Aalto University
School of Economics

Some Approaches for Assessing the Sustainability of Public Finances

Economics

Master's thesis

Tuukka Sarvi

2011

Department of Economics
Aalto University
School of Economics

ABSTRACT

The sustainability of public finances is currently a hot topic in economic policy debate. This is because of the on-going sovereign debt crises in Europe and the long-term public spending pressures caused by the impeding demographic change in developed countries. This thesis examines some of the approaches that have been used to assess the public finance sustainability in the literature. Also, theoretical criteria for sustainability are examined. The study is conducted by a way of literature review.

There is no consensus among economists about the correct theoretical criterion for public finance sustainability. Rather, each approach to assess sustainability introduces its own, sometimes differing, definitions. Government's inter-temporal budget constraint (IBC) is the most commonly used theoretical criterion for sustainability. We find that it is not theoretically waterproof and even invalid in some cases. We suggest an alternative criterion from the literature, Bohn's model-based sustainability, to be used in place of the IBC in theoretical settings.

In the thesis, six different approaches to assess public finance sustainability are examined. These are: summary indicators of sustainability, econometric tests, Value-at-Risk framework, fiscal limits and fiscal space, general equilibrium models and generational accounting. Each approach is described and analysed based on research found in the literature.

Summary indicators are the most commonly used practical tool used in sustainability assessments. They are based on projections of future public debt and give the budgetary adjustment required to satisfy the IBC or reach a target debt level. Econometric tests are statistical tests for various theoretical sustainability criteria that can be used to determine whether a given criterion holds in the data. Value-at-Risk framework uses stochastic simulations of the public sector balance sheet to study the degree of public sector solvency. It gives an estimate of a probability distribution for government's future net asset position. Fiscal limits and fiscal space attempt to estimate a public debt ceiling for a country based on assumed constraints to government's fiscal policies. General equilibrium models are detailed large-scale frameworks which assess sustainability based on comprehensive modelling of the whole economy. Generational accounting analyses sustainability by comparing the net tax burden of current and future generations.

We analyse the strengths and weaknesses of each of the approaches and tentatively compare them with each other. We find that each approach has its uses. Approaches should be viewed as complementary. Availability of data and modelling resources, goals of the analysis and other case-specific constraints affect relative suitability of the approaches in different situations. From purely theoretical perspective, general equilibrium models and the Value-at-Risk approach appear most attractive. We conclude that theoretical accuracy of the models doesn't guarantee the accuracy of the future forecasts and thus sustainability estimates.

Key words: public finance, sovereign debt, fiscal sustainability, public debt sustainability, fiscal policy, inter-temporal budget constraint, sustainability indicators

TIIVISTELMÄ

Julkisen talouden kestävyys on tällä hetkellä ajankohtainen aihe taloustieteellisessä keskustelussa johtuen yhtäältä EU-maita ravistelevista velkakriiseistä, ja toisaalta väestön ikääntymisestä kehittyneissä talouksissa, minkä ennakoidaan johtavan merkittävään julkisten menojen kasvuun tulevaisuudessa. Tämä tutkielma käy läpi joukon kirjallisuudesta löytyviä lähestymistapoja julkisen talouden kestävyyden arviointiin. Tämän lisäksi tarkastellaan teoreettisia kriteerejä julkisen talouden kestävyydelle. Tutkimusmenetelmä on kirjallisuuskatsaus.

Ekonomistien keskuudessa ei ole konsensusta siitä, mikä on oikea määritelmä tai kriteeri julkisen talouden kestävyydelle. Eri tutkimukset käyttävät usein hieman erilaisia kriteerejä. Julkisen sektorin intertemporaalinen budjettirajoite on yleisimmin käytetty teoreettinen kriteeri kestävyydelle. Kirjallisuuskatsauksesta käy kuitenkin ilmi, että se ei ole teoreettisesti vedenpitävä ja tiettyjen oletusten pätiessä jopa virheellinen. Bohnin (2005) malliperusteinen määritelmä kestävyydelle vaikuttaa teoreettisesta näkökulmasta paremmalta kriteeriltä kestävyydelle kuin intertemporaalinen budjettirajoite.

Tässä tutkielmassa tarkastellaan kuutta eri lähestymistapaa julkisen talouden kestävyyden arviointiin. Ne ovat: kestävyysindikaattorit, ekonometriset testit, Value-at-Risk malli, fiscal limits ja fiscal space –lähestymistapa, yleisen tasapainon mallit ja sukupolvilaskenta. Kutakin lähestymistapaa kuvaillaan ja analysoidaan sitä käsittelevän kirjallisuuden perusteella.

Kestävyysindikaattorit perustuvat ennusteisiin tulevasta valtion velan kehityksestä. Ne ilmoittavat tavoitevelkatason saavuttamiseksi tai intertemporaalisen budjettirajoitteen toteuttamiseksi vaadittavat muutokset valtion budjettisuureissa. Ekonometriset testit testaavat tilastollisesti eri kestävyyskriteereiden pätevyyttä historiallisen datan perusteella. Value-at-Risk -lähestymistavassa käytetään stokastisia simulaatioita julkisen sektorin kokonaistaseesta arvioimaan julkisen talouden kestävyyttä. Fiscal limits ja fiscal space – lähestymistavassa pyritään määrittämään katto julkiselle velalle valtion finanssipolitiikkaa rajoittavien tekijöiden perusteella. Yleisen tasapainon mallit ovat yksityiskohtaisia ja laajoja malleja, jossa kestävyyttä analysoidaan mallintamalla koko kansantalouden toiminta. Sukupolvilaskennassa analysoidaan kestävyyttä vertaamalla nykyisten ja tulevien sukupolvien nettovelkataakkaa.

Eri lähestymistapojen vahvuuksia ja heikkouksia analysoidaan. Käy ilmi, että tilanteesta riippuen eri lähestymistavat ovat soveltuvia. Lähestymistapoja tulisi ajatella toisiaan täydentävinä. Pelkästään teoreettisesta näkökulmasta yleisen tasapainon mallit ja Value-at-Risk –lähestymistapa vaikuttavat parhailta. Lähestymistavan teoreettinen tarkkuus ei kuitenkaan takaa sen ennustetarkkuutta, eikä siis myöskään kestävyysarvioiden tarkkuutta.

Avainsanat: julkinen talous, valtion maksukyky, julkisen talouden kestävyysvaje, finanssipolitiikka, intertemporaalinen budjettirajoite, kestävyysindikaatorit

CONTENTS

1. INTRODUCTION

1.1 BACKGROUND AND MOTIVATION

At the moment sustainability of public finances is a timely topic in many advanced economies. Questions have been raised by various commentators, investors and analysts whether public finances in the EU countries and in the US are on a sustainable track. After the financial crisis of 2008-2009, the public debt of many countries has been on a steep upward trajectory due to implementation of various stimulus and relief packages directed to financial sector and the economy as a whole. A rising debt combined with long-term issues like the demographic change which affects the balance between number of people in the labour force and number of retirees, have alerted fiscal authorities to study the problem in detail. In fact, some European countries like Greece, Ireland, Portugal, Spain and Italy are currently in serious trouble with their public finances which is reflected in the high yield demanded from the government bonds of these countries. As of this moment, Greece, Ireland and Portugal have already received aid from other member states. It has been argued that the underlying problem in Europe is the fragility of banks and the financial sector, not the public sector itself. From the perspective of public finance sustainability this is not a valid point since problems of any sector, like the financial sector, become problems of the public sector once they get big enough.

Study of public finances is not important only in the current situation in US and in the EU. It has been continuously pertinent issue in less developed countries. In less developed countries, the public sector is usually more fragile and prone to shocks than in developed countries. This is because the public sector of these economies is more vulnerable to exchange rate fluctuations, commodity price fluctuations (like the price of oil), changes in interest rate on government debt, sprees of high inflation and political turmoil. In the past, many emerging market economies have experienced crises that have been closely tied to problems in public finances. Some of these crises have led to debt restructuring efforts spearheaded by the IMF and some have led to outright default.

The knowledge whether public finances are on a sustainable track is important in many respects. Fiscal authorities of a country want to keep finances sustainable in order to give a healthy ground for economic growth in the country for years to come. If public finances are not sustainable, this affects the economy as a whole. This is evidenced by the fact that public sector accounts for a large part of the total production in most countries. Furthermore, it is the public sector that provides the institutions and services which are prerequisite for the normal function of corporations. Thus, in order to avoid any crises in the public sector and the economy at large, it is valuable for fiscal authorities to monitor the sustainability of public finances and inform politicians of any significant developments in that area.

Also, the creditors of the government bodies follow the sustainability of public finances closely. After all, their goal is to make profit and therefore they don't want to pay too much for government bonds. In order to have idea of the risk premium they require for the bonds, these creditors, which are usually big banks, have to analyse the risks present in the public sector. Therefore, the study of sustainability of public finances is crucial for them.

1.2 Objectives and limitations of the thesis

This thesis has three goals or research questions. First goal is to critically examine various theoretical criteria that have been proposed for sustainability in the literature. Second goal is to study and go through different approaches which have been employed to measure the sustainability of public finances. The study doesn't attempt to be comprehensive: not all approaches to evaluate public finance sustainability found in the literature are covered. Third goal is to analyse the strengths and weaknesses of these approaches and compare them with one another. The method of study is literature review.

1.3 Central definitions

In this thesis, a broad definition of public sector is used. It is defined to comprise of central government, local governments, public corporations, central bank and social security funds.

The general definition of sustainability of public finances in this paper is the following: public finances are sustainable if consolidated public sector is solvent given current policies. That is, public sector is able to honour all its obligations (outlays, transfers, debt service, etc.) now and in the future without adjusting its policies (tax rate, promised expenditures, etc.). This concept is synonymous with fiscal sustainability and the two are used interchangeably. This general definition elaborated later in Chapter 4 when various theoretical criteria of sustainability are examined. Models and approaches to assess public finance sustainability often use slightly different definitions for sustainability. However, the intent behind these definitions is the same and it is described by the general definition given above.

1.4 STRUCTURE OF THE THESIS

The structure of the thesis is the following. First, in the second chapter, challenges to public finance sustainability are outlined in the light of the existing debt burdens and large projected costs due to the period of rapid demographic change many advanced economies are entering. In the third chapter, public sector balance sheet and income statement are defined and described. In the fourth chapter, several alternative theoretical criteria for sustainability are examined. In the fifth chapter, summary indicators, the first approach to assess sustainability, are examined. In the sixth chapter, econometric tests of fiscal sustainability are studied. Seventh chapter defines and analyses a Value-at-Risk measure for sustainability. Eighth chapter introduces the concepts of fiscal limit and fiscal space and their relation to sustainability. In the ninth chapter, some general equilibrium models intended for analysis of sustainability are examined. Tenth chapter defines and analyses generational accounting approach to public finance sustainability. Eleventh chapter compares different approaches presented and examines their strengths and weaknesses. The last chapter concludes.

2. CHALLENGES TO SUSTAINABILITY

Public finance sustainability is a current topic for two main reasons. Firstly, in the aftermath of the financial crisis of 2008-2009 public debt of many countries has rocketed to levels that have never been seen before. Markets recognize this and require higher yields to compensate for the additional risk. This has resulted in sovereign debt crises, especially in Europe. Secondly, the long-run sustainability of public finances in many advanced economies is threatened by the impeding large-scale demographic change. In the coming 50 years, population ageing is projected to lead to significant increases in public expenditures due to higher pension payments and health care costs. Furthermore, the demographic change leads to lower expectations of economic growth because of reduced growth of the workforce. This chapter offers a glance at the debt burden faced in economies around the world, describes the extent of the coming demographic transition in advanced economies and records some estimates of the long-run sustainability of public finances prepared by governmental institutions.

2.1 THE DEBT BURDEN AT A GLANCE

The extent of the debt burden varies greatly internationally. Some countries have little debt while others have clearly too much for them to bear. Using gross debt per GDP as the benchmark for indebtedness, it is seen that many advanced economies have significant stocks of public debt while big emerging economies like China and Russia do not have much debt. Figure 1 shows IMF estimates of general government gross debt per GDP in selected economies in 2011.

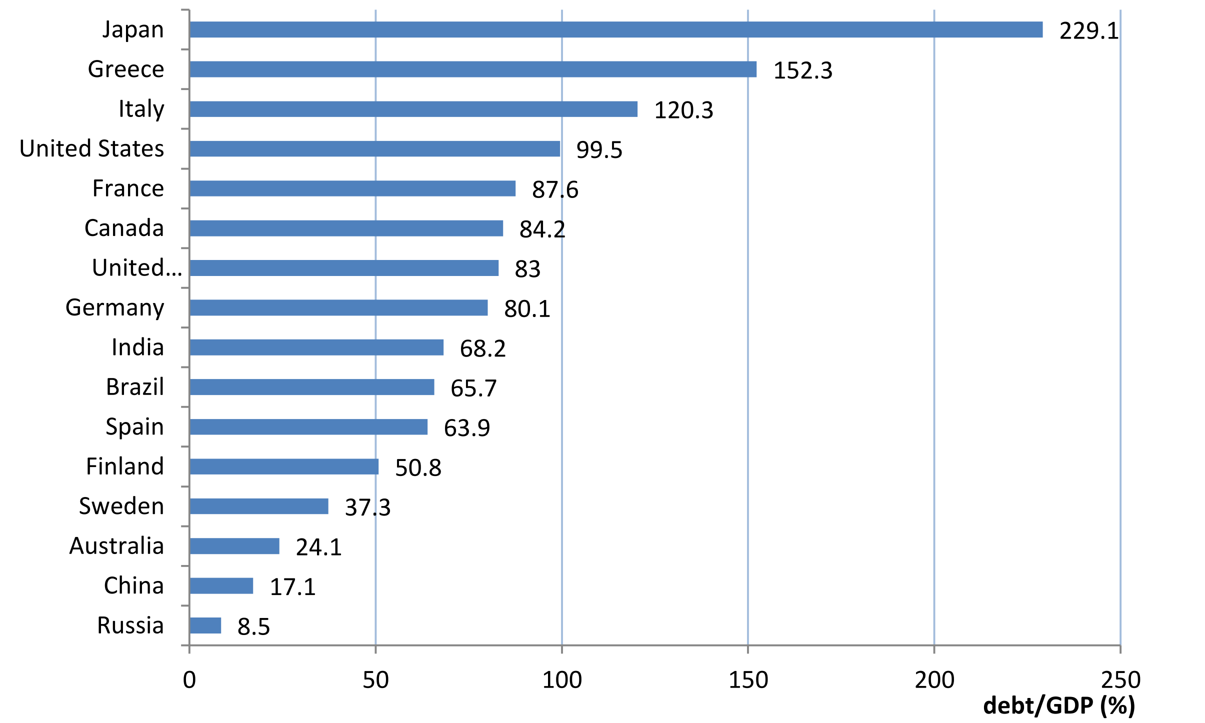

Figure 1. General government gross debt as percentage of GDP in 2011. Source: IMF World Economic Outlook estimates (April 2011).

Japan is projected to have the largest debt burden by a wide margin. Greece and Italy have debt levels above 100 % of GDP. Greece is nearly in default and markets are suspicious of Italy's ability to pay back its loans which is reflected in recent hikes of bond yields. Despite its massive debt stock, Japan is not facing a credit crisis because interest rates of government bonds are low due to continued expectations of deflation and regulatory pressures that cause Japanese domestic institutions to finance major portion of the government debt. While United States and big European economies have taken a lot of new debt during recent years, they are still below the 100 % of GDP boundary.

Figure 2 below shows the IMF 2011 estimates of net interest expense (interest expense minus interest revenue) relative to general government revenues in the same set of countries as in Figure 1 with the exception that China and India are missing due to data unavailability. Net interest expense per revenue measures the proportion of yearly revenues that governments have to utilise just to pay the interest on existing debt.

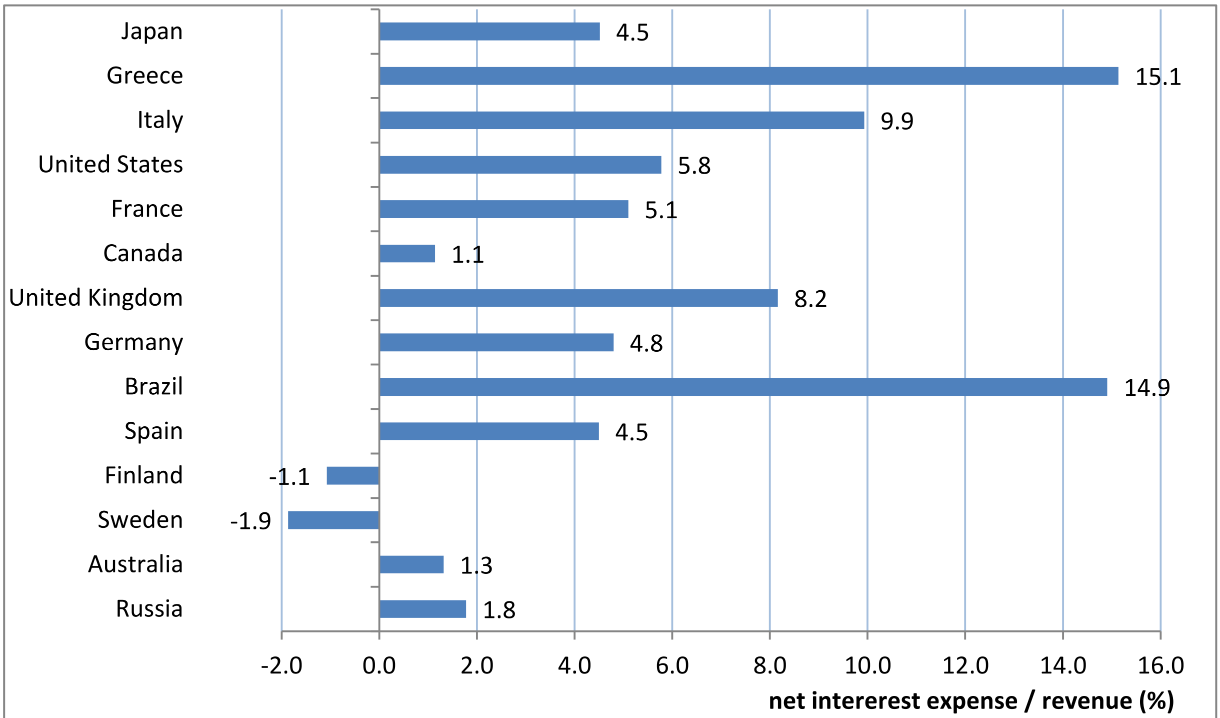

Figure 2. Net interest expense per general government revenue in 2011.
Source: : IMF World Economic Outlook estimates (April 2011).

It seems that in general, those countries that have big gross debt relative to GDP (Figure 1) also spend large amount of their revenues on interest expenses. However, there are also significant differences between the stories told by Figures 1 and 2 because what matters for net interest payments is net debt, not gross debt. Furthermore, some countries have larger revenue incomes relative to GDP and therefore for a given debt-to-GDP ratio spend less interest expenses relative to revenues. Also, interest rates for different countries differ. In Figure 2, Japan, which holds the largest stock of gross debt relative to GDP in the world, doesn't seem to especially burdened by interest payments. This is because according to IMF statistics net debt in Japan is about half of the gross debt. Secondly, Japan faces very low interest rates currently. IMF projects Finland and Sweden having net interest revenue in 2011. This results from the fact that net debt in Finland and Sweden is negative (stock of assets is greater than stock of debt) because IMF calculations of net debt count pension funds as government assets. Brazil is projected to have a large interest expense burden probably because it has higher interest rates than many advanced economies.

2.2 DEMOGRAPHIC TRANSITION

Major demographic changes are unfolding in the coming decades at the global level. Fertility has been declining and longevity has been and is expected to keep increasing primarily due to drop in the mortality rates at higher ages. Projections indicate that world total population will keep on increasing and the population age composition will change significantly. At the global level population is getting older. As a result of this, there is an upward trend in the dependency[1], although there is variation in the trend in different regions. Figure 3 shows the projected trajectory of old-age dependency ratio in developed countries, less developed countries and least developed countries.

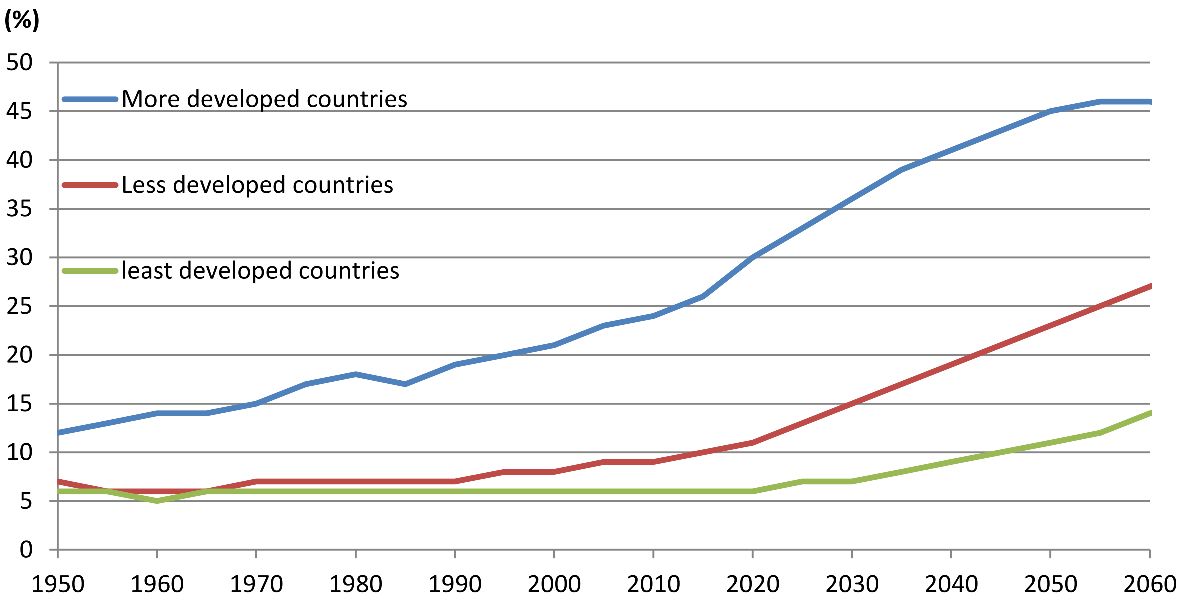

Figure 3. Old-age dependecy ratio. The old-age dependency ratio is the ratio of the population aged 65 years or over to the population aged 15-64. Source: United Nations, 2010 revision of World Population Prospects.

The message of the figure is clear: during the next 50 years, the proportion of elderly in the population of developed countries will approximately double.

[1]Dependency measures the number of people outside work force relative to number of people in work force.

The two main drivers behind the trend shown in Figure 3 are decrease in fertility and increase in longevity. Fertility has been decreasing after peaking in the middle of the 20th century. Currently, it has converged to a level of about 2 children per woman. Most of the developed countries are now at the point when the large generations reach retirement age and new comparatively smaller generations enter the job market which increases the dependency ratio. Longevity has been on an increasing trend since 1950 with gains in life expectancy amounting to 0.1 to 0.2 years per year. The continuing upward trend in longevity increases the old-age dependency ratio because people spend longer times in retirement and thus the age cohort 65+ increases in size. (United Nations 2011).

2.3 Some sustainability estimates

The demographic change leads to increases in age-related public expenditures such as pensions and health care in advanced economies. Many research institutes and governing bodies such as the IMF, the European Commission and national governments have made estimates of public finance sustainability that take into account both the debt burden and the projected long-run developments.

According to European Commission (2009), age-related public expenditure relative to GDP is projected to rise on average 4.3 percentage points by 2060 in the EU. The change is mainly due to increases in pension, healthcare and long-term care spending. European commission assesses the long-run sustainability of public finances by calculating so-called sustainability gaps. These gaps measure the required adjustment by a shift in the projected path of primary balances[2]. The size of the adjustment is such that the long-run solvency of the government is re-established. Figure 4 shows the S2 sustainability gaps calculated by the European Commission in 2009[3].

[2] Primary balance equals government revenues minus expenditures excluding interest expenditure.

[3] The S2 and other similar indicators are mathematically derived later in chapter 5 "Summary indicators of sustainability".

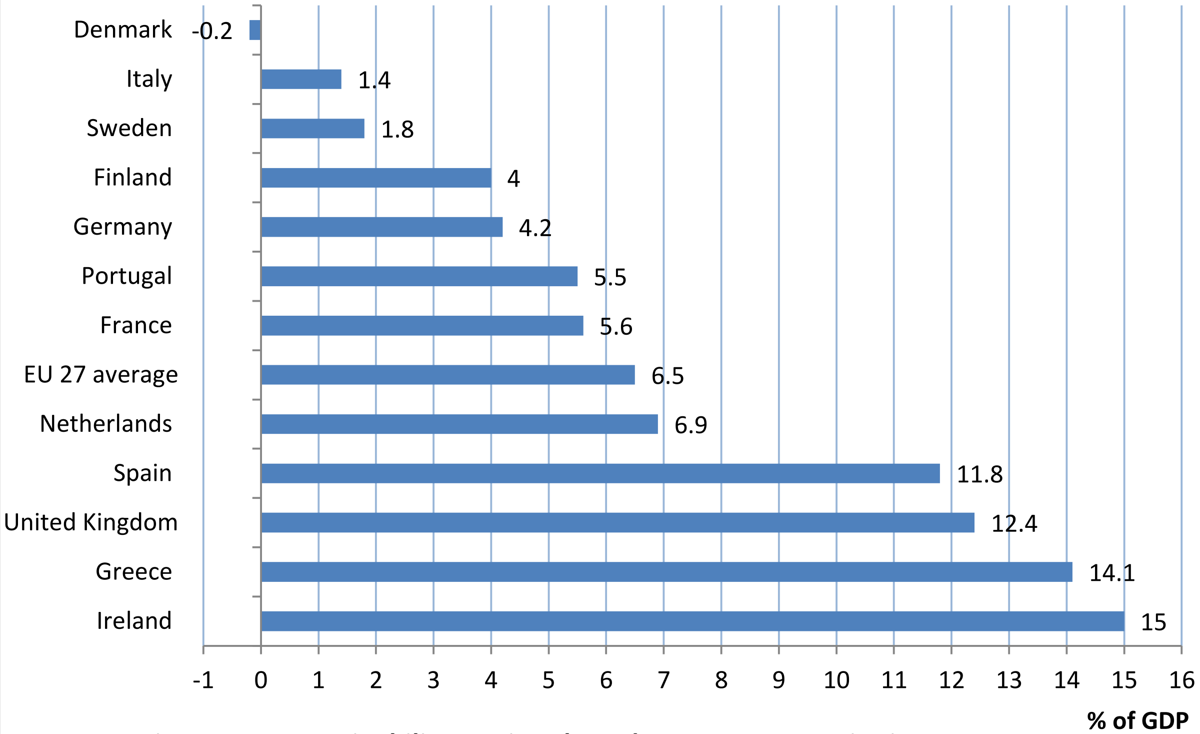

Figure 4. S2 sustainability gap in selected European countries in 2009. Source: European Commission (2009).

The S2 sustainability gap for the EU as a whole was 6.5 % of GDP. This means EU countries should immediately raise their tax ratios (tax income divided by GDP) by 6.5 % percentage points in order to become solvent and be make fiscal policy sustainable. Alternatively, the sustainability gap could be closed by making an expenditure cut of the same magnitude. According to European Commission (2009) about half of the gaps are due to the initial budgetary positions (too much debt compared to projected primary surpluses) and half are due to spending pressures arising from long-term demographic changes. There is significant variation in degrees of sustainability problems in different EU countries. The gaps as measured by S2 and S1 indicators range between -0.2% (Denmark) and 15% (Ireland). It is notable that almost all countries have a positive sustainability gap as measured by both indicators – this reflects the weak fiscal position due to economic crisis and the intensive phase of demographic transition that most countries are entering. Most countries have gaps near the EU average of 6.5 %.

In the United States, the Congressional Budget Office prepares long-term fiscal projections and estimates of fiscal sustainability. These projections are made for two scenarios: the extended-baseline scenario and the alternative scenario. Of these two the alternative scenario is perhaps the more realistic one. The alternative fiscal scenario extends Congressional Budget Office's 10 year baseline estimate further in time incorporating several changes to the current law that are widely expected to occur or which alter some provisions which might be hard to sustain in the long run. CBO projects that the ageing of the population and the rising cost of health care will cause spending on major mandatory health care programs (Medicaid and Medicare) and Social Security to grow roughly by 12 % of GDP by 2060 under the alternative fiscal scenario respectively. CBO uses fiscal gap indicators to measure long-term sustainability of fiscal policies[4]. Under the alternative fiscal scenario the gaps were 4.8 %, 6.9 % and 8.7 % for time horizons of 25, 50 and 75 years. (Congressional Budget Office 2010).

In Canada, Office of the Parliamentary Budget Officer prepares long-term fiscal sustainability estimates. Like in the U.S, two scenarios are used: a baseline scenario and an alternative scenario. The difference between the two is that under the alternative scenario healthcare costs are projected to keep growing at a high rate whereas in baseline scenario the growth is assumed to be much more moderate. Due to population ageing, spending pressures in health care and elderly benefits are projected to intensify. Under the alternative scenario, Canada Health Transfer program, the largest health care –related expenditure of Government of Canada, is projected to grow by about 3.5 % per GDP by 2060. Elderly benefits are estimated to grow roughly by 1.2 % of GDP by 2060. PBO estimates fiscal gaps that are identical to those estimated by CBO in the U.S. Under the alternative scenario fiscal gaps were 0.74 %, 1.38 % and 1.89 % of GDP for time horizons 25, 50 and 75 years. (Parliamentary Budget Officer 2010).

3. Public sector balance sheet and income statement

In the study of the sustainability of a country's public finances it is important to consider the public sector assets, liabilities, revenues and expenditures as comprehensively as possible. Traditionally, researchers have focused mainly on flow variables that appear in the budget such as tax revenues, revenues from natural resources, government consumption and transfer payments. In the past years, however, more attention has

[4] Fiscal gap measure used by CBO is similar to S2 gap used by the European Commission. It is defined as the one-time permanent adjustment to primary balance per GDP (tax ratio or spending ratio or both) in order to stabilize debt-to-GDP ratio to its initial level in the end of the forecast horizon. For details see Chapter 5.

been devoted to the analysis of stocks of assets and liabilities that constitute the government or public sector balance sheet. It has been recognized that changes in values of these assets and liabilities are and have been an important factor affecting sustainability.

In this thesis, as all-encompassing as possible definition of public sector is used: public sector consists of a central government, local governments, public corporations, a central bank and social security funds. Basically all entities whose assets or liabilities can be turned to government's ownership should be included in the balance sheet analysis. Explicit and implicit guarantees and contingent liabilities that affect government net worth (difference between value of assets and liabilities) can have important implications for sustainability. Examples of these "hidden debts" are unfunded social security programs, deposit insurance schemes and implicit bail out guarantees to e.g. the financial sector.

This chapter goes through detailed versions of the public sector balance sheet and income statement (or equivalently, budget constraint). The goal is to give the reader a broad perspective of the factors that may affect public sector net worth.

3.1 Public sector balance sheet

The 2001 IMF GFS Manual presented in International Monetary Fund (2001) sets out detailed definitions for concepts and guidelines for accounting of public sector transactions and assets and liabilities. In other words, it gives guidelines for implementation of the public sector balance sheet. The 2001 IMF GFS Manual defines public sector net worth as the difference between assets and liabilities at market prices. Both financial and nonfinancial assets and liabilities are included. Changes in net worth could occur as a result of (1) budgetary transactions like tax collection, grants and asset returns or government expenditures, interest expenses, payment of subsidies and depreciation; (2) price effects, i.e. changes in the value of assets or liabilities; and (3) changes in the volumes of assets and liabilities other than those resulting from normal transactions, e.g. natural resource discoveries and disasters like floods and earthquakes which can destroy assets.

An alternative definition of the public sector balance sheet is presented in Easterly and Yuravlivker (2000). It is broadly in line with the definition of 2001 IMF GFS Manual except in that contingent contracts and value of the social security system is recorded directly in the balance sheet. In 2001 IMF GFS Manual they are recorded as memorandum items outside balance sheet. The public sector balance sheet and its constituents are presented in Table 1. In this view of the public sector balance sheet, government's (= public sector) net worth is analogous to book value of equity in balance sheets of private corporations.

TABLE 1.THE PUBLIC SECTOR BALANCE SHEET AND ITS CONSTITUENTS.

Assets	Liabilities
Government-owned public goods (infrastructure, schools, health clinics, etc. that generate an adequate ERR and an indirect ERR through tax collection)	Public external debt
Government-owned capital that is financially profitable (anything for which government can charge user fees to generate adequate ERR)	Public domestic debt
Value of government-owned natural resource stocks (oil, minerals, etc.)	Domestic contingent liabilities (e.g. bank deposit guarantees, net present value of pension scheme, guarantees of private debt)
Expected present value of loans to private sector	Government's net worth

Source: Easterly and Yuravlivker (2000).
1/ ERR stands for economic rate of return.

Budgetary flows like tax revenues and government expenditures do not enter the balance sheet but affect its evolution in time in a similar way as net sales and production costs affect balance sheets of private corporations. Naturally, the future public sector revenues and expenditures affect sustainability considerations significantly. Having said that, here the expected present values of those flows do not enter the public sector balance sheet. The balance sheet is consolidated from entities constituting the public sector so that liabilities between these entities are disregarded and duplicate accounts are merged.

3.2 Public sector income statement

Public sector's income statement accounts for the various flows that affect the evolution of its balance sheet. IMF GFSM 2001 divides income statement into two parts: statement of government operations and statement of other economic flows. The former includes normal budgetary items like revenues and expenses and net acquisition of financial and nonfinancial assets and the net incurrence of debt. The latter records changes in public sector net worth that are not a result of government transactions. These are changes in values and volumes of assets and liabilities. Here these two statements are merged to yield an income statement comparable to those of private corporations similarly as in Da Costa and Juan-Ramón (2006). The evolution of public sector balance sheet is described by the following income statement (all variables are recorded at market prices):

$$NW_t = NW_{t-1} + NOB_t + OEF_t$$

$$NOB_t = T_t - E_t$$

$$OEF_t = NGPC_t + NGVC_t$$

$$NW_t = (A_F + A_{NF})_t - D_t - OL_t.$$

TABLE 2. TERMS IN THE PUBLIC SECTOR INCOME STATEMENT.

NW_t	public sector net worth
NOB_t	net operating balance (statement of government operations)
OEF_t	other economic flows (statement of other economic flows)
T_t	public sector revenues
E_t	public sector expenditures including depreciation and interest expenses
$NGPC_t$	net gains due to changes in prices of assets and liabilities
$NGVC_t$	net gains due to changes in volumes of assets and liabilities
A_F, A_{NF}	stocks of financial assets and nonfinancial assets owned by public sector
D_t	stock of debt
OL_t	stock of non-debt, non-contingent liabilities

4. THEORETICAL CRITERIA FOR PUBLIC FINANCE SUSTAINABILITY

There is a lack of clear consensus among economists about the definition of public finance sustainability. In fact, many research papers in the area of sustainability introduce their own criteria for sustainability that are in many ways similar but not identical. Sustainable policies, as mentioned earlier, are such that they can be continued into the projected future without any changes in taxation or spending patterns. The analysis of public finance sustainability boils down to questions of public sector solvency and public debt sustainability. This chapter examines the different theoretical criteria that have been proposed for fiscal sustainability.

4.1 INTER-TEMPORAL BUDGET CONSTRAINT

A large part of the literature concerning fiscal sustainability takes the inter-temporal budget constraint of public sector as a starting point for the analysis. It is often assumed that the constraint is binding to government. In many cases, it is taken as the definition of sustainability. This choice, however, has been subjected to a fair amount of criticism.

The derivation of the inter-temporal budget constraint (IBC) starts with simple version of public sector income statement, that is, one-period budget constraint which describes the evolution of net debt:

$$B_{t+1} = (1+r)B_t - PB_{t+1},$$

where B_t is the stock of public sector net debt, r is the interest rate[5], PB_t is the primary balance of the public sector which equals revenues minus expenditures excluding interest expenditure.

Solving the budget constraint recursively forwards in time gives:

$$B_t = (1+r)^{-1}B_{t+1} + (1+r)^{-1}PB_{t+1}$$

$$B_t = (1+r)^{-2}B_{t+2} + (1+r)^{-2}PB_{t+2} + (1+r)^{-1}PB_t$$

$$\dots$$

$$B_t = (1+r)^{-n}B_{t+n} + \sum_{i=1}^{n}(1+r)^{-i}PB_{t+i}.$$

Taking the limit as n tends to infinity:

$$B_t = \lim_{n\to\infty}(1+r)^{-n}B_{t+n} + \sum_{i=1}^{\infty}(1+r)^{-i}PB_{t+i}.$$

The crucial assumption behind the inter-temporal budget constraint is that the first term giving the present value of the government debt in infinity is assumed to be zero:

$$\lim_{n\to\infty}(1+r)^{-n}B_{t+n} = 0.$$

[5] Here, constant interest rate is assumed for simplicity. Similar results can be derived in the case of time-dependent interest rate.

This assumption is called the transversality condition (TC) or no-Ponzi-game condition (NPG). By substituting it to the above equation, the inter-temporal budget constraint is received:

$$B_t = \sum_{i=1}^{\infty} (1+r)^{-i} PB_{t+i}.$$

The transversaility condition and inter-temporal budget constraint are equivalent in this context. The inter-temporal budget constraint tells that the present value of the flow of primary balances must equal the present stock of net debt. That is, government's total net liability must be equal to its total assets (flow of primary balances).

The transversality condition is sometimes called the no-Ponzi-game condition meaning that government is not allowed to run a Ponzi game and that government doesn't finance Ponzi games. A Ponzi game or scheme is a system in which return to the principal of previous investors is paid by new investments by subsequent investors. In the case of debt, the debtor is running a Ponzi-game at the expense of the creditors when she always pays the interest by issuing more debt.

Inter-temporal budget constraint is often analysed in the context of per GDP measures. The underlying assumption is that GDP grows at a constant exponential rate[6] g so that $Y_{t+n} = (1+g)^n Y_t$. Substituting this gives:

$$B_t = (1+r)^{-n} B_{t+n} + \sum_{i=1}^{n} (1+r)^{-i} PB_{t+i}$$

$$\frac{B_t}{Y_t} = \frac{(1+r)^{-n}(1+g)^n B_{t+n}}{Y_{t+n}} + \sum_{i=1}^{n} \frac{(1+r)^{-i}(1+g)^i PB_{t+i}}{Y_{t+i}}$$

$$b_t = \left(\frac{1+g}{1+r}\right)^n b_{t+n} + \sum_{i=1}^{n} \left(\frac{1+g}{1+r}\right)^i pb_{t+i},$$

where lower-case symbols refer to the per GDP versions of net debt and primary balance.

Now there is a new variable to be considered in the analysis of sustainability: the growth rate of GDP g. Taking limits as n tends to infinity:

[6] Similar results can be derived in the case of time dependent growth rates.

$$b_t = \lim_{n\to\infty}\left(\frac{1+g}{1+r}\right)^n b_{t+n} + \sum_{i=1}^{\infty}\left(\frac{1+g}{1+r}\right)^i pb_{t+i}.$$

The new version of the transversality condition states that

$$\lim_{n\to\infty}\left(\frac{1+g}{1+r}\right)^n b_{t+n} = 0.$$

This is equivalent with the earlier transversality condition. For per GDP variables the interpretation is just different: at the limit debt per GDP must grow at a gross rate slower than $\left(\frac{1+r}{1+g}\right)$. For r>g, this allows exponentially growing debt per GDP trajectories. For r=g, debt per GDP must be constant. For r<g, debt per GDP must converge to zero in an exponential rate.

The corresponding inter-temporal budget constraint is:

$$b_t = \sum_{i=1}^{\infty}\left(\frac{1+g}{1+r}\right)^i pb_{t+i}.$$

As noted earlier, there is disagreement among economists of whether IBC is a binding constraint for governments.

O'Connel and Zeldes (1988) show that Ponzi-games do not exist or equivalently the IBC must hold in a credit market with a finite number of rational non-satiable participants over time. This means that for IBC not to constrain government lending, the economy must consist of an infinite number of agents entering it over time. This seems plausible, given infinite time horizon of the analysis. However, the government constantly violating the IBC must be infinitely-lived as well which seems less plausible.

Bagnai (2004) finds it unreasonable that the IBC allows for explosive trajectories of debt-to-gdp ratio. He argues that the IBC is not a fact of nature but rather it is a constraint imposed on the behaviour of debtors by the rational creditors in a well-defined class of inter-temporal equilibrium models. Therefore, he argues, if the class of models to which IBC is based is true, "unsustainable" debt paths will never be observed. This is because the IBC must be respected in these models in equilibrium and thus, observed violations of the constraint must be temporary and hence irrelevant as far as the infinite-horizon asymptotic IBC is concerned. Thus, IBC may appear to be violated only because 1) it is not binding in the economy for some 2) it is a temporary violation.

Furthermore, Bagnai (2004) seems to argue that assessment of IBC in economies in which it is binding is a futile exercise: the possible violations are only temporary and thus don't require any actions. However, we would argue that examining whether IBC holds or how big adjustment is required for it to hold in the projected future is useful in these economies precisely because governments position as a debtor is grounded on the fact that IBC holds: hence, fiscal authorities need to periodically adjust policies so that creditors can reasonably expect IBC to hold.

There are several theoretical models in which IBC doesn't have to hold. Diamond (1965) overlapping generations model may generate competitive equilibria in which growth rate of the labour force exceeds the long-run return on capital. In these cases the economy is said to be dynamically inefficient and government debt can increase at rate higher than the interest rate violating the IBC. Counter-argument to this is that empirical evidence indicates that most of the advanced economies are dynamically efficient (Abel et al. 1989). However, de la Croix and Michel (2002, 192) show that in an overlapping generations model based on Diamond (1965) IBC doesn't have to hold given that government can tax both the young and the old generation even if the economy is dynamically efficient. Other examples of where IBC doesn't constrain government in dynamically efficient economy are Persson (1985) and Wigger (2009).

Fiscal theory of the price level developed by Leeper (1991), Sims (1994) and Woodford (1995) among others also touches upon the meaning of inter-temporal budget constraint. According to this somewhat controversial theory, the IBC can be made to hold by changes in the general price level $\mathrm{P_t}$. In this case, the government's inter-temporal budget constraint states that the real value of government debt $\frac{\mathrm{B_t}}{\mathrm{P_{t+1}}}$ must equal to the present value of future cash flows:

$$\frac{B_t}{P_{t+1}} = \sum_{i=1}^{\infty}(1+r)^{-i} PB_{t+i}.$$

Fiscal theory of the price level claims that in some situations the general price level can be affected by fiscal policy via the inter-temporal budget constraint. In other words, in addition to changes in tax or expenditure policies, changes in the price level can serve as means of satisfying the IBC and re-establishing sustainability.

4.2 BOHN'S MODEL-BASED SUSTAINABILITY

Bohn (2005) criticizes the use of IBC in sustainability analyses and introduces criterion for sustainability which he terms Model-Based Sustainability (MBS). The MBS criterion generalises the traditional deterministic IBC to a world with uncertainty. Bohn argues that the question which policies are sustainable is a general equilibrium question about the behaviour of potential government creditors. Different assumptions about the behaviour of creditors lead to different conclusions about sustainability of fiscal policies. Under the assumptions that potential creditors are infinitely-lived optimizing agents, government doesn't run a negative debt in the long-run and that financial markets are complete Bohn (2005) argues that the inter-temporal budget constraint takes the form (Model-Based Sustainability criterion)

$$B_t = \sum_{n=0}^{\infty} E_t[u_{t,n} PB_{t+n}],$$

where $u_{t,n}$ is the economy's pricing kernel for contingent claims on period (t+n) (the state-contingent discount factor). The MBS criterion is derived from optimizing creditor behavior. It differs from the usual IBC in that the discount rates for future surpluses depend on the distribution of primary surpluses across states of nature. Bohn (2005) shows that the above IBC condition can lead to government facing constraints other than the traditional IBC.

4.3 CONVERGENCE OF THE DEBT TO OUTPUT RATIO

A common criterion for fiscal sustainability is the convergence of the debt-to-GDP ratio to a finite value (the boundedness criterion):

$$\lim_{t\to\infty} \frac{B_t}{Y_t} = \lim_{t\to\infty} b_t \leq k.$$

This condition was first proposed as a sufficient condition for sustainability in Domar (1994). A stricter form of this criterion is that debt-to-output ratio must eventually converge back to its initial value. It is analysed for example in Blanchard et al. (1990).

The above condition requires that eventually debt cannot grow at a rate greater than growth rate g of the economy. Inter-temporal budget constraint and the transversality condition requires that debt cannot grow at a rate greater than the interest rate r. Thus, when r > g, as is usually assumed, this criterion is stricter than the inter-temporal budget constraint. When r < g, the situation is the opposite.

The asymptotic boundedness condition of debt to output ratio compares the governments collateral, fraction of output taxed, to the stock of debt. It can be argued that the boundedness condition is always a looser condition that the inter-temporal budget constraint. It is reasonable to assume that primary balance cannot exceed output in any period. Thus, assuming IBC holds implies

$$b_t \leq \sum_{i=1}^{\infty} \left(\frac{1+g}{1+r}\right)^i pb_{t+i} \leq \sum_{i=1}^{\infty} \left(\frac{1+g}{1+r}\right)^i = \frac{1+g}{r-g}$$

for all t, assuming r>g. Therefore, if r>g, the assumption that primary balance can never exceed output requires the boundedness of debt-to-output ratio for all t. This means that the boundedness criterion is looser also when r>g.

4.4 OTHER CRITERIA

Roubini (2001) argues that the inter-temporal budget constraint is too loose criterion for fiscal sustainability. According to the inter-temporal budget constraint a government could run very large primary deficits for a long time provided that it could commit to run primary surpluses in the long run to satisfy the IBC. Roubini argues that this is not realistic for three reasons. Firstly, government cannot credibly commit to such a path. Secondly, adjustment required to run large enough primary surpluses in the long run would be highly costly and inefficient given distortionary taxation – i.e. it doesn't make sense to have marginal tax rates of 70 % in the long run to compensate for low marginal tax rates of 10 % in the short run. Thirdly, if the adjustment falls on government consumption rather than taxes it may again be unfair and inefficient to cut government spending and services to low levels in the long run to allow large spending in the short run. Roubini (2001) suggest that a very practical criterion for sustainability: public debt can be viewed as sustainable as long as the public debt to GDP ratio is non-increasing. This means that primary balance to output ratio should fulfil the following condition:

$$pb_t \geq \frac{r-g}{1+g} b_{t-1}.$$

Artis and Marcellino (2000) distinguish between solvency and sustainability. They define that a government is solvent if the IBC is fulfilled, that is, governement is capable during an infinite horizon to service its debt. On the other hand, according Artis and Marcellino sustainability is a more imprecise concept referring ability of government under current policies to achieve a pre-specified debt ratio in a finite time horizon.

The Treaty of Maastricht sets explicit conditions for fiscal sustainability of the EMU countries. Article 109 (1) of the treaty requires "sustainability of the governments financial position" for a country's eligibility to EMU. The treaty further defines the criteria to assess the sustainability by setting ceiling values of 3 % and 60 % for deficit and debt to GDP ratios respectively. The issue of sustainability is also elaborated in The Stability and Growth Pact by introducing the medium target of a fiscal position close to balance or in surplus thus tightening the deficit rule of the Treaty of Maastricht. It is unclear what "a position of close to balance or surplus in medium term" actually constitutes. In the EU framework sustainability is thus defined as non-violation of predefined conditions which include an arbitrary debt to output ceiling which is consistent with the boundedness criterion of sustainability.

5. SUMMARY INDICATORS OF PUBLIC FINANCE SUSTAINABILITY

After discussing the theoretical criteria for sustainability it is now possible to turn to the different approaches employed in sustainability assessments. Firstly, summary indicators which employ rather simple discounted cash flow calculations are studied.

Summary indicators are perhaps the most common approach to analyse sustainability in practice. These indicators are derived from the government budget constraint governing the evolution of debt as a function of interest rates, growth rate of the economy and future primary balances. These derivations do not use an explicit economic model to account to for the various interactions between the model variables and hence the indicators derived can only be regarded as giving an approximation of the degree of sustainability. They take as an input the projected primary balances, interest rates and growth rates of the economy. The indicators are widely used by for example European commission, national governments and the International monetary fund to assess sustainability.

5.1 Finite horizon tax gap indicator

The derivation is based on the equation describing the evolution of net debt per output that was derived earlier:

$$b_t = \left(\frac{1+g}{1+r}\right)^n b_{t+n} + \sum_{i=1}^{n} \left(\frac{1+g}{1+r}\right)^i pb_{t+i}.$$

Compounding the equation to period t+n gives

$$b_{t+n} = \left(\frac{1+r}{1+g}\right)^n b_t - \sum_{i=1}^{n} \left(\frac{1+r}{1+g}\right)^{n-i} pb_{t+i}.$$

Defining a specific time horizon (t, T) and a debt-to-output target level b_T to be achieved at the end of the time horizon gives the condition for permanent constant adjustment to primary balance relative to output $FTGAP$ required to achieve the target:

$$b_T = \left(\frac{1+r}{1+g}\right)^{T-t} b_t - \sum_{i=1}^{T-t} \left(\frac{1+r}{1+g}\right)^{T-t-i} (pb_{t+i} + FTGAP).$$

Using the formula for geometric series gives the size of the gap:

$$FTGAP = \frac{\left(b_t - b_T\left(\frac{1+g}{1+r}\right)^{T-t} - \sum_{i=1}^{T-t}\left(\frac{1+g}{1+r}\right)^i pb_{t+i}\right)(r-g)}{\left(1-\left(\frac{1+g}{1+r}\right)^{T-t}\right)(1+g)}.$$

This indicator is equivalent to the S1 indicator used by the European Commission. In S1 indicator the end of the time horizon is the year 2060 and the target debt-to-output ratio is 60 %. Congressional Budget Office in the US and Office of the Parliamentary Budget Officer in Canada use the same type of indicator which they term fiscal gap. They use time horizons of 25, 50 and 75 years. They require that at the end of time horizon debt-to-output ratio returns to its initial value.

Finite horizon tax gap indicator determines a target level for debt-per-GDP ratio to be reached at the end of the specified time horizon. It is called tax gap indicator because the adjustment is measured in terms of change in primary balance to output ratio so that the indicator gives the permanent change required in the total tax ratio, total tax revenues per GDP, to reach the target debt-per-GDP ratio at the end of the time horizon. For example, if FTGAP = 3 %, this means a permanent raise of 3 % in the tax income per GDP ratio would close the sustainability gap if the tax hike didn't have any dynamic effects. However, the required permanent adjustment in the primary balances per output can be realised equally well by cutting expenditures relative to GDP or by any combination of tax revenue increase and expenditure cut that permanently increase primary balance relative to output by the amount specified by the indicator.

Finite horizon tax gap indicator (and any other tax gap indicator for that matter) divides the burden of adjustment equally (as a proportion of GDP) to the years belonging to the time horizon. This means that the change in net taxes paid relative to total output is equal in every period. Thus, the finite horizon tax gap indicator assumes that the required adjustment is smoothed over the time horizon.

5.2 Infinite horizon tax gap indicator

The infinite horizon tax gap indicator is based on the inter-temporal budget constraint. It measures the permanent constant adjustment to primary balance to output ratio required to satisfy the IBC:

$$b_t = \sum_{i=1}^{\infty} \left(\frac{1+g}{1+r}\right)^i (pb_{t+i} + ITGAP)$$

$$\mathrm{ITGAP} = \frac{(\mathrm{r}-\mathrm{g})\left(\mathrm{b_t} - \sum_{i=1}^{\infty}\left(\frac{1+g}{1+r}\right)^i pb_{t+i}\right)}{1+g}.$$

The sustainability indicator S2 used for example by the European Commission is equivalent to the above indicator. Assuming $r > g$, infinite horizon tax gap can be derived from the formula of finite horizon tax gap by taking the limit T tends to infinity. This just means that if debt converges to a finite level at infinity it is consistent with IBC when $r>g$.

ITGAP indicator divides the required adjustment relative to GDP evenly to infinite time horizon. Similarly as the finite horizon indicator, ITGAP is a tax gap indicator which means that its value can be interpreted as immediate permanent change to tax ratio required for IBC to hold. The change to primary balances can come equally well from expenditure cuts.

5.3 FINANCING GAP

The needed adjustment as measured by finite or infinite horizon tax gaps can be divided to the time horizon in ways other than smoothing the adjustment evenly as a percentage of GDP. Financing gap takes the flow of predicted future primary balances and compares it with the current level of net debt. In the case of infinite horizon, it gives the immediate adjustment to the debt per GDP level needed to satisfy the inter-temporal budget constraint:

$$IFGAP = \sum_{i=1}^{\infty} \left(\frac{1+g}{1+r}\right)^{i} pb_{t+i} - b_t.$$

The indicator measures the adjustment required in present value terms relative to output unlike tax gaps in which the adjustment was given in a form of an annuity. Measuring the gap in present value terms might be useful in some cases. At minimum, it is a good way to demonstrate the extent of the sustainability problem. A similar indicator can be defined for finite horizons. In that case, the financing gap measures the immediate required change in debt per GDP level to attain the target level of debt per GDP at the end of the chosen time horizon. Financing gap is analysed e.g. in Giammarioli et al. (2007).

5.4 PRIMARY GAP

Unlike previous indicators, the primary gap assumes the constancy of primary balances in the future. It determines the constant primary balance that would satisfy the required infinite or finite horizon sustainability conditions. In the infinite horizon case, primary gap indicator is defined as the difference between the required constant primary balance to satisfy the government's inter-temporal budget constraint and the projected primary balance. It was proposed by Buiter et al. (1985). Assuming r>g and substituting $pb_i = pb$ to the IBC and solving for pb gives:

$$\mathrm{pb} = \frac{(\mathrm{r}-\mathrm{g})\mathrm{b_t}}{1+\mathrm{g}}.$$

This is called the "debt stabilising primary balance". It means that if $pb_i > pb$, debt ratio decreases and if $pb_i < pb$, debt ratio increases. The only stable long-run steady state is $pb_i = pb$ because other values of primary balance lead either to unbounded increase or decrease of the debt-to-output ratio. The condition $pb_i \geq pb$ is equivalent with the criterion of sustainability proposed by Roubini (2001) which requires the debt-to-output ratio to be non-increasing.

The primary gap is the difference between debt stabilising primary balance and the projected primary balance:

$$\text{primary gap} = \text{pb} - \text{pb}_0 = \frac{(\text{r} - \text{g})\text{b}_0}{1 + \text{g}} - pb_0.$$

While primary gap gives a direct target for the adjustment in fiscal policy, it comes with a caveat: primary gap doesn't take into account the projected future of government revenues and expenditures and therefore is not useful in guiding fiscal policy when there is significant pressure coming from the revenue or expenditure side of the budget in the future. Assuming a constant primary balance places inordinate burden to those years in which expenditures are projected to exceed revenues by the largest margin.

5.5 Notes about the summary indicators

A key issue with summary indicators of public finance sustainability is whether to use infinite or finite horizon indicators. The problem with infinite horizon indicators is that predictions about the far future have to be made. Furthermore, they require the assumption that the inter-temporal budget constraint is de facto a constraint to government behaviour. The fact is that the IBC is not unanimously accepted in the literature. However, having said that there are grave problems with finite horizon indicators as well. According to Andersen (2010) finite horizon analyses are extremely sensitive to the particular end-point imposed. This also makes the analysis somewhat arbitrary because a target debt level has to be specified. It is hard to define a public debt ceiling for a country, let alone an optimal public debt level. In addition, the end-year of the finite horizon indicators has to be changed from time to time which may cause large changes in the values of the indicator that may be hard to communicate to policy makers. A solution to this might be that the length of the time horizon is fixed, like in sustainability reports of the CBO in the U.S and PBO in Canada.

Andersen (2010) emphasises the need to look beyond the infinite horizon tax gap indicator, or any single indicator for that matter, to the projected path of primary balances underlying the single value of the indicator. Depending on the profile of projected primary balances different policies may be optimal.

The presented summary indicators of sustainability do not take into account the interactions between the variables: for example those between debt level and interest rates and tax rate and economic growth. Thus, the results can be taken only as an approximation of the extent of the sustainability problem.

The values received from the sustainability indicators depend crucially on the inputs: flow of predicted primary balances, current debt level, interest rates and growth rates. The values of the inputs have to be determined by using some economic and econometric forecasting models. Therefore, these models and their assumptions, whatever they may be, represent an underlying framework behind any estimates of sustainability given by the summary indicators. These underlying economic models are an essential factor behind the results. Hence, the analysis of any values of the summary indicators of sustainability will be incomplete without careful analysis of underlying economic forecasting models.

Haaparanta (2011) emphasises the need of using internally consistent projections of the key inputs to the summary indicators. He notes that in Ministry of Finance (2010), which is a standard application sustainability gap indicators, the projections about the future public spending and economic growth are contradictory and thus cannot simultaneously be realised in standard growth models. To avoid these kinds of errors, Haaparanta recommends making the predictions using a consistent economic model instead of utilising simple ad-hoc predictions or assumptions.

One problem with the summary indicators is that there is not any explicit consideration for uncertainty of the estimated values of indicators. Many sustainability reports like European Commission (2009) include scenario and sensitivity analyses in which the various inputs to the calculation of summary indicators, such as the interest rates and growth rates, are varied and new values for the indicators are determined. These types of sensitivity analyses are essential feature of the practical use of the indicators because in order to draw any policy implications, it is important to get a sense of the uncertainty inherent in the point estimates of summary indicators. However, it would be even better, if models (the underlying framework) that yield the inputs to calculations and the indicators itself would explicitly account for uncertainty. There have been some extensions to the presented indicators that account for uncertainty, e.g. in Giammarioli et al. (2007). Ideally, the result of sustainability analyses with uncertainty would be a set of estimated distributions for the output variables like the sustainability gaps.

Policy implications of a sustainability gaps as measured by the summary indicators are not straightforward. According to Barro (1979) tax smoothing argument, tax rates should be kept constant over time to minimize tax distortions, allowing government budget to absorb variations in net expenditures. Hence, temporary expenditure variations would be absorbed through budget, while permanent expenditure changes require a permanent change in the tax rate. Thus, different underlying reasons causing the sustainability gaps may lead to different conclusions about needed policy actions. For instance, in the case of demographic change due to permanent lower level of fertility and permanent higher longevity which are the primary causal factors behind the sustainability problems in developed countries, it would seem that the correct policy response would be an immediate permanent change in the tax rate. However, this argument disregards the issues of intergenerational equity.

6. ECONOMETRIC TESTS OF PUBLIC FINANCE SUSTAINABILITY

This section examines the various econometric tests that have been proposed as tests of fiscal sustainability. In this strand of literature, different theoretical conditions for sustainability are tested with historical data of, among other things, debt and deficits. This section is divided into two parts: unit root and cointegration tests of sustainability and test for Bohn's model-based sustainability. Definitions of central mathematical concepts used in the Chapter can be found in Appendix 1.

6.1 UNIT ROOT AND COINTEGRATION TESTS OF SUSTAINABILITY

One of the first attempts to test sustainability by econometric means is Hamilton and Flavin (1986). In that paper, Hamilton and Flavin derive a statistical test to determine whether the inter-temporal budget constraint or equivalent transversality condition holds in empirical data. They use a detailed framework accounting for the general debt structure of government bonds and possibility of deficit monetising (money printing). Their null hypothesis which is equivalent with the IBC is the following:

$$B_t = \sum_{i=1}^{\infty}(1+r)^{-i}PB_{t+i},$$

where B_t is the real market value of government debt, r is average real return earned on government bonds, and $PB_t = T_t + S_t - G_t$ is real primary balance: a sum of tax revenue and seigniorage revenue minus government spending.

Specifically, they test whether $A_0 = 0$ in the formulation:

$$B_t = A_0(1+r)^t + E_t\sum_{j=1}^{\infty}(1+r)^{-j}PB_{t+j} + n_t,$$

where n_t is regression disturbance term and E_t denotes the expectations of creditors which Hamilton and Flavin assume to be formed rationally.

Hamilton and Flavin (1986) find that the IBC holds (i.e. $A_0 = 0$) in the empirical data for US during the years 1960-1984 and conclude that IBC seems to have been a restriction on government borrowing during that period. It is noteworthy that Hamilton and Flavin (1986) test whether IBC holds in the data but take no stance of whether it should hold. Thus, a negative result wouldn't have meant that government policy had not been sustainable but rather that IBC has not been a relevant constraint for government during the period in question.

Wilcox (1989) uses the same data set as Hamilton and Flavin and finds that if the hypothesis of constant interest rate is relaxed and stochastic violations to the inter-temporal budget constraint are considered as well, then the null hypothesis that IBC holds in the data must be rejected.

Trehan and Walsh (1988) show that if real revenues, real spending and real debt have unit roots, a stationary deficit (including interest payments) is sufficient to guarantee that IBC holds. Trehan and Walsh test these statements in US data spanning years 1890-1986 and find that it is consistent with IBC.

Trehan and Walsh (1991) generalizes results of their previous paper. Firstly, with variable discount rates, IBC holds if debt is difference-stationary and if the discount rate is strictly positive. This statement can be understood by noting that if debt B_t is difference-stationary with mean δ the expected debt n-periods forward is:

$$E_t[B_{t+n}] \approx B_t + n\delta.$$

In other words, expected growth of debt is linear. This implies that the expected present value of debt tends to zero in accordance with the transversality condition and the inter-temporal budget constraint because discounting factor decays exponentially.

Secondly, Trehan and Walsh (1991) show that IBC holds if a quasi-difference of debt $B_t - \lambda B_{t-1}$ is stationary for some $0 \leq \lambda < 1 + r$ and if debt and primary surpluses are cointegrated, covering the case of non-stationary with-interest deficits. The intuition behind this condition is the fact that $B_t - \lambda B_{t-1}$ is stationary with $0 \leq \lambda < 1 + r$ implies that asymptotic rate of growth of debt is $\lambda - 1$ which is in turn less than interest rate r. Hence, the asymptotic expected present value of debt tends to zero in line with the transversality condition.

Using these tests described above, Trehan and Walsh (1991) find that postwar US fiscal policy is consistent with the IBC.

Bohn (2005) applies unit root tests to a longer sample of US data spanning the time period 1792-2003. He finds that there is no credible evidence of unit roots in the U.S. debt-GDP and deficit-GDP ratios and that Trehan and Walsh (1988) condition for IBC to hold is satisfied in the data. Absence of unit roots means that debt-GDP ratio is stationary and thus policies are sustainable.

6.2 Test for Bohn's model-based sustainability

Bohn (1998) introduces an approach to sustainability testing which he names Model-Based Sustainability (MBS). The condition for model-based sustainability which is a generalization of the usual inter-temporal constraint was referred to earlier (MBS criterion):

$$B_t = \sum_{n=0}^{\infty} E_t[u_{t,n} PB_{t+n}].$$

Under the MBS framework sustainability analysis reduces to testing whether or not the primary balance-output ratio responds positively to increases in public debt-output ratio. The regression to test this is:

$$pb_t = \lambda b_t + \mu_t,$$

where λ is a constant and μ_t is a composite of determinants of the primary balance other than the initial stock of debt. Bohn (1998) shows that if $\lambda > 0$ in the equation above, μ_t is bounded as a share of GDP and if the present value of GDP is finite, then fiscal policy satisfies the MBS criterion.

Bohn (1998) estimates above regression for US data encompassing the period 1916-1995. He finds strong evidence in favour of $\lambda > 0$ in US data for the period 1916-1995. International Monetary Fund (2003, 113-152) applies Bohn's MBS test to a sample of industrial and developing countries. The results signal that the sustainability condition holds for industrial countries and developing countries with low debt ratios. It fails for developing countries with high debt ratios. Bohn (2005) continues in this track and finds robust positive response of primary surpluses to fluctuations in the debt-GDP ratio in the US during the period 1792-2003 and hence concludes that US fiscal policy has been sustainable. Mendoza and Ostry (2008) use MBS test for 34 emerging market and 22 industrial countries over the period 1990-2005. They find that the MBS condition for fiscal solvency is consistent with the data for both emerging market and industrial countries.

While Mendoza and Ostry (2008) consider MBS framework a powerful and tractable tool to find out if government policies have been sustainable, they note two caveats with the framework. Firstly, the test assumes complete markets. In the case of incomplete asset markets where there are not sufficient state-contingent claims to fully hedge against possible shocks, Mendoza and Ostry (2008) argue that tighter debt limits than those imposed by the MBS criterion are required. Secondly, they note that it is unclear how to interpret results when λ is statistically insignificant or when $\lambda < 0$. One interpretation is that failure of the test indicates that MBS criterion is not a relevant constraint for government borrowing in the economy. Alternatively, such a failure could mean that either MBS is a relevant constraint but market anticipates a policy change or government policies are unsustainable.

6.3 NOTES ABOUT ECONOMETRIC TESTS FOR SUSTAINABILITY

Bohn's model based sustainability approach to econometric testing seems robust and preferable to the earlier unit root tests. However, the problem with traditional application of econometric tests is that they provide information about the sustainability of past policies only. This is fine if the purpose of the research is to examine sustainability in retrospect. However, usually a more interesting question is the sustainability of the current fiscal policy as projected into the future. If econometric tests were applied to projected future data, they could give information about the sustainability of current fiscal stance, that is, the expected future fiscal policy. The usefulness of such an exercise is not clear, however, because the researcher can build in any feedback relations into the projected data. If projected data and sustainability testing were done in credibly separate processes, and the projections were trustworthy, applying econometric tests to the projected data might be useful. We were unable to find any papers doing this.

7. VALUE-AT-RISK MEASURE OF SUSTAINABILITY

Every assessment of public finance sustainability contains considerable amount of uncertainty because in order to assess sustainability uncertain predictions into the far future have to be made. This chapter goes through a Value-at-Risk framework of sustainability by Barnhill and Kopits (2003) which attempt explicitly to account for the uncertainty in predictions of future values of relevant variables such as budget balances, growth rates and interest rates.

In financial mathematics and risk management Value-at-Risk (VaR) is a commonly used risk measure of the risk of loss on a portfolio of financial instruments. It measures the worst possible loss over a target horizon with a given level of confidence. For example, if one-month 95 % Value-at-Risk for a portfolio of stocks is 1 million , this means that the monthly loss on the portfolio is greater than or equal to 1 million with probability 5 %. VaR methodology is commonly used in the financial industry to manage risk exposure.

Barnhill and Kopits (2003) apply VaR methodology to analyse the public finance sustainability. The approach simulates a distribution of possible future financial conditions for the government and assesses the probability of financial failure given this distribution. The analysis of Barnhill and Kopits is based on the income statement of the consolidated public sector encompassing both monetary and fiscal authorities:

$$Y_t = T_t + N_t + S_t - G_t - rB_{t-1} = Z_t + S_t - rB_{t-1}.$$

Public sector revenue consists of tax receipts less government transfers (T), determined by the level of economic activity, the tax structure and administrative efficiency; net revenue from resource sales (N), a function of production level, production cost and resource price; and income from seigniorage (S). Expenditure is comprised of government consumption (G), made up of mandatory and discretionary payments on wages, goods and services; and interest payments given by the product of average interest rate (r) and the net stock of public debt outstanding (B). The net debt consists of domestic liabilities less assets and of foreign liabilities less assets. The term $Z_t = T_t + N_t - G_t$ refers to public sector primary balance excluding seigniorage revenue. Furthermore, the public sector may hold a stock of net unfunded contingent liabilities (C) relating to social security programs, deposit insurance schemes, insurance for natural disasters or other government guarantees that may realized and converted into B. Realization of C is affected by the level of economic activity, demographic trends, effectiveness of bank supervision, bank capitalization and occurrence of natural disasters.

In the framework, the net value V of the public sector at t=0 is the sum of the present values of the terms in the income statement[7]:

$$V_0 = PV(Z') - PV(\Delta C) - B_0$$

$$V_0 = \sum_{t=0}^{\infty}(1+r)^{-t}Z'_t - \sum_{t=0}^{\infty}(1+r)^{-t}\gamma_t\Delta C_t - \sum_{t=0}^{\infty}(1+r)^{-t}\Delta B_t,$$

where Z' is the primary balance generated by the existing fiscal system (tax structure and mandatory spending programs in existence at t=0), γ_t is the probability of realizing contingency ΔC_t at time t and ΔB_t is the discounted net amortization schedule of the net debt so that $B_0 = \sum_{t=0}^{\infty}(1+r)^{-t}\Delta B_t$. Calculation of the net value assumes that there is no discretionary adjustment in tax structure or government spending in the future. Also, income from seignorage S_t is assumed to equal zero at all times.

If net value is non-negative, the public sector is deemed solvent and policies are sustainable[8]. In traditional assessments of sustainability, the expected value of net value

[7] Barnhill and Kopits (2003) use the term net worth instead of net value. Net value is used here because net worth has a different definition in this thesis.

[8] The net value is related to inter-temporal budget constraint in the following way: $V = 0 \iff IBC\ holds$. Having a positive net value is equivalent to saying that government doesn't run a Ponzi scheme.

is studied. Here V is explicitly recognized as a stochastic variable and its distribution estimated.

Barnhill and Kopits (2003) model the public sector net value as a function of underlying risk variables:

$$V = PV(q, r_H, r_F, f, p_N, p).$$

The equation tells that the public sector net value depends on the present and future level of output (q) , interest rates at home and abroad (r_H, r_F), the exchange rate (f) world commodity prices (p_N) and the domestic price level (p).

The risk variables determine the financial environment thus and the asset prices and present values of future revenues and expenditures. Therefore, by simulating the risk variables, distributions for government assets, liabilities and net value can be estimated. From the estimated distributions, relevant Value-at-Risk measures can be calculated.

In order to simulate the behaviour of the risk variables, Barnhill and Kopits assume that they follow known stochastic processes. Barnhill and Kopits use different stochastic processes for description of different variables. For example, they use Brownian motion for rates of return, output and prices and time-dependent mean reversion process for interest rates. Parameters of the stochastic processes are estimated from historical data. By estimating the historical correlations between the risk variables, they can reconstruct the correlation structure inherent in the variables. In practice this means that imposing the correlation structure of the risk variables on the error terms of the stochastic procesess. This way Barnill and Kopits can combine the stochastic processes describing the time-evolution of the risk variables and the correlation structure to yield simulated values of the risk variables.

Barnhill and Kopits apply their Value-at-Risk framework to study the sustainability of public sector finances in Ecuador. Based on data from year 2000 they simulate distribution of public sector net value in year 2001. They estimate that one-year 95 % VaR of Ecuador was 29 US$ billion. Because, absent risk, net value of the public sector was 8 US$ billion, it is equivalent to say that net value of the public sector with 5 % probability was -21 US$ billion. It is noteworthy that assessment focused expected value of public sector would have found no sustainability problem because expected public sector net value was 8 US$ billion. Insolvency, that is negative net value of the public sector, is predicted to occur with 35 % probability. Their simulations indicate that the principal sources of fiscal risk for Ecuador were volatility in the interest yield spread, the exchange rate, and oil prices together with their comovements.

The VaR approach of Barnhill and Kopits is a valuable tool for analysis of public finance sustainability. It attempts to account for most of the complexities involved in the analysis of sustainability. Specifically, it is an improvement to traditional approaches because uncertainty is explicitly and more realistically modelled. In addition, a comprehensive view of the public sector net value is undertaken with due recognition to the fact that changes in values of government assets and liabilities significantly affect public sector solvency and policy sustainability.

At heart of the VaR approach is the estimation of distribution of government net value and thus solvency. Both effects from changes in asset values and present values of future budgetary flows are accounted for. Usage of the VaR approach is not limited to calculation of specific VaR estimates. Other risk measures such as conditional Value-at-Risk can be calculated as easily. By estimating the distribution of government net value, the VaR approach allows for the complete risk management of public sector assets and liabilities.

Barnhill and Kopits (2003) note that the VaR approach is open to question on several grounds. Firstly, the variances and covariances of the risk variables estimated from past data may not provide a reliable picture of future risks. This shortcoming is mitigated by the fact that VaR approach flexible is in that the risk models can be readjusted on the basis of expert opinion. Secondly, Barnhill and Kopits point out that the contingent liability modelling is lacking because risks stemming from e.g. financial system defaults are not accounted for in a comprehensive manner. Thirdly, the approach needs quite rich data about public sector assets and liabilities which is not available in every country. Public sector balance sheets are a minimum input requirement for VaR analysis.

8. FISCAL LIMITS, FISCAL SPACE AND SUSTAINABILITY

Recently a few novel approaches have been developed for estimating public debt ceiling for a country. Knowledge of an upper limit for a country's public debt would be very useful for sustainability assessments since it would give indication of how much more debt the country is able to sustain at maximum. Here two approaches are analysed. The first tries to determine so-called fiscal limit for a country. Fiscal limit is the point beyond which taxes and government expenditures can no longer adjust to stabilize the value of government debt. This approach stems from the literature on monetary and policy interactions and appears in for example in Bi (2010), Cochrane (2010) and Leeper and Walker (2011a). The second approach attempts to estimate a debt ceiling for a country based on the country's past record of fiscal policy. It has been developed by Ostry et al. (2010).

8.1 FISCAL LIMITS

The concept of fiscal limit appears in novel and growing strand of literature on monetary and fiscal policy interactions. Our treatment of the subject is based on Bi (2010).

The fiscal limit of a country gives the maximum level of debt that the country's government can accommodate solely by fiscal instruments. After an economy hits the fiscal limit, the government has to stabilise debt by monetary policy: seigniorage revenue generates enough surplus to stabilise the level of debt. Alternatively, if this kind of money printing is not possible, government can default on some of its obligations such as the debt or promised expenditures. Thus, determining the fiscal limit of a country and comparing it to the present and projected future levels of debt gives indication of how much room the government has left for fiscal policy adjustments. This, in turn, is valuable piece of information in public finance sustainability assessments.

The most common economic rationale for existence of fiscal limits follows from the Laffer curve. The Laffer curve depicts an inverse U-shaped between the tax rate and collected tax revenue. Under distortionary taxation, higher tax levels lead to diminishing incentives to work, save and invest and thus to lower levels of realised tax bases. As the tax rate is increased, at some point the marginal relative decrease in the tax base will exceed the marginal relative increase in tax rate and from that point on the collected tax revenue will fall. Thus, there is some level of the tax rate which maximizes tax revenue.

Given a level of promised government expenditures, the existence of Laffer curve implies some maximum stream of primary surpluses that the government can generate. Because government debt is valuable only to the extent it is backed up by flow of future primary surpluses (assuming seigniorage revenues are zero), and the present values of this flow is bounded, the size of government debt relative to output must be bounded as well[9]. Fiscal limit B* can be defined (Bi 2010):

$$B_t^* = E_t \sum_{i=t}^{\infty} Q_i^{max} \left(T_i^{\max} - G_i\right),$$

where Q is the discount factor, T^{max} is the maximum tax revenue and G is government spending and transfers. The actual fiscal limit is usually below the economic fiscal limit defined above because political considerations prevent tax rates reaching revenue maximising levels.

The fiscal limit is a model-dependent measure like most of the measures related to sustainability. Next we describe the way Bi (2010) attempts to estimate the fiscal limit.

Bi (2010) constructs an infinite-horizon model of a closed economy in which fiscal limits arise endogenously from Laffer curves. The economy has a simple linear production technology and productivity follows an exogenous stochastic process. Government is modelled as an automaton which finances purchases and lump-sum transfers to households by collecting tax revenue and issuing bonds. Purchases are modelled by an exogenous stochastic process and transfers follow productivity counter-cyclically. Government follows a simple tax rule in which tax rate rises linearly with increases in debt. Bi models government default by random draw from distribution of fiscal limit. If the current level of debt surpasses the draw, government defaults on a fraction of its outstanding debt. If not, government fully honours all its obligations. By modelling default by a random draw from the distribution of fiscal limit, Bi (2010) abstracts from strategic concerns of default related to models of political economy.

In Bi (2010), a representative household chooses consumption and leisure and government bond purchases to maximize expected present value of utility. Proportional

[9] The Laffer curve is not the only rationale for fiscal limits. Sargent and Wallace (1981) analyse a type of fiscal limit in a model in which seigniorage revenues are explicitly accounted for. In their model, private sector's demand for government bonds imposes an upper bound on the debt-GDP ratio.

tax on labour distorts household's choice and induces a Laffer curve in the economy. The Laffer curve, given level of government purchases and lump-sum transfers, results in a ceiling for surplus in any period and gives rise to a fiscal limit. In Bi (2010), fiscal limit is also affected by political willingness to raise taxes near revenue maximising rates. This is modelled by the parameter θ in the following expression for fiscal limit

$$B_t^* = E_t \sum_{i=t}^{\infty} Q_{i,t}^{\max} \theta_t (T_i^{\max} - z_i - g_i),$$

where $Q_{i,t}^{\max}$ is a discount factor depending on household's subjective discount factor and marginal utility of consumption when tax rate is set at the peak of the Laffer curve, θ is political risk factor, T_i^{max} is maximal revenue at time i and z_i and g_i are government transfers and purchases respectively.

The distribution of the fiscal limit is country-specific and depends on underlying parameters of the model. However, Bi (2010) emphasises that the fiscal limit is independent of equilibrium conditions of the model. Bi (2010) estimates distribution of the fiscal limit by using Markov Chain Monte Carlo simulation.

Bi (2010) calibrates the model to economies of Canada, New Zealand, Belgium, Japan and Sweden and observes that the model predicts distributions of fiscal limits consistent with the sovereign credit ratings on these countries. The model also produces a nonlinear "S-shaped" relationship between sovereign risk premia and the level of government debt which is consistent with empirical evidence.

The model by Bi (2010) is one attempt to estimate fiscal limits and endogenise the relationship between interest rates on government debt and the debt level. Even though the model is fairly simple it produces encouraging qualitative results that match empirical observations. The model has many assumptions that are not very realistic such as the simplistic logarithmic utility function and linear production function, assumption that government is an automaton and the deterministic tax rule followed by government. In spite of all this, our opinion is that Bi (2010) and other recently published papers on fiscal limits represent a new and promising way to look at public finance sustainability.

8.2 Fiscal space

Ostry et al. (2010) introduce another approach to estimate the degree to which a country has room for fiscal manoeuvring, that is the fiscal space, by looking at the historical record of the country's fiscal policy. The intent of the approach is similar to that of fiscal limits though methodology differs significantly. Ostry et al. (2010) define fiscal space as the difference between the current level of public debt and the debt limit implied by the country's historical record of fiscal adjustment. Their approach to estimating an upper limit for public debt is based on the fiscal reaction function pioneered by Bohn (1998) which determines country's primary balance as a function of the debt level.

First component of the analysis of Ostry et al. (2010) involves determination of the fiscal reaction function. Ostry et al. observe that governments usually behave responsibly increasing primary surplus in response to rising debt service in order to stabilize the debt ratio. However, this cannot always be true because at sufficiently high levels of debt this would require primary balance to exceed GDP. They plot primary balance ratio against public debt ratio for a sample 23 advanced economies over the period 1970-2007 and observe that there seems to be a S-shape or loosely sigmoid-shape relationship between the two. In other words, while fiscal effort is increasing in debt level, the effort eventually diminishes as tax increases and spending cuts become less and less feasible politically. This relationship is depicted by the solid line in Figure 5.

The second component of the analysis by Ostry et al. is the determination of the interest rate schedule which gives the ratio of primary balance to GDP that leads to unchanged debt ratio as a function of debt ratio. This is equivalent with the "debt-stabilising primary balance" defined in chapter 5:

$$\mathrm{pb} = \frac{(\mathrm{r}(\mathrm{b_t}) - \mathrm{g})\mathrm{b_t}}{1 + \mathrm{g}} \approx (\mathrm{r}(\mathrm{b_t}) - \mathrm{g})\mathrm{b_t}.$$

The dashed line in Figure 5 depicts the interest rate schedule. Given that interest rate r doesn't depend on the debt level, which is the case for low levels of debt, the interest rate schedule is a linear function of the debt level. However, as the debt level rises and country's solvency comes under question, creditors start to demand increased yields because of the default risk. This means the interest rate schedule turns steeply upward at some point ($\hat{d}$ in Figure 5).

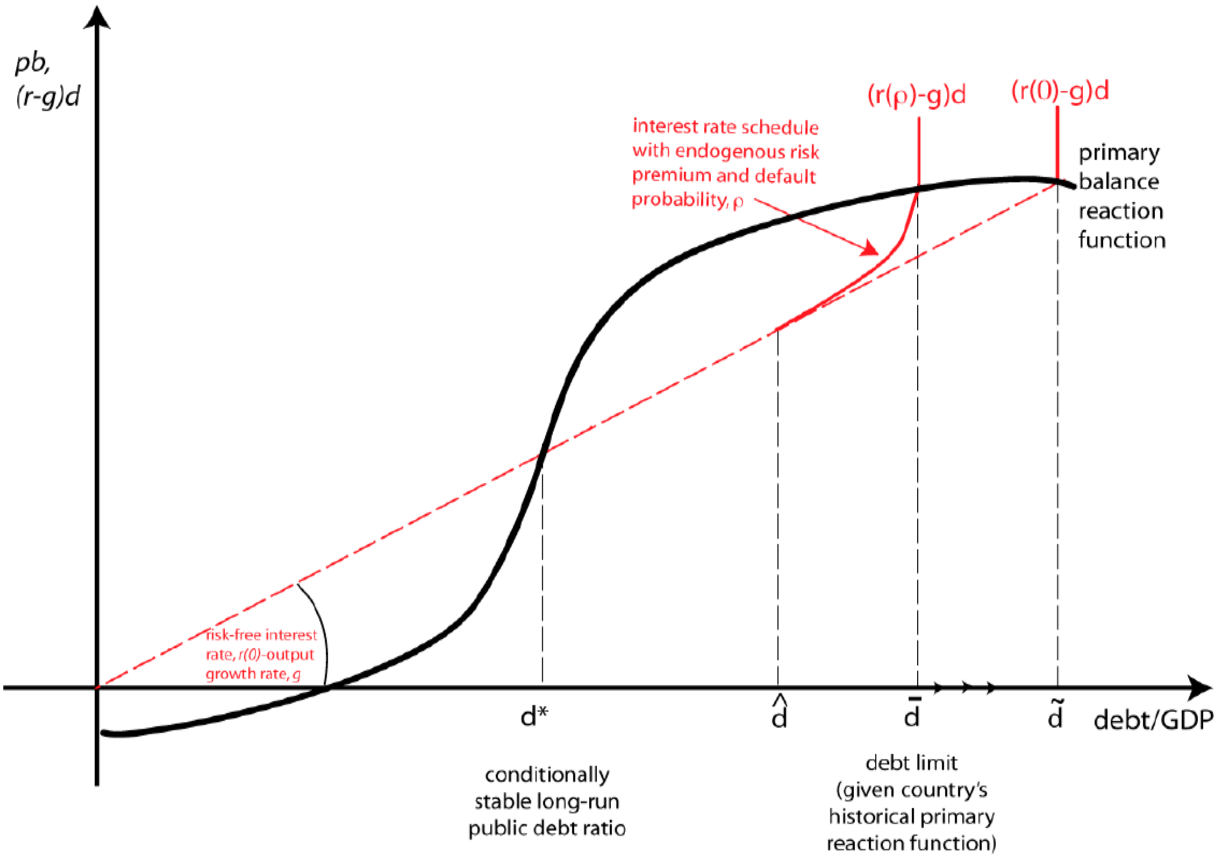

Figure 5. Determination of the debt limit. Source: Ostry et al. (2010).

Figure 5 presents a heuristic treatment of the determination of the debt limit according to Ostry et al. (2010). The two curves, primary balance reaction function (solid line) and interest rate schedule (dashed line), intersect at two points. The intersection at a lower level of debt ratio corresponds to the conditionally stable long-run public debt ratio (d^*). The intersection at a higher level of debt ratio corresponds to the debt limit ($\tilde{d}$ and $\bar{d}$). Given that debt ratio is below the debt limit, absent shocks, the country's debt ratio will eventually converge to the long-run stable ratio because below d^* primary balance response is such that debt ratio increases and above d^* it is such that debt ratio decreases. Ostry et al. define two debt limits: $\tilde{d}$ and $\bar{d}$. The former is based on the interest rate schedule which assumes that interest rate doesn't increase with the debt ratio. The latter is based on assumption that due to the possibility of stochastic shocks to primary balance markets will require a risk premium on top of the risk-free rate after some point $\hat{d}$ which causes the interest rate schedule to turn upwards and to intersect the primary balance reaction function earlier than in a riskless world. Beyond the debt limit the country's debt ratio will increase without bounds in absence of historically extraordinary fiscal effort.

Ostry et al. (2010) estimate fiscal reaction function relating primary balances to lagged debt and various economic, structural and institutional variables, as well as country-specific fixed effects using a sample of 23 advanced economies over the period 1970-2007. The function is cubic in lagged debt allowing for two apparent inflection points in the fiscal reaction function. Ostry et al. assume that coefficients of the debt ratio in this function are the same for all countries. Varying fiscal reactions for different countries are allowed for by the country-specific fixed effects other independent explanatory variables.

Ostry et al. (2010) determine interest rate schedule using two techniques. Firstly, they use a linear form of the interest rate schedule with either historical or projected interest rate-growth rate differential. Secondly, they calculate the interest rate schedule endogenously by using country-specific histories of shocks to primary balances. This second technique accounts for the fact that interest rate rises as country gets nearer its debt limit.

Using the estimated fiscal reaction functions and interest rate schedules, Ostry et al. (2010) calculate values of long-run debt ratio (d^*), debt limit ($\bar{d}$) and fiscal space (difference between debt ratio projected for 2015 and the debt limit) for different advanced economies. Furthermore, in order to take into account the uncertainty, they report the estimates of fiscal space in terms of probability that a country has a given amount (0, 50 or 100 % of GDP) of remaining fiscal space. For example, they estimate the probability that a country has any fiscal space left is 6.3 %, 34.4 %, 71.8 % and 96.2 % for Greece, Portugal, United States and Finland. Their estimates of debt limits, fiscal spaces and the associated probabilities vary greatly between the countries in the sample.

The approach to estimate debt limits and resulting fiscal space by Ostry et al. (2010) appears promising. The debt limit is calculated based on a country's past record of fiscal policies. The framework with fiscal reaction function and interest rate schedule is both simple and quite easy to practically implement. The crucial interaction between the level of debt and interest rates is explicitly modelled. Furthermore, the debt limit and fiscal space are pretty intuitive concepts and thus can be relatively easily communicated.

Weakness of the approach is that it needs quite a lot of data for determination of the fiscal reaction function. Such data is not usually available for countries for sufficiently broad range of debt ratio levels. For this reason Ostry et al. had to assume common regression coefficients and use in part common data for estimation of the reaction functions. In addition, the approach doesn't look at the public sector balance sheet as a whole: it doesn't explicitly account for government assets, expected spending pressures e.g. due to aging and contingent liabilities. It would be preferable to use net worth or net debt to GDP ratios instead of gross debt to GDP ratio.

9. GENERAL EQUILIBRIUM MODELS

This section describes three papers that analyse public finance sustainability by using general equilibrium models: GE-OLG model by Moraga and Vidal (2004) application of AGE-OLG model by van Ewijk et al. (2006) and application of CGE-OLG model by Andersen and Pedersen (2006). Many papers that use general equilibrium models to study sustainability are very theoretical. These three papers were chosen for further study because in them, these models are applied for practical policy analysis.

9.1 GE-OLG MODEL BY MORAGA AND VIDAL

Moraga and Vidal (2004) investigate fiscal sustainability in a general equilibrium overlapping generations model with endogenous growth resulting from human capital formation through educational spending. Interest rate and growth rate of the economy which have a crucial effect on long-run sustainability are determined endogenously in the model. In the model fiscal sustainability means that economic equilibrium exists in every period.

The model's economy has three sectors: households, firms and the public sector. Household sector consists of individuals that live through three periods: childhood, adulthood and old-age. These individuals maximize utility which is derived from private and public consumption in adulthood and old-age and their children's expected wage level. Consumption, saving and educational spending are financed by labour income which depends on individual's human capital. An individual's human capital is determined in their childhood by her parents' human capital and educational spending. This process of human capital formation is the engine of growth in the economy.

Production occurs in firms which use physical and human capital as inputs. Each period, firms choose demand for labour to maximize profits in a competitive setting. The public sector consists of the government which levies taxes, pays pension benefits to retirees, finances public consumption and reimburses existing stock of debt along with interest payments.

Moraga and Vidal (2004) derive a system of two differential equations which determine the dynamics of the economy. They describe the evolution of stock physical capital and government debt relative to stock of human capital. No general analytical solution exits for the system. Moraga and Vidal calibrate the model roughly to European Union data. By means of simulations they study the effect of a demographic shock which is projected to occur in the EU during the next 50 years. They show that the demographic change leads to unsustainable situation unless fiscal policies are changed in response. Proper fiscal rules guarantee convergence to a new equilibrium and thus sustainability.

The strengths of the model by Moraga and Vidal (2004) concerning sustainability assessments lie on theoretical analysis of an economy. It allows for study of the impact of various shocks and fiscal rules to sustainability. However, it is not so well suited to practical sustainability assessments concerning actual countries. The overlapping generations framework only allows for a crude description of the dynamics because length of the period is 1/3 of a lifetime, i.e. very long (Moraga and Vidal use 30 years). Furthermore, it is hard to calibrate. To assess sustainability more realistically, adjustments to the model such as shorter time period, more detailed generational structure and richer description of the tax and pension policies are needed.

9.2 Application of AGE-OLG model by van Ewijk et al.

Van Ewijk et al. (2006) use a large scale applied general equilibrium model with overlapping generations of households to study ageing and its effect on sustainability of public finances in Netherlands. The model they use is called GAMMA and it is constructed for the purpose of analysing long-term public policy issues.

The model consists of the following agents: households, government, firms, pension funds and the foreign sector. Households are divided up into 100 age cohorts. Households decide on labour supply and private saving, firms decide on demand for labour and capital and pension funds decide on pension contributions and benefit levels.

Agents are rational and forward-looking and optimise in a consistent microeconomic framework. The model includes a comprehensive set of generational accounts for all current and future generations. The Dutch economy is modelled as a small open economy. Interest rate determined in world capital markets and it is not affected by domestic policies. Domestic and foreign goods are assumed to be perfect substitutes. Perfect labour and capital markets are assumed. Furthermore, the model is deterministic.

Van Ewijk et al. (2006) construct a baseline projection which forecasts the evolution of public finances and the economy under unchanged policies from year 2006 onwards. Detailed demographic projections up to year 2100 are employed. In the baseline scenario, they assume that age-specific expenditures increase at the rate of productivity of private sector productivity and aggregate non-age-related expenditures increase at the rate of GDP growth. They assume annual rate of labour-augmenting technological change of 1.7 %, average real market rate of return of 3 % and inflation of 2 %. Government revenues are determined endogenously from household and firm behaviour except for natural resource revenues which evolve in line with separate projections. Van Ewijk et al. also conduct sensitivity tests around this baseline projection by varying parameter values and assumptions.

Van Ewijk et al. (2006) find that Dutch public finances are unsustainable in the long run. This is primarily due to population ageing which leads to significant increases in pension and health care expenditures and projected depletion of natural gas reserves and the associated revenue flow. They estimate that total debt, the sum of net debt (official debt minus financial assets of the government) and implicit debt that is due to projected deficits, is 2 times GDP in 2006. This indicator is equivalent with financing gap derived earlier in chapter 5. The annuity value of this total debt amounts to 2.6 % GDP which is the sustainability gap equivalent with infinite horizon sustainability gap. They conduct sensitivity tests around the baseline scenario and estimate a confidence interval of (0.9%, 5.1%) for the sustainability gap.

The study by van Ewijk et al. (2006) is a successful application of generational equilibrium framework to assessment of public finance sustainability. Model is detailed and structurally rich and therefore should be able to produce accurate projections concerning the Dutch economy. Interactions between the various forces in the economy are better accounted for than if separate models were used for projecting the inputs to sustainability calculations. Weakness of this approach is that a lot of detailed data about the economy under analysis is required. Secondly, a lot of effort needs to be put in to build such a large scale model. Finally, despite the structural richness of the model, there is no practical guarantee about its predictive accuracy.

9.3 Application of CGE-OLG model by Andersen and Pedersen

Andersen and Pedersen (2006) use a large scale dynamic computable general equilibrium overlapping generations model to study the long-term sustainability of fiscal policies in Denmark. The model they use is called DREAM, the Danish Rational Economic Agents Model, developed for the specific purpose of evaluating medium- to long-term effects of fiscal policy in Denmark.[10]

DREAM model represents a small open economy with a fixed exchange rate regime, perfect mobility of capital and imperfect substitutability of Danish and foreign products in consumption and production. The model uses a detailed household structure based on projections of Danish population. The adult population has 85 generations and a representative household is generated for each of the generations. Each household optimizes its labour supply, consumption and savings decisions given perfect foresight. The labour market modelled as having unionized behaviour. There are two private production sectors: a construction sector and a sector for other goods and services. The model assumes an exogenous productivity growth rate of 2 % per annum and an exogenous foreign inflation rate of 2 % per annum.

Public sector produces goods for public consumption and levies taxes and pays transfers and subsidies to firms and households. Taxes, transfers and subsidies are modelled with great detail in order to match the actual rules and regulations in Denmark as closely as possible. There are numerous different types of taxes and transfers in the model. The expenditures for individual public consumption (education, health and social expenditures) are forecasted to increase with rate of inflation and the exogenous productivity growth.

Andersen and Pedersen (2006) use two indicators to study the long-term sustainability of public finances in Denmark. Pay-as-you go indicator calculates the path of yearly

[10] Detailed information can be found at www.dreammodel.dk.

changes in the base tax required to balance the budget each year. Therefore, this indicator imposes a constant public debt level on the economy. The second indicator calculates the difference between the constant level of base tax rate required for inter-temporal budget constraint to hold (the sustainable tax rate) and the current base tax rate. Thus, the level of debt is allowed to fluctuate while the tax rate remains constant. This second indicator is reminiscent of the infinite horizon tax gap indicator (ITGAP) derived earlier in this paper. However, it is notable that due to flexibility of the DREAM model, these indicators can be calculated for the case of any tax or expenditure variables in the model.

Andersen and Pedersen (2006) calculate the sustainability indicators with respect to an "unchanged policy" scenario which represents the state of existing policies and their expected continuation in Denmark in year 2003. Projections from the model show that in the "unchanged policy" scenario expenditures increase faster than revenues and the former surpasses the latter around 2020 with the gap growing larger from then on. This is mostly due to projected permanent change in the demographic dependency ratio. The PAYG sustainability indicator in which consequences of the demographic changes are finances in a period-by-period basis shows a similar pattern: needed increases from the initial base tax rate accumulate in a constant manner increase being in the year 2040 around 4.5 %. The sustainable tax indicator finances the consequences of the demographic changes in using a pre-funding strategy: tax rate is immediately raised to a level which is needed to satisfy the inter-temporal budget constraint of the government. The increase in the base tax rate required for this hold was 7.9 %. Thus, both indicators point to the existence of a sizable sustainability problem in Denmark. Alternative changes that would re-establish sustainability were a permanent reduction in public expenditures of 3.2% of GDP and permanent increase in employment by 10 %. Andersen and Pedersen also perform detailed sensitivity analyses to estimate the sensitivity of the sustainability assessments to various changes in policy assumptions. Furthermore, using the model, they assess the effects of retirement and labour market reforms.

Andersen and Pedersen (2006) represents an application of detailed large scale structural model to study the sustainability of public finances. The benefits of this kind of approach are apparent: realistic modelling of the interactions in the economy should enable better and more detailed analysis of needed policy reforms. There are some caveats too, however. Firstly, a lot of data needs to be amassed to calibrate the model parameters. Secondly, despite the structural accuracy of the model there is not much assurance about the predictive accuracy of the model at least in the case of long-run predictions. Thirdly, the structural description of the economy with utility maximising representative agents, perfect mobility of capital and other modelling assumptions is open to question. Fourthly, the model as we understood it was not stochastic, i.e. didn't account for effects of uncertainty in the various variables which might compromise some of the results. Finally, a large effort is required to build such a detailed model.

10. Generational accounting and sustainability

Generational accounting (see e.g. Auerbach et al. 1991) measures sustainability from a little bit different perspective than traditional evaluations based on yearly budget surpluses. In the context of generational accounts, public finance sustainability means that the fiscal burden, i.e. the payment of taxes net of received benefits, is distributed evenly among different generations. If a lot of fiscal burden is shifted to future generations fiscal policy is deemed unsustainable. This definition of sustainability is closely related to the usual definitions of public finance sustainability which associate sustainability to non-excessive debt accumulation. This is because the way in which fiscal burden is shifted to future generations is by taking debt. A large national debt or big deficits therefore signals that future generations are made to pay for the current and past government consumption in addition to future government consumption.

The basis of sustainability treatment using generational accounts is the inter-temporal budget constraint:

$$\mathrm{B_t} = \sum_{s=\mathrm{t}+1}^{\infty} (1+r)^{-(s-t)}\mathrm{PB_s},$$

where B_t is the present stock of net debt, PB_t is the primary balance at time t, and r is the interest rate at time t.

Generational accounts approach differs from the standard surplus-based treatment of national accounts in that it disaggregates the surplus to contributions made by different generations:

$$PB_{\mathrm{s}} = \sum_{i=0}^{M} O_{s,s-i} - C_s,$$

where $O_{s,s-i}$ denotes the net payments (payments made minus benefits received) to government at time s by generation born at time s-i, M is the maximum length of life and C_s is government consumption at time s that is collective in the sense that it cannot be allocated to different generations.

Combining the two equations above gives:

$$B_t = \sum_{s=\mathrm{t}}^{\infty}\left(\left(\sum_{i=0}^{M} O_{s,s-i} - C_s\right)(1+r)^{-(s-t)}\right)$$

$$B_t = \sum_{s=\mathrm{t}}^{\infty}\sum_{i=0}^{M} O_{s,s-i}(1+r)^{-(s-t)} - \sum_{s=\mathrm{t}}^{\infty} C_s(1+r)^{-(s-t)}\,.$$

Separating generations born prior to time t from those born thereafter:

$$B_t = \sum_{i=0}^{M-1}\sum_{s=\mathrm{t+1}}^{t+M-i} O_{s,t-i}(1+r)^{-(s-t)} + \sum_{i=1}^{\infty}\sum_{s=\mathrm{t+i}}^{t+M+i} O_{s,t+i}(1+r)^{-(s-t)} - \sum_{s=\mathrm{t+1}}^{\infty} C_s(1+r)^{-(s-t)}$$

$$B_t = \sum_{i=0}^{M-1} \mathrm{N_{t,t-i}} + \sum_{i=1}^{\infty} N_{t,t+i} - \sum_{s=\mathrm{t+1}}^{\infty} C_s(1+r)^{-(s-t)}\,,$$

where $N_{t,k}$ is the value at time t of the net payments to government to be made by a generation born at time k.

In the above equation, first term is the net payments to government to be made by generations living at time t, i.e. currently. The second term is the net payments to government to be made by future generations. The last term is the present value of all future collective government consumption.

The generational account $GA_{t,k}$ means the net payments per capita to be made by members of generation k from time t onwards valued at time t. In other words, it is the present value of the net tax burden facing average member of generation k. The relationship between $GA_{t,k}$ and the total payment of the generation $N_{t,k}$ is:

$$N_{t,k} = P_{\max(t,k),k} GA_{t,k},$$

where $P_{t,k}$ is the number of people of generation k alive at time t.

Sustainability analysis using generational accounts is based on comparison between the net tax burden facing currently living generations and future generations. The total net payments to be made by future generations are:

$$\sum_{i=1}^{\infty} N_{t,t+i} = B_t + \sum_{s=\mathrm{t}+1}^{\infty} C_s(1+r)^{-(s-t)} - \sum_{i=0}^{M-1} \mathrm{N}_{\mathrm{t,t-i}}$$

$$\sum_{i=1}^{\infty} P_{t+i,t+i} GA_{t,t+i} = B_t + \sum_{s=\mathrm{t}+1}^{\infty} C_s(1+r)^{-(s-t)} - \sum_{i=0}^{M-1} P_{t,t-i} GA_{t,t-i}.$$

The first term on the right side, net debt, can be calculated using government balance sheets. The second term, present value of collective government consumption, can estimated using projections of government purchases of goods and services. The last term on the right side, net tax burden of current generations, can be determined using projections of annual aggregate taxes and transfers, population forecasts, micro-data surveys of tax payments and transfer receipts by age and cohort-specific mortality rates. Thus, the total present value tax burden facing future generations, the term on the left side, can be determined as a residual of the IBC.

To make net payments between generations comparable, per capita net payments, that is, generational accounts of different generations are calculated. The generational accounts of current generations can be estimated from the abovementioned data sources. In order to estimate the per capita net payments of future generations assumptions about the distribution of the payments have to be made. The usual assumption is that the net tax burden measured at the time generation is born grows at the rate of GDP growth:

$$GA_{t+i,t+i} = GA_{t,t+1}(1+r)(1+g)^{i-1}$$

$$GA_{t,t+i} = GA_{t,t+1}\left(\frac{1+g}{1+r}\right)^{i-1}.$$

Thus, provided that r>g, present value of payments is lower for generations farther in the future. Substituting this yields an estimate for the net tax burden facing the generation t+1:

$$GA_{t,t+1} = \frac{B_t + \sum_{s=\mathrm{t}+1}^{\infty} C_s(1+r)^{-(s-t)} - \sum_{i=0}^{M-1} P_{t,t-i} GA_{t,t-i}}{\sum_{i=1}^{\infty} P_{t+i}\left(\frac{1+g}{1+r}\right)^{i-1}}.$$

The sustainability of public finances can then be assessed by comparing the projected net payments to be made the by the newborn generation $GA_{t,t}$ to the calculated net payments to be made by future-born generation $GA_{t,t+1}$. This can be done for example by calculating the indicator (generational balance):

$$GB = \frac{GA_{t,t+1}}{GA_{t,t}}.$$

If the indicator value is greater than one, there exists an inter-generational sustainability problem and too much tax burden is shifted to future generations. If the value is less than one, policies are sustainable but too much tax burden is borne by the living generations and inter-generational equity would be better served by cutting taxes and issuing more debt. If the value is less than one, policies are sustainable and there is no inter-generational equity problem according to generational accounting.

The table below shows an example of generational accounts (Gokhale 2008). It can be seen from the table that per capita net payments faced by generation born in 2005 (333 200$) is considerably larger than per capita net payments faced by the generation born 2004 (104 300$). Thus, according to generational accounts approach US faces a sizable sustainability problem.

TABLE 3. GENERATIONAL ACCOUNTS FOR THE UNITED STATES (THOUSANDS OF CONSTANT 2004 DOLLARS).

Year of birth	Age in 2004	Male	Female
2005 (future-born)	−1	333.2	26.0
2004 (newborn)	0	104.3	8.1
1989	15	185.7	42.0
1974	30	201.3	30.2
1959	45	67.8	− 54.1
1944	60	− 162.6	− 189.4
1929	75	− 171.1	− 184.1
1914	90	− 65.0	− 69.2

Source: Gokhale (2008).

In our opinion, generational accounting approach to fiscal sustainability seems to be interesting and fruitful way to analyse sustainability. The method of comparing present generations' net tax burden to those of future generations sheds light to questions of intergenerational equity and fairness as well. However, some criticism towards the methodology of generational accounting has been made since its birth in early 1990s.

First, the assessments of generational accounting do not take into account the wide margins of uncertainty in the predictions and possible interactions between variables such as tax rates, interest rates, growth rates and predicted revenues and expenditures. Uncertainty part of the critique can be answered by conducting robust sensitivity analyses by varying central parameters in the calculations and estimating confidence intervals to the derived indicators instead of point estimates. Second, the usual way of allocating the total residual tax burden to future generations by assuming that generational payments grow at the rate of economic growth seems like quite narrow an approach. This problem can be solved by using other assumptions. Furthermore, alternative fiscal and generational imbalance measures developed by Gokhale and Smetters (2003) that do not involve assumptions about hypothetical future policies can be used. Third, because of the forward-looking nature of generational accounts, the accounts of currently living generations cannot be compared directly. Finally, generational accounting does not explicitly distribute some government expenditures such as purchases of public goods and services and government insurance provision to different age groups – they are either measured as a part of collective government consumption or not measured at all. This may distort the results.

11. STRENGTHS AND WEAKNESSES OF THE APPROACHES

In this study, six distinct approaches to assess sustainability have been described and analysed. They are: summary indicators of sustainability, econometric tests, Value-at-Risk framework, fiscal limits and fiscal space, general equilibrium models and generational accounting. Strengths and weaknesses of the approaches were already touched upon when they were presented in earlier chapters. This chapter summarizes the strengths and weaknesses and discusses them.

The table below summarises the strengths and weaknesses of the different approaches. Discussion follows after the table.

TABLE 4. SUMMARY OF STRENGTHS AND WEAKNESSES OF THE SIX APPROACHES.

Approach	Strengths	Weaknesses	Key references
Summary indicators	- simple to use - good first approximation - can be used with different modelling frameworks - easy to communicate - results between studies easy to compare	- require inputs from other models - do not explicitly account for uncertainty - do not explicitly account for interactions between variables	Buiter et al. (1985), Blanchard et al. (1990)
Econometric tests	- derived directly from theory - useful in study of past policies	- mostly retrospective; hard to conduct prospective analysis - no quantitative measure of sustainability (answer either accept or reject)	Hamilton and Flavin (1986), Bohn (1998; 2005)
Value-at-Risk approach	- explicitly accounts for interactions and uncertainty - public sector balance sheet is analysed as a whole - can be used with different modelling frameworks	- a lot of data needed (public sector balance sheet etc.) - large effort to build the model needed - long-run analysis hard	Barnhill and Kopits (2003)
Fiscal limits and fiscal space	- different perspective - explicitly accounts for interactions and uncertainty - easy to communicate	- very model-dependent (fiscal limits in Bi 2010) - a broad sample of data needed (fiscal space in Ostry et al. 2010)	Bi (2010), Cochrane (2010), Leeper and Walker (2011a), Ostry et al. (2010)
General equilibrium models	- explicitly accounts for interactions - structurally detailed and accurate description of the economy - country-specific features can be modelled	- very large effort to build a model - a lot of parameter values need to be calibrated - predictive accuracy of the model not guaranteed	van Ewijk et al. (2006), Andersen and Pedersen (2006)
Generational accounting	- different perspective - inter-generational equity also considered	- do not explicitly account for interactions or uncertainty - hard to allocate benefits of expenditures accurately to age groups	Auerbach et al. (1991), Gokhale and Smetters (2003)

Summary indicators are probably the most common method to analyse sustainability in practice. They have been applied for example in European Commission (2009), Congressional Budget Office (2010), Parliamentary Budget Officer (2010), Krejdl (2006), Ministry of Finance (2010) and International Monetary Fund (2003). A key strength in these indicators is that they are easy to calculate and they require relatively few inputs. Calculation of the primary gap requires only data of current debt ratio, long-run average interest rate and growth rate. For tax and financing gaps projections of future primary balances are also required. These indicators can serve as first approximations in sustainability assessments. Furthermore, they can be communicated relatively easily to policy makers.

Once summary indicators are employed to produce more accurate assessments of sustainability, simple steady state assumptions need to be relaxed. Projections of primary balances, interest rates and growth rates for each year in the future are needed. To make these predictions, some predictive tools have to be used. Thus, the assessment of sustainability by these indicators comes to encompass the whole of the predictive framework. This is a strength because these indicators can be used with different modelling frameworks. On the other hand, this is a weakness because the approach is not self-sufficient in that it requires inputs from other models. Another notable weakness of summary indicators is that they do not explicitly account for uncertainty in the input variables or for interactions between them.

Econometric tests of sustainability are primarily useful for testing whether various sustainability criteria hold in historical data. They are theoretically well grounded in that they are based on explicit derivations from various economic and econometric assumptions. It is not easy to apply these tests to study the sustainability of current and future policies because estimation of inputs (projections of future debt levels and primary balances) would be needed to be done in a credibly separate process from testing; otherwise, the researcher can build any interrelations to data to satisfy any sustainability condition. Furthermore, these econometric tests either reject or accept sustainability of policies with some confidence level and thus do not provide a quantitative measure of the degree of sustainability. The tests do not directly give goals for policy adjustments in the case of rejection of sustainability.

Value-at-Risk approach by Barnhill and Kopits (2003) attempts to explicitly estimate the distribution of public sector net value (net worth + present value of all future cash flows). It takes into account the evolution of the whole public sector balance sheet and contingent government liabilities. This is good because changes in values of government assets and liabilities can affect solvency and sustainability considerations significantly. The approach explicitly models uncertainty in future asset values and cash flows. Also, interactions between variables like interest rates and output can be modelled by using historical correlations. Another good side of the approach is that it can be used with many modelling frameworks. A researcher is not limited to the specification employed by Barnhill and Kopits (2003) to study the economy of Ecuador but can freely choose a set of risk variables and the stochastic processes that describe their evolution.

A weakness of the Value-at-Risk approach is that long-run analysis of sustainability is not easy because the evolution of the public sector balance sheet has to be projected into the far future. That is, evolution of asset and liability values 50 years into the future has to be described stochastically. In addition, the approach requires a lot of data about the public sector: current balance sheets are a minimum requirement. Finally, one weakness of this approach is that quite large modelling effort is needed to construct a satisfactory framework for stochastic simulations.

Estimating a debt ceiling is the crux of the analysis of sustainability with fiscal limits and fiscal space. Bi (2010) estimates a debt ceiling by using structural models in which the Laffer curve gives rise to maximum primary budget surplus in each period. Ostry et al. (2010) estimates debt ceilings using the fiscal reaction function which gives the historical response of primary balance to changes in debt level. These approaches are valuable because they look sustainability from a bit different angle: instead of estimating the current public sector net value they attempt to estimate an upper limit for debt. Both Bi (2010) and Ostry et al. (2010) model the feedback interaction between the debt level and the interest rate. In addition, both approaches explicitly allow for uncertainty in the estimates. The weakness of the approach by Bi is that it relies on a specific model which is quite theoretical and simplistic. Thus, the estimated debt ceilings cannot be regarded as very reliable. The weakness of the approach by Ostry et al. is that it requires data of the response of primary surplus to wide range debt levels which is not available for most countries. Thus, data from other countries fiscal responses are needed to supplement the estimation of fiscal reaction functions.

Using general equilibrium models to analyse public finance sustainability is perhaps theoretically the most accurate way to prepare sustainability estimates, but not necessarily in practice. GE models are good because they model the economy and its structure in detail. This means country-specific features like tax and pension systems can be described realistically as a part of the model. Also, interactions between the economic variables are modelled in a realistic and comprehensive manner. Weakness of this approach is that a very large effort is needed to build such a detailed model of the economy that is typical for applied general equilibrium models. Secondly, a lot of parameter values have to be estimated which increases the possibility of error. Thirdly, a weakness of the GE models is that they are in a sense "black box" –models for the average policy maker because of their complexity. This makes it hard to communicate and built trust in the results. Finally, despite structurally accurate description of the economy, the predictive accuracy of these models cannot be taken for granted.

Generational accounting analyses sustainability from yet another perspective. It compares the net tax burden of current generations to that of future generations. If the tax burden of future generations is much heavier than that of current generations, policies are considered unsustainable. This approach is good also because it gives information about inter-generational equity in addition to policy sustainability. Weakness of generational accounting is that uncertainty and interactions between model variables are not explicitly accounted for. In addition, the estimation of benefits that accrue to different age groups from public policies is not easy and may cause errors in results.

12. CONCLUSIONS

This chapter summarizes the central findings of the thesis. In addition, some general points about conducting sustainability assessments are made which are relevant regardless of the specific approach chosen.

In this study, several alternative theoretical criteria for public finance sustainability have been examined. Most important of these are government's inter-temporal budget constraint, Bohn's model-based sustainability criterion and the boundedness of debt-to-GDP ratio. The inter-temporal budget constraint is clearly the most commonly used of these criteria. There exists some well-founded criticism against the use of the IBC as de-facto condition for public finance sustainability. It seems to us that Bohn's model-based sustainability criterion, which is a generalization of the traditional IBC, is better as a general theoretical condition for sustainability than the IBC.

Six approaches to assess sustainability of public finances have been analysed in this thesis. These approaches are: summary indicators of sustainability, econometric tests, Value-at-Risk framework, fiscal limits and fiscal space, general equilibrium models and generational accounting. These approaches and their application in sustainability assessments have been described. Furthermore, the strengths and weaknesses of these approaches from the viewpoint of practical policy analysis have been analysed.

It seems that summary indicators of sustainability based on steady state analysis can be used as first approximations in sustainability assessments. Also, they can be used to effectively communicate results to policy makers. In our opinion, a more detailed and broad modelling framework is preferable for detailed assessments of sustainability. Such approaches are for example general equilibrium models and the VaR approach. Strength of general equilibrium models lies in that they can be tailored for country-specific tax systems, pension schemes and other policies and thus can be expected provide better estimates of long-run sustainability than less detailed models. The VaR approach, on the other hand, can be useful in situations where there are significant uncertainties and variations in government cash flows and asset and liability values. Such is the case, for example, in many emerging market economies. Estimation of debt ceilings (fiscal limits and fiscal space) and generational accounting can give a different viewpoint on sustainability: debt ceilings by considering the amount of fiscal manoeuvring space government has and generational accounting by considering inter-generational equity.

In general, no single approach can be singled out as a preferable to others in every situation. Each of the approaches has its uses. The approaches are complementary rather than rivalrous, each looking sustainability from somewhat different angle. From purely theoretical perspective, general equilibrium models and the Value-at-Risk approach appear most attractive. However, it is worth to bear in mind that theoretical complexity and finesse are no guarantees for ability to forecast accurately and that sustainability estimates crucially depend on accurate projections.

The approaches analysed in this thesis do not explicitly consider seigniorage or money printing by central bank as a means to combat rising level of debt. However, it is not ruled out: the approaches just assume that seigniorage revenue is zero. This means that some or possibly the entire sustainability gap observed can be covered by large-scale money printing by central bank. This option is of course available only for countries which have an independent central bank. There have been some work to this direction in the area of research on monetary and fiscal policy interactions: for instance, Davig et al. (2011) and Leeper and Walker (2011b). However, it would be interesting to see more models and sustainability assessments which explicitly take this option and its costs (high inflation) into consideration.

When making sustainability assessments, it is very important to take into account the various sources of uncertainties in the calculations. Any projections of future cash flows or other variables such as those pertaining to population forecasts should be accompanied with a measure of the degree of uncertainty in the projections: for example, the variance of the future forecasts around the estimated mean. When the uncertainties from different sources are identified and quantified, it makes the assessment of uncertainty in the end results, such as sustainability gaps, much easier. Any sustainability assessment should not only produce a point estimate of the needed adjustment to re-establish sustainability but also give a measure of the variance around the point estimate: confidence interval, standard deviation or ideally a probability distribution. Uncertainty assessments can be done for example by conducting sensitivity analyses. However, if possible, they should not be ad-hoc in nature but be based on the probability distributions of inputs and the structure of the model used to produce the sustainability estimates.

Results of sustainability assessments can be sensitive to the assumptions about the initial state of the economy. Thus, if economy is assumed to be in recession, a great part of the sustainability problem can be solved just by waiting the economy to get out of the slump. If, on the other hand, the assumption is that the economy resides in a midway of the business cycle or is booming, none of the sustainability problem can be solved by waiting the things to normalise. Corrections for the cyclical factor (e.g. using cyclically adjusted inputs) have to assume some initial cyclical state for the economy. This uncertainty about the initial cycle point, i.e. the near-term economic developments, feeds right into uncertainty of the sustainability estimates. This factor of uncertainty can be accounted for like any other: by using sensitivity tests or other methods derive a distribution of values of the sustainability indicator. Another option is try to separate the effects of long-term developments from the effect of the initial position in a manner similar to European Commission (2009) which decomposes the sustainability gap to two parts: a part caused by initial budgetary position and a part caused by long-term demographic developments.

One important aspect which may affect public finance sustainability in a country is the net foreign asset position of the country. If the country runs persistent current account deficits thus accumulating foreign liabilities, this may signal an external imbalance which may lead to large-scale private debt deleveraging, banking crises and to eventually to a soverign debt crisis. As noted by Chalk and Hemming (2000), there is a concept of current account sustainability or external sustainability that is closely parallel to the concept of public finance sustainability that can be utilised to examine the sustainability of the external debt position of an economy. Analysis of current account sustainability and private sector debt sustainability is beyond the scope of this thesis. However, it would be very interesting to see research explicitly linking current account sustainability and public finance sustainability.

Finally, it is worth noting that regardless of the method used to analyse sustainability, it is paramount to take a comprehensive view of the public sector finances. That is, construct an estimate of the balance sheet, contingent liabilities and evolution of asset prices and volumes in addition to the usual projections of future government revenues and expenditures. Results of the sustainability assessment can be seriously compromised if only budget balances and gross debt is studied. In general, this calls for a more detailed, transparent and explicit treatment of government assets and liabilities.

This study was not a fully comprehensive literature review on sustainability assessments. The bulk of literature that relates to public finance sustainability is large and growing. It seems to us that during the last few years a lot of new papers have been published in response to long-run sustainability problems in developed countries due to population ageing and the on-going sovereign debt crisis in Europe. Therefore, we expect that the analysis of public finance and public debt sustainability continues to develop significantly in the coming years.

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APPENDIX 1: TIME SERIES ANALYSIS

Some basic definitions related to stochastic processes and time series analysis.

STATIONARY STOCHASTIC PROCESS

A stochastic process $\{X_t\}_{t=1}^{\infty}$ is stationary if its joint distribution function for times $t_1, t_2, \ldots, t_n$ $F(X_{t_1}, X_{t_2}, .., X_{t_n})$ doesn't change when time is shifted, i.e. :

$$F(X_{t_1}, X_{t_2}, .., X_{t_n}) = F(X_{t_1+\tau}, X_{t_2+\tau}, .., X_{t_n+\tau})$$

for all n, τ and $t_1, t_2, \ldots, t_n$.

Covariance stationarity is a commonly used form of stationarity in which only mean and autocovariances are required to be time invariant:

$$E[X_t] = \mu$$

$$Cov(X_t, X_{t+\tau}) = \lambda_\tau.$$

DIFFERENCE-STATIONARY STOCHASTIC PROCESS

A stochastic process $\{X_t\}_{t=1}^{\infty}$ is said to be difference-stationary if it is not stationary but its first difference $X_t - X_{t-1}$ is stationary.

UNIT ROOT

A stochastic process that has a unit root is not stationary. Unit root means that 1 is a root of the process's characteristic equation.

COINTEGRATION

A time series is said to be integrated of order d, denoted I(d), if it has a stationary, invertible, non-deterministic ARMA representation after differencing d times. Differencing means forming a new series from the original by taking the first difference $X_t - X_{t-1}$. When differencing d times, this is operation is repeated recursively d times.

Take two time series x_t and y_t which are both I(d). Usually any linear combination of x_t and y_t will be I(d). However, if there exists a coefficient β such that the linear combination $z_t = x_t + \beta y_t$ is I(d-b), then the series x_t and y_t are said to be cointegrated of order (d,b) with β as a cointegration coefficient.

If two time series are co-integrated they share a common stochastic drift and thus cannot evolve into opposite directions for very long.