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Fundamental uncertainty and unconventional monetary policy: an info-gap approach

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ABSTRACT

This paper applies the info-gap approach to the unconventional monetary policy of the Eurosystem and so takes into account the fundamental uncertainty on inflation shocks and the transmission mechanism. The outcomes show that a more demanding monetary strategy, in terms of lower tolerance for output and inflation gaps, entails less robustness against uncertainty, particularly if financial variables are taken into account. Augmenting the Taylor rule with a financial variable leads to a smaller loss of robustness than taking into account the effect of financial imbalances on the economy. However, in some situations, the augmented model is more robust than the baseline model. A conclusion from our framework is that including financial imbalances in the monetary policy objective does not necessarily increase policy robustness, and may even decrease it.

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1. Introduction

Central banks have used different types of unconventional monetary policy measures to support monetary transmission and raise inflation. As pointed out by Borio and Disyatat (2010), the distinguishing feature of these measures is that the central bank actively uses its balance sheet to affect market prices and conditions beyond a short-term interest rate. A crucial question is to what extent these measures are effective in supporting the central bank objectives. This depends on the working of the transmission channels of unconventional balance sheet policies and on the drivers of inflation.

Both factors are surrounded by an unusual dose of uncertainty, in particular in the euro area. It faces major external shocks in a changing global macroeconomic and financial environment, a highly fragmented and changing monetary transmission mechanism, and an unprecedented combination of high private and public debt. The severe uncertainty on inflation dynamics and the untested impacts of new monetary tools may imply that the system no longer operates under measurable probabilistic uncertainty (risk) but under Knightian (fundamental) uncertainty where probability distributions are less informative or even lacking. In such a situation, an approach for managing Knightian uncertainty is more appropriate than aiming at an optimal outcome (eg a specific inflation target) based on probabilistic models.

Two uncertainty management strategies for managing Knightian uncertainty that have emerged in the literature are robust control and info-gap. The former insures against the maximally worst outcome as defined by the policy maker (min-max, see Hansen *et al*, 2006; Sargent and Hansen, 2008 and Williams, 2007). Typically policies derived are more aggressive by comparison to those under no uncertainty. The European Central Bank's approach looks like a min-max strategy (to quote Draghi, 2016: *"there are no limits to how far we are willing to deploy our instruments within our mandate to achieve our objective …"*). With little prior knowledge about mechanisms at work, proponents of this approach justify such aggressive action as the only way to learn. Two objections have been raised about such an approach; (i) policymakers do not like experimenting for the purposes of learning; (ii) worst events are rare and hence poorly known. It is odd therefore, to design a policy that is focussed precisely on those events about which one knows least (Sims, 2001).

However, the most important drawback in our view is that robust control does not account for the fundamental choice between robustness against uncertainty on the one hand, and aspiration for high-value outcomes, on the other. This is where the alternative approach, info-gap, makes an important contribution by mapping explicitly this trade-off (Ben-Haim, 2006, 2010). If the central bank adopts an ambitious inflation target, it needs to compromise on the degree of confidence in achieving it (Ben-Haim and Demertzis, 2008). Conversely, if the central bank requires high confidence in achieving specified goals, it needs to moderate how ambitious these goals are. Info-gap theory quantifies this intuitive trade off.

In this paper we apply the info-gap approach to the unconventional monetary policy as pursued by the Eurosystem. Since the effectiveness of these policy measures depends to a large extent on financial channels, the reaction function of central banks has become increasingly oriented to financial variables. Estimations of a Taylor rule for the euro area indicate that since the 2007-08 crisis, unconventional monetary policy has significantly reacted to financial developments, next to the inflation and output gap (Pattipeilohy *et al*, 2016). The influence of balance sheet policies on financial markets also implies that monetary policy potentially has adverse side effects in the financial sphere. By encouraging financial risk taking for instance, quantitative easing may contribute to financial imbalances and excessive asset price developments (Van den End, 2015). Since such effects may become manifest in the long run, they are particularly uncertain.

We model both aspects of unconventional monetary policy in a macroeconomic framework. For that purpose, we define four semi-structural models. Rule 0, our benchmark model, is a standard macro model that has a Phillips curve, an aggregate demand curve and a traditional Taylor rule. Rule 1 extends the benchmark model by augmenting the Taylor rule with a financial variable, by which monetary policy reacts to financial stress. In Rule 2 the concept of financial imbalances is introduced by including a debt variable in the demand curve. This takes into account the long-term implications of unconventional monetary policy for the economy that become manifest through the debt channel. Rule 3 is the full model that includes both the augmented Taylor rule and financial imbalances. The models are used to simulate shocks to inflation that are uncertain in magnitude and time. Both the fundamental uncertainty on the shocks and on the transmission of unconventional monetary policy (as reflected in the model parameters) is assessed by the info-gap approach. This reveals which rule is most robust to uncertainty.

Our paper relates to the ongoing debate in the literature on whether monetary policy should take into account financial stability objectives, or should leave these to macroprudential policy (see Smets, 2014, for an overview). The different positions in this debate have been defended on theoretical and empirical grounds by augmenting macro-economic models, monetary policy rules in particular, with financial variables (eg by Svensson, 2016; Gambacorta and Signoretti, 2014; Gourio *et al*, 2016). While according to Stein (2014), measures of risk premiums may be useful inputs into the monetary policy framework, he concludes that there is a way to go – in terms of modelling and calibration – before it can be used to make quantitative statements. This comes close to the starting point in our paper that the process that is being modelled is prone to Knightian uncertainty. Ajello *et al* (2016) follow a similar reasoning in their standard new-Keynesian model augmented with an endogenous financial crisis event. They assume fundamental uncertainty on the model parameters (with regard to monetary transmission) and the shock (severity of crises). Based on a robust-control approach they conclude that optimal policy can call for larger adjustments to the policy rate than in a situation without financial stability concerns. This more aggressive policy is consistent with the min-max strategy as explained above. But the question that arises is whether policies derived are also robust to uncertainty, ie to a wide spectrum of driving events. Can these types of policies be shown to also do well under different circumstances to the ones assumed?

We address this research question by the info-gap approach, to take into account the fundamental choice between robustness against uncertainty versus the aspiration for high-value outcomes. This method accounts for the fact that more demanding performance (smaller acceptable output and inflation gaps) entails less robustness against uncertainty. Our results show that for all four policy rules, the cost of robustness increases significantly if the simulation horizon is extended, causing much lower robustness. This is natural because uncertainty propagates and magnifies over time. Comparing the rules, it appears that the benchmark model is most robust to uncertainty. Its parsimonious specification, that excludes the uncertain effects of financial stress and debt on the macroeconomy and monetary policy, reduces the vulnerability to modelling uncertainty. Augmenting the Taylor rule with a financial variable (Rule 1) leads to a loss of robustness, but less than in Rule 2, which includes financial imbalances in the demand equation. The preference reversals between the policy rules are generally similar with and without shocks and considering only parameter uncertainty. This holds for the preference reversal between Rules 2 and 3, and the robust dominance of the benchmark model.

For central banks these outcomes imply that including financial stability considerations in the monetary policy framework is challenging, given the Knightian uncertainty on the dynamics of financial variables and their interaction with the real economy. Modelling such complexities requires deep knowledge of the underlying structures. However, taking into account uncertainty means that such knowledge by definition is incomplete or missing. So while complicated models may seem better in an optimal sense, more simple rules that manage uncertainty in small and concrete steps are preferable in a robust sense, in line with the literature on heuristics (eg Gigerenzer *et al*, 2011). For monetary policy this means that price stability should remain the primary objective of monetary policy, particularly given the uncertainties on the long run effects of financial imbalances on the economy. However, the central bank could consider including financial stress its reaction function, which de facto has been done by the Eurosystem since the crisis started in 2007. The simulations show that the loss of robustness is limited in that case, which may be acceptable for the policymaker given the gain in reaching his objectives. The clear loss of robustness in case a debt measure is taken into account calls for macroprudential policies that should address financial imbalances.

Beyond these specific policy implications, this paper demonstrates a methodology that can be applied to policy evaluation and selection in a wide range of further studies. For example, one can explore the advantages (in terms of enhanced robustness to uncertainty) of different coefficients in the Taylor rule. Or, one can examine the implications of uncertain temporal behaviour of shocks to inflation or to other variables. One can examine the robustness of larger and more comprehensive models. In short, the contribution is both policy-oriented and methodological.

In the next section the framework underlying the rules is specified. In section 3 the putative outcomes of the model estimations are presented. Section 4 formulates the info-gap method, including the performance requirement, definitions of uncertainty and robustness. Section 5 shows the model simulations and robustness curves for the benchmark model, including uncertainty on the value of the coefficients. This is extended in section 6 for all four Rules. Section 7 introduces uncertainty on the shocks to inflation in addition to parameter uncertainty. Section 8 discusses the results and section 9 concludes.

2. Model framework

The benchmark Rule 0 is a standard macroeconomic model that summarises the behaviour of producers, households and the central bank. The model has a Phillips curve, an aggregate demand curve and a traditional Taylor rule. We assume that the policy rate is not bounded by zero and that below zero it reflects a shadow rate that captures the effect of unconventional monetary policy measures. The benchmark model is extended with an augmented Taylor rule (Rule 1), with debt in the demand equation (Rule 2) and with a combination of both extensions (Rule 3). The latter – most extended model - is presented by the following four basic equations,

$$\hat{\pi}_t = \alpha_1 + \beta_1 \hat{\pi}_{t-1} + \beta_2 \hat{y}_t \tag{1}$$

$$\hat{y}_t = \alpha_2 + \beta_3 \hat{y}_{t-1} + \beta_4 r_{t-6} + \beta_5 \omega_{t-2} + \beta_6 D\omega_{t-4} \tag{2}$$

$$i_t = \alpha_3 + \beta_7 \hat{\pi}_{t-1} + \beta_8 \hat{y}_{t-1} + \beta_9 f_t \tag{3}$$

$$\omega_t = \alpha_4 + \beta_{10} \left(i_{t-1} - \bar{i}_t \right) \tag{4}$$

These equations are augmented with the following supplemental equations to complete the full dynamic specification,

$$r_t = i_t - \pi_t - i^n \tag{5}$$

$$\pi_t = \hat{\pi}_t + \pi^* \tag{6}$$

$$\bar{i}_t = \frac{1}{16} \sum_{j=t-16}^{t-1} i_j \tag{7}$$

$$D = \begin{cases} 1 & \text{if} \quad |\omega_t| \geq 2 \\ 0 & \text{else} \end{cases} \tag{8}$$

$$f_t = \alpha_5 + \beta_{11} f_{t-1} \tag{9}$$

where $i^n = 1$ and $\pi^* = 2$.

Eq. 1 is the Philips curve, with $\hat{\pi}_t$ the inflation gap and $\hat{y}_t$ the output gap. The former is the inflation rate π_t as deviation from its target π^*. Eq. 2 links aggregate demand to the real interest rate r_t (being the nominal policy rate i_t – minus the inflation rate π – as deviation from the natural rate i^n as in eq. 5). We take a 6-quarters lagged interest rate, taking into account the usual lags in the monetary transmission process. A downward deviation of the policy rate from the natural rate has an expansionary effect on output and vice versa for an upward deviation. Beyond the lower bound, the policy rate *i* equals a shadow rate, as determined by Pattipeilohy *et al* (2016). The shadow rate is an indicator for the effect of unconventional monetary policy on the monetary stance (eg Krippner, 2013; Lombardi and Zhu, 2014). It measures the effect of quantitative easing and forward guidance on the expectations component and the term premium component of bond yields.

In Rules 2 and 3 the demand equation includes debt (ω_t), as defined in eq. 4 by the debt-to-GDP gap (deviation of debt-to-GDP ratio from trend). An increasing debt relaxes the budget constraint (reflecting the credit channel of monetary transmission as in Guerrieriy and Iacoviello, 2015), but when the debt ratio rises beyond a critical level it can become a (non-monotonic) constraint on spending. Both features are included in eq. 2, with $D\omega_t$ representing a critical high debt gap. Dummy variable *D* = 1 when the absolute value of the credit-to-GDP gap is larger than a threshold value as in eq. 8. Our approach is based on Borio *et al* (2011), who find that at a certain threshold of the credit-to-GDP gap, this variable performs best as early warning indicator of financial crises[1]. The early warning property is reflected in the 4-quarter lag of $D\omega_t$. Borio *et al* (2016) also include the credit-to-GDP gap in an extended demand equation and find – in estimates for the US – that this variable is informative for the output gap.

Eq. 3 is a backward-looking Taylor rule, which assumes that the Eurosystem reacts to the inflation gap and the output gap. In Rules 1 and 3, the Taylor rule is augmented with a financial stress variable (*f*) to which the central bank is assumed to react with its policy rate or with unconventional measures (variable *f* is modelled as a naïve autoregressive process of the financial sectors' credit spread in eq. 9). Those measures are reflected in the shadow rate *i*. It is likely that monetary policy is relaxed when financial stress is high (eg non-performing loans or credit spreads in financial markets) and vice versa. In terms of the model, this represents the short-term response of the central bank to financial variables. The reason for the central bank to respond to financial stress is that stress may negatively impact on inflation through confidence or wealth effects. So the Taylor rule reflects the response of the central bank to cyclical, more short-term, developments in the economy and financial markets. In augmenting the Taylor rule with a credit spread variable we follow Woodford (2010) and Taylor and Zilberman (2016).

Rules 2 and 3 also include eq. 4, describing the potential side-effects of monetary policy as summarised in the credit-to-GDP gap ω_t (an alternative proxy would be an asset price gap). This reflects the debt overhang, as driven by the deviation of the nominal policy rate (or shadow rate) from its trend rate ($\bar{\iota}$, which is defined in eq. 7). This deviation reflects the monetary policy stance, measured over a longer-term horizon. Following Borio and Disyatat (2011) and Borio *et al* (2016), we assume that a large and persistent gap of the policy rate from the equilibrium rate gives rise to an unsustainable expansion in credit in the longer run (i.e. the undesired effects of unconventional monetary policy). The central bank may want to prevent a large debt overhang, given its indicator properties for financial crisis. A sudden unwinding of such imbalances may go in tandem with financial stress, as reflected by variable *f* in the Taylor rule. In Rules 2 and 3 the financial imbalances are a determinant of the output gap in eq. 2, through which the central bank indirectly takes the side-effects of monetary policy into account in its strategy, as the output gap is part of the loss function in eq. 5. Here we depart from models that include debt in the Taylor rule (as in Gourio *et al*, 2016), assuming that monetary policy reacts foremost to debt developments that would affect the real economy.

[1] Based on a sample of 36 countries, Borio *et al* (2011) find that a threshold of the credit-to-GDP gap is 2 to 2.5 percentage points has the highest prediction power for financial crises (credit-to-GDP gap based on deviation from HP filtered trend with lamda 1,600).

Based on the model, we specify the loss function that evaluates economic outcomes,

$$L = \lambda_\pi \hat{\pi}_t^2 + \lambda_y \hat{y}_t^2 \qquad (10)$$

The central bank fulfils its mandate by choosing an interest rate (and/or unconventional monetary policy measures as reflected in the shadow rate) to obtain adequately low values for the quadratic loss function. It penalises the inflation gap and the output gap, with coefficient λ being the relative weight that the central bank gives to the objectives. Monetary policy takes into account financial developments through its response to financial stress in the Taylor rule and through the effect of financial imbalances on the output gap, which enters the loss function.

These equations, together with known values for $\hat{\pi}_t$, $\hat{y}_t$, ω_t and i_t for quarters $t = 1, \ldots, 16$ enable computation of $\hat{\pi}_t$, $\hat{y}_t$, i_t and ω_t for $t = 17, 18, 19, \ldots$. This is the basis for evaluating the robustness to uncertainty of an interest rate rule. The coefficient values used in eq. 10 are $\lambda_\pi = 1$ and $\lambda_y = 0.5$.

3. Putative estimation outcomes

We estimate the model assuming no fundamental uncertainty on the parameters ('putative model'). Each model variant, Rules 0 to 3, is estimated as a system of equations by GMM. The system estimator uses more information than a single equation estimator (ie the contemporaneous correlation among the error terms across equations) and therefore will produce more precise estimates. We do not impose cross-equation restrictions. GMM takes into account the interdependencies among the equations in the model, while controlling for the endogeneity of regressors and for the correlation between the lagged dependent variables and the error terms[2]. The model is estimated for the euro area over the period 1990-2015 (quarterly observations), see Appendix 1 for a detailed description of the data.

3.1 Rule 0: benchmark model

The estimation outcomes of Rule 0 (benchmark model) in column 1 of Appendix 2 show that most coefficients are significant and have the expected sign. The low J-statistic indicates that the model is well specified. In the Phillips curve, the inflation gap has a significant relationship with its own lag and

[2] The model is estimated by heteroskedasticity and autocorrelation consistent GMM (HAC), applying pre-whitening to soak up the correlation in the moment conditions.

with the output gap (the positive sign of the coefficient means that a more positive output gap leads to higher inflation and vice versa). The coefficient of the real interest rate in the demand curve (the interest rate channel) is negative as would have been expected, although not significant. The Taylor rule estimate shows that monetary policy reacts stronger to the inflation gap than to the output gap (given that the coefficient of the former is larger than the coefficient of the latter, which is insignificant). This is in line with the single inflation mandate of the Eurosystem. Parameter β_7 is somewhat higher and parameter β_8 is somewhat smaller than is generally assumed in the literature (coefficient for inflation gap 1.5 and for the output gap 0.5).

3.2 Rule 1: augmented Taylor rule

Rule 1 has a Taylor rule augmented with financial variable *f*. So in this model version the central bank reacts to the inflation gap, the output gap and to financial stress. Column 2 in Appendix 2 provides empirical evidence that the central banks indeed reacts to financial stress. The coefficient of variable f_t is significantly negative, meaning that the interest rate (or shadow rate) is reduced in response to rising stress and vice versa. This reflects the short-term response of monetary policy to financial developments.

3.3 Rule 2: financial imbalances

Rule 2 includes the costs of side-effects, as captured by the credit gap ω_t. Column 3 in Appendix 2 shows that the debt variable ω_t has a significant positive effect on the output gap - in line with the credit channel - meaning that higher borrowing (driven by a lower interest rate in eq. 4) positively affects output and vice versa. The significant negative coefficient of the interaction term $D\omega_t$ in the demand equation indicates that an excessively high debt ratio constrains spending. Compared to the benchmark model, the coefficient of the real interest rate in the demand curve is significant. The justification for this is that the impact of monetary policy on aggregate demand to an important extent runs through the credit channel. By including the debt variable ω_t in the model, the interest rate significantly affects aggregate demand. In eq. 4, the credit-to-GDP gap (ω_t) is significantly related to the interest rate gap, implying important side effects. The coefficient is negative, implying that a looser monetary stance goes in tandem with an increasing debt and vice versa. The credit-to-GDP gap affects the output gap in eq. 2, which captures the credit channel.

3.4 Rule 3: augmented Taylor rule and financial imbalances

Rule 3 is the most extended model, including a Taylor rule augmented with the financial variable and debt in the demand equation. The estimation outcomes in column 4 in Appendix 2 show that all coefficients in the demand equation and the Taylor rule are significant. The latter provides empirical evidence that the central bank reacts to the inflation and output gaps, as well as to financial stress. Through the shadow interest rate, this monetary policy also drives financial imbalances in eq. 4 (i.e. the debt ratio), which in their turn determine aggregate demand in eq. 2. Similar to Rule 2, the debt variable and the interaction term both have a significant effect on aggregate demand.

4. Info-Gap formulation

We use the model in the linear difference eqs. (1)-(4), with the supplemental eqs. (5)-(9), with uncertain inputs (shocks) on the inflation equation only. The output of the dynamic model is the loss function L, eq.(10), which depends on the coefficients, the shocks and the policy choice.

4.1 Performance requirement

The current time, in quarterly increments, is $t = t_1$, and we require that the loss function in eq.(10) does not exceed a critical value, L_c, at specified later time t_2 in the future:

$$L(t_2) \leq L_c \tag{11}$$

In other words, given the current and past values of the state variables, our time horizon of interest is $t = t_1, \ldots, t_2$ where $t_1 = 17$.

We note that eq.(11) is a satisficing requirement: it specifies a policy goal that must be reached (loss no greater than L_c). This is different from an outcome optimisation, which would seek to minimise the loss. Simon (1997) stressed the importance of satisficing when facing uncertainty. Furthermore, solutions that satisfice are usually under-determined, precisely because they are sub-optimal. This means that multiple satisficing policies are available. The policy selection strategy advocated here is to choose the satisficing policy that maximises the robustness. This robust-satisficing strategy thus optimises something (the robustness) rather than optimising the substantive outcome (the loss).

4.2 Info-gap models of uncertainty

We consider uncertainty both in some of the coefficients of the difference equations, and in the time and magnitude of the shocks to the inflation equation. We first consider uncertainty in the coefficients of the difference equations. Let c denote a vector of N uncertain coefficients (eg α_1, α_4, etc.). Let $\tilde{c}_n$ denote the putative best estimate of c_n, where σ_n is the standard error of the estimate of $\tilde{c}_n$ (the standard errors as shown in Appendix 2). A fractional-error info-gap model for uncertainty in the coefficients is:

$$V(h) = \left\{ c: \quad \left| \frac{c_n - \tilde{c}_n}{\sigma_n} \right| \le h, \text{ for all } n = 1:N \right\}, \quad h \ge 0 \tag{12}$$

This info-gap model is an unbounded family of nested sets, $V(h)$, containing possible values of the vector c of uncertain coefficients. In Rules 2 and 3 the coefficients α_1, α_2 and α_4 (the elements of vector c) are treated as info-gap uncertain and in Rule 0 and 1 the coefficients α_1 and α_2 are treated like that. These coefficients are chosen because they have the highest standard error in the estimation. As an alternative to this data-driven approach, one could opt for choosing parameters that have the highest uncertainty from an economic viewpoint.

Like all info-gap models, the model in eq. 12 displays the properties of contraction and nesting. Contraction asserts that $V(0) = \{\tilde{c}\}$. This means that, in the absence of uncertainty, namely when $h = 0$, the only possible coefficient vector is the best estimate, $\tilde{c}$. Nesting means that the uncertainty sets, $V(h)$, become more inclusive as the horizon of uncertain, h, gets larger: $h < h' \quad \Rightarrow \quad V(h) \subseteq V(h')$. The property of nesting endows h with its meaning as the horizon of uncertainty.

Now we consider uncertainty in the inflation shocks. Our approach will be to consider a shock as an input of uncertain magnitude and time but of duration of only a single time step. The number of such shocks is also uncertain. The overall strategy will be to evaluate the robustness for a single shock of uncertain time and magnitude, and then to find the least-robust shock-times for this shock. A future shock occurring at time t_c is represented as a function of t of the form:

$$\varepsilon_{t|t_c} = \begin{cases} \varepsilon^0, & \text{if } t = t_c \\ 0, & \text{else} \end{cases} \tag{13}$$

That is, $\varepsilon_{t|t_c} = \varepsilon^0$ at time $t = t_c$ and equals 0 at all other times.

The putative best estimate of the shock amplitude is $\varepsilon^0 = \tilde{\varepsilon}^0$. This would presumably be a typical historical value for shock amplitudes. For instance, it could equal the standard deviation of the shock in the inflation equation (0.278 in Table 1), or it could be taken as the historical standard deviation of the inflation series (1.13 in our series, which is the value we use subsequently).

The info-gap model for uncertainty in the amplitude of a single inflation shock at time t_c is:

$$U_1(h, t_c) = \left\{ \varepsilon_{t|t_c} : \quad \left| \frac{\varepsilon^0 - \tilde{\varepsilon}^0}{s} \right| \le h \right\}, \quad h \ge 0 \tag{14}$$

where $\varepsilon_{t|t_c}$ refers to the function in eq.(13) and s is an error estimate of the shock amplitude. For example, s could be a measure of typical deviation of historical inflation shock amplitudes, such as the standard deviation of the inflation series. We assume s to be one third of the estimated shock amplitude.

We now consider M shocks at times $t_{c,1},\ldots,t_{c,M}$ where ε_m^0 is the uncertain amplitude of the mth shock. The info-gap model for uncertainty in the amplitudes of these shocks is the following direct extension of eq.(14):

$$U_M(h, t_{c,1}, \ldots, t_{c,M}) = \left\{ \varepsilon_{t|t_{c,m}},\, m = 1, \ldots, M : \quad \left| \frac{\varepsilon_m^0 - \tilde{\varepsilon}^0}{s} \right| \le h \right\}, \quad h \ge 0 \tag{15}$$

4.3 Robustness functions

We now define several different robustness functions. First consider a single inflation shock at time t_c. The robustness with respect to uncertainty in the coefficients, eq.(12), and uncertainty in the amplitude of a shock at time t_c, eq.(14), is the greatest horizon of uncertainty, h, up to which the requirement in eq.(11) is always satisfied:

$$\hat{h}_1(L_c, t_c) = \max\left\{ h : \ \left(\max_{c \in V(h),\, \varepsilon_{t|t_c} \in U_1(h, t_c)} L(c, \varepsilon_{t|t_c}) \right) \le L_c \right\} \tag{16}$$

The time of the inflation shock is uncertain, so the robustness to a single shock at some unknown time is the minimum of $\hat{h}_1(L_c, t_c)$ over the time horizon of concern:

$$\hat{h}_1(L_c) = \min_{t_1 \le t_c \le t_2} \hat{h}_1(L_c, t_c) \tag{17}$$

We now consider the robustness to M inflation shocks at times $t_1 \le t_{c,1} < t_{c,2} < \cdots < t_{c,M} \le T$. In analogy to eq.(16) but using the info-gap model in eq.(15), we define the robustness as:

$$\hat{h}_M(L_c, t_{c,1}, \ldots, t_{c,M}) = \max\left\{ h : \left(\max_{c \in V(h),\, \varepsilon_{t|t_{c,m}} \in U_M(h)} L(c, \varepsilon_{t|t_{c,1}}, \ldots, \varepsilon_{t|t_{c,M}}) \right) \le L_c \right\} \quad (18)$$

The times of the M inflation shocks are uncertain, so the robustness to these shocks at unknown times $t_{c,1}, \ldots, t_{c,M}$ is the minimum of $\hat{h}_M(L_c, t_{c,1}, \ldots, t_{c,M})$ over the time horizon of concern:

$$\hat{h}_M(L_c) = \min_{t_1 \le t_{c,1} < t_{c,2} < \cdots < t_{c,M} \le t_2} \hat{h}_M(L_c, t_{c,1}, \ldots, t_{c,M}) \quad (19)$$

Finally, we define the robustness to uncertainty in the coefficients assuming that no shock occurs. This is obtained from $\hat{h}_1(L_c, t_c)$ in eq.(16) by removing the shock:

$$\hat{h}_0(L_c) = \max\left\{ h : \left(\max_{c \in V(h)} L(c, \varepsilon_{t|t_c} = 0) \right) \le L_c \right\} \quad (20)$$

5. No-shock robustness with uncertain coefficients

We evaluate the robustness in eq.(20), using the full model (Rule 3), with estimated coefficients given in column 4 of Appendix 2, taking into account that the estimates are susceptible to Knightian uncertainty.

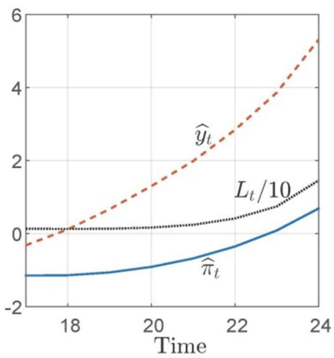

Figure 1: Output and inflation gap, and loss function: putative values (Rule 3)

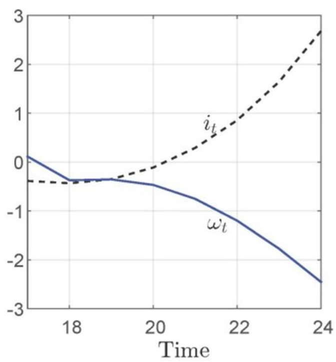

Figure 2: Inflation and debt: putative values (Rule 3)

Figure 1 shows the output and inflation gaps, $\hat{y}_t$ and $\hat{\pi}_t$, and the loss function reduced by a factor of 10, $L_t/10$, vs. time, based on the putative coefficient values of Rule 3 and no shocks. These, and all subsequent figures in this section, are calculated for eight quarters, so $t_1 = 17$ and $t_1 + 7 = 24$. Figure 2 shows the putative interest rate and debt functions for eight quarters. These functions show substantial divergence from the initial values after the sixth quarter.

The simulation outcomes indicate that the inflation and output gaps become positive over the simulation horizon, in response to very accommodative monetary conditions. At t_1 -1 the policy (or shadow) rate is around -0.8 percent and the rate of inflation 0.9 percent. This implies a real interest rate of -1.7 percent, which is substantially below the assumed level of the natural rate of 1 percent (the policy or shadow rate is low because it responds to the negative output and inflation gaps at t_1 -1 via the Taylor rule (eq. 3)). The accommodative monetary policy stance stimulates output via the demand curve (eq. 2). As output rises, the inflation gap $\hat{\pi}_t$ is positively affected through the Philips curve (eq. 1). The central bank reacts via the Taylor rule to the positive developments in output and inflation, by raising the interest rate from t_1 + 3 onward. This closes the gap between the policy rate and its trend level and so exerts downward pressure on credit gap ω_t via eq. 4.

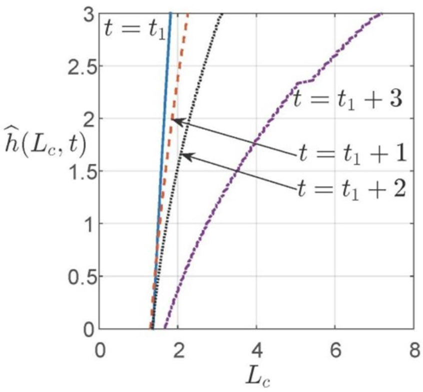

Figure 3: No-shock robustness for 1st four quarters (Rule 3)

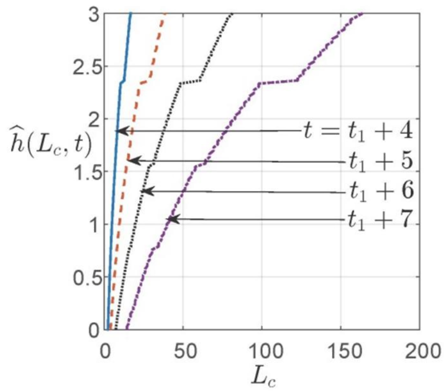

Figure 4: No-shock robustness for last 4 quarters (Rule 3)

Figures 3 and 4 show the robustness against uncertainty in the dynamic coefficients with no shocks $\hat{h}_0(L_c)$, vs. the maximum acceptable loss, L_c. The time at which the performance requirement is imposed is specified in each curve.

The positive slopes of the curves in Figures 3 and 4 express the irrevocable trade-off between robustness and performance. More demanding performance (smaller acceptable loss L_c, meaning smaller output and inflation gaps) entails less robustness against uncertainty (smaller $\hat{h}_0(L_c)$). We also note that the robustness becomes zero precisely at the putative best estimate of the loss function. That is, $\hat{h}_0(L_c)=0$ when L_c is chosen as the value of the loss function, L, evaluated with the putative coefficients, as shown in Figure 1. The significance of this 'zeroing' phenomenon is that the putative best estimate of the loss is not a reliable basis for policy selection because the robustness-against-uncertainty for obtaining that value of loss is zero.

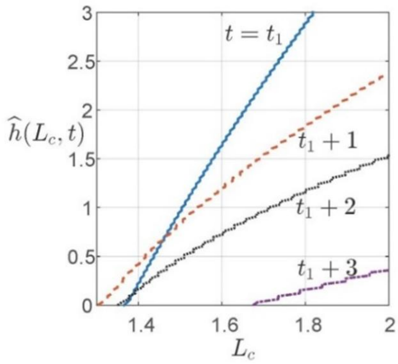

Figure 5: Expansion of Figure 3 (Rule 3)

Figure 5 shows an expanded version of Figure 3, illustrating crossing of robustness curves of the first four quarters. The putative value of the loss is lowest in quarter t_1+1 and equals 1.30, which is the horizontal intercept of the red curve. One might obtain the impression, based on the putative best-estimates of the loss function, that quarter t_1+1 is better than quarter t_1; that is, the situation seems to be improving. However, we stress two points. First, these best-estimates have no robustness against uncertainty so these predictions are unreliable; this is the zeroing property. Second, the slopes of the robustness curves decrease from quarter t_1 to quarter t_1+3.

Based on the trade-off property, the slope can be understood as a cost of robustness: low slope implies high cost of robustness. For the first quarter, t_1, for which the robustness curve is relatively steep, we see that an increase in robustness from 0 to 2 entails increasing the acceptable loss, L_c, from 1.37 to 1.66. In contrast, for quarter t_1+1, increasing the robustness from 0 to 2 entails increasing L_c from 1.30 to 1.86. The cost of robustness is even higher for the latter two quarters. The intuition here is that robustness against uncertainty is more costly in the distant future than in the immediate future because of greater intervening uncertainty. The slopes of these curves quantify this intuition.

Finally, from Figure 4 we see that the robustness rapidly decreases as we consider quarters t_1+4 to t_1+7. Figure 3 shows that the robustness at quarter t_1+3 reaches a value of 3 at $L_c=7.2$. In contrast, we see from Figure 4 that the robustness equals 3 at $L_c = 16.9$ in quarter $t_1 + 4$, and at $L_c = 164$ in quarter $t_1 + 7$. This seems to support the claim that predictions are not reliable, or even particularly useful, after about four or five quarters. It implies that a macroeconomic model including financial variables (as Rule 3) is increasingly less robust to parameter uncertainty if the forecasting horizon is extended. While this is a common feature of empirical models, in the next sections we test to what extent this holds for the other model specifications.

6. Policy rules for no-shock robustness with uncertain coefficients

We continue the example from the previous section and now compare the four policy rules as given in Appendix 2. We consider only the first four quarters because we saw in the previous section that prediction becomes quite unreliable thereafter.

6.1. Putative dynamics without shocks

Figures 6 and 7 show the putative dynamics without shocks, based on the best estimates of the coefficients, for the three different policy rules beyond the baseline model (notice that the right panels - Rule 3 - are equal to Figures 1 and 2 up to $t_1 + 3$). Compared to Rules 1 and 3, the dynamics of Rule 2 are driven by a higher interest rate *i*. In Rule 2, the central bank does not react to financial stress via the Taylor rule (eq. 3). Since financial stress has been relatively high before t_1, the policy rate is lower in Rules 1 and 3 than in Rule 2. As a consequence, $\hat{y}_t$ develops less favorably in Rule 2 and ω_t is decreasing faster. The declining debt ratio also has a downward effect on the output gap via the demand equation (eq. 2). Note that in Rule 1 the debt function is identically zero, as defined by this rule. Moreover, the improvement of the output gap in Rule 1 lags behind its dynamics in Rule 3, which can be explained by the much lower coefficient of *r* the in the demand equation (eq. 2) in Rule 1.

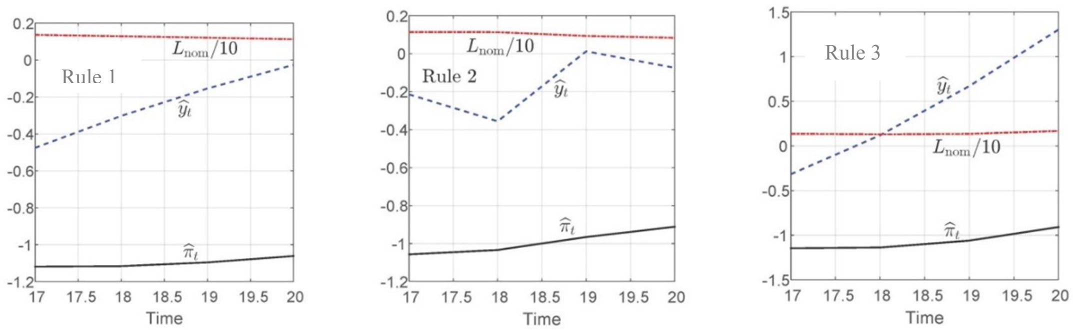

Figure 6: Putative dynamics: inflation and output gaps, and loss function

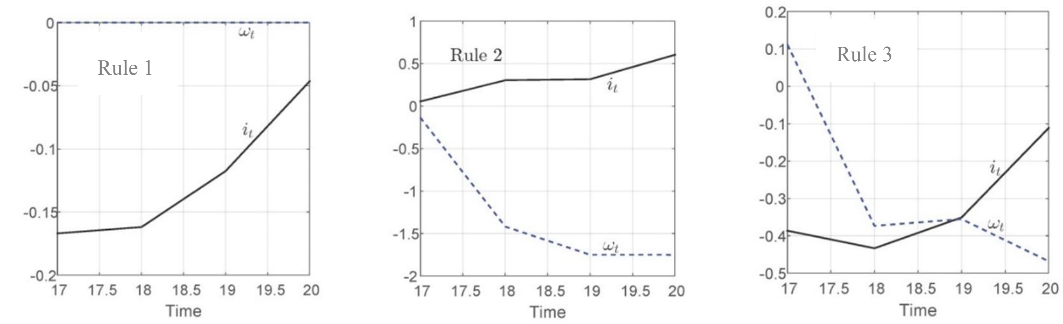

Figure 7: Putative dynamics: interest rate and debt

6.2 Robustness

The four panels of Figure 8 show robustness functions for the four policy rules, taking into account uncertainty in the model parameters, each panel at a different quarter. That is, we are evaluating the robustness in eq. (20) based on the info-gap model in eq. (12). In the first quarter, t_1, we see that Rule 2 is putatively better than the other rules, as implied by the zeroing property (horizontal intercept of Rule 2 is to the left of that for the other rules). While Rule 2 is putatively better, its cost of robustness is greater resulting in crossing of the robustness curve of Rule 0. The curve crossing implies that the central bank should exclude debt (ω_t) in its monetary policy framework – and choose Rule 0 - if it prefers a high level of robustness. Rules 1 and 3 are very nearly the same, both putatively and in terms of robustness, although they are less robust than the benchmark model (Rule 0).

In the second step, t_1+1, we see that Rule 2 is putatively better than Rules 1 and 3 but its cost of robustness is greater resulting in crossing of the other robustness curves. For L_c exceeding approximately 1.6 we see that both Rules 1 and 3 are clearly more robust than Rule 2. It shows that only including debt in the model makes the framework less robust to parameter uncertainty compared to a set up in which the central bank (also) reacts to financial stress models (models 1 and 3). However, not including any financial variables in the framework (Rule 0, the benchmark model) is always the most the robust rule at this time step. A similar picture emerges in quarter t_1+2 and again in quarter t_1+3, though in the latter case Rule 1 is substantially robust dominant over Rules 2 and 3. This underlines that including debt (ω_t) comes with the cost of lower robustness.

In summary, the benchmark model (Rule 0) is robust-preferred almost always for the specific market situation of the first 16 quarters. Its parsimonious specification, that excludes the uncertain effects of financial stress and debt on the macroeconomy and monetary policy, reduces the vulnerability to parameter uncertainty. Augmenting the Taylor rule with a financial variable (Rules 1 and 3) leads to a loss of robustness, but less than in Rule 2, which includes financial imbalances in the demand equation. Rules 1 and 3 are quite similar in robustness (until $t_1 + 3$) even though the model of Rule 3 has more parameter uncertainty due to the debt function, ω_t, whose coefficient, α_4, is highly uncertain. So, while this parsimony of Rule 1 impugns that realism of the model, it also reduces its vulnerability to modelling uncertainty. This perhaps explains the robust dominance of Rule 1 over Rule 3 in the fourth quarter. Finally, we note that the robustness decreases from the first to the fourth quarter, as reflected by the shifting of the robustness curves to the right.

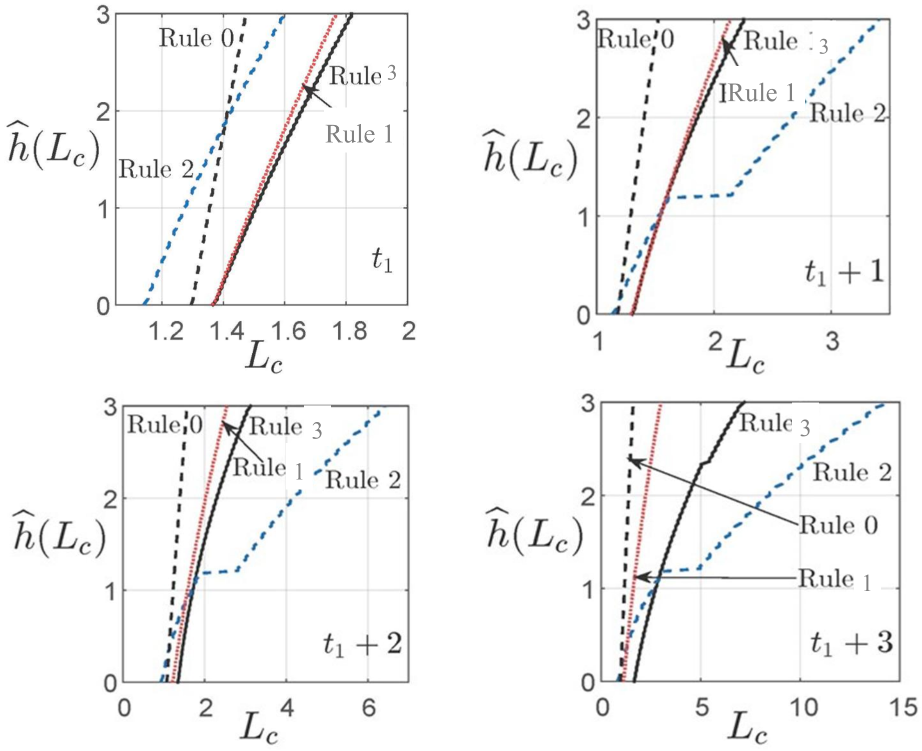

Figure 8: No-shock robustness functions in 1st 4 quarters

7. Policy rules for robustness to uncertain shock with uncertain coefficients

7.1 Dynamics of economic variables

We continue the previous section and introduce a single shock to inflation, where the amplitude of the shock (ε_t) is uncertain. We also consider three uncertain model coefficients, and evaluate the robustness in eq.(16) as discussed in Appendix 4. The putative amplitude of the shock, $\tilde{\varepsilon}^0$ in the info-gap model of eq. (14), is 1.13 (i.e. the standard deviation of the inflation series π). The uncertainty weight in the info-gap model is $s = \tilde{\varepsilon}^0 / 3$. In this section we present results for a single inflation shock of uncertain amplitude at time $t_c = t_1$. Since the system is causal, meaning that the future does not influence the past, a shock occurring at time t_1+k does not alter the robustness function evaluated at an earlier time. Uncertainty in the underlying data process of inflation is both interesting and topical, given the 'unknowns' with regard to the macroeconomic dynamics in a low inflation environment.

Figures 9 and 10 show that the shock to the inflation gap reduces the loss function for Rules 1 - 3. For example, compare the putative dynamics under Rule 3, with a shock at t_1 (Figures 9-10) to the no shock dynamics (Figures 6-7). With no shock, the inflation gap is about -1 percentage point throughout the first four quarters, while with a shock it is near zero for the first three quarters, rising slightly in the fourth quarter. The (positive) inflation shock thus brings inflation closer to its target rate, which improves the economic outcome and so reduces the loss. The result is that the loss function is lower with the shock for the first three quarters (a less negative inflation gap results in a lower loss). The effect is even more dramatic with Rules 1 and 2, for which the no-shock loss is much larger than with a shock at t_1.

The upward shock to $\hat{\pi}_t$ at t_1 leads to a rise in the policy rate at t_1+1 according to the backward looking Taylor rule. This counteracts the downward effect of the (upward) inflation shock on the real interest rate r_t in demand eq.2. The increase of the policy rate i_t following the inflation shock causes much stronger downward effects on debt variable ω_t than in the no-shock simulations and so drags on output via the credit channel (which is part of the demand equation in Rules 2 and 3). Similar effects are generated by a shock at $t_c=t_1+1$.[3]

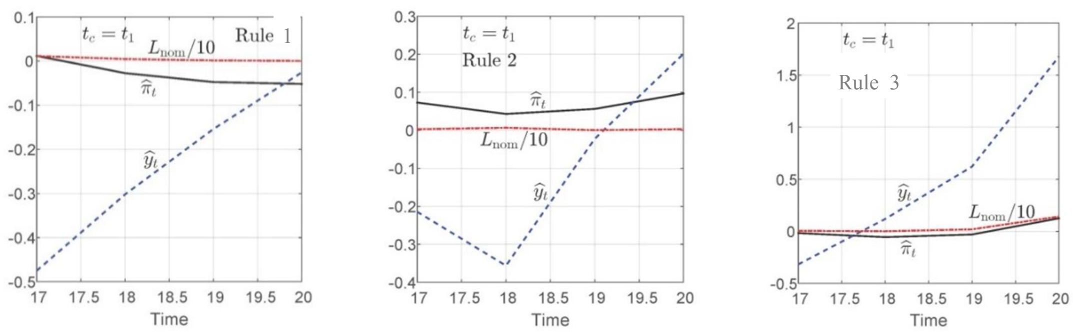

Figure 9: Putative dynamics: inflation and output gaps, and loss function after shock $\tilde{\varepsilon}^0$ at $t_c=t_1$

[3] The plots for $t_c=t_1+1$ are not shown to economize on the number of figures, but are available from the authors on request.

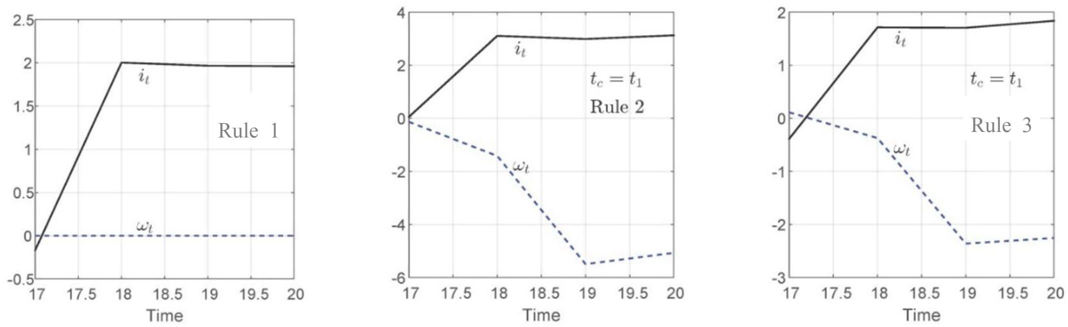

Figure 10: Putative dynamics: interest rate and debt after shock $\tilde{\varepsilon}^0$ *at* $t_c = t_1$

7.2 Robustness

Uncertainty in the model coefficients is represented by the info-gap model of eq. (12) and uncertainty in the amplitude of a single shock, occurring at time t_c, is represented by the info-gap model of eq. (14). We now consider robustness to uncertainty in the coefficients and uncertainty in the amplitude of a single shock at time t_1, defined in eq. (16).

The robustness curves in the case of a shock (Figure 11) show that robustness is zero at the putative value of the loss function, and increases monotonically as the critical value of loss (L_c) increases (these are the zeroing and trade off properties, and they hold for all info-gap robustness functions as discussed earlier). We recall from our earlier discussion that a shock to the inflation gap, in the first quarter, tends to reduce the loss function. This, together with the zeroing property, implies that the robustness curve sprouts off the horizontal axis further left with a shock than without (compare Figures 11 and 8). The costs of robustness (slopes of the robustness curves) are similar in these two cases, or sometimes slightly greater with the shock. This holds for Rules 2 and 3 in particular; their relatively flatter robustness curves imply that the improvement of robustness is relatively small if a higher loss is tolerated. It shows that including financial imbalances (i.e. debt) in the model goes with a deteriorating trade-off between performance and robustness. In other words, uncertainty on the inflation process exacerbates the consequences of parameter uncertainty for the robustness of the model framework.

The preference reversals between the four policy rules (as expressed by intersecting robustness curves) are generally similar with and without shocks. Compare, for instance, the robustness curves at time $t_1 + 1$ without a shock (Figure 8) and with a shock at $t_1 + 1$ (Figure 11). The relative distance between robustness curves for Rules 1 and 3 are comparable in these two cases, though the robustness for Rule 3 is somewhat flatter and the putative (zero-robustness) values differ. However, the preference reversal between Rules 2 and 3 at $t_1 + 2$ and $t_1 + 3$, and the robust dominance of Rule 0 (and Rule 1 over Rules 2 and 3), is much the same in both cases.

Consider a shock in the first quarter, $t_c = t_1$. For all four policy rules, the cost of robustness increases significantly from t_1 to $t_1 + 3$, causing much lower robustness at moderate and large values of critical loss L_c, and an expansion of the horizontal scale in the robustness plots for these four quarters at time t_1. This phenomenon is observed also when the shock occurs at a later period.[4] It is also significant that Rule 0 and to a lesser extent Rule 1 have the lowest cost of robustness - i.e. the steepest robustness curve – especially in the third and fourth quarters. This gives both Rules their robust dominance in later quarters, and presumably results from their modeling parsimony. A complementary interpretation is that the impact of the Knightian uncertainty on the economic dynamics, both with regard to the estimated relationships and the shock dynamics, is especially large when debt (ω_t) is taken into account in the model framework.

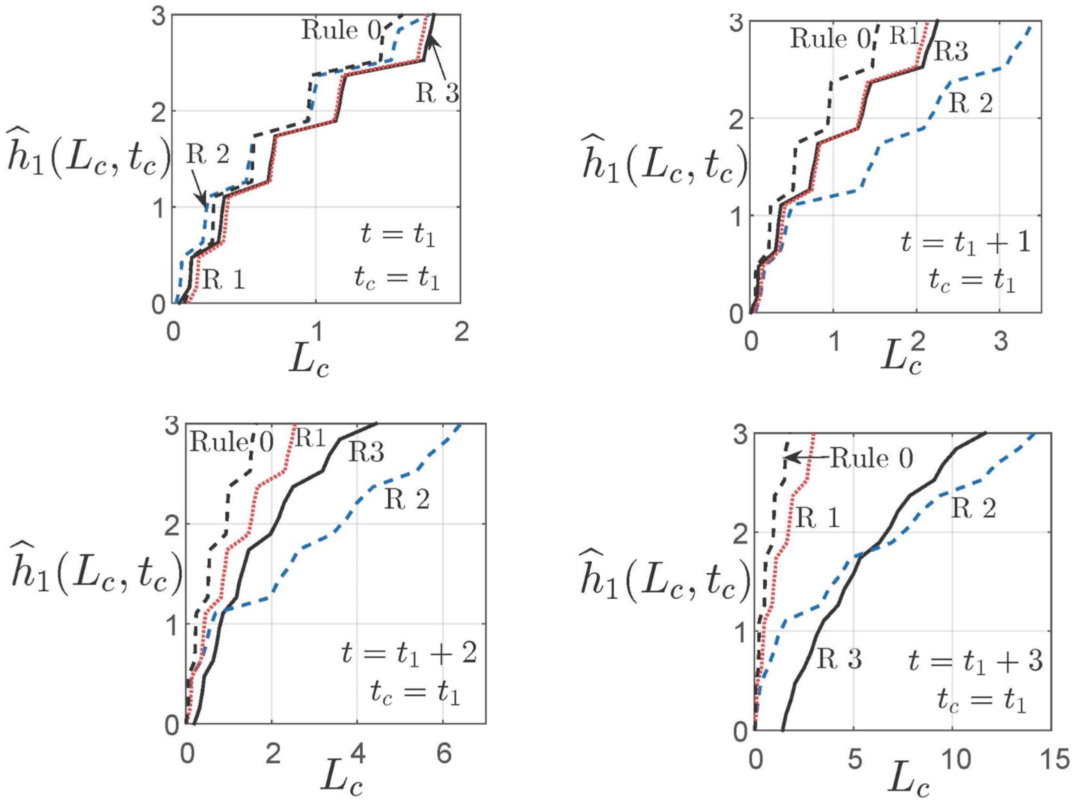

Figure 11: Robustness functions in 1st 4 quarters with shock in 1st quarter

4 The robustness curves for shocks at times $t_1 + 1$, $t_1 + 2$ and $t_1 + 3$ are not shown to economize on the number of figures, but are available from the authors on request.

8. Discussion

There are several explanations for our main finding that a macroeconomic model excluding financial variables (i.e. Rule 0) is, in most but not all circumstances, most robust to parameter and shock uncertainty (for the specific initial market conditions we examined). The info-gap robust satisficing analysis identifies those situations in which financial variables can be included without loss of robustness. A technical or structural, but not substantively economic, explanation is that Rule 0 is simpler: it has fewer parameters that can err. Even though Rule 0 starts out 'more wrong' than Rule 2, which is putatively better, the benchmark model nonetheless manages uncertainty better than the other rules, in almost all cases. Hence Rule 0 is more robust. Robustness (often) is a proxy for probability of success and so the greater robustness of Rule 0 makes it more likely to lead to policy success.

Another explanation of the outperformance of Rule 0 relates to the literature on heuristics, which asserts that simple rules turn out to be more robust and accurate in complex environments (eg Simon, 1995; Gigerenzer *et al*, 2011). In such conditions agents have limited capacity to understand the system and are not able to describe the statistical distribution of economic shocks. As a result they use simple rules ('heuristics') to guide their behaviour. De Grauwe (2011) introduces heuristics into macroeconomic models and monetary policy. He concludes that very simple rules fit the data process well. Aikman *et al* (2010) show that this also holds for modelling the financial system, which has become more complex due to interactions between the real and financial sphere.

Adding complexity may seem to be a good thing as it makes the models more 'realistic'. However, modelling such complexities requires deep knowledge of the underlying structures. Taking into account uncertainty means that such knowledge by definition is incomplete or missing. So while complicated models may seem better in a putative modelling sense, simpler rules are preferable in a robust sense. Simple rules tend to be better at managing uncertainty. More complex, augmented models – like Rules 1, 2 and 3 – can break down once uncertainty is introduced into the system. This explains why our simpler benchmark model is most robust and preferable.

9. Conclusion

Based on info-gap theory we assess the impact of Knightian uncertainty, both with regard to model parameters and the shock (uncertainty on the inflation process), on macroeconomic models that include financial variables. The info-gap approach takes into account the fundamental choice between robustness against uncertainty versus the aspiration for high-value outcomes.

The main conclusions are that a macroeconomic model including financial variables is increasingly less robust to parameter and shock uncertainty as the forecasting horizon is extended. We found that excluding all financial variables from the framework is usually, but not always, the most robust policy rule. The info-gap analysis of robustness allows the analyst to identify those situations in which financial variables can be included without loss of robustness. Knightian uncertainty on the economic dynamics, both with regard to the estimated relationships and the shock dynamics, is especially large when a measure of debt is taken into account in the model framework. Less robustness is sacrificed if the central banks reacts to financial stress, by augmenting the Taylor rule with a credit spread variable.

For central banks these outcomes imply that including financial stability considerations in the monetary policy framework is challenging, given the Knightian uncertainty on the dynamics of financial variables and their interaction with the real economy. This would call for keeping price stability the primary objective of monetary policy, particularly given the uncertainties on the long run effects of financial imbalances on the economy. However, the central bank could consider including financial stress in the reaction function of the central bank. The info-gap simulations show that the loss of robustness is limited in that case, which may be acceptable for the policymaker given the gain in reaching his objectives. The clear loss of robustness when a debt measure is taken into account indicates that including financial imbalances in the monetary policy objective does not necessarily increase policy robustness, and may even decrease it. This does not say that the central bank should not include financial variables in its information set, to gauge risks to price stability from a wide spectrum of factors.

Finally, our contribution is methodological. The analysis can be applied to explore other uncertainties, and other policy interventions can be evaluated with respect to their robustness against Knightian uncertainty.

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Appendix 1: Definition of model variables

$\hat{\pi}_t$ = inflation gap (realised inflation rate – inflation objective of 2 percent)

$\hat{y}_t$ = output gap (realised real GDP – HP filtered trend of real GDP)

i_t = relevant policy rate (Eonia rate until 2008, shadow rate from 2008 onward)

i^n = natural real interest rate (proxied by a constant value of 1 percent)

$\bar{\imath}$ = trend rate of interest (4 years moving average of i_t)

r_t = real interest rate (i_t - π- i^n)

f_t = financial stress (proxied by CDS spread financials (log))

ω_t = credit-to-GDP ratio (total credit euro area, households, firms, governments) as deviation from HP filtered trend

D = dummy, which equals 1 if the absolute value of the deviation of the credit-to-GDP ratio from trend is higher than 2 percentage points (critical threshold level)

Appendix 2: Estimation outcomes

	Rule 0	**Rule 1**	**Rule 2**	**Rule 3**
	Benchmark model	Model with augmented Taylor rule	Model with financial imbalances	Full model
Philips curve				
α_1	0.000	-0.003	0.018	-0.054**
	(0.023)	(0.027)	(0.024)	(0.026)
β_1	0.963***	0.963***	0.953***	0.958***
	(0.026)	(0.025)	(0.025)	(0.026)
β_2	0.100***	0.118***	0.125***	0.124***
	(0.024)	(0.026)	(0.020)	(0.020)
IS curve				
α_2	0.051	0.058	-0.031	-0.252**
	(0.110)	(0.081)	(0.140)	(0.112)
β_3	0.892***	0.904***	1.230***	1.312***
	(0.044)	(0.047)	(0.095)	(0.060)
β_4	-0.030	-0.015	-0.131***	-0.187***
	(0.026)	(0.034)	(0.047)	(0.035)
β_5			0.383***	0.045***
			(0.098)	(0.056)
β_6			-0.183***	-0.252***
			(0.065)	(0.055)
Taylor rule				
α_3	3.299***	8.575***	2.995***	8.251***
	(0.750)	(1.228)	(0.552)	(0.839)
β_7	2.280***	1.915**	2.480***	1.899***
	(0.550)	(0.792)	(0.424)	(0.253)
β_8	0.421	0.383	0.322*	0.213**
	(0.270)	(0.325)	(0.178)	(0.105)
β_9		-1.262***		-1.267***
		(0.249)		(0.167)
Debt equation				
α_4			-0.919**	-0.436
			(0.464)	(0.443)
β_{10}			-1.425***	-0.997***
			(0.328)	(0.367)
N	337	302	357	322
J stat	0.024	0.022	0.057	0.067

Outcomes of GMM estimation of system of equations 1, 2,3 and 4, with data for the euro area. Sample period 1990-2015 (quarterly observations). The model is estimated by heteroskedasticity and autocorrelation consistent GMM (HAC), applying prewhitening to soak up the correlation in the moment conditions. Standard errors in parentheses, where *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level.

Appendix 3: Efficient computation of no-shock robustness with 3 uncertain coefficients

The robustness $\hat{h}_0(L_c)$, defined in eq.(20), is a function of the time, t_2, at which the performance requirement, eq.(11), must be satisfied. We now formulate the computation of the inverse of $\hat{h}_0(L_c)$, denoted $m(h,t)$, which is the inner maximum in eq.(20). We consider 3 uncertain coefficients, so $N = 3$ in the info-gap model of eq.(12).

In order to evaluate the inner maximum in eq.(20) we need to sample the values of the uncertain coefficients in the info-gap model. Let $h_k, k=1,\ldots,K$ denote the horizon of uncertainty values at which we will calculate $m(h,t)$, where $h_k < h_{k+1}$, and define $h_{\max} = h_K$. Let J_c denote the number of evenly spaced values of c_n that we will sample from $V(h_{\max})$. The step size for increments of each c_n is:

$$d_n = \frac{2\sigma_n h_{\max}}{J_c - 1} \tag{21}$$

The sampled values of c_n are:

$$c_{n,j} = \tilde{c}_n - \sigma_n h_{\max} + (j-1)d_n \tag{22}$$

for $n = 1,\ldots,N$ and $j=1,\ldots,J_c$. We require that J_c be an odd integer to assure that the value of $c_{n,j}$ at the middle value of j equals the putative estimate, $\tilde{c}_n$.

The basic procedure is that, for each value of h_k, we calculate the system dynamics from time step from t_1 to t_2. At each time step, t, we calculate the inner maximum in eq.(20), $m(h_k,t)$, which is the maximum on $V(h_k)$. This uncertainty set is the hypercube of coefficients satisfying:

$$\tilde{c}_n - h_k\sigma_n \le c_{n,j} \le \tilde{c}_n + h_k\sigma_n \tag{23}$$

which, using eq.(22), implies:

$$-h_k\sigma_n \le -\sigma_n h_{\max} + (j-1)d_n \le h_k\sigma_n \tag{24}$$

and this implies:

$$\frac{(h_{\max} - h_k)\sigma_n}{d_n} + 1 \le j \le \frac{(h_{\max} + h_k)\sigma_n}{d_n} + 1 \tag{25}$$

Denote the quantities on the left and right of eq.(25) by $j_{n,\min}$ and $j_{n,\max}$, respectively. Thus the coefficient samples from $V(h_k)$ are the coefficients $c_{n,j}$ in eq.(22) for $n=1,\ldots,N$ and for

$j_{n,\min} \le j \le j_{n,\max}$.

However, each time we increment the value of h_k we don't have to re-calculate the dynamics for coefficient values that are in the interior of the info-gap model (corresponding to horizons of uncertainty $h_1,\ldots,h_{k-1}$). We only need to compare the maximum obtained with coefficients in the new "outer layer" that has been added to the hypercube $V(h_k)$, to the maximum obtained previously. An *N*-tuple of coefficients, $c_{1,j_1},\ldots,c_{N,j_N}$, is on the new outer layer of $V(h_k)$ if and only if at least one of these coefficients is at an extreme value. That is, the *N*-tuple $c_{1,j_1},\ldots,c_{N,j_N}$ is on the boundary of $V(h_k)$ if and only if there is an *n* such that $j_n = j_{n,\min}$ or $j_n = j_{n,\max}$.

We can now summarise the algorithm for evaluating the inverse of $\hat{h}_0(L_c)$.

Loop on horizon of uncertainty, $h_k = h_1,\ldots,h_{\max}$.

Loop on $j_1 = j_{1,\min},\ldots,j_{1,\max}$.

Loop on $j_2 = j_{2,\min},\ldots,j_{2,\max}$

Loop on $j_3 = j_{3,\min},\ldots,j_{3,\max}$

If the triplet $c_{1,j_1},\ldots,c_{3,j_3}$ is on the boundary of $V(h_k)$ then:

- Loop on t from t_1 to t_2.
- Calculate the performance function, L in eq.(10), for each t and for the triplet of coefficients. Call this value $L(t,c_{1,j_1},\ldots,c_{3,j_3})$.

Find the maximum of the values of $L(t,c_{1,j_1},\ldots,c_{3,j_3})$ on the boundary triplets. Call this $m_{\text{temp}}(h_k,t)$.

Calculate $m(h_k,t)$ by comparing $m_{\text{temp}}(h_k,t)$ against $m(h_{k-1},t)$. Specifically:

$$m(h_k,t) = \max\left(m_{\text{temp}}(h_k,t),\ m(h_{k-1},t)\right) \tag{26}$$

Appendix 4: Efficient computation of 1-shock robustness with 3 uncertain coefficients

The algorithm of the previous section is directly extendable when adding a shock at a single time step, t_c. We need only define analogs of the quantities in eqs.(21)-(25), as follows, and then add a loop on the shock-amplitude index j_ε.

Let J_ε denote the number of evenly spaced values of the shock amplitude, ε, that we will sample from $U_1(h_{\max}, t_c)$ in eq.(14). The step size for increments of ε is:

$$d_\varepsilon = \frac{2sh_{\max}}{J_\varepsilon - 1} \tag{27}$$

The sampled values of ε are:

$$\varepsilon_{j_\varepsilon} = \tilde{\varepsilon}^0 - sh_{\max} + (j_\varepsilon - 1)d_\varepsilon \tag{28}$$

for $j_\varepsilon = 1, \ldots, J_\varepsilon$. We require that J_ε be an odd integer to assure that the middle value of $\varepsilon_{j_\varepsilon}$ equals the putative estimate, $\tilde{\varepsilon}^0$.

The basic procedure is that, for each value of h_k, we calculate the system dynamics from time step from t_1 to t_2. At each time step, t, we calculate the inner maximum in eq.(16), $m(h_k, t)$, which is the maximum on $V(h_k)$ and $U_1(h_k, t_c)$. The second of these uncertainty sets is the interval of shock amplitudes satisfying:

$$\tilde{\varepsilon}^0 - h_k s \le \varepsilon_{j_\varepsilon} \le \tilde{\varepsilon} + h_k s \tag{29}$$

which, using eq.(28), implies:

$$-h_k s \le -sh_{\max} + (j_\varepsilon - 1)d_\varepsilon \le h_k s \tag{30}$$

and this implies:

$$\frac{(h_{\max} - h_k)s}{d_\varepsilon} + 1 \le j_\varepsilon \le \frac{(h_{\max} + h_k)s}{d_\varepsilon} + 1 \tag{31}$$

Denote the quantities on the left and right of eq.(31) by $j_{\varepsilon,\min}$ and $j_{\varepsilon,\max}$, respectively. Thus the shock-amplitude samples from $U_1(h_k, t_c)$ are the amplitudes ε_j in eq.(28) for $j_{\varepsilon,\min} \le j_\varepsilon \le j_{\varepsilon,\max}$.

We are considering uncertainty in both the coefficients and the shock amplitude, so the info-gap model is actually the Cartesian product of the info-gap models in eqs.(12) and (14):

$$U(h) = V(h) \times U_1(h, t_c) \tag{32}$$

This is a 4-dimensional hypercube. Thus, each time we increment the horizon of uncertainty, we don't need to re-calculate the dynamics for the coefficients and shock amplitudes that are in the interior of this hypercube. We add a loop to the algorithm in the previous section and check if we are on a boundary. Thus the algorithm for calculating the inverse of $\hat{h}_1(L_c, t_c)$ is:

Loop on horizon of uncertainty, $h_k = h_1, \ldots, h_{\max}$.

Loop on $j_1 = j_{1,\min}, \ldots, j_{1,\max}$.

Loop on $j_2 = j_{2,\min}, \ldots, j_{2,\max}$

Loop on $j_3 = j_{3,\min}, \ldots, j_{3,\max}$

Loop on $j_\varepsilon = j_{\varepsilon,\min}, \ldots, j_{\varepsilon,\max}$

If the quadruplet $c_{1,j_1}, \ldots, c_{3,j_3}, \varepsilon_{j_\varepsilon}$ is on the boundary of $U(h)$ in eq.(32) then:

- Loop on t from t_1 to t_2.
- Calculate the performance function, L in eq.(10), for each t and for the quadruplet. Call this value $L(t, c_{1,j_1}, \ldots, c_{3,j_3}, \varepsilon_{j_\varepsilon})$.

Find the maximum of the values of $L(t, c_{1,j_1}, \ldots, c_{3,j_3}, \varepsilon_{j_\varepsilon})$ on the boundary quadruplets. Call this maximum $m_{\text{temp}}(h_k, t)$.

Calculate $m(h_k, t)$ by comparing $m_{\text{temp}}(h_k, t)$ against $m(h_{k-1}, t)$. Specifically:

$$m(h_k, t) = \max\left(m_{\text{temp}}(h_k, t),\ m(h_{k-1}, t)\right) \tag{33}$$