for the downlink of single-user systems, wherein only one user receives data over a given slot. Firstly, an overview of existing methods is presented. Thereafter, the contributions of these thesis are introduced. Generally speaking, the proposed methods are designed to approximate the full-CSI BER performance in slowly-fading channels, and to minimize the performance loss at moderate fading rates, while keeping a low computational complexity. The contributions presented in this chapter can be grouped as a) feedback mechanisms for closed-loop eigenbeamforming (CSIT consists of one or more of the right singular vectors of the channel matrix). This is the topic of Sections 3.3.1 to 3.3.4, b) feedback mechanisms for transmitter-assisted interference rejection. This is covered in Section 3.4, and c) single-bit closed-loop enhancements for quasi-orthogonal space-time codes, as described in Section 3.5.

The performance of the proposed algorithms is benchmarked against the associated full-CSIT performance, e.g., uncoded BER under full-CSIT assumptions, and existing feedback methods operating at the same feedback rate. The performance degradation for a fixed feedback rate and increasing mobile speeds is also assessed.

3.1 Introduction

Single-user closed-loop MIMO techniques are of interest because exploiting CSI with linear receivers provides significant gains for single-stream transmissions, and enables multiple-stream communications with low computational complexity requirements.

Consider the transmission of N_b streams and the use of an N_r-antenna linear receiver $\boldsymbol{\Omega}(k,l) = [\boldsymbol{\omega}_1(k,l) \ \ldots \ \boldsymbol{\omega}_{N_b}(k,l)] \in \mathbb{C}^{N_r \times N_b}$. During slot l, the filtered

received signal is

$$\begin{aligned}\mathbf{z}(k,l) &= \mathbf{\Omega}^{\dagger}(k,l)\mathbf{H}(k)\mathbf{W}(l)\mathbf{x}(k) + \mathbf{\Omega}^{\dagger}(k,l)\mathbf{n}(k) \quad k = (l-1)L, \ldots, lL-1 \\ &= \mathbf{G}(k,l)\mathbf{x}(k) + \mathbf{n}'(k,l)\end{aligned} \tag{3.1}$$

where $\mathbf{G}(k,l) = \mathbf{\Omega}^{\dagger}(k,l)\mathbf{H}(k)\mathbf{W}(l)$ and $\mathbf{n}'(k,l) = \mathbf{\Omega}^{\dagger}(k,l)\mathbf{n}(k)$ define the linear system in $\mathbf{x}(k)$ after the linear combiner.

For single data stream communications ($N_b = 1, \mathbf{W}(l) = \mathbf{w}(l)$), the matched filter combiner $\boldsymbol{\omega}_{mf}(k,l) = \mathbf{H}(k)\mathbf{w}(l)$ produces an equivalent system

$$z_{mf}(k,l) = \mathbf{w}^{\dagger}(l)[\mathbf{H}^{\dagger}(k)\mathbf{H}(k)]\mathbf{w}(l)x(k) + \mathbf{w}^{\dagger}(l)\mathbf{H}^{\dagger}(k)\mathbf{n}(k)$$

If the noise is spatially white and the channel matrix $\mathbf{H}(k)$ remains constant within the slot, then the maximal SNR gain with respect to $\mathbf{w}(l)$ is given by the dominant eigenvalue of $\mathbf{H}^{\dagger}(k)\mathbf{H}(k)$, and the optimal weight vector $\mathbf{w}_{opt}(l)$ is the associated eigenvector, which is the dominant right singular vector of $\mathbf{H}(k)$ [37]. This solution maximizes the mutual information of the single stream system by maximizing the SNR at the receiver.

On the other hand, for $N_b > 1$ and assuming spatially white Gaussian noise, the capacity-optimal solution is to use the N_b dominant singular vectors of $\mathbf{H}(k)$ and distribute the power among the streams according to a water-filling procedure, which determines whether or not the channel can support the transmission of N_b streams [125].

Let the singular-value decomposition (SVD) [38] of $\mathbf{H}(k)$ be

$$\mathbf{H}(k) = \mathbf{U}(k)\mathbf{D}(k)\mathbf{V}^{\dagger}(k) \tag{3.2}$$

where $\mathbf{U}(k) \in \mathbb{C}^{N_r \times N_r}$, $\mathbf{V}(k) \in \mathbb{C}^{N_t \times N_t}$ are unitary and called the left and right singular matrices of $\mathbf{H}(k)$, respectively, and $\mathbf{D}(k) \in \mathbb{R}^{N_r \times N_t}$ is a diagonal matrix containing the singular values of $\mathbf{H}(k)$. If $\mathbf{H}(k)$ is full-rank, then it will have $\min(N_r, N_t)$ non-zero singular values.

The channel can be diagonalized with an MF receiver if the beamforming matrix is $\mathbf{W}(l) = [\boldsymbol{v}_1(l) \ldots \boldsymbol{v}_{N_b}(l)]$ [125] (we can absorb the power adjustment coefficients into the symbol vector $\mathbf{x}$). Assuming that the channel does not change abruptly from slot to slot, the closed-loop algorithms are concerned with tracking the last channel sample of the slot, that is, the optimal beamforming matrix with perfect (unquantized) feedback and the associated

matched filter are

$$\begin{aligned}\mathbf{W}_{opt}(l+1) &= \mathbf{V}(lL-1)\mathbf{I}_{N_t,N_b} = [\boldsymbol{v}_1(lL-1)\ \ldots\ \boldsymbol{v}_{N_b}(lL-1)] \\ \boldsymbol{\Omega}_{mf,opt}(k,l) &= \mathbf{H}(k)\mathbf{W}_{opt}(l) \\ &= \mathbf{U}(k)\mathbf{D}(k)\mathbf{V}^{\dagger}(k)\mathbf{V}(lL-1)\mathbf{I}_{N_t,N_b} \\ &\approx \mathbf{U}(k)\mathbf{D}(k)\mathbf{I}_{N_t,N_b}\end{aligned} \tag{3.3}$$

where $\mathbf{I}_{N_t,N_b}$ is a size-N_t identity matrix truncated to N_b columns. The approximation in (3.3) is due to possible channel variations within the slot, namely $\mathbf{H}(k)-\mathbf{H}(lL-1) \neq \mathbf{0}, k = lL, \ldots, (l+1)L-1$. The approximation is in general valid for the fading and feedback rates considered throughout this thesis.

Assuming that the feedback processing delays and the channel fluctuations within the transmission slot are small, the received signal for the matched filter receiver with unquantized feedback has negligible interference between the components of $\mathbf{x}(k)$:

$$\begin{aligned}\mathbf{z}_{mf,opt}(k,l) &= \mathbf{G}(k,l)\mathbf{x}(k) + \mathbf{n}'(k,l) \\ &\approx \begin{bmatrix} D_1^2(k) & & \\ & \ddots & \\ & & D_{N_b}^2(k)\end{bmatrix}\mathbf{x}(k) + \mathbf{n}'(k,l)\end{aligned} \tag{3.4}$$

where $D_1(k) \geq \ldots \geq D_{N_b}(k)$ are the N_b dominant singular values of $\mathbf{H}(k)$. Since $\mathbf{U}(k)$ is unitary, it is clear from (3.4) that the power gains for the components of $\mathbf{x}(k)$ after the linear combiner are approximately $D_1^2(k), , \ldots, D_{N_b}^2(k)$.

More generally, the performance of the system is determined by the signal-to-interference-plus-noise ratios (SINRs) for each stream. Conditioned on the channel matrices, the beamforming weights and given the noise-plus-interference statistics, the SINRs $\gamma_i(k,l)$ are:

$$\gamma_i(k,l) = \frac{G_{ii}(k,l)^2}{\sum_{m\neq i}|G_{im}(k,l)|^2 + \boldsymbol{\omega}_i^{\dagger}(k,l)\mathrm{E}\left\{\mathbf{n}(k)\mathbf{n}^{\dagger}(k)\right\}\boldsymbol{\omega}_i(k,l)} \tag{3.5}$$

where a unit average transmit power per stream is assumed.

Equation (3.5) defines the impact of the transmission weights on the SINRs for the current channel and interference statistics, given a linear receiver structure. It therefore enables optimization techniques to be applied to adjust the beamforming matrix $\mathbf{W}(l)$ for a scalar function of the SINRs $\gamma_i(k)$, such as capacity or uncoded bit error rate. More specifically, transmit weight optimization schemes based on uncoded BER are proposed in Section 3.4, where the adaptation of $\mathbf{W}(l)$ at the transmitter is based on feedback messages, and the algorithm enhances the link-level reliability in the presence of inter-cell interference.

3.2 Overview of existing methods

Feedback mechanisms for the right singular vectors of the channel matrix have been extensively studied in literature. They concern the use of the feedback message $\mathbf{b}(l) \in \{0,1\}^{n_b}$ for the transmission of the N_b dominant singular vectors of the last channel sample of slot l, that is, the orthonormal matrix $[\boldsymbol{v}_1(lL-1) \ldots \boldsymbol{v}_{N_b}(lL-1)]$ where N_b is the number of streams to be transmitted.

Recursive methods are concerned with feeding back some update information, which enables the transmitter to compute $\mathbf{W}(l+1)$ from $\mathbf{W}(l)$. In contrast, non-recursive methods assemble $\mathbf{W}(l+1)$ at the transmitter based only on $\mathbf{b}(l)$ and disregarding $\mathbf{W}(l)$. Therefore, a well designed recursive method can exploit the channel temporal correlation, and achieve better accuracy compared to non-recursive counterparts, assuming the same feedback rate. This is possible in scenarios where more than one feedback message can be sent before the channel decorrelates temporally. This, in turn, depends on the receiver speed, the frequency band used for transmission and indirectly on the antenna array sizes, the feedback rate and the antenna correlation properties.

The limited-rate characteristics of the feedback channel typically preclude the use of independent scalar quantization. For example, feeding back the dominant eigenbeam amounts to transmit a unit-norm vector of size N_t, where the first component is real-valued [49] and therefore involves $2(N_t-1)$ real-valued parameters. Using scalar quantization and three bits per real-valued parameter would already exceed values of n_b currently considered practical. For example, the two WCDMA closed loop modes [5] allow only one bit per feedback channel use, and 3GPP LTE wideband precoding [1] allows up to four bits to signal the chosen codebook index. Research articles consider values between four and ten. Only early works dealing with channel feedback considered independent scalar quantization. For example, in [34] channel feedback is proposed to enable a zero-forcing MU-MISO, where the feedback overhead amounted to 4.2 bits per real-valued component of each channel coefficient.

A typical assumption is that the CSIR is perfect. However, this is only a simplification used to isolate the effects of the feedback quality and transmission strategies. The impact of the imperfect CSIR based on training is treated for MISO systems in [19].

3.2.1 Covariance matrix of the transmitted data

From a capacity point of view, the optimal number of streams and the allocated power to each stream are determined from the waterfilling procedure, when the CSIT is perfect. In the limited-feedback case, one possibility is to let the receiver compute the optimal transmit covariance matrix, and feed it back to the transmitter. In [72], generic feedback symbols from an alphabet obeying the feedback capacity restriction (bits per channel use) are considered. The problem is stated as optimizing the rule for computing the data covariance matrix given the CSIT symbol, and optimizing the rule for choosing the CSIT symbol given the channel matrix. The proposed solution is a codebook containing pairs consisting of one beamforming matrix and one power control matrix determining the power poured into each eigenmode, or equivalently, it is a codebook of transmit covariance matrices. The codebook is constructed based on training data, where an iterative procedure clusters the data into quantization cells and finally sets the codewords to be the cell centroids. The centroids are computed to optimize an approximation of a distortion measure. This aims to maximize the mutual information under transmit power constraints, and results in setting the centroids to the dominant eigenvector of the sample covariance matrix of each cell. These codebooks depend on the SNR, and the study does not reveal whether the codebooks favor beamforming at low SNR, instead of multiple-stream transmission.

From an Information Theory perspective, the problem of finding the optimal transmission strategy given partial side information can be addressed. For example, Visotsky [133] derives optimal transmission strategies for the cases of 1) the channel has non-zero mean, but is spatially uncorrelated and the channel average has been fed back ("mean feedback") and 2) the channel is assumed to have zero mean, but it is not spatially white, and the covariance matrix has been fed back ("covariance feedback"). In both cases the transmitter must decide between using beamforming (single stream), or space-time coding diversity techniques. For the mean feedback case, the suggested strategy is to use beamforming when the feedback reliability is above a given threshold, and diversity techniques otherwise. In the case of covariance feedback, which is deemed more reliable because it can be formed through long term averaging, parallel streams should be employed, including water-pouring to adjust the power allocation and the number of streams. The mean feedback has also been addressed by Narula [88].

Asymptotic results for a fixed N_t/N_r ratio are given in [110], where random

vector quantization (RVQ) techniques are considered. In RVQ, the codebook is changed randomly for every feedback channel use, and both the transmitter and receiver need to use the same codebook (e.g., a common random seed). The paper considers the optimization of the covariance matrix rank, depending on the number of feedback bits employed. The coarser the feedback, the lower the rank. Furthermore, the power is allocated evenly among the chosen number of streams, and it is argued that once the number of streams is optimized, the power allocation is not critically detrimental. The use of RVQ for transmit covariance quantization has been further studied in [26], where a bound for the capacity loss associated with the limited feedback has been derived, showing an exponential decrease on the codebook size, as well and a dependency on N_t and N_b. The random codebooks of covariance matrices are generated randomly based on the outer product of reshaped random vectors, which produce covariance matrices with randomized rank.

In [154], transmission adaptation in terms of bit and power loading over the data streams is designed, in order to compensate for the effects of outdated CSIT. Practical issues arise relating to the calculation of the correlation coefficient between CSIR and CSIT, which depends on the fading rate and the fading capacity.

3.2.2 Vector quantization techniques

The non-recursive transmission of the first N_b columns of $\mathbf{V}$ can be treated as a vector/matrix quantization problem. The main idea is that both transmitter and receiver know a fixed set of matrices (a "codebook"), from which a matrix is selected by the receiver, who sends the chosen index to the transmitter. When the matrices to be quantized are assumed to have a fixed distribution, several techniques can be employed to design a fixed codebook. A brief overview is given next.

Vector quantization (VQ) techniques are well-known from developments in audio coding theory, where blocks of audio samples would be quantized prior to transmission. For example, linear predictive coding (LPC) techniques parametrize a block of speech samples, and the resulting parameters can be quantized and transmitted through a communications channel. The parametrization is done through the use of linear prediction (LP) strategies, where a model of the signal is built in terms of a digital filter. An overview of LPC systems is given in [81], including a discussion of different parametrizations for the sample blocks, e.g., filter impulse response, autocorrelation coefficients, zeroes and poles of the predictor, etc. A taxonomy of early data

compression techniques including optimum linear prediction can be found in [11]. The use of digital filter structures in time series analysis appears as early as 1927 in the work by Yule [143], 1941 (Kolmogorov) [70] and 1964 (Wiener) [137].

A prominent algorithm for training vector (equivalently block) quantizers is the Linde-Buzo-Gray (LBG) algorithm [74]. The LBG generalizes the scalar quantizers by Lloyd, which date from 1957 in Bell Labs but were not published until 1982 in [77]. The LBG method allows training vector quantizers for signals with fixed distribution, based on a training sample set. The original paper from 1980 shows an application to joint quantization of LPC speech parameters, which were until then typically coded independently using optimum scalar quantizers. The joint quantization from LBG allowed to lower the number of transmitted bits while maintaining similar levels of some distortion measures. This implies that the non-recursive feedback methods should not quantize vector or matrix elements independently. However, the situation is different for recursive methods, where partial updates can be allowed to lower the computational complexity, as described in Section 3.3.3 and Chapter 4.

Static codebooks

More formally, let us define a codebook $\mathcal{C}$ of orthonormal matrices as a set

$$\mathcal{C} \triangleq \{\mathbf{C}_m \in \mathbb{C}^{N_t \times N_b}, \mathbf{C}_m^\dagger \mathbf{C}_m = \mathbf{I}_{N_b}\}_{m=1}^{2^{n_b}}$$

The receiver employs a selection criterion to choose a codebook element that should be used by the transmitter, and reports the index of selected matrix through the feedback channel. The criterion is application-dependent. For example, mutual information can be used to optimize the performance of coded communications and the minimum singular value can be used to optimize the error rate of the uncoded communication [78]. The latter assumes the form

$$m^* = \underset{m}{\operatorname{argmax}}\ \lambda_{\min}(\mathbf{H}\mathbf{C}_m)$$

where $\lambda_{\min}(\cdot)$ represents the minimum singular value of the matrix argument.

The codebook $\mathcal{C}$ is typically designed off-line, depending on the antenna array sizes and the statistics of the channel matrix $\mathbf{H}$. The computational complexity of choosing a matrix element is $\mathcal{O}(2^{n_b})$, which reflects a trade-off between the codebook accuracy and a combined cost of feedback overhead and computational operations. It is possible to reduce this complexity by structuring the data into a tree-structure, at the cost of increased storage requirements and decreased performance. If the codebook size can be expressed in terms of integers y and u such that $2^{n_b} = y^u$, then the data can be arranged into

a $y-$ary tree with u stages, which incurs in a search complexity proportional to yu rather than y^u [35]. It was shown in [104] that for $N_t = 4$, a six-bit codebook using full search achieves similar capacity than a seven-bit codebook using the binary tree with seven stages. The reduction in the computational complexity order is, however, significant, since $2 \cdot 7/2^6 \approx 22\%$.

An analysis of the capacity loss due to quantizing the optimal beamforming vector in MISO systems is given in [103], based on the works presented in [101, 104]. In [101], the LBG algorithm is employed to maximize the mean squared inner product (MSIP), similar to Narula's work [88] but noting that in correlated channels, the norm of the channel vector is not independent of the normalized channel vector. Therefore, quantizing the normalized channel vector can perform differently than quantizing the channel vector without normalization, in spatially correlated channels.

Codebooks based on subspace packing

It has been shown that for channel matrices with i.i.d. Gaussian components, the right singular matrix is isotropically distributed in the group of unitary matrices [78]. This means that the codebook design should maximize a distance measure between its elements, and therefore the problem is related to the Grassmannian subspace packing problem [16]. This allows codebook design methodologies more specialized than the LBG algorithm. For example, explicit constructions are given in [121] for small codebooks satisfying $2^{n_b} \leq N_t^2$. This has been used in the single-stream Grassmannian beamforming design [80] to produce two-bit codebooks for $N_t = 3$. For larger codebook sizes, the method by Hochwald [45] can be used, which requires the optimization of N_t real-valued parameters. This procedure provides flexibility to design codebooks for different configurations, i.e., there is no restriction on the antenna size or N_b. When the DFT matrices are used as starting point, the vector codebooks generated in [45] have unit-norm entries, i.e., they implement equal gain transmission (EGT) codebooks and generalize the work in [44]. Furthermore, the reduced power variation arising from the lack of per-antenna power control can alleviate the requirements on the transmitter power amplifier.

LBG-based codebooks

The mean squared inner product (MSIP) criterion of [101] designs the quantizer function $\mathcal{Q}(\cdot)$ to maximize $\mathrm{E}\left\{| < \boldsymbol{v}, \mathcal{Q}(\boldsymbol{v}) > |^2\right\}$, where the expectation is over the channel distribution. This contrasts with the Grassmannian beamforming design criterion [80], which minimizes the maximum inner product between codebook elements. The simulations give similar performance for

both approaches, and Roh argues that the VQ method can be applied to a broader range of channel distributions and antenna array sizes, compared to the method in [80].

In [104], on the other hand, it is argued that using the LBG algorithm to design a codebook that minimizes the quantization loss is difficult. This is due to the fact that the optimal centroid of each cell cannot be solved analytically, unlike the case where an inner product distortion measure is used. Thus, it is claimed that the monotonicity of the algorithm cannot be asserted. Instead, an alternative method is proposed, where the weighted sample covariance matrix of the cells are computed, and the cell centroids are set to the dominant eigenvector of the respective covariance matrix. This is the mean squared weighted inner product (MSwIP) design criterion and it is based on a large codebook approximation for the capacity loss of the quantized beamforming system and an iterative solution for computing the cluster centroids. This design is now SNR-dependent and reduces to the MSIP at high SNR. Furthermore, it is shown that for i.i.d. Rayleigh fading the MSIP, MSwIP and minimizing the inner product on unnormalized channels are equivalent, because the singular values and singular vectors of the channel are statistically independent. The difference comes, however, in correlated channels, where this is not true. The MSwIP is shown to uniformly outperform both MSIP and VQ codebooks of un-normalized channels in correlated channels.

Sequential vector quantization (SVQ)

On the other hand, a sequential vector quantization (SVQ) scheme is also proposed in [101]. The main idea is to represent the eigenbeams in terms of unit-norm vectors of decreasing size, e.g., $N_t, N_t-1, \ldots, N_t-N_b+1$. For i.i.d. Gaussian fading, these vectors are independent and uniformly distributed over the associated hyper-sphere, thus allowing to quantize them with a codebook for MISO systems. This means that the receiver needs only to compute the N_b unit vectors, quantize them independently with codebooks of the proper dimension and feed back the resulting indices. The receiver can then rebuild an approximation of the original orthonormal matrix.

Let $\mathbf{V} = [\boldsymbol{v}_1 \ldots \boldsymbol{v}_{N_b}]$ be the N_b dominant right singular vectors of the channel matrix $\mathbf{H}$ (the channel eigenbeams), and $\mathbf{q}_i \in \mathbb{C}^{[N_t-i+1]\times 1}, i = 1, \ldots, N_b$ be the N_b unit-norm vectors to be computed by the receiver from $\mathbf{V}$. The calculation is done as follows. The first vector $\mathbf{q}_1$ equals $\boldsymbol{v}_1$ by definition. An auxiliary matrix $\mathbf{G}_1$ is built such that that the first column is $\mathbf{q}_1$ and the rest are arbitrary unit-norm vectors that are orthogonal to $\mathbf{q}_1$. By construction, the

product $\mathbf{G}_1^\dagger \mathbf{V}$ has the first column and row equal to that of an identity matrix of the same size, and the remaining lower-right block is also an orthonormal matrix. Let this $N_t - 1 \times N_t - 1$ block be $\mathbf{V}^{(2)}$ as in [101]. The same strategy can be applied again, and $\mathbf{q}_2$ is set to be the first column of $\mathbf{V}^{(2)}$. A new auxiliary matrix $\tilde{\mathbf{G}}_2$ is created, with $\mathbf{q}_2$ as the first column and the rest to satisfy orthonormality. This is used in the following step to determine $\mathbf{q}_3$ from the lower-right sub-block of $\tilde{\mathbf{G}}_2^\dagger \mathbf{V}^{(2)}$. The procedure continues until all the $\mathbf{q}$ vectors have been computed. In other words, the matrix $\mathbf{V}$ is processed such that

$$\begin{aligned} \mathbf{G}_{N_b}^\dagger \dots \mathbf{G}_1^\dagger \mathbf{V} &= \mathbf{I}_{N_t \times N_b} \\ \mathbf{G}_i &= \begin{bmatrix} 1 & 0 & \dots \\ 0 & \tilde{\mathbf{G}}_i & \\ \vdots & & \end{bmatrix} \end{aligned} \tag{3.6}$$

The transmitter can build transmit-side versions of the matrices $\tilde{\mathbf{G}}_i$ from the quantized versions of $\mathbf{q}_i$ (e.g., with Householder reflections), and compute an approximation of $\mathbf{V}$ based on (3.6).

Further application of the SVQ algorithm from [101] was proposed in [73] for application in subcarrier clustering in MIMO OFDM. Furthermore, large unit-norm vectors are further split and quantized using codebooks for smaller sizes and interpolation of the beamforming vectors was discussed.

Recursive codebook-based update

In some scenarios, on the other hand, the feedback frequency can be high enough compared to the fading rate, so that the same codeword is signaled in two or more consecutive feedback messages. This indicates that the coherence time of the channel allows for the transmission of a refinement of the codeword. Hierarchical codebook schemes such as those in [67] exploit this, where a "coarse" codebook is used to give a non-recursive estimate of the first N_b columns of $\mathbf{V}$, and in subsequent slots a "fine" codebook is employed, where further refinements to the coarse fed-back vector/matrix are provided. Let the coarse codebook be of size 2^{N_c} and the fine codebook of size 2^{N_f}. Further,let N be the number of slots between uses of the coarse codebook, i.e., one usage of the coarse codebook is followed by $N-1$ uses of the fine codebook. As long as the true vector/matrix remains within the same decision region of the coarse codebook, the 2-layer structure effectively uses a codebook of $N_c + N_f$ bits with an average feedback of $(N_c+(N-1)N_f)/N$. For example, if $N_c = 2, N_f = 4, N = 5$, then a 40% reduction in average bits per feedback channel use is obtained. A simple two-layer codebook approach for two-dimensional real-

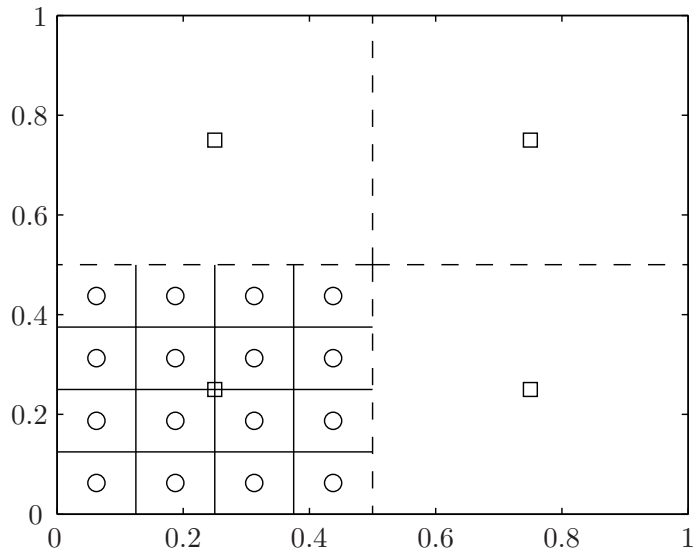

Figure 3.1. Illustration of a two-layer codebook for real-valued vectors with uniformly i.i.d. components in $[0, 1]$. First, a two-bit codebook is used. Thereafter, a four-bit codebook is used. This allows to reduce the feedback rate from six to four bits per feedback channel use, provided that the vector to be quantized remains inside the same quadrant. The representation resembles the hierarchical M-QAM constellation of the DVB-T standard [31].

valued vectors is illustrated in Fig. 3.1. This case resembles the hierarchical (non-uniform) M-QAM constellations that have been adopted in the DVB-T standard [31], in the sense that groups of points are clustered around a center which can be seen as a member of a smaller constellation, equivalently a smaller codebook. Applications and performance analysis of these non-uniform QAM constellations can be found, e.g., in [23, 117].

Another approach for temporally correlated channels is based on Markov chain models for the chosen codebook index [51–54]. This allows to analyze the effect of the feedback delay, and provide substantial feedback rate reductions for large codebooks. The main idea is to signal the codeword index transition, given the last chosen index. The feedback reduction comes from truncating the unlikely transitions. This can be related to the hierarchical codebook structures, where after using a coarse codebook a nested codebook is used. In both approaches the next codeword is "close" to the previous, depending on the fading rate.

Alternatively, adaptive codebooks implementations include scaling of a mother codebook [97, 98, 109], switched codebooks [86] and the successive beamforming scheme [76].

3.2.3 Decomposition-based approaches

Another line of research exploits different parameterizations of orthonormal matrices, in order to utilize vector quantization and real-valued tracking techniques.

One type of parametrization is based on real-valued Givens rotations [36, 38] and described in [102]. The procedure processes an orthornormal matrix column-wise, first premultiplying with a diagonal matrix that converts the entries of the column into real-valued quantities, and then applying a cascade of rotations that null the column elements below the diagonal. The decomposition of an orthonormal matrix $\mathbf{V} \in \mathbb{C}^{N_t \times N_b}$ can be written as [102]

$$\mathbf{V} = \left[\prod_{m=1}^{N_b} \begin{bmatrix} \mathbf{I}_{m-1} & 0 & & \\ 0 & e^{j\phi_{m,1}} & 0 \dots & 0 \\ & 0 & \ddots & \\ & 0 & & e^{j\phi_{m,N_t}} \end{bmatrix} \prod_{n=1}^{N_t-m} \mathbf{J}^{N_t-n\, N_t-n+1}(\alpha_{mn}) \right] \mathbf{I}_{N_t \times N_b} \tag{3.7}$$

where $\mathbf{I}_m$ is the identity of size m, ϕ_{mn} are angles used to eliminate the imaginary parts column-wise and $\mathbf{J}^{mn}(\alpha)$ denotes a real-valued Givens or Jacobi transformation [36, 38] with the angle α, operating on the coordinate plane m, n. This decomposition is different than the one used throughout this thesis, in that the Jacobi or Givens matrices used in the proposed algorithms are complex-valued, and therefore the real-valued parameters are computed differently.

It was shown in [102] that the resulting parameters are statistically independent in case of spatially white Rayleigh channels, and the corresponding PDFs were given. Thus, one can either use VQ techniques for joint quantization, or use scalar tracking techniques to recursively feed back each parameter to the transmitter. This has been summarized in [105], but the application of VQ for non-recursive feedback of the parameters was not considered.

When the complex-valued Givens rotors are used, the resulting parameters are also statistically independent, and the PDFs are known [107], which allows to use vector quantization techniques to design efficient quantizers [106, 107].

3.2.4 Single-bit stochastic recursive methods

Recursive algorithms have been proposed, based on a single-bit feedback message $b(l) \in \{0, 1\}$. The original work by Banister and Zeidler [14] dealt with $N_b = 1$. This method was based on a stochastic signed approximation of the gradient of the received power, and resembles the Sign Error technique from

adaptive filtering [27]. The update at the transmitter is given by

$$\mathbf{w}(l+1) = \frac{\mathbf{w}(l) + b(l)\mu\mathbf{p}(l)}{||\mathbf{w}(l) + b(l)\mu\mathbf{p}(l)||}$$

where $\mathbf{p}(l)$ is a perturbation that allows the signed gradient approximation, μ is a step size parameter and $b(l)$ is the single-bit feedback message. This perturbation can be either transmitted in the downlink, or be chosen synchronously based on, e.g., a common random seed.

This method can be used in frequency fading channels and it yields a "wideband" precoding, by attempting to obtain the dominant eigenvalue of the matrix

$$\mathbf{R}_{wb}(k) = \sum_{d=1}^{D} \mathbf{H}(d,k)\mathbf{H}^{\dagger}(d,k)$$

where the sum includes D symbol-spaced channel taps $\mathbf{H}(d,k)$.

An improved 2-bit feedback message and enhanced stochastic gradient construction were analyzed in the MU-MISO case in [15].

The extension to multiple substreams and MIMO systems was presented in [13], although the design was restricted to a frequency flat channel. Now the objective function attempts to maximize the trace of $\mathbf{W}^{\dagger}\mathbf{H}^{\dagger}\mathbf{H}\mathbf{W}$, which indirectly diagonalizes $\mathbf{H}^{\dagger}\mathbf{H}$. The update is similar to the MISO case, except for the random perturbation dimensions and the necessity to orthonormalize the resulting matrix:

$$\tilde{\mathbf{W}}(l+1) = \mathbf{W}(l) + b(l)\mu\mathbf{P}(l) \quad \mathbf{W}(l+1) = GS(\tilde{\mathbf{W}}(l+1))$$

where $GS(\cdot)$ denotes the Gram-Schmidt orthonormalization [38] or an equivalent procedure that produces an orthonormal matrix based on the argument.

An alternative recursion avoiding the orthonormalization step and also based on probing diametrical directions was proposed in [142]. The update candidates $\mathbf{W}_{\pm}$ are built as:

$$\begin{aligned} \mathbf{W}_{\pm} &= [\mathbf{W}_k \ \mathbf{W}_{\perp}]\mathbf{\Phi}_{\pm}\mathbf{W}_0 \\ \mathbf{\Phi}_{\pm} &= e^{\pm\mathbf{B}} \\ \mathbf{B} &= \begin{bmatrix} \mathbf{0}_{N_b} & -\mathbf{A}^{\dagger} \\ \mathbf{A} & \mathbf{0}_{(N_t-N_b)} \end{bmatrix} \\ \mathbf{A} \in \mathbb{C}^{(N_t-N_b)\times N_b} & \qquad \mathbf{A}_{m,n} \text{ i.i.d. } \sim \mathcal{N}(0,\mu) \end{aligned} \tag{3.8}$$

where $\mathbf{W}_{\perp}$ is the orthogonal complement of $\mathbf{W}_k$, $\mathbf{W}_0$ contains the first N_b columns of the identity of size N_t, $e^{\pm\mathbf{B}}$ denotes the matrix exponential function, and μ is similar to a step size parameter. This was later analyzed in [141].

3.2.5 Other approaches

Alternative methods include a decentralized channel tracking algorithm proposed in [87]. This method enables the transmitter to track the channel directly, by using a set membership filtering (SMF) algorithm, where the scalar required input is fed back by the receiver. This input to the update formula is the received symbol, which was obtained during the transmission of a reference symbol known to both transmitter and receiver. Thus, the feedback capacity is used to give more resolution to single quantity, rather than having a coarse joint quantization of the channel vector. Moreover, the receiver can determine when the selective update is not required according to a given threshold, and skip the feedback message transmission. This makes the scheme a variable feedback rate algorithm.

Early closed-loop eigenbeamforming algorithms are also WCDMA closed-loop modes 1 and 2 [5], which are specified for $N_t = 2$. Mode 1 is an EGT scheme employing bit filtering, where a single feedback bit specifies one of the real-valued components of the second antenna weight, thus allowing this weight to move between neighbouring elements of a QPSK constellation. In Mode 2, one bit is used for adjusting the power ratio between the antennas, and three bits are used for quantizing the phase adjustment of antenna two to eight levels. As only one feedback bit is allowed per feedback channel use, this mode requires buffering the bits until all have been sent to the transmitter. Enhanced filtering techniques for Mode1, which allow increasing the weight resolution and may also improve the feedback reliability have been proposed in [50].

Other limited-feedback approaches to adaptive transmission include antenna selection [115, 139], whereby one or a subset of antennas are selected based on feedback. In [140], adaptive beamforming based on a predictor acting on uplink measurements was proposed.

3.2.6 SU-MIMO in LTE

The 3GPP LTE standard defines two types of SU-MIMO precoding for spatial multiplexing ($N_b > 1$):

- Static codebook based precoding for $N_t = 2, 4$. The specification [2] (release eight version) defines 2 and 4-bit codebooks based on Householder transformations, for the case of two and four transmit antennas, respectively. The feedback message is called the precoding matrix indicator (PMI), and is sent

independently of the per-stream channel quality indicator (CQI) and the rank indicator (RI).

- Precoding with receiver-specific reference signals. Releases eight and nine include the possibility of user-specific reference signals in downlink, which are employed for the estimation of the beamformed channel $\mathbf{HW}$ at the receiver. In release eight the transmission rank is restricted to $N_b = 1$, while $N_b = 2$ is supported in release nine. These transmission modes are to be implemented first in TDD systems, where the CSIT is obtained from uplink measurements through channel reciprocity.

It is possible to configure the frequency granularity of the feedback mechanisms, since the large system bandwidths (up to 20 MHz in a single "carrier") are subject to frequency selective fading. Thus, the precoding feedback calculation and transmission is executed independently for every frequency sub-band configured for reporting.

Recursive feedback precoding techniques such as the algorithms contributed in this thesis could be relevant for future releases of the standard, although up to now only non-recursive techniques have been standardized.

3.3 Contributions on recursive methods

This section describes the contributions of the thesis in the field of recursive feedback methods in MIMO systems. Sections 3.3.1, 3.3.2, 3.3.3 and 3.3.4 deal with closed-loop eigenbeamforming problems (closed-loop subspace tracking). Section 3.5 describes a closed-loop extension to space-time quasi-orthogonal block codes. Section 3.4 introduces closed-loop interference-tolerant MIMO communications.

3.3.1 Gradient reuse

This algorithm constitutes an extension of the signed stochastic gradient algorithm [14]. A system with $N_b = n_b = 1$ is considered, where the transmit weights are optimized to maximize the total received power. The adaptation is implemented through a signed stochastic approximation of the gradient of the combined power at the receiver. Optimizing the instantaneous received power minimizes the uncoded BER and maximizes the mutual information of the single-stream system.

The received power for a maximal ratio combining (MRC) receiver is given by

$$P_r(k,l) = ||\mathbf{H}(k)\mathbf{w}(l)||^2 = \mathbf{w}^\dagger(l)\mathbf{H}^\dagger(k)\mathbf{H}(k)\mathbf{w}(l) \tag{3.9}$$

which acts as a cost function for the adaptation of $\mathbf{w}$. In [14], a random perturbation vector $\mathbf{p}(l) \in \mathbb{C}^{N_t \times 1}$ with i.i.d. circular Gaussian entries is used to generate two candidate vectors $\mathbf{w}_+, \mathbf{w}_-$, which are tested against the current channel matrix and determine the feedback message as follows

$$\begin{aligned} \mathbf{w}_+(l) =& \frac{\mathbf{w}(l) + \mu\mathbf{p}(l)}{||\mathbf{w}(l) + \mu\mathbf{p}(l)||} \quad \mathbf{w}_-(l) = \frac{\mathbf{w}(l) - \mu\mathbf{p}(l)}{||\mathbf{w}(l) - \mu\mathbf{p}(l)||} \\ b(l) =& \text{sign}\{||\mathbf{H}(lL-1)\mathbf{w}_+(l)||^2 - ||\mathbf{H}(lL-1)\mathbf{w}_-(l)||^2\} \end{aligned} \tag{3.10}$$

The beamforming vector is updated at both transmitter and receiver, according to the feedback message $b(l)$:

$$\mathbf{w}(l+1) = \frac{\mathbf{w}(l) + b(l)\mu\mathbf{p}(l)}{||\mathbf{w}(l) + b(l)\mu\mathbf{p}(l)||} \tag{3.11}$$

This single-bit technique uses $b(l)\mathbf{p}(l)$ as a coarse estimate of the gradient of the received power $P_r(k,l)$ defined in (3.9). If the true gradient does not change abruptly from slot to slot, then the gradient estimate can be improved by filtering, which in turn produces a tracking performance enhancement that can be observed in moderate to fast fading channels.

Let $\mathbf{g}(l)$ and $\lambda_G \geq 0$ denote the filtered gradient estimate and a gradient reuse controlling parameter, respectively. The gradient reuse equation is given as

$$\mathbf{g}(l) = b(l-1)\tilde{\mathbf{p}}(l-1) + \lambda_G\mathbf{g}(l-1)$$

where the perturbation $\tilde{\mathbf{p}}(l)$ is chosen orthogonal to the previous gradient estimate:

$$\tilde{\mathbf{p}}(l) = \mathbf{p}(l) - \frac{\mathbf{g}^\dagger(l-1)\mathbf{p}(l)}{||\mathbf{g}(l-1)||^2}\mathbf{g}(l-1) = \left[\mathbf{I} - \frac{\mathbf{g}(l-1)\mathbf{g}^\dagger(l-1)}{||\mathbf{g}(l-1)||^2}\right]\mathbf{p}(l)$$

with $\mathbf{p}(l) \in \mathbb{C}^{N_t \times 1}$ is the random Gaussian perturbation employed in the original algorithm.

The update is then computed as

$$\mathbf{w}(l) = \frac{\mathbf{w}(l-1) + \mu\mathbf{g}(l)}{||\mathbf{w}(l-1) + \mu\mathbf{g}(l)||}$$

This extension enables convergence rates exceeding the capabilities of the original algorithm, or otherwise achievable only through very large convergence steps that optimize the performance at a given speed, but compromise it at other mobilities. The proposed algorithm is shown to outperform the original stochastic gradient scheme uniformly for speeds larger than 15 km/h, when

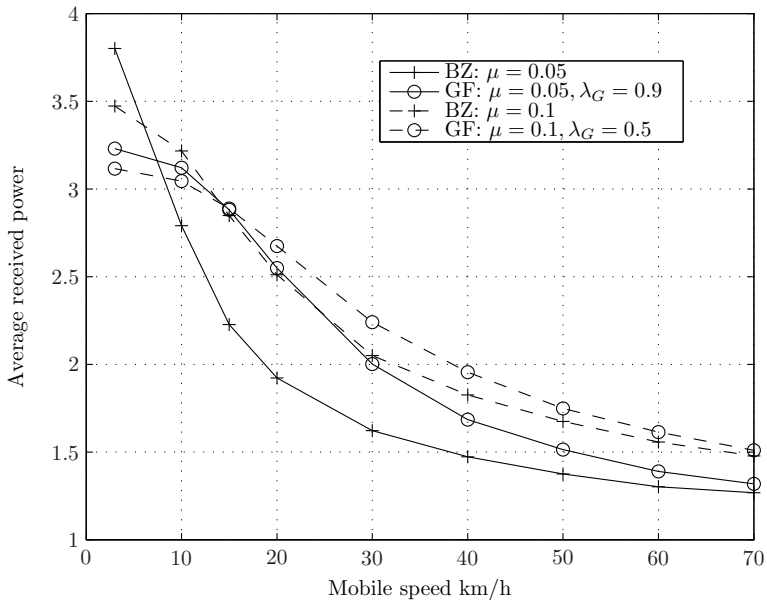

Figure 3.2. Performance of gradient filtering algorithm as a function of the mobile speed. SU-MISO case with $N_t = 4, N_r = 1$, single-bit feedback in spatially white Rayleigh channels. "BZ" refers to the original algorithm of [14] and "GF" refers to the gradient filtering strategy proposed in Section 3.3.1.

the carrier frequency of 2.1 GHz and single-bit feedback messages are sent with frequency $f_{\mathbf{b}} = 1500$ Hz. Figure 3.2 shows the average received power as a function of the mobile speed, for the case of $\mu = 0.05, \lambda_G = 0.9$.

The original algorithm can be obtained by setting $\lambda_G = 0$ and generating the perturbations independently, i.e., $\tilde{\mathbf{p}}(l) = \mathbf{p}(l)$.

3.3.2 Single bit methods for more than one stream

The original single beam algorithm [14] was subsequently extended to handle a general case $N_b > 1$ in [13]. However, the candidate generation required a matrix orthonormalization step, in order to enforce the orthonormality conditions. This motivated the methods described in this section, where the conditions do not need to be explicitly enforced. A different recursion that also guaranteed the orthonormality of the candidates was proposed in [142].

The proposed methods probe around the current beamforming matrix $\mathbf{W}(l) \in \mathbb{C}^{N_t \times N_b}$ to form update candidates $\mathbf{W}_{\pm}(l)$ and inform the best candidate to the transmitter. This is conceptually similar to the single beam case described in Section 3.2.4. This section, however, deals with tracking of orthonormal matrices, as opposed to the tracking of unit-norm vectors. Consequently, the procedures for building the update candidates are also different. Two different recursions will be presented.

The IGREB algorithm

Since the Stiefel manifold [29] is connected, premultiplication with a unitary matrix is a natural way to update $\mathbf{W}$ without enforcing the orthonormality conditions explicitly. One way to define a single-bit update to $\mathbf{W}$ is to choose between two unitary matrices, where the update is obtained by premultiplying $\mathbf{W}$ with the chosen matrix. Thus, the incremental Givens rotations eigenbeamforming (IGREB) algorithm features update candidates $\mathbf{W}_+, \mathbf{W}_-$ defined as

$$\mathbf{W}_\pm(l) = \mathbf{U}_\pm(l)\mathbf{W}(l) \tag{3.12}$$

where $\mathbf{U}_-(l) = \mathbf{U}_+^\dagger(l)$ and $\mathbf{U}_+(l)$ is a random unitary matrix including a step size parameter, which allows to control the adaptation rate. The choice of $\mathbf{U}_-$ is intuitively "the opposite direction of rotation".

The random unitary matrix is generated from a vector of angles with i.i.d. Gaussian entries with zero mean and variance μ^2, as follows. Any unitary matrix of size N_t can be built from N_t real-valued angles through a cascade of Givens rotors [36, 38] and a diagonal matrix with unit-modulus complex-valued scalings. This can be written as [9, 132]:

$$\begin{aligned} \mathcal{U}(\boldsymbol{\lambda}, \boldsymbol{\psi}) &= \boldsymbol{\Psi} \left[\prod_{m=N_t-1}^{1} \prod_{n=m+1}^{N_t} \mathbf{J}^{mn}(\theta_{mn}, \beta_{mn}) \right] \\ \boldsymbol{\Psi} &= \begin{bmatrix} e^{j\psi_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & e^{j\psi_{N_t}} \end{bmatrix} \end{aligned} \tag{3.13}$$

where $N_t(N_t-1)/2$ angle pairs $(\theta_{mn}, \beta_{mn})$ are stacked in $\boldsymbol{\lambda} \in \mathbb{R}^{[N_t(N_t-1)]\times 1}$ and $\boldsymbol{\psi} = [\psi_1 \dots \psi_{N_t}]^T \in \mathbb{R}^{N_t \times 1}$.

The complex-valued Givens rotors or Jacobi transformation $\mathbf{J}^{mn}(\theta_{mn}, \beta_{mn})$ equals an identity of size N_t, except for the entries (m,m), (m,n), (n,m) and (n,n). The matrix is defined element-wise as [36, 38]

$$J_{pq}^{mn} = \begin{cases} \cos(\alpha_{mn}) & \text{if } p = q \in \{m,n\} \\ -\sin(\alpha_{mn})e^{-j\beta_{mn}} & \text{if } p = n \text{ and } q = m \\ \sin(\alpha_{mn})e^{j\beta_{mn}} & \text{if } p = m \text{ and } q = n \\ 1 & \text{if } p = q \notin \{m,n\} \\ 0 & \text{otherwise} \end{cases} \tag{3.14}$$

The matrix $\mathbf{U}_+(l)$ is then built as $\mathbf{U}_+(l) = \mathcal{U}(\boldsymbol{\lambda}, \boldsymbol{\psi})$, with $\boldsymbol{\lambda}, \boldsymbol{\psi}$ chosen randomly. All the angles are i.i.d. and follow a Gaussian distribution of zero mean and variance μ^2. Therefore, smaller step-size values of μ generate unitary matrices closer to the identity, as the Givens rotors themselves become

closer to identity matrices. In slowly fading channels, the number of random parameters can be further reduced with the approximation $e^{j\beta_{mn}} \approx 1$, in which case $\mathbf{U}_+(l) = \mathcal{U}(\boldsymbol{\lambda}, \mathbf{0})$.

It is also possible to generate $\mathbf{U}_+(l)$ as the matrix exponential of a skew-Hermitian random matrix, but the Givens rotors cascade of (3.13) has been chosen for its simplicity of implementation.

The SCGAS algorithm

Another way to generate the candidate matrices is to move in the underlying angle space. Any orthonormal matrix can be built from a set of angles through a cascade of Givens rotors and a diagonal matrix with unit-modulus scalings. This mapping between $2N_tN_b - N_b^2$ real-valued angles and a complex-valued orthonormal matrix of size $N_t \times N_b$ can be written as [9, 132]:

$$\mathcal{T}(\boldsymbol{\theta}, \boldsymbol{\xi}) = \left[\prod_{m=N_b}^{1} \prod_{n=m+1}^{N_t} \mathbf{J}^{mn}(\alpha_{mn}, \beta_{mn}) \right] \boldsymbol{\Gamma}(\boldsymbol{\xi})$$

$$\boldsymbol{\Gamma}(\boldsymbol{\xi}) = \begin{bmatrix} e^{j\xi_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & e^{j\xi_{N_b}} \\ 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix} \tag{3.15}$$

where $(2N_tN_b - N_b^2 - N_b)/2$ angle pairs $(\alpha_{mn}, \beta_{mn})$ are stacked in $\boldsymbol{\theta} \in \mathbb{R}^{[N_tN_b - N_b^2 - N_b] \times 1}$, $\boldsymbol{\xi} = [\xi_1 \dots \xi_{N_b}]^T \in \mathbb{R}^{N_b \times 1}$ and $\mathbf{J}^{m,n}$ is the Givens rotor defined in (3.14). We will restrict our attention to the special case $\boldsymbol{\Gamma}(\boldsymbol{\xi}) = \mathbf{I}_{N_t \times N_b}$ since the optimal beamforming matrix is unique up to unit-norm scalings on each column, and therefore define

$$\mathcal{M}(\boldsymbol{\theta}) = \mathcal{T}(\boldsymbol{\theta}, \mathbf{0}) \tag{3.16}$$

The stochastic gradient search over an angle space (SCGAS) algorithm keeps an angle vector $\boldsymbol{\theta}(l)$ such that $\mathbf{W}(l) = \mathcal{M}(\boldsymbol{\theta}(l))$. The update candidates are then built based on random perturbations of the current angle vector, that is

$$\mathbf{W}_\pm(l) = \mathcal{M}\{\boldsymbol{\theta}(l) \pm \mu \mathbf{p}(l)\}$$

where $\mathbf{p}(l) \in \mathbb{R}^{[2N_tN_b - N_b^2 - N_b] \times 1}$ is chosen with i.i.d. zero-mean unit-variance Gaussian variables. Similar to the case of the IGREB algorithm, μ controls the adaptation rate and no adaptation occurs if $\mu = 0$, since all the rotors become equal to identity matrices.

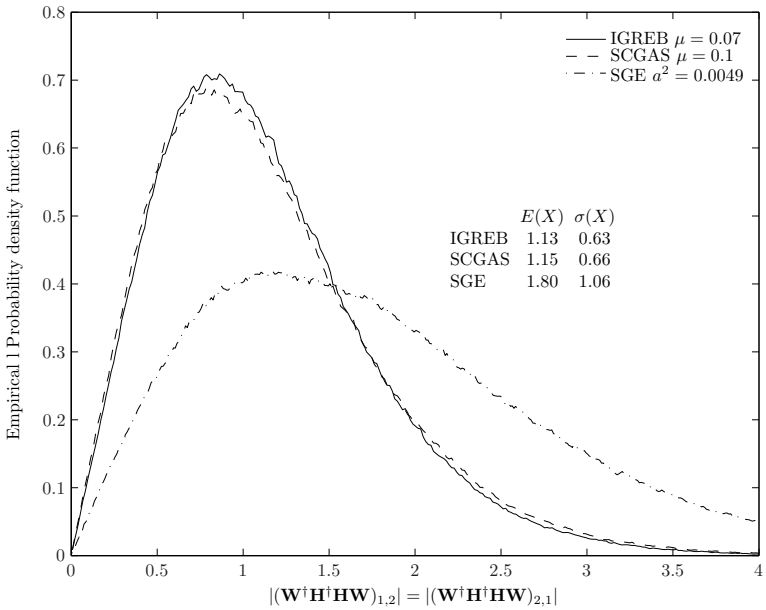

Figure 3.3. Statistics for non-diagonal power of $\mathbf{W}^\dagger\mathbf{H}^\dagger\mathbf{HW}$, with mobile speed of 3 km/h and single-bit algorithms closed-loop algorithms, in MU-MIMO with $N_t = 4, N_r = 2, N_b = 2$ spatially white Rayleigh channels.

In order to keep the search within a bounded space, the inverse mapping is used to find $\boldsymbol{\theta}(l+1)$ within the nominal ranges:

$$\boldsymbol{\theta}(l+1) = \mathcal{M}^{-1}\{\mathbf{W}(l+1)\}$$

where $\mathcal{M}^{-1}(\cdot)$ involves sequential nulling of the elements under the diagonal of $\mathbf{W}(l+1)$.

The performance of the IGREB and SCGAS algorithms is illustrated in Figs. 3.3 and 3.4, for a $N_t = 4, N_r = 2$ system with carrier frequency 2.1 GHz and single bit feedback messages sent with frequency $f_b = 1500$ Hz, and a mobile moving at 3 km/h. It is seen from Fig. 3.3 that the tracking performance is somewhat poor, as evidenced by the large residual interference between streams that can be seen after the matched filter. However, both algorithms outperform existing single-bit proposals like [142]. The corresponding BER performance curves are shown in Fig. 3.4.

3.3.3 Partial-update eigenbeamforming

This section deals with single-beam systems $N_b = 1$, but differs from the previous section, in that it features deterministic tracking, as opposed to the feedback assisted stochastic probing described in the single-bit methods. The feedback requirements are different as well. The algorithms presented here will typically use $n_b = 6, 8$ bits per feedback message. This results in nearly-optimal performance in low mobility scenarios and a better performance in moderate speeds over the pedestrian ranges.

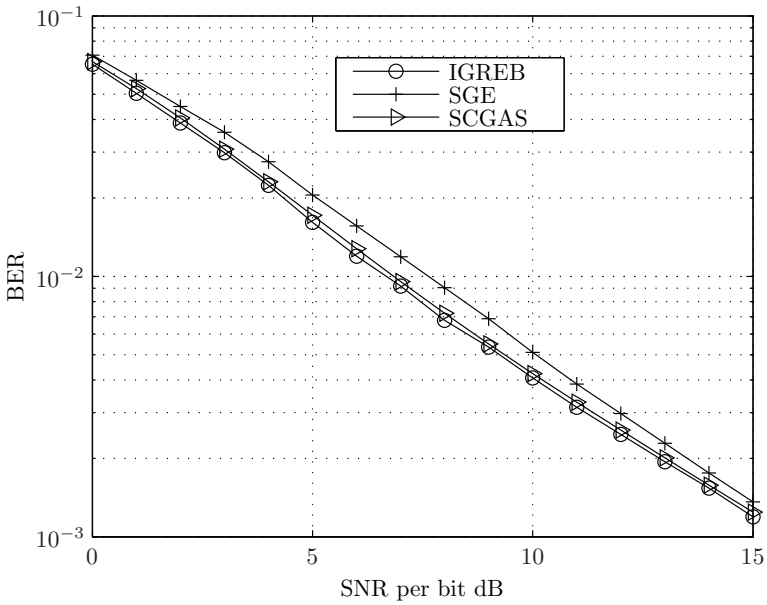

Figure 3.4. BER performance for single-bit closed-loop algorithms with mobile speeds of 3 km/h, double-stream MIMO transmission with uniform power allocation in $N_t = 4, N_r = 2, N_b = 2$ i.i.d. channels and linear MMSE detection. The proposed IGREB and SCGAS algorithms from Section 3.3.2 give power gains statistically similar to the channel eigengains, and therefore employ uneven bit loading (16-QAM and 4-QAM modulation symbols on stream one and two, respectively). The "Stochastic Geodesic" (SGE) algorithm of [142] employs 8-PSK constellations on both streams.

The partial-update algorithms aim to provide low-complexity weight adaptation while achieving precision in the update of one element of $\mathbf{w}$ at each slot. The partial-update concept is known in adaptive filtering theory, where the motivation is to decrease the computational complexity associated to the full update of the filter.

In the following sections, the partial-update algorithms are described in their two variants: the sequential, where the coefficients are updated in a round-robin fashion and the "ranked", where part of the feedback message is destined to signal which coefficient will be updated. The latter provides better performance than the first, for mobility ranges outside the pedestrian ranges.

Partial update in MISO systems

In MISO systems, the optimal beamformer is given by $\boldsymbol{v}(l) = \mathbf{h}^{\dagger}(k)/||\mathbf{h}(k)||$. The update formula for coefficient $a \in \{1, \ldots, N_t\}$ is

$$w_a(l+1) = Q_\rho\{|v_a(l)|\}e^{jQ_\phi\{\angle v_a(l)\}}$$

where $Q_n\{\cdot\}$ quantizes the argument uniformly to n bits within a given interval and $\angle(\cdot)$ denotes the argument of a complex number.

The ranked version of the algorithm chooses a as the coefficient that has the

largest weighted error

$$a = \underset{i}{\operatorname{argmax}}\{|h_i(lL-1)|^2|v_i(lL-1) - w_i(l)|^2\}$$

Partial update in MIMO systems

The previous section presented a very low computational-complexity algorithm, which is limited to MISO systems. In this section, we present the alignment enhanced partial update beamforming (ALE-PUB) algorithm, which allows the partial concept in a MIMO system with $N_b = 1$.

The ALE-PUB algorithm solves the partial-update problem by maximizing the received power that is obtained with the updated weights. Let $P_r(k,l)$ be the combined received power at symbol period k on slot l

$$P_r(k,l) = \mathbf{w}^\dagger(l)\mathbf{H}^\dagger(k)\mathbf{H}(k)\mathbf{w}(l)$$

The updated weight $w_a(l+1)$ is parametrized in polar coordinates as $\rho e^{j\theta}$ and the unquantized solution solves

$$\underset{\rho,\theta}{\operatorname{argmax}}\, P_r(lL-1, l+1)$$

The solution has an intuitive interpretation in the MISO case, which is illustrated in Fig. 3.5.

The ranked version chooses a by selecting the partial-update giving the largest combined power, and the optimal weight is quantized uniformly according to a predetermined bit budget for power/phase. When operating in sequential mode, the coefficients are updated in round-robin fashion, and therefore the transmitter knows the index a on each slot. When using the ranked mode, on the other hand, the chosen coefficient index a has to be fed back to the transmitter, which decreases the available bits for quantizing the optimal coefficient $w_a(l+1)$. Moreover, this overhead depends on the number of transmit antennas and equals $\lceil \log_2(N_t) \rceil$. The transmitter extracts the quantized power and phase of $w_a(l+1)$ from the feedback message and uses them to perform the update of the beamforming weights. In ranked mode, the index a is also extracted from the feedback message bits.

The performance of the proposed ALE-PUB algorithms is illustrated in Figs. 3.6 and 3.7 for the carrier frequency of 2.1 GHz and the feedback message frequency of 1500 Hz, with six bits per feedback channel use. In Fig. 3.6 the BER performance as a function of the mobile speed is given, for the different variants of the algorithm, when the SNR per bit is 7 dB. It can be seen that the ranked algorithms perform worse than the sequential variants at pedestrian

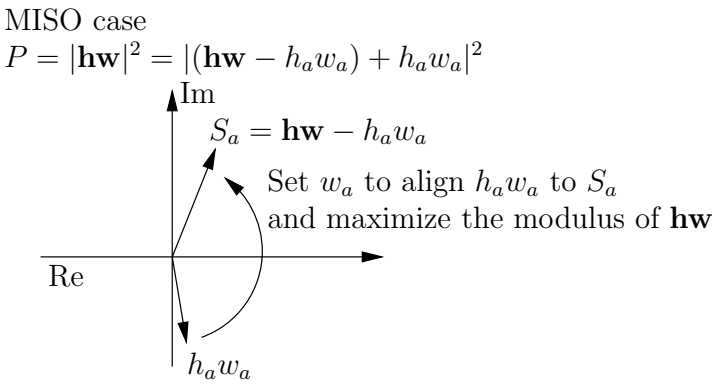

Figure 3.5. Geometrical interpretation of the partial-update eigenbeamforming algorithms ALE-PUB in the case of $N_r = 1$. The single beamforming coefficient is selected to align the adjustable term to the fixed term, thus maximizing the combined power at the receiver.

speeds. This is due to the decreased resolution in the feedback of the optimal weight, which arises from the overhead of signalling which coefficient will be updated. As the mobile speed increases, on the other hand, the round-robin update of the weights becomes too slow and the selective update has better tracking performance, despite the lower quantization resolution. The full BER v/s SNR curve is given in Fig. 3.7 for the sequential version when the mobile speed is 3 km/h. It can be seen that the algorithm performs close to the perfect CSI case, where the full unquantized update is available on every slot.

3.3.4 Coordinate plane unitary transformations: the D-JAC algorithm

The previous sections have presented both stochastic and deterministic closed-loop eigenbeamforming methods. In this section, the distributed Jacobi (D-JAC) eigenbeamforming algorithm is introduced. This is a deterministic recursive closed-loop subspace tracking method, which allows tracking a set of N_b orthonormal vectors.

The D-JAC algorithm is based on an auxiliary orthonormal matrix $\mathbf{\Phi} \in \mathbb{C}^{N_t \times N_t}$, which starts as an identity of size N_t and is updated with a single Givens rotor on each update. Both transmitter and receiver have their own copy of the matrix, which are identical if the feedback channel is error-free. The receiver computes the angle pair defining the rotor for update, quantizes the angles and sends them in the feedback message. The transmitter recreates the rotor and applies it to its own copy of $\mathbf{\Phi}$, thus applying the same update as the receiver. In a static channel and without quantization of the angle pair, the first N_b columns of the matrix $\mathbf{\Phi}$ converge to the dominant N_b eigenvectors of the channel. Furthermore, the rotors after convergence become identity

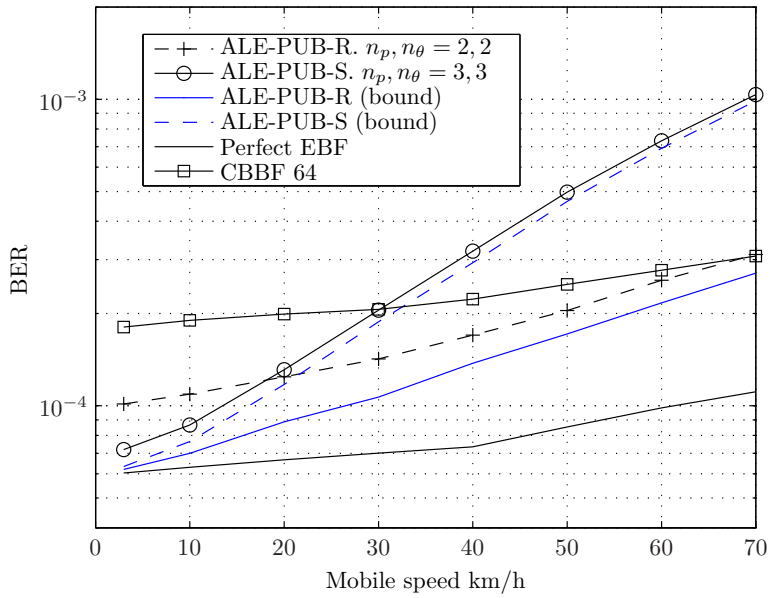

Figure 3.6. Performance of the PU algorithm ALE-PUB as function of the mobile speed, in spatially white $N_t = 4, N_r = 2$ channels with a single stream of 16-QAM symbols. ALE-PUB-R and ALE-PUB-S denote the ranked and sequential variants and "bound" denotes the performance of the single weight update with unquantized values. The limited-feedback cases use $n_b = 6$ bits per feedback message. "CBBF 64" refers to the $n_b = 6$ bits Grassmannian codebook of [80] and "Perfect EBF" is the performance of the transmitter with the unquantized dominant eigenbeam of the channel. The ranked version uses two bits to indicate the weight to be updated.

matrices, up to machine precision.

More formally, the following steps are involved:

1. Receiver: determine which coordinate plane will be used for the update. The planes are selected in round-robin fashion, and the list of planes is defined by $(m, n) : n = 1, 2, \ldots, N_b; m = n + 1, n + 2, \ldots, N_t$.

2. Receiver: given the most recent channel sample $\mathbf{H}$ and the current matrix $\mathbf{\Phi}$, compute an angle pair α, β for the coordinate plane (m, n).

3. Receiver: quantize the angles according to bit budget and feed them back to the transmitter. The nominal ranges for α, β are $[0, \pi/2)$ and $[-\pi, \pi)$, respectively. The quantization is uniform over the nominal ranges, and the n_b feedback bits are split evenly between the angles. Since transmitter and receiver share the list of coordinate planes and operate on them in round-robin fashion, signalling which plane will be updated on each slot is not needed.

4. Receiver: use the quantized angles to update $\mathbf{\Phi}$ and $\mathbf{W}$.

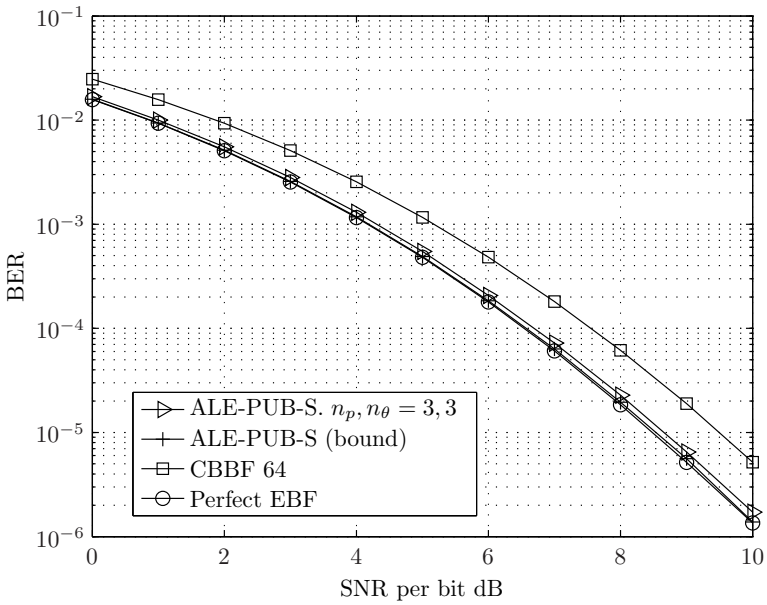

Figure 3.7. BER performance of the sequential PU algorithm ALE-PUB in i.i.d. $N_t = 4, N_r = 2$ channels, $n_b = 6$, single stream with 16-QAM symbols and mobile speed of 3 km/h. "bound" refers to the case when the single weight update is done without any quantization effects and "Perfect EBF" refers to the transmitter using the unquantized dominant channel eigenbeam. "CBBF 64" refers to the $n_b = 6$ bits Grassmannian codebook of [80].

5. Transmitter: determine the plane (m, n) as the receiver did.

6. Transmitter: retrieve the fed-back angles and use them to assemble a rotor in plane (m, n).

7. Transmitter: update the transmit-side version of $\mathbf{\Phi}$ and $\mathbf{W}$ with the rotor so computed.

The method mimics the Jacobi method for diagonalization of Hermitian matrices [38] by treating the current channel conditions as an intermediate step in the diagonalization algorithm. Only one coordinate plane transformation is used per update, as opposed to $2N_tN_b - N_b(N_b + 1)$ transformations contained in a full Jacobi sweep. This is to keep the feedback requirements low. Furthermore, the proposed update solves the angle rotors directly, as opposed to previously proposed methods found in [43].

The performance of theD-JAC algorithm is illustrated in Figs. 3.9, 3.10 and 3.8. For mobile speeds of 3 km/h and uncorrelated Rayleigh fading, the D-JAC algorithm diagonalizes the $N_t = 4, N_r = 2$ almost perfectly, when six feedback bits are sent with feedback frequency $f_{\mathbf{b}} = 1500$ Hz. It can be seen in Fig. 3.8 that the diagonal entries of $(\mathbf{WH})^\dagger(\mathbf{WH})$ follow closely the distributions of the squared singular values of $\mathbf{H}$, while the off-diagonal

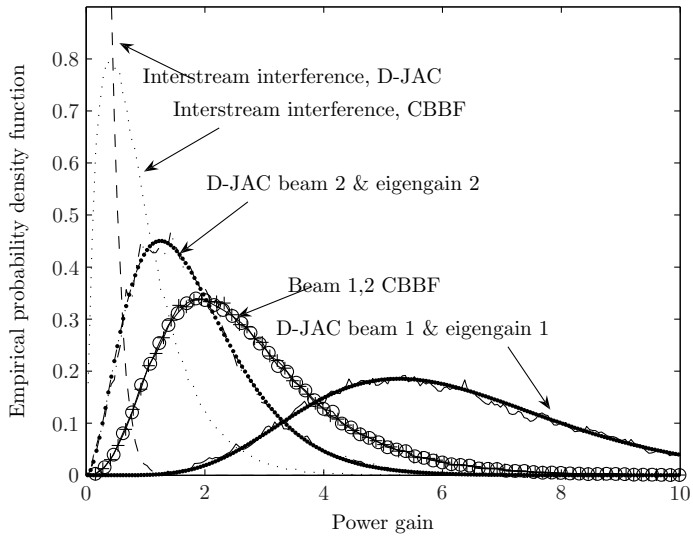

Figure 3.8. Statistics of the power gains of the equivalent linear system after matched filter reception for the D-JAC algorithm, compared to the full CSI (perfect eigenbeamforming) case. "Eigengain 1,2" refer to the two largest squared singular values of the channel, respectively. "Interstream interference" refers to $|(\mathbf{WH})^{\dagger}\mathbf{WH}|_{12}$, and "Beam 1,2" refer to $|(\mathbf{WH})^{\dagger}\mathbf{WH}|_{11}$ and $|(\mathbf{WH})^{\dagger}\mathbf{WH}|_{11}$, respectively.

term is in general very small. The BER performance for the same scenario is given in Fig. 3.10. It can be seen that the D-JAC algorithm operates close to the optimal performance, which is obtained when the transmitter has the unquantized dominant singular vectors of $\mathbf{H}$. The performance advantage compared to the non-recursive Grassmannian codebook of [78] is about 3 dB for BER levels of 0.01, with both algorithms operating at the same feedback rate. The BER performance degradation as the mobile speed increases is given in Fig. 3.9 for the case of SNR per bit equal to 5 dB. It is shown that the D-JAC obtains a performance advantage over the codebook of size 64, up to mobile speeds of 30 km/h when operating at the same feedback rate, and up to 25 km/h when operating at a lower feedback rate of $n_b = 4$.

3.4 Interference-tolerant MIMO transmissions

This section deals with closed-loop techniques, where the transmit weights are adapted to aid the interference rejection capabilities of the receiver. This extends the classical receive-only diversity interference rejection combining receivers to a MIMO setup, where the transmit weights assist the spatial noise whitening and channel gain for the received streams.

Given a short-term estimate of the noise-plus-interference signals affecting a receiver, the link-level performance is dictated by the SINRs, as given by (3.5). The following sections will describe how the transmit weights can be adapted,

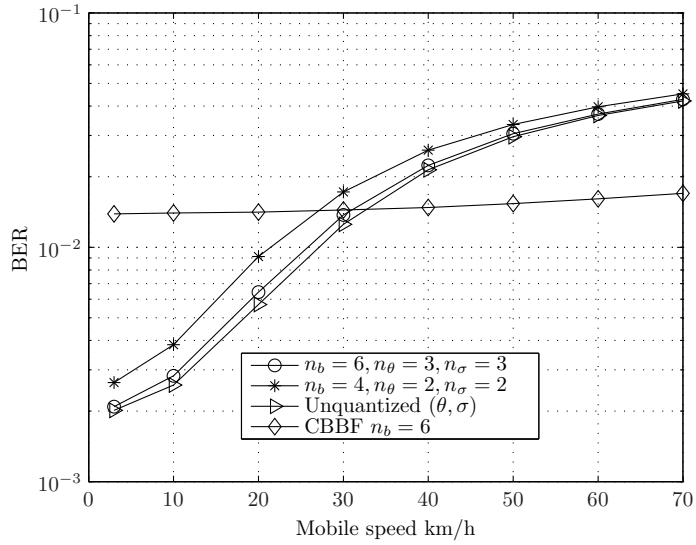

Figure 3.9. D-JAC: BER performance of the D-JAC algorithm as a function of the mobile speed and bits per feedback message in uncorrelated $N_t = 4, N_r = 2$ channels, and a single rotor applied per update. "Unquantized (θ, σ) refers to the update employing an unquantized rotor angle pair, n_θ, n_σ refer to the number of bits used to encode the θ and σ angles, respectively. "CBBF" refers to the Grassmannian codebook from [78].

in order to optimize the link performance.

3.4.1 Single stream IRC-MIMO

For the special case of $N_b = 1$, the optimal combiner [138] can be defined as

$$\boldsymbol{\omega}(k,l) = \mathbf{Q}^{-1}(k,l)\mathbf{H}(k)\mathbf{w}(l)$$

where $\mathbf{Q}(k,l)$ is the covariance matrix of the interference plus noise signals from (2.3).

Conditioned on $\mathbf{H}(k)$, $\mathbf{Q}(k,l)$ and given $\mathbf{w}(l)$, the combiner yields the following SINR (in the following we assume $\mathcal{P} = 1$)

$$\gamma(k,l) = \mathbf{w}^\dagger(l)[\mathbf{H}^\dagger(k)\mathbf{Q}^{-1}(k,l)\mathbf{H}(k)]\mathbf{w}(l)$$

which shows that the transmit weights that optimize the SINR are given by the dominant eigenvector of $\mathbf{H}^\dagger(k)\mathbf{Q}^{-1}(k,l)\mathbf{H}(k)$, and that the optimal SINR is the corresponding eigenvalue.

Assuming some temporal correlation in the interference sources, it is possible to feed back the optimal transmit weight to the transmitter by means of a closed-loop eigenbeamforming algorithm. In particular, the D-JAC algorithm described in Section 3.3.4 can be applied directly, if the rotor angle pair is computed upon $\mathbf{H}^\dagger(k)\mathbf{Q}^{-1}(k,l)\mathbf{H}(k)$ instead of $\mathbf{H}^\dagger(k)\mathbf{H}(k)$. A substantial performance enhancement is obtained, when compared to the use of eigenbeamforming and the IRC receiver, which can be implemented with the same

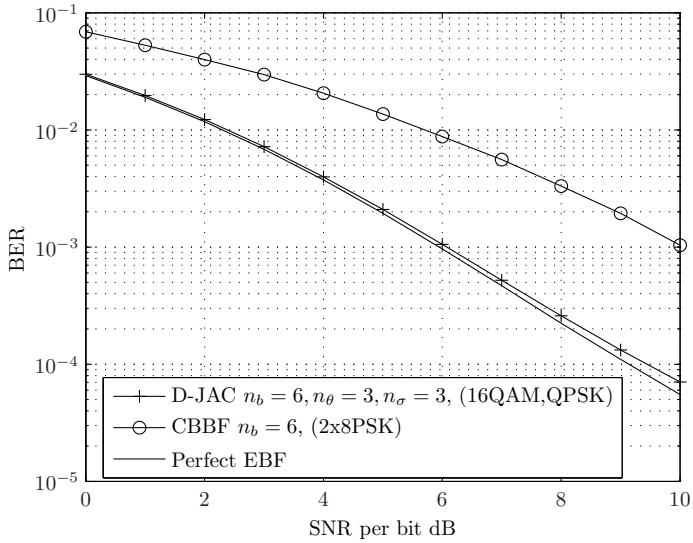

Figure 3.10. BER performance of the D-JAC algorithm in uncorrelated $N_t = 4, N_r = 2$ channels and mobile speed of 3kmh, with two streams employing 16-QAM and QPSK symbols, respectively. Single rotor update with $n_b = 6$ bits per feedback message. "CBBF" refers to the Grassmannian codebook from [78] and "Perfect EBF" uses the unquantized channel eigenbeams at the transmitter.

feedback rate, but does not include the interference statistics in the adaptation of $\mathbf{w}(l)$.

The performance of the single stream transmit-assisted IRC algorithm is shown in Fig. 3.11. A system with $N_t = 4, N_r = 2$ is considered with carrier frequency of 2.1 GHz and mobile speed of 3 km/h. The feedback channel sends six bits every slot, where the slot frequency is $f_{\mathbf{b}} = 1500$ Hz. We assume a single interferer which is an i.i.d. complex-valued Gaussian vector $\boldsymbol{u}_{11} \in \mathbb{C}^{N_r \times 1}$ with the same temporal autocorrelation properties of $\mathbf{H}$, that is, the relative motion of the receiver and the interference source is also of 3 km/h. The interfering signal arrives at the receiver with the same SNR that the data signal has. Under this harsh interference conditions, the transmission by eigenbeamforming alone is not feasible. It is seen that the traditional IRC receiver of [138] in conjunction with eigenbeamforming has some interference cancellation capabilities, but it is clearly outperformed by the proposed transmit-assisted IRC algorithm, with either the D-JAC weight recursion from Section 3.3.4 or the SCGAS algorithm extended to operate on $n_b > 1$ bits (as described in Section 3.4.2). Furthermore, the advantage of using recursive methods over static Grassmannian codebooks is also given.

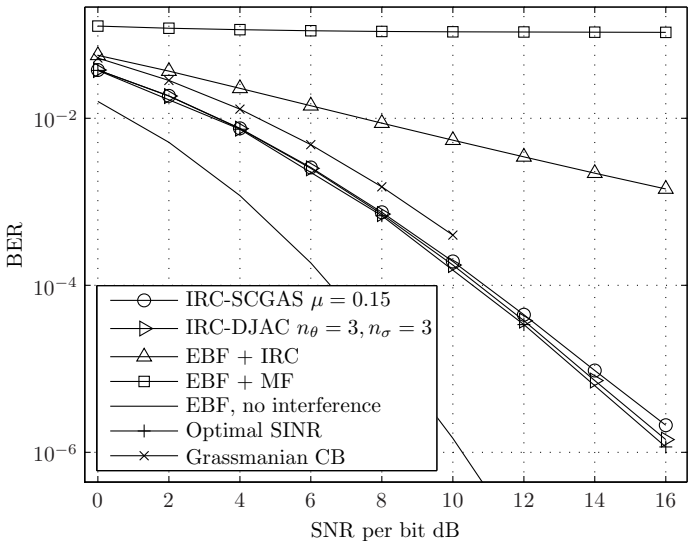

Figure 3.11. BER performance of 16-QAM single-stream closed-loop transmit-assisted IRC algorithms for SU-MIMO, under strong interfering signals. A slowly-varying interference signal arrives at the receiver with the same SNR of the data signal. "EBF" denotes beamforming along the unquantized dominant channel eigenbeam, "IRC-DJAC" and "IRC-SCGAS" denote the proposed algorithms of Section 3.4.1 using feedback messages of $n_b = 6$ bits, with $N_t = 4, N_r = 2$ uncorrelated channels and mobile speed of 3 km/h. "Grassmannian CB" denotes the 6-bit codebook of [80].

3.4.2 Multiple-stream IRC-MIMO

In a general case $N_b > 1$, the N_b combiners are stacked as columns of $\mathbf{\Omega} \in \mathbb{C}^{N_r \times N_b}$ and computed as

$$\mathbf{\Omega}(k,l) = \mathbf{Q}^{-1}(k,l)[\mathbf{H}(k)\mathbf{W}(l)]$$

which features combiners $\boldsymbol{\omega}_i(l) = \mathbf{Q}^{-1}(k,l)[\mathbf{H}(k)\mathbf{w}_i(l)]$ as in the single stream case, but also exhibits a strong coupling between the vectors $\mathbf{w}_i(l)$ and the SINRs $\gamma_i(k,l)$, as defined by (3.5). This is clear, for example, by considering streams one and two. The transmit weights $\mathbf{w}_1(l)$ define the forward gain for the stream, $\mathbf{w}_1^\dagger(l)\mathbf{H}^\dagger(k)\mathbf{Q}^{-1}(k,l)\mathbf{H}(k)\mathbf{w}_1(l)$ and also the interference that stream two suffers from stream one, namely $\mathbf{w}_2^\dagger(l)\mathbf{H}^\dagger(k)\mathbf{Q}^{-1}(k,l)\mathbf{H}(k)\mathbf{w}_1(l)$. This motivates the optimization of a joint (scalar) metric of the SINRs.

In this sense, the average conditional BEP of the streams is a possible metric. Given the conditional SINRs from (3.5), the conditional BEP across the streams is

$$P(\mathbf{W}(l)|\mathbf{H}(k)) = \sum_{p=1}^{N_b} \frac{\mathcal{B}_i}{\sum_n \mathcal{B}_n} P_p\left(\frac{\gamma_p(k,l)}{\mathcal{B}_p}\right) \tag{3.17}$$

where $\mathcal{B}_p$ is the number of bits per symbol in the stream p and $P_p(\cdot)$ can be any suitable SINR to BEP mapping for the constellation in use in stream p.

Other metrics could be considered, such as a mapping between the conditional SINRs and the average coded BER under a given channel coding scheme.

This can be produced with either laboratory measurements or link-level simulations.

In order to optimize the beamforming matrix $\mathbf{W}(l)$, an extension of the signed stochastic gradient algorithm is employed, in which the receiver chooses one out of 2^{n_b} update candidates, which are built as perturbations to the current matrix $\mathbf{W}(l)$. Thus, the feedback message contains an index m^*, where

$$m^* = \underset{m}{\operatorname{argmin}}\ P(\mathbf{W}_m(l)|\mathbf{H}(lL-1))$$

and the set of perturbations $\{\mathbf{W}_m(l)\}_{m=1}^{2^n_b}$ can be generated around $\mathbf{W}(l)$ in different manners, each of which define a variant of the proposed IRC-MIMO technique. Upon reception of the feedback message, both the transmitter and the receiver execute the synchronous update

$$\mathbf{W}(l+1) = \mathbf{W}_{m^*}(l)$$

This thesis considers two different assumptions about the structure of $\mathbf{W}$. The first keeps the columns of $\mathbf{W}$ orthonormal, and generates the perturbed matrices by perturbing the angle vector representing $\mathbf{W}(l)$, as in the SCGAS algorithm, but employing 2^{n_b-1} random vectors used to build diametrical perturbations. In other words, the perturbations are generated as

$$\{\mathbf{W}_{2n-1}(l) = \mathcal{M}(\boldsymbol{\theta}(l) + \mu\boldsymbol{\Delta\theta}_n), \mathbf{W}_{2n}(l) = \mathcal{M}(\boldsymbol{\theta}(l) - \mu\boldsymbol{\Delta\theta}_n)\}_{n=1}^{2^{n_b-1}}$$

where 2^{n_b-1} vectors $\{\boldsymbol{\Delta\theta}_n\}$ are generated from zero mean, unit variance i.i.d. Gaussian variables, the mapping $\mathcal{M}(\cdot)$ is defined in (3.16) and $\mathbf{W}(l) = \mathcal{M}(\boldsymbol{\theta}(l))$.

The second variant of the algorithm relaxes the orthonormality of the columns and restricts only the Frobenius norm of $\mathbf{W}$, thus only controlling the average transmit power. In this case, the candidates are generated as

$$\left\{\begin{aligned}\mathbf{W}_{2n-1}(l) &= \sqrt{\frac{N_b}{||e^{\mu\mathbf{K}_m}\mathbf{W}(l)||_F^2}}e^{\mu\mathbf{K}_m}\mathbf{W}(l),\\ \mathbf{W}_{2n}(l) &= \sqrt{\frac{N_b}{||e^{-\mu\mathbf{K}_m}\mathbf{W}(l)||_F^2}}e^{-\mu\mathbf{K}_m}\mathbf{W}(l)\end{aligned}\right\}_{n=1}^{2^{n_b-1}}$$

The performance of the two- stream transmit-assisted IRC algorithm is shown in Fig. 3.12. A system with $N_t = 4, N_r = 3, n_b = 6$ and $f_{\mathbf{b}} = 1500$ Hz is considered, with carrier frequency of 2.1 GHz and mobile speed of 3 km/h. The interfering signal is modeled as a complex-valued Gaussian vector with the same temporal autocorrelation properties of $\mathbf{H}$, as in Section 3.4.1. It is seen that using non-orthogonal columns in $\mathbf{W}$ ("IRC-EXPM") can improve the

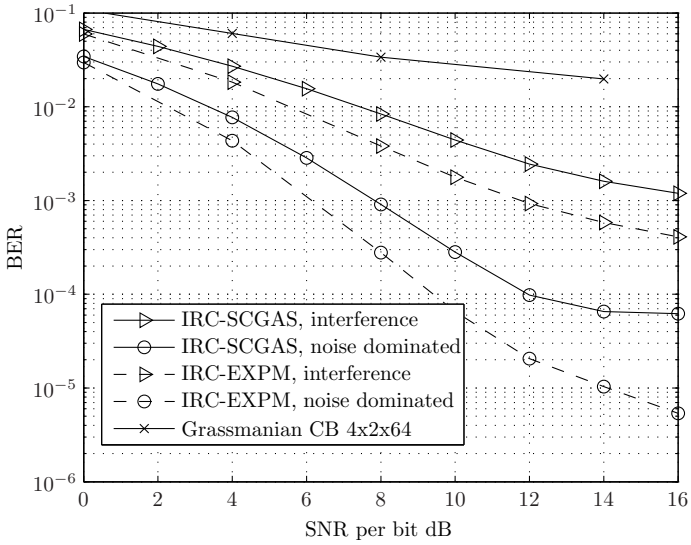

Figure 3.12. BER performance of double-stream closed-loop transmit-assisted SU-MIMO under strong interfering signals, $N_t = 4, N_r = 3, n_b = 6$, 16QAM symbols on each stream and mobile speed of 3 km/h. A slowly-varying interference signal arrives at the receiver with the same SNR of the data signal. The "IRC-SCGAS" adaptation enforces orthonormality on $\mathbf{W}$, whereas "IRC-EXPM" can have non-orthogonal transmit beams. "Grassmannian $4 \times 2 \times 64$" denotes the usage of a 6-bit codebook from [78] and the cost function from (3.17)

performance, when compared to the strictly orthonormal precoder of the "IRC-SCGAS" method. Unlike the single stream case, the performance advantage of the recursive methods over the use of static codebooks is much larger.

3.5 Closed-loop enhancements for quasi-orthogonal space-time codes

It is known that limited feedback closed-loop techniques may outperform space-time codes when the feedback latencies are not too large [49]. This suggests that whenever timely CSIT is available, the system should use closed-loop transmission techniques instead of space-time codes. Nevertheless, performance enhancements to space-time codes have been studied in the literature, for example in [10, 62, 71, 75, 85].

This section addresses transmit-weight optimization for systems employing quasi-orthogonal space-time codes (QOSTBC). The purpose of the feedback is to allow the transmitter to modify its transmit weights, in order to optimize the link-level performance.

The proposed algorithm is tailored for the case of linear receivers, unlike constellation rotations which restore the full-diversity of QOSTBC under non-linear maximum-likelihood receivers [114]. Furthermore, the design considers only one set of N_t weights, which contrasts with more general frameworks such

as [62, 75], where higher-dimension beamforming matrices may be employed.

Consider a single beam system where the associated data stream is processed in blocks of T symbols. Assuming that the channel stays constant over the T symbol periods, the following equivalent system for the block $k' = \lfloor k/T \rfloor$ can be written:

$$\tilde{\mathbf{y}}(k') = \boldsymbol{\mathcal{H}}(k', l)\mathbf{s}(k') + \tilde{\mathbf{n}}(k') \tag{3.18}$$

where $\tilde{\mathbf{y}}(k'), \tilde{\mathbf{n}}(k') \in \mathbb{C}^{N_t N_r \times 1}$ are the respective vectorized versions of the T received and noise vectors, and $\boldsymbol{\mathcal{H}}(k', l)$ is the equivalent channel matrix for the STBC which includes the channel matrix $\mathbf{H}(k')$, the beamforming weights $\mathbf{w}(l) = [w_1^*(l), \ldots, w_{N_t}^*(l)]^\dagger$ and the encoding strategy of the STBC. This system is illustrated in Fig. 3.13 and can be written for a broad class of STBC, in particular for some high-performance full-rate QOSTBC, which are considered here.

Assuming a linear combiner $\boldsymbol{\Omega}(k', l) \in \mathbb{C}^{N_t N_r \times T} = [\boldsymbol{\omega}_1(k', l) \ldots \boldsymbol{\omega}_T(k', l)]$, the filtered received signal $\mathbf{z}(k', l) \in \mathbb{C}^{T \times 1}$ reads

$$\begin{aligned} \mathbf{z}(k', l) &= \boldsymbol{\Omega}^\dagger(k', l)\tilde{\mathbf{y}}(k') \\ &= \boldsymbol{\Omega}^\dagger(k', l)\boldsymbol{\mathcal{H}}(k', l)\mathbf{s}(k') + \boldsymbol{\Omega}^\dagger\tilde{\mathbf{n}}(k') \\ &= \mathbf{G}(k', l)\mathbf{s}(k') + \mathbf{n}'(k', l) \end{aligned} \tag{3.19}$$

where the equivalent gain matrix $\mathbf{G}(k', l) = \boldsymbol{\Omega}^\dagger(k', l)\boldsymbol{\mathcal{H}}(k', l)$ includes both the transmit weights $\mathbf{w}(l)$ and the channel $\mathbf{H}(k')$ for the block under consideration. The matrix $\mathbf{G}$ can be written explicitly, given a choice of QOSTBC and linear receiver. For example, the ABBA scheme [127] with a linear minimum mean square error (LMMSE) receiver and $N_r = 1$ give

$$\begin{aligned} \boldsymbol{\mathcal{H}}_{ABBA} &= \begin{bmatrix} w_1h_1 & w_2h_2 & w_3h_3 & w_4h_4 \\ w_2^*h_2^* & -w_1^*h_1^* & w_4^*h_4^* & -w_3^*h_3^* \\ w_3h_3 & w_4h_4 & w_1h_1 & w_2h_2 \\ w_4^*h_4^* & -w_3^*h_3^* & w_2^*h_2^* & -w_1^*h_1^* \end{bmatrix} \\ \boldsymbol{\Omega}^\dagger(k', l) &= [\boldsymbol{\mathcal{H}}^\dagger(k', l)\boldsymbol{\mathcal{H}}(k', l) + \sigma^2\mathbf{I}]^{-1}\boldsymbol{\mathcal{H}}^\dagger(k', l) \end{aligned} \tag{3.20}$$

A link-level metric can be computed upon the equivalent system (3.19). For example, the average BEP can be computed as

$$\begin{aligned} P(\mathbf{w}(l)|\mathbf{H}(k'), \mathbf{Q}(k', l)) &= \frac{1}{T}\sum_{i=1}^{T} P\left(\frac{\gamma_i(k', l)}{\mathcal{B}}\right) \\ \gamma_i(k', l) &= \frac{|G_{ii}(k', l)|^2}{\boldsymbol{\omega}_i^\dagger(k', l)\mathbf{Q}(k', l)\boldsymbol{\omega}_i(k', l) + \sum_{m \neq i} |G_{im}(k', l)|^2} \\ & \quad i = 1, \ldots, T \\ \mathbf{Q}(k', l) &= \mathrm{E}\left\{\mathbf{n}'(k', l)\mathbf{n}'(k', l)^\dagger\right\} \end{aligned} \tag{3.21}$$

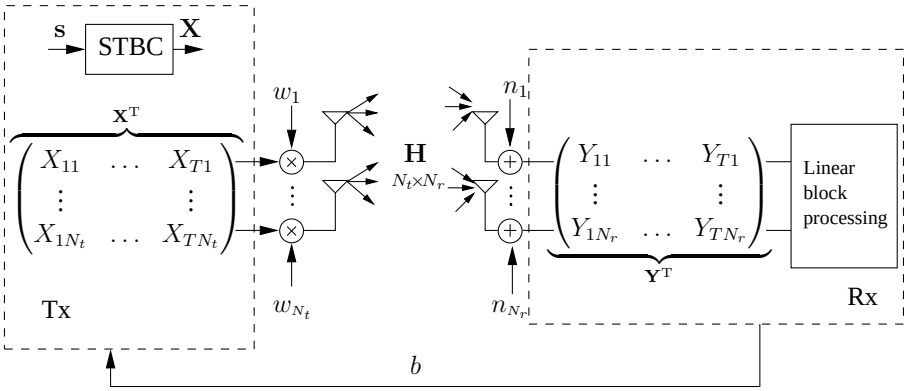

Figure 3.13. System model for closed-loop STBC.

where $P(\cdot)$ denotes the conditional BEP expression for the symbol constellation in use, and $\mathcal{B}$ is the number of bits per symbol.

Since the transmitter has no knowledge of $\mathbf{H}$, the weight adaptation proceeds based only on the feedback message $\mathbf{b}(l)$ and the current value $\mathbf{w}(l)$. A simple recursive procedure is to probe the cost function in diametrical directions, and inform the result through a single feedback bit. This is conceptually similar to the single-bit method [14] and the thesis contributions presented in Section 3.3.2. Different ways to build the candidates give origin to variants of the algorithm. For example, one can use the parametrization of the SCGAS algorithm given in Section 3.3.2. Alternatively, one can use the additive perturbation followed by vector normalization, as in [14].

The performance of the proposed algorithm is depicted in Figs. 3.14 and 3.15. We denote two variants of the algorithm as "ABBA-SCGAS" and "ABBA-BZ", when the weight updates are done as in the SCGAS algorithm of Eq. (3.14) and as in Banister and Zeidler's stochastic gradient [14]. The sensitivity of the BER performance with respect to the step size parameter μ is found to depend on both the SNR conditions and the recursion type. This is shown in Fig. 3.14. The BER performance of the optimized step size parameter is shown in Fig. 3.15, where a $N_t = 4, N_r = 1$ SU-MISO system with carrier frequency 2.1 GHz and single-bit feedback messages sent with frequency 1500 Hz is simulated, when the mobile speed is 3 km/h. The performance is compared to that of existing single-bit closed-loop algorithms for STBC, such as the group-coherent codes of Akhtar and Gesbert [10], which is based on OSTBC blocks.

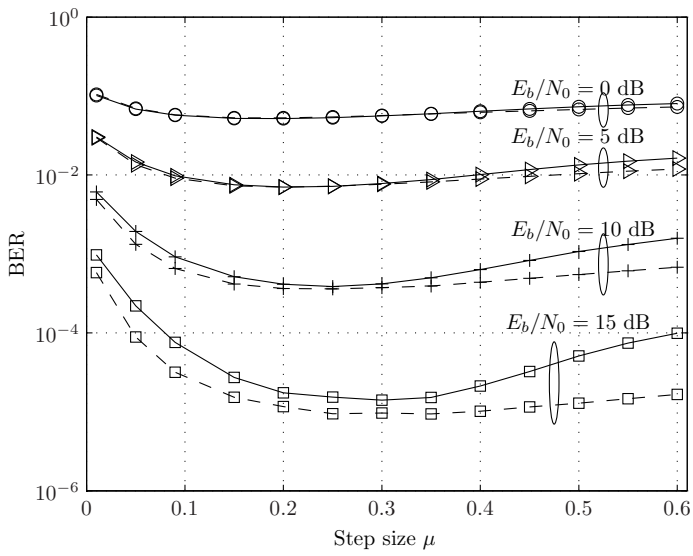

Figure 3.14. Sensitivity of the closed-loop assisted ABBA QOSTBC to the convergence step parameter μ, in $N_t = 4$ SU-MISO with mobile speeds of 3 km/h, and QPSK symbols. Dashed and solid lines represent the ABBA-BZ and ABBA-SCGAS recursions, respectively.

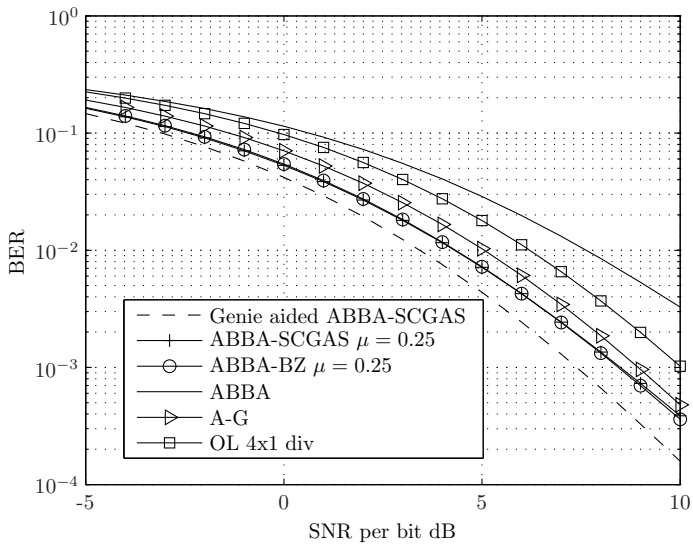

Figure 3.15. BER performance of single-bit closed-loop QOSTBC methods with QPSK symbols, for $N_t = 4, N_r = 1$ spatially white Rayleigh channels and mobile speed of 3 km/h. "A-G" refers to the group-coherent codes of [10], ABBA is the QOSTBC from [127], and "ABBA-BZ", "ABBA-SCGAS" denote the proposed method under different weight recursions. The "genie aided" method has no feedback capacity restriction and can update the transmit weights to optimize Eq. (3.21) on every channel use. "OL-4x1" refers to the fourth-order open-loop diversity performance.

4. Channel feedback methods for MU-MIMO

This chapter considers channel feedback methods for usage within MU-MIMO transmission. We depart from the feedback of orthonormal matrices in Chapter 3, and consider the quantization and feedback of the complete channel matrix $\mathbf{H}$. This allows the proposed feedback methods to be used with any existing MU-MIMO technique, such as those outlined in Chapter 5.

We propose low computational complexity partial update (PU) feedback methods, and analyze their performance in terms of tracking error and BER loss incurred by MU-MIMO techniques, compared to the full-CSI case. Similarly to the PU eigenbeamforming methods proposed in Section 3.3.3, the sequential variants are suitable for slowly-fading channels and moderate antenna array sizes, while the ranked variants provide better tracking gain in some conditions, depending on the feedback and fading rates, and the antenna array sizes.

4.1 Introduction

In this thesis, closed-loop channel tracking and channel feedback methods refer to the transmission of digital information that enable the transmitter to keep an estimate of the forward channel experienced by a given user.

Let $\mathbf{H}_t$ be the channel matrix available at the transmitter. In order to study and compare different methods, we will assume that the receiver knows its channel matrix $\mathbf{H}$ exactly. Therefore, there is no channel estimation error at the receiver, and the mismatch between $\mathbf{H}_t$ and $\mathbf{H}$ stems from limitations in the feedback frequency $f_{\mathbf{b}}$ and the number of bits per feedback message, n_b.

The closed-loop tracking error is defined as

$$e(k) = ||\mathbf{H}(k) - \mathbf{H}_t(k)||_F^2 \tag{4.1}$$

where channel variations within a slot imply that this error is in general not zero. We are interested, however, on scenarios where the intra-slot variations

are negligible, and that the channel can be tracked, producing statistics of $e(k)$ that are "small". In this thesis, the quality of the channel feedback algorithm is evaluated in terms of the BER performance loss incurred by MU-MIMO solutions using $\mathbf{H}_t(k)$ as inputs, compared to the case when $\mathbf{H}(k)$ is employed (full CSI case).

Furthermore, the tracking considered here refers to the tracking of the instantaneous channel $\mathbf{H}(k)$, as opposed to the feedback of channel statistical measures, such as mean and covariance channel feedback methods.

It should be stressed that the channel feedback methods are motivated by providing CSI for MU-MIMO methods designed for full-CSI, where the complete channel matrix is required. If only the right singular vectors of the matrix are required, then closed-loop subspace tracking methods should be used, because no extraneous information is transmitted. This is the case of single-user systems employing adaptive transmit eigenbeamforming as described in Chapter 3, and also for MU-MIMO systems with partial-CSI block diagonalization, as presented in Chapter 5.

4.2 Overview of existing methods

For TDD systems, the forward channel can be measured based on uplink transmission. This can, however, become outdated in case of delays due to multiuser scheduling. For FDD systems such as those considered in this thesis, on the other hand, the reciprocity only applies to long-term statistical properties of the channel. Quantization methods found in literature typically discard some features of the channel matrix, depending on the specific purpose of the feedback. For example, for MU-MIMO based on the block diagonalization technique [118, 120], only a quantized version of the null-space basis of the channel is fed back. Alternatively, a vectorized and normalized version of the channel matrix can be fed back. This has been proposed in conjunction with static codebooks in [116]. Generic approaches that aim to convey the whole matrix, however, are somewhat rare. One approach has been proposed for MISO systems in [63], where the instantaneous channel is parametrized as coefficients in the eigenbasis of the long-term covariance matrix, and it is these coefficients that are independently quantized and fed back, with the underlying assumption that the channel covariance matrix changes very slowly and thus it can be transmitted and updated with negligible feedback overhead.

On the other hand, the use of predictive vector quantization (PVQ) techniques such as [11, 25, 48, 81] can also be considered. These techniques employ

a linear filter to form an estimate of the instantaneous channel based on previous filter outputs, which can also be computed at the transmitter, provided that the difference between the actual channel and the filter output is fed back, and that the filter stability is not compromised by the quantization errors. However, specific codebook selection criteria and design criteria have not been proposed for wireless channels, to the best of our knowledge. It is also noteworthy that the computational complexity of the channel predictor and codebook selection and lookup can become burdensome, as the channel matrix sizes increase.

Explicit channel feedback techniques for MU-MIMO are still not part of the 3GPP LTE standard (at least up to release ten). One possibility for MU-MIMO in release ten is to use zero-forcing MU-MIMO (similar to the BD technique [118]) based on CSIT provided by the SU-MIMO precoding codebooks as defined in [2] (release ten).

4.3 Channel feedback based on single-bit tracking structures

In this section, we present a channel tracking method based on the use of single-bit tracking structures for real-valued quantities, in conjunction with partial-update filtering concepts. First, the single-bit tracking structure employed is introduced, and thereafter the proposed methods are described.

4.3.1 Tracking of real-valued quantities based on single-bit messages

A single-bit real-valued tracker (SBRVT) is defined as a structure containing a real-valued number that represents the current state of the tracked quantity, and a positive real-valued number representing the current value of the step size. The tracker interprets a single binary quantity as its input, updating the current value according to the step size, and automatically adjusting the latter according to a predetermined rule. The structure is known from pioneering works in digital coding of voice signals [39, 59]. Further elaboration on the subject is found in [153], where the step size adaptation is improved by introducing a delay and allowing the feedback decision to be based in a "future" sample. The step size adaptation rules have been employed later in the context of adaptive filtering by Harris [42], who employs a variable step size for each coefficient in a modified LMS filter. The binary variables for adaptation are signed gradient estimates, as opposed to 1-bit quantization of

the error between the quantized and true speech waveform.

More formally, let the real-valued quantity of interest be $s(l)$, the tracked value $\hat{s}(l)$, the value at the transmitter $\hat{s}_t(l)$ and $\Delta(l)$ the step size to be used on slot l. Let the error between the incoming sample and the current value of the tracker be $e_{sbrvt}(l) = s(l) - \hat{s}(l)$. We define auxiliary counters n_e and n_d, where n_e is the number of consecutive equal signs of e_{sbrvt} that have occurred and n_d is the number of consecutive different signs. For example, if $\text{sign}[e_{sbrvt}(l)] = \text{sign}[e_{sbrvt}(l-1)] \neq \text{sign}[e_{sbrvt}(l-2)]$, then $n_e = 2$ and $n_d = 0$.

The tracking performed by the SBRVT is defined as

1. Compute $e_{sbrvt}(l) = s(l) - \hat{s}(l)$.

2. Examine the sign change counters: if $\text{sign}[e_{sbrvt}(l)]$ equals $\text{sign}[e_{sbrvt}(l)]$, increase n_e by one and set n_d to zero. Otherwise, increase n_d by one and set n_e to zero.

3. Apply the step size control: if $n_e \geq m_1$, then set $\Delta(l+1)$ to $\max\{\alpha_u \Delta(l), \Delta_{\max}\}$, where $\alpha_u > 1$ is a fixed parameter. Otherwise, check if $n_d \geq m_0$. If so, then set $\Delta(l+1)$ to $\min\{\alpha_l \Delta(l), \Delta_{\min}\}$ with $\alpha_l < 1$ fixed beforehand.

4. Do the update: set $\hat{s}(l+1)$ to $\hat{s}(l) + \text{sign}[e_{sbrvt}(l)]\Delta(l+1)$. Encode the binary decision $\text{sign}[e_{sbrvt}(l)]$ in the feedback message

5. Transmitter: upon receiving the feedback message, extract the single bit associated to $\text{sign}[e_{sbrvt}(l)]$.

6. Transmitter: apply step size control for the transmit-side step size $\Delta_t(l+1)$.

7. Transmitter: update $\hat{s}_t(l+1)$ to $\hat{s}_t(l) + \text{sign}[e_{sbrvt}(l)]\Delta_t(l+1)$.

We restrict ourselves to the case of $m_0 = m_1 = 1$ for the tracking configuration. Furthermore, we consider the special case $\alpha_l = 1/\alpha_u$ for simplicity and denote α_l as α. While it has been shown in [153] that lifting this restriction can be beneficial in a static case, it is not clear if this benefit is also present in a tracking scenario.

In the following sections, we will make use of $2N_tN_r$ SBRVT structures, to track the real and imaginary parts of the elements of the channel matrix $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$. For systems where $n_b < 2N_tN_r$, not all the trackers can be up-

dated on a given slot, and a partial update rule is employed, which determines which trackers receive update information through the feedback message. This is conceptually similar to partial update filtering [7, 136], where only some coefficients of a digital filter are updated. The partial-update strategy has also been applied in closed-loop eigenbeamforming [P. IV], [P. III], where the goal is to enable good tracking of the dominant eigenbeam under limited feedback conditions.

Let us index the real-valued components of the channel matrix by defining $\hbar_j, j = 1 \ldots 2N_t N_r$ as

$$\begin{aligned} \hbar_j &= \begin{cases} \mathrm{Re}\,(H_{mn}) & j \text{ odd} \\ \mathrm{Im}\,(H_{mn}) & j \text{ even} \end{cases} \\ m &= 1 + \left\lfloor \frac{j-1}{2} \right\rfloor \bmod N_r \\ n &= 1 + \left\lfloor \frac{j-1}{2N_r} \right\rfloor \end{aligned} \tag{4.2}$$

that is, the real and imaginary parts of H_{mn} are listed consecutively, and the enumeration proceeds along the rows of $\mathbf{H}$, from the leftmost to the rightmost column.

4.3.2 Convergence of SBRVT on static channels

Given a fixed real-valued quantity to be fed back, the SBRVT mechanism converges, up to a precision given by the minimum step size parameter. The following summarizes the convergence analysis presented in[P. VIII].

Let us consider the output of the tracker in response to a fixed input $\hbar$, drawn from a known distribution $F_\hbar(\cdot)$. Let $\hat{\hbar}(l)$ be the output of the algorithm at update instant l. We say that the algorithm converges to $\Delta_{\min}$ if there exists an integer $v_t > 0$ such that $\Delta(l) \leq \Delta_{\min}$, $\forall l > v_t$. Without loss of generality, we assume that $\hbar > 0$, and therefore the algorithm traces a monotonically increasing curve until it surpasses the value of $\hbar$. The following three-branch function models the rise of the algorithm output under a stream of positive input bits, which is related to the aforementioned first segment of $\hat{\hbar}(\cdot)$

$$\begin{aligned} q(n) &= \begin{cases} n\Delta_{\min} & n \leq m_1 \\ m_1\Delta_{\min} + \left(\frac{1-\alpha^{n-m_1+1}}{1-\alpha} - 1\right)\Delta_{\min} & m_1 < n \leq m_1 + p \\ q(m_1+p) + \Delta_{\max}(n-p-m_1) & m_1 + p < n \end{cases} \\ p &= \left\lfloor \ln\left\{\frac{\Delta_{\max}}{\Delta_{\min}}\right\} \frac{1}{\ln(\alpha)} \right\rfloor \end{aligned} \tag{4.3}$$

where $\alpha = \alpha_u = 1/\alpha_l$.

We characterize the learning curve $\hat{\hbar}(\cdot)$ by t monotonic segments and t ver-

tices, where a vertex is a pair $v, \hbar(v)$ defined as

$$\text{sign}\{\hat{\hbar}(v-1) - \hat{\hbar}(v)\} = \text{sign}\{\hat{\hbar}(v+1) - \hat{\hbar}(v)\} \tag{4.4}$$

In other words, the curve increases monotonically up to the first vertex, after which the sign of the error changes between vertices, up to vertex v_t, after which the error is bounded by $\Delta_{\min}$. In the following, propositions will summarize the results of the analysis.

The following establishes a sufficient condition for convergence, assuming $m_0 = 1$.

Proposition 1. *Given a static channel $\hbar$, a sufficient condition for the SBRVT algorithm to reach a vertex v_t such that $\Delta(l) \leq \Delta_{\min}, l > v_t$ is $m_1 \leq \lceil \alpha \rceil$, $m_0 = 1$, where $\alpha \equiv \alpha_u = 1/\alpha_l$.*

Assuming the sufficient condition for convergence $m_1 \leq \lceil \alpha \rceil$, $m_0 = 1$, the location of the first vertex can be computed as follows.

Proposition 2. *Given a static channel $\hbar$ and the conditions $m_1 \leq \lceil \alpha \rceil$, $m_0 = 1$, the SBRVT algorithm reaches the first vertex at the update time v_1 given by*

$$v_1 = \begin{cases} \left\lceil \frac{\hbar}{\Delta_{\min}} \right\rceil & \hbar \leq m_1 \Delta_{\min} \\ m_1 + \left\lfloor \ln\left\{ \left(\frac{\hbar - m_1 \Delta_{\min}}{\Delta_{\min}} + 1 \right)(\alpha - 1) + 1 \right\} \frac{1}{\ln(\alpha)} \right\rfloor & m_1 \Delta_{\min} < \hbar < q(m_1 + p) \\ m_1 + p + \left\lceil \frac{\hbar - q(m_1+p)}{\Delta_{\max}} \right\rceil & q(m_1 + p) < \hbar \end{cases} \tag{4.5}$$

where the function $q(\cdot)$ and the number p are given in (4.3).

Now, the number of vertices and their locations can be determined. This gives the convergence time.

Proposition 3. *The number of vertices t such that $\Delta(l) \leq \Delta_{\min}, l > v_t$ is*

$$t = \begin{cases} 1 & \hbar \leq m_1 \Delta_{\min} \\ v_1 - m_1 & m_1 \Delta_{\min} < \hbar < q(m_1 + p) \\ \left\lceil \ln\left\{ \frac{\Delta_{\max}}{\Delta_{\min}} \right\} \frac{1}{\ln(\alpha)} \right\rceil & q(m_1 + p) < \hbar \end{cases} \tag{4.6}$$

Furthermore, the vertices and their associated outputs are

$$v_i = v_{i-1} + \left\lceil \frac{r_{i-1}}{s} \right\rceil \quad r_i = \left| r_{i-1} - \left\lceil \frac{r_{i-1}}{s} \right\rceil s \right| \quad i = 2, \ldots, t$$

$$r_1 = q(v_1) - \hbar, \quad s = \max(\Delta_0/\alpha^{i-1}, \Delta_{\min}) \quad \hat{\hbar}(v_i) = \hbar + (-1)^{i+1} r_i$$

$$\Delta_0 = \begin{cases} \Delta_{\min} & \hbar \leq m_1 \Delta_{\min} \\ \Delta_{\min} \alpha^{v_1 - m_1} & m_1 \Delta_{\min} < \hbar < q(m_1 + p) \\ \Delta_{\max} & q(m_1 + p) < \hbar \end{cases} \tag{4.7}$$

with v_1 computed from (4.5) and $q(\cdot)$, p defined in (4.3).

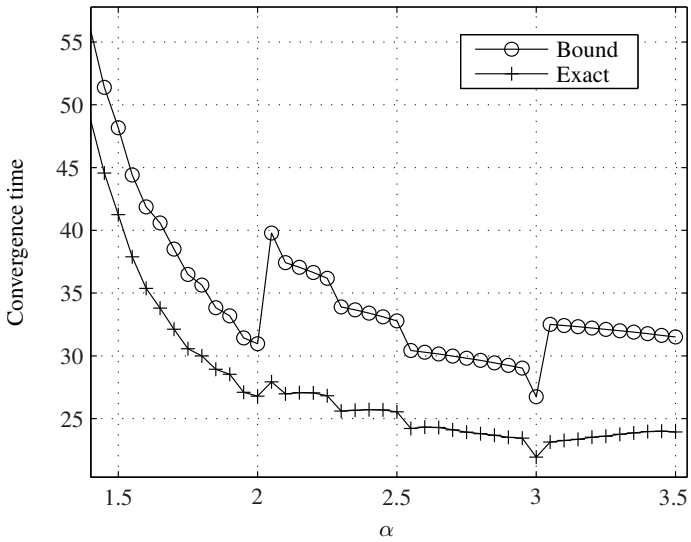

Figure 4.1. Expected convergence time for single-bit real-valued trackers in real-valued static channels as a function of α. The upper bound is computed according to (4.8), with Gaussian distributed channels of zero mean and variance 0.5.

To conclude the analysis in static channels, we give an upper-bound for the convergence time, and the expected value of this bound, assuming that the cumulative distribution function (CDF) is known.

Proposition 4. *The convergence time is upper-bounded by* $N(\hbar) = 1 + v_t \leq 1 + v_1 + \lceil \alpha \rceil (t-1)$. *Given a symmetric PDF of* $\hbar$ *with CDF* $F_\hbar(\cdot)$, *the expected value of the convergence time bound is given by*

$$\begin{aligned}
\int_{-\infty}^{\infty} N(x) f_\hbar(x) dx \leq & 2\sum_{i=1}^{m_1}(i+1)\{F_\hbar[i\Delta_{\min}] - F_\hbar[(i-1)\Delta_{\min}]\} \\
& + 2\sum_{i=1}^{p}[i(m_1+1)+1]\{F_\hbar[g_2(i)] - F_\hbar[g_2(i-1)]\} \\
& + 2[(p+1)(m_1+1)+1]\{F_\hbar[q(m_1+p)] - F_\hbar[g_2(p)]\} \\
& + 2\sum_{i=1}^{\infty}[i + m_1(p+1) + p + 1]\{F_\hbar[g_3(i)] - F_\hbar[g_3(i-1)]\} \\
g_2(i) = & m_1\Delta_{\min} + \frac{\Delta_{\min} e^{(i+1)\ln(\alpha)} - \alpha\Delta_{\min}}{\alpha - 1} \\
g_3(i) = & i\Delta_{\max} + q(m_1+p)
\end{aligned} \tag{4.8}$$

with the auxiliary function and number $q(\cdot)$, p *defined in (4.3).*

Note that the infinite summation term can be safely truncated in many cases at a value $i_{\max}$ such that $F_\hbar[g_3(i_{\max})] \approx 1$. Care is needed in case of heavy-tailed distributions such as the Cauchy-Lorentz distribution. An example of the expected convergence time is shown in Fig. 4.1 for the case of h being Gaussian-distributed with zero mean and variance 0.5.

4.3.3 Sequential partial update channel feedback (SPUCF)

The simplest partial-update rule is based on a round-robin update of the trackers, depending on the number of available feedback bits. This fixed sequence is known at transmitter and receiver, and therefore the feedback message does not need to identify the trackers for which the update bits are intended.

Let $\mathcal{I}$ represent the index of the tracker that was updated last in the previous slot. The current update considers n_b trackers with indices

$$\{1 + \quad (\mathcal{I} + n) \bmod 2N_r N_t\}_{n=1}^{n_b}$$

which implements a circular list from which n_b elements are read at the time.

In case n_b is allowed to be larger than $2N_tN_r$, the update becomes $\lfloor n_b/(2N_tN_r) \rfloor$ full updates, followed by a partial update of $n_b \bmod 2N_tN_r$ trackers. This resembles a step in data reuse filtering [90], and constitutes an alternative for scenarios where a partial-update does not provide sufficient tracking performance.

4.3.4 Ranked partial update channel feedback (RPUCF)

The performance of the SPUCF mechanism deteriorates whenever the effective time between tracker updates compromises the tracking ability of the SBRVT structures. The associated effective tracker update frequency is approximately

$$f_{tr} \approx f_{\mathbf{b}} \left\lfloor \frac{n_b}{2N_tN_r} \right\rfloor$$

in other words, a given tracker belongs to a group of n_b trackers that is updated once every $2N_tN_r/n_b$ slots if the ratio is an integer number.

One interpretation of the performance degradation mechanism is that errors accumulate in the trackers because the ratio of f_{tr} to the fading rate is not high enough. While we assume that the autocorrelation function of all the elements of $\mathbf{H}$ is the same, this does not imply that a given time all the channel coefficients have exactly the same variation rate (the average variation rate is assumed to be the same, though). This suggests that at a given update instance, some trackers have larger error than others, and should be updated before those with smaller mismatch. This leads to the idea of a selective update of the trackers, which consists of a tracker ranking, followed by a partial-update. The resulting method is called the ranked partial-update channel feedback (RPUCF). Another view is to consider the trackers as a real-valued adaptive filter, where the filter coefficients with larger lag are prioritized for update.

A simple metric for ranking is the squared error of the tracker, where the error is the difference between the true value and the current value of the tracker. It must be noted, however, that due to the distributed nature of the tracking, it is not possible to update the n_b highest ranked trackers with n_b feedback bits. Doing so would require $\lceil \log_2 \binom{2N_rN_t}{n_b} \rceil$ bits for signaling which trackers are to be updated, along with n_b information bits with the update information.

To keep the signaling overhead controlled, the choice of trackers to be updated is limited to choosing among N_g groups of N_{tr} trackers. The feedback message is therefore composed of N_{tr} update bits and $\lceil \log_2 N_g \rceil$ bits denoting the chosen group. Let the N_g groups be represented by a set $\{\mathbf{c}_n \in \{0,1\}^{2N_tN_r \times 1}\}_{n=1}^{N_g}$ of binary vectors with Hamming weight N_{tr}, where the non-zero entries of vector $\mathbf{c}_n$ indicate the trackers that are to be updated, should the group n be chosen.

The group error is chosen as a ranking criterion, and is defined as

$$e_n = \sum_{m=1}^{2N_tN_r} (\hbar_m - \hat{\hbar}_m)^2 \delta(1, c_{n,m}) \quad , \quad n = 1, \ldots, N_g \tag{4.9}$$

where $c_{n,m}$ is the element m of $\mathbf{c}_n$ and $\delta(\cdot,\cdot)$ is one if both arguments are equal, and zero otherwise.

The RPUCF algorithm proceeds then as follows:

1. The receiver ranks the N_g groups by computing the errors $\{e_n\}_{n=1}^{N_g}$ from (4.9).

2. The group n^* is chosen, which has the largest error. The group index is binary encoded in the feedback message $\mathbf{b}$.

3. The receiver computes the updates for the trackers in group n^* and encodes them in $\mathbf{b}$.

4. The transmitter retrieves n^* and the update bits from $\mathbf{b}$, and applies the update to the trackers in group n^*.

An alternative metric is the error after update, which consists of the error in the trackers after an hypothetical update. This is conceptually similar to the ranked partial-update eigenbeamforming presented in [P. IV], where the received power after the update is used as a ranking metric. Whether the increased complexity provides a significant benefit for the channel feedback problem, is a topic for further research.

The choice of n_b, N_{tr} and N_g is system dependent. On one hand, very short feedback messages may not allow for any signaling overhead. On the other, increasing the antenna array sizes increases the effective time between updates, which increases the likelihood of the round-robin update missing the most critical tracker updates. A final consideration is that too large signaling overhead decreases the performance of RPUCF compared to SPUCF. Indeed, it is possible to find settings where using $n_b = \lceil \log_2 \binom{2N_rN_t}{N_{tr}} \rceil + N_{tr}$ performs worse than SPUCF at the same feedback rate. In this thesis, finding sets $\{\mathbf{c}_n\}_{n=1}^{N_g}$ of N_g binary vectors yielding good performance for some antenna array sizes and feedback frequency has been done by numerical search procedures. We consider sets of binary vectors of size $2N_tN_r$ with "large" minimum Hamming distance, and choose according to the resulting distribution of the total error $e(k)$ as defined in (4.1). The requirement for a large minimum Hamming distance is justified intuitively by the fact that each group should represent a set of trackers as different as possible from the others, and thus provide a selection range close to the case of "genie-aided" signaling of the highest ranked N_{tr} trackers. This problem resembles a binary vector quantization problem. An example of the improved tracking performance due to the ranked partial update is shown in Fig. 4.2, where an $N_t = N_r = 4$ matrix is tracked using feedback message frequency $f_{\mathbf{b}} = 1500$ Hz and each message carrying 13 bits, with a carrier frequency of 2.1 GHz and a mobile speed of 3 km/h. The reference is taken as the perfect ranking for the update of $N_{tr} = 8$ trackers per feedback message, which requires $\lceil \log_2 \binom{32}{8} \rceil + 8 = 32$ bits per feedback channel use.

4.3.5 Computational complexity

In the following, we give a brief outline of the computational complexity of the SPUCF and RPUCF, in comparison with that of a solution based on PVQ on the $N_tN_r \times 1$ vectorized channel.

The computational complexity of the SPUCF algorithm equals n_b times the computational complexity of the update of one tracker. This can be considered a fixed complexity, which is furthermore low. Based on the description in Section 4.3.1, we have a) error sign calculation: one real-valued subtraction, and b) automatic step size control: update the counters n_e, n_o (two real-valued additions), increase or decrease the step size $\Delta(l)$ and compare to the bounds (one real-value multiplication, one real-valued subtraction). Thus, the computational complexity is $\mathcal{O}(n_b)$, where in general $n_b < N_tN_r$. In case of RPUCF, the cost can be larger, as the error and step size control have to be evaluated

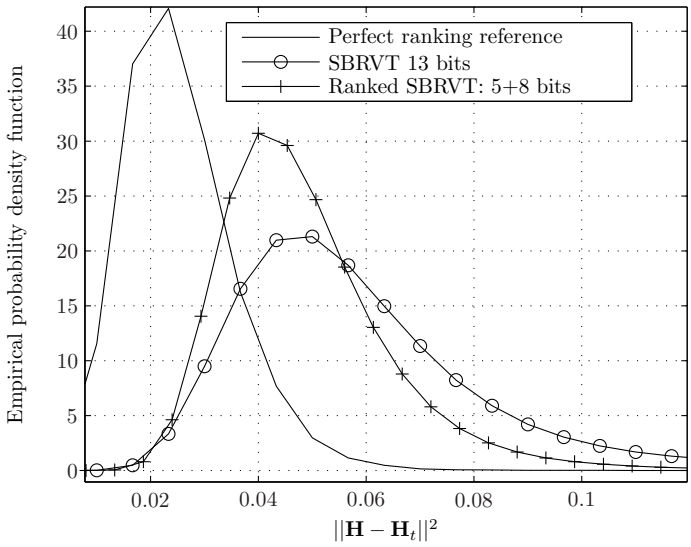

Figure 4.2. Channel tracking error performance of partial update channel feedback algorithm: sequential versus ranked PU operating at 13 bits per feedback channel use in SU-MIMO with $N_t = N_r = 4$ i.i.d. channels and mobile speed of 3 km/h. The ranked version employs 5 bits for signaling the trackers to be updated, and the perfect ranking reference for the case of eight trackers per update has a feedback requirement of 32 bits per feedback message.

for $N_g N_{tr}$ trackers.

On the other hand, the complexity of a PVQ scheme operating on a vectorized channel has two main components (c.f., e.g., [25]):

- Calculation of the predicted channel vector. This is the cost of one or more (up to the predictor order) matrix-vector multiplications, with sizes $N_t N_r$, i.e., $\mathcal{O}(N_t^2 N_r^2)$

- The quantization of the $N_t N_r \times 1$ residue. This depends on the codeword selection strategy and the codebook size, which is 2^{n_b}.

Therefore, it is clear that SPUCF has a lower complexity than a PVQ-based approach. On the other hand, it is likely that the performance of SPUCF and RPUCF will deteriorate faster with increasing mobile speeds and antenna array sizes, compared to higher computational-complexity PVQ schemes operating at the same feedback rate.

4.4 Connections to closed-loop eigenbeamforming

This section presents channel feedback algorithms based on single-beam eigenbeamforming methods. This approach decouples the channel feedback problem into scalar feedback and vector feedback problems. For the first problem,

the tracking can be performed with the single-bit SBRVT technique of Section 4.3.1. For the second, on the other hand, any closed-loop eigenbeamforming can be used, but special care needs to be taken for the cases where the phase ambiguity of the dominant eigenbeam is exploited.

Let the normalized vectorized channel $\tilde{\mathbf{h}} \in \mathbb{C}^{N_t N_r \times 1}$ and its norm $n_{\tilde{\mathbf{h}}} > 0$ be

$$\tilde{\mathbf{h}} \triangleq \frac{\mathrm{vec}(\mathbf{H})}{||\mathbf{H}||_F}, \quad n_{\tilde{\mathbf{h}}} \triangleq ||\mathbf{H}||_F^2 \tag{4.10}$$

where $\mathrm{vec}(\cdot)$ converts the argument to a single column vector by stacking the columns vertically.

The norm of the channel is tracked with a SBRVT structure, as any of the coefficients $\hbar_j$ of Section 4.3.1.

Let the tracked version of $\tilde{\mathbf{h}}$ be $\hat{\tilde{\mathbf{h}}}$. Several alternatives exist to implement the closed-loop tracking:

1. Define a pseudo channel correlation matrix $\mathbf{R} \triangleq \tilde{\mathbf{h}}\tilde{\mathbf{h}}^\dagger$, whose dominant eigenvector can be written as $e^{j\theta}\tilde{\mathbf{h}}, \theta \in \mathbb{R}$. This approach requires to compensate for the factor $e^{j\theta}$, so that the transmitter can compute $\tilde{\mathbf{h}}_t = e^{j\alpha_t}\hat{\tilde{\mathbf{h}}} \approx \tilde{\mathbf{h}}$, where α_t is the transmit side version of the phase correction term defined as $\alpha = -\arg(\tilde{\mathbf{h}}^\dagger\hat{\tilde{\mathbf{h}}})$. The SBRVT structure can be used to track α with single-bit feedback messages. The closed-loop eigenbeamforming algorithms apply directly to $\mathbf{R}$, for example the contributions described in Section 3.3.

2. Employ stochastic perturbations to track $\tilde{\mathbf{h}}$. The receiver generates 2^{n_b} perturbations around $\hat{\tilde{\mathbf{h}}}$, and chooses the one closest in Euclidean norm to $\tilde{\mathbf{h}}$. The perturbed vectors can be generated for example through premultiplication with matrix exponentials (c.f. Section 3.4.2) or by adding perturbation vectors and normalizing, as in [14].

The performance of the proposed channel feedback algorithms is illustrated by comparing the BER performance of MU-MIMO solutions with perfect CSIT and that of imperfect CSIT provided by the channel feedback mechanisms. Figure 4.3 shows the case of $N_u = 2$ users, each with two receive antennas, and a transmitter employing $N_t = 4$ transmit antennas to send one stream of 16-QAM symbols to each user. The carrier frequency is 2.1 GHz and both receivers are assumed to move at 3 km/h and have the same SNR conditions. The MU-MIMO solutions are the block diagonalization (BD) scheme of [118] and a recursion minimizing the total BER obtained from the SINRs given by (3.5). The CSIT is provided by the SPUCF algorithm of Section 4.3.3 and

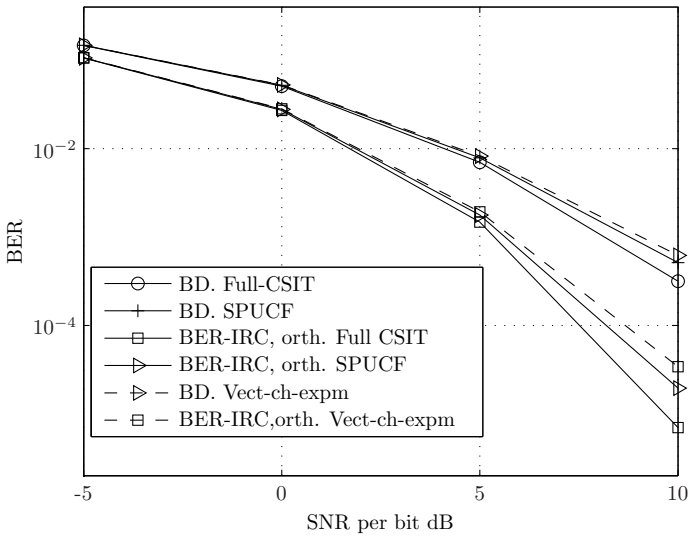

Figure 4.3. BER performance of $N_u = 2$ users in $N_t = 4, N_r = 2$ MU-MIMO at 3 km/h with $n_b = 7$ bits per feedback message per user. "BD" denotes the block diagonalization technique of [118] and "BER-IRC, orth" minimizes the average BER based on the SINRs from (3.5), with orthonormal precoder $\mathbf{W}$. "SPUCF" is the method proposed in Section 4.3.3. "Vect-ch-expm" denotes channel feedback of the vectorized channel by means of perturbations through matrix exponentials, as described in Section 4.4.

the vectorized channel matrix of (4.10), when matrix exponentials are used as described in (II) above, with a single bit used to track the norm $n_{\tilde{\mathbf{h}}}$ from (4.10). Both schemes send seven feedback bits per slot. It is seen that the performance degradation due to the channel feedback mechanisms is not significant for uncoded BER levels above 0.01. At higher mobile speeds, however, the performance degradation can be significant and MU-MIMO based on SPUCF may perform worse than MU-MIMO based on a vectorized channel. This can be alleviated in some scenarios, where more efficient recursive methods can be designed. For example, when only one stream per user is transmitted $N_b = N_u$, a decentralized weight recursion avoiding explicit channel feedback can be used. This contribution of the thesis is described in Section 5.2.

5. Limited-feedback MU-MIMO methods

Multiuser MIMO techniques aim to increase the spectral efficiency of the communications system by transmitting to more than one user, over the same frequency and time period. In TDD systems, the required CSIT can be acquired through feedback links, as described in Chapter 4.

Multiuser multiplexing techniques utilizing spatial filtering techniques can be classified as linear and non-linear precoding techniques. This chapter gives an introduction to both types of solutions. Thereafter, the contribution of the thesis to limited-feedback linear precoding MU-MIMO with linear receivers is described: we propose a decentralized solution where the feedback is used to adjust each user's access point transmit weights, rather than to transmit the channel matrix or a related orthogonal matrix.

5.1 Overview of existing methods

5.1.1 Linear precoding full-CSI MU-MIMO techniques

This section reviews some existing linear precoding techniques for MU-MIMO. These algorithms are of practical interest because they typically have low computational complexity requirements for the mobile receiver, which can employ linear filters to process the received signals.

In a MU-MIMO scenario, the transmitter employs the precoding matrix to separate the streams transmitted to the N_u receivers. For each user i, the signals intended for users $m \neq i$ form a disturbance signal known as multiuser interference (MUI). Let MUI_i denote the MUI that affects the signal intended for user i. This can be written as

$$MUI_i(k) = \mathbf{H}_i(k) \left(\sum_{m \neq i} \mathbf{W}_m(l) \mathbf{x}_m(k) \right) \quad i = 1 \ldots N_u, l = k \bmod L \tag{5.1}$$

where N_u is the number of users and L is the number of symbol periods per

slot. In the following, we discuss how the MUI is mitigated by different MU-MIMO solutions.

The MMSE-based solutions, for example, consider the MUI as additional noise that depends on the precoders, but do not treat it separately from the thermal Gaussian noise. Given knowledge of the channel matrices $\mathbf{H}_i$ and the noise levels σ_i^2, the precoding matrix and the receive filters $\mathbf{\Omega}_i$ can be designed to optimize a certain criterion, e.g., the sum-MMSE which is given by

$$\sum_{i=1}^{N_u} \mathrm{E}\left\{||\mathbf{x}_i - \mathbf{\Omega}_i^\dagger \mathbf{y}_i||^2\right\} = \sum_{i=1}^{N_u} \mathrm{E}\left\{||(\mathbf{I} - \mathbf{\Omega}_i^\dagger \mathbf{H}_i \mathbf{W}_i)\mathbf{x}_i - \mathbf{\Omega}_i^\dagger \mathbf{n}_i'||^2\right\} \tag{5.2}$$

where $\mathbf{n}_i'$ from (2.2) includes the MUI and depends explicitly on the precoding and channel matrices. Optimization techniques can be applied to minimize a function of the per-user MSEs (e.g., the sum-MSE or a weighted sum of them). All of these approaches arrive at a solution by means of iterative algorithms. Typically only the precoder design is addressed, since the optimal combiners $\mathbf{\Omega}_i$ are the linear MMSE receivers for the channel $\mathbf{H}_i\mathbf{W}_i$, with the noise including the MUI [12, 155]. When using downlink formulations such as (5.2), the optimization variables (the precoder matrices) are tightly coupled and the iterations may converge to local minima [12, 155]. On the other hand, formulations based on the downlink (broadcast) and uplink (multiple access channel) duality [37, 84] can avoid this ambiguity and iteratively obtain the true MMSE solution, see for example [84, 111].

Other approaches consider the MUI more explicitly in the precoder design. For example, Spencer's sum-capacity Block Diagonalization (BD) scheme [118] decouples the system into N_u SU-MIMO subsystems. The precoders are designed in a factored fashion: one term nulls the MUI towards other users, and the other optimizes the MIMO transmission of the own SU-MIMO subsystem. The precoders are expressed as $\mathbf{W}_i = \mathbf{W}_{i,a}\mathbf{W}_{i,b}\mathbf{D}_i$, with $\mathbf{D}_i$ a diagonal matrix with positive entries, which balances powers between the streams and constrains the average transmitted power. The first term is an orthonormal matrix such that the MUI from (5.1) is nulled:

$$\mathbf{H}_{m\neq i}\mathbf{W}_{i,a} = \mathbf{0}$$

The other factors are chosen to optimize the own channel after the separation factor has been applied, that is $\mathbf{W}_{i,b}\mathbf{D}_i$ can be designed like a SU-MIMO precoder for the channel $\mathbf{H}_i\mathbf{W}_{i,a}$.

It is known that the null-MUI or zero-forcing condition imposes a performance penalty, specially at low to medium SNR conditions. For example, SNR-dependent solutions like the MMSE precoders of [12] can vastly outperform the BD. On the other hand, however, the BD solution lends itself to

limited-feedback implementation by means of orthonormal matrices. Indeed, any feedback method for closed-loop eigenbeamforming can be used for feeding back an orthonormal basis for the null space of the own channel matrix $\mathbf{H}_i$. Full-CSI MU-MIMO work derived from the BD can be found in the Regularized Block Diagonalization (RBD) [120], which optimizes the precoders while taking into account both the MUI and the SNR conditions.

Earlier work by Peel proposed and analyzed the idea of inverting the channel at the precoding stage in MU-MISO systems [94]. Consider the case $N_r = 1$ and let $\mathbf{H}_p = [\mathbf{h}_1 \dots \mathbf{h}_{N_u}]^T$ be the matrix containing the MISO channels of all the users. The ZF precoder then uses the Moore-Penrose pseudoinverse [38] of $\mathbf{H}_p$ to form the transmitted signal. For example, if $N_t > N_u$ the received signals are

$$\mathbf{H}_p \left[\frac{1}{\mathbf{x}^\dagger (\mathbf{H}_p \mathbf{H}_p^\dagger)^{-1} \mathbf{x}} \mathbf{H}_p^\dagger (\mathbf{H}_p \mathbf{H}_p^\dagger)^{-1} \right] \mathbf{x} + \mathbf{n} = \frac{1}{\mathbf{x}^\dagger (\mathbf{H}_p \mathbf{H}_p^\dagger)^{-1} \mathbf{x}} \mathbf{x} + \mathbf{n}$$

which shows the well-known SINR degradation in case of minimum eigenvalue of $\mathbf{H}_p$. Furthermore, Peel showed that for $N_t = N_u$, the capacity of this channel inversion scheme does not scale up with N_t.

One way to improve the performance is to use the regularized channel inversion, where the precoder becomes

$$\frac{1}{\mathbf{x}^\dagger (\mathbf{H}_p \mathbf{H}_p^\dagger + \alpha \mathbf{I}_{N_u})^{-1} \mathbf{x}} \mathbf{H}_p^\dagger (\mathbf{H}_p \mathbf{H}_p^\dagger + \alpha \mathbf{I}_{N_u})^{-1}$$

for some $\alpha > 0$

This overcomes the aforementioned capacity floor for growing N_t, and improves the performance significantly [94] even though the MUI is now not zero.

5.1.2 Non-linear precoding full-CSI MU-MIMO techniques

Non-linear precoding techniques can outperform the linear precoding counterparts, similarly to non-linear receiver techniques outperforming the linear ones ([64], Ch. 20). One line of research in this area stems from an Information Theory result by Costa [24] known as the "dirty paper coding" (DPC) theorem. The other follows the works of Tomlinson [128] and Harashima [41], who devised transmission strategies to pre-equalize intersymbol interference (ISI) in early digital transmission lines.

Costa showed that the capacity of an additive Gaussian channel does not change due to the presence of an interfering signal, as long as the transmitter has perfect non-causal knowledge of it. Furthermore, his paper suggested

that the transmitter should adapt its signal to the interference, instead of attempting to cancel the interfering signal explicitly. Indeed, the transmitted signal is computed as an auxiliary signal minus a fraction of the known interference, where the auxiliary signal is computed based on the input signal and the interference, so that the transmitter output is approximately orthogonal to the known interference. This has inspired successive encoding transmission schemes, where signals for different users are encoded in a given order, and each signal can be adapted to those that have been encoded before - very much like DPC system: the previously encoded signals are interference for the one currently being encoded. For a given encoding order $\pi_1, \ldots, \pi_{N_u}$ and transmitted vectors $\mathbf{s}_1, \ldots, \mathbf{s}_{N_u}$ with positive semi definite covariance matrices $\mathbf{R}_i = \mathrm{E}\left\{\mathbf{s}_i \mathbf{s}_i^\dagger\right\}$, the achievable rates are [37]

$$R_{\pi_k} = \log_2 \frac{|\mathbf{I} + \mathbf{H}_{\pi_k}(\sum_{j \geq k} \mathbf{R}_{\pi_j})\mathbf{H}_{\pi_k}^\dagger|}{|\mathbf{I} + \mathbf{H}_{\pi_k}(\sum_{j > k} \mathbf{R}_{\pi_j})\mathbf{H}_{\pi_k}^\dagger|} \quad k = 1, \ldots, N_u$$

where the noise is assumed to have unit variance per component and the total transmit power is constrained to be $\mathrm{tr}\{\mathbf{R}_1 + \mathbf{R}_2 + \ldots\} \leq P$. Due to non-concavity and non-convexity of the expression above, the capacity region can be found a) by exhaustive search over the space of covariance matrices that meet the power constraint or b) by exploiting the duality between the broadcast and the multiple-access channels [37]. An extensive literature on the Information Theory aspects of the MU-MIMO channel exists, see, e.g., [20, 135].

Tomlinson-Harashima precoding, on the other hand, employs interference pre-subtraction in conjunction with modular arithmetic. The transmitted signal can be viewed as the data signal, plus the known interference and perturbation vector that can be removed at the receiver through modular arithmetic. Connections between DPC and THP can be established, see for example [30] and the references therein. The order of the user encoding is important in both schemes.

The independent original work by Tomlinson [128] and Harashima [41] solved the problem of transmit pre-equalization of the ISI. The idea is to presubtract the ISI of the transmitted signal, to yield an overall SISO channel transfer function of unity. This assumes that the channel can be modeled as a linear time invariant filter and that the transmitter has perfect knowledge of the filter characteristics. The transmit power is controlled by the usage of a modulo device at both transmitter and receiver. At the transmitter, the device adds an integer multiple of the modulo parameter, to keep the signal within nominal ranges. This shift, unknown to the receiver, is eliminated after the modulo

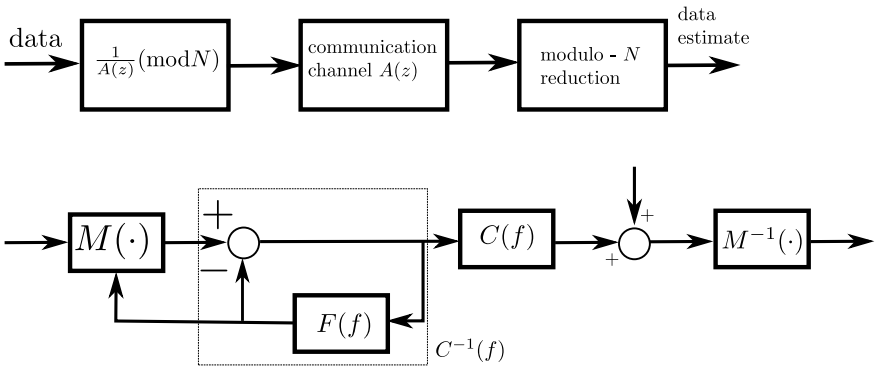

Figure 5.1. System model for the original work of Tomlinson (above) and Harashima (below) for pre-equalization of ISI in early digital modems. $A(z)$ represents the $\mathcal{Z}$ transform of the discrete-time input data signal, $C(f)$ and $F(f)$ represent the Fourier transform of the channel and feedback filters. $M(\cdot)$ represents the modulo devices introduced to constraint the output power after the presubtraction of the inter-symbol interference.

device has been applied to the received signal, provided that the noise does not push the signal across the non-linear boundaries of the modulo function. The systems of Tomlinson and Harashima are depicted in Fig. 5.1

The TH principles can be applied to the MU-MIMO case. The basic idea is that the vectors for different users are encoded sequentially, and the interference from the previously encoded data is presubtracted. Therefore, the encoding order is of paramount importance. Indeed, the users with the weakest channels should be encoded towards the end, so that they experience smaller MUI. The precoders can be designed by iterative duality-based algorithms such as those in [83]. Explicit designs for the case of MU-MISO can be found in [64], Ch. 20. The system model for THP MU-MIMO in downlink is depicted in Fig. 5.2. The modulo devices $M_\tau(\cdot)$ play a very important role in this scheme, as they prevent the transmit power to grow uncontrolled and they further randomize the transmitted signal vectors $\boldsymbol{v}_i$ [64, 83].

Consider a two-user case with ordering $\pi_1 = 1$ and $\pi_2 = 2$. In this case, user 1 will suffer from MUI due to the signal intended for user 2, depending on how much the precoder decouples the transmission for each user. Intuitively, the pre-subtraction adds one more degree of freedom, since the MUI suffered by user 2 needs not to be cancelled by the linear precoding. Let $M_\tau(\cdot)$ denote the modulo device and be defined as in (5.4). The symbol vector estimate for user 2 reads:

$$\begin{aligned}\hat{\mathbf{x}}_2 &= M_\tau\left(\mathbf{\Omega}_2^\dagger\mathbf{H}_2\mathbf{W}_2\boldsymbol{v}_2 + \mathbf{\Omega}_2^\dagger\mathbf{H}_2\mathbf{W}_1\boldsymbol{v}_1 + \boldsymbol{\nu}_2\right)\\ &= M_\tau\left(\mathbf{\Omega}_2^\dagger\mathbf{H}_2\mathbf{W}_2 M_\tau(\mathbf{x}_2 - \mathbf{\Omega}_2^\dagger\mathbf{H}_2\mathbf{W}_1\boldsymbol{v}_1) + \mathbf{\Omega}_2^\dagger\mathbf{H}_2\mathbf{W}_1\boldsymbol{v}_1 + \boldsymbol{\nu}_2\right)\\ &= M_\tau\left(\mathbf{G}_2\mathbf{x}_2 + (\mathbf{I}-\mathbf{G}_2)\mathbf{\Omega}_2^\dagger\mathbf{H}_2\mathbf{W}_1\boldsymbol{v}_1 + \mathbf{G}_2\mathbf{a}_2 + \boldsymbol{\nu}_2\right)\\ &= M_\tau\left(\mathbf{G}_2\mathbf{x}_2 + \mathbf{a}_2 + \boldsymbol{\nu}_2'\right),\ \boldsymbol{\nu}_2' = (\mathbf{I}-\mathbf{G}_2)\mathbf{\Omega}_2^\dagger\mathbf{H}_2\mathbf{W}_1\boldsymbol{v}_1 + \boldsymbol{\nu}_2 + (\mathbf{G}_2-\mathbf{I})\mathbf{a}_2\end{aligned} \tag{5.3}$$

where $\mathbf{G}_2 = \mathbf{\Omega}_2^\dagger\mathbf{H}_2\mathbf{W}_2$, $\mathbf{a}_2 = \tau(\mathbf{a}_{2r} + j\mathbf{a}_{2i})$ is the term added by the modulo device at the transmitter with $\mathbf{a}_{2r}, \mathbf{a}_{2i}$ integer vectors, and $\boldsymbol{v}_1, \boldsymbol{v}_2$ are the output of the modulo-τ devices, as illustrated in Fig. 5.2.

It can be seen from (5.3) that the presubtraction of the MUI contributed by the first user is only fully effective if $\mathbf{I} = \mathbf{G}_2 = \mathbf{\Omega}_2^\dagger\mathbf{H}_2\mathbf{W}_2$. It is also clear that the modulo device at the receiver experiences additional noise in case $\mathbf{I} \neq \mathbf{G}_2$.

Both Tomlinson and Harashima described the modulo device as adding or subtracting an integer multiple of the modulus parameter, so that the result lies within given bounds. The function is commonly defined in literature as [46, 64]

$$M_\tau(s) := s - \left\lfloor \frac{s}{\tau} + \frac{1}{2} \right\rfloor \tau \tag{5.4}$$

and it is therefore clear that if $|s| < \tau/2$, then $M_\tau(s + m\tau) = s$ when m is an integer. However, if noise ϵ is added to s, then $M_\tau(s + \epsilon + m\tau) = s + \epsilon$ as long as $\epsilon \in [-\min(s) - \tau/2, \tau/2 - \max(s))$. Larger noise samples will cause an error of absolute value at most equal to τ.

With this in mind, let us assume that the entries of $\mathbf{G}_2\mathbf{x}_2$ have real and imaginary parts still within $[-\tau/2, \tau/2)$, and similarly the I/Q parts of the components of $\boldsymbol{\nu}_2$ are within $[-s_{\min} - \tau/2, \tau/2 - s_{\max})$. Here, $s_{\min}, s_{\max}$ represent the minimum and maximum value of the real and imaginary parts of the symbol constellation entries. Therefore, the symbol vector estimate from (5.3) becomes

$$\hat{\mathbf{x}}_2 = \mathbf{G}_2\mathbf{x}_2 + \boldsymbol{\nu}_2'$$

In contrast, user one does not benefit from MUI-presubtraction. Its signal vector estimate reads:

$$\begin{aligned}\hat{\mathbf{x}}_1 &= M_\tau\left(\mathbf{\Omega}_1^\dagger\mathbf{H}_1\mathbf{W}_1\boldsymbol{v}_1 + \mathbf{\Omega}_1^\dagger\mathbf{H}_1\mathbf{W}_2\boldsymbol{v}_2 + \boldsymbol{\nu}_1\right)\\ &= M_\tau\left(\mathbf{\Omega}_1^\dagger\mathbf{H}_1\mathbf{W}_1\mathbf{x}_1 + \mathbf{\Omega}_1^\dagger\mathbf{H}_1\mathbf{W}_2 M_\tau(\mathbf{x}_2 - \mathbf{\Omega}_2^\dagger\mathbf{H}_2\mathbf{W}_1\boldsymbol{v}_1) + \boldsymbol{\nu}_1\right)\\ &= M_\tau\left(\mathbf{G}_1\mathbf{x}_1 + \mathbf{C}_{1,2}\mathbf{x}_2 - \mathbf{C}_{1,2}\mathbf{C}_{2,1}\mathbf{x}_1 + \mathbf{C}_{1,2}\mathbf{a}_2 + \boldsymbol{\nu}_1\right)\\ &= M_\tau\left((\mathbf{G}_1 - \mathbf{C}_{1,2}\mathbf{C}_{2,1})\mathbf{x}_1 + \mathbf{C}_{1,2}(\mathbf{x}_2 + \mathbf{a}_2) + \boldsymbol{\nu}_1\right)\end{aligned} \tag{5.5}$$

where $\mathbf{G}_1 = \mathbf{\Omega}_1^\dagger \mathbf{H}_1 \mathbf{W}_1$, $\mathbf{C}_{1,2} = \mathbf{\Omega}_1^\dagger \mathbf{H}_1 \mathbf{W}_2$ and $\mathbf{C}_{2,1} = \mathbf{\Omega}_2^\dagger \mathbf{H}_2 \mathbf{W}_1$. In this case, the MUI could include a term in the own signal vector $\mathbf{x}_1$. However, the MUI towards user one can be cancelled with the precoder, i.e., $\mathbf{C}_{1,2} = \mathbf{0}$. Since the presubtraction is used, using the precoder to cancel the MUI of user one towards user two is not necessary. This gives additional degrees of freedom in choosing the precoders.

Hochwald [46], on the other hand, proposed a deterministic perturbation to the data vector in conjunction with the modulo devices, thus extending the MU-MISO work from [94]. The perturbation can be applied to both the channel inversion or regularized channel inversion precoder described in Section 5.1.1. This perturbation is selected to balance the received SINRs, taking into account the small singular values of the inverse channel. The perturbation takes the form of a vector with integer multiples of a certain number $\tau > 0$ which can then be removed at the receivers by applying the modulo-τ operation. For example, the received signal in the case of the perturbation and the inversion precoder is

$$\frac{1}{(\mathbf{x} + \tau\boldsymbol{\ell})^\dagger (\mathbf{H}_p \mathbf{H}_p^\dagger)^{-1} (\mathbf{x} + \tau\boldsymbol{\ell})} \mathbf{x} + \boldsymbol{\eta}$$

where the complex-valued integer vector $\boldsymbol{\ell}$ is designed to minimize the term $(\mathbf{x} + \tau\boldsymbol{\ell})^\dagger (\mathbf{H}_p \mathbf{H}_p^\dagger)^{-1} (\mathbf{x} + \tau\boldsymbol{\ell})$ and maximize the SINRs, and $\boldsymbol{\eta}$ is the Gaussian noise after the modulo device.

The perturbation is thus chosen jointly for all the users and it is unknown to the receivers. The use of modular arithmetic at the receivers allows to discard the perturbation vector, as long as the noise levels do not to push the received signal over the non-linear boundaries of the modulo functions (see [46] for details). The perturbation vector depends on the data vector and needs to be found numerically for each channel use. Since the perturbation cannot be expressed as a linear combination of the data symbols, the transmission scheme is non-linear. The search over integer vectors can be done with a "sphere encoder" algorithm, for example based on [47]. Hochwald mentions the possibility to apply lattices as in [30] and related work, but this is not explored.

While it is known that the non-linear precoding techniques can yield large performance advantage over linear precoding schemes under full-CSI assumptions, the performance in presence of imperfect CSI can deteriorate severely. Several aspects of this performance deterioration have been studied in the available literature, and efforts have been made to overcome these shortcomings. For example, [82] analyzes the impact of imperfect CSI on the per-

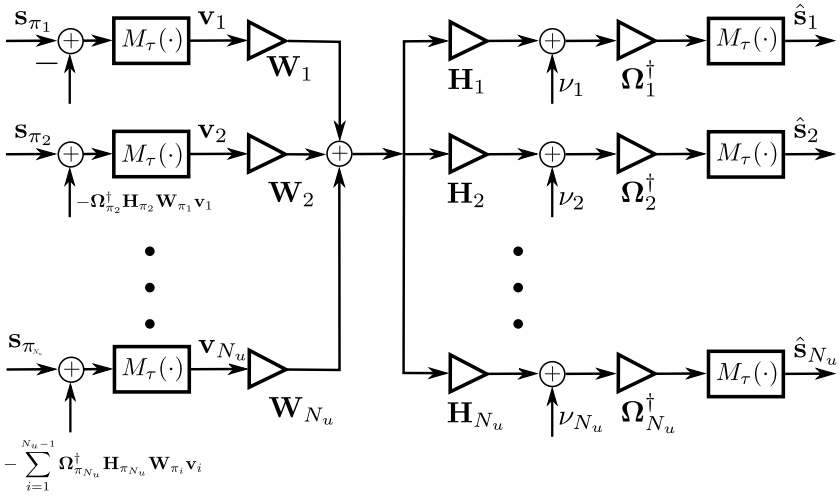

Figure 5.2. System model for Tomlinson-Harashima precoding applied to MU-MIMO by Mezghani et al. [83].

formance of the vector perturbation technique [46], and proposes a variation which is effective even in the case of finite dynamic ranges at the receivers.

5.1.3 Existing limited-feedback MU-MIMO techniques

The solutions described in the previous sections are designed under the assumption that the transmitter knows the channel matrices of all the users perfectly. In FDD systems, the quality of the CSI is limited by the feedback channel characteristics and the fading rates.

One type of limited-feedback MU-MIMO found in literature is based on the BD and its variations. Indeed, the minimum requirement for implementing BD is that each user feeds back an orthonormal matrix which is an approximation of the null-space basis of its channel matrix [118]. Another possibility is to vectorize and normalize each channel matrix and feed back quantized versions of the N_tN_r-size vectors. This has the advantage of preserving some power information in the form of power ratio between receive antennas, but it loses the relative SNR conditions among the users because of the normalization step. Rate analysis show that for static codebooks, the codebook size must be proportional to the SNR, in order to avoid error and capacity floors [99, 116]. Alternatively, recursive structures can be implemented, as long as the channel coherence time allows for convergence and tracking. One possibility is to design hierarchical codebook structures [67]. Other possibilities include switched codebooks [86] and rotation and scaling adaptive codebooks [97].

Based on the opportunistic beamforming scheme [134], another codebook-based solution for single stream per user is presented in [129]. This is used in combination with scheduling. Each receiver feeds back both a chosen code-

book vector and the expected SINR that would be obtained with the chosen precoding vector. The receiver does not know which vectors the other users would choose and therefore can only compute an expected SINR, given the other vectors in the codebook. In this case, the receive filter is computed to optimize the SINR, and the transmitter does not need to know it. This approach can be extended as a hierarchical codebook, to exploit temporal correlation of the channels.

Another limited-feedback MU-MIMO scheme is the per user unitary rate control (PU2RC) [66]. This is based on a unitary precoder and a scheduling algorithm. In a related scheme [65], the receivers feed back a chosen unitary matrix from a codebook, a chosen column index within the matrix and the expected SINR. This paper studies the codebook design problem for such a system.

Currently, the most advanced MU-MIMO precoding in the 3GPP FDD-LTE standard (up to release ten) is the zero-forcing precoder (similar to the BD technique [118]). This is furthermore based on the SU-MIMO static codebooks for four and eight transmit antennas. Thus, recursive channel feedback techniques or more sophisticated MU-MIMO solutions could still provide enhanced performance for future releases of the standard.

5.2 Contribution: decentralized limited-feedback MU-MIMO solution

In the following, we use the term "centralized solutions" to represent any MU-MIMO solution solely computed by the transmitter, and which is computed with either full or partial CSI. In contrast, the proposed algorithm in [P. IX] is a *decentralized* solution, where the transmitter is used as an information exchange node. The solution is tailored for the case of a single stream per user, makes use of the D-JAC algorithm of Section 3.3.4 as means of tracking the optimal beamforming vectors of each user.

The proposed decentralized alternating optimization distributed Jacobi (DAO-D-JAC) algorithm is based on a variation of the alternating optimization (AO) [18] technique. In AO, an iterative optimization is done by grouping the variables into groups, and solving a single cost function alternately for each group, while treating the others as constants. The central idea in the DAO-D-JAC is to operate in cycles: the receivers determine their beamforming vectors to maximize their own SINR, and the transmitter implements the changes. The procedure is called decentralized because each receiver cannot

include the SINRs of the other users into the optimization. Thus, there is no single objective function as in the case of AO. This implies that the convergence properties need to be analyzed separately.

The receivers may employ the IRC filter from (2.5). In the following, we assume that the same power is allocated to each user, and without loss of generality set $\mathrm{E}\left\{|x_i(k)|^2\right\} = \mathcal{P} = 1, i = 1, \ldots, N_u$. Given the channel matrices $\mathbf{H}_i$ and the noise powers σ_i^2, the SINRs of each user are

$$\gamma_i(k,l) = \mathbf{w}_i^\dagger(l)[\mathbf{H}_i^\dagger(k)\mathbf{Q}_i^{-1}(k,l)\mathbf{H}_i(k)]\mathbf{w}_i(l) \tag{5.6}$$

with the covariance matrices $\mathbf{Q}_i(k,l)$ reflecting the intra-cell interference and the thermal noise:

$$\mathbf{Q}_i(k,l) = \sum_{m \neq i} \mathbf{H}_i(k)\mathbf{w}_m(l)\mathbf{w}_m^\dagger(l)\mathbf{H}_i^\dagger(k) + \sigma_i^2 \mathbf{I}_{N_r} \tag{5.7}$$

Furthermore, a low-pass filtered covariance matrix is defined slot-wise ($k = lL - 1$) as

$$\begin{aligned} \tilde{\mathbf{Q}}_i(l) &= (1-\epsilon)\tilde{\mathbf{Q}}_i(l-1) + \epsilon\hat{\mathbf{Q}}_i(lL-1,l) \\ \tilde{\mathbf{Q}}_i(0) &= \mathbf{I}_{N_r}, \qquad \epsilon \in [0,1) \end{aligned} \tag{5.8}$$

with $\hat{\mathbf{Q}}(lL-1,l)$ an estimate of $\mathbf{Q}_i(lL-1,l)$ defined in (5.7) and $\epsilon \in [0,1)$ a parameter controlling the filter memory.

The DAO-D-JAC is summarized in Table 5.1.

Static channel simulations show that the algorithm converges for small systems, e.g., two users with two receive antennas each, and four transmit antennas. However, increasing the number of users to four shows that the DAO-D-JAC is suboptimal when compared to an MMSE solution. Nevertheless, the strength of the algorithm is that the limited-feedback implementation requires tracking only one N_t-size vector per user, as opposed to feeding back the $N_r \times N_t$ channel matrix or the orthonormal basis of the channel's null-space.

Estimation errors for the covariance matrices have not been studied, as they are implementation-dependent. This can be circumvented if the users track the weights of the other users through the forwarded angles, so that the structured part of $\mathbf{Q}_i(k,l)$ can be computed exactly at each receiver.

The performance of the proposed decentralized MU-MIMO solution is shown in Figs. 5.3 and 5.4 in a system with $N_u = 2, N_t = 4, N_r = 2, n_b = 6, f_{\mathbf{b}} = 1500$ Hz, with carrier frequency 2.1 GHz. Figure 5.3 shows that the DAO-D-JAC performs comparably to the full-CSI RBD and outperforms limited-feedback RBD implementations, specially the existing codebook-based channel feedback [116]. It is also shown that the 7-bit static codebook of [116] performs

Table 5.1. Pseudo-code for the DAO-D-JAC algorithm. This process occurs at the end of each slot $l > 0$. "U. i" denotes processing at the ith receiver, "Tx." denotes processing at the transmitter.

U. i:	Set $\mathbf{H} = \mathbf{H}_i(lL-1)$, the most recent channel sample
U. i:	Set $\hat{\mathbf{Q}}(lL-1, l)$ to the most recent estimate of $\mathbf{Q}_i(lL-1, l)$
U. i:	Set $\tilde{\mathbf{Q}}$ to the updated filtered covariance matrix $\tilde{\mathbf{Q}}_i(l)$ from (5.8)
U. i:	compute $\mathbf{R} = \mathbf{H}^\dagger \tilde{\mathbf{Q}}^{-1} \mathbf{H}$
U. i:	For each rotor in the update: compute the angle pair (α, β) based on $\mathbf{R}$ and the receive-side auxiliary matrix $\mathbf{\Phi}_i$, as in the D-JAC algorithm. Update $\mathbf{\Phi}_i$ and the own weights $\mathbf{w}_i$. Quantize (α, β) and include it in the feedback message $\mathbf{b}_i(l)$
Tx. :	For each rotor in the update of user i: retrieve (α, β) from $\mathbf{b}_i(l)$, assemble the corresponding Givens rotor $\mathbf{J}_i^{1,q}$. Update the transmit-side $\mathbf{\Phi}_i(l), \mathbf{w}_i(l)$ as in the D-JAC algorithm.

worse than a 6-bit, two-layer nested codebook (c.f. the hierarchical codebooks described in Section 3.2.2) designed with the same distortion function. Both codebook-based solutions are outperformed by recursive channel feedback schemes based on closed-loop eigenbeamforming (c.f. Section 4.4) operating at $n_b = 6$ bits per message. Note that the norm of the vectorized channel is not fed back, to keep the schemes consistent with [116]. The performance degradation as a function of the mobile speed is given in Fig. 5.4. It can be seen that the effect of the filter is significant only at high SNRs, and that the proposed DAO-D-JAC performs significantly better than the eigenbeamforming-based channel feedback methods, as the mobile speed increases.

The feedback error compensation mechanism of Section 2.4.1 can be used when the transmitter forwards the rotor angles of each user. In this case, the receiver detects when a feedback error has occurred, reverts the previous up-

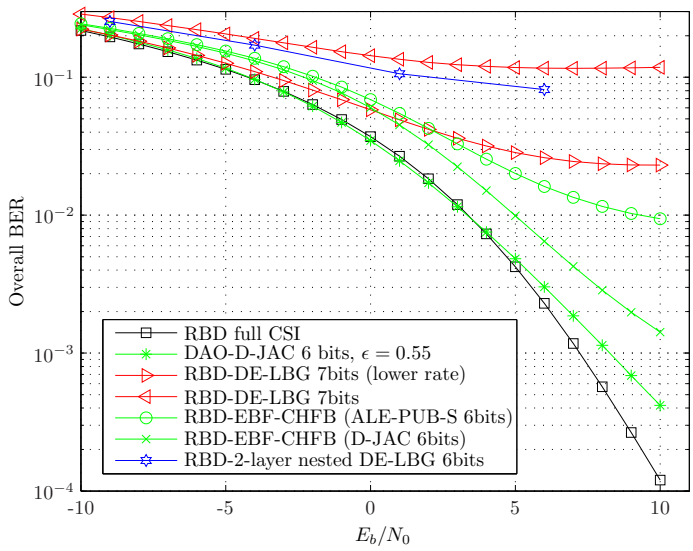

Figure 5.3. Performance of decentralized MU-MIMO with $N_u = 2, N_t = 4, N_r = 2, n_b = 6$ in i.i.d. Rayleigh at mobile speed of 3 km/h.

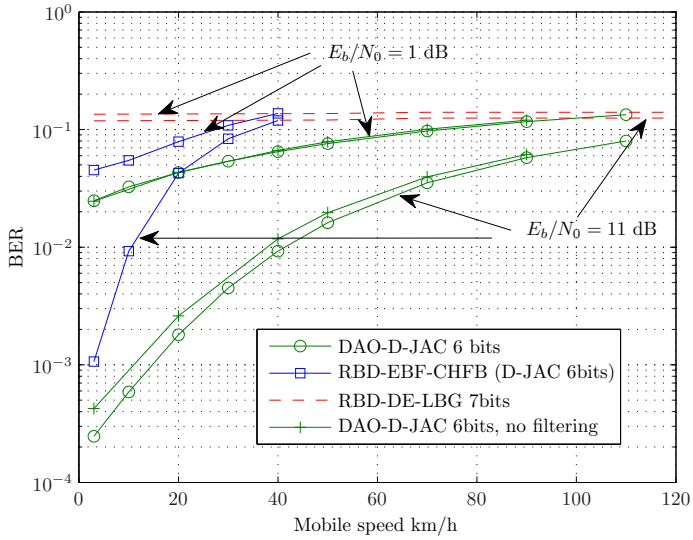

Figure 5.4. BER performance of DAO-D-JAC as a function of mobile speed

date and applies an update with a rotor built upon the forwarded angles. This keeps the transmit and receive version of the beamforming weights synchronized, but introduces random perturbations in the convergence and tracking of the alternating optimization algorithm. The proposed DAO-D-JAC can operate under moderate error rates in the feedback link. For example, the performance for uncoded BER levels of 10% is practically unchanged when the feedback link has an error rate of 1%, and becomes similar to that of the two-layer nested codebook without feedback errors (c.f. Fig. 5.3), when the feedback error rate is 10%.

Finally, we note that as a MU-MIMO solution in static channels, the DAO-D-JAC yields suboptimal performance in terms of SINRs compared to a solution optimizing the sum-MMSE defined in (5.2), in the two and four users case

with $N_t = 4$ and $N_r = 2$ antennas per user. Furthermore, this loss is larger in the case of four users. For more details, please refer to [P. IX]. We also note that the MMSE-based solution outperforms the RBD in both cases. Nevertheless, as a limited-feedback MU-MIMO mechanism, the proposed DAO-D-JAC provides BER performance advantage compared to the RBD technique with CSIT based on static and two-layer hierarchical codebooks, operating at the same feedback rate (c.f. Figs. 5.3 and 5.4).

6. Summary

Feedback methods are an important part of multiantenna transceiver techniques in systems employing frequency division duplex. They are the only means to provide timely and accurate channel side information to the access point transmitter in a point to point, or point to multipoint mobile wireless network when frequency-division duplexing is used. In order to minimize the feedback overhead, the feedback mechanisms are designed to feed back only the most relevant CSI for the application at hand. For example, a transmit beamformer requires only one complex-value weight per transmit antenna, irrespective of the number of receive antennas at the mobile. The CSIT is important in multiantenna systems, because it allows multiple data streams per user, with each user employing low-complexity linear receivers. Similarly, multiuser multiplexing is enabled when each user feeds back its channel matrix, or an approximation of it.

One of the most important limiting factors in the performance of closed-loop systems is the channel fading rate, compared to the feedback frequency and payload. This imposes a limit to the accuracy that can be achieved in the CSIT. Furthermore, the choice of recursive versus non-recursive feedback depends on whether the channel exhibits sufficient temporal coherence between two consecutive feedback messages. The effect of the outdated CSIT, on the other hand, is application-dependent.

This thesis contributes several recursive feedback schemes in the context of multiantenna communications. Emphasis has been put on the design of low computational complexity algorithms, which can outperform non-recursive alternatives operating at the same feedback frequency and payload, and incurring in a substantially higher number of operations. The performance of all the proposed algorithms is analyzed and compared to existing solutions through extensive link level simulations. Basic feedback error compensation mechanisms have been considered and shown to allow the proposals to operate

under moderate probabilities of feedback errors.

The first part of the thesis deals with single user MIMO systems. A total of eight novel algorithms are proposed, five are specific for single stream transmission, and three are for a general case of more than one stream. Variations of these three algorithms are developed, which provide performance enhancements for quasi-orthogonal space-time codes (STC) and transmit-assisted interference cancellation.

The second part of the thesis deals with the channel feedback problem. The use of partial updates (PU) in conjunction with recursive single-bit adaptive differential updates (referred to simply as "trackers") is applied to the feed back of a general MIMO matrix. A convergence analysis is given, establishing the convergence time of the trackers as function of the step size adaptation parameters. An expected value given an input channel distribution is also derived. Thereafter a selective partial update is considered. It is shown that this can provide a more efficient update, compared to the sequential PU in cases where the feedback bit budget is not so severely limited. However, the design of the tracker subsets is an open problem.

The last part of the thesis proposes a decentralized multiuser MIMO solution for the case of a single data stream per user. The proposed algorithm avoids the explicit feedback of the channel matrices, or of associated orthogonal subspaces by feeding back the optimal precoding vector of each user. This is accomplished by taking an alternating optimization approach, in which each user has access to the precoding vectors of the other users through an estimate of the interference statistics but has no access to the channel matrices of the other users. A basic feedback error compensation scheme is simulated, and shown to enable the algorithm to operate under moderate probabilities of feedback bit errors.

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Errata

Publication II

"MRC" in Figure 1 should read "Linear receiver".

Publication III

"TS25.202" in citation [8] should read "TS25.201".

Publication VIII

a) "log" in Proposition 4 should read "ln". b) In Proposition 1, "such that $\Delta(n > v_t) = \Delta_{\min}$" should read "such that $\Delta(n) \leq \Delta_{\min}, n > v_t$". c) Similarly, in the paragraph above (24), "the algorithm converges to $\Delta_{\min}$ if there exists an integer $v_t > 0$ such that $\Delta(n) = \Delta_{\min}$" should read "the algorithm converges to $\Delta_{\min}$ if there exists an integer $v_t > 0$ such that $\Delta(n) \leq \Delta_{\min}$".

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