

New Multiple Target Tracking Strategy Using Domain Knowledge and Optimisation

Runxiao Ding, Miao Yu, Hyondong Oh and Wen-Hua Chen

Abstract— This paper proposes an environment-dependent vehicle dynamic modelling approach considering interactions between the noisy control input of a dynamic model and the environment in order to make best use of domain knowledge. Based on this modelling, a new domain knowledge-aided moving horizon estimation (DMHE) method is proposed for ground moving target tracking. The proposed method incorporates different types of domain knowledge in the estimation process considering both environmental physical constraints and interaction behaviours between targets and the environment. Furthermore, in order to deal with a data association ambiguity problem of multiple target tracking in a cluttered environment, the DMHE is combined with a multiple hypothesis tracking structure. Numerical simulation results show that the proposed DMHE-based method and its extension could achieve better performance than traditional tracking methods which utilise no domain knowledge or simple physical constraint information only.

Index Terms—**Multiple target tracking, domain knowledge, force-based model, moving horizon estimation (MHE), multiple hypothesis tracking (MHT).**

I. INTRODUCTION

Tracking multiple road users plays an important role in various applications such as surveillance, advanced driver assistance systems (ADAS) and autonomous vehicles. Many model-based state estimation methods have been proposed for target tracking. However, the majority of current methods assume an open field environment in which the tracked target(s) could move freely. This contradicts with the realistic scenario where the motion of the ground target(s) movement is often affected by its operational environment such as road and terrain. This information could be taken as domain knowledge and exploited in the development of tracking algorithms in order to enhance tracking quality and continuity.

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The most apparent domain knowledge for ground vehicle tracking is the road constraint information such as the constrained region imposed by a road map. The studies on the road network-aided ground vehicle tracking have been reported in [1-6]. In these papers, the road network is taken as physical constraint information. Although comprehensive studies have been made for dealing with constraint information, limitations still exist. In particular, for a realistic tracking scenario, in addition to above physical road constraints, there are interactions between the target and its surrounding environment which need to be considered. For instance, the driver behaviours are affected by the surrounding environment and tend to obey the traffic rules. Drivers typically try to keep away from the road boundary while following the road/lane centre and the speed limit. They also anticipate potential collision risks with incoming cars and make avoidance manoeuvres whenever necessary.

An accurate dynamic model reflecting the aforementioned realistic movement of a vehicle is vital to obtain good tracking performance, especially when limited or even no measurements are available. However, most of the current vehicle dynamic models for target tracking ([10, 11]) predict the target's location from its past trajectory without fully taking into account the environmental interaction information. Recently, a social force model [12, 13] has been applied to model the interactions between pedestrians and environmental objects (building and walls) by using forces introduced by a potential field. These forces reflect different motion behaviours, for example, targets may be attracted to other objects or pushed away from them. However, the applications of the social force are limited to pedestrian tracking in the context of surveillance rather than vehicle tracking.

With this background, we propose a new vehicle dynamic modelling approach and its application to the multiple target tracking (MTT) problem. The proposed modelling extends the traditional methods by incorporating the environmental information into the noisy control input of a dynamic model. The interaction between the target and the environment is modelled by virtual forces constructed by the target state, target dynamics and environment information. Compared with existing social force model used for pedestrian tracking [12, 13], the proposed model is more suitable for ground vehicle tracking involving much faster manoeuvres as it utilises the entire vehicle dynamic states (e.g. position and speed) and the predicted future position rather than using current position information only.

Among various target estimation algorithms [7, 8], the optimisation-based moving horizon estimation (MHE) has a promising capability of being able to accommodate different types of constraints [9, 14, 25]. Thus, this study proposes a domain knowledge-aided MHE method (denoted as DMHE) by using the aforementioned vehicle dynamic model, which incorporates both the physical environmental constraints and interaction information into the tracking process in a comprehensive manner. The DMHE is further extended by combining with multiple hypothesis tracking (MHT), denoted as the DMHE-MHT, to deal with miss detection and false alarm considering a data association problem in a realistic multiple target tracking scenario. Note that, although miss detection and false alarms frequently occur in a cluttered environment, they have not been fully considered in most domain knowledge-aided tracking works [1-6]; only miss detection is considered in [2, 3]. In the proposed DMHE-MHT strategy, tracking association ambiguity is handled by the MHT concept which associates measurements with corresponding targets. The association results are then applied for the DMHE

optimisation function construction.

This remaining part of this paper is structured as follows. The literature review on the associated problems in the domain knowledge aided MTT field is presented in Section II. The domain knowledge dependent dynamic and measurement model are proposed in Section III. Section IV explains the DMHE-based target tracking algorithm, as well as its extension by combining it with the MHT to solve the MTT problem. In order to verify the benefit and efficiency of the proposed algorithm, numerical simulation results are presented in Section V. Finally, conclusions are given in Section VI.

II. LITERATURE REVIEW

In this section, we review the MTT problem from its related three aspects: vehicle dynamic models, state estimation aided by domain knowledge (mainly the constraint information in the current works) and the data association:

A. Vehicle dynamic models

Various dynamic models can be generally divided into three categories: macroscopic, mesoscopic and microscopic models [28], [29]. In macroscopic models, the dynamics of the whole group of moving objects is described as an aggregate flow. Mesoscopic models determine the state of the system by the position or velocity distribution of each entity on the basis of aggregate relationships. Microscopic models refer to entities individually. In this case, the dynamics of every individual is considered by incorporating the social behaviour of each target taking into account the interaction between the target and environmental moving/static objects. This paper focuses on incorporating the environmental information in a target tracking problem using the concept of the microscopic model. Examples of microscopic models include car-following model [28], cellular automata model [29], optimisation-based models [30] and force-based models [12]. Among them, the force-based models have a great advantage of incorporating the environmental information as different sources of forces deterministically in a continuous model.

B. Constrained state estimation

One of effective approaches to solve a road-constrained multiple target tracking (MTT) problem is to incorporate the constraint-related information into a standard filtering algorithm (i.e. state estimation process) as state constraints. Kalman filtering and its variations can be used to estimate the state of a target based on its dynamic and measurement models while considering limited constraint information. However, when the road state constraint types cannot fit easily into the structure of the Kalman filter (i.e., non-linear and inequality constraints), they are often ignored or dealt with heuristically [25]. Recently, some other methods, for instance the constrained Particle filters (PFs) and the constrained Gaussian mixture filters (GMF) (see [8], [9], [31], [32], for more details), are also developed based on optimisation and truncation approaches. The majority of filters proposed to solve the constrained estimation problems focus on linear (in)equality or nonlinear equality constraints. A little research has been conducted on nonlinear inequality constraints so far. However, nonlinear inequality constraints

(e.g. curved road boundary) are important and necessary for most tracking scenarios in ground vehicle tracking problems.

To address above issue, the moving horizon estimation (MHE) can be adopted, which proposed by Rao et al. [9] as a constrained state estimation method for nonlinear discrete-time systems. The basic strategy of the MHE is to reformulate the estimation problem as an optimisation problem using a fixed-size estimation window. Theoretically, for a linear system without constraints and with a quadratic cost, the MHE becomes the same as the Kalman filter when an infinite horizon window is considered. This method has been widely used in chemical engineering. Other applications include hybrid systems, distributed, network systems, and large-scale systems. However, the implementation of the moving horizon approach in vehicle target tracking is still relatively an uncharted area.

Advantages of the MHE for target tracking are manifolds. Firstly, since the method is based on optimisation, road or similar constraints can be naturally handled by the MHE as additional (non)linear and/or (in)equality constraints. In addition to state constraints, the MHE is also able to incorporate constraints on the process and/or observation noises for modelling bounded disturbance or truncated distribution/density representing the influence of the operation environment on vehicle movement such as acceleration and deceleration. Another advantage of using the MHE in target tracking is that it always considers a certain number of latest measurements. Such feature is very meaningful in target tracking especially when targets are occluded by each other or static obstacles which leads to no reliable measurement during certain time steps. Simulation results in [7], [8], [14] showed that MHE achieves the smallest estimation error for nonlinear systems and nonlinear constraints. It was also shown that the constrained MHE filter outperforms most of the other constrained algorithms (e.g. constrained particle filter, constrained unscented Kalman filter, etc.) in terms of estimation error while keeping an acceptable computational time for target tracking applications with a relatively simple implementation process.

C. Data association

Different techniques can be used to deal with the miss detections and false alarms for MTT data association problems; for example, Nearest Neighbour Standard Filter (NNSF) [15], Global Nearest Neighbour (GNN) approach [16], Joint Probabilistic Data Association (JPDA) [17], Multiple Hypothesis tracking (MHT) algorithm [18] and finite set statistics (FISST) [19]. Among them, the MHT is a complex approach that considers data association across multiple scans and multiple hypotheses. The MHT essentially keeps a set of multiple hypotheses, and the assignment ambiguity will be resolved in future when subsequent new observations are arrived. Hard decisions are not made until they are needed with sufficient information rather than just the current data frame; thus possible error association could be corrected when more evidences are obtained. Such features along with the dramatic increases in computational capabilities have made the MHT a preferred data association method for modern tracking systems [20].

III. ENVIRONMENTAL INFORMATION AIDED DYNAMIC AND MEASUREMENT MODEL

A. Environmental Information-Aided Dynamic Model

First let us review the general dynamic model for target tracking problem:

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k) + \boldsymbol{\omega}_k \quad (1)$$

where \mathbf{x}_k represents the state vector, which usually includes the position and velocity for tracking problem. $\boldsymbol{\omega}_k$ is generally known as the process noise and more specifically considered as noisy acceleration components that controls the dynamic evolution of \mathbf{x}_k and follows a certain type of distribution to represent uncertainty of a driver's behaviour. $f(\cdot)$ represents the system dynamic function which reflects a desired target dynamic type representing the state transition between consecutive time steps. According to [10], in most of the target tracking problems, the control term $\boldsymbol{\omega}_k$ is modelled as a Gaussian distribution with zero mean and constant covariance matrix representing target movement uncertainty irrespective to the surrounding environment. However, in realistic tracking scenarios, targets' movements are affected by the surrounding environment (e.g. road boundary, road centreline or speed limit). In other words, the vehicle noisy control input $\boldsymbol{\omega}_k$ from uncertain driver behaviours is related with the environment.

Therefore, this section proposes a new vehicle dynamic modelling approach which incorporates environmental information into the vehicle control input, inspired by the social force model [12, 21]. In the original social force model as in [12, 21], pedestrians are assumed moving with low and constant velocity in a short time interval and force is considered to be only related to the relative distance between pedestrian's current position and other environmental objects. Compared with the human tracking scenario, our problem exhibits much more complex vehicle movements with high velocity. In this case, the force (control) term needs to consider not only position but velocity information and the desired dynamics of the vehicle. As illustrated in Fig. 1(a), the object j is assumed to perform a turning manoeuvre should receive a higher repulsive force than the object i because it will get closer to the boundary. Besides, forces should also relate to the magnitude of the velocity; for instance, if the velocity \mathbf{v}_j of the object j towards the road boundary becomes larger, a larger repulsive force should be imposed on the object.

In the proposed dynamic model, both repulsive and attractive effects from the environment are considered where the repulsive (or attractive) force is modelled as a monotonously decreasing (or increasing) exponential function. According to the current state \mathbf{x}_k (including both position and velocity states) of the vehicle i , the predicted position $x_i^{predict}$ is first calculated from the dynamic model determined by $f(\mathbf{x}_k)$. In this way, the entire state and dynamic model information are incorporated. Then, the relative Euclidean distance $d_{ij}^{prediction}$ between $x_i^{predict}$ and position of the object j (e.g. road boundary, road centreline or other vehicles) is estimated. The repulsive/attractive force between target i and object j can then be represented as:

$$\mathbf{f}_{i,j}^{repulsive} = A \cdot \exp\left(\frac{-d_{ij}^{prediction}}{B}\right) \mathbf{n}_{ji} \quad (2)$$

$$\mathbf{f}_{i,j}^{attractive} = A \cdot \left(1 - \exp\left(\frac{-d_{ij}^{prediction}}{B}\right) \right) \mathbf{n}_{ij} \quad (3)$$

where A and B are positive constants representing the magnitude and range of the force, respectively. \mathbf{n}_{ij} is the normalised vector pointing from i to j .

As shown in Fig. 1(b), it is assumed that there exist different forces acting on ego vehicle i generated by the surrounding environmental objects, such as the repulsive force $\mathbf{f}_{i,o}$ from road boundary o , attractive force $\mathbf{f}_{i,c}$ to the centreline c and the repulsive force $\mathbf{f}_{i,j}$ from another moving vehicle j to avoid a collision,

These forces are summed to a net environmental force \mathbf{f}^e acting on the vehicle i , which can be incorporated into the dynamic model (1) as:

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k) + I(\mathbf{a}_k^e) + \boldsymbol{\omega}_k \quad (4)$$

where $\mathbf{a}_k^e = \frac{\mathbf{f}_k^e}{m}$ represents the acceleration introduced by the environmental force. $I(\mathbf{a}_k^e)$ is the function representing the influence of the acceleration \mathbf{a}_k^e on the vehicle dynamic model, which has different forms according to different representations of (1).

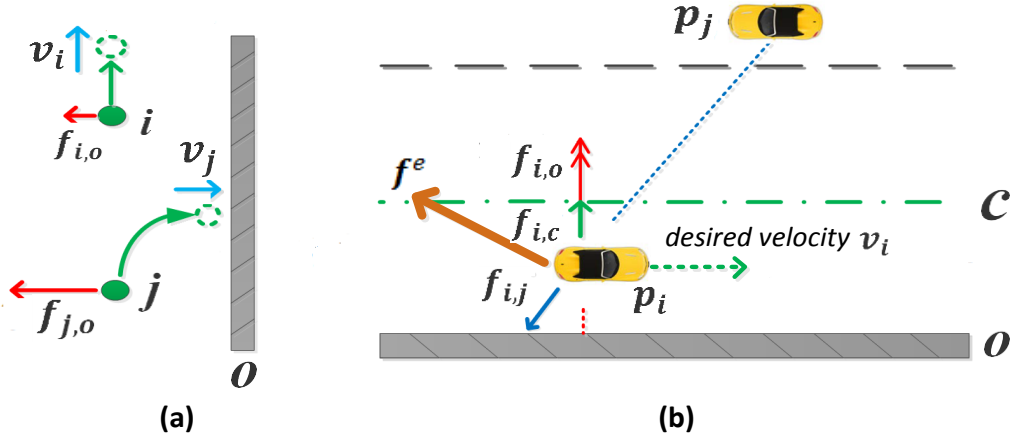


Fig. 1 The influence of the environment on a moving target by forces: (a) different repulsive forces $\mathbf{f}_{i,o}$ and $\mathbf{f}_{j,o}$ on objects i and j with different dynamics between $T=t$ (when objects position are marked as green circles) and $T=t+\Delta t$ (where objects position are marked as dash circles) (b) p_i receives interaction force $\mathbf{f}_{i,j}$ from another vehicle, attractive force $\mathbf{f}_{i,c}$ from the centreline and repulsive force $\mathbf{f}_{i,o}$ from the road boundary.

B. Environmental Information-Aided Measurement Model

For the model based tracking problem, usually measurements are associated with a measurement model which can be generally represented as:

$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{v}_k \quad (5)$$

where \mathbf{y}_k is a measurement vector, $h(\mathbf{x}_k)$ is the measurement function and \mathbf{v}_k is zero mean

Gaussian noise of the measurement with the covariance R .

We assume that the ground vehicles only move within the road network region. This matches with a realistic scenario where road boundaries are considered as physical constraints and all drivers are supposed to move within the constraint region. Due to the limited tracking sensor's capability, the received measurements usually contain noises as in (5), which make them not always stay on the road network and far away from the ground truth values. Such noisy measurements are usually known as false alarms in MTT which make data association process really difficult with tracking ambiguity problems. To this end, a pre-processing approach is used in this paper to project the raw measurements onto the constrained surface (road network) at each time step.

Assuming that target vehicles are traveling on linear road following the centre line, the raw measurement data \mathbf{y}_k could then be projected by the following linear equality constraint as:

$$D\tilde{\mathbf{y}}_k = \mathbf{d}_k \quad (6)$$

where D is a full-rank constraint matrix and \mathbf{d}_k is the constraint vector. $\tilde{\mathbf{y}}_k$ is the projected (constrained) Cartesian measurement. Following [25], the expression of deriving constrained measurement $\tilde{\mathbf{y}}_k$ by directly projecting the unconstrained Cartesian coordinate measurement \mathbf{y}_k onto the constraint surface is by solving the problem:

$$\min_{\tilde{\mathbf{y}}_k} (\tilde{\mathbf{y}}_k - \mathbf{y}_k)^T W (\tilde{\mathbf{y}}_k - \mathbf{y}_k) \quad s. t. \quad D\tilde{\mathbf{y}}_k = \mathbf{d}_k \quad (7)$$

where W is a symmetric positive definite weighting matrix. In this work, it is chosen as $W = R$ following the mean square method, where R is a measurement error covariance matrix of the original measurements. The solution of this problem is then given by:

$$\tilde{\mathbf{y}}_k = \mathbf{y}_k - R^{-1}D^T(DR^{-1}D^T)^{-1}(D\mathbf{y}_k - \mathbf{d}_k) \quad (8)$$

According to [25], the projected measurement error covariance matrix \tilde{R} can be expressed as:

$$\tilde{R} = (I - R^{-1}D^T(DR^{-1}D^T)^{-1}D)R(I - R^{-1}D^T(DR^{-1}D^T)^{-1}D)^T \quad (9)$$

In this way, the measurement model is modified as:

$$\tilde{\mathbf{y}}_k = \begin{bmatrix} x_k \\ y_k \end{bmatrix} + \tilde{\mathbf{v}}_k \quad (10)$$

where $\tilde{\mathbf{y}}_k$ is the projected measurement, $\begin{bmatrix} x_k \\ y_k \end{bmatrix}$ represents the target position and $\tilde{\mathbf{v}}_k$ is the measurement noise for the projected measurement with zeros mean and covariance \tilde{R} .

For nonlinear road constraint $g(\tilde{\mathbf{y}}_k) = \mathbf{d}_k$, the development of the constrained measurement (10) as given above is still valid with a linearisation process. We can perform a first order Taylor series expansion of the constraint equation around $\tilde{\mathbf{y}}_k$ to obtain:

$$g(\tilde{\mathbf{y}}_k) = \mathbf{d}_k \approx g(\tilde{\mathbf{y}}_k) + g'(\tilde{\mathbf{y}}_k)(\mathbf{y}_k - \tilde{\mathbf{y}}_k) \quad (11)$$

which indicates that:

$$g'(\tilde{\mathbf{y}}_k)\tilde{\mathbf{y}}_k \approx \mathbf{d}_k - g(\tilde{\mathbf{y}}_k) + g'(\tilde{\mathbf{y}}_k)\tilde{\mathbf{y}}_k \quad (12)$$

We now have an approximated nonlinear constraint that is equivalent to the linear constraint $D\tilde{\mathbf{y}}_k = \mathbf{d}_k$ where D is replaced with $g'(\tilde{\mathbf{y}}_k)$ and \mathbf{d}_k is replaced with $\mathbf{d}_k - g(\tilde{\mathbf{y}}_k) + g'(\tilde{\mathbf{y}}_k)\tilde{\mathbf{y}}_k$.

In a realistic ground vehicle tracking scenario, each vehicle may undergo different regions with different environmental conditions. Thus, one single model might not be able to accurately describe various movement types in different environmental conditions. Considering that multiple state models are involved, before performing the FMHE-MHT algorithm, it is required to determine the particular state model which best fits to describe the vehicle movement. To this end, this work adopts the Bayesian inference framework which utilises the posterior model probability to decide the appropriate state model at each time step. The implementation of the Bayesian inference is done by interactive multiple model particle filtering method (IMMPF) among others. The details for the IMMPF can be found in [26].

IV. MHE-BASED TARGET TRACKING WITH ENVIRONMENTAL INFORMATION

Based on the domain knowledge aided dynamic modelling and measurements as mentioned in the previous section, the MHE based optimization scheme is applied for the state estimation, which is detailed as follows:

A. General Moving Horizon Estimation

By the original state model (1) and measurement model (5) without considering any environmental information, the standard MHE is defined as an optimisation problem by the following form [9]:

$$\min_{\{\mathbf{x}_{k-N}, \{\omega_j\}_{j=k-N}^{k-1}\}} \sum_{j=k-N}^{k-1} \left(\|\mathbf{x}_j - f(\mathbf{x}_{j-1})\|_{Q^{-1}}^2 + \|\mathbf{y}_j - h(\mathbf{x}_j)\|_{R^{-1}}^2 \right) + \Gamma_{k-N}(\mathbf{x}_{k-N}),$$

$$\text{for } \{\mathbf{x}_{k-N}, \dots, \mathbf{x}_k\} \in C_x \quad (13)$$

where C_x represents the constrained region which represents the feasible road region determined by the road network in the ground vehicle tracking problem. $\{\mathbf{x}_{k-N:k}\}$ is the ensemble of states from time instance $k - N$ to k which solves the quadratic programming problem (13) while giving the optimal estimate solution. N is a moving horizon length which is chosen to give a trade-off between the estimation accuracy and the computational cost. $\Gamma_{k-N}(\mathbf{x}_{k-N}) = -\log(p(\mathbf{x}_{k-N}|Y_{0:k-N-1}))$ represents an arrival cost which plays an important role in summarising the effect of the past measurement as a *priori* information on the initial state \mathbf{x}_{k-N} .

$$\Gamma_{k-N}(\mathbf{x}_{k-N}) \approx \|\mathbf{x}_{k-N} - \hat{\mathbf{x}}_{k-N}^{mh}\|_{P_{k-N}^{-1}}^2 \quad (14)$$

where $\hat{\mathbf{x}}_{k-N}^{mh}$ and P_{k-N} represents the previous moving horizon state estimate and covariance at $k - N$, respectively. The unconstrained extended Kalman filter (EKF) [9] is adopted as the approximate method for calculating the arrival cost error covariance matrix P_{k+1} by using the following update rule:

$$P_{k-N} = Q + FP_{k-N-1}F^T - FP_{k-N-1}H^T(R + HP_{k-N-1}H^T)^{-1}HP_{k-N-1}F^T \quad (15)$$

where F and H represent the Jacobian matrix of the function $f(\mathbf{x}_k)$ and $h(\mathbf{x}_k)$ respectively.

The state estimate of the MHE optimisation function (13) at time k is denoted as $\mathbf{x}(k; \hat{\mathbf{x}}_{k-N}^*, \{\hat{\omega}_j^*\}_{j=k-N}^{k-1})$, including the optimised initial state $\hat{\mathbf{x}}_{k-N}^*$ and the optimised process noise sequence $\{\hat{\omega}_j^*\}_{j=k-N}^{k-1}$. The optimised estimated state $\hat{\mathbf{x}}_k^*$ at time instance k considering a linear dynamic system can be expressed as:

$$\hat{\mathbf{x}}_k^* = \mathbf{x}(k; \hat{\mathbf{x}}_{k-N}^*, \{\hat{\omega}_j^*\}_{j=k-N}^{k-1}) = F^k \hat{\mathbf{x}}_{k-N}^* + \sum_{j=k-N}^{k-1} F^{k-j-1} \hat{\omega}_j^*. \quad (16)$$

B. Domain Knowledge aided Moving Horizon Estimation

Although the aforementioned MHE method could incorporate the constraint information for the state estimation, it cannot exploit the environmental information in a comprehensive way:

- i. The interaction between the target and surrounding environment (e.g. a vehicle keeps away from static/moving environmental objects, such as road boundary, another vehicle, etc.) is not considered in the original MHE framework; and
- ii. Domain knowledge is not considered in the measurement model.

To this end, a new framework of the MHE which fully exploits the domain knowledge (denoted as DMHE for short) is proposed. Both the proposed state model (4), which considers the interaction information and the projected measurement model (10) are used to construct a new MHE optimisation function as:

$$\min_{\{\mathbf{x}_{k-N}, \{\omega_j\}_{j=k-N}^{k-1}\}} \sum_{j=k-N+1}^k (\|\mathbf{x}_j - f(\mathbf{x}_{j-1}) - I(\mathbf{a}_j^e)\|_{Q^{-1}}^2 + \|\tilde{\mathbf{y}}_j - h(\mathbf{x}_j)\|_{\tilde{R}^{-1}}^2) + \Gamma_{k-N}(\mathbf{x}_{k-N}), \quad (17)$$

for $\{\mathbf{x}_{k-N}, \dots, \mathbf{x}_k\} \in C_x$.

Compared with the MHE function in (13), besides the road constraint based information the domain knowledge is better exploited from two aspects:

- i. a new $I(\mathbf{a}_j^e)$ term is introduced, which is related to the environmental force modelling the interaction as mentioned previously. In this way, the interaction information is considered in the MHE process; and
- ii. the projected measurements $\tilde{\mathbf{y}}_k$ and associated error covariance \tilde{R} are exploited to better model measurement information

By solving the DMHE cost function (17), the optimised estimated state at time k can be obtained as:

$$\hat{\mathbf{x}}_k^* := \mathbf{x}(k; \hat{\mathbf{x}}_{k-N}^*, \{\hat{\omega}_j^*\}_{j=k-N}^{k-1}, \{\mathbf{f}_j^{e,*}\}_{j=k-N}^{k-1}) = F^k \hat{\mathbf{x}}_{k-N}^* + \sum_{j=k-N}^{k-1} F^{k-j-1} (\hat{\omega}_j^* + I(\mathbf{a}_j^{e,*})) \quad (18)$$

Note that, $\mathbf{a}_j^{e,*}$ is a function of $\hat{\mathbf{x}}_{k-N}^*$ and $\{\hat{\omega}_j^*\}_{j=k-N}^{k-1}$, according to the force terms defined in (2) and (3). The covariance required for the arrival cost computation as in (15) is modified by considering the

influence of the term $I(\mathbf{a}_k^e)$ at time instance k by:

$$P_{k-N} = Q + \left(F + \nabla_{\hat{\mathbf{x}}_{k-N-1}^*} I(\mathbf{a}_{k-N-1}^e) \right) P_{k-N-1} \left(F + \nabla_{\hat{\mathbf{x}}_{k-N-1}^*} I(\mathbf{a}_{k-N-1}^e) \right)^T - \left(F + \nabla_{\hat{\mathbf{x}}_{k-N-1}^*} I(\mathbf{a}_{k-N-1}^e) \right) P_{k-N-1} H^T (R + H P_{k-N-1} H^T)^{-1} H P_{k-N-1} \left(F + \nabla_{\hat{\mathbf{x}}_{k-N-1}^*} I(\mathbf{a}_{k-N-1}^e) \right)^T \quad (19)$$

where $\nabla_{\hat{\mathbf{x}}_k^*} I(\mathbf{a}_k^e)$ represents the gradient of the term $I(\mathbf{a}_k^e)$ with respect to $\hat{\mathbf{x}}_k^*$ at time instance k .

C. DMHE-based Multiple Hypothesis Tracking (DMHE-MHT)

The proposed DMHE algorithm is further extended to address the data association problem by incorporating it into a multiple hypothesis tracking (MHT) structure, which constructs a DMHE-MHT framework for MTT in a more complicated scenario with both miss detections and false alarms. Comparing with other data association algorithms, the multiple hypothesis tracking (MHT) has the advantages of being able to deal with track creation, confirmation, occlusion and deletion in a probabilistically consistent way and keeping a multiple number of past hypotheses between consecutive time steps [22]. In this combined DMHE-MHT strategy, tracking association ambiguity is handled by MHT data association. After measurements are associated with proper targets, vehicle states are then estimated by the DMHE algorithms by exploiting the domain knowledge. The block diagram of the DMHE-MHT algorithm is presented in Fig. 3.

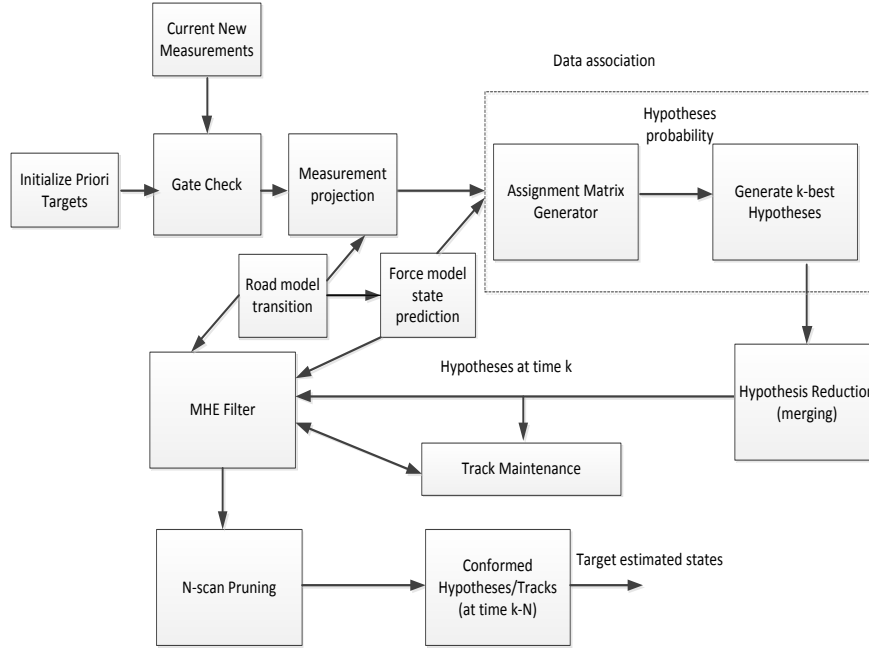


Fig. 3 Flow diagram of DMHE-MHT algorithm.

Initially, let $Y_k = \{y_i^k\}_{i=1}^{m_k}$ denote the set of m_k measurements received at time k . Each measurement has three possible hypotheses: i) the measurement starts a new target, ii) the measurement is a false alarm, and iii) the measurement belongs to an existing target. The procedures of the DMHE-MHT are divided into following steps.

1) Gate Check: First, the distance between the predicted (prior) target position and the current measurements is calculated. The prediction of target position is done by the Kalman filter time update, and the distance is defined as the Mahalanobis distance as:

$$(y_m^k - \bar{y}_{k/k-1})^T S_{k/k-1} (y_m^k - \bar{y}_{k/k-1}) \quad (20)$$

where y_m^k is the m -th measurement at time k , $\bar{y}_{k/k-1}$ is the predicted target position and $S_{k/k-1}$ is the covariance of innovation vector corresponding to the position. Note that the predicted position is calculated by the forced-aided model (4) considering environmental interaction information. Gating is a matrix of binary values which indicates the maximum possible distance between measurements and targets. The measurements whose Mahalanobis distances with particular targets are smaller than a particular threshold are used for the further data association.

2) Measurement projection: After the gating process, the candidate measurements within the gating region are projected to the road using the process described in section IV considering road constraints. The state dependent road model transition process is used to determine on which road the target is moving. $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{R}}$ are calculated for the data association process.

3) State prediction: After determining on which road the target is moving, the corresponding target-environment interaction force is calculated. The force-based state dynamic prediction is then calculated which will be used in both data association and the MHE process.

4) Data association: The FDHE-MHT implements a similar data association process as the Reids algorithm [18] with the projected measurement $\tilde{\mathbf{y}}$, the constrained measurement error covariance $\tilde{\mathbf{R}}$ and the forced-based state prediction. The assignment matrix is generated to represent all possible target-to-measurement associations. Then, each new hypothesis contains a set of potential target-to-measurement assignments, leading to an exhaustive process of enumerating all the possible assignment combinations. To address this issue, the Murty's algorithm [24] is used to find the k -best assignment and new hypotheses generated from each parent hypothesis. To further reduce the computational cost, a merging algorithm is also implemented to prevent hypotheses from being considered if the ratio of their probability to the best hypothesis becomes too small.

5) Target Maintenance: In ground target tracking scenarios, vehicles may enter or leave the region of interest during the tracking process. Moreover, occlusion or miss detection is also possible when a vehicle is hidden behind other objects. Based on the data association results, we implement target maintenance to identify targets which are entering, staying or leaving the tracking scene, by considering three possible statuses for a set of targets: target initiation, confirmation/deletion and maintenance.

- Target initiation: If the measurement is associated with a new target, then the new target hypothesis appears in the current k -best hypotheses. A target *lifetime* index is added to the target with value 1.
- Target confirmation/deletion: The new target is confirmed only if the detected target appears along the same track over a consecutive iteration of C_t (confirmation threshold) times. The *lifetime* index

is accumulated by 1 whenever the tentative target is detected and will become C_t when confirmed. On the contrary, the *lifetime* index for any existing target is reduced by 1 whenever the target is not associated with the current measurement and will be permanently deleted from the target list when the *lifetime* is 0.

- Target maintenance: The confirmed target may be temporally occluded or undetected by the sensor without measurements being associated. For this situation, the track is updated according to the predicted position of the target last associated states.

The high level logic for the DMHE-MHT target maintenance process is summarised in Table 1.

Table 1 High level logic for DMHE-MHT target maintenance.

-- At time k , for $nExistedTarg$ number of existing target in a hypothesis

For $k=1$: $nExistedTarg$

(Case one: permanent deleted targets)

If $Lifetime == 0$

Continue; (the target is permanently deleted/already disappeared)

End

(Case two: target maintenance—target updating with measurement or temporarily miss detection)

If $Targ \neq asso$ (Target not associated with current measurement)

$Lifetime = Lifetime - 1$;

If $LifePoint > 0$

Implement force model based state prediction as DMHE estimation result

End

Else (Target associated with current measurement)

Implement DMHE update;

If $Lifetime < MaxLifetime$

$Lifetime = Lifetime + 1$;

End

End

(Case three: target initialisation)

For $k=1$: $nNewTarg$ (measurement is associated to a new target)

Use current measurement as initial position;

$Lifetime = 0$;

End

6) DMHE filter: The details about implementing the DMHE for constrained target tracking based on associated measurements have been discussed in previous section. Note that, in the original MHT, the ‘filter’ process is based on the KF consisting of: time update (i.e. prediction) and measurement update. However, these two steps are combined in the DMHE and solved concurrently by the optimisation process. The state estimation is determined online by solving a state estimation problem for a finite horizon window.

7) N-scan pruning: An N-scan pruning technique [24] is applied in the MHT structure to control the complexity of the algorithm. The growing number of hypotheses is controlled by N-scan pruning technique by keeping only the ones with the largest probability values (details as in [24]), while other hypotheses with low probability are deleted after N-scan pruning. In the DMHE-MHT, the number of N scans is chosen as the same value for the horizon length in the MHE.

V. NUMERICAL SIMULATION RESULTS

In this section, two simulation examples are presented in the context of ground vehicle tracking. The first example is single target tracking, aiming at illustrating the proposed DMHE with both linear and nonlinear inequality road constraint. The second one is a complex multiple vehicle tracking scenario incorporating road inequality constraints from real world map data for the DMHE-MHT.

A. Single Target Tracking

The proposed DMHE algorithm is evaluated by single target tracking scenario for both linear (position) and nonlinear (bearing/range) measurement models with road boundary constraints. The first one is a linear trajectory, considering a single carriageway with road width of 4 meters and an angle of 45 degrees anticlockwise to the horizontal axis. The vehicle dynamics is described by a constant velocity model with the noisy acceleration:

$$x_{k+1} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \end{bmatrix} \omega_k \quad (21)$$

where the state vector $x_k = [x_{1,k}, x_{2,k}, \dot{x}_{1,k}, \dot{x}_{2,k}]^T$ consists of the vehicle position and velocity in x and y directions, and $T = 1$ is the sampling interval, ω_k is a two-dimensional Gaussian process noise with zero mean and covariance matrix $Q = \text{diag}\{5, 2\}$ in a local coordinate where $\text{diag}\{\cdot\}$ represents a diagonal matrix. This covariance represents higher motion uncertainty along the centre line direction and smaller uncertainty orthogonal to the road. The vehicle measurement model is a linear matrix in x and y position with a Gaussian measurement noise v_k and covariance matrix $R = \text{diag}\{20/\sqrt{2}, 20/\sqrt{2}\}$ in a global Cartesian coordinate as:

$$z_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x_k + v_k. \quad (22)$$

The vehicle has a centre line direction velocity of 10m/s with no initial lateral velocity and the initial state is $x_0 = [0, 1502.83, 7.0711, -7.0711]^T$.

The movement of the target is constrained by road boundaries and supposed to follow the centre line of the road. Different environmental forces are considered including lateral forces orthogonal to the road as:

- Repulsive force generated from lower road boundary

$$\mathbf{f}_{i,j}^{rep1} = A \cdot \exp\left(\frac{-d_{lb}}{B}\right) \mathbf{n}_{ji} \quad (23)$$

- Repulsive force generated from upper road boundary

$$\mathbf{f}_{i,j}^{rep2} = A \cdot \exp\left(\frac{-d_{ub}}{B}\right) \mathbf{n}_{ji} \quad (24)$$

- Attractive force to centre line of the road

$$\mathbf{f}_{i,j}^{att} = A \cdot \left(1 - \exp\left(\frac{-d_{center}}{B}\right)\right) \mathbf{n}_{ij} \quad (25)$$

where i and j represents the target and the environment (road boundary, centre line, and speed limit where applicable), respectively. d_{lb} and d_{ub} represent the Euclidean distance between lower and upper boundary of the road and the predicted vehicle position $x_i^{predict}$ calculated from the dynamic model (11) base on the current location, respectively. Similarly, d_{center} represents the distance between centre line and predicted vehicle position. Note that the closer (further) the vehicle gets to the road boundaries (away from the centre line), the larger the repulsive (attractive) force will be generated.

Beside above lateral forces, a velocity-based breaking (repulsive) force is also considered along the centre line direction so as to present the road speed limitation:

$$\mathbf{f}_{i,j}^{rep3} = -A \cdot \exp\left(\frac{-(v_{limit} - v_{heading})}{B}\right) \mathbf{v} \quad (26)$$

where $v_{heading} = \sqrt{\dot{x}_k^2 + \dot{y}_k^2}$ is the speed of the vehicle towards heading direction. The speed limitation v_{limit} is defined as a valid speed interval around a specific speed value. And \mathbf{v} represents a unit velocity vector. Although the relative speed difference can be either positive or negative, the repulsive effect is much larger when the vehicle exceeds the speed limit, as illustrated in Fig. 4.

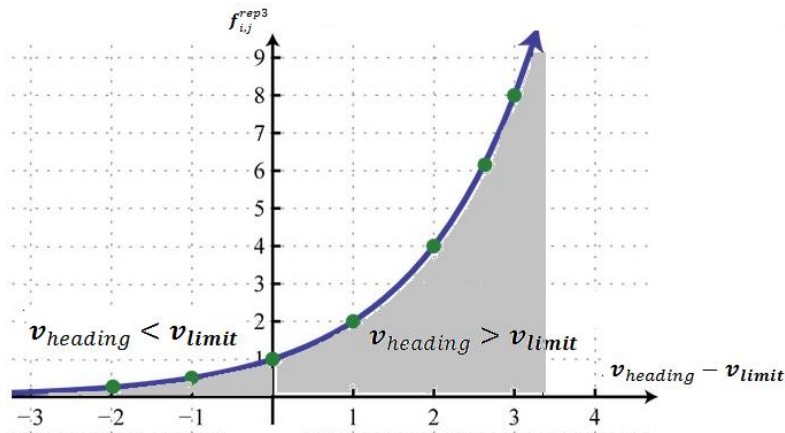


Fig. 4 Force generated from the speed limitation.

To evaluate the performance, four different tracking models are compared: i) general MHE without considering any environmental information (MHE), ii) force based MHE without

considering physical constraints (FMHE), iii) general MHE with inequality physical constraints (road boundaries) (CMHE), and iv) the proposed DMHE approach. In Table 2, the performance of different models is compared in terms of mean-square error (MSE) in three different aspects: i) position MSE, ii) centre line tangential direction position MSE and iii) orthogonal position MSE to the road with a horizon length of $N=4$. It is shown that road physical constraint is of great importance when comparing the CMHE with the MHE and the DMHE with the DMHE, especially in orthogonal direction where road boundary is considered. In addition to physical constraints, environmental forces further improve the estimation accuracy. Both the FMHE and the DMHE have shown a significant improvement for target's position estimate compared with their relative MHE and CMHE.

Table. 2 Estimation performance comparison of MHE, FMHE, CMHE, and DMHE.

MSE	MHE	FMHE	CMHE	DMHE
Position	7.0255	5.8592	6.1888	5.3900
Centre line direction	9.7958	8.7652	9.7223	8.6519
Orthogonal direction	8.3707	6.3303	2.6552	2.1281

In the second scenario, a vehicle is simulated to move along the quarter of a circular road with an angular velocity of 0.1 rad/s along the road centreline for 15 seconds. Small noises are added to the simulated vehicle position to represent the disturbance of the vehicle movement. The road has a width of 4 meters and is defined by two arc boundaries of $r_1=96$ m and $r_2=100$ m, respectively, centred at the origin of a Cartesian coordinate system, as shown in Fig. 5. The speed limit of this road segment for the vehicle to keep is assumed to be 30 miles/hour.

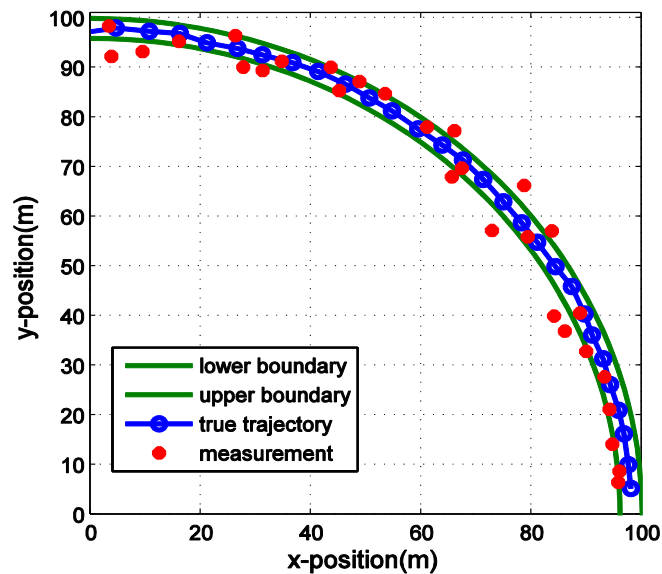


Fig. 5 The simulated circular road tracking scenario.

Regarding the range and bearing measurement model in (28), it is assumed that a radar sensor is positioned at the origin. The corresponding measurement noise \mathbf{v}_k follows a Gaussian distribution with zero mean and covariance $R = \text{diag}\{36, 10^{-2}\}$.

$$\mathbf{z}_k = \begin{bmatrix} \sqrt{x_{1,k}^2 + x_{2,k}^2} \\ \arctan\left(\frac{x_{2,k}}{x_{1,k}}\right) \end{bmatrix} + \mathbf{v}_k. \quad (27)$$

Three algorithms are chosen for comparison for this simulated scenario including the EKF, the constrained MHE (CMHE) which considers the road boundary constraint and the proposed DMHE. For the EKF and the CMHE, the system dynamic model for tracking is the same as the previous scenario. For the proposed DMHE method, additional interactions between the target and environment are considered by using two forces: i) road repulsive forces generated by the road upper and lower boundary and ii) force acting in the opposite of movement tangential direction to prevent the vehicle from exceeding the speed limitation. For a fair comparison, all the algorithms are set to have the same initial condition with mean $\mathbf{x}_0 = [75, 0, 10, 10]^T$ and covariance $P_0 = \text{diag}\{10, 10, 1, 1\}$.

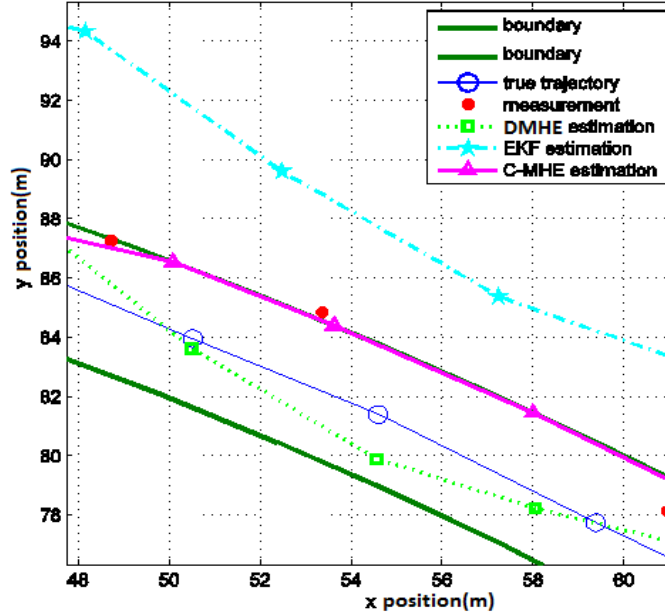


Fig. 6 True and estimated results for EKF, CMHE and DMHE.

Firstly, a sample tracking performance of three different algorithms is illustrated in Fig. 6. It can be observed that the estimation result of the EKF is outside the road boundary. The performance is improved in the CMHE with the tracking results being projected on the road boundary. However, it is still quite different from the true trajectory. The most accurate and reasonable tracking result is obtained by the DMHE. Next, we perform numerical evaluations on three algorithms using the root mean square errors (RMSEs) through a hundred Monte Carlo simulations for the same scenario. Figure 7 presents the averaged RMSE time history of the estimated position of each filter (the sampling interval is 0.5s). It can be seen that the DMHE approach achieves the minimum

RMSEs during the majority of times. Besides, the averaged RMSEs for the whole target trajectory by different methods are presented in Table 3. Again, the DMHE achieves the most accurate tracking performance. In comparison to the EKF and the CMHE, the averaged RMSE for position estimation by the DMHE is improved by 66.8% and 27.7%, respectively.

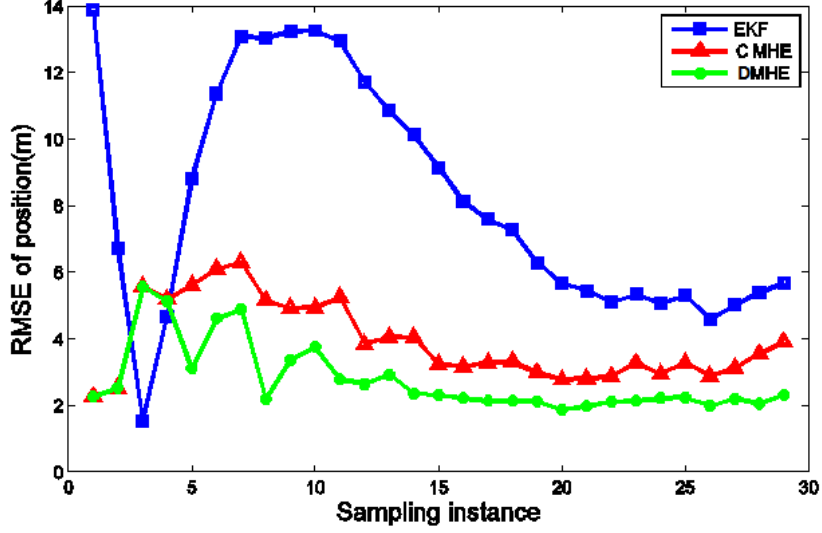


Fig. 7 RMSE of estimated position of EKF, CMHE and DMHE.

Table 3 Averaged RMSEs for position using EKF, CMHE and DMHE.

	EKF	CMHE	DMHE
RMSE (m)	8.8261	4.0494	2.9281

B. Multiple target tracking

1) *Simulation scenario*: The performance of the DMHE-MHT is compared against the MHT, and the constraint MHE-MHT for multiple target tracking. Three vehicles are simulated to move in a realistic region (near Loughborough town in the UK, and the region's geographic information is obtained from the GIS). As shown in Fig. 8, we consider a road intersection scenario with a rectangular region of surveillance, with an unknown and time varying number of targets observed in a clutter environment. The vehicle dynamics is described the same as (21). The two-dimensional Gaussian process noise has covariance matrix Q of 25 m/s^2 . Initially, two targets start moving in the environment: vehicle 1 (shown as the red point) heads to the southwest direction with an initial speed along road one of 12 m/s , it then crosses the intersection and travel on road 3; vehicle 2 (shown as the black point) starts from road 4 heading to the northwest direction with an initial speed along the road network of 8 m/s , it then crosses the intersection and travel on road 2. A new vehicle 3 starts to move three seconds later from road 2 with initial speed of 8 m/s heading to southeast direction and then change its direction at the intersection heading to northeast on road 1. As shown in Fig. 8, tracking

ambiguity occurs during the process around the intersection and on road 1 and 2, which makes the problem challenging.

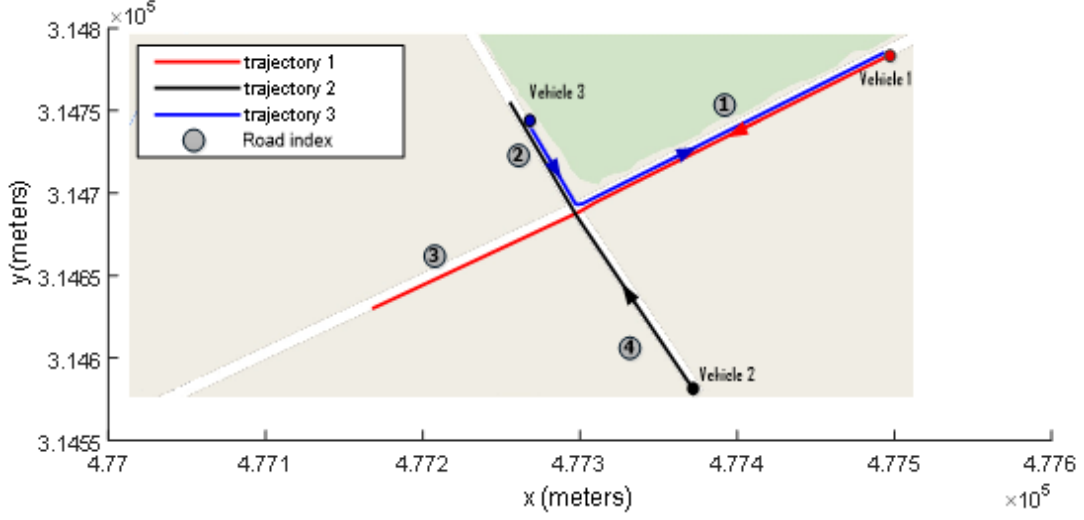


Fig. 8 Multiple target tracking scenario.

The target initial covariance is defined as $P_0 = \text{diag}\{100,100,25,25\}$ for all three targets. Each target is detected with a probability of $P_d = 0.98$. Regarding the range and bearing measurement model in (28), it is assumed that a radar sensor positioned at the bottom right corner. The corresponding measurement noise v_k follows a Gaussian distribution with zero mean and covariance $R = \text{diag}\{25, 2.5^{-3}\}$. The detected measurements are immersed in a high clutter environment that can be modelled as a Poisson distribution with clutter density of $\beta_{FA} = 7.3 * 10^{-5}$ (false alarms/area/scan) over the $1.375 * 10^5 m^2$ region (i.e., clutter returns over the region of interest).

2) *Domain knowledge exploitation*: The speed limitations of the main road (road one and road three along the east-west direction) and side road (road two and road four along the north-south direction) are 40 miles/hour and 30 miles/hour, respectively. The road constraints are applied to constrain the vehicle positions and measurements. In addition to physical constraints, different target interactions with the environment are considered including interaction between: i) the vehicle and road boundary, ii) the vehicle speed and speed limitation, and iii) vehicle in the minor road (2 and 4) and the junction (the vehicle in the minor road will slow down when it approaches the junction). Besides, the interactions between moving vehicles are also considered. These interactions are represented by forces, which is defined below:

$$f_{i,j}^{rep4} = \begin{cases} A \cdot \exp\left(\frac{-d_{ij}}{B}\right) n_{ji}, & \text{if } d_{ij} \leq D_t \\ 0, & \text{otherwise} \end{cases} \quad (28)$$

where d_{ij} represents the relative distance between vehicle i and vehicle j in a Cartesian coordinate. A threshold value D_t is defined for interaction force so that repulsive behaviour is activated only if the relative distance d_{ij} is less than D_t .

3) *Parameters setting for the MHE and the MHT*: The lifetime threshold is defined as 5 in the MHT implementation, which means any new target can only be confirmed if successfully detected in 5

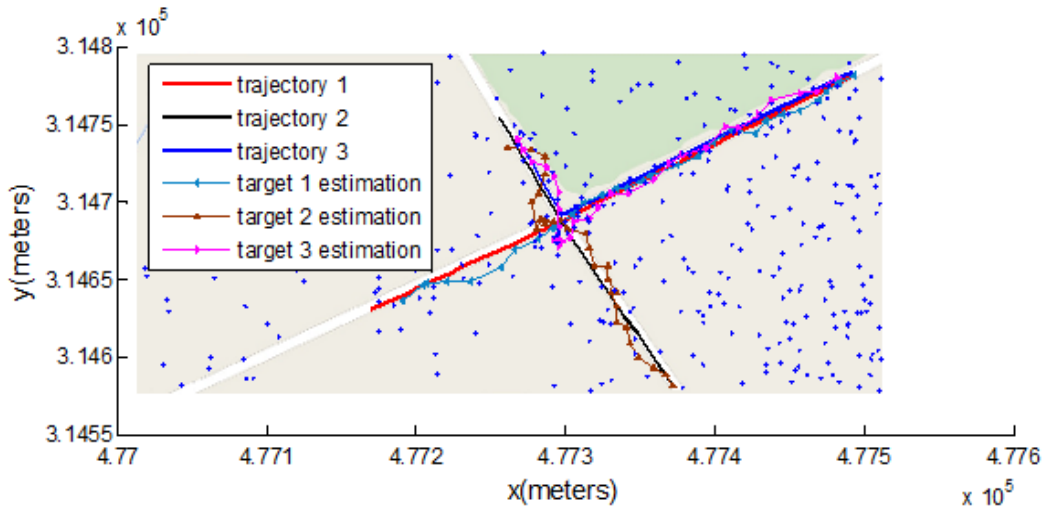
consecutive time steps. Similarly, tracking any existing target will be terminated after miss detection of 5 sequential time steps. The horizon length used in the MHE is set as 4 and so as for N-scan pruning. Since only a small number of targets are considered in this study, at each time step, 3 new hypotheses generated from one existing parent hypothesis are kept so as to reduce the computational cost.

The position estimates are shown in Fig. 9, and it can be shown that the road constraint and force based interaction play a significant part for improving the tracking accuracy. By comparing Fig. 9(a) and (b), we can find that map-based road boundary constraints improves the overall tracking results significantly. Due to the inequality state constraints, the vehicle positions are constrained within the road. The results get even better after introducing the force-based interaction information. In this case, the estimated vehicle trajectories are not only within the road boundaries but also get closer to the real trajectories. For further comparison between different algorithms, 50 trials of Monte-Carlo simulations are performed. The performances of algorithms are measured using the root mean-square error (RMSE). As shown in Table 4, the DMHE-MHT gives the best tracking results for all three targets by considering both road boundary constraint and force-based interaction. A more remarkable performance improvement is obtained for target 3 as it has the most interactions with the road and other incoming vehicles.

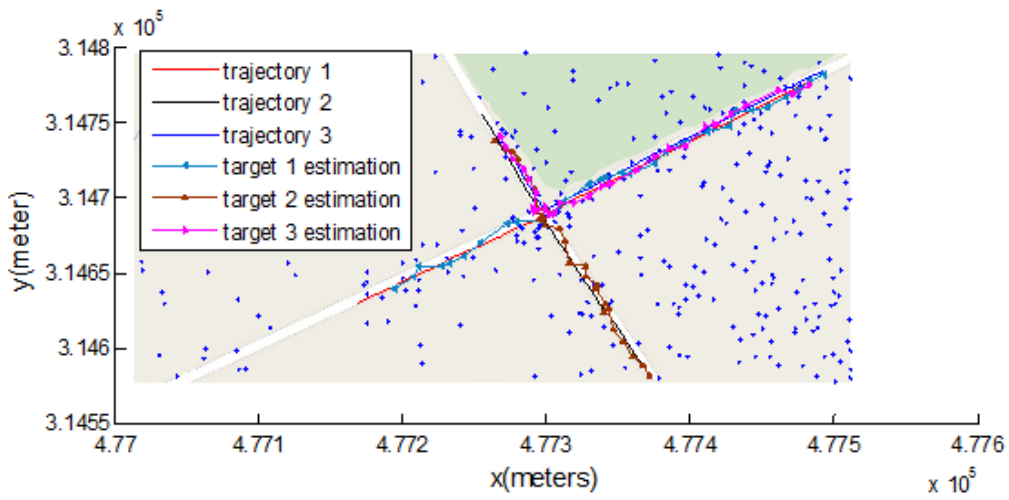
The MHT, the constrained MHE-MHT and the proposed DMHT-MHE are also compared using the optimal sub-pattern assignment metric (OSPA) [27]. The OSPA is proposed for evaluating the performance of multiple target tracking algorithms, which considers not only the estimation performance but also association accuracy. The OSPA metric computes the distance between two sets of tracks by adding the error between target labels (or target indices) to the spatial distance. As can be seen in Fig. 10, the DMHE-MHT has the smallest OSPA value, which represents the smallest estimation error and least amount of incorrect data association. Besides, the proposed DMHE-MHT algorithm performance is more stable than the others by observing the variation of the OSPA distance over time, which presents the smoothest OSPA results.

Table 4 Averaged RMSEs for three vehicles by different approaches.

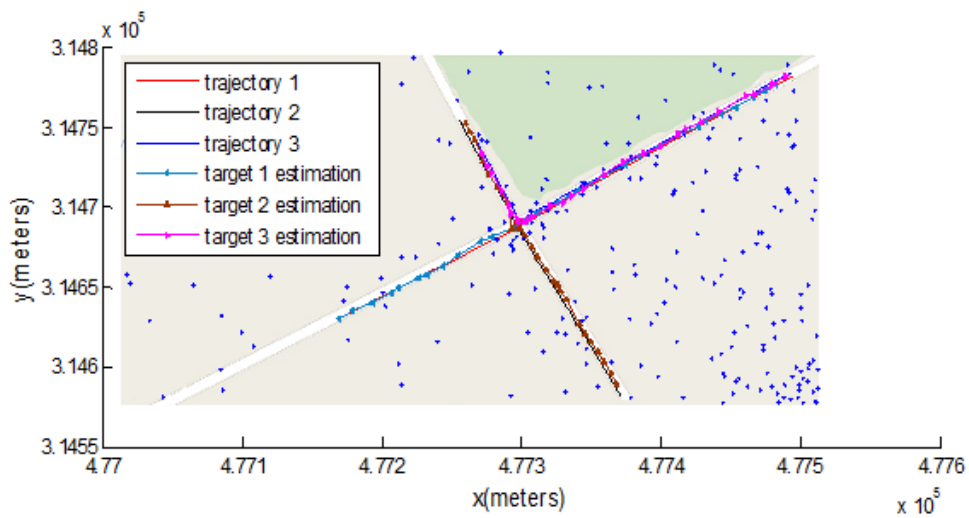
	EKF-MHT	MHE-MHT	DMHE-MHT
Overall MSE position	8.9004	5.6353	5.0077
MSE for Target 1	6.9271	5.4747	5.1271
MSE for Target 2	8.7000	5.3629	4.8760
MSE for Target 3	11.0740	6.0683	5.0200



(a)



(b)



(c)

Fig. 9 Multiple target tracking using EKF-MHT (a); MHE-MHT with road constraint (b); DMHE-MHT with force interaction model and road constraint (c).

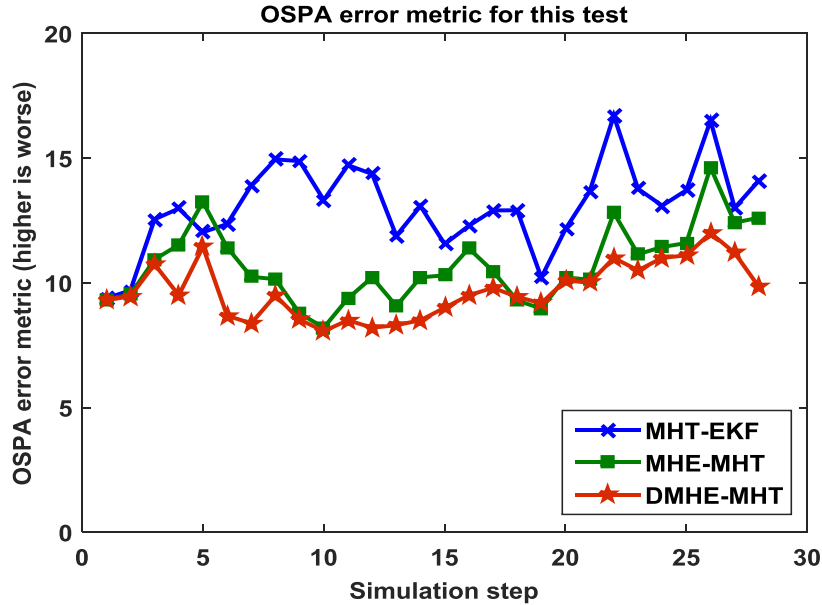


Fig. 10 OSPA for different algorithms.

VI. CONCLUSIONS

This paper has proposed a new model-based ground vehicle tracking method considering domain knowledge in a comprehensive way. In particular, the physical road constraint together with a force-based dynamic model representing interactions between the target and the environment is used in the DMHE target tracking approach. This DMHE is further extended to the DMHE-MHT to deal with target association ambiguity, noisy measurements and multiple road model transition in multiple target tracking. By comparing the DMHE-based approach with traditional constrained state estimation methods using numerical simulation studies, it was shown that a significant improvement can be obtained in terms of target position estimate. Besides, the simulation results also showed that the proposed DMHE-MHT algorithm provides the most accurate tracking performance and robustness for an unknown and time varying number of targets observed in clutter environment using real road map constraint information and force-based target interaction information.

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