Total Impact of Periodic Terms and Coloured Noise on Velocity Estimates

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INTRODUCTION

The uncertainties of velocity estimates for position time series of Global Navigation Satellite System (GNSS) stations are mainly affected by a misfit of the deterministic model applied to this data. Insufficiently modelled seasonal signals will propagate into the stochastic model and falsify the results of the noise analysis besides the velocity estimates and their uncertainties. In this presentation we derived the General Dilution of Precision (GDP) of velocity uncertainties. We define this dilution as the ratio between the uncertainties of velocities determined when different deterministic and stochastic models are applied. In this way we discuss, referring to previously published results, how insufficiently modelled seasonal signals influence station velocity uncertainties with white and coloured noise. Using simulated and real data from selected (115) IGS (International GNSS Service) stations we show that the noise character affects GNSS data more than seasonals for time series longer than 9 years.

DILUTION OF PRECISION (DP)

Blewitt and Lavallée (2002) developed a model to calculate the bias level while one may not account for annual signals:

$$DP=\frac{\sigma_{v_1}}{\sigma_{v_2}}=\left[1-\frac{6}{(\pi\cdot f\cdot\tau)^2}\cdot\frac{\cos(\pi\cdot f\cdot\tau)-\frac{\sin(\pi\cdot f\cdot\tau)}{\pi\cdot f\cdot\tau}}{1-\frac{\cos(\pi\cdot f\cdot\tau)\cdot\sin(\pi\cdot f\cdot\tau)}{\pi\cdot f\cdot\tau}}\right]^{-\frac{1}{2}}$$

where τ is the time span, v_1 and v_2 denotes velocity determined without and with accounting for periodic terms of frequency f, respectively.

Blewitt and Lavallée (2002) considered only white noise, assuming that the stochastic character of residuals ε has no or little impact on the estimated uncertainties. We expect that when a power-law process is added, the time when the difference between both variances is below 5% will increase, as power-law noise provides a better representation of correlation.

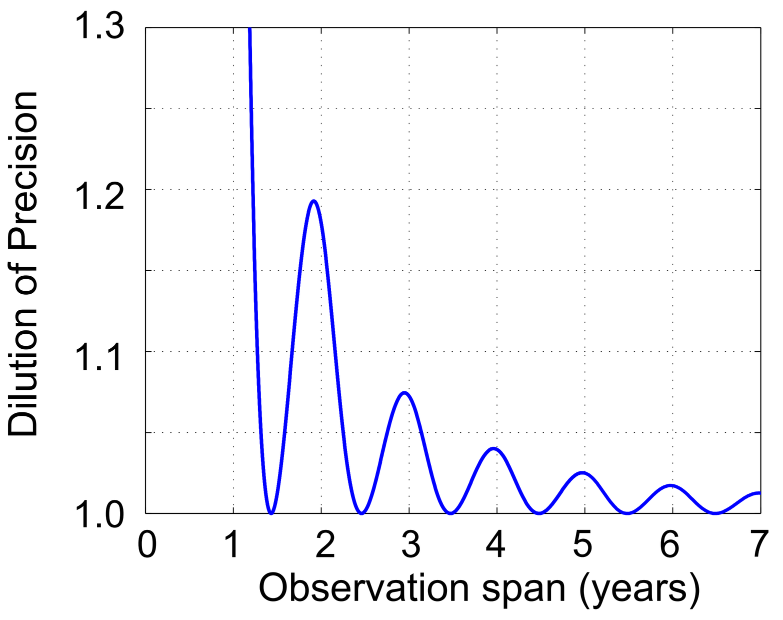


Figure 1: Dilution of precision for models: with linear velocity and linear velocity plus periodic signals (reproduced from Blewitt and Lavallée, 2002):

Bos et al. (2010) discussed the results of Blewitt and Lavallée (2002). They concluded, that the choice of character of stochastic part is much more important than the seasonal part.

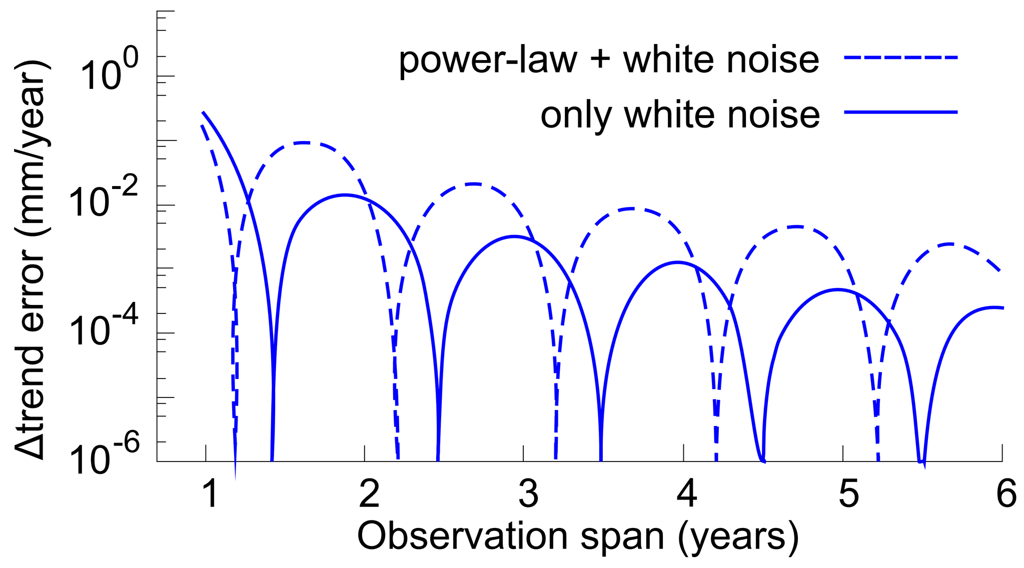


Figure 2: Dilution of precision for models: with linear velocity and linear velocity plus periodic signals with certain noise process applied (reproduced from Bos et al., 2010):

GENERAL DILUTION OF PRECISION (GDP)

A difference between two uncertainties estimated with and without periodic terms can be better understood by computing a ratio between them. We called it the **General Dilution of Precision** (GDP) to be consistent with Blewitt and Lavallée (2002), but taking into consideration power-law noise in the stochastic part. We have adopted two approaches: widely used annual and semi-annual terms to be subtracted (n=2) and the approach consistent with Bogusz and Klos (2015): tropical and draconitic up to their 9th harmonics plus 1st and 2nd Chandlerian (n=20) in:

$$x(t)=x_0+v_x\cdot t+\sum_{i=1}^{n}\left[AC^i\cdot\cos(\omega_i^T\cdot t)+AS^i\cdot\sin(\omega_i^T\cdot t)\right]+\varepsilon(t)$$

The parameters of model are determined by means of Maximum Likelihood Estimation (MLE) where $\mathbf{A}$ is the model matrix, $\boldsymbol{\theta}$ is a vector with parameters of model and $\boldsymbol{\varepsilon}$ is a vector of residuals:

$$\hat{\boldsymbol{\theta}}=\left(\mathbf{A}^T\cdot\mathbf{C}_{\varepsilon\varepsilon}^{-1}\cdot\mathbf{A}\right)^{-1}\cdot\mathbf{A}^T\cdot\mathbf{C}_{\varepsilon\varepsilon}^{-1}\cdot\mathbf{x}$$

The covariance matrix of determined parameters with the covariance matrix $\mathbf{E}$ resulting from power-law process and σ_{pl} and σ_{wn} being the standard deviations of power-law and white noise:

$$\mathbf{C}_{\hat{\theta}\hat{\theta}}=\sigma_{pl}^2\cdot\mathbf{E}(\kappa)+\sigma_{wn}^2\left[\mathbf{A}^T\cdot\mathbf{A}\right]^{-1}$$

SIMULATED TIME SERIES

The simulations were performed up to a maximum length of time series equal to 26 years. We estimated the relative differences in variances of trend between two deterministic models as:

$$\Delta\sigma_v^2=\frac{\sigma_{v1}^2-\sigma_{v2}^2}{\sigma_{v1}^2}$$

σ_{v1}^2 - variance of trend for model with linear velocity,
σ_{v2}^2 - variance of trend for model with linear velocity and periodic terms.

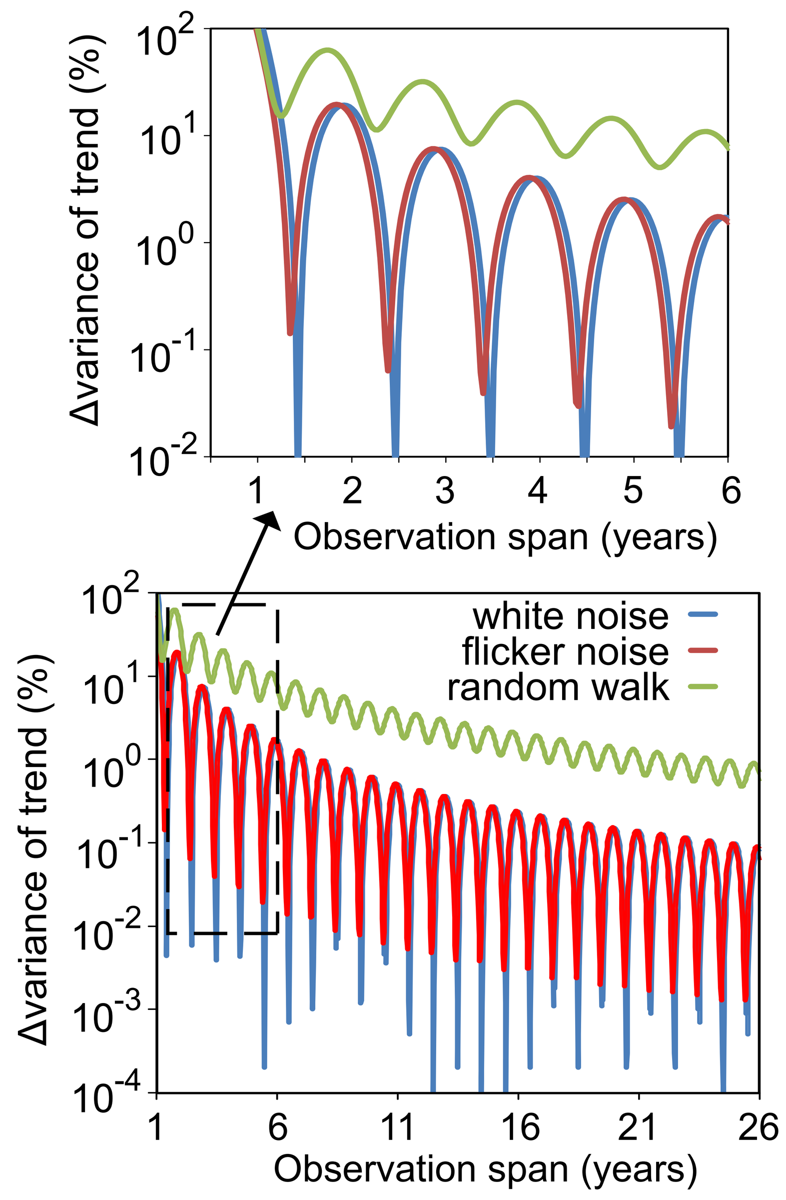
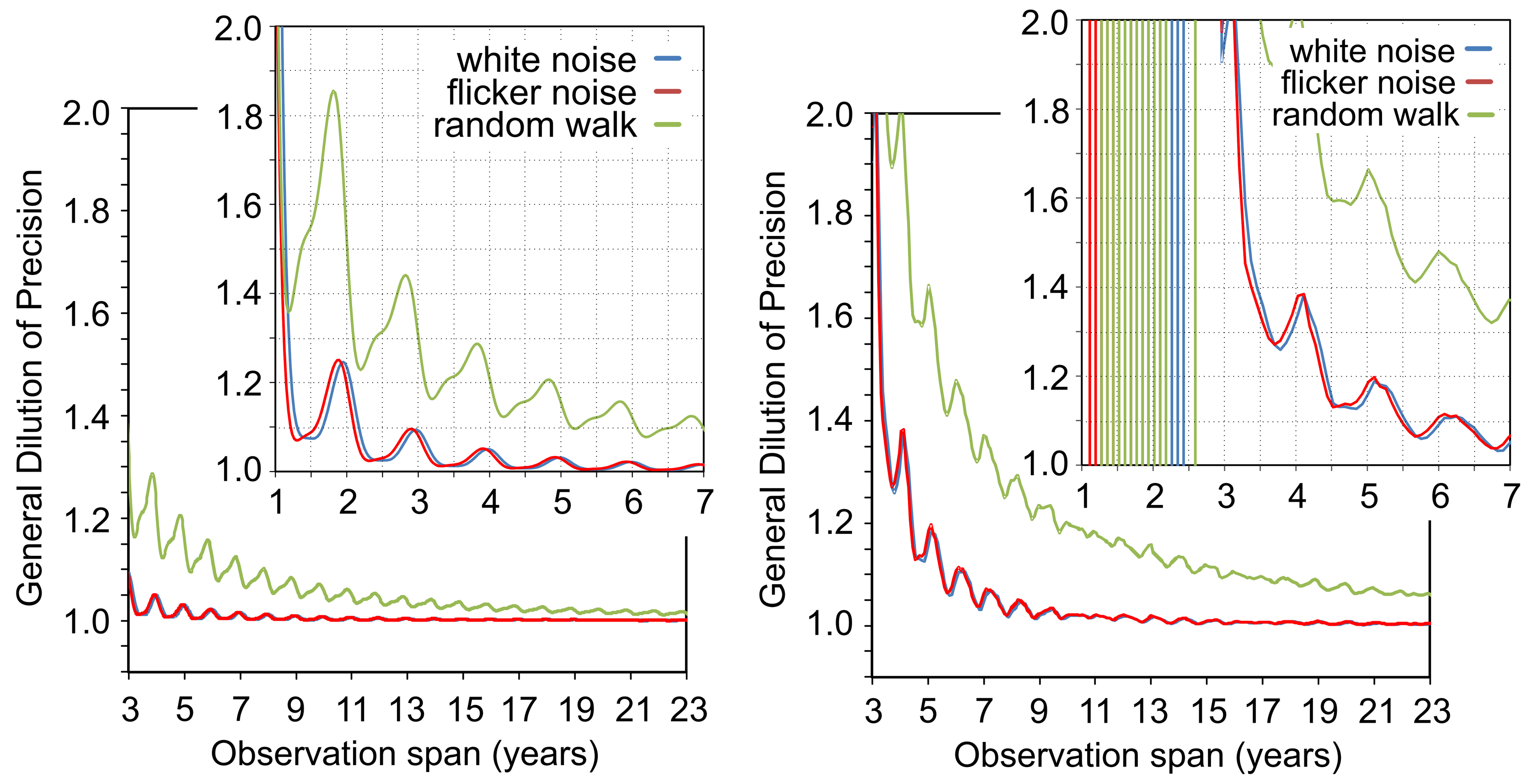


Figure 3: Variance of the slope $\Delta\sigma_v^2$ (in %) for different lengths of time series. The integer spectral indices of white (blue), flicker (red) and random walk (green) processes are examined. Two deterministic models are considered: with linear velocity (σ_{v1}^2) and with linear velocity plus periodic terms (σ_{v2}^2). We assumed that σ_{pl}^2=1 and σ_{wn}^2=1.

Figure 4: A General Dilution of Precision (GDP) for white noise (blue), flicker noise (red) and random walk (green) plotted for deterministic model containing a linear trend plus:

annual and semiannual *annual and draconitics up to their 9th harmonics plus 1st and 2nd Chandlerian (Bogusz and Klos, 2015):*



GNSS DATA

Blewitt and Lavallée (2002) theoretically predicted the minimum velocity bias at integer-plus-half years, while Bos et al. (2010) found minimal influence closer to integer-plus-a-quarter year by considering coloured noise. Following those two conclusions we started a simulation with a minimum length of 2 years and extended it with an increment of 0.25 years. We used real data from selected (115) IGS (International GNSS Service) stations.

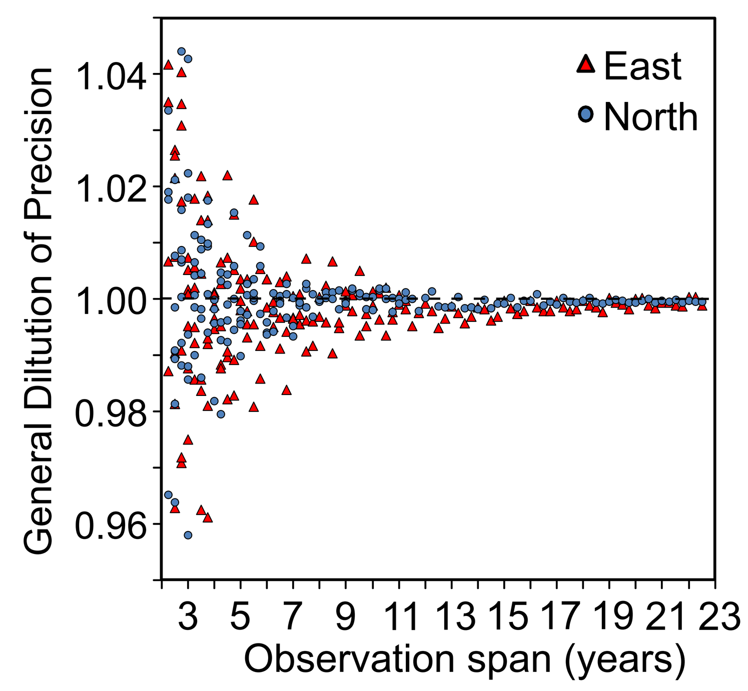


Figure 5: The General Dilution of Precision as a function of the observation span for MATE (Matera, Italy), North (blue dots) and East (red triangles) components:

DISCUSSION

1. Along with the increasing spectral index, the amplitudes of oscillations also increase. This arises from the fact that any power-law process with κ<0 brings a correlation between amplitudes of seasonal terms and velocity. In this way, the GDP value is much higher for any time series length considered.
2. The strong peaks of oscillations may be indicated for short time scales, especially for random-walk case. On the other hand, the assumed oscillations play here a significant role, even much more important than assumed noise character. The noise character starts to become important for data longer than 9 years.
3. The local minima and maxima are also being enlarged together with a change of spectral index into -2. This shows, that the GDP differs from integer-plus-half years by Blewitt and Lavallée (2002), who considered only white noise. This is clearly noticed for random-walk noise.
4. With increasing spectral index, the General Dilution of Precision decreases more slowly.
5. Blewitt and Lavallée (2002) used the value of 5% to calculate the minimum velocity bias. However, this value is disputable. With the ever increasing demands on velocities, we argue that even a change of 2% in GDP could be considered as significant. The value of 2% results from the median ratio of the horizontal velocity error to the velocity itself as derived from real GNSS data. This means, that 13 years of continuous observations is enough to make GDP to decrease below 2% when white and flicker noise were assumed. However, when random walk is present, this time is as long as 48 years. This span of observations enables to omit periodic oscillations in the GNSS-derived time series and take into consideration only appropriate noise model.

REFERENCES

Blewitt G., Lavallée D., (2002): Effect of annual signals on geodetic velocity, J. Geophys. Res. 107 (2002) 2145.
Bogusz J., Klos A. (2015): On the significance of periodic signals in noise analysis of GPS station coordinates time series. GPS Solutions, doi:10.1007/s10291-015-0478-9.
Bos, M., Bastos, L. and Fernandes, R.M.S. (2010): The influence of seasonal signals on the estimation of the tectonic motion in short continuous GPS time-series. J. Geodyn, 49, 205-209, doi:10.1016/j.jog.2009.10.005.
Bos, M.S., Fernandes, R.M.S., Williams, S.D.P. and Bastos L. (2008): Fast Error Analysis of Continuous GPS Observations. J. Geod (2008) 82, pp. 157-166, doi:10.1007/s00190-007-0165-x.

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