AALTO UNIVERSITY
SCHOOL OF ELECTRICAL ENGINEERING

Department of Electronics
Lighting Unit

Rajendra Dangol

Colour Rendering Index and colour rendering of LEDs

Thesis for the degree of Master of Science
Espoo 16.02.2011

Supervisor Professor Liisa Halonen

Instructor D.Sc. Pramod Bhusal

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Supervisor: Professor Liisa Halonen

Instructor: D.Sc. Pramod Bhusal

Abstract text:
The purpose of this study was to understand CIE (Commission Internationale del´Eclairage) Colour Rendering Index (CRI) and its deficiencies. Another aim was to find out limitation of CIE CRI for LEDs. Finally, current works on colour rendering of LEDs was examined in the study.

CIE (Commission Internationale de l´Eclairage) Colour Rendering Index (CRI) is the only internationally recognized colour rendering metric. This metric expresses the colour rendering properties of light sources based on colour shift of test objects when illuminated by reference illuminant and test source. The CIE method to obtain CRI is called CIE test colour (sample) method. Despite of prominence, CIE test colour method has numerous deficiencies.

The problems of CIE test colour method became more serious when applied to white light emitting diode based sources. Various studies have indicated that even LEDs with low values of CIE CRI can produce visually appealing, vivid, and natural light. CIE technical committee TC1-62 concluded that the CIE CRI is generally not applicable to predict the colour rendering rank order of white LEDs light sources.

The colour quality of light sources is not just a colour difference. Colour discrimination, colour harmony, colour preference, colour acceptability, visual clarity, and brightness are some known dimensions of light source colour quality. For different application different light sources are suitable and relevant dimensions of light source colour quality should be used to select appropriate light sources as per the application. The universal colour rendering metric should be able to define all the dimensions of light source colour quality.

Keywords: Colour Rendering Index, CIE test sample method, colour rendering of LEDs, Colour quality of light sources.

Preface

The study has been conducted at the Lighting Unit of Aalto University School of Electrical Engineering. I would like to acknowledge and thank several people who supported me during this research.

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Lastly, I offer my regards and blessings to all of those who supported me in any respect during the completion of my thesis.

Espoo 16.02.2011 Rajendra Dangol

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List of symbols and abbreviations

Symbols

$\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$	CIE 1931 RGB colour matching functions
$\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$	colour matching functions of the CIE 1931 standard observer
$\phi_{\lambda}(\lambda)$	colour stimulus function of light
ΔC	chromaticity difference in u', 2/3v' diagram
ΔE	colour difference
ΔE^{*}_{ab}	CIELAB Colour difference
ΔE^{*}_{uv}	CIELLUV Colour difference
ΔH^{*}_{ab}	CIELAB hue-difference
ΔH^{*}_{uv}	CIELUV hue-difference
Δh_{ab}	CIELAB hue-angle
Δh_{uv}	CIELUV hue-angle
C^{*}_{uv}	CIELUV Chroma
C^{*}_{uv}	CIELAB Chroma
DC	Chromaticity difference
DC	Chromaticity difference in CIE u, v diagram
E	illuminance
h_{ab}	CIELAB hue angle
I	luminous intensity
K	a normalizing constant
L	luminance
L*	lightness
L*,a*,b*	coordinates of CIELAB colour space
L*u*v*	coordinates of CIELUV colour space
M_{CCT}	multiplication CCT factor
Q_a	general Colour Quality Scale
$Q_{a,rms}$	rms average colour quality scale
R, G, B	tristimulus values of RGB Space
$R96_a$	CIE TC 1-33 general colour rendering index
$R96_i$	CIE TC 1-33 special colour rendering indices
R_a	general colour rendering index
R_i	special Colour rendering indices
$S_A(\lambda)$	relative spectral power distribution
S_{ab}	CIELAB saturation
S_{uv}	CIELUV saturation
U*, V*	nonlinear transformation of u and v
u,v	chromaticity coordinates of CIE 1960 (u,v) chromaticity diagram
u',v'	chromaiticity coordinates of CIE 1976 uniform chromaticity diagram
u_0, v_0	uv coordinates of the white point
W*	lightness index
x, y	chromaticity coordinates of CIE x,y chromaticity diagram.
X, Y, Z	tristimulus values of CIE XYZ trichromatic system

X_w, Y_w, Z_w	tristimulus values of reference white
λ	wavelength in nanometer
ρ, γ, β	cone responses of given stimulus
ρ', γ', β'	cone responses in reference state of adaptation
ρ'_w, γ'_w, β'_w	cone responses for reference white
Φ	luminous flux

Abbreviations

CAM02	Colour Appearance Model 2002
CAT	Chromatic Adaptation Transform
CCRI	Categorical Colour Rendering Index
CCT	Correlated Colour Temperature
CHF	Colour Harmony Formula
CIE	Commission Internationale de L'eclairage (International Commision on Illumination)
CIE CRI	CIE Colour Rendering Index
CIELAB	CIE 1976 (L*a*b*)
CMF	colour matching function
CIELch	cylindrical version of CIELUV colour space
CIELUV	CIE 1976 (L*u*v*)
CQS	Colour Quality Scale
CRI	Colour Rendering Index
FCI	Feeling of contrast Index
HRI	Harmony Rendering Index
IEC	International Electrotechnical Commission.
IESNA	Illuminating Engineering Society of North America
LED	Light Emitting Diode
LER	Luminous efficacy of radiation
MCRI	Memory Colour Rendering Index
NIST	National Institute of Standards and Technology
RCRI	Rank-order based Colour Rendering Index
RGB	Red, Green, and Blue
SPD	Spectral Power Distribution
TC	Technical Committee
UCS	Uniform Colour Space

1 Introduction

Colorimetry is the science and technology for mixing and investigating colour systematically. Colour is a perception, and colour perception is not a physical quantity that can be measured. In other words colour perception is not accessible to engineering measurement. There are several systems in use to describe the colour of a source or object for e.g. CIE (Commission Internationale de L'eclairage) system, Plochere colour system, Munsell system, etc. (Murdoch 1985). In CIE system, colour is expressed in terms of three tristimulus values or in terms of two chromaticity coordinates and a measure of luminance. The result is a chromaticity diagram on which a given chromaticity coordinates can be located. This system will be discussed in details in Chapter 2.

Colour rendering is the effect of a light source on the appearance of objects in comparison with their colour appearance under a reference light source (CIE 1995). The colour rendering properties of a light source can not be assessed by visual inspection of the source or by knowledge of its colour. For this purpose, full knowledge of its spectral power distribution curve is required (Rea 2000). The objects, which look red, yellow, green, blue, or purple in daylight, might look quite different under low-pressure sodium and yellow fluorescent lamps. Under the low pressure sodium lamp objects appear more or less as one hue[1], from light to very dark. Under the yellow fluorescent lamp, more hues can be recognized, but the colour of objects will still differ considerably from their daylight colour (Rea 2000).

CIE recommended method of measuring and specifying colour rendering properties of light sources based on resultant colour shifts of test samples, named Test Colour Method (CIE 1995). It rates lamps in terms of CIE Colour Rendering Index (CIE CRI). The resultant colour shift is the combination of illuminant perceived colour shift and adaptive perceived colour shift. Illuminant perceived colour shift is the change in the perceived colour of an object caused solely by change of illuminant in the absence of any change in the observer's state of chromatic adaption[2]. Adaptive perceived colour shift is the change in the perceived colour of an object caused solely by change of chromatic adaptation (CIE 1987).

The CIE CRI has been used over 30 years and has been widely accepted (Rea 2010). Despite its prominence, CIE CRI has a number of shortcomings and problems. CIE CRI uses outdated colorimetric techniques (see Section 3.2.3 for details). However, the problems became serious enough to revise the metric after the emergence of the white light from light emitting diodes (LEDs) (CIE 2007).

A large number of visual experiments have been conducted to investigate the colour rendering properties of LED based light sources together with other conventional light sources (CIE 2007). The CIE CRI has been found extremely problematic for LEDs as the ranking given by the CIE CRI for white LEDs often contradicts the visual rankings (CIE 2007). Meanwhile, various aspects of colour rendering properties such as colour fidelity, colour discrimination, colour preference, colour harmony, etc. are being studied (CIE 2007, Bodrogi 2010, Sándor 2005, Szabó 2007).

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[1] Hue is the attribute of visual sensation according to which an area appears to be similar to one of the perceived colours, red, yellow, green, and blue, or to a combination of two of them (CIE 1987)

[2] Chromatic adaption is the adaptation by stimuli in which the dominant effect is that of different relative spectral distribution (CIE 1987) (see section 2.3 for details)

1.1 Aim of the work

The overall aim of the work was to understand CIE colour rendering index, its properties and limitations, and colour rendering of LEDs. The main focus was to find out limitation of CIE CRI for LEDs and also to study the current CIE work on colour rendering of LEDs.

2 Background

Colour is a perception. Colour can be perceived when there exist light, object, and observer. The white light which seems to be colourless, actually contains all the colours of visible spectrum. The visible spectrum is small part of electromagnetic radiation spectrum. It ranges in wavelength from about 400 to 700 nm. Electromagnetic radiation with wavelength between 400 to 700 nm is visible to human eyes. These wavelengths (400 to 700 nm) are associated with different colours. The electromagnetic radiation with wavelengths from approximately 400 to 450 nm appears violet, 450 to 490 nm appears blue, 500 to 575 nm appears green, 575 to 590nm appears yellow, 590 to 620 appears orange, and 620 to 700 appears red. Therefore, colour is closely linked to the spectral characteristics of light.

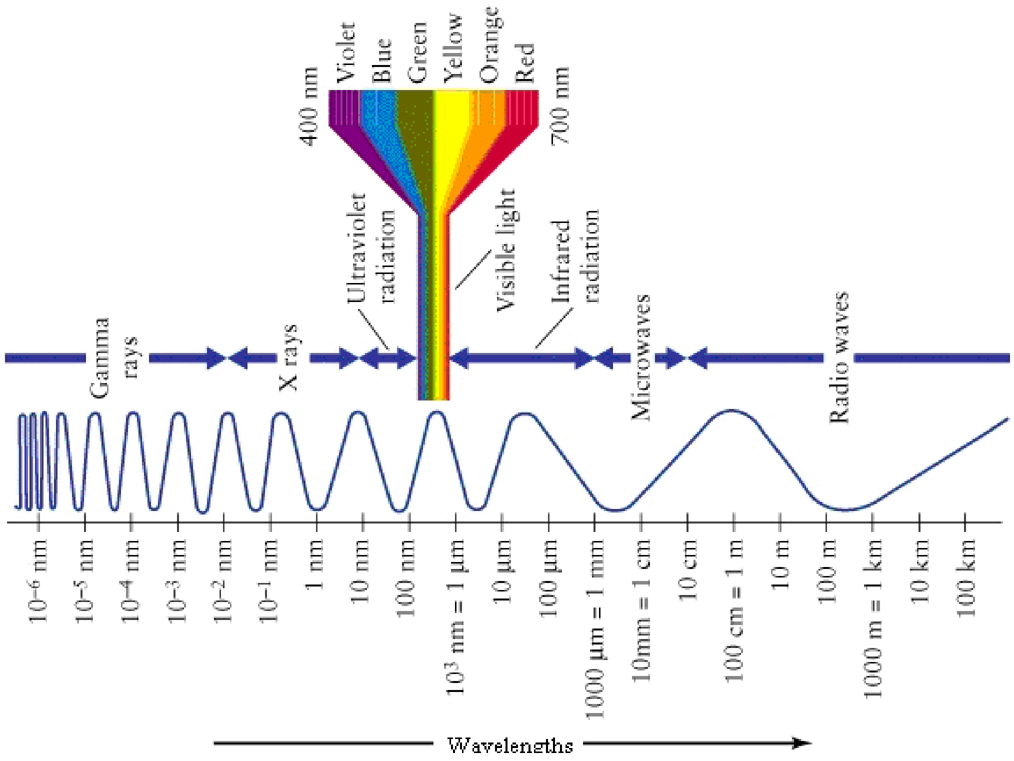

Figure 1. Electromagnetic radiation spectrum (UPEI 2010).

When an object is illuminated by light, it absorbs certain wavelengths and reflects others. The reflected light is detected by the photoreceptor cells (roods and cones) in the retina of the eye of the observer. The rods are sensitive to low light levels and do not participate in colour vision. There are three types of cones, which are different on how they react to different wavelengths. The colour data from the retina is transmitted to the visual cortex in the brain via the optic nerve. The actual perception of colour happens in the brain.

All the light sources whether they are natural or artificial have unique spectral power distribution (SPD) curves. The SPD curve shows the radiant power that is emitted by the source at each wavelength or band of wavelengths. The spectral power distribution curve of different light sources is shown in Figure 2.

When a red object is struck by daylight, it will absorb all wavelengths except red, which is reflected, and hence we see the object as red. Whereas if an object is struck by the light produced from low-pressure sodium lamp, the object will appear yellow, black or a shade of gray. This is because that low-pressure lamp produce light saturated in yellow as shown in Figure 2.

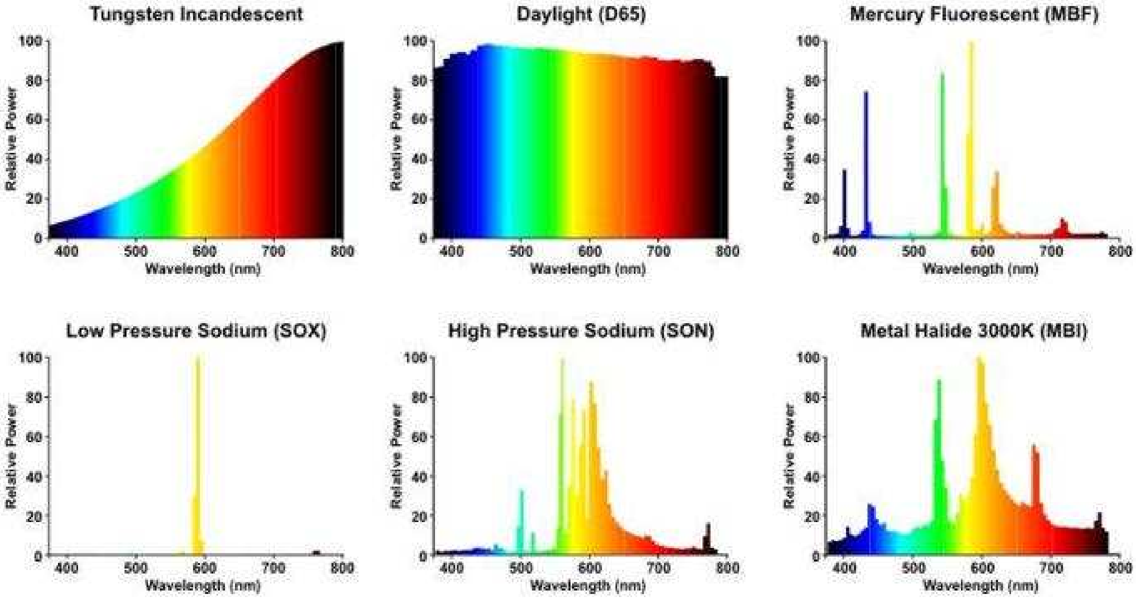

Figure 2. Spectral Power distribution curves of various light sources (Lamptech 2003).

The shape of SPD curve determines the chromaticity of the light. However, light sources with different SPD curve can have the same colour. This phenomenon is called metamerism (see Section 2.1.7for details).

In the past, when artificial illumination was not much developed, spectral power distribution curve and colour temperature was used to describe how light of a lamp will affect the colour of objects (CIE 2006). Colour temperature of a light source is the temperature of a Planckian radiator whose radiation has the same chromaticity as that light source (CIE 1987). Colour temperature is a specification of chromaticity only (Rea 2000). If the chromaticity of the test source does not exactly match the chromaticity of blackbody radiator then correlated colour temperature (CCT) is used (for details see Section 2.1.8). Chromaticity refers to the colour appearance of light source, “warm” for low CCT values and “cool” for high CCT values (Rea 2000).

With the development of artificial light sources with different SPD but equal correlated colour temperature, the problem of colour rendering became serious (CIE 2006). People need colour metrics to know how well the light source can render the colour. Therefore, the first method to characterize the colour rendering of light sources called the spectral band method was introduced (CIE 2006). It was based on spectral power distribution. Later, experts realized that colour appearance should be based on colour difference between the test source and a reference source. After long investigation, CIE committee introduced the CIE test-sample method to calculate the colour rendering index of light sources. The first edition of the CIE recommended method of measuring and specifying colour rendering prosperities of light sources was published in 1965. It was based on test colour sample method and published as publication CIE 13-1965 (CIE 1965). The 2nd edition of CIE 13 (Method of measuring and specifying colour rendering properties of light sources) was published in 1974 (CIE 1974) and 3rd edition in 1995 (CIE 1995).

After the invention of tri-band fluorescent lamps, which were designed to give high efficacy and high CIE CRI, the complaint about the failure of CIE CRI to describe the visual colour rendering was well increased (CIE 2006). CIE Technical Committee TC

1-33 was formed but the committee was dissolved with a status report and without final recommendation (for detail see Section 3.3). After the white LEDs became available level of perfection of CIE CRI has again been questioned.

2.1 CIE Colorimetry

CIE Colorimetry is the metric of psychophysical colour stimulus (CIE 2006). Colour stimuli can be produced by two fundamental methods: additive and subtractive colour mixing. Basic colorimetry is built on additive colour mixture (CIE 2006).

There are four basic empirical laws of additive colour mixing formulated by HG grassman in 1853 and used to obtain the colour match (CIE 2006). They are:

1. The three mathematically determinable elements, the hue, the brightness of colour, and the brightness of the intermixed white can be used to analyse the every impression of colour.
2. If one of two mingling lights is continuously altered (while the other remains unchanged), the impression of the mixed light is also continuously changed.
3. The two colours that have same hue and the same proportion of intermixed white, give identical mixed colours regardless of the composition of colours.
4. The total intensity of any mixture is the sum of the intensities of the lights mixed.

A CIE standard colorimetric observer is an efficient way to describe the colour stimulus (CIE 2006). To obtain colour match test stimulus and three matching stimuli are placed on either side of bipartite field. The observer obtains colour match between test stimulus and matching stimuli by adjusting the intensity of the three matching stimuli. The three matching stimuli should be selected in such a way that no two stimuli on additive mixing produces third stimulus. The standardized observation conditions are: foveal vision, 2^{o} or 10^{o} field size, dark surrounding.

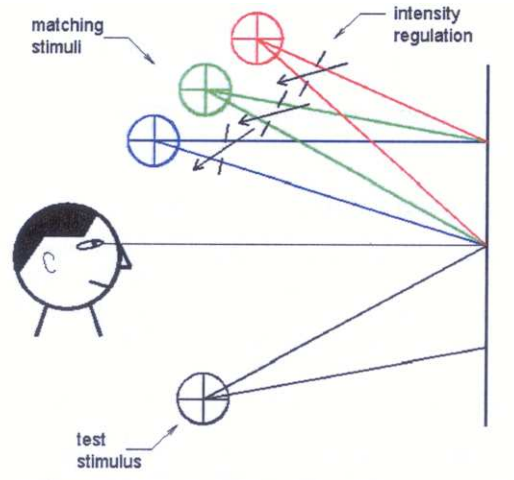

Figure 3. Basic experiment of colour matching (CIE 2006).

Then, the colour match is described as:

$$[C] = R[R] + G[G] + B[B] \tag{1}$$

where	[C]	is unknown colour stimulus
	[R],[G],[B]	are the units of the matching stimuli
	R, G, B	represent the amounts of matching stimuli used

2.1.1 Tristimulus values

The above mentioned R, G, and B are the tristimulus values of the CIE RGB space. These values are obtained from CIE standard Colorimetric observers using 2 degree foveal field of observation and a dark surround. If $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ represent colour matching functions then for any colour of radiance $S(\lambda)$ or $P(\lambda)$, the tristimulus value of CIE RGB space is given by (Murdoch 1985):

$$\mathrm{R} = K\int_{380}^{770} \bar{r}(\lambda)S(\lambda)d\lambda \quad \textbf{(2)}$$

$$\mathrm{G} = K\int_{380}^{770} \bar{g}(\lambda)S(\lambda)d\lambda \quad \textbf{(3)}$$

$$\mathrm{B} = K\int_{380}^{770} \bar{b}(\lambda)S(\lambda)d\lambda \quad \textbf{(4)}$$

where

R, G, B	are the tristimulus values of the CIE RGB space
$\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$	are the colour matching functions
S(λ)	is spectral power distribution
K	is a normalizing constant

Colour matching functions are the values corresponding to monochromatic stimuli of equal radiant power. It is a wavelength dependent quantity. The graph of colour matching functions (CMFs) of the CIE 1931 standard colorimeter observer is shown in Figure 4.

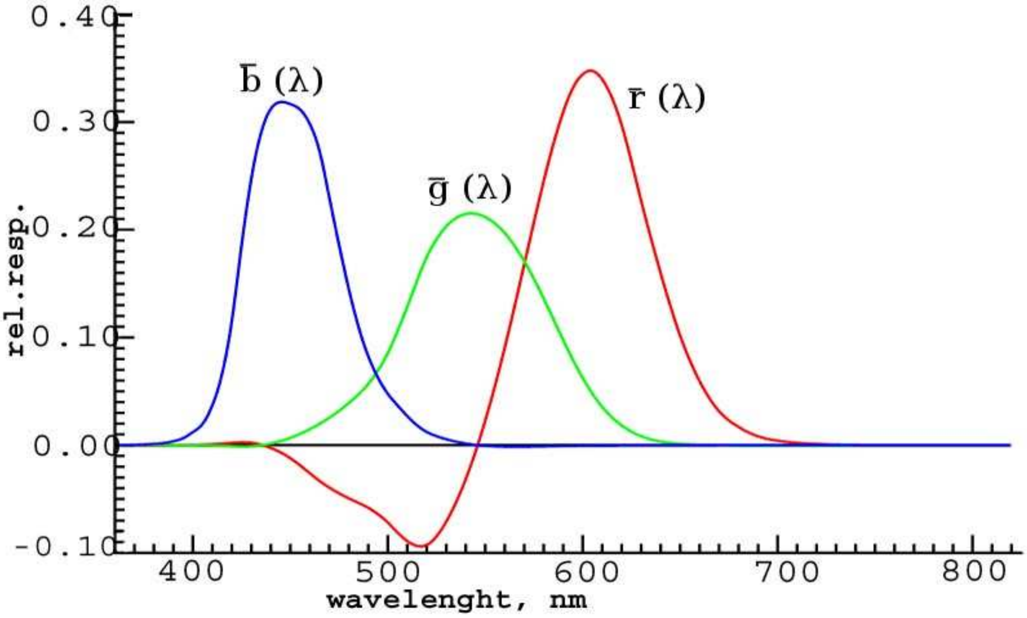

Figure 4. CIE 1931 RGB colour matching functions (CIE 2006).

The negative lobes or values less than zero in Figure 4 indicate that in some part of spectrum, one of the matching stimuli had to be added to the test stimulus to make the match (CIE 2006).

At the time when there were no computers available the negative lobes in the CMFs made calculation more difficult. Therefore, to remove those negative lobes, CIE in 1931 decided to transform [R], [G],[B] primaries to a set of non-physical primaries [X], [Y], [Z] such that the tristimulus values of an equi-energy stimulus should be equal(X=Y=Z) and one of the tristimulus values should provide photometric quantities(Φ,I, L, E etc) (CIE 2006).

The transformation is straightforward but mathematically complicated. The final result is:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2.768892 & 1.751748 & 1.130160 \\ 1.000000 & 4.590700 & 0.060100 \\ 0 & 0.056508 & 5.594292 \end{bmatrix} \cdot \begin{bmatrix} R \\ G \\ B \end{bmatrix} \tag{5}$$

where

X, Y, Z	are the new tristimulus values
R, G, B	are the tristimulus values of the CIE RGB space

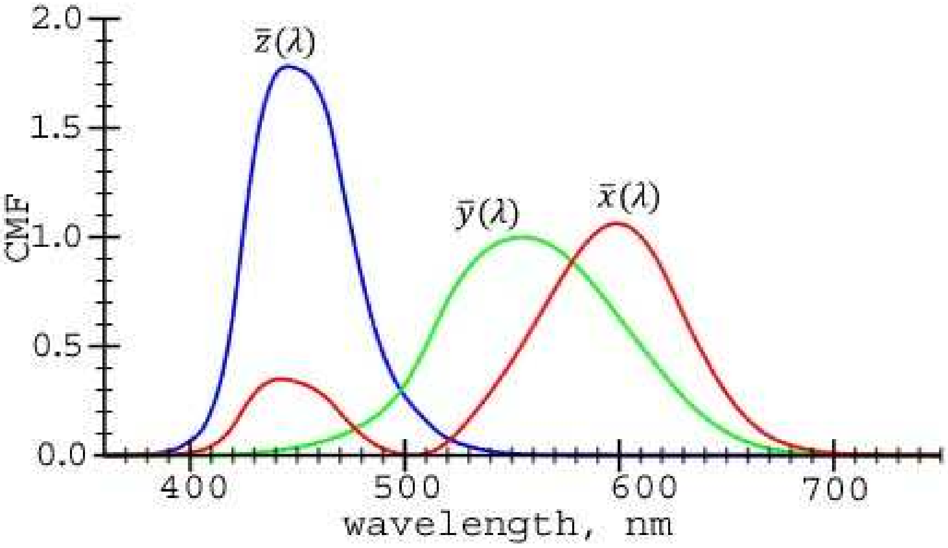

Figure 5. CIE 1931 XYZ colour matching functions (CIE 2006).

In CIE XYZ trichromatic system the tristimulus values (X, Y, Z) are defined as

$$\mathrm{X} = K \int_{380}^{770} \phi_{\lambda}(\lambda)\bar{x}(\lambda)d\lambda \tag{6}$$

$$\mathrm{Y} = K \int_{380}^{770} \phi_{\lambda}(\lambda)\bar{y}(\lambda)d\lambda \tag{7}$$

$$\mathrm{Z} = K \int_{380}^{770} \phi_{\lambda}(\lambda)\bar{z}(\lambda)d\lambda \tag{8}$$

where

X, Y, Z	are the tristimulus values of CIE-XYZ trichromatic system
$\phi_{\lambda}(\lambda)$	is the colour stimulus function of the light seen by the observer or spectral distribution of light stimulus
K	is a normalizing constant
$\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$	are the colour matching functions (CMF) of the CIE 1931 standard observer

Later in 1964, CIE defined colour matching functions for a 10^{o} field of view as the supplement of 1931 standard observer. It is denoted as $\bar{x}_{10}(\lambda), \bar{y}_{10}(\lambda), \bar{z}_{10}(\lambda)$ and called the CIE 1964 standard colorimetric observer.

2.1.2 Chromaticity coordinates and chromaticity diagram

The chromaticity coordinates are the ratio of each of a set of three tristimulus values to their sum (CIE 1987). The three chromaticity coordinates are mathematically defined in equations (9), (10), & (11). The sum of the three chromaticity coordinates equals 1 as a result of which two of them are sufficient to define a chromaticity. Chromaticity is the property of a colour stimulus defined by its chromaticity coordinates, or by its dominant or complementary wavelength and purity taken together (CIE 1987). Chromaticity specifies the colour's hue and saturation but not lightness. Hue is the attribute of a colour perception denoted by blue, green, yellow, red, purple, and so on. Saturation is the attribute of a visual sensation which permits a judgement to be made of the degree to which a chromatic stimulus differs from an achromatic stimulus regardless of their brightness (Wyszecki 2000). Lightness is the brightness of an area judged relative to the brightness of a similarly illuminated area that appears to be white or highly transmitting (CIE 1987). Brightness is the attribute of a visual sensation according to which an area appears to exhibit more or less light. Chromaticity expresses colour in two dimensional planes. The chromaticity coordinates are defined as:

$$x = \frac{X}{X+Y+Z} \tag{9}$$

$$y = \frac{Y}{X+Y+Z} \tag{10}$$

$$z = \frac{Z}{X+Y+Z} \tag{11}$$

where

x, y, z	are the chromaticity coordinates such that x + y + z = 1
X,Y, Z	are the tristimulus values

The plot of x,y chromaticity coordinates in a rectangular coordinate system results the diagram as shown in Figure 6. This diagram is referred as the CIE 1931 chromaticity diagram or the CIE (x,y) chromaticity diagram.

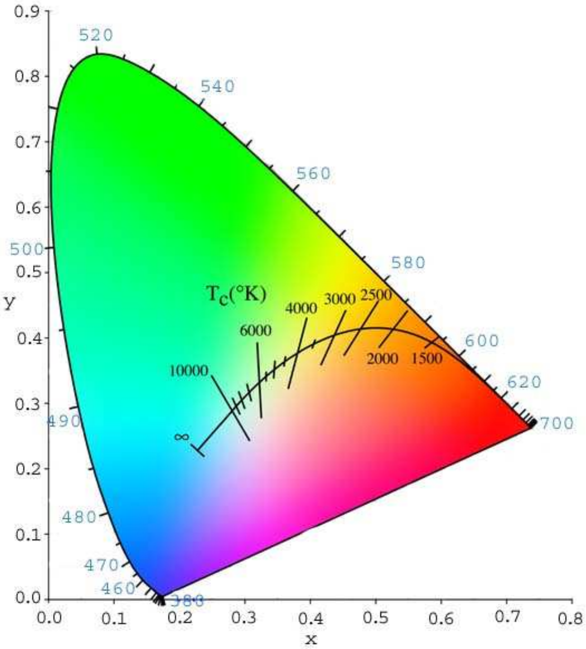

Figure 6. CIE x,y chromaticity diagram (GhO, 2010).

In 1942, David MacAdam (1942) showed colour difference in different area of the CIE x,y chromaticity diagram in elliptical form as shown in Figure 7. These ellipses are called MacAdam ellipses. They contain all colours which are indistinguishable, to normal human eyes, from the colour at the centre of the ellipse.

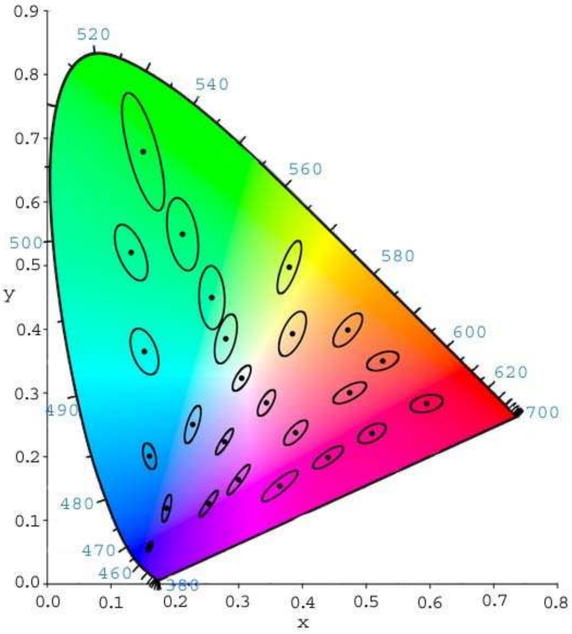

Figure 7. MacAdam Ellipses (CIE 2006).

The contour of the ellipse is just a noticeable difference of chromaticity. As shown in the Figure 7, small difference of chromaticity coordinates for blue-purple colours is perceptible but for the green colours 10 times higher difference in chromaticity coordinates is hardly perceived. MacAdam Ellipses show that the CIE x, y chromaticity diagram is not uniform.

2.1.3 Uniform Chromaticity diagram

To make chromaticity diagram more uniform, in 1960, CIE defined an improved diagram known as CIE 1960 (u,v) chromaticity diagram purposed by MacAdam (1937)(Wyszecki 2000). The chromaticity coordinates *u* and *v* are defined as (MacAdam 1937):

$$u = \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2x + 12y + 3}$$
$$v = \frac{6Y}{X + 15Y + 3Z} = \frac{6y}{-2x + 12y + 3} \quad \textbf{(12)}$$

where

X, Y, Z	are the tristimulus values
x,y	are the chromaticity coordinates of CIE x,y chromaticity diagram

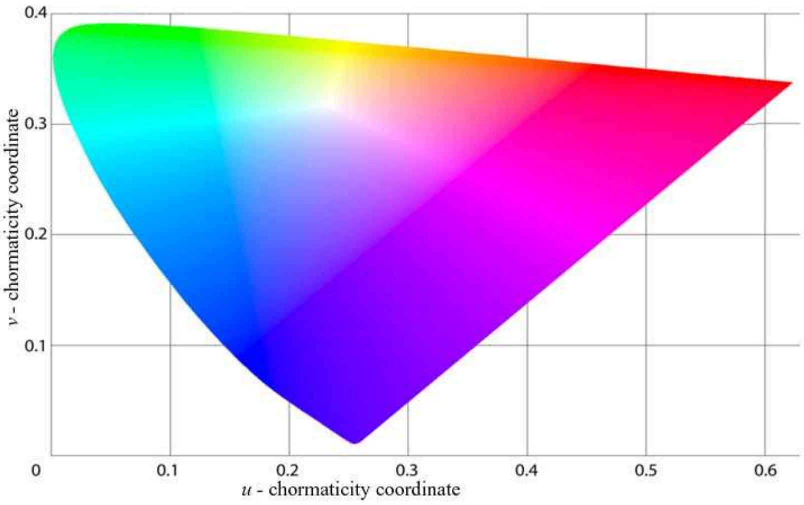

Figure 8. CIE 1960 (u, v) chromaticity diagram (MacAdam 1937).

In 1976, CIE defined further improved diagram known as CIE 1976 uniform chromaticity scale (UCS) diagram. The 1976 UCS diagram is noticeably more uniform than CIE x,y chromaticity diagram. The big advantage of 1976 UCS diagram is that the distance between points on the diagram is approximately proportional to the perceived colour difference (HyperPhysics 2010).

The chromaticity coordinates in CIE 1976 uniform chromaticity or (u',v') diagram is given by:

$$u' = \frac{4X}{X+15Y+3Z}\ ;\ v' = \frac{9Y}{X+15Y+3Z} \qquad \textbf{(13)}$$

where

u',v'	are chromaticity coordinates in CIE 1976 uniform chromaticity diagram
X, Y, Z	are the tristimulus values

Also, the x, y values can be transformed to u', v' by using following formulae:

$$u' = \frac{4x}{-2x+12y+3}\ ;\ v' = \frac{9y}{-2x+12y+3} \qquad \textbf{(14)}$$

Equation (14) can be used to obtain chromaticity diagram as shown in Figure 9. If the field of view is greater than 4^{o}, CIE 1964 standard colorimetric observer should be used.

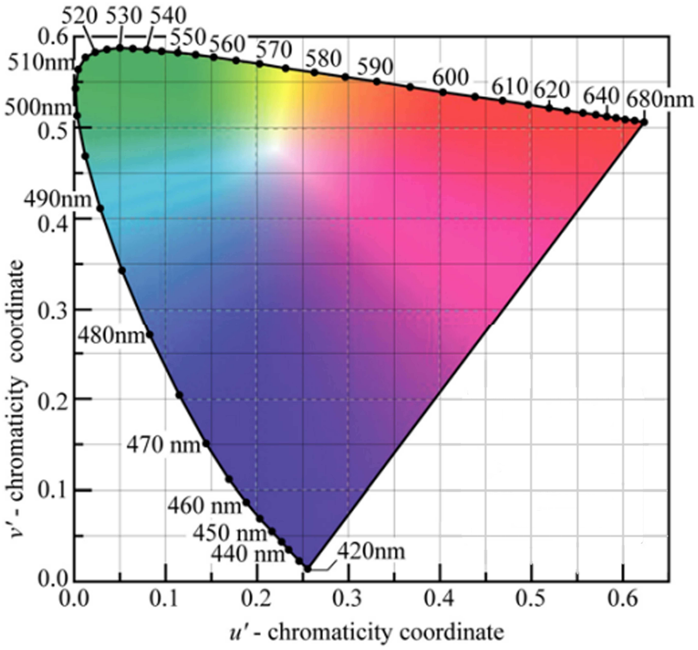

Figure 9. CIE 1976 UCS (uniform chromaticity scale) diagram (Schubert 2006).

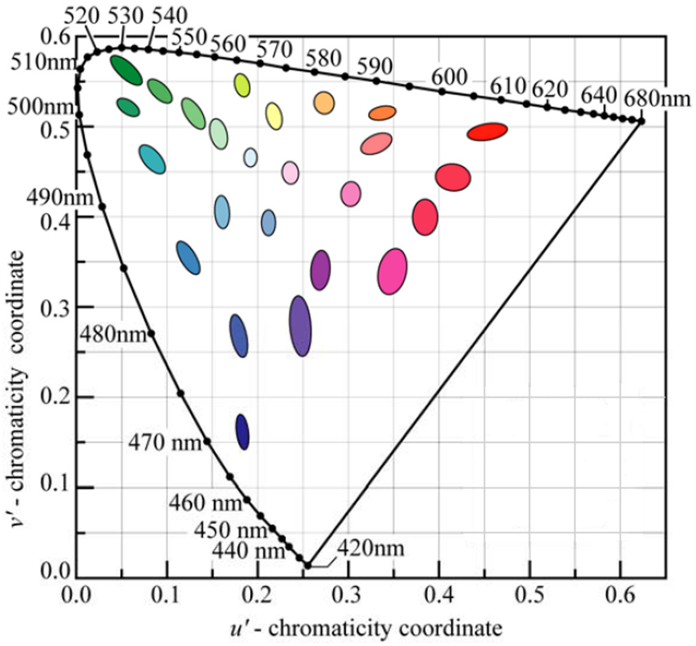

Figure 10. MacAdam’s ellipses in CIE 1976 Chromaticity diagram (Schubert 2006).

The MacAdam’s ellipses in CIE 1976 chromaticity diagram are shown in Figure 10. It shows that the colour difference has significantly decreased which leads to the fact that CIE 1976 UCS diagram is more uniform than the CIE x,y chromaticity diagram, although ellipses are not totally converted to circle.

2.1.4 Uniform colour spaces

The colour space is method by which colour can be specified, created, and visualised. The three attributes of colour are brightness, hue, and colourfulness (Ohno, 2000). Colour space expresses these three attributes in a three dimensional space. The most useful colour spaces are CIE 1976 L*U*V*, abbreviated by CIELUV and CIE 1976 L*a*b*, abbreviated by CIELAB. Although CIELUV and CIELAB colour spaces are widely useful, the CIE test sample method to evaluate the colour rendering index uses CIE 1964 (U*,V*,W*) colour space abbreviated as CIEUVW.

2.1.4.1 CIEUVW Colour space

The CIEUVW colour space is based on CIE 1960 UCS diagram with the co-ordinates U*, V* and W*. W* is a lightness index and is a function of luminance factor Y. U* and V* are nonlinear transformation of u and v (u & v are obtained by using equation (12)). The transformation is done using the equations:

$$\begin{aligned} W^* &= 25Y^{1/3} - 17 \\ U^* &= 13W^*(u - u_0) \\ V^* &= 13W^*(v - v_0) \end{aligned} \qquad \textbf{(15)}$$

where

W*, U*, V*	are the coordinates of CIEUVW colour space
u_0 and v_0	are the uv coordinates of the white point and are placed at the origin of the U*V*W* system

This arrangement has the benefit of being able to express the loci of chromaticities with constant saturation simply as $(U^*)^2 + (V^*)^2 = C$ for a constant C. This colour space was invented by G. Wyszecki to calculate the colour difference (Wyszecki 1963).

2.1.4.2 CIE L*a*b* Colour space

The CIELAB colour space is based directly on CIE XYZ system. It is organized in a cube form. L* shows the lightness. The maximum value for L* is 100 which indicate white and minimum value is 0, which represents black. The axes a* and b* do not have specific numerical limit. The axis a* represents red to green with + numbers indicating increased redness and – numbers indicating increased greenness. The axis b* represents yellow to blue with + numbers indicating increased yellowness and – numbers indicating blueness. The three-dimensional orthogonal coordinates are calculated as:

$$\begin{aligned} L^* &= 116 f(Y/Y_n) - 16 \\ a^* &= 500\left[f(X/X_n) - f(Y/Y_n)\right] \\ b^* &= 200\left[f(X/X_n) - f(Y/Y_n)\right] \end{aligned} \qquad \textbf{(16)}$$

where

L*, a*, b*	are the coordinates of CIE L*a*b* Colour space
X, Y, Z	are the tristimulus values of the test object colour stimulus considered
X_n, Y_n, Z_n	are the tristimulus values of a specified white object colour stimulus

$$\begin{aligned} &f(X/X_n) = (X/X_n)^{1/3} && if \quad (X/X_n) > (24/116)^3 \\ &f(X/X_n) = (841/108)(X/X_n) + 16/116 && if \quad (X/X_n) \le (24/116)^3 \end{aligned}$$

and

$$\begin{aligned} &f(Y/Y_n) = (Y/Y_n)^{1/3} && if \quad (Y/Y_n) > (24/116)^3 \\ &f(Y/Y_n) = (841/108)(Y/Y_n) + 16/116 && if \quad (Y/Y_n) \le (24/116)^3 \end{aligned}$$

and

$$\begin{aligned} &f(Z/Z_n) = (Z/Z_n)^{1/3} && if \quad (Z/Z_n) > (24/116)^3 \\ &f(Z/Z_n) = (841/108)(Z/Z_n) + 16/116 && if \quad (Z/Z_n) \le (24/116)^3 \end{aligned}$$

The correlates of lightness, chroma and hue defined by CIE LAB space are:

Lightness: $L^* = 116 f(Y/Y_n) - 16$

Chroma: $C^*_{ab} = (a^{*2} + b^{*2})^{1/2}$ **(17)**

Hue: $h_{ab} = \arctan(b^*/a^*)$

It does not define saturation. The CIELAB colour space is shown in Figure 11

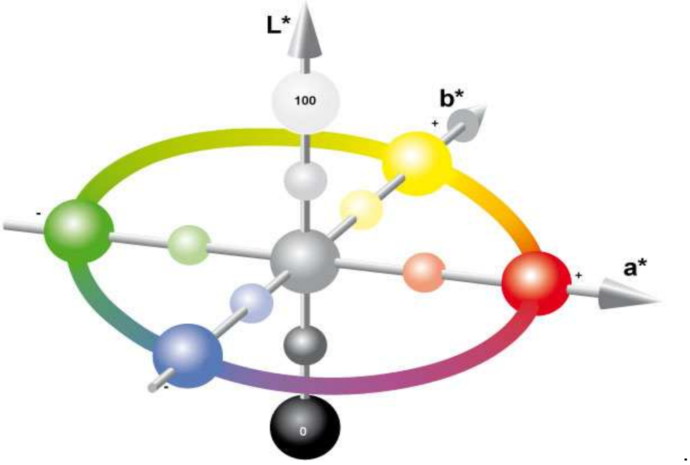

Figure 11. CIELAB colour space (GM 2010).

2.1.4.3 CIE L*u*v* Colour space

This colour space is based on CIE 1976 (u', v') system. The calculation of coordinate L* is same as that of CIELAB colour space but u* and v* are calculated by using following formula:

$$u^* = 13L^*(u' - u_n) \quad and \quad v^* = 13L^*(v' - v_n) \qquad \textbf{(18)}$$

where

u*, v*	are the coordinates of the CIE L*u*v* colour space
u', v'	are the CIE 1976 UCS coordinates of the test stimulus
$u'_{n,}$ v'_n	are the CIE 1976 UCS coordinates of the reference

The correlates of saturation, chroma and hue are defined as:

Saturation: $S_{uv} = 13\left[(u' - u'_n)^2 + (v' - v'_n)^2\right]^{1/2}$

Chroma: $C^*_{uv} = (u^{*2} + v^{*2})^{1/2} = L^*.S_{uv}$ **(19)**

Hue: $h_{ab} = \arctan(v^*/u^*)$

The cylindrical version of CIELUV colour sapce is abbreviated as CIELch and is shown in Figure 12. Many other uniform colour spaces have been proposed recently including Frei space, OSA, ATD, LABHNU, Hunter LAB, SVF space, Nayatani Space, Hunt space, IPT spaces, J'a'b' space. Among these colour space J'a'b' colour space is most recent and is based on CIE colour space appearance model CIECAM97s (Li 2001a).

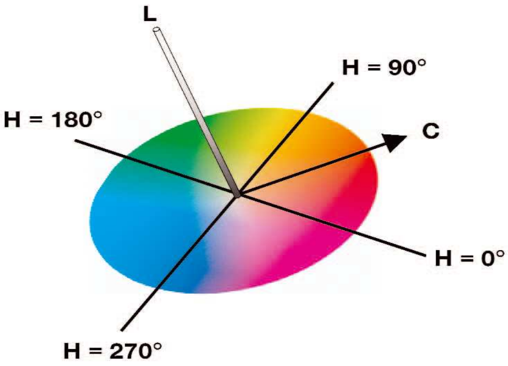

Figure 12. CIELch colour system (Ingraham 2010).

2.1.5 Colour difference

The colour difference is a metric, which define the difference of two colours. A colour difference formula is a mathematical expression providing a non-negative value ΔE, which try to be well correlated with identical size and shape, simultaneously viewed under the same experimental conditions (light source, background etc.) (Melgosa 2008). Wyszecki (1963) defined the colour difference formula by introducing lightness index (W) in 1960 UCS diagram with the assumption that lightness difference (ΔW*) of 1 corresponds to chromaticness difference $(\Delta U^{*2}+\Delta V^{*2})^{1/2}$ of 13. The formula is as follows:

$$\Delta E = \sqrt{\Delta W^{*2} + \Delta U^{*2} + \Delta V^{*2}} \quad \textbf{(20)}$$

where

ΔE	is CIE 1960 UCS colour difference
$\Delta W^*, \Delta U^*, \Delta V^*$	are the arithmetic difference between the coordinates of test and reference sources

Later CIE adopted this formula as CIEUVW colour difference formula.

In 1976 after the recommendation of uniform colour spaces CIELAB and CIELUV, CIE recommended the colour difference formulas. In CIELAB, two equivalent equations ((21) & (22)) can be used to describe the colour difference.

$$\Delta E^*_{ab} = \sqrt{\Delta L^{*2} + \Delta a^{*2} + \Delta b^{*2}} \quad \textbf{(21)}$$

or,

$$\Delta E^*_{ab} = \sqrt{\Delta L^{*2} + \Delta C_{ab}{}^{*2} + \Delta H_{ab}{}^{*2}} \quad \textbf{(22)}$$

where

ΔE^*_{ab}	is CIELAB colour difference
$\Delta L^*, \Delta a^*$ and Δb^*	are the arithmetic difference between the coordinates of the test and reference sources ($\Delta L^* = L_1{}^* - L_2{}^*, \Delta a^* = a_1{}^* - a_2{}^* and\ \Delta b^* = b_1{}^* - b_2{}^*$, the indices 1 and 0 referring to test and reference sources respectively)
$\Delta C_{ab}{}^*$	is CIELAB chroma difference i.e.($\Delta C_{ab}{}^* = C_{ab,1}{}^* - C_{ab,0}{}^*$, indices 1 and 0 referring to test and reference sources respectively)
$\Delta H_{ab}{}^*$	CIELAB hue difference and should be calculated using equation(23)

$$\Delta H_{ab}{}^* = 2 \times \sqrt{(C_{ab,1}{}^* - C_{ab,0}{}^*)} \times \sin({}^{\Delta h_{ab}}\!/_{2}) \quad \textbf{(23)}$$

where

$\Delta h_{ab} = h_{ab,1} - h_{ab,0}$, indices 1 and 0 refer to test and reference sources respectively

Similarly, in CIELUV, the colour difference is defined as follows:

$$\Delta E^*_{uv} = \sqrt{\Delta L^{*2} + \Delta u^{*2} + \Delta v^{*2}} \quad \textbf{(24)}$$

where

ΔE^*_{uv}	is CIELUV colour difference
$\Delta L^*, \Delta u^*$ and Δv^*	are the arithmetic difference between the coordinates of the test and reference sources

Continuous research in the field has lead to the proposal of many successful CIELAB based colour-difference formulae such as CIE94 and CIEDE2000. DIN99 formula (which start from CIELAB colour space) and CAM02 formula (based on CIECAM02 colour appearance model) has also been developed. For various applications different type of colour formula has been developed and more to be seen in future

2.1.6 CIE Illuminants

CIE illuminants are numerical representation of spectral power distribution of different types of white light sources (HunterLab, 2008). In 1931 the CIE established three illuminant A, illuminant B and illuminant C. Illuminant A resembles the SPD of an average incandescent light, whereas illuminant B and illuminant C resembles the SPD of direct sunlight and average daylight respectively. In 1964, CIE recommended a new set of daylight illuminants (illuminant D50, illuminant D55, standard illuminant D65, etc.), where the SPD was defined in the ultraviolet (UV) part of the spectrum (CIE 2006). At present illuminant A and illuminant D65 are called CIE standard illuminates (CIE 2004).

CIE standard illuminant A has a correlated colour temperature of approximately 2856 K (CIE 2006). CIE standard illuminate A should be used in all applications of colorimetry involving the use of incandescent lighting unless there are specific reasons for using a different illuminant (CIE 2004). CIE standard illuminant A is defined over the spectral region from 300 nm to 830 nm to six significant digits (CIE 2004). The relative spectral power distribution of illuminant is defined by equation (25).

$$S_A(\lambda)=100\left(\frac{560}{\lambda}\right)^5\times\frac{\exp\frac{1{,}435\times10^7}{2848\times560}-1}{\exp\frac{1{,}435\times10^7}{2848\lambda}-1} \qquad \textbf{(25)}$$

where

$S_A(\lambda)$ is the relative spectral power distribution

λ is the wavelength in nanometres

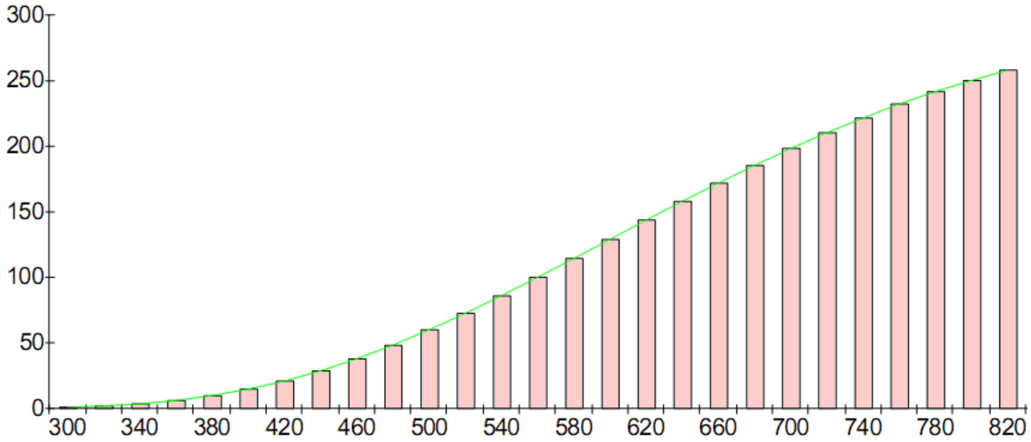

Figure 13. CIE Standard Illuminant A (DiCosola 1995).

CIE standard illuminant D65 is defined to represent average daylight. It has correlated colour temperature of approximately 6500 K. For the condition where D65 is not applicable, CIE selected D50, D55, and D75 as preferred daylight illuminants (CIE 2006). The spectral power distribution of D50, D55, and D75 are shown on Figure 15.

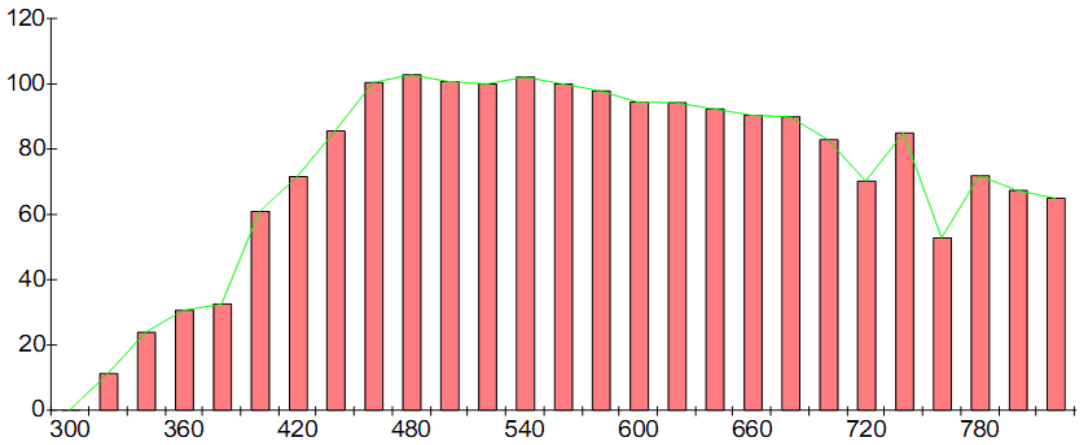

Figure 14. CIE Standard Illuminant D65 (DiCosola, 1995).

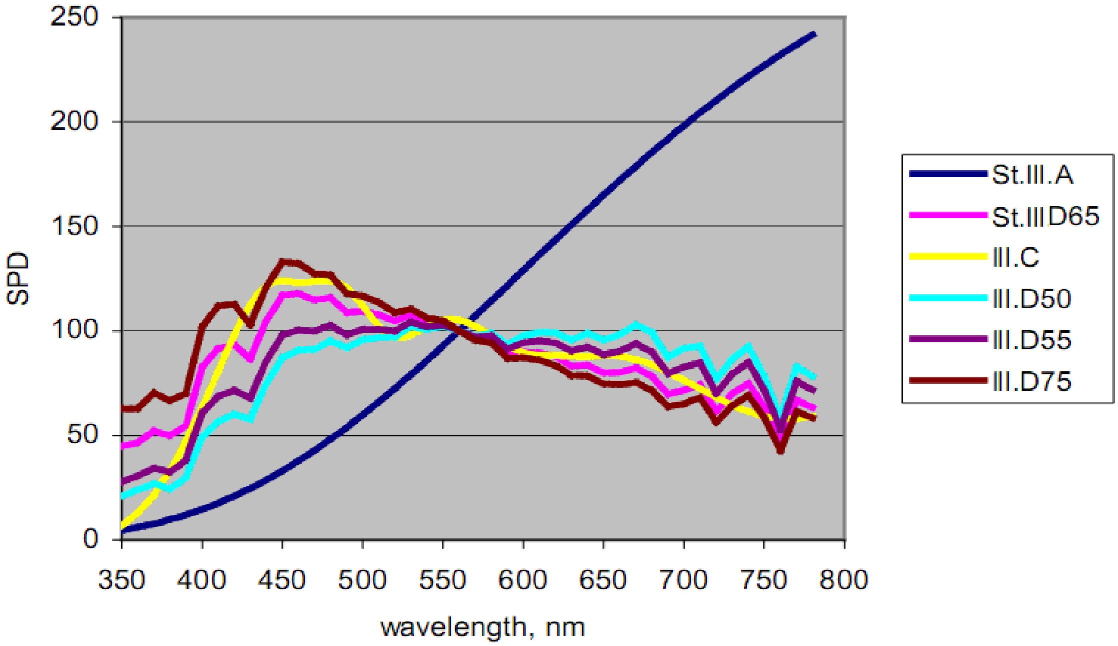

Figure 15. Relative spectral power distribution of the CIE standard illuminants and a further 3 daylight illuminants (D50, D55, & D75) and illuminant C (CIE 2006). (St.Ill.: Standard illuminant; Ill.: Illuminant)

Furthermore, CIE published SPDs of some other illuminants that represent fluorescent lamps (FL). These illuminants representing fluorescent lamps are: FL 1 to 6 for standard fluorescent lamps, FL 7 to 9 for broad-band and FL 10-12 for narrow band fluorescent lamps.

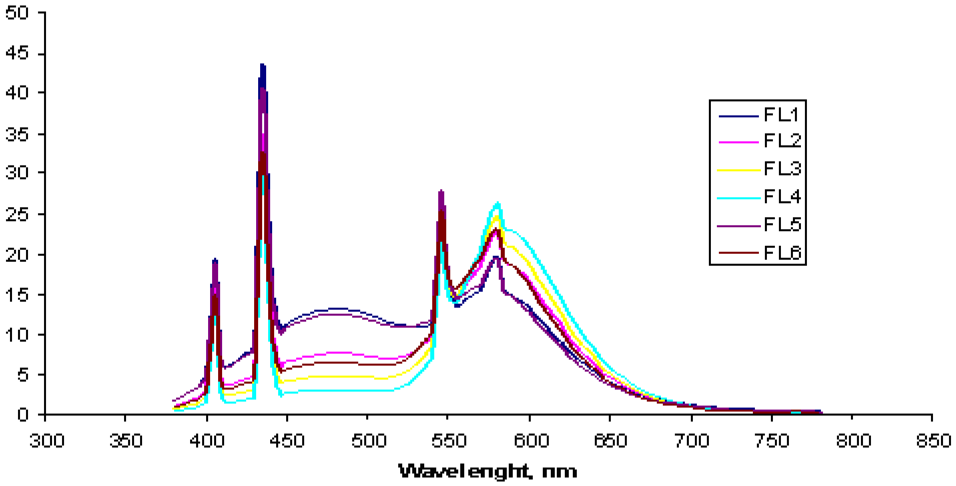

Figure 16. Illuminant FL 1-6: Standard fluorescent lamp (CIE 2004).

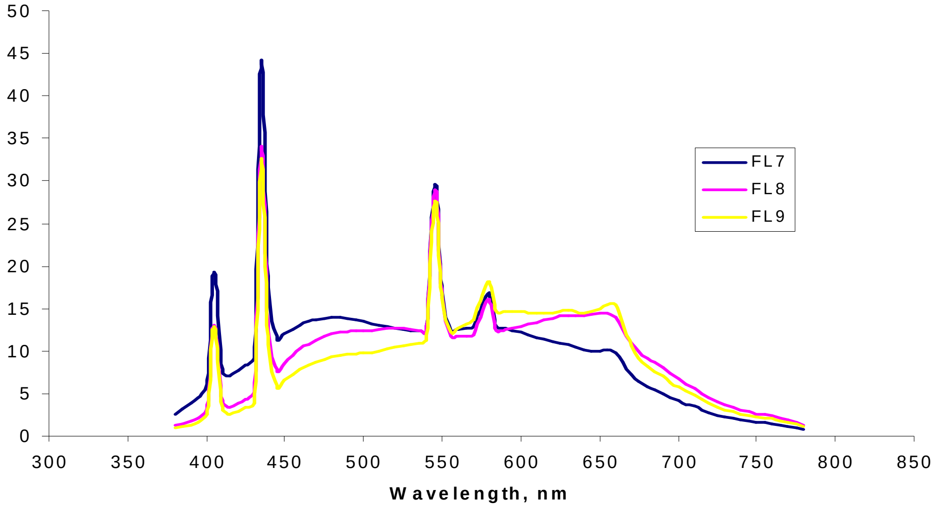

Figure 17. Illuminant FL 7-9: Broad-band fluorescent lamps (CIE, 2004).

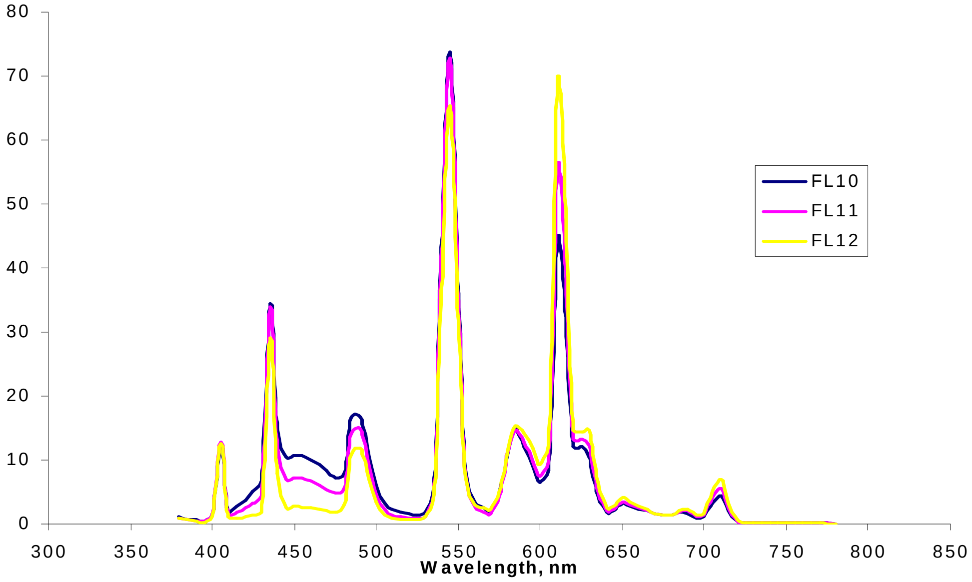

Figure 18. Illuminant FL 10-12 Narrow band fluorescent lamps (CIE, 2004).

2.1.7 Metamerism

Metamerism is the phenomenon in which two spectrally different coloured materials appear to have same colour under one illuminant, but not under other illuminants (HunterLab, 2008a). The colours that match this way are called metamers. The official definition of metamerism is:

Two specimens having identical tristimulus values for a given reference illuminant and reference observer are metameric if their spectral radiance distributions differ within the visible spectrum. (CIE 2006)

The tristimulus value will no longer be identical if illuminant or observer is changed. Therefore, metamerism is broke down into illuminant metamerism and observer metamerism.

2.1.8 Correlated Colour Temperature

Colour temperature of light source is the temperature of blackbody radiator or Planckian radiator at which colour of blackbody radiator is exactly same as that of light source. It is expressed in Kelvin. Blackbody radiator or Planckian radiator is theoretical perfect radiator. It radiates energy in the visible range (red, orange, yellow and bluish white) when its temperature is raised. The colours of blackbody radiation when plotted on the CIE chromaticity diagram form a curved line known as blackbody locus or Planckian locus.

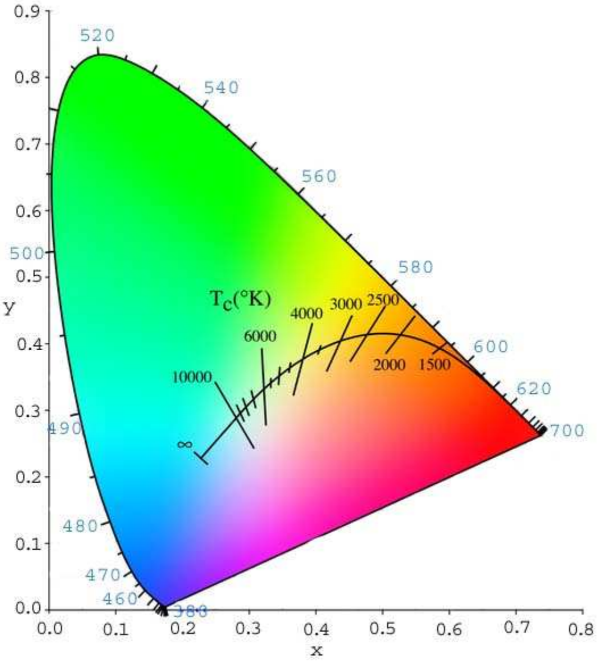

Figure 19. The CIE chromaticity diagram with blackbody locus (GhO, 2010).

If the chromaticity of the test source does not have a perfect match on the blackbody locus (if deviates slightly), then chromaticity of that test source can be compared to a Planckian radiator using correlated colour temperature (CCT). The CIE definition of CCT is:

The correlated colour temperature is the temperature of a Planckian radiator having the chromaticity nearest the chromaticity associated with the given spectral distribution on a diagram where the (CIE 1931 standard observer based) u', 2/3v' coordinates of the Planckian locus and the test stimulus are depicted. (CIE 2006)

Test sources with equal CCT lie on the straight lines perpendicular to the Planckian locus on the u', 2/3v' diagram. These are called iso-temperature lines (CIE 2006). Iso-temperature lines in the u', 2/3v' diagram describe the smallest visual chromaticity distance between test source and Planckian radiator. According to the CIE recommendation the CCT should not be used if the chromaticity ΔC between test source and Planckian radiator is more than 5×10^{-2}. ΔC is calculated using equation (26).

$$\Delta C=\left[(u'_t-u'_p)^2+\frac{4}{9}(v'_t-v_p)^2\right]^{1/2} \tag{26}$$

where

ΔC	is the chromaticity difference between test source and Planckian radiator in u', 2/3v' coordinate system
u'_t, v'_t	are chromaticity coordinate of test source
u'_p, v'_p	are chromaticity coordinate of Planckian radiator

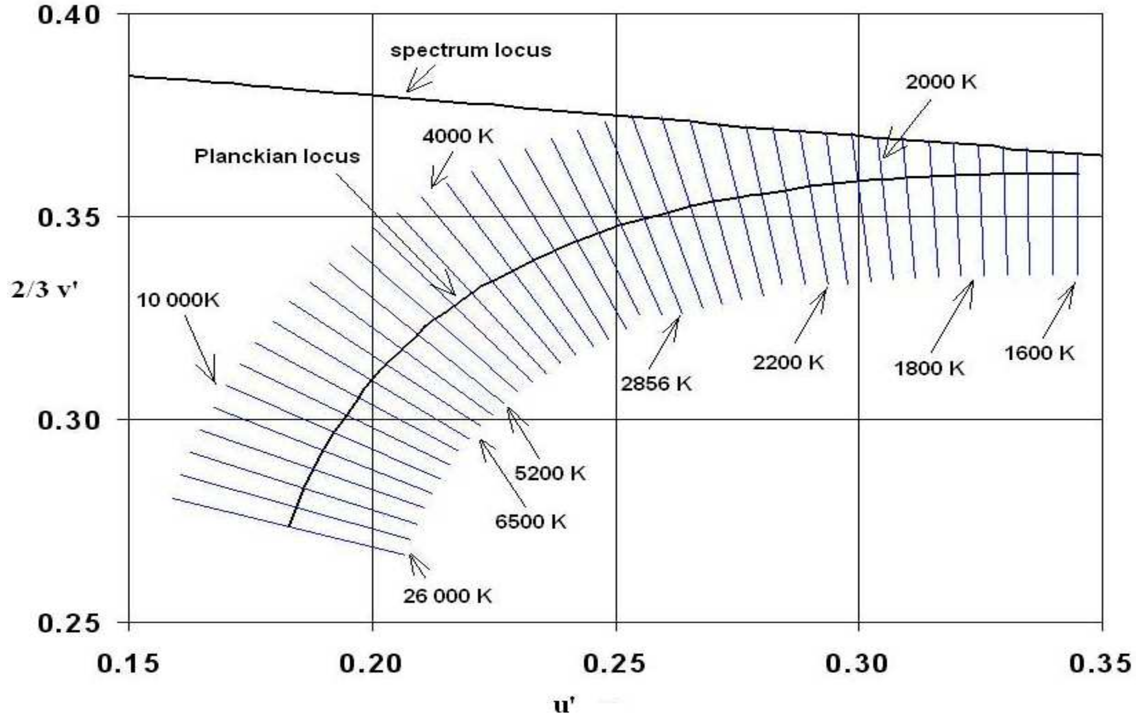

Figure 20. u', 2/3v' chromaticity diagram with Planckian locus and a few iso-temperature lines (CIE 2006).

2.2 Munsell colour system

Munsell colour system is a colour space with a three colour dimensions hue, value and chroma. Munsell defines hue as the quality by which we distinguish one colour from another (Kang 2006). Hue has five principle colours (red, yellow, green, blue and purple and five) intermediate colours (yellow-red, green-yellow, blue-green, purple-blue, and red-purple). Munsell further divides principle colours and intermediate colours into other 10 intervals which are arranged in the form of wheel as shown in Figure 21.

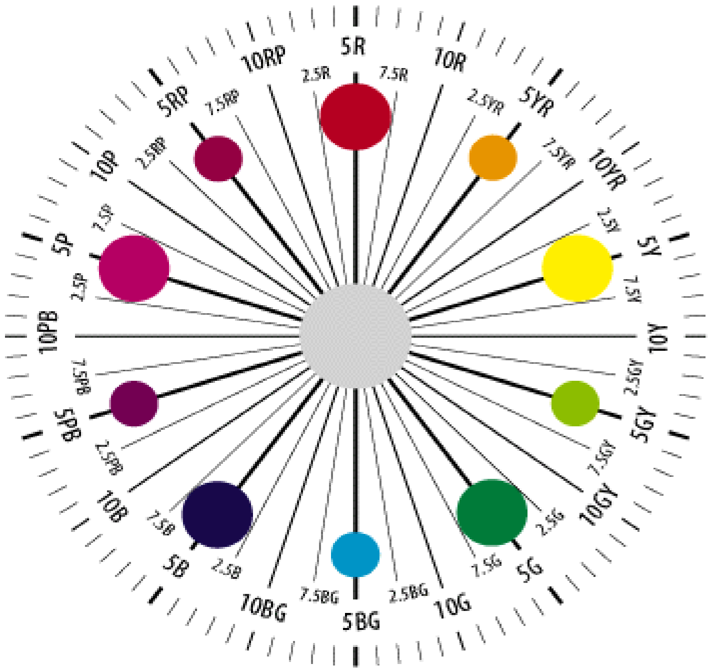

Figure 21. Hue of Munsell colour system (Adobe 2000).

The value is defined in the Munsell system as the quality by which we distinguish a light colour from a dark one. It represents grey level of colour and range from white to black. The white is designated by 10 and black by 0.

Chroma in the Munsell system is a measure of the saturation of a colour. Colour saturation decreases as grey contain in colour increases and vice versa. Chroma axis is horizontal axis perpendicular to value axis. The chroma scale starts at zero, for neutral colours, but there is no arbitrary end to the scale. The three-directional solid representation of Munsell colour system is shown in Figure 23. The complete Munsell notation for a colour is written symbolically as H V/C, e.g. for a vivid red having hue of 5R, a value of 6 and chroma of 14, the complete notation is 5R 6/14.

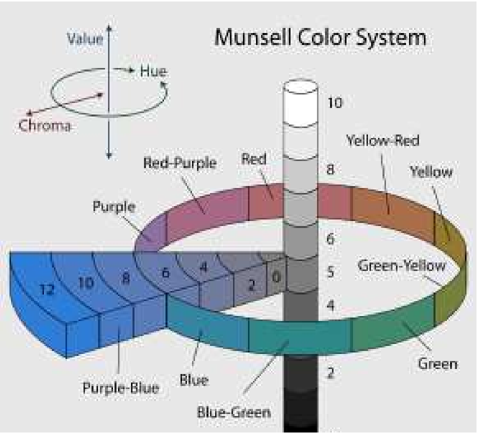

Figure 22. Munsell Colour System, showing: a circle of hues at value 5 chroma 6; the neutral values from 0 to 10; and the chromas of purple-blue (5PB) at value 5 (Makeshwar 2010).

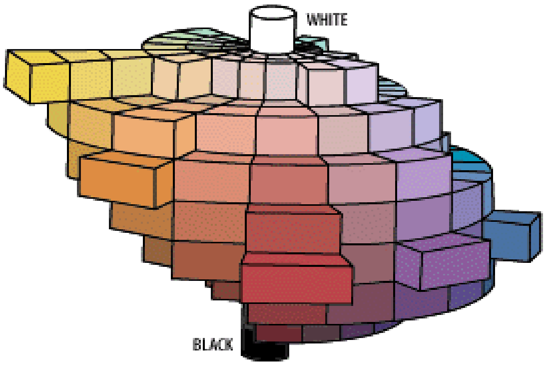

Figure 23. The three-directional solid representation of Munsell colour system (Adobe 2000).

2.3 Chromatic Adaptation

Chromatic adaptation is the ability of the human visual system to discount the colour of a light source to approximately preserve the appearance of an object (Finlayson 2000). For example, a photographic scene viewed under tungsten lamp and daylight D_{65} looks pretty much the same although the measured tristimulus values are very different under these two illuminants (Kang 2006). This is because our eyes have adapted under each condition to discount the illuminant difference.

Image capturing systems, such as digital cameras and scanners, do not have the ability to adapt to the light source (Finlayson 2000). Scanners usually use fluorescent light sources, whereas light sources for digital camera vary with the scene, and sometimes within a scene. Therefore, to faithfully reproduce the appearance of original scene under different display conditions such as computer monitor or a light booth, the tristimulus values of captured image have to be transformed to take into account the light source of the viewing conditions. Such transformations are called chromatic adaptation transforms (CATs). There are numerous methods developed for the CATs to accurately predict colour appearance across a change in illumination. Most of them are based on the von Kries model (Finlayson 2000). This chapter only reviews von Kries chromatic adaptation.

2.3.1 Von Kries chromatic adaptation transformation

In the von Kries chromatic adaption transformation, it is assumed that chromatic adaptation can be represented by the cone responses being multiplied (or divided) by factors that result in reference whites giving rise to the same signals in all states of adaptation (Hunt 1991). If a stimulus gives rise to cone response ρ, γ, β, the visual signals will depend on $\rho/\rho_w \quad \gamma/\gamma_w \quad \beta/\beta_w$, where $\rho_w, \gamma_w, \beta_w$ are the cone responses for the reference white. For a stimulus in the reference state of adaptation to have the same colour appearance in the test state of adaptation, it is necessary that:

$$\rho/\rho_w = \rho'/\rho'_w \qquad \gamma/\gamma_w = \gamma'/\gamma'_w \qquad \beta/\beta_w = \beta'/\beta'_w \tag{27}$$

where

ρ', γ', β' represents cone responses in reference state of adaptation

$\rho'_w, \gamma'_w, \beta'_w$ represents cone responses for the reference white

It follows that:

$$\rho' = (\rho'_w/\rho_w)\,\rho \qquad \gamma' = (\gamma'_w/\gamma_w)\,\gamma \qquad \beta' = (\beta'_w/\beta_w)\,\beta \tag{28}$$

A stimulus having tristimulus values X, Y, Z, in a state of adaptation such that the reference white has tristimulus values X_w, Y_w, Z_w, it is possible to calculate the corresponding colour stimulus, X', Y', Z' which has the same appearance in the reference state. The steps involved in this procedure are then as follows:

Step 1: Determine the tristimulus values X'_w, Y'_w, Z'_w of reference white in the reference state and calculate $\rho'_w, \gamma'_w, \beta'_w$ using equation (29) .

$$\begin{aligned} \rho'_w &= 0.40024X'_w + 0.70760Y'_w - 0.08081Z'_w \\ \gamma'_w &= -0.22630X'_w + 1.16532Y'_w + 0.04570Z'_w \\ \beta'_w &= 0.91822Z'_w \end{aligned} \tag{29}$$

where

$\rho'_w, \gamma'_w, \beta'_w$ are cone responses of reference white in the reference state

X'w, Y'_w, Z'_w	are the tristimulus values of reference white in the reference state

Step 2: From X_w, Y_w, Z_w calculate ρ_w, γ_w, β_w.

$$\begin{aligned} \rho_w &= 0.40024X_w + 0.70760Y_w - 0.08081Z_w \\ \gamma_w &= -0.22630X_w + 1.16532Y_w + 0.04570Z_w \\ \beta_w &= 0.91822Z_w \end{aligned} \tag{30}$$

where

ρ_w, γ_w, β_w	are cone responses for reference white
Xw, Y_w, Z_w	are the tristimulus values of reference white

Step 3: Calculate ρ'_w/ρ_w γ'_w/γ_w β'_w/β_w.

Step 4: For the test colour, from X, Y, Z, calculate ρ, γ, β as done in Step 1 and Step 2.

Step 5: Calculate ρ', γ', β' using equation (28).

Step 6: From ρ', γ', β' calculate X', Y', Z' using equation (31).

$$\begin{aligned} X' &= 1.85995\rho' - 1.12939\gamma' + 0.21990\beta' \\ Y' &= 0.36119\rho' + 0.63881\gamma' \\ Z' &= 1.08906\beta' \end{aligned} \tag{31}$$

where

ρ', γ', β'	are cone responses in reference state
X', Y', Z'	are corresponding colour stimulus that has same appearance in the reference state

Step 7: Use X', Y', Z' in the chosen colour difference formula.

Von Kries transformations are very useful but they are not always accurate (Hunt 1991).

2.4 Summary

In this chapter the fundamentals of CIE colorimetry was introduced. The CIE standard colorimetric observers, illuminantes, fundamentals of the recommended measuring geometries, and the basic equations for determining colour differences and other colorimetric descriptors were introduced. The task of this chapter was to provide knowledge of CIE system which will be beneficial to understand the upcoming chapters.

3 CIE Colour Rendering Index

CIE Colour rendering index (CRI) is a metric which describes the ability of a light source to reproduce the colours of various object in comparison with an ideal or natural light source. In short, CIE CRI defines the colour rendering properties of light sources. The light sources with higher CIE CRI ratings have better colour rendering abilities.

3.1 CIE definition of colour rendering

The international commission of illumination (CIE) defined the colour rendering as *Effect of an illuminant on the colour appearance of objects by conscious or subconscious comparison with their colour appearance under a reference illuminant.*(CIE 1987)

The colour appearance and reference illuminant are most difficult problem of this definition. At the time when this definition was given there was no colour appearance model. Therefore experts decided to use colour difference for the evaluation. Also, the selection of the reference illuminant was left open (CIE 2006). However, later it was decided to use reference illuminant of equal correlated colour temperature (CCT). If the test illuminant's CCT is below 5000 K then the reference illuminant should be selected from the pool of black body radiators otherwise from a phase of daylight. The use of equal CCT was recommended because the available chromatic adaption transformation does not work well for large chromaticity difference (Schanda 2003).

The final detail description of colour rendering index calculation method was published in CIE 13.3-1995: Method of measuring and specifying colour rendering properties of light sources.

3.2 CIE Evaluation Method for CRI (Test colour method)

The CIE Test colour method to evaluate the colour rendering index of light source is based on resultant colour shift of test objects. This method was approved by CIE in 1974 (CIE 1974) and reprinted in 1995 with some corrections. The main purpose of CIE to introduce Test colour method is to make this method as the fundamental method for appraisal of colour rendering properties of light sources (CIE 1995).

The flowchart to determine the colour rendering indices using test colour method is shown in Figure 24. This method requires following procedures to be followed to assign CRI rating for different light sources:

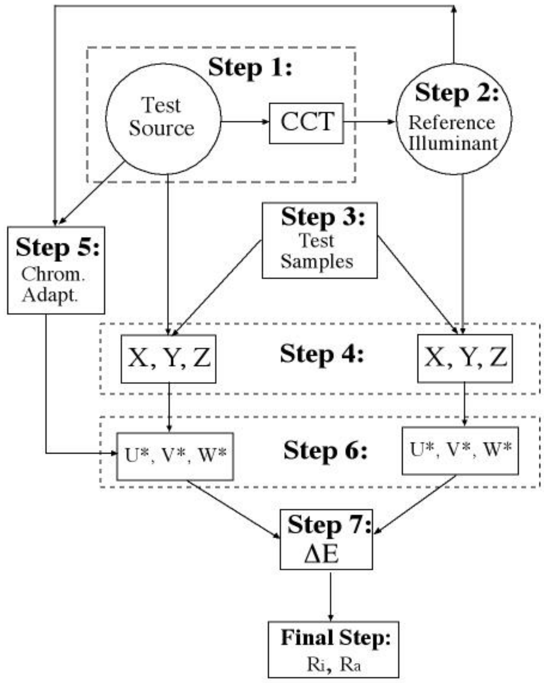

Figure 24. Flowchart for determining the colour rendering indices (CIE 2006).

Step 1: Determine chromaticity coordinate of test source in the CIE 1960 UCS diagram using 2^{o} standard colorimetric observers and then the CCT of test source.

Step 2: Depending upon the CCT of test source select the reference illuminant with the criteria that if CCT of test source is less than 5000 K then select the reference source from the pool of black body radiators otherwise from the a phases of daylight. The reference illuminant and test source must have same or nearly the same chromaticity so that like quantities are compared. The chromaticity difference (DC[3]) between the lamp to be tested (u_k, v_k) and the reference illuminant (u_r, v_r) should be smaller than 5.4×10^{-3}. It should be calculated using equation (32). If the tolerance DC is greater than 5.4×10^{-3}, the resulting colour rendering indices may be less accurate.

$$DC = \sqrt{(u_k - u_r)^2 + (v_k - v_r)^2} \tag{32}$$

where

DC	is chromaticity difference
u_k, v_k	are the chromaticity coordinate of test source
u_i,v_i	are the chromaticity coordinate of reference illuminant

Step 3: Choose the 14 test colour samples from the Munsell Book of Colours. The first eight test colour samples should cover the hue circle, moderate in saturation, and are approximately the same in lightness. The other six colour samples should represent a strong red, yellow, green and blue and represent complexion and foliage. The CIE-1974 test colour samples are listed in Table 1. The CIE-1974 test-colour samples are specified by the spectral radiance factors in appendix 1.

Table 1. CIE test colour samples (CIE 1995).

No.	Approximate Munsell notation	Colour appearance under daylight	
1	7,5 R 6/4	Light greyish red	🟥
2	5 Y 6/4	Dark greyish yellow	🟫
3	5 GY 6/8	Strong yellow green	🟩
4	2.5 G 6/6	Moderate yellowish green	🟩
5	10 BG 6/4	Light bluish green	🟦
6	5 PB 6/8	Light blue	🟦
7	2.5 P 6/8	Light violet	🟪
8	10 P 6/8	Light reddish purple	🟪
9	4.5 R 4/13	Strong red	🟥
10	5 Y 8/10	Strong Yellow	🟨
11	4.5 G 5/8	Strong green	🟩
12	3 PB 3/11	Strong blue	🟦
13	5 YR 8/4	Light yellowish pink (human complexion)	⬜
14	5 GY 4/4	Moderate olive green (leaf green)	🟫

Step 4: Calculate the tristimulus values of the test samples when illuminated by the test source and the reference illuminant by using CIE 1931 standard colorimetric observers. Transform these tristimulus values into coordinates of the CIE 1960 UCS diagram using equation (12).

---

[3] DC is the distance on the CIE 1960 (u,v) chromaticity diagram. It is also denoted by duv but not to be confused with colour difference (ΔE).

Step 5: The adaptive colour shift due to the different state of chromatic adaptation under the lamp to be tested (k) and under the reference illuminate (r) should be taken into account using the equation (33).

$$u'_{k,i} = \frac{10.872 + 0.404\frac{c_r}{c_k}c_{k,i} - 4\frac{d_r}{d_k}d_{k,i}}{16.518 + 1.481\frac{c_r}{c_k}c_{k,i} - \frac{d_r}{d_k}d_{k,i}}$$

$$v'_{k,i} = \frac{5.520}{16.518 + 1.481\frac{c_r}{c_k}c_{k,i} - \frac{d_r}{d_k}d_{k,i}} \qquad \textbf{(33)}$$

where

$u'_{k,i}$ and $v'_{k,i}$ are the chromaticity co-ordinates of a test-colour sample (i) after consideration of the adaptive colour shift.

The functions (*c)* and (*d)* of equation (33) can be calculated for the light source to be tested u_k, v_k (giving c_k, d_k) and the test colour samples (i) under the light source to be tested $u_{k,i}$, $v_{k,i}$ (giving $c_{k,i}$, $d_{k,i}$) according to the following formulae.

$$c = \frac{1}{v}(4 - u - 10v)$$

$$d = \frac{1}{v}(1.708v + 0.404 - 1.481u) \qquad \textbf{(34)}$$

Step 6: The colorimetric data thus obtained is transformed into 1964 Uniform space co-ordinates using equation (15). Re-writing equation (15), we get:

$$W^*_{r,i} = 25(Y_{r,i})^{1/3} - 17 \qquad W^*_{k,i} = 25(Y_{k,i})^{1/3} - 17$$

$$U^*_{r,i} = 13W^*_{r,i}(u_{r,i} - u_r) \qquad U^*_{k,i} = 13W^*_{k,i}(u'_{k,i} - u_r) \qquad \textbf{(35)}$$

$$V^*_{r,i} = 13W^*_{r,i}(v_{r,i} - v_r) \qquad V^*_{k,i} = 13W^*_{k,i}(v'_{k,i} - v_r)$$

The values $Y_{r,i}$ and $Y_{k,i}$ must be normalised so that $Y_r = Y_k = 100$.

Step 7: In this step, the resultant colour shift is calculated. The CIE 1964 Colour difference formula (equation (20)) should be used for the purpose. The resulting equation is:

$$\Delta E_i = \sqrt{(U^*_{r,i} - U^*_{k,i})^2 + (V^*_{r,i} - V^*_{k,i})^2 + (W^*_{r,i} - W^*_{k,i})^2} \qquad \textbf{(36)}$$

where

ΔE_i is the colour difference between the colour coordinates determined for the same test colour sample illuminated by the test and reference illuminant

i refers to the test sample number

The sections 3.2.1 and 3.2.2 describe the final step.

3.2.1 Special Colour rendering indices

The Special colour rendering indices are designated by symbol R_i (where i= 1, 2, 3…..n, where n denotes number of test-colour sample that are studied) and derived by the use of following equation:

$$R_i = 100 - 4.6\Delta E_i \tag{37}$$

where

R_i	is special colour rendering indices
ΔE_i	is colour difference of test colour samples under test source and reference illuminant

The value obtained from equation (37) is rounded to the nearest whole number. Equation (37) is derived with the assumption that the colour rendering index of reference source is 100. The constant value 4.6 is derived from general colour rendering index by assigning R_a = 50 to a standard white fluorescent lamp at 3000 K i.e. FL-4 illuminant.

3.2.2 General Colour Rendering Index

The general colour rendering index is the arithmetical mean of the eight special colour rendering indices (R_i) of the CIE test colour samples 1 to 8. The general colour rendering index is CIE CRI. It is symbolically denoted by R_a and mathematically defines as:

$$R_a = \frac{1}{8}\sum_{i=1}^{8} R_i \tag{38}$$

where

R_a	is general colour rendering index or CIE CRI
R_i	represent special colour rendering indices

It is assumed that higher the general colour rendering index better the light source can render the colour.

3.2.3 Deficiencies and limitation of CIE CRI method

The CIE CRI method to predict the colour rendering properties of light source has many deficiencies. Some of the limitations of CIE CRI are:

1. Reference Source: In CIE Test sample method the reference source should be selected either from black body radiator or from a phase of day light. Only the criterion present was correlated colour temperature (CCT). If CCT of test source is greater or less than 5000 K then reference source should be selected from a series of SPD of phases of day light or black body radiator respectively. Therefore, there are many reference sources that can be selected and lead to the confusion. There is nothing apparent which says that Planckian radiators and phase of daylight are perfect in a colour-rendering sense. Also, no light source can render colour better than reference source. It limits the innovation of new light sources.

2. Test samples: CIE test sample method uses 14 (8+6) Munsell test samples. These samples are not available anymore (CIE 1999) and only metameric samples can be obtained. Also, Munsell colour sample does not cover the colour gamut sufficiently as Macbeth Colour Checker test sample does (McCamy 1976).
3. Chromatic adaptation: Von Kries chromaticity transformation was used to cope with chromatic adaption in CIE test sample method. Von Kries Chromaticity transformation is not good choice as it is not applicable for larger chromatic difference condition.
4. Colour space and colour difference formulae: The U*V*W* colour space is visually not uniform compared to other colour spaces such as CIELAB colour space. CIE test sample method uses CIE U*V*W* colour space and corresponding colour difference formulae which is obsolete and inadequate.
5. The single averaged number can not define the colour rendering properties of light sources. The light sources with same CRI may render colour differently if they have different special CRI.
6. For some lamps like low-pressure sodium general colour rendering index (R_a) is negative (Rea 2000, Ohno 2010) and it is difficult to interpret.

3.3 CIE TC 1-33 closing remark to update CIE test sample method

A number of new recommendations on colorimetric methods have been introduced after the introduction of the CIE Test sample method (CIE 2006). The two colour spaces CIELAB and CIELUV in 1976 and new chromatic adaption transformation in 1994 (CIE 1994) are most important recommendations. The new colour spaces are more equidistant than the CIE U*V*W* space and new chromatic adaption transformation is better than von Kries transformation. This discrepancy was soon realised by CIE colorimetric Committee. CIE established the technical committee several times to tackle the discrepancy but every TCs were closed in five to ten years as they could not find a solution that every members would agree upon. CIE TC 1-33 is the last such committee, established in 1991 and closed down in 1999.

The Terms of Reference of CIE TC 1-33 was formulated as:

Study indices for evaluation of colour rendering properties of light sources based on a colour appearance model. Prepare a technical report on a proposed method that will replace CIE publication 13.2, this model shall be consistent with all the official recommendations on colorimetry. (CIE 1999)

3.3.1 Work of CIE TC 1-33

The members of CIE TC 1-33 concluded that the original Munsell test samples do not cover the colour gamut sufficiently. It was replaced by easily available set of Macbeth Colour Checker test samples. This is a set of 18 coloured and six achromatic samples used in photography. The committee selected eight samples of this set and for the very important skin tones agreed on a typical Caucasian and Oriental complexion reflectance spectra.

The next issue was the question of what reference illuminant should be used. The traditional system uses the concept of CCT to select the reference illuminant. However, the CCT concept is based on the outdated CIE 1960 u, v diagram. Firstly, the committee agreed to abandon the concept of equal correlated colour temperature between test and reference illuminant completely and suggested to use one of those target chromaticites used by IEC (International Electrotechnical Comission). This allows using reference illuminant whose chromaticity lies nearest to the chromaticity of the test lamp in CIELAB space. Later, some TC Members objected to this solution as it gave slightly lower colour rendering indices for some of the lamps they produced. The views of the committee members differed on choosing reference illuminant so gravely that this was one issue where no consensus for a new method could be obtained.

For the chromatic adaptation, TC committee agreed to used new chromatic adaptation transformation formula (CIE 1994) introduced by CIE in 1994. CIE recommended using this formula for colour rendering. Both set of data from test lamp chromaticity and from reference illuminant chromaticity should be transformation into a D65 illuminant adaptation, as the CIELAB space has been most thoroughly tested under D65 illumination.

The TC Members also agreed that in a new method, CIELAB space should be used to calculate colour difference between the test colour samples illuminated once by the test lamp and then by the reference illuminant.

The TC committee decided that the principle of transforming colour differences into colour rendering indices should not be changed. The equation (39) should be used to calculate the special colour rendering indices

$$R_i = 100 - c\Delta E_{ab,i} \tag{39}$$

where

R_i is special colour rendering indices
c is constant
$\Delta E_{ab,i}$ is CIELAB colour difference for test colour sample i

The general colour rendering index is calculated as the arithmetic mean of the 10 special colour rendering indices.

The reasonable agreement can be seen from a comparison between the traditional method (R_a) and the new proposed one (R96) (see Figure 25). The rank orders of only some tested light sources were changed. This was annoying to some experts (CIE 2006).

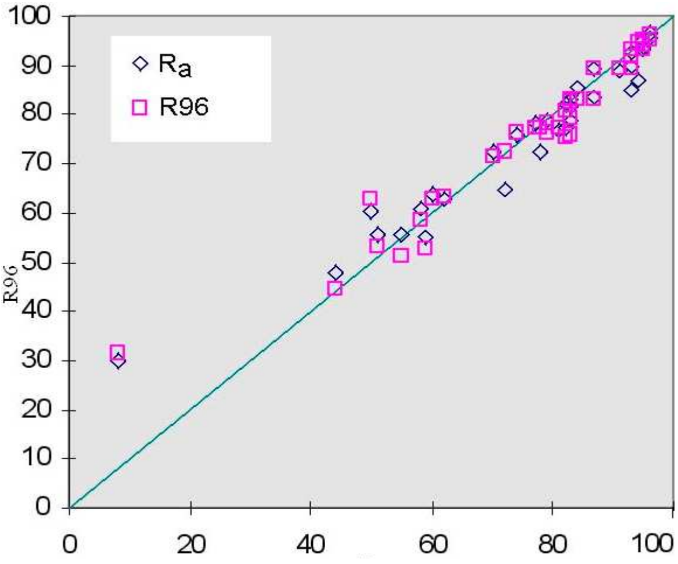

Figure 25 Correlation Chart depicting the R96 values versus the R_a (CIE 2006). R96 denotes CIE TC 1-33 general colour rendering index and Ra denotes general colour rendering index calculated using the current method.

3.4 Summary

CIE CRI describes the ability of a light source to reproduce the colours of various coloured samples in comparison with an ideal or natural light source. CIE recommended CIE test sample method described in CIE 13.3-1995 (CIE 1995) to calculate CIE CRI. The method is based on obsolete colorimetric techniques. For example, the CIE 1964 U*V*W* uniform colour space used to calculate colour difference is outdated, and the von Kries chromatic adaptation transform performs poorer than other available models, such as CMCCAT2000 or CIECAT02. CIE test sample method fails to explain the meaning of negative colour rendering index (R_a = -44 for low pressure sodium lamp (Rea 2000)).

CIE technical committee TC 1-33 tried to update CIE test sample method but failed to do so and finally closed down with closing remarks. Deadlocks in the work of the CIE TC 1-33 were more political nature than of technical character (CIE 1999). The questions that produced the deadlock are:

- Selection of the six IEC (International Electrotechnical Committee) target chromaticities or a continuous set of daylight/blackbody reference illuminants (advantage of adhering to target lamp-light chromaticities versus somewhat higher colour rendering indices if the lamp chromaticity is further away from the IEC target chromaticity). This will influence also the selection of constant c.

- The selection of those lamp spectra on which the transition from the old system to the new system should be based.

4 Colour Rendering of LEDs

Light Emitting Diode (LED) is a solid state light source. The term “solid state” is used because light is emitted from solid material (i.e. semiconductor) and not generated in vacuum or gas filled tubes (Martin 1997). Today LEDs are in wide use. They are used in signal lights, exterior and interior lighting, displays, and in automotive lighting. Recently, LEDs have also been started to be used in general lighting. LEDs can produce white light mainly in two ways: phosphor conversion and colour mixing. In phosphor conversion a phosphor material is used to convert monochromatic light from a blue or UV LED. In colour mixing light from multiple monochromatic LEDs (red, green, and blue) is mixed, resulting in white light (EERE 2008).

The spectral power distribution of RGB LED (a cluster of red, green, and blue LED) has peaks in red, green, and blue region of visible spectrum, whereas phosphor based LEDs have peaks in blue and yellow regions. Unlike the traditional light sources, LEDs have greater freedom in spectral design as white light from LEDs is realized by mixture of multi-colour LEDs or by a combination of phosphors excited by blue or UV LED emission.

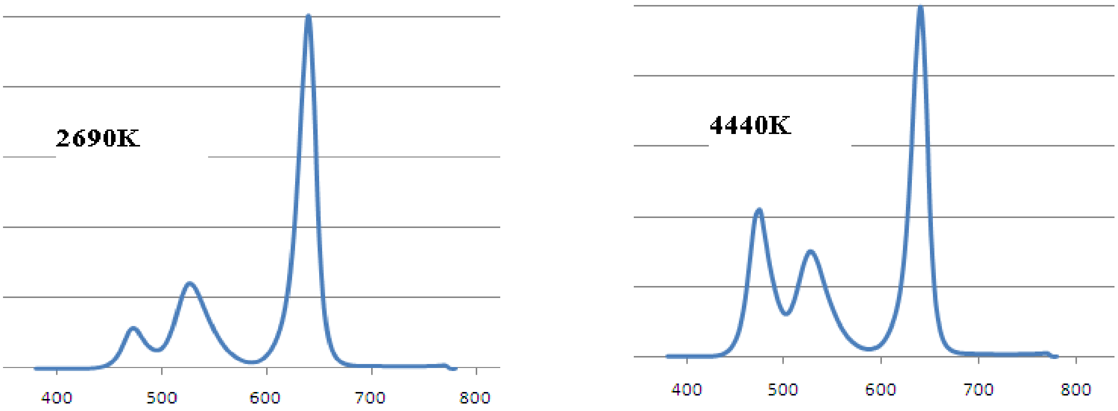

Figure 26. SPDs of RGB LEDs, 2690 K and 4400 K (Khanh 2010).

During the early days when white light from LEDs was achieved scientists were more concerned about luminous efficacy. The lighting scientists were able to get increasing luminous efficacy but with low colour rendering index. Many questions have also arisen about whether or not CIE CRI should be considered as design aspect of white LEDs, because of many flaws in the CIE Test sample method (see Section 3.2.3). In 2007, CIE concluded that CIE CRI is not a good means to judge the colour rendering properties of white LEDs (CIE 2007). The CIE Technical Committee TC 1-62 recommended developing new colour rendering index. However, it should not replace the current CIE colour rendering index immediately. The replacement should be considered only after successful integration of the new index. Until new metrics are developed, colour rendering properties of LEDs should be compared with traditional light sources by using CIE CRI.

4.1 Why CIE CRI fail for LEDs?

White RGB LED light sources have peculiar SPDs with three significant peaks in R, G and B region and small amount of power between certain neighbouring peaks. The experiment conducted by Bodrogi et al. (Bodrogi 2004) found that RGB LEDs have wide range of perceived colour difference compared to reference illuminant or traditional light sources. This wide range of perceived colour differences can not be characterized by obsolete colour difference formulae (used in CIE Test Method) and especially not by one single average number (R_a) (Bodrogi 2004).

It is possible to tune the spectrum to get the high CIE CRI value but with little change in colour rendering as perceived by human eye. Narendran et al. (2001) found CIE CRI for light source with combination of 465, 525 and 640 nm RGB LED to be 18 at 2800 K. In the same experiment when 640nm red LED was replaced by 615 nm red LED, CIE CRI improved significantly to 71. However, visual experiment showed that the RGB LED with low value of CIE CRI was more preferred than incandescent lamp or tungsten halogen lamps (Narendran 2002).

The light sources with broadband SPD (e.g. incandescent lamps) have high CIE CRI but the light sources with narrowband SPD (e.g. low pressure sodium lamps) have low CIE CRI. LEDs can be designed to have broad SPD curve but the luminous efficacy of that light source will be low (Ohno 2005). The luminous efficacy of radiation (LER) and Colour rendering are generally in a trade-off relationship. Therefore, compromise should be made between LER and CIE CRI. But what if the metric to calculate colour rendering of light sources is improper?

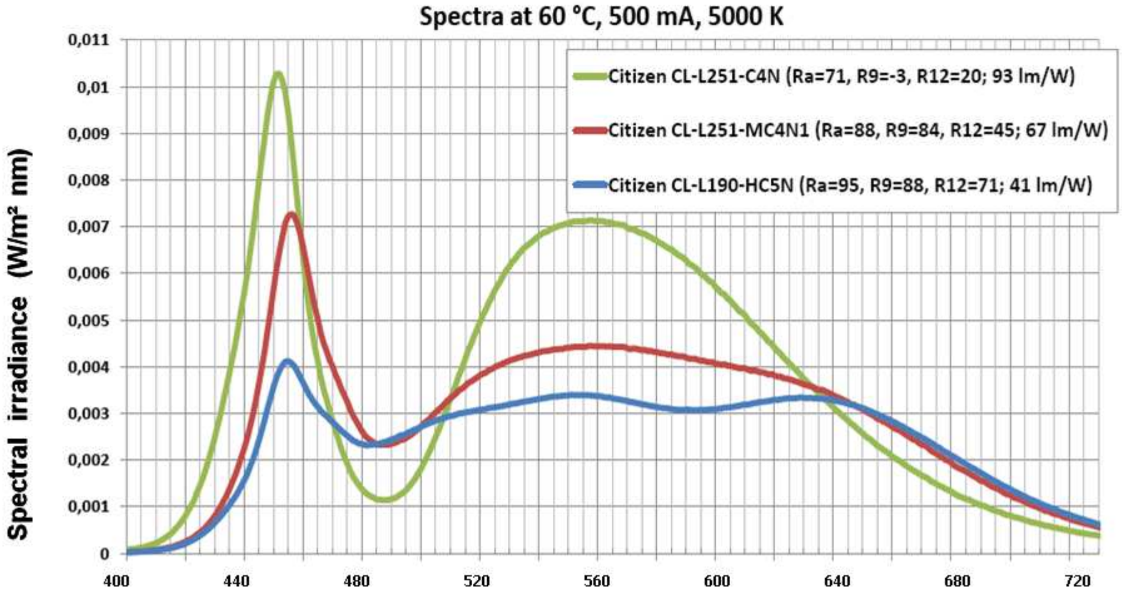

Figure 27. Typical white phosphor LEDs spectra (Khanh 2010).

Figure 27 shows the SPDs of Citizen CL-L251-C4N, CL-L251-MC4N1 and CL-L190-HC5N LEDs, on which the Citizen CL-L251-C4N LED has general colour rendering index R_a = 71. If this LED is judged only with CIE CRI value it should render the colour fairly. However, if we see special colour rendering index R_9 (-3) and R_{12} (20), this LED fails to render strong red and strong blue colours. The Figure also shows that LEDs with higher CIE CRI have lower luminous efficacy and vice versa.

Technical report CIE 177:2007 (CIE 2007) presented three case studies related to the visual experiments and simulations on colour rendering. They were conducted in Hiroshima City University, National Institute of Standards and Technology (NIST), and University of Pannonia. Results from those case studies showed that the CIE CRI is problematic. In the experiment at the Hiroshima city University semantic differential method was applied (consisting of 18 adjective pairs like high fidelity-low fidelity, warm-cool, beautiful-ugly, etc.) to evaluate the visual impression of 5 natural scenes under nine RGB LED clusters and one blue LED with a yellow phosphor (Nakano, 2005). Principle component analysis of the results showed two relevant factors. The first factor was associated with the visual impression of colourfulness, and the second factor with colour fidelity. The second factor correlated well with the CIE CRI but the correlation of the first factor was low (CIE 2007).

NIST has been conducting a series of colorimetric simulations examining the appearance of many different reflective samples when illuminated by various light spectra (CIE 2007). The CIE CRI was compared with visual observations from simulations as well as with the colour shifts of samples on CIELAB coordinates. It was observed that several RGB LED models render the colour samples used in the calculation of CIE CRI well, and have high CIE CRI value but render the samples of higher chroma poorly. The peak spectra of LED models were particularly vulnerable to poor rendering of a small range of hues. The eight test colour samples used to calculate CIE CRI were found to be insufficient to cover all the possible hues. RGB LEDs produced only one or two colour samples with large colour differences and all other colours showed very small colour differences, resulting high CIE CRI. Some RGB LEDs increase the chroma of objects, which is considered a possible important feature of LED lighting but is penalized by the CIE CRI (CIE 2007).

Sándor and Schanda (2005) carried out series of visual experiments to check the CIE CRI method. The Visual experiments showed that for white phosphor LED light sources, correlation coefficients between visual colour difference and calculated using the CIE Test colour method were not significant. For RGB LED clusters, the correlation coefficients were significant. From this study the authors concluded that visual colour rendering is not well described by the CIE Test colour method.

4.2 Recent CIE TC on Colour rendering of white light sources

In 2006 CIE formed a new technical committee CIE TC 1-69 Colour Rendition by White Light Sources. The terms of reference of this committee is:
To investigate new methods for assessing the colour rendition properties of white-light sources used for illumination, including solid-state light sources, with the goal of recommending new assessment procedures. (CIE 2007a)

The working plan of CIE TC 1-69 are:

1. To agree on some basic criteria for new metric (or system of metrics) such that it (or they) could be developed to be scientifically sound, acceptable to lighting industry, and useful
2. To solicit, share, and discuss proposals for new assessment procedures for colour rendition properties of white light sources
3. To evaluate proposed assessment procedures with visual experiments and compatibility with basic criteria (in # 1)
4. To recommend a new metric(or system of metrics) based on evaluation (in # 3)
5. To prepare a CIE Technical Report on recommended new metric (or system of metrics), including calculation procedures and justification for recommendation

The First meeting of CIE TC 1-69 was held at the 26th session of the CIE in Beijing, China. After discussion in first meeting it was agreed:

- to design new metric assuming that CIE CRI may not exist in the future.
- new metric to have one number output, with optional supplementary indices.
- to scale new metric to be comparable to CIE CRI for traditional lamps but exact ordering of certain existing lamps may change.

In the same meeting it was also agreed that colour fidelity would be an important component of new metrics. After the Beijing meeting three other meetings had held in 2008 (Stockholm, Sweden), 2009 (Budapest, Hungary) and 2010 (Princeton, USA). Till

January 2010, about 17 research reports were prepared from 10 labs/groups from 7 different countries. The research topics cover colour memory for real objects, chromatic discrimination, attractiveness and naturalness of fruits and vegetables, estimations of colour differences, rendering of human skills, and colour harmony. During the Princeton meeting held in 2010, seven new metrics were purposed:

1. Rank- order based colour rendering index (RCRI)
2. Feeling of contrast index (FCI)
3. CRI-CAM02UCS,
4. Colour quality scale (CQS)
5. Harmony rendering index (HRI)
6. Memory CRI (MCRI)
7. Categorical colour rendering index (CCRI)

The final metric should be chosen from these 7 metrics. The new metric was expected to be introduced in February 2010 but the decision has been delayed due to extra work on defining terminology.

4.2.1 Rank- order based colour rendering index (RCRI)

Peter Bodrogi et al (2010) introduced a new colour rendering index called rank-order based colour rendering index (RCRI) also known as ordinal scale based colour rendering index. The aim was to provide easy interpretation value for a nonprofessional user to assess the equality of the colour rendering properties of the test light sources or to assess the ranking among the different test light sources with respect to colour rendering properties.

RCRI is calculated by using seventeen test colour samples (1-12 from Macbeth Colour Checker chart and 13-17 from NIST Colour Set). The reference illuminants are the same as those used by the CIE CRI (see section 3.1). The RCRI uses CAM02 –UCS (colour appearance model) to calculate the colour difference. The colour difference ΔE_{calc} for each one of the seventeen test colour samples should be calculated by using CAM02-UCS formula (Moroney 2002) for a given test light source and its reference source (as defined in CIE CRI). The mean values of ΔE_{calc} should be calculated using the CAM02-UCS formula in each rating category ($\Delta E_{calc,mean,R}; R = 1-5$) by considering the whole experimental dataset (see (Bodrogi 2010a) for details). Table 2 shows the $\Delta E_{calc,mean,R}$ values computed in each rating category (R = 1-5) together with the number of cases, standard deviations, and 95% confidence intervals.

Table 2. Mean $\Delta E_{calc,mean,R}$ values for each rating category (R=1-5), number of cases (No.), standard deviations (STD), and 95% confidence intervals(Cl).

	Computed colour differences (CAM02-UCS)				
R	1	2	3	4	5
No.	163	390	361	300	104
Mean	2.01	2.37	2.58	6.53	11.28
STD	1.25	1.72	2.58	4.03	5.13
95% Cl	0.19	0.17	0.27	0.46	0.99

Ranking (1 for excellent, 2 for good, 3 for acceptable, 4 for not acceptable, and 5 for very bad) for every test colour sample (k=1-17) is predicted using $\left|\Delta E_{calc,mean,i} - \Delta E_{calc,k}\right|$= minimum criterion. For example, if absolute difference $\left|2.01 - \Delta E_{calc,k}\right|$ (for R = 1, $\Delta E_{calc,mean,i}$ = 2.01) is the smallest for kth test colour sample (any of seventeen test colour samples) then ranking of that kth test colour sample is predicted as 1, and if $\left|2.37 - \Delta E_{calc,k}\right|$ (for R=2) is smallest for kth test colour sample then ranking of that kth test colour sample is predicted as 2. Similarly rank 3, 4 and 5 are predicted. The number of test colour samples that have a predicted rank 1 and 2 are counted, and represented by N_1 and N_2 respectively. Finally, Rank-order Colour Rendering Index is calculated using equation:

$$RCRI = 100\left[(N_1 + N_2)/17\right]^{1/3} \tag{40}$$

where

RCRI is Rank-order Colour Rendering Index

4.2.2 Feeling of contrast index (FCI)

Feeling of contrast index (FCI) is the index that describes quantitative degree of the feelings of contrast for the test lamp to the reference illuminate (Hashimoto, 2007). Based on the studies made by Hashimoto et al, the FCI is calculated using equation:

$$FCI = \left[G(T, Et = 1000\,lx) / G(D_{65,} Er = 1000\,lx)\right]^{1.5} \times 100 \tag{41}$$

where

FCI	is Feeling of contrast index
$G(T, Et = 1000\,lx)$	corresponds to the gamut-area value on the four-colour combination under the test illuminant (T) at illuminance 1000 lx
$G(D_{65,} Er = 1000\,lx)$	corresponds to the gamut-area value on the four-colour combination under the reference illuminant (D65) at reference illuminance 1000 lx

Red (5R4/12), Blue (4.5PB3.2/g), Yellow (5Y&2/10), and Green (5.5G5/8) are selected as four-colour combination (the values inside parentheses are their Munsell notations). These four-colour combinations are selected with the consideration that they represents almost all the hues used in the actual environment (Hashimoto, 1994). The tristimulus values of the corresponding colour of each component colour (R, Y, G, B) of the four-combinations are converted into CIELAB coordinates and plotted in CIELAB space. The gamut areas are computed by the area sum of the two triangles: one consisting red, yellow and green; and the other of red, blue, and green. Red is taken as reference because the red component colour was most important in the assessment of the feeling of contrast. (Hashimoto, 1994)

4.2.3 CRI-CAM02UCS

The CRI-CAM02UCS (Li, 2009) follows the fundamentals of the original calculation of the CIE CRI. This method is based on the variation in colour appearance of test samples illuminated under the test source and the reference illuminant. The calculation steps for CRI-CAM02UCS is exactly same as that of CIE CRI except CRI-CAM02UCS uses CIECAM02 (Moroney 2002) colour appearance model and includes both chromatic adaptation transform of 2002 (CAT02) and colour difference equation. The difference equation in CRI-CAM02UCS is equally weighted for shifts in lightness, colourfulness and hue for the test samples between the test light source and the reference illuminant. Figure 28 shows the workflow for calculating CRI-CAM02UCS. In this method, new scaling factor (8) is determined so that the average score of the CRI-CAM02UCS for the CIE standard fluorescent lamps (F1-F12) is equal to the average score of the CIE CRI (R_a=75) for these sources (i.e. F1-F12). The new CRI-CAM02UCS is calculated using equation:

$$CRI - CAM02UCS = \frac{1}{8}\sum_{1}^{8}(100 - 8 \times \Delta E_{iCAM02-UCS}) \tag{42}$$

where

CRI-CAM02UCs is Colour rendering index based on CAM02UCs

$\Delta E_{iCAM02-UCS}$ is the colour difference for 8 test colour sampled in the CAM02-UCS

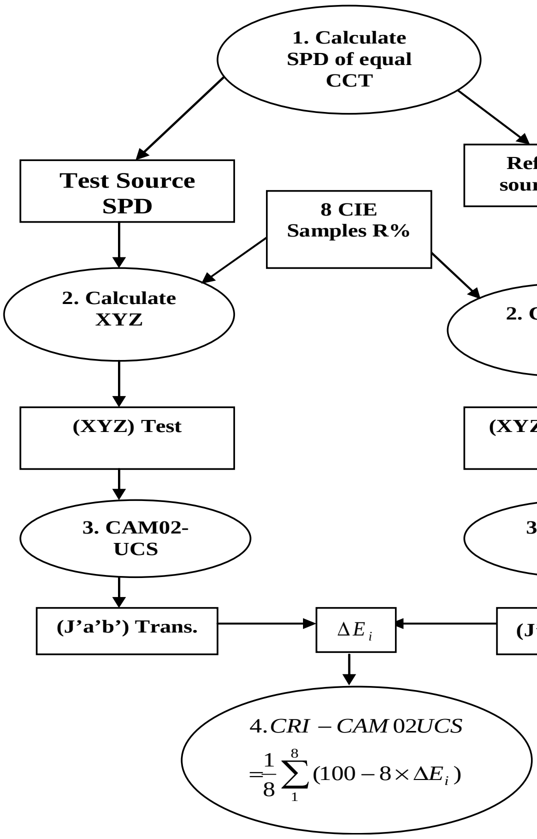

Figure 28. Flowchart for Calculating the CRI-CAM02UCS (Li 2009). (J'a'b') Trans. denotes chromatic adaptation transform of 2002 (CAT02).

4.2.4 Colour quality scale (CQS)

Colour quality scale is being developed in National Institute of Standard and Technology (NIST). This metric describes several aspects of colour quality including colour rendering, chromatic discrimination, and observer preferences (Davis 2010). The method for calculating the CQS is derived from CIE Test Sample Method with some modifications. Unlike CIE CRI, CQS uses 15 Munsell samples covering low to high chromatic saturation. The reference illuminant is selected as in CIE CRI. The tristimulus values of illuminated samples are corrected for chromatic adaptation using CMCCAT2000 (Li 2001b) and CIELAB colour space is used instead of CIE 1964 W*U*V* colour space. The saturation factor (Davis 2010) is introduced in calculating the colour difference of each reflective sample in CQS. The saturation factor is introduced to neutralize the colour difference that arises from an increase in object chroma from test source relative to the reference illuminant. To ensure the influence of large hue shifts of any sample on the results, the root-mean-square (RMS) of colour shifts of each individual sample is used on the CQS rather than arithmetic mean. The RMS colour differences of the 15 samples are calculated using equation (43).

$$\Delta E_{\mathrm{RMS}} = \sqrt{\frac{1}{15}\sum_{i=1}^{15}\left(\Delta E^{*}{}_{ab,i,sat}\right)^{2}} \tag{43}$$

where

ΔE_{RMS} is RMS colour difference

$\Delta E^{*}{}_{ab,i,sat}$ represents colour difference for each sample (i) illuminated by test source and reference illuminant with the intergration of the saturation factor

The rms average colour quality scale score is calculated as:

$$Q_{a,rms} = 100 - 3.1 \times \Delta E_{RMS} \tag{44}$$

where

$Q_{a,rms}$ is rms average colour quality scale

The constant value 3.1 in equation (44) is the scaling factor. The scaling factor was selected such that the average output of the CIE CRI (R_a= 75.1) for a set of CIE standard fluorescent lamp spectra (F1-F12) is equal to the average of the General Colour Quality Scales for these sources. Since CIE CRI can give negative values, CQS implemented a mathematical function ($Q_{a,0-100}$) given by equation (45) to avoid occurrences of negative numbers.

$$Q_{a,0-100} = 10\ln\left\{\exp(Q_{a,rms}/10) + 1\right\} \tag{45}$$

where

$Q_{a,0-100}$ is a mathematical function of CQS to avoid occurrences of negative numbers

To overcome the limitation of reference illuminant seen in CIE CRI, CQS uses a multiplication factor called as CCT factor. The multiplication CCT factor (M_{CCT}) is calculated from the gamut area of the reference source:

$$M_{CCT} = T^{3}(9.2672\times10^{-11}) - T^{2}(8.3959\times10^{-7}) + T(0.00255) \tag{46}$$

$$-1.612(\mathit{for}\, T < 3500\, K)$$

$$M_{CCT} = 1\ (\mathit{for}\, T \geq 3500\, K) \tag{47}$$

where

M_{CCT} is multiplication CCT factor

Finally, the General Colour Quality Scale (*Q_a*) is calculated using the equation:

$$Q_a = M_{CCT} Q_{a,0-100} \tag{48}$$

where

Q_a is general Colour Quality Scale

4.2.5 Harmony rendering index (HRI)

The Harmony rendering index was purposed by Szabó et al (2009). Harmony rendering index (HRI) is based on colour harmony predictions. When two or more colours seen in neighbouring areas produce a pleasing effect, they are said to produce a colour harmony (Judd 1975). The HRI is determined by calculating the difference in colour harmony predictions for a set of test colour sample combinations between the appearance of test samples under the test light source and the reference light source. These differences characterize the colour harmony rendering property of the test light source by calculating the extent of distortion of the perceived colour harmony of a set of test colour combinations that are harmonious under the reference illuminant.

The Harmony rendering index (HRI) is calculated as:

$$HRI = 100 - k * \sum \left| CHF_{i_{ref}} - CHF_{i_{test}} \right| \qquad \textbf{(49)}$$

where

HRI is the Harmony rendering index
$CHF_{i_{ref}}$ is the colour harmony formula under the reference light source
$CHF_{i_{test}}$ is the colour harmony formula under the test light source
n indicates the number of test colour pairs
k is scaling factor or constant whose optimised value is 5

The value of (k) is determined by equating average HRI of CIE standard fluorescent lamps and average score of CIE CRI of those sources. The four colour harmony formulae CHF_{2M}, CHF_{2D}, CHF_{3M} and CHF_{3T} (see appendix of Szab*ó* 2009) give colour harmony predications and can be inserted in equation (49) depending on the set of 2 or 3 test colour combinations used.

4.2.6 Memory Colour rendering index (MCRI)

The MCRI is the colour rendering metric based on memory colours (Smet 2010a). Memory colours are those colours that are associated with familiar objects in long-term memory (Smet 2010b). The proposed MCRI is based on the perceived similarity between an object’s apparent colour and its memory colour. The test sources will have better colour rendering properties if the degree of similarity is higher. Development of MCRI involved visual experiments taking a set of nine familiar real objects having colours distributed around the hue circle. The nine familiar objects were a green apple, a banana, orange, dried lavender, a smurf figurine, strawberry yoghurt, a sliced cucumber, a cauliflower, and caucasian skin. In the experiment the group of observers were asked to rate the colour appearance of the objects with respect to what they thought the object looked like in reality. The pooled ratings of each object were modelled by a modified bivariate Gaussian distribution in IPT colour space (Hashimoto 2007). The degree of similarity with the memory colours were calculated using the similarity distributions. The general MCRI was obtained by taking geometric mean of the set of calculated similarities. The general MCRI describes the colour rendering properties of a light source in general. Higher the number better the light source can render the colour.

4.2.7 Categorical colour rendering index (CCRI)

The categorical colour rendering index is based on categorical colour name and uses the colour appearance model CIECAM97s (Yaguchi 2010). To calculate categorical colour rendering Yaguchi et al. (Yaguchi 2010) carried out the simple experiment with 9 subjects. The D65 fluorescent lamp and four white phosphorous LEDs (with CCTs; 2800 K, 4000 K, 5100 K and 7500 K) were used as reference and test light sources respectively. These five light sources were used to illuminate 292 colour chips at illuminace level of 500 lx. The subjects were asked to sort samples into eleven basic colour categories specified by Berlin and Kay (Berlin, 1969). The evaluation method for the categorical colour rendering developed by Yaguchi (Yaguchi 1999) has an idea similar to percent overlap developed by Boynton et al (Boynton 1990). The comparison was made between the colour name region obtained from the reference source and four LED light sources. The samples that were sorted into the same colour category consistently for three trails under the reference source were selected. Among these selected samples (N_{D65}), the numbers (Ns) of samples under each LED illumination sorted into the same category as in reference source were counted. Finally, the categorical colour rendering index (CCRI) was calculated as:

$$CCRI = \frac{100\times(N_s - N_d)}{N_{D65}} \tag{50}$$

where

CCRI	is the categorical colour rendering index
N_{D65}	is total number of samples sorted into the same colour category consistently for three tails under the reference source (D65)
N_s	represents numbers of samples under each LED illumination sorted into the same category as in reference source
N_d	represents number of samples sorted into a different category consistently for three trials

4.3 Summary

White LED light sources are the new innovations in the field of lighting. Unlike the traditional light sources, LEDs have greater freedom in spectral design as white light from LEDs is realized by mixture of multi-colour LEDs or by a combination of phosphors excited by blue or UV LED emission. High luminous efficacy or high CIE CRI value can be achieved by tuning the spectrum of LEDs (Narendran, 2001). However, luminous efficacy and CIE CRI have trade off relationship (Narendra 2001, Ohno 2005). Many questions have arisen about the whether or not CIE CRI should be considered as design aspect of white LEDs because of many flaws in the CIE test colour method.

Many visual experiments with LED light sources indicate that even LEDs with a low value of CIE CRI can produce visually appealing, vivid, and natural light (Davis 2010, Sándor 2005, Narendran 2002). The limitations of CIE test colour method to calculate CRI of white LED light sources has been realized by CIE. CIE technical committee TC 1-62 concluded that current CIE CRI is generally not applicable to predict the colour rendering rank order of a set of light sources when white LED light sources are involved in the set and hence recommended the development of new colour rendering index (CIE 2007). The new technical committee CIE TC 1-69: Colour Rendition by White light sources is currently working to find new metric(s). It is expected that new metric will be chosen from seven new metrics purposed during Princeton meeting held in 2010 (CIE 2010).

5 Colour rendering characteristics of LEDs: Case studies

5.1 Case 1: Colour Rendering Properties of LED light sources

Nadarajah Narendran and Lei Deng (Narendran 2002)

The aim of the study was to understand the colour rendering and colour preference properties of various LED-based aircraft reading lights and compare them with conventional halogen and incandescent light sources. The hypothesis was that subjective response for colour preference of objects illuminated by two mixed colour white LED light sources will be almost same, although the CIE CRI value changes dramatically. The authors conducted a psychophysical experiment to verify the hypothesis.

<u>Experiment Setup</u>

Two identical cabinets were used and each had an opening of 38 cm by 38 cm. The walls of cabinets were painted with matte finish white paint. To ensure subjects viewed the scenes from a constant distance, 50cm, from the scene, a forehead holder was fixed in the middle frame of the apparatus. The light sources were mounted above the display area pointing upwards and light reached the displayed objects after being reflected from the domed ceiling area and the walls. The viewing objects inside the two cabinets were magazine, two soda cans, and a text card with various font sizes. One cabinet was equipped with six test light sources and the other with two reference light sources. Table 3 shows the different types of light sources used in the experiment.

Table 3. Light sources used in the experiment.

	Test sources	*Reference Sources*
LED aircarft Reading Lights	RGB-Mix Low CRI (R_a=25)	RGB-Mix Low CRI (R_a=23)
	RGB-Mix High CRI (R_a=63)	
	Phosphor/Amber (R_a=81)	
	5-mm Phosphor White (R_a=82)	
	High-power, Two-phosphor White (R_a=83)	
Conventional Reading Lights	Incandescent (R_a=98)	Halogen (R_a=98)

The experiment was conducted in the context of an aircraft reading light. Two experiments were conducted with 30 subjects: 15 male and 15 female. In the first experiment twenty subjects took part and in the second ten subjects took part. Prior to the experiment, subjects were screened for colour deficiencies. The subjects viewed the object sitting in a chair in front of the display cabinets with their forehead in the forehead holder. The first experiment was a side-by-side evaluation. The lights in the two cabinets were turned on and off randomly by the experimenter. After viewing the scene, subjects were asked to complete a questionnaire to rate their preference for a given scene illuminated by a test source compared to an identical scene illuminated by reference source. The second experiment was the individual evaluation where one cabinet was illuminated and the subjects were asked to express their preferences for lighting on a -3 to +3 scale. The +3 rating indicating that the subject strongly liked the lighting, -3 strongly disliked, and zero indicating that lighting was just acceptable for the aircraft reading light application. The subjects were also asked to rate their preference for skin tone colour appearance by viewing their own hand inside the illuminated areas.

Results:

Side-by-side evaluation results:

Figure 29 illustrates the results of side-by-side evaluation experiment for general preference and skin tone preference. The graphs show the percentage of people who preferred the test light source as compared to the reference halogen source.

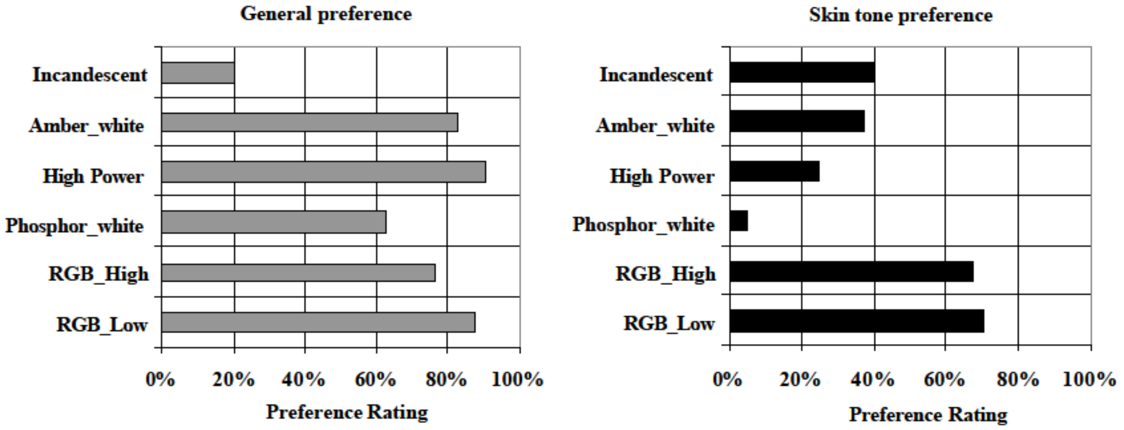

Figure 29. Side-by-side evaluation results for general preference and skin tone preference for reference halogen source.

The results showed that over 50% people preferred LED-based light sources compared to the halogen light source in terms of general preference for object colour appearance. In terms of skin tone preference, amber white LED, high power LED, and phosphor white LED performed much worse than the standard halogen reading lights.

Individual evaluation results:

Figure 30 shows the results of individual evaluation results for general preference and skin tone preference. In terms of both general and skin tone preference, the RGB based white LED reading lights were more liked by the subjects than other test light sources used.

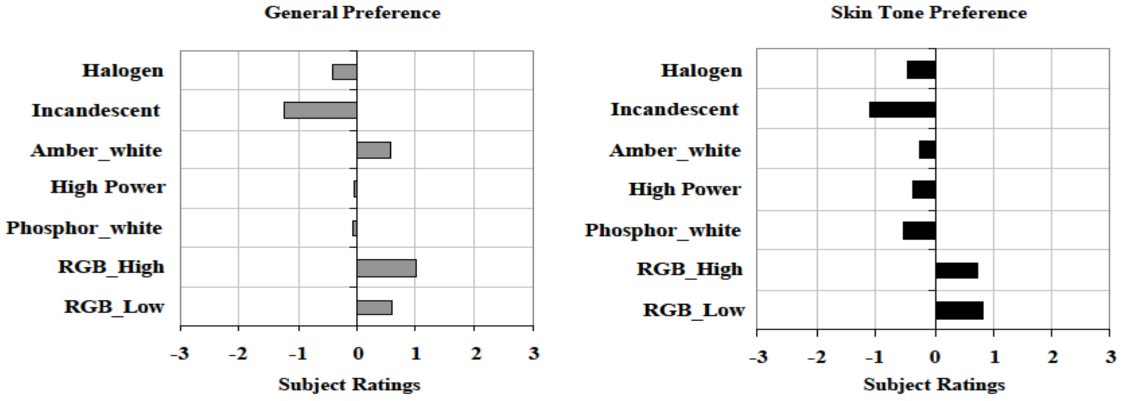

Figure 30 Individual evaluation results for general preference and skin tone preference.

Both experiments gave very similar results. In both experiments RGB based white LEDs were preferred the most. The overall colour preference results for the different light sources are illustrated in Figure 31. It shows that the CIE CRI value of the source does not correlate with people's preference for colour, although the CIE CRI values at different CCT values in general cannot be compared.

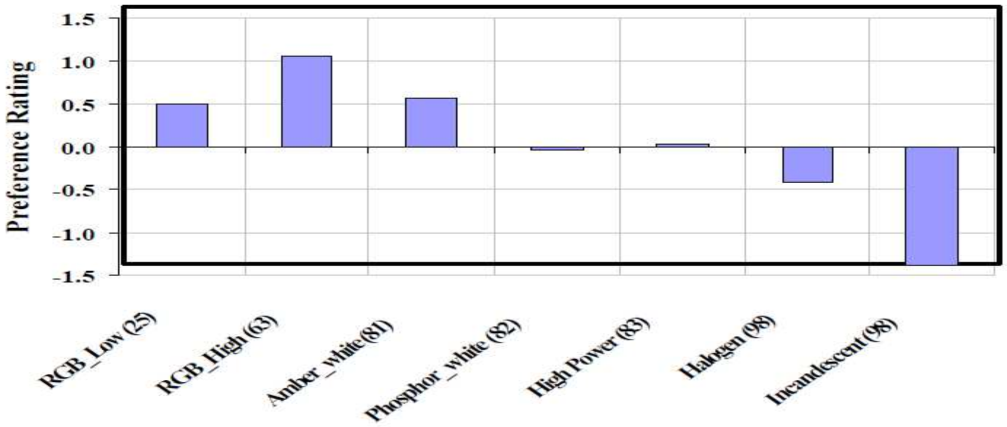

Figure 31. Average preference rating for the different light sources when viewed individually with CIE CRI values inside the parenthesis (Narendran, 2002).

Conclusion

The results showed that LED light source with low CIE CRI was more preferred than a halogen or incandescent light source. In addition the study has shown that CIE CRI has no correlation to people's colour preference. The authors do not recommend CIE CRI as metric for evaluating solid-state light sources because it could negatively impact overall performance.

5.2 Case II: Colour appearance under LED illumination - The visual judgement of observers

F. Viénot, E. Mahler, J.-J. Ezrati, C. Boust, A. Rambaud and A. Bricoune J. (Viénot 2008)

The main objective of this study was is to find out the visual judgement of observers under different LED illumination based on colour discrimination and on colour appearance.

Experiment Setup:

A simple lighting booth was constructed where several LED clusters were housed. The tungsten-halogen lamp with a CCT of 4000 K was taken as reference. The various LED configurations were:

- Solux incandescent lamp (tungsten-halogen lamp)
- a cluster of red, green, and blue LEDs (RGB)
- a cluster of red, green, blue, and amber LEDs (RGBA)
- a cluster of two phosphor cool white plus red LEDs (WWR)
- an "enriched" mixture of cool white, warm white, red, green, blue, and amber LEDs (WWRGBA)

Table 4. Description of light sources.

	Solux	RGB	RGBA	WWR	WWRGBA
E	653	666	657	657	669
x	0.385	0.381	0.385	0.379	0.382
y	0.386	0.385	0.387	0.369	0.381
Tcp	3960	4030	4030	3990	4000
Ra	97	22	67	91	97
***Ra*14**	96	6	60	89	94

The CCT of all these combinations was 4000 K.

Colour discrimination test

Forty observers with normal colour vision took part in the test in which they had to arrange 32 colour caps in order under each illumination. The colour caps were equally distributed along a colour circle in the CIELAB colour space. The hypothesis of experiment was that if many observers fail the test, it reflects a defective quality of the illumination.

Colour appearance test

Twenty normal colour observers were invited to rate the apparent colourfulness of 38 samples from the Natural Colour System (NCS) atlas. The observers were instructed to rate the apparent colourfulness of samples on 0-10 scale.

Hue naming test

In this test the observers were asked to name the hue of individual samples (38 samples from NCS). They were instructed to use two out of the four elementary hue names: Red, Yellow, Green, and Blue. The first named hue was considered as the dominant hue of the sample.

Results and Discussions

The results of colour discrimination test showed that lighting with RGB and the RGBA clusters produce about twice as many errors compared to the WWR cluster, the WWRGBA, and the Solux incandescent lamp. The errors were not evenly distributed along the colour circle. Most of the errors were around red and bluish green shades.

The results of colour appearance test showed that yellowish-red and bluish-green samples were more colourful under RGB or RGBA illumination than under WWR, WWRGBA or Solux illumination. The test results were compared with chroma value of the 38 NCS colour samples computed in CIECAM02 colour appearance model. The comparison showed that the apparent colourfulness apprised by the observers and the computed chroma follow the same pattern of variation over the colour circle. Both approaches showed an increase of colourfulness for yellowish-red and bluish-green samples under the RGB illumination.

The results of hue naming showed that under RGB illumination, hues are apparently attracted toward the yellowish-red or the greenish-blue categories.

Conclusion

The configurations RGB or RGBA have large colour gamut, enhancing the colourfulness around yellowish-red and bluish-green hues but fail to provide faithful colour rendering of environment and colour discrimination. Colour discrimination is a major issue. The authors concluded that the RGB or RGBA configuration as used in their experiment should be avoided and also suggested that any new colour rendering quotation should take colour discrimination into account.

5.3 Case III: A Comparative study of new solid state light sources

Ferenc Szabó, János Schanda, Peter Bodrogi, Emil Radkov (Szabó 2007)

In this study, a series of experiments were carried out to investigate the colour quality of modern light sources. The aim was to characterize the visual colour rendering properties along with three related visual properties (colour discrimination, colour preference, and colour harmony) of various light sources.

Method:

A double booth was setup for the experiments where reference lamp illuminated one section and the test light sources illuminated the other section of the booth. A filtered tungsten halogen lamp and a compact fluorescent lamp were used as reference light sources. Three white phosphor LEDs with different CIE CRI and an RGB LED cluster were used as test lamps. Nine observers with good colour vision took part in the experiments.

Part A. Rendering: In this test a MacBeth Colour Checker Chart (MCC) was placed in each section of the booth. The observers were asked to scale the visual colour difference between the corresponding samples of MCC. To aid the observer, a grey scale was placed into the reference chamber so that the observers could estimate the observed colour difference by comparing it to a given lightness difference on the gray scale.

Part B. Discrimination: Observers had to order the Farnsworth-Munsell 100 hue test under the test and reference light sources. The same observer did the experiment twice: (1) observers had a fixed time, and the error rate was measured; and (2) observers had unlimited time, and the time of completion and the error rate were measured.

Part C. Harmony: Observers had to rate the perceived harmony under the reference source and the test source, on the scale -5, -4….0….4, 5. Ten different combinations of test samples from the Munsell atlas were used.

Part D. Preference: Observers were asked to place each colour sample of set (including natural objects, a Caucasian skin tone, deep red, yellow, a white business card paper, and the MCC) into the middle of both sections of the booth and had to decide which colour they preferred. The other task was preference ranking under one light source at a time, on a rank scale 1, 3, 5.

Results:
The result of colour rendering test showed that the largest colour differences were obtained for the RGB LED cluster. The visual colour difference data correlates best with CIECAM based colour difference formulae.

In the discrimination test, the LED lamp with high CIE CRI showed the lowest error rates and produced lowest time of completion. The RGB LED cluster produced the highest error rates.

For colour harmony, the LED lamps with low, medium, and high CIE CRI provided better perceived harmony than the reference lamps. The RGB LED cluster showed significantly worse harmony.

The results of colour preference test showed that the appearance of test colour samples under the white phosphor LED lamps were most preferred. The visual whiteness was highest under the LED lamp with low CIE CRI and lowest in CFL light source.

Conclusion:
Results of the visual experiments suggested that for different applications different light sources are suitable. The relevant colour quality factors of light sources such as colour rendering, colour preference, colour discrimination and colour harmony should be used to select the appropriate light sources depending upon the application.

5.4 Case IV: Dimensions of light source colour quality

Peter Bodrogi, Stenfan Brückner, and Tran Quoc Khanh (Bodrogi 2010)

The aim of the study was to scale the different dimensions of colour quality under incandescent, fluorescent as well as RGB and white phosphor LED lamps through visual experiments. Colour fidelity, colour discrimination, visual clarity, colour preference, colour harmony, colour acceptability and brightness are some aspects (dimensions) of colour quality. The authors highlighted the importance of quantifying every dimension of colour quality separately.

Experiments were conducted to study the characteristic changes in typical object (e.g. flowers, fruits, vegetables, etc) colour distributions when a different light source illuminates the same objects.

Experiment Setup

Two still lifes (or table top arrangements) were constructed in a viewing booth illuminated diffusely. In the first still life five light sources having CCT 2900 K were used. The second still life was illuminated with three light sources having CCT 2600 K. Only one light source was on in each still life at a time. The five light sources included a tungsten halogen lamp (TUN, reference), a fluorescent lamp (FL), a high colour rendering white LED lamp (HC3L), a low colour rendering white LED lamp (C3L), and an RGB LED lamp (RGB). The three light sources include Tungsten lamp (TUN, reference), compact fluorescent lamp (CFL) and a white phosphor retrofit LED lamp (LED).

For both still life experiments, the questions related to fidelity, harmony, acceptability, visual clarity, and brightness were asked. In second still life experiment there was a further question concerning the general preference judgement of all object colours by considering each object separately.

The observers had to evaluate the still life under every light source separately. The answer had to be indicated on an open continuous scale. For reference light, the value of the scale was fixed at 100 for all questions. In the observation of the first still life, four observers of normal colour vision took part and for second still life five observers of normal colour vision took part.

Result and Discussion

The mean visual scale values and their 95% confidence intervals showed that colour fidelity, colour acceptability, colour harmony and visual clarity (related to colour contrasts) for a HC3L were better than for FL, C3L, and RGB lamps. However, RGB LED lamp had better brightness and larger colour gamut than HC3L. The colour harmony rating of HC3L and colour gamut rating of RGB only exceeded the reference value of 100 (TUN).

The correlations among the different dimensions were also studied. Harmony(H), acceptability (A), and local colour contrast (V1) exhibit significant positive correlations with fidelity (F) ($r^2_{H,F}$=0,98, $r^2_{A,F}$=0,86, $r^2_{A,F}$=0,91)and also with each other ($r^2_{H,A}$=0,90, $r^2_{H,V1}$=0,92, $r^2_{A,V1}$=0,98). The fidelity (F) does not correlate with colour gamut (V2) ($r^2_{V2,F}$=0,00) or with brightness (B) ($r^2_{B,F}$=0,10). There was significant positive correlations between colour gamut and brightness ($r^2_{V2,B}$=0,73).

The visual assessment of the second still life by five observers showed that LED had higher harmony rating than CFL and tungsten halogen lamp. However, CFL had best brightness ratings higher than tungsten halogen lamp and LED. On the other hand tungsten halogen lamp obtained the best acceptability, colour contrast, colour gamut, and preference ratings.

Conclusion
Although colour quality at present is judged by colour rendering index, it has several visual dimensions. The study showed the importance of quantifying every dimension of colour quality separately. The experiments carried out to scale the different dimensions of colour quality under variety of light sources showed that the visual ranking among these light sources depend on the particular dimension of colour quality.

5.5 Summary

The case studies showed that CIE CRI has no correlation to people’s colour preference. In some studies, the LED light source with low CIE CRI was more preferred than tungsten halogen or incandescent light source having high CIE CRI (Case I and III). The CIE CRI is unable to define all the colour qualities of light source such as colour discrimination, colour harmony, colour preference, colour acceptability, brightness etc. Results of the visual experiments suggested that for different application different light sources are suitable and relevant dimensions of light source colour quality should be used to select appropriate light sources as per the application.

One study also showed that LED light source with high CIE CRI have good colour quality and vice versa (Case III). However, for LEDs with medium CIE CRI some dimensions of light source colour quality are very good and some are worst. CIE CRI fails to give this information and hence colour quality of LED light sources can not be judged by CIE CRI

6 Discussion and Conclusion

CIE Colour Rendering Index is the only internationally recognized colour rendering metric to judge the colour rendering properties of artificial light sources, and the sole measure of the colour rendering of light sources used by the industry today (Rea 2010). It is based on the CIE system of colorimetry. CIE CRI expresses the colour rendering properties of light sources based on colour shift of test objects when illuminated by reference illuminant and test source. CIE CRI is a continuous scale from 100 (maximum) to less than zero (Rea 2010). The light sources with CIE CRI ratings above 90 are considered to have excellent colour rendering properties.

The CIE method to obtain Colour rendering index based on resultant colour shifts of test samples is called CIE test colour (sample) method (CIE 1995). The CIE test colour method to calculate CIE CRI has numerous drawbacks. The CIE test colour method is based on obsolete colorimetric techniques. The CIE 1964 U*V*W* uniform colour space used to calculate colour difference is outdated. Moreover, the von Kries chromatic adaptation transform performs poorer than other available models, such as Colour Measurement Committee’s Chromatic Adaptation Transform of 2000 (CMCCAT2000) and the CIE’s Chromatic Adaptation Transform (CIECAT02)(CIE 2004a). The CIE test colour method combines the eight special colour rendering indices by a simple averaging to obtain the general colour rendering index. This makes possible for a lamp to score quite well, even it renders one or two colour samples very poorly. The eight Munsell test sample used in CIE test colour method are not highly saturated (Davis 2010). The lamp which possess very poor colour rendering of saturated colours can still have high value of CIE CRI if rendering of desaturated colours is good (Davis 2010). The CIE test colour method can generate CIE CRI with negative values for very poor test sources, for e.g. low pressure sodium lamp, R_a= -44 (x = 0.569, y = 0.421) (Rea 2000). However, CIE CRI method fails to give useful information of negative values. Colour quality of light is more than the simple colour differences determined on a number of test samples (CIE 2006). Despite of limitations of CIE CRI, it is still used to describe the colour rendering properties of light sources.

White LED light sources are the new innovative inventions in the field of lighting. They are energy efficient, long-lasting, and compact. Unlike the traditional light sources, LEDs have greater freedom in spectral design as white light from LEDs is realized by mixture of multi-colour LEDs or by a combination of phosphors excited by blue or UV LED emission. Many questions have arisen about whether or not CIE CRI should be considered as design aspect of white LEDs, because of many flaws in the CIE test colour method. In past, the compact fluorescent lamp phosphors was designed to get high colour rendering index but the visual colour rendering was not good (Schanda 2003). Such mistakes should not be repeated again with LED light sources. LEDs can be tuned to get high CIE CRI but these will not necessarily look pleasing and provide good visual colour rendering and also luminous efficacy would be compromised (Ohno 2010).

Many visual experiments with LED light sources indicate that even LEDs with low values of CIE CRI can produce visually appealing, vivid, and natural light (Davis 2010, Sándor 2002, Narendran 2002). The case studies presented in Chapter 5 have shown that CIE CRI has no correlation to colour preferences. In some studies, the LED light source with low CIE CRI was more preferred than tungsten halogen or incandescent light source. The CIE CRI is unable to define all the colour qualities of light source such as colour discrimination, colour harmony, colour preference, colour acceptability, brightness etc. Different light sources are suitable for different application and relevant dimensions of light source colour quality should be used to select appropriate light sources according to the application.

The limitations of the CIE test colour method to calculate the CIE CRI of white LED light sources has been realized by CIE. CIE technical committee TC 1-62 concluded that current CIE CRI is generally not applicable to predict the colour rendering rank order of a set of light sources when white LED light sources are involved in the set and hence recommended the development of new colour rendering index (CIE 2007). The new technical committee CIE TC 1-69 is currently working to find new metric(s) to define colour rendering of white light sources (CIE 2010).

Until a flawless, more appropriate system to calculate the colour rendering index (or indices) is (or are) recommended by CIE, lighting designers, specification organizations and engineers should consider all the properties of prospective light sources before deciding project specification.

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Appendix 1

Table 1 : Spectral radiance factor $\beta_i(\lambda)$ of CIE-1974 test-colour samples Nos. 1...8 (TCS01...08), to be used in calculating the General Colour Rendering Index.

lambda	TCS01	TCS02	TCS03	TCS04	TCS05	TCS06	TCS07	TCS08
360	0,116	0,053	0,058	0,057	0,143	0,079	0,150	0,075
365	0,136	0,055	0,059	0,059	0,187	0,081	0,177	0,078
370	0,159	0,059	0,061	0,062	0,233	0,089	0,218	0,084
375	0,190	0,064	0,063	0,067	0,269	0,113	0,293	0,090
380	0,219	0,070	0,065	0,074	0,295	0,151	0,378	0,104
385	0,239	0,079	0,068	0,083	0,306	0,203	0,459	0,129
390	0,252	0,089	0,070	0,093	0,310	0,265	0,524	0,170
395	0,256	0,101	0,072	0,105	0,312	0,339	0,546	0,240
400	0,256	0,111	0,073	0,116	0,313	0,410	0,551	0,319
405	0,254	0,116	0,073	0,121	0,315	0,464	0,555	0,416
410	0,252	0,118	0,074	0,124	0,319	0,492	0,559	0,462
415	0,248	0,120	0,074	0,126	0,322	0,508	0,560	0,482
420	0,244	0,121	0,074	0,128	0,326	0,517	0,561	0,490
425	0,240	0,122	0,073	0,131	0,330	0,524	0,558	0,488
430	0,237	0,122	0,073	0,135	0,334	0,531	0,556	0,482
435	0,232	0,122	0,073	0,139	0,339	0,538	0,551	0,473
440	0,230	0,123	0,073	0,144	0,346	0,544	0,544	0,462
445	0,226	0,124	0,073	0,151	0,352	0,551	0,535	0,450
450	0,225	0,127	0,074	0,161	0,360	0,556	0,522	0,439
455	0,222	0,128	0,075	0,172	0,369	0,556	0,506	0,426
460	0,220	0,131	0,077	0,186	0,381	0,554	0,488	0,413
465	0,218	0,134	0,080	0,205	0,394	0,549	0,469	0,397
470	0,216	0,138	0,085	0,229	0,403	0,541	0,448	0,382
475	0,214	0,143	0,094	0,254	0,410	0,531	0,429	0,366
480	0,214	0,150	0,109	0,281	0,415	0,519	0,408	0,352
485	0,214	0,159	0,126	0,308	0,418	0,504	0,385	0,337
490	0,216	0,174	0,148	0,332	0,419	0,488	0,363	0,325
495	0,218	0,190	0,172	0,352	0,417	0,469	0,341	0,310
500	0,223	0,207	0,198	0,370	0,413	0,450	0,324	0,299
505	0,225	0,225	0,221	0,383	0,409	0,431	0,311	0,289
510	0,226	0,242	0,241	0,390	0,403	0,414	0,301	0,283
515	0,226	0,253	0,260	0,394	0,396	0,395	0,291	0,276
520	0,225	0,260	0,278	0,395	0,389	0,377	0,283	0,270
525	0,225	0,264	0,302	0,392	0,381	0,358	0,273	0,262
530	0,227	0,267	0,339	0,385	0,372	0,341	0,265	0,256
535	0,230	0,269	0,370	0,377	0,363	0,325	0,260	0,251
540	0,236	0,272	0,392	0,367	0,353	0,309	0,257	0,250
545	0,245	0,276	0,399	0,354	0,342	0,293	0,257	0,251
550	0,253	0,282	0,400	0,341	0,331	0,279	0,259	0,254
555	0,262	0,289	0,393	0,327	0,320	0,265	0,260	0,258
560	0,272	0,299	0,380	0,312	0,308	0,253	0,260	0,264
565	0,283	0,309	0,365	0,296	0,296	0,241	0,258	0,269
570	0,298	0,322	0,349	0,280	0,284	0,234	0,256	0,272
575	0,318	0,329	0,332	0,263	0,271	0,227	0,254	0,274
580	0,341	0,335	0,315	0,247	0,260	0,225	0,254	0,278
585	0,367	0,339	0,299	0,229	0,247	0,222	0,259	0,284
590	0,390	0,341	0,285	0,214	0,232	0,221	0,270	0,295
595	0,409	0,341	0,272	0,198	0,220	0,220	0,284	0,316
600	0,424	0,342	0,264	0,185	0,210	0,220	0,302	0,348
605	0,435	0,342	0,257	0,175	0,200	0,220	0,324	0,384
610	0,442	0,342	0,252	0,169	0,194	0,220	0,344	0,434
615	0,448	0,341	0,247	0,164	0,189	0,220	0,362	0,482
620	0,450	0,341	0,241	0,160	0,185	0,223	0,377	0,528
625	0,451	0,339	0,235	0,156	0,183	0,227	0,389	0,568
630	0,451	0,339	0,229	0,154	0,180	0,233	0,400	0,604

Table 1 cont.

lambda	TCS01	TCS02	TCS03	TCS04	TCS05	TCS06	TCS07	TCS08
635	0,451	0,338	0,224	0,152	0,177	0,239	0,410	0,629
640	0,451	0,338	0,220	0,151	0,176	0,244	0,420	0,648
645	0,451	0,337	0,217	0,149	0,175	0,251	0,429	0,663
650	0,450	0,336	0,216	0,148	0,175	0,258	0,438	0,676
655	0,450	0,335	0,216	0,148	0,175	0,263	0,445	0,685
660	0,451	0,334	0,219	0,148	0,175	0,268	0,452	0,693
665	0,451	0,332	0,224	0,149	0,177	0,273	0,457	0,700
670	0,453	0,332	0,230	0,151	0,180	0,278	0,462	0,705
675	0,454	0,331	0,238	0,154	0,183	0,281	0,466	0,709
680	0,455	0,331	0,251	0,158	0,186	0,283	0,468	0,712
685	0,457	0,330	0,269	0,162	0,189	0,286	0,470	0,715
690	0,458	0,329	0,288	0,165	0,192	0,291	0,473	0,717
695	0,460	0,328	0,312	0,168	0,195	0,296	0,477	0,719
700	0,462	0,328	0,340	0,170	0,199	0,302	0,483	0,721
705	0,463	0,327	0,366	0,171	0,200	0,313	0,489	0,720
710	0,464	0,326	0,390	0,170	0,199	0,325	0,496	0,719
715	0,465	0,325	0,412	0,168	0,198	0,338	0,503	0,722
720	0,466	0,324	0,431	0,166	0,196	0,351	0,511	0,725
725	0,466	0,324	0,447	0,164	0,195	0,364	0,518	0,727
730	0,466	0,324	0,460	0,164	0,195	0,376	0,525	0,729
735	0,466	0,323	0,472	0,165	0,196	0,389	0,532	0,730
740	0,467	0,322	0,481	0,168	0,197	0,401	0,539	0,730
745	0,467	0,321	0,488	0,172	0,200	0,413	0,546	0,730
750	0,467	0,320	0,493	0,177	0,203	0,425	0,553	0,730
755	0,467	0,318	0,497	0,181	0,205	0,436	0,559	0,730
760	0,467	0,316	0,500	0,185	0,208	0,447	0,565	0,730
765	0,467	0,315	0,502	0,189	0,212	0,458	0,570	0,730
770	0,467	0,315	0,505	0,192	0,215	0,469	0,575	0,730
775	0,467	0,314	0,510	0,194	0,217	0,477	0,578	0,730
780	0,467	0,314	0,516	0,197	0,219	0,485	0,581	0,730
785	0,467	0,313	0,520	0,200	0,222	0,493	0,583	0,730
790	0,467	0,313	0,524	0,204	0,226	0,500	0,585	0,731
795	0,466	0,312	0,527	0,210	0,231	0,506	0,587	0,731
800	0,466	0,312	0,531	0,218	0,237	0,512	0,588	0,731
805	0,466	0,311	0,535	0,225	0,243	0,517	0,589	0,731
810	0,466	0,311	0,539	0,233	0,249	0,521	0,590	0,731
815	0,466	0,311	0,544	0,243	0,257	0,525	0,590	0,731
820	0,465	0,311	0,548	0,254	0,265	0,529	0,590	0,731
825	0,464	0,311	0,552	0,264	0,273	0,532	0,591	0,731
830	0,464	0,310	0,555	0,274	0,280	0,535	0,592	0,731

Table 2: Spectral radiance factor $\beta_i(\lambda)$ of CIE -1974 test-colour samples Nos. 9...14

lambda	TCS9	TCS10	TCS11	TCS12	TCS13	TCS14
360	0,069	0,042	0,074	0,189	0,071	0,036
365	0,072	0,043	0,079	0,175	0,076	0,036
370	0,073	0,045	0,086	0,158	0,082	0,036
375	0,070	0,047	0,098	0,139	0,090	0,036
380	0,066	0,050	0,111	0,120	0,104	0,036
385	0,062	0,054	0,121	0,103	0,127	0,036
390	0,058	0,059	0,127	0,090	0,161	0,037
395	0,055	0,063	0,129	0,082	0,211	0,038
400	0,052	0,066	0,127	0,076	0,264	0,039
405	0,052	0,067	0,121	0,068	0,313	0,039
410	0,051	0,068	0,116	0,064	0,341	0,040
415	0,050	0,069	0,112	0,065	0,352	0,041
420	0,050	0,069	0,108	0,075	0,359	0,042
425	0,049	0,070	0,105	0,093	0,361	0,042
430	0,048	0,072	0,104	0,123	0,364	0,043
435	0,047	0,073	0,104	0,160	0,365	0,044
440	0,046	0,076	0,105	0,207	0,367	0,044
445	0,044	0,078	0,106	0,256	0,369	0,045
450	0,042	0,083	0,110	0,300	0,372	0,045
455	0,041	0,088	0,115	0,331	0,374	0,046
460	0,038	0,095	0,123	0,346	0,376	0,047
465	0,035	0,103	0,134	0,347	0,379	0,048
470	0,033	0,113	0,148	0,341	0,384	0,050
475	0,031	0,125	0,167	0,328	0,389	0,052
480	0,030	0,142	0,192	0,307	0,397	0,055
485	0,029	0,162	0,219	0,282	0,405	0,057
490	0,028	0,189	0,252	0,257	0,416	0,062
495	0,028	0,219	0,291	0,230	0,429	0,067
500	0,028	0,262	0,325	0,204	0,443	0,075
505	0,029	0,305	0,347	0,178	0,454	0,083
510	0,030	0,365	0,356	0,154	0,461	0,092
515	0,030	0,416	0,353	0,129	0,466	0,100
520	0,031	0,465	0,346	0,109	0,469	0,108
525	0,031	0,509	0,333	0,090	0,471	0,121
530	0,032	0,546	0,314	0,075	0,474	0,133
535	0,032	0,581	0,294	0,062	0,476	0,142
540	0,033	0,610	0,271	0,051	0,483	0,150
545	0,034	0,634	0,248	0,041	0,490	0,154
550	0,035	0,653	0,227	0,035	0,506	0,155
555	0,037	0,666	0,206	0,029	0,526	0,152
560	0,041	0,678	0,188	0,025	0,553	0,147
565	0,044	0,687	0,170	0,022	0,582	0,140
570	0,048	0,693	0,153	0,019	0,618	0,133
575	0,052	0,698	0,138	0,017	0,651	0,125
580	0,060	0,701	0,125	0,017	0,680	0,118
585	0,076	0,704	0,114	0,017	0,701	0,112
590	0,102	0,705	0,106	0,016	0,717	0,106
595	0,136	0,705	0,100	0,016	0,729	0,101
600	0,190	0,706	0,096	0,016	0,736	0,098
605	0,256	0,707	0,092	0,016	0,742	0,095
610	0,336	0,707	0,090	0,016	0,745	0,093
615	0,418	0,707	0,087	0,016	0,747	0,090
620	0,505	0,708	0,085	0,016	0,748	0,089
625	0,581	0,708	0,082	0,016	0,748	0,087
630	0,641	0,710	0,080	0,018	0,748	0,086

Table 2. cont.

lambda	TCS9	TCS10	TCS11	TCS12	TCS13	TCS14
635	0,682	0,711	0,079	0,018	0,748	0,085
640	0,717	0,712	0,078	0,018	0,748	0,084
645	0,740	0,714	0,078	0,018	0,748	0,084
650	0,758	0,716	0,078	0,019	0,748	0,084
655	0,770	0,718	0,078	0,020	0,748	0,084
660	0,781	0,720	0,081	0,023	0,747	0,085
665	0,790	0,722	0,083	0,024	0,747	0,087
670	0,797	0,725	0,088	0,026	0,747	0,092
675	0,803	0,729	0,093	0,030	0,747	0,096
680	0,809	0,731	0,102	0,035	0,747	0,102
685	0,814	0,735	0,112	0,043	0,747	0,110
690	0,819	0,739	0,125	0,056	0,747	0,123
695	0,824	0,742	0,141	0,074	0,746	0,137
700	0,828	0,746	0,161	0,097	0,746	0,152
705	0,830	0,748	0,182	0,128	0,746	0,169
710	0,831	0,749	0,203	0,166	0,745	0,188
715	0,833	0,751	0,223	0,210	0,744	0,207
720	0,835	0,753	0,242	0,257	0,743	0,226
725	0,836	0,754	0,257	0,305	0,744	0,243
730	0,836	0,755	0,270	0,354	0,745	0,260
735	0,837	0,755	0,282	0,401	0,748	0,277
740	0,838	0,755	0,292	0,446	0,750	0,294
745	0,839	0,755	0,302	0,485	0,750	0,310
750	0,839	0,756	0,310	0,520	0,749	0,325
755	0,839	0,757	0,314	0,551	0,748	0,339
760	0,839	0,758	0,317	0,577	0,748	0,353
765	0,839	0,759	0,323	0,599	0,747	0,366
770	0,839	0,759	0,330	0,618	0,747	0,379
775	0,839	0,759	0,334	0,633	0,747	0,390
780	0,839	0,759	0,338	0,645	0,747	0,399
785	0,839	0,759	0,343	0,656	0,746	0,408
790	0,839	0,759	0,348	0,666	0,746	0,416
795	0,839	0,759	0,353	0,674	0,746	0,422
800	0,839	0,759	0,359	0,680	0,746	0,428
805	0,839	0,759	0,365	0,686	0,745	0,434
810	0,838	0,758	0,372	0,691	0,745	0,439
815	0,837	0,757	0,380	0,694	0,745	0,444
820	0,837	0,757	0,388	0,697	0,745	0,448
825	0,836	0,756	0,396	0,700	0,745	0,451
830	0,836	0,756	0,403	0,702	0,745	0,454