An Analysis of the Fish Pool Market in the Context of Seasonality and Stochastic Convenience Yield

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Abstract

On the basis of a popular two-factor approach applied in commodity market, we develop a model featuring seasonality and study future contracts written on fresh farmed salmon, which have been actively traded at the Fish Pool Market in Norway since 2006. The model is estimated by means of Kalman filtering, using a rich data set of contracts with different maturities traded at Fish Pool between 01/01/2010 and 24/04/2014. The results are then discussed in the context of other

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commodity markets, specifically live cattle which acts as a substitute. We show that the seasonally adjusted model proposed in this paper can describe the behavior of salmon price very well. More importantly we show that seasonality persists in the salmon futures market. This is highly important in pricing of contingent claims, designing hedging strategies and making real investment decisions in marine assets.

Keywords: Aquaculture, Commodities, Futures, Risk Management, Seasonality

JEL Subject Classification: G13, Q20, Q22

Introduction

Fish Pool, located in Bergen (Norway), is a marketplace established for trading derivatives on fish and seafood, where futures and options on fresh farmed salmon have been offered as pioneering products since 2006. During 2014, contract values traded at this market have reached 4.3 billion NOK, equivalent to 97,000 tons. The average weekly trading volume is 1,775 tons over 2010 - 2014. Currently, Oslo Børs ASA owns 94.3% of the shares in Fish Pool ASA and Nasdaq offers clearing of salmon derivatives traded there. In this paper, we analyze futures contracts on fresh farmed salmon traded on the Fish Pool exchange, in the context of seasonality and stochastic convenience yield. Unlike Ewald, Nawar, Ouyang, and Siu (2016) who connect the Schwartz (1997) multi-factor model to the classical literature in aquaculture in micro economic terms, the model proposed in this paper is innovative in the way of extending the original Schwartz (1997) two-factor model by incorporating the seasonal behavior of salmon prices, and is the first model capable of testing for seasonality through salmon futures prices.[1]

Nowadays, about 70% of the world's salmon production is farmed and most of the cultured salmon comes from Norway, Chile, Scotland and Canada (Asche & Bjørndal, 2011). According to Food and Agricultural Organization (2015), "aquaculture is understood to mean the farming of aquatic organisms including fish, molluscs, crustaceans and aquatic plants. Farming implies some form of intervention in the rearing process to enhance production, such as regular stocking, feeding, protection from predators, etc. Farming also implies individual or corporate ownership of the stock being cultivated." In the aquaculture industry, regardless of different species of fish and different farming technologies, the general process is similar: the farmer releases juvenile fish (recruits) into pens or ponds, feeds them until they reach a certain level, and then harvests for sale; after that, pens or ponds become available for a new generation and a new rotation may begin. These features make aquaculture share lots of common characteristics with agriculture.

Similar to many agricultural commodities, salmon prices show seasonal pattern.[2] As discussed in Bjørndal, Knapp, and Lem (2003), Asche and Bjørndal (2011) and Asche, Misund, and Oglend (2016a), the seasonal behaviour of salmon spot prices is due to several factors. Generally speaking, on one hand, the availability and production of different weight classes of salmon for market follows a seasonal pattern because of salmon growth being affected by the water temperature; on the other hand, major social events or holidays and changes in salmon's quality can cause seasonal fluctuation in salmon consumption. Considering the front-month futures price as a proxy of spot price, Figure 1 plots the average price for each month over the years 2007-2013.[3] We can observe that the price peaks in May and hits bottom in October and a lower peak occurs in July. It is further worthwhile to find out the effects of seasonality on futures prices. We obtain the pattern of futures contracts by grouping data into expiration months,[4] see Figure B1 in Appendix B. Although patterns are similar, futures prices (Figure B1) seasonally fluctuate within a more narrow range, compared to the spot/front-month-futures prices (Figure 1).

[Figure 1 about here.]

The identification of the dynamic process underlying a commodity price is vital in valuing financial derivatives as well as designing hedging and investment strategies, which applies to marine assets. As shown in the groundbreaking work of Schwartz (1997), futures prices for any expiry and more specifically knowledge about the shape of the whole futures curve are essential to make optimal decisions. Models which focus on the spot price or closest to maturity futures only are generally incapable to produce realistic terms structures which are essential. How to model seasonality of commodity prices has been addressed by several authors. Inspired by Schwartz and Smith (2000), Sørensen (2002) include the seasonality by modelling the dynamics of the spot price as the sum of a deterministic seasonal component, a non-stationary state-variable, and a stationary state-variable. West (2012) adopted a multi-factor seasonal Nelson-Siegel model to obtain estimates for seasonal commodity prices. Mirantes, Población, and Serna (2013) mainly focus on the convenience yield and use the four-factor model proposed by Mirantes, Población, and Serna (2012) to capture mean-reversion and stochastic seasonality of convenience yield. In our model, the seasonality factor is embedded in the drift term of convenience yield as a function of calendar time. Convenience yield can be understood as the benefit or premium associated with holding an underlying product or physical good, rather than the contract or derivative product. Several papers have indicated that the convenience yield is economically significant, e.g., Brennan (1958), Deaton and Laroque

(1992), Routledge, Seppi, and Spatt (2000), Casassus and Collin-Dufresne (2005) and Wei and Zhu (2006). They point out that the convenience yield arises endogenously as a result of the interaction among supply, demand, and storage decisions. According to the theory of storage, there is a negative relationship between supply/inventories and convenience yields, see Brennan (1958). Fama and French (1987) find reliable seasonal elements in the basis for most agricultural and animal products.[5] Asche, Oglend, and Zhang (2015) also demonstrate that the convenience yield of salmon depends on expected growth which is highly seasonal. These previous studies provide an economic rationale for allowing the drift term of the convenience yield to capture the seasonality as in our model.

Other authors have focused on the volatility of fish and salmon prices in particular. In a very general context Dahl and Oglend (2014) looked at price volatility of fish, across species and regions and also differentiating between farmed, wild catch and frozen fish. Solibakke (2012) and Bloznelis (2016) discuss price volatility in the context of the fishpool market. Seasonality does not feature in their models; nevertheless an interesting avenue for expanding our research would be the identification of seasonal patterns in the volatility in addition to seasonal patterns in the convenience yield. Asche, Misund, and Oglend (2016b) investigate in how far salmon futures can provide unbiased estimators of spot prices. Their conclusion is that the potential of salmon futures to guide price discovery is limited. However, their model does not account for seasonal stochastic convenience yield and it would be interesting to further discuss this issue in the context of our new model. Misund and

Asche (2016) study the hedging effectiveness of salmon futures, concluding that the best hedging results are obtained through a simple one-to-one hedge. Issues relevant to hedging and price formation in the salmon futures market are also discussed in Ankamah-Yeboah, Nielsen, and Nielsen (2017). Again, it would be interesting to explore the issue of hedging in the context of our model.

The large body of literature emerging in the context of the salmon futures market across the disciplines of fisheries, aquaculture, agricultural economics, finance and risk management is evidence for its importance, both from a scientific and institutional point of view. Fishpool provides a vital function to efficiently price and manage marine assets and to guide investment into the marine environment. A good understanding of this market is necessary to manage aquaculture economically efficient. So far, fishpool has been a success story which is strongly tied to the booming aquaculture industry. Its future success however is far from self-evident, in fact previous futures market on marine resources have mainly failed, compare Martínez-Garmendia and Anderson (1999).

The rest of the paper is structured as follows. In section 2, we provide a description of our model. In section 3, data and empirical study will be discussed. Following that, in section 4, we draw a comparison between the futures contracts written on live cattle and salmon. Our conclusions are summarized in the final section.

Models

In this section, we demonstrate a valuation model for contingent claims on commodity prices featuring seasonality and derive the corresponding pricing formula for futures price. Further, by transforming the valuation model into the state space form, our empirical model is presented.

Valuation Model

The valuation model is based on the Schwartz (1997) two-factor model, by adding a seasonality feature to the mean-level of the convenience yield (α). The spot price of the commodity (P) and the instantaneous convenience yield (δ) are assumed to follow the joint stochastic process:

$$
\begin{aligned}
dP(t) &= (\mu - \delta(t))P(t)dt + \sigma_1 P(t)dZ_1(t) && (1) \\
d\delta(t) &= \kappa(\alpha(t) - \delta(t))dt + \sigma_2 dZ_2(t), && (2)
\end{aligned}
$$

where

$$
\alpha(t) = \alpha_0 + \sum_{k=1}^{N} (\gamma_k \cos(2k\pi \cdot t) + \gamma_k^* \sin(2k\pi \cdot t)) \quad (3)
$$

and $Z_1(t)$ and $Z_2(t)$ are Brownian motions under the real world probability $\mathbb{P}$ and $dZ_1(t)dZ_2(t) = \rho dt$. The parameters α_0, γ_k and γ_k^* are constant while N determines the number of trigonometric coefficients. Fackler and Roberts (1999), Sørensen (2002), Richter and Sørensen (2002), Lin and Roberts (2006) use a similar trigonometric function as in (3) to describe seasonality.

The stochastic convenience yield described in (2) reflects the benefits received by agents who hold commodities or physical goods other than derivative contracts. It follows a mean-reverting Ornstein-Uhlenbeck process, where $\alpha(t)$ represents the mean reversion level and $\kappa > 0$ represents the mean reversion speed. The seasonality feature, embedded in the convenience yield process by a truncated Fourier series, can further influence the price dynamics. If $\rho > 0$, $P(t)$ is positively correlated with $\delta(t)$ which implicitly creates a mean reversion feature. More specifically, $P(t)$ is likely to be large when $\delta(t)$ is large and $\delta(t)$ may then exceed μ. In this case, the drift term in (1) is negative, pushing $P(t)$ downwards. The opposite happens if $P(t)$ is small, pushing $P(t)$ upwards.[6]

Under the pricing measure $\mathbb{Q}$ which takes the market price of convenience yield risk (λ) into account, the dynamics are in the form of

$$
\begin{aligned}
dP(t) &= (r - \delta(t))P(t)dt + \sigma_1 P(t) d\widetilde{Z}_1(t) && (4)\\
d\delta(t) &= [\kappa(\alpha(t) - \delta(t)) - \lambda]dt + \sigma_2 d\widetilde{Z}_2(t), && (5)
\end{aligned}
$$

where $\widetilde{Z}_1(t)$ and $\widetilde{Z}_2(t)$ are $\mathbb{Q}$-Brownian motions and $d\widetilde{Z}_1(t)d\widetilde{Z}_2(t) = \rho dt$. The mean-level of the convenience yield under $\mathbb{Q}$ can be defined as

$$\tilde{\alpha}(t) = \alpha(t) - \lambda/\kappa, \tag{6}$$

which leads to the dynamics

$$dP(t) = (r - \delta(t))P(t)dt + \sigma_1 P(t)d\widetilde{Z}_1(t) \tag{7}$$

$$d\delta(t) = \kappa(\tilde{\alpha}(t) - \delta(t))dt + \sigma_2 d\widetilde{Z}_2(t). \tag{8}$$

Equation (6) can also be expressed as

$$\tilde{\alpha}(t) = \bar{\alpha} + \sum_{k=1}^{N} \left(\gamma_k \cos(2k\pi \cdot t) + \gamma_k^* \sin(2k\pi \cdot t)\right), \tag{9}$$

where

$$\bar{\alpha} = \alpha_0 - \lambda/\kappa \tag{10}$$

Futures Price

Since the interest rate is constant in our model, we do not need to distinguish between futures and forward prices. Therefore, out statements on futures contracts also hold for forward contracts. Let the futures price at time t with given and fixed expiration date T be $F(P, \delta, t; T)$. Under the no-arbitrage condition, the futures price satisfies the partial differential equation

$$\frac{1}{2}\sigma_1^2 P^2 F_{PP} + \sigma_1\sigma_2\rho P F_{P\delta} + \frac{1}{2}\sigma_2^2 F_{\delta\delta} + (r - \delta)PF_P + (\kappa(\tilde{\alpha}(t) - \delta))F_\delta + F_t = 0, \tag{11}$$

subject to the terminal boundary condition $F(P, \delta, T; T) = P(T)$. Note, $\tilde{\alpha}(t)$ in (11) is the mean-level of convenience yield defined in (9). The solution is given as follows:

$$F(P,\delta,t;T) = \mathbb{E}_{\mathbb{Q}}\left(P(T)|\mathcal{F}_t\right). \tag{12}$$

$$= P(t)e^{A_1(t;T)+A_2(t;T)+B(t;T)\delta(t)} \tag{13}$$

with

$$\begin{aligned} A_1(t;T) &= \left(r-\tilde{\alpha}+\frac{1}{2}\frac{\sigma_2^2}{\kappa^2}-\frac{\sigma_1\sigma_2\rho}{\kappa}\right)(T-t)+\frac{1}{4}\sigma_2^2\frac{1-e^{-2\kappa(T-t)}}{\kappa^3} \\ &\quad+\left(\kappa\tilde{\alpha}+\sigma_1\sigma_2\rho-\frac{\sigma_2^2}{\kappa}\right)\frac{1-e^{-\kappa(T-t)}}{\kappa^2} \qquad (14) \\ A_2(t;T) &= \sum_{k=1}^{N}\gamma_k\left(\frac{\sin(2k\pi\cdot t)-\sin(2k\pi\cdot T)}{2k\pi}-\frac{\kappa e^{-\kappa(T-t)}\cos(2k\pi\cdot t)-\kappa\cos(2k\pi\cdot T)}{\kappa^2+(2k\pi)^2}\right. \\ &\quad\left.-\frac{2k\pi e^{-\kappa(T-t)}\sin(2k\pi\cdot t)-2k\pi\sin(2k\pi\cdot T)}{\kappa^2+(2k\pi)^2}\right) \\ &\quad+\sum_{k=1}^{N}\gamma_k^*\left(\frac{\cos(2k\pi\cdot T)-\cos(2k\pi\cdot t)}{2k\pi}-\frac{\kappa e^{-\kappa(T-t)}\sin(2k\pi\cdot t)-\kappa\sin(2k\pi\cdot T)}{\kappa^2+(2k\pi)^2}\right. \\ &\quad\left.+\frac{2k\pi e^{-\kappa(T-t)}\cos(2k\pi\cdot t)-2k\pi\cos(2k\pi\cdot T)}{\kappa^2+(2k\pi)^2}\right) \qquad (15) \\ B(t;T) &= -\frac{1-e^{-\kappa(T-t)}}{\kappa}, \qquad (16) \end{aligned}$$

where the symbol $\mathcal{F}_t$ denotes the information available at time t and $(T-t)$ is the time-to-maturity. In the absence of seasonality, we have $A_2(t;T)=0$, and the solution is the same as the solution of the classic Schwartz (1997) two-factor model.

Empirical Model

In our model, both the commodity price (P) and the convenience yield (δ) are assumed to be unobservable, and only the futures price (F) can be observed.[7]

The state space approach is a powerful way to deal with situations in which the state variables are not observable. Once the model has been cast in state space form, model parameters can be estimated by the Kalman filter. For details about state space modelling and the Kalman filter, we refer to Harvey (1990), Harvey, Koopman, and Shephard (2004), Commandeur and Koopman (2007). Let $\mathbf{y}_t$ denote an $(n \times 1)$ vector of futures prices observed at time t and $\mathbf{\Phi}_t$ denote a (2×1) vector of state variables, i.e., the log spot price (X) and the convenience yield (δ). The state space representation can be written as

$$\begin{aligned} \mathbf{y}_t &= \mathbf{d}_t + Z_t \mathbf{\Phi}_t + \boldsymbol{\epsilon}_t \qquad (17) \\ \mathbf{\Phi}_{t+1} &= \mathbf{c}_t + Q_t \mathbf{\Phi}_t + \boldsymbol{\eta}_t, \qquad (18) \end{aligned}$$

where (17) is the measurement equation with components

$$\mathbf{y}_t = \begin{pmatrix} \ln F(t;T_1) \\ \vdots \\ \ln F(t;T_n) \end{pmatrix}, \quad \mathbf{d}_t = \begin{pmatrix} A(t;T_1) \\ \vdots \\ A(t;T_n) \end{pmatrix}, \quad \mathbf{Z}_t = \begin{pmatrix} 1 & B(t;T_1) \\ \vdots & \vdots \\ 1 & B(t;T_n) \end{pmatrix} \qquad (19)$$

and $\boldsymbol{\epsilon}_t$ is a $(n \times 1)$ vector of serially uncorrelated disturbance with

$$\mathrm{E}(\boldsymbol{\epsilon}_t) = 0, \qquad \mathrm{Var}(\boldsymbol{\epsilon}_t) = H. \qquad (20)$$

Equation (18) is the transition equation with components

$$\mathbf{\Phi}_t = \begin{pmatrix} X(t) \\ \delta(t) \end{pmatrix} \tag{21}$$

$$\mathbf{c}_t = \begin{pmatrix} \left(\mu - \frac{1}{2}\sigma_1^2 - \alpha_0\right)\Delta t + \frac{1-e^{-\kappa\Delta t}}{\kappa}\left(\alpha_0 + L(t)\right) - \left(M(t+\Delta t) - M(t)\right) \\ \alpha_0\left(1 - e^{-\kappa\Delta t}\right) + \left(L(t+\Delta t) - e^{-\kappa\Delta t}L(t)\right) \end{pmatrix} \tag{22}$$

$$\mathbf{Q}_t = \begin{pmatrix} 1 & \frac{1}{\kappa}\left(e^{-\kappa\Delta t} - 1\right) \\ 0 & e^{-\kappa\Delta t} \end{pmatrix} \tag{23}$$

and $\boldsymbol{\eta}_t$ represents serially uncorrelated disturbances with

$$\mathrm{E}(\boldsymbol{\eta}_t) = 0, \qquad \mathrm{Var}(\boldsymbol{\eta}_t) = \begin{pmatrix} \sigma_X^2(\Delta t) & \sigma_{X\delta}(\Delta t) \\ \sigma_{X\delta}(\Delta t) & \sigma_\delta^2(\Delta t) \end{pmatrix} \tag{24}$$

where $\Delta t = t_{k+1} - t_k$ represents the time interval of discretization and T_i denotes the given and fixed maturity of the i-th closest-to-maturity futures contract. The functions $A(\cdot)$ and $B(\cdot)$ are defined in (14) - (16); while $L(\cdot)$ and $M(\cdot)$ are defined in (30) and (35) respectively in Appendix A. Moreover, the derivation of the joint distribution of $X(t)$ and $\delta(t)$ can be found in Appendix A.

Data and Empirical Results

In this section, we briefly describe the data set of salmon futures prices used and present our empirical results and conclusions.

Data

Our data set consists of 1126 daily observations of futures prices on Fish Pool ASA from 01/01/2010 to 24/04/2014. We use a similar notation as in Schwartz (1997) and denote with F1 the contract closest to maturity (with average maturity of 0.040 year) counting up to F29 which represents the contract farthest to maturity (with average maturity of 2.389 years). Table 1 describes the data features of sample contracts. Unlike Ewald, Ouyang, and Siu (2017) who use a different combination of contracts, i.e., short-term, medium-term, long-term and mixed-term to emphasize different parts of the forward curve, we do not consider the medium-term and long-term contracts individually in this paper. We would expect that on top of lower liquidity of these contracts, over the long time that it takes until these contracts mature, seasonal effects wash out and become blurred in a way, which also negatively effects the filtering process.[8] Therefore, taking factors such as liquidity and representativeness into consideration, two panels with 5 contracts each are considered in our empirical study.[9] More precisely, Panel A consists of F1, F3, F5, F7 and F9, having a relatively short and narrow range of maturities; Panel B contains F1, F7, F14, F20, and F25, having longer and a wider range of maturities. Descriptive statistics for selected contracts in both panels are given in Table 2. The *last trading day* of contract is chosen to represent the expiration date, for the reason that it is actually the final day that a contract can be traded or closed out at the market. In other words, contracts outstanding by the end of the *last trading day* must be settled in cash or by delivery of the underlying asset.[10] For each contract, its time-to-maturity fluctuates within a certain narrow range as time progress during the sample period.

[Table 1 about here.]

[Table 2 about here.]

Empirical Results

Once the model has been cast in the state space form as introduced in the previous Section , the Kalman filter can be applied to estimate parameters in the model. We compared estimates using different values for N in (3), i.e., the number of trigonometric terms describing seasonality in the model, and selected $N = 2$ based on the log-likelihood ratio test. This leads to

$$\alpha(t) = \alpha_0 + [\gamma_1 \cos(2\pi \cdot t) + \gamma_1^* \sin(2\pi \cdot t) + \gamma_2 \cos(4\pi \cdot t) + \gamma_2^* \sin(4\pi \cdot t)] \,. \quad (25)$$

By using sample data ranging from 01/01/2010 to 24/04/2014 and choosing the average rate of the 3-month Norwegian Treasury Bill as the risk-free rate r (1.81%), the estimates are obtained as shown in Table 3. Most importantly, significance of γ_1 and γ_1^* in both panels suggest that there is seasonality in the market. In each panel, parameters are all highly significant at 1% level, except γ_2 in Panel A and γ_2^* in Panel B; the correlation coefficient ρ is positive and large as expected; the expected return on the spot price μ, the mean-reversion speed κ and the market price of convenience yield risk λ are all positive and reasonable. Due to containing contracts with relatively short term, Panel A has lower expected return on spot commodity μ but higher mean-reversion speed κ, compared to Panel B. It is also worth to note that the volatility of convenience yield σ_2 decreases as the term of contracts increases while the volatility of spot price σ_1 is relatively stable. This implies that the convenience yield is more sensitive to changes in maturities. The estimates are generally good in both panels as indicated by Table 4. Particularly, F7 in Panel A and F20 in Panel B are nearly perfectly fitted by the model.

[Table 3 about here.]

[Table 4 about here.]

Figure 2 depicts the state variables, i.e., spot price (P) and convenience yield (δ), filtered by the model, from which we can observe a strong positive correlation not only between state variables but also between spot price and futures price. As we would expect, the ability of futures contracts to proxy spot prices becomes weaker when maturity increases. We cam also see a clear seasonal pattern for each variable, which is consistent with the pattern shown in Figure 1 and Figure B1. Moreover, as shown in Figure 3 and Figure 4, the spot prices filtered from Panel A and Panel B are almost the same; while the filtered convenience yields share similar pattern but have different bounds due to different selection of futures contracts. Figure 5 shows the term structures for each panel, where in each sub-figure, the left part displays the actual term structures and the right part displays the model generated term structures. Overall, the model makes a good prediction for each panel, namely the model generated forward curves match the actual forward curves and the filtered spot price is near the price of closest-to-maturity futures. It is obvious that term structures of Panel A and Panel B are different, for they consist of different futures contracts, but as mentioned before, the plots of filtered spot prices are nearly the same. Since Kalman filter based estimation is an iterative procedure, we also include the figure of parameter evolution in Appendix B. Figure B2 shows that the convergence is good in all cases.

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

[Figure 5 about here.]

Comparison between Cattle and Salmon

How do salmon futures compare to futures traded on other related commodities? Live cattle seems to reflect some of the properties of farmed salmon as a commodity and futures on live cattle are traded in high volume on the Chicago Mercantile Exchange. Based on data availability for both the Fish Pool market and the live-cattle futures market, we have chosen 6 cattle contracts covering almost the same period as for the salmon contracts, i.e., from 04/01/2010 to 24/04/2014. With regard to the risk-free rate, we use the average rate of the 3-month Norwegian Treasury Bill and the 3-month U.S. Treasury Bill during the sample period for the salmon and cattle contracts accordingly, i.e., 1.81% and 0.08%, based on the place where they are traded. Six salmon contracts S2, S4, S6, S8, S10 and S12 [11] as listed in Table 1 are chosen for they have similar maturities as the first six cattle contracts, where the first contract is referred to as C1 and the sixth contract as C6. The average maturities of these contracts are 0.124 years, 0.292 years, 0.460 years, 0.627 years, 0.795 years and 0.963 years respectively. The empirical results of our analysis are shown in Table 5. We observe that in general, cattle has higher expected returns on the spot commodity μ and mean-reversion speed κ, but lower volatilities of both spot price and convenience yield, compared to salmon. In addition, the market price of convenience yield risk in the case of cattle is notably higher during the sample period. Some parameters related to seasonality, as γ_2 and γ_2^* for the cattle and γ_1 and γ_1^* for the salmon, are not statistically significant. However others are, hence seasonality is presented in both markets.

[Table 5 about here.]

[Table 6 about here.]

Figure 6 shows the filtered state variables, i.e. the spot price and the instantaneous convenience yield, along with selected futures prices. We observe from Figure 6 that the convenience yields are notably different in cattle than in salmon. To have a better view of the results, we also plot the filtered spot prices and convenience yields separately in Figure 7 and Figure 8. Not surprisingly, the spot price and convenience yield obtained from the live cattle and salmon are quite different. The convenience yield for cattle fluctuates in a more narrow range compared to salmon, see Figure 8. This may be attributed to storage issues and costs reflecting that fresh salmon is a highly perishable good, more so than cattle. It may also point towards liquidity issues and the fact that salmon farming is still far less developed than cattle farming, which may affect supply. In this case, the benefits for holding salmon in storage in the short term and hence being able to provide liquidity are higher than for cattle. Looking at the term structures in Figure 9 as well as root-mean-square error (RMSE) and mean-absolute error (MAE) in Table 6, it appears that the model captures both the salmon and the cattle contracts well but slightly better for the cattle.

[Figure 6 about here.]

[Figure 7 about here.]

[Figure 8 about here.]

[Figure 9 about here.]

Conclusion

The accurate modelling of marine commodity price behavior is highly important in pricing contingent claims, designing dynamic hedging strategies, and making investment decisions into marine assets. In this paper we investigated the issue of seasonality in spot and futures prices for salmon at the Fish Pool market through a seasonally adjusted Schwartz (1997) model featuring a seasonal stochastic convenience yield. Specifically, we added the seasonality factor as a truncated Fourier series to the mean-level of the convenience yield (α_t) and derived a formula for the futures price. Our empirical analysis has been based on futures contracts of salmon with different maturities traded at Fish Pool between 01/01/2010 and 24/04/2014 facilitating Kalman filtering. Our results statistically show that their is indeed seasonality in the salmon futures market and spot market. This confirms previous results of other authors, which had been based on the analysis of the spot price or closest futures only. We showed that our model can describe the behavior of salmon futures prices well, providing a good fit to the whole term structure of contracts. We further compared our results on salmon futures with those obtained for the same model fitted to live cattle-futures, an agricultural commodity which functions as a substitute. We identified seasonality in live-cattle as well, however some of the estimated model parameters had been clearly distinguished from those for the salmon futures market, possibly in relation to higher liquidity for live-cattle, the fact that the live-cattle market is a more mature market and storage issues that take account of the freshly harvested salmon being a highly perishable product.

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Appendix A:

Derivation of the joint distribution

The derivation follows the idea proposed by Erb, Lüthi, and Otziger (2014). The joint dynamics of the commodity log-price $X_t = \ln(P_t)$ and the spot convenience yield δ_t can be expressed as [12]

$$dX_t = \left(\mu - \delta_t - \frac{1}{2}\sigma_1^2\right) dt + \sigma_1\sqrt{1-\rho^2}dZ_t^1 + \sigma_1 \rho dZ_t^2 \tag{26}$$

$$d\delta_t = \kappa(\alpha_t - \delta_t)dt + \sigma_2 dZ_t^2. \tag{27}$$

By using the substitution $\tilde{\delta}_t = e^{\kappa t}\delta_t$ and Itô's lemma, (27) can be solved as

$$\delta_t = e^{-\kappa t}\delta_0 + \kappa e^{-\kappa t}\int_0^t e^{\kappa u}\alpha_u du + \sigma_2 e^{-\kappa t}\int_0^t e^{\kappa u} dZ_u^2, \tag{28}$$

with

$$\kappa e^{-\kappa t}\int_0^t e^{\kappa u}\alpha_u du = \alpha_0(1 - e^{-\kappa t}) + L_t, \tag{29}$$

where

$$L_t = \sum_{k=1}^{N} \frac{\kappa}{\kappa^2 + (2k\pi)^2}\Big\{\gamma_k\left[\kappa\cos(2k\pi\cdot t) + 2k\pi\sin(2k\pi\cdot t) - \kappa e^{-\kappa t}\right] + \gamma_k^*\left[\kappa\sin(2k\pi\cdot t) - 2k\pi\cos(2k\pi\cdot t) + 2k\pi e^{-\kappa t}\right]\Big\}. \tag{30}$$

Plugging (28) into (26) gives

$$
\begin{aligned}
X_t &= X_0 + \int_0^t dX_u & (31)\\
&= X_0 + \left(\mu - \frac{1}{2}\sigma_1^2\right) t - \int_0^t \delta_u du + \int_0^t \sigma_1 \sqrt{1-\rho^2} dZ_u^1 + \int_0^t \sigma_1 \rho dZ_u^2, & (32)
\end{aligned}
$$

where

$$
\int_0^t \delta_u du = \int_0^t e^{-\kappa u} \delta_0 du + \int_0^t \left(\alpha_0 (1 - e^{-\kappa u}) + L_u\right) du + \int_0^t \sigma_2 e^{-\kappa u} \left(\int_0^u e^{\kappa s} dZ_s^2\right) du. \tag{33}
$$

With regards to the integral $\int_0^t \left(\alpha_0(1-e^{-\kappa u}) + L_u\right) du$, we have

$$
\int_0^t \left(\alpha_0 (1 - e^{-\kappa u}) + L_u\right) du = \alpha_0 \left(t - \frac{1 - e^{-\kappa t}}{\kappa}\right) + M_t, \tag{34}
$$

where

$$
\begin{aligned}
M_t = \sum_{k=1}^{N} \frac{\kappa}{\kappa^2 + (2k\pi)^2} \Bigg\{ & \gamma_k \left[\frac{\kappa \sin(2k\pi \cdot t)}{2k\pi} - \cos(2k\pi \cdot t) + e^{-\kappa t}\right] \\
& -\gamma_k^* \left[\frac{\kappa \cos(2k\pi \cdot t)}{2k\pi} + \sin(2k\pi \cdot t) - \frac{\kappa}{2k\pi} - \frac{2k\pi\,(1 - e^{-\kappa t})}{\kappa}\right] \Bigg\}. & (35)
\end{aligned}
$$

According to Fubini's theorem, the order of integration of $\int_0^t e^{-\kappa u} \left(\int_0^u e^{\kappa s} dZ_s^2\right) du$ can be interchanged as

$$
\begin{aligned}
\int_0^t \left(\int_0^u e^{-\kappa u} e^{\kappa s} dZ_s^2\right) du &= \int_0^t \left(\int_s^t e^{-\kappa u} e^{\kappa s} du\right) dZ_s^2 & (36)\\
&= \int_0^t \frac{1}{\kappa} \left(1 - e^{-\kappa(t-s)}\right) dZ_s^2. & (37)
\end{aligned}
$$

Plugging (34) and (37) into (33) and solving the integrals yields

$$\int_0^t \delta_u du = \frac{\delta_0}{\kappa}\left(1-e^{-\kappa t}\right) + \alpha_0\left(t - \frac{1-e^{-\kappa t}}{\kappa}\right) + M_t + \sigma_2 \int_0^t \frac{1}{\kappa}\left(1-e^{-\kappa(t-s)}\right) dZ_s^2. \tag{38}$$

Therefore, X_t can be further expressed as:

$$\begin{aligned} X_t &= X_0 + \left(\mu - \frac{1}{2}\sigma_1^2 - \alpha_0\right) t + (\alpha_0 - \delta_0)\frac{1-e^{-\kappa t}}{\kappa} - M_t \\ &\quad + \int_0^t \sigma_1\sqrt{1-\rho^2}dZ_u^1 + \int_0^t \left\{\sigma_1\rho + \frac{\sigma_2}{\kappa}\left(e^{-\kappa(t-u)} - 1\right)\right\} dZ_u^2. \end{aligned} \tag{39}$$

The log-price X_t and the convenience yield δ_t are jointly normal distributed with expectations

$$\mathrm{E}(X_t) = \mu_X = X_0 + \left(\mu - \frac{1}{2}\sigma_1^2 - \alpha_0\right) t + (\alpha_0 - \delta_0)\frac{1-e^{-\kappa t}}{\kappa} - M_t \tag{40}$$

$$\mathrm{E}(\delta_t) = \mu_\delta = e^{-\kappa t}\delta_0 + \alpha_0\left(1-e^{-\kappa t}\right) + L_t. \tag{41}$$

and variances can be obtained by using expectation rules for Itô integrals and the Itô isometry.

$$\mathrm{Var}(X_t) = \sigma_X^2 = \frac{\sigma_2^2}{\kappa^2}\left\{\frac{1}{2\kappa}\left(1-e^{-2\kappa t}\right) - \frac{2}{\kappa}\left(1-e^{-\kappa t}\right) + t\right\} + 2\frac{\sigma_1\sigma_2\rho}{\kappa}\left(\frac{1-e^{-\kappa t}}{\kappa} - t\right) + \sigma_1^2 t \tag{42}$$

$$\mathrm{Var}(\delta_t) = \sigma_\delta^2 = \frac{\sigma_2^2}{2\kappa}\left(1-e^{-2\kappa t}\right) \tag{43}$$

$$\mathrm{Cov}(X_t, \delta_t) = \sigma_{X\delta} = \frac{1}{\kappa}\left[\left(\sigma_1\sigma_2\rho - \frac{\sigma_2^2}{\kappa}\right)\left(1-e^{-\kappa t}\right) + \frac{\sigma_2^2}{2\kappa}\left(1-e^{-2\kappa t}\right)\right]. \tag{44}$$

The mean-parameters given in (40) and (41) refer to the $\mathbb{P}$-dynamics. To obtain the parameters under $\mathbb{Q}$ we can simply replace μ by r and α_0 by $\bar{\alpha}$

defined in (10).

Appendix B: Additional Figures

Figure B1 plots the pattern of futures prices by grouping contracts into expiration months during the sample period. We can observe a similar pattern as shown in Figure 1 but inclusion of longer term futures reduces the seasonality effect. Figure B2 and Figure B3 shows the parameter evolution, due to space limits, not all parameters are included. We can observe that the convergence of parameter is generally good.

[Figure B1 about here.]

[Figure B2 about here.]

[Figure B3 about here.]

Notes

[1]Asche et al. (2015) detect seasonality in salmon spot prices, but the analysis there is not informed by the rich set of futures prices.

[2]Seasonality of many agricultural commodities prices can be naturally caused by the market supply, e.g., harvesting pattern, and demand, e.g., consumer preferences. See Brennan (1958), Fama and French (1987), Milonas (1991), Sørensen (2002) and Richter and Sørensen (2002).

[3]We first transform the observed daily prices into monthly data, then standardize the data by the annual mean value, and finally take the average of each month over the time period.

[4]Unlike plotting time-series data of the spot/front-month-futures prices, the seasonal pattern of futures is investigated via term structure.

[5]As mentioned in Fama and French (1987), under the theory of storage, inventory seasonals generate seasonals in the marginal convenience yield and in the basis.

[6]Schwartz (1997) illustrates that in an equilibrium setting, supply will increase when prices are relatively high, since higher cost producers of the commodity will enter the market putting a downward pressure on prices and vice versa. This is known as the mean reversion in commodity prices.

[7]As indicated by Schwartz (1997), one major difficulty in the implementation of commodity price models arises from the indirectly observable state variables. In most cases, the spot price is quite uncertain and published at irregular intervals, and the instantaneous convenience yield is hardly observable at all. The futures contracts traded on exchanges are more attainable.

[8]Seasonality becomes less prominent the larger the time-to-maturity, see Figure B1. Why this is the case is indeed an interesting research question. It might be linked to the so called 'Samuelson effect', which indicates that the volatility of futures prices declines with the maturity (Samuelson, 1965).

[9]Comparison can be made between different selections of futures contracts.

[10]No physical delivery and only financial settlement occurs at the Fish Pool.

[11]S- instead of F- is used as the prefix to represent the salmon futures contract in this section.

[12]We indicate time dependence via sub-indices here, e.g. $P_t = P(t)$, which is common in literature.

Table 1. Contracts Features, 01/01/2010-24/04/2014

Contract	Mean Price (Standard Deviation)	Mean Maturity (Standard Deviation)
F1	34.67 (7.58) *NOK*	0.040 (0.024) *year*
F3	33.76 (6.27)	0.208 (0.025)
F5	33.27 (5.51)	0.376 (0.025)
F7	32.79 (5.03)	0.543 (0.025)
F9	32.49 (4.50)	0.711 (0.026)
F12	32.24 (4.14)	0.963 (0.026)
F14	31.91 (3.81)	1.131 (0.026)
F16	31.56 (3.50)	1.298 (0.026)
F18	31.38 (3.15)	1.466 (0.026)
F20	31.27 (2.95)	1.634 (0.026)
F24	30.76 (2.73)	1.969 (0.026)
F25	30.56 (2.56)	2.053 (0.026)
F26	30.40 (2.43)	2.137 (0.026)
F28	30.20 (2.19)	2.305 (0.026)
F29	30.08 (2.07)	2.389 (0.026)

Note: We use a similar notation as in Schwartz (1997) and denote with F1 the contract closest to maturity counting up to F29 which represents the contract farthest to maturity.

Table 2. Descriptive Statistics in Panels, 01/01/2010-24/04/2014

	F1	F3	F5	F7	F9	F14	F20	F25
Mean	34.67	33.76	33.27	32.79	32.49	31.91	31.27	30.56
Median	36.00	34.75	33.75	32.90	32.75	33.00	32.03	30.35
Maximum	50.25	47.40	44.50	43.20	40.90	40.50	37.50	36.25
Minimum	20.53	23.30	23.85	24.10	23.80	24.15	25.70	25.45
Std. Dev.	7.58	6.27	5.51	5.03	4.50	3.81	2.95	2.56
Skewness	-0.02	-0.03	0.02	-0.03	-0.28	-0.25	-0.35	0.07
Kurtosis	1.88	1.87	2.08	2.14	1.96	2.36	2.13	2.61
Observations	1126	1126	1126	1126	1126	1126	1126	1126

Note: Descriptive statistics of daily futures prices in Panel A & B for the whole sample.

Table 3. Results of Whole Sample, Avg. Rate 1.81%, 01/01/2010-24/04/2014

Parameter	Panel A	Panel B
	F1, F3, F5, F7, F9	F1, F7, F14, F20, F25
μ	0.419 (0.150)***	0.528 (0.173)***
κ	2.885 (0.128)***	0.958 (0.046)***
α_0	0.801 (0.257)***	0.742 (0.217)***
σ_1	0.299 (0.019)***	0.236 (0.016)***
σ_2	1.228 (0.094)***	0.290 (0.023)***
ρ	0.855 (0.026)***	0.908 (0.020)***
λ	1.286 (0.620)**	0.676 (0.222)***
γ_1	0.332 (0.128)***	0.837 (0.121)***
γ_2	-0.215 (0.133)	-1.024 (0.180)***
γ_1^*	-0.586 (0.218)***	-0.425 (0.054)***
γ_2^*	-0.562 (0.179)***	0.143 (0.137)
ξ_1	0.009 (0.001)***	0.009 (0.001)***
ξ_2	0.057 (0.001)***	0.082 (0.002)***
ξ_3	0.049 (0.001)***	0.033 (0.001)***
ξ_4	0	0
ξ_5	0.055 (0.001)***	0.039 (0.001)***
Log-Likelihood	-11409.86	-12452.29

Note: Standard errors in parentheses. [***] Significant at 1% level; [**] Significant at 5% level; [*] Significant at 10% level. μ is the expected return on the spot commodity; κ is the speed of mean-reversion of the convenience yield; α_0 is the constant term in the mean level of the convenience yield; σ_1 is the volatility of the spot price; σ_2 is the volatility of the convenience yield; ρ is the correlation coefficient of spot price and convenience yield; λ is the market price of the convenience yield risk; γ_1, γ_2, γ_1^* and γ_2^* are the coefficients of trigonometric terms in the mean level of the convenience yield; ξ_1 - ξ_5 are the measurement errors.

Table 4. RMSE and MAE of Log Price: Salmon, 01/01/2010-24/04/2014

Panel A

	F1	F3	F5	F7	F9	ALL
RMSE	0.0046	0.0575	0.0496	0.0000	0.0552	0.0420
MAE	0.0020	0.0458	0.0375	0.0000	0.0400	0.0251

Panel B

	F1	F7	F14	F20	F25	ALL
RMSE	0.0050	0.0817	0.0336	0.0000	0.0410	0.0436
MAE	0.0021	0.0664	0.0248	0.0000	0.0294	0.0246

Note: The root-mean-square error (RMSE) and mean-absolute error (MAE) are used to evaluate the model fit.

Table 5. Estimation Results: Comparison between Live Cattle and Salmon, 01/01/2010-24/04/2014

Parameter	Live Cattle	Salmon
	C1, C2, C3, C4, C5, C6	S2, S4, S6, S8, S10, S12
μ	0.992 (0.106)***	0.447 (0.141)***
κ	1.988 (0.120)***	1.000 (0.131)***
α_0	1.058 (0.117)***	0.988 (0.291)***
σ_1	0.162 (0.006)***	0.258 (0.016)***
σ_2	0.347 (0.020)***	0.582 (0.054)***
ρ	0.784 (0.021)***	0.902 (0.015)***
λ	2.211 (0.251)***	1.037 (0.333)***
γ_1	0.025 (0.009)***	-0.007 (0.036)
γ_2	-0.010 (0.033)	-0.175 (0.083)**
γ_1^*	-0.126 (0.013)***	0.041 (0.036)
γ_2^*	0.028 (0.020)	0.244 (0.111)**
ξ_1	0.003 (0.000)***	0.078 (0.002)***
ξ_2	0.022 (0.000)***	0.003 (0.000)***
ξ_3	0.028 (0.001)***	0.043 (0.001)***
ξ_4	0.018 (0.000)***	0.044 (0.001)***
ξ_5	0.002 (0.000)***	0.001 (0.001)**
ξ_6	0.019 (0.000)***	0.053 (0.001)***
Log-Likelihood	-18153.09	-14243.34

Note: Standard errors in parentheses. [***] Significant at 1% level; [**] Significant at 5% level; [*] Significant at 10% level. μ is the expected return on the spot commodity; κ is the speed of mean-reversion of the convenience yield; α_0 is the constant term in the mean level of the convenience yield; σ_1 is the volatility of the spot price; σ_2 is the volatility of the convenience yield; ρ is the correlation coefficient of spot price and convenience yield; λ is the market price of the convenience yield risk; γ_1, γ_2, γ_1^* and γ_2^* are the coefficients of trigonometric terms in the mean level of the convenience yield; ξ_1 - ξ_6 are the measurement errors.

Table 6. RMSE and MAE of Log Price

Live Cattle							
	C1	C2	C3	C4	C5	C6	ALL
RMSE	0.0010	0.0232	0.0269	0.01789	0.0009	0.0182	0.0179
MAE	0.0007	0.0199	0.0214	0.01357	0.0006	0.0139	0.0117
Salmon							
	S2	S4	S6	S8	S10	S12	ALL
RMSE	0.0754	0.0010	0.0438	0.0448	0.0002	0.0529	0.0455
MAE	0.0589	0.0005	0.0332	0.0320	0.0001	0.0406	0.0276

Note: The root-mean-square error (RMSE) and mean-absolute error (MAE) are used to evaluate the model fit.

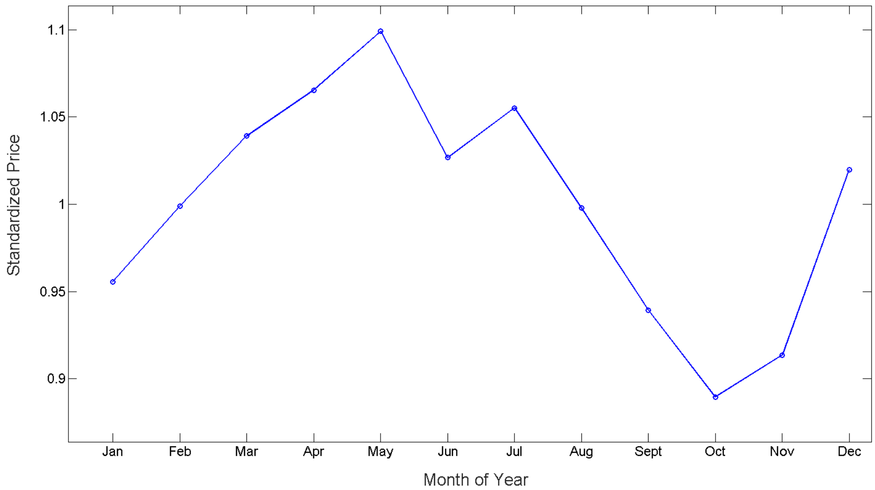
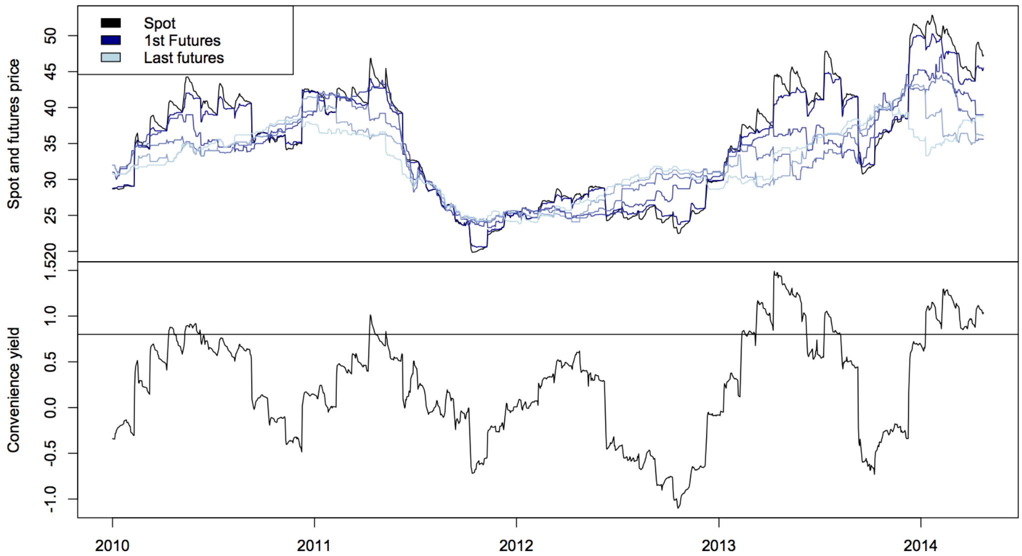


Figure 1. Price pattern: spot price

Note: The line is obtained by using the front-month futures price as a proxy of spot price, Jan 2007 - Dec 2013.

(a) Panel A



(b) Panel B

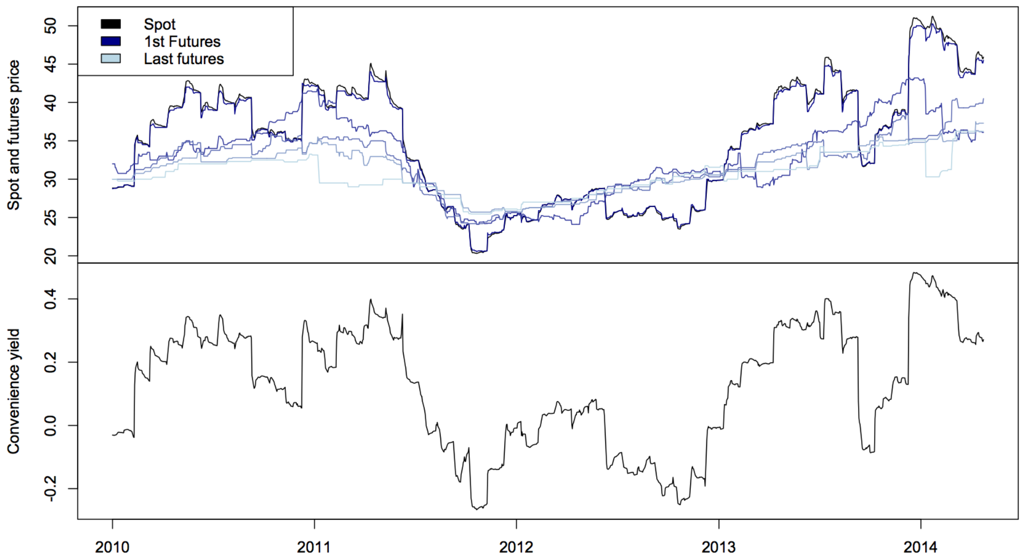


Figure 2. State variables: (a) Panel A; (b) Panel B; spot and futures prices on the top of convenience yield

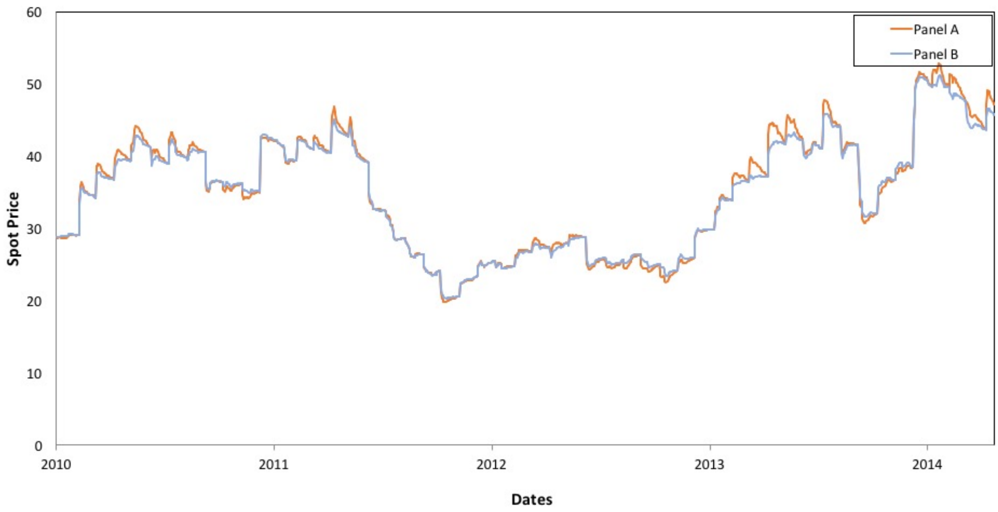
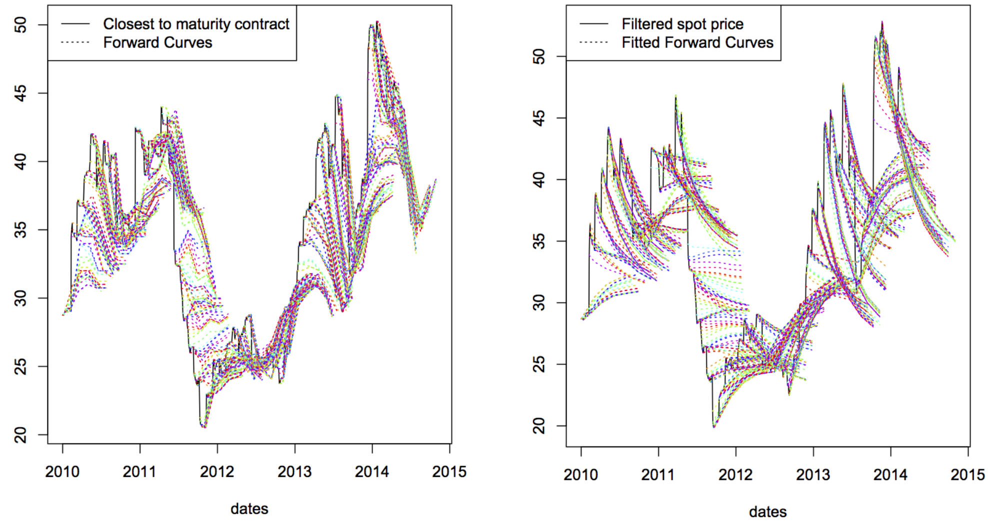


Figure 3. Filtered spot prices: Panel A and Panel B

Figure 4. Filtered convenience yields: Panel A and Panel B

(a) Panel A



(b) Panel B

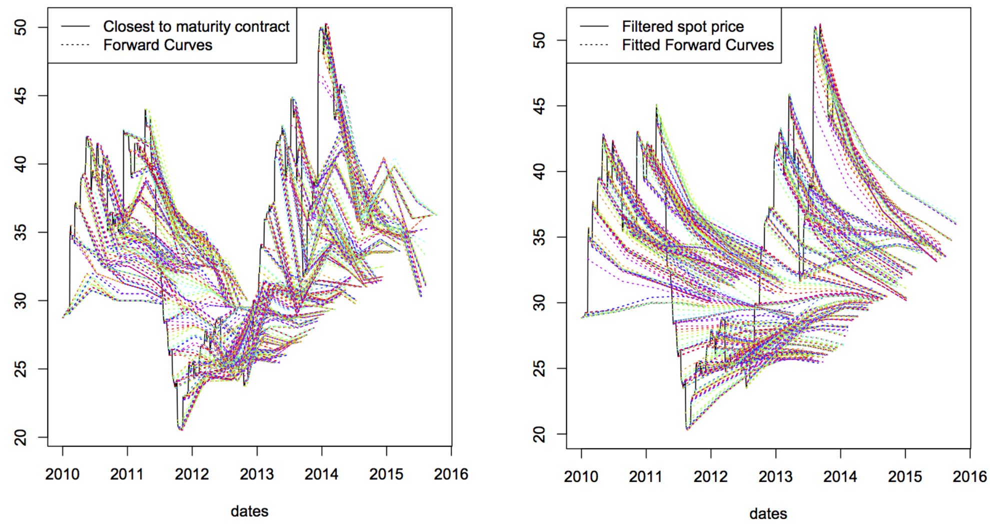
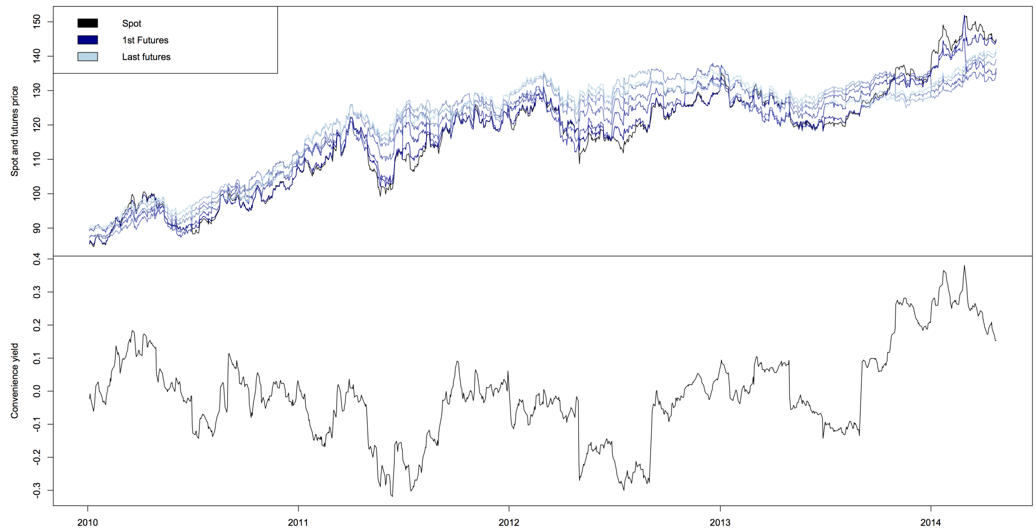


Figure 5. Term structures: (a) Panel A; (b) Panel B; actual forward curves on the left, model generated forward curves on the right

Note: Each colored curve is a static picture of futures prices (y-axis) against contract maturities (x-axis), which is analogous to a plot of the term structure of interest rates. On the left side of the figure, the solid line represents the price of the closest-to-maturity futures contract, i.e., $F1$ in this case; while the dashed line consists of the actual prices of other futures contracts with different maturities in this panel. On the right side of the figure, the solid line is the filtered spot price obtained through the estimation procedure; while the dashed line consists of the estimated futures prices given by the pricing formula.

(a) Live Cattle



(b) Salmon

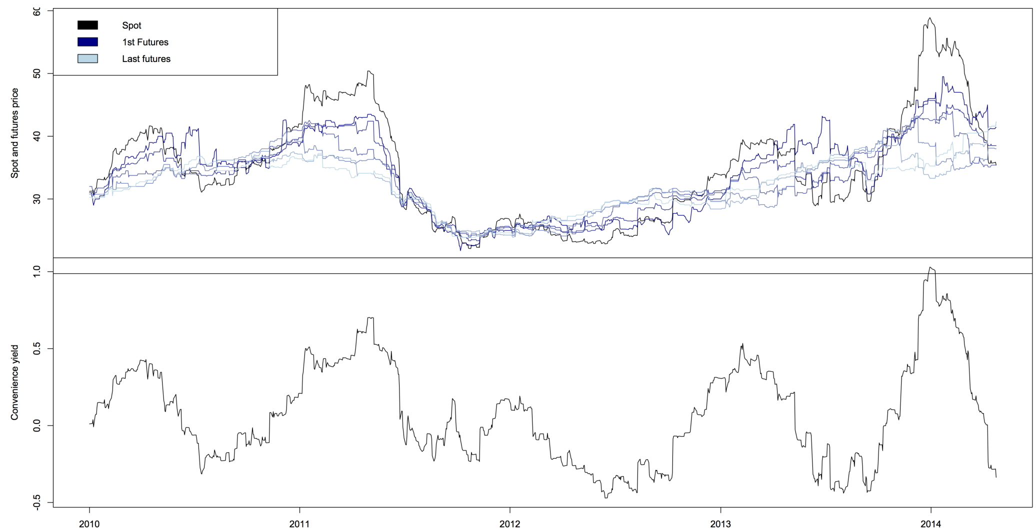


Figure 6. State variables: (a) live cattle; (b) salmon; spot and futures prices on the top of convenience yield

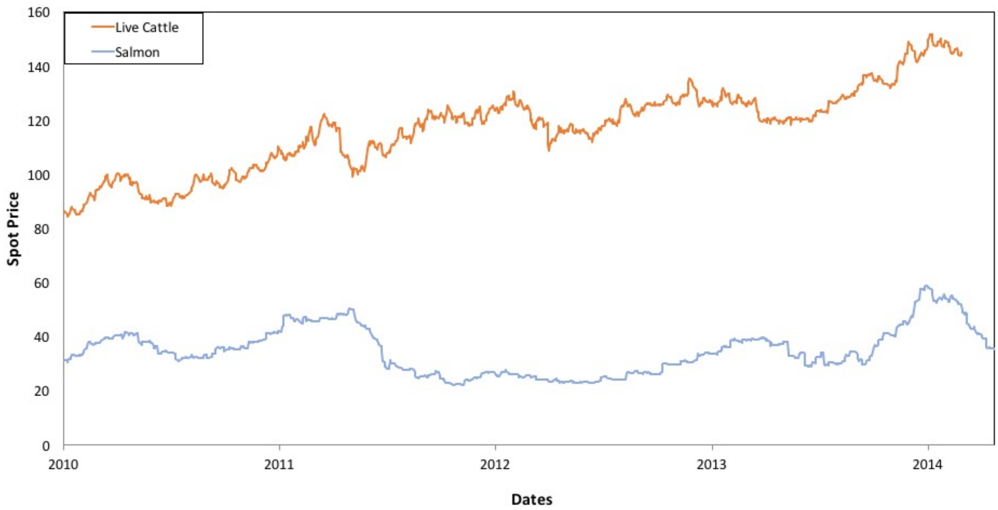


Figure 7. Filtered spot prices: live cattle and salmon

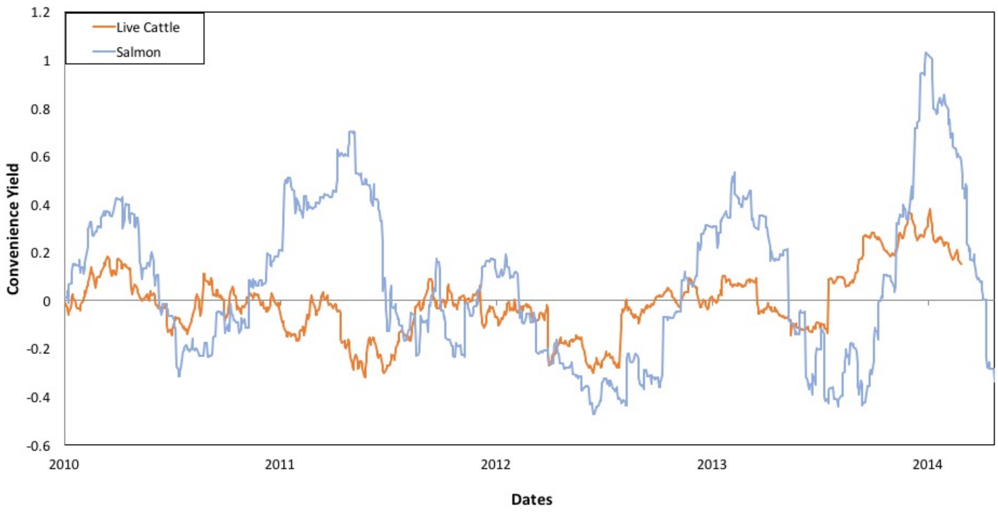
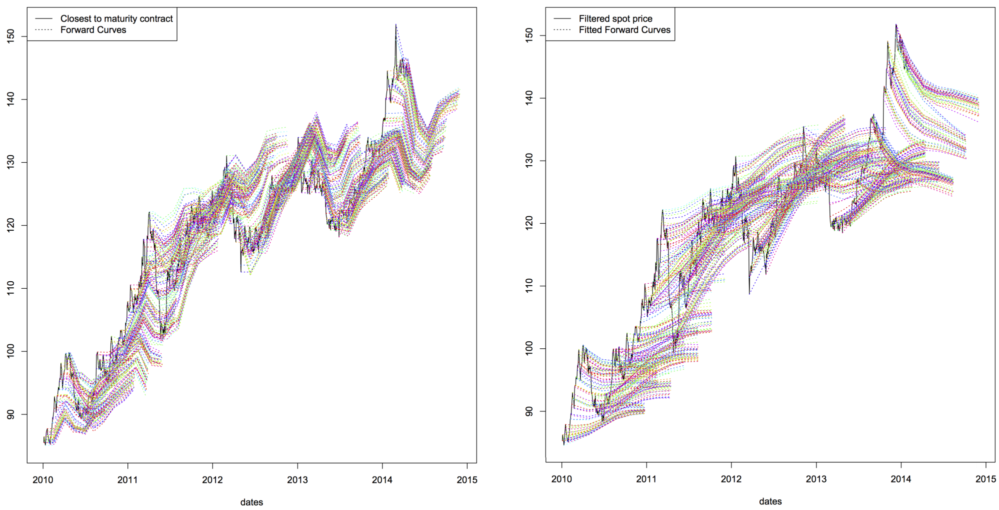


Figure 8. Filtered convenience yields: live cattle and salmon

(a) Live Cattle



(b) Salmon

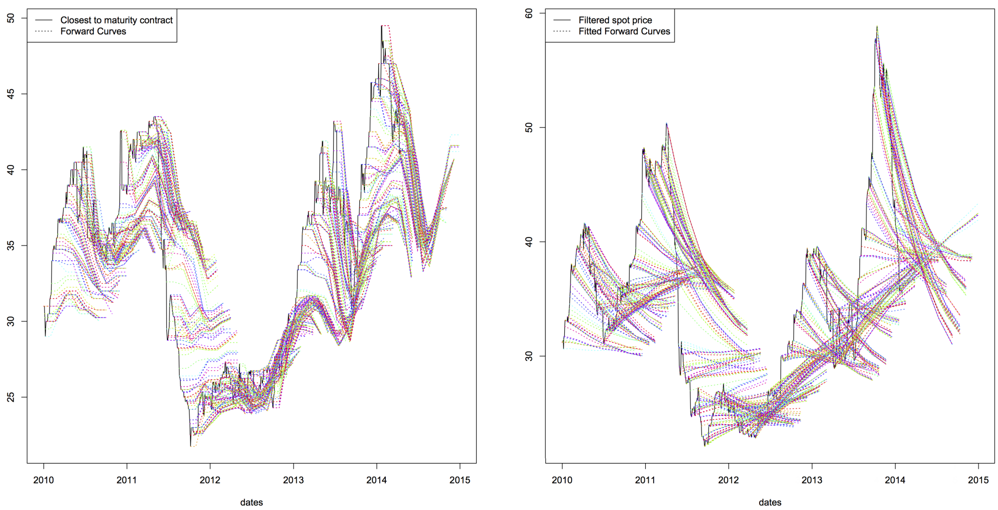


Figure 9. Term structures: (a) live cattle; (b) salmon; actual forward curves on the left, model generated forward curves on the right

Note: Each colored curve is a static picture of futures prices (y-axis) against contract maturities (x-axis), which is analogous to a plot of the term structure of interest rates. On the left side of the figure, the solid line represents the price of the closest-to-maturity futures contract, i.e., $C1$ and $S2$ in this case; while the dashed line consists of the actual prices of other futures contracts with different maturities in this panel. On the right side of the figure, the solid line is the filtered spot price obtained through the estimation procedure; while the dashed line consists of the estimated futures prices given by the pricing formula.

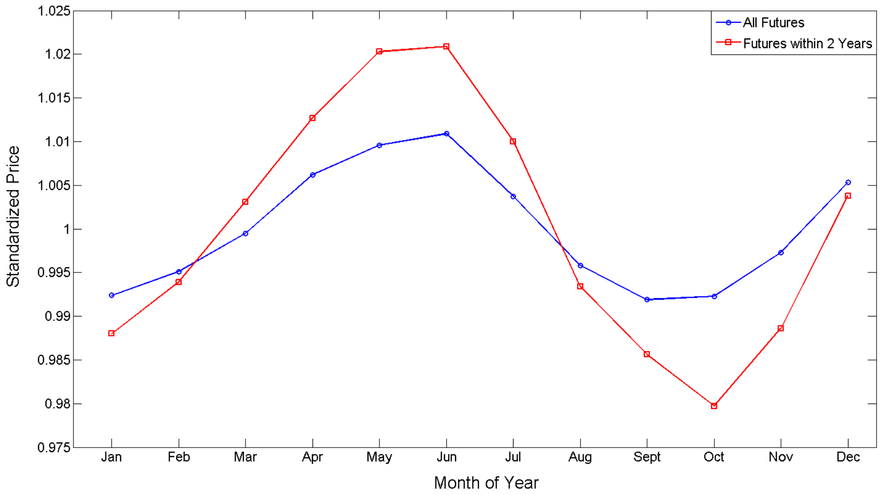
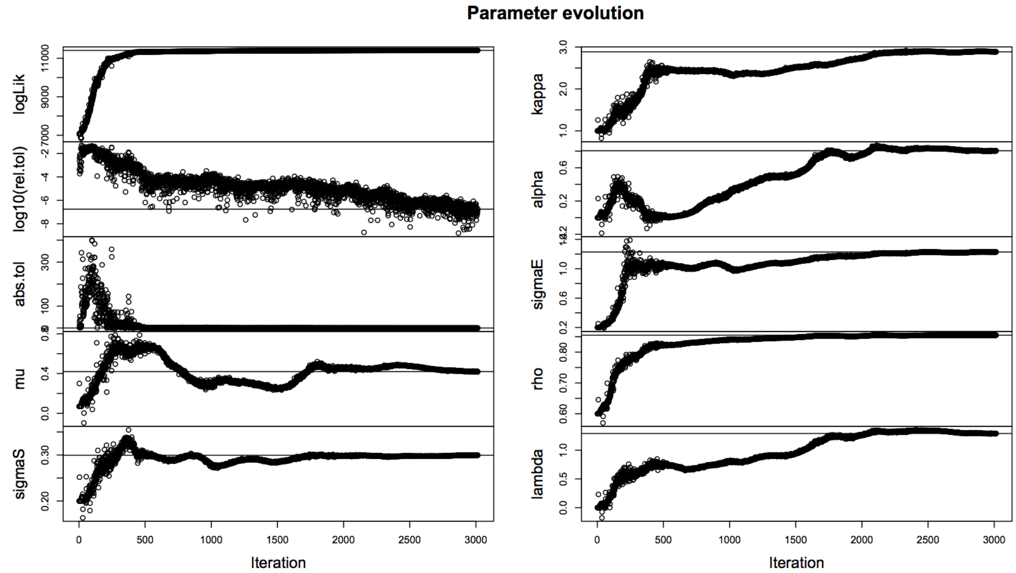


Figure B1. Price pattern: futures price

Note: Blue line is obtained by grouping all available futures contracts into expiration months; red line is obtained by grouping futures contracts spanned no longer than 2 years into expiration months. The data ranges from Jan 2007 to Dec 2013.

(a) Panel A



(b) Panel B

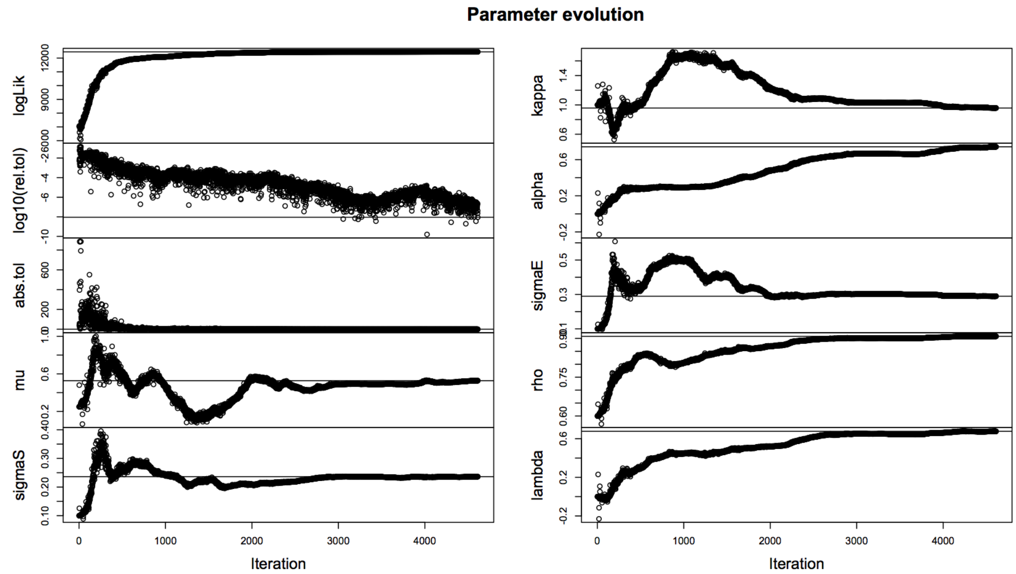
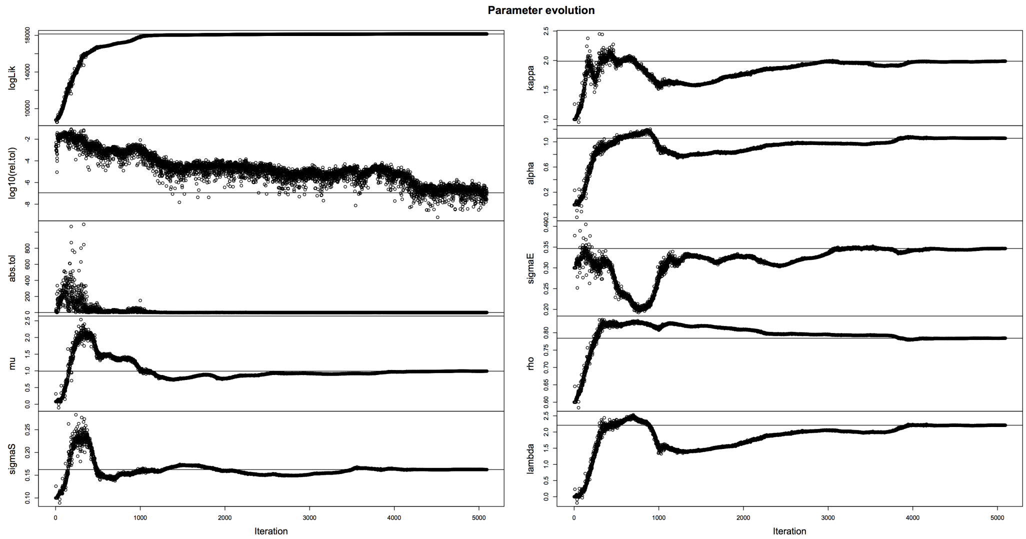


Figure B2. Parameter evolution: (a) Panel A; (b) Panel B

(a) Live Cattle



(b) Salmon

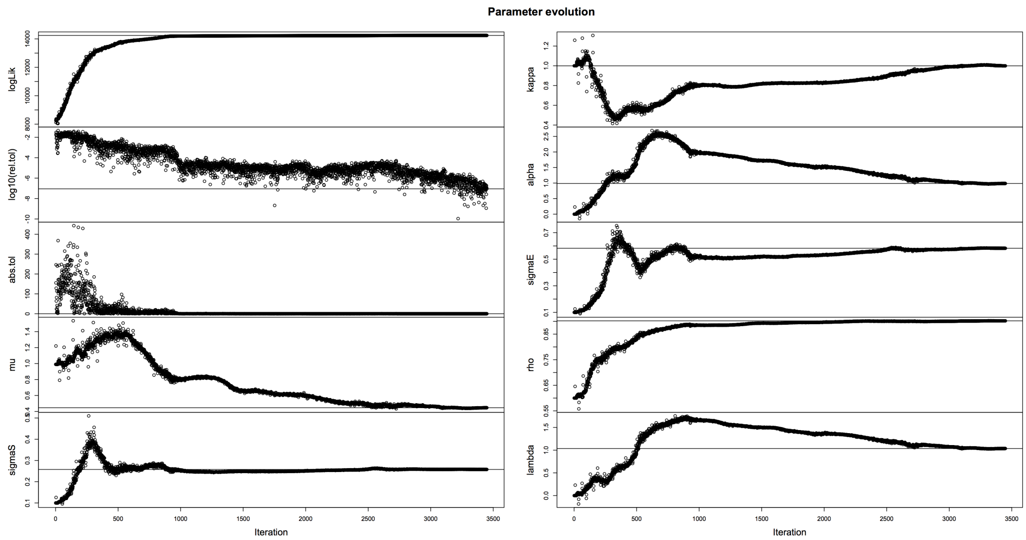


Figure B3. Parameter evolution: (a) live cattle; (b) salmon