Markowitz Portfolio Theory

Prof. Salvatore Perdichizzi

Economics of Financial Intermediation

March 27, 2017

Markowitz Portfolio Selection

There is no single portfolio that is best for everyone.

- The life Cycle - *different consumption preference*
- Time Horizons - *different terms preference*
- Risk Tolerance - *different risk aversion*
- Limited Variety of Portfolio - Limited *finished products* in markets

The trade-off between Expected Return and Risk

Markowitz's contribution 1: The measurement of return and risk

- Portfolio of two assets

	Expected Return	Risk	Weight
Asset 1	$E(r_1)$	σ_1	W_1
Asset 2	$E(r_2)$	σ_2	W_2

$$E(r) = wE(r_1) + (1-w)E(r_2)$$
$$\sigma^2 = W^2\sigma_1^2 + (1-W)^2\sigma_2^2 + 2w(1-w)\rho\sigma_1\sigma_2$$
ρ *is the correlation coefficient* $-1 \leq \rho + 1$
$$\rho = \frac{Cov(r_1, r_2)}{\sigma_{r_1}\sigma_{r_2}}$$

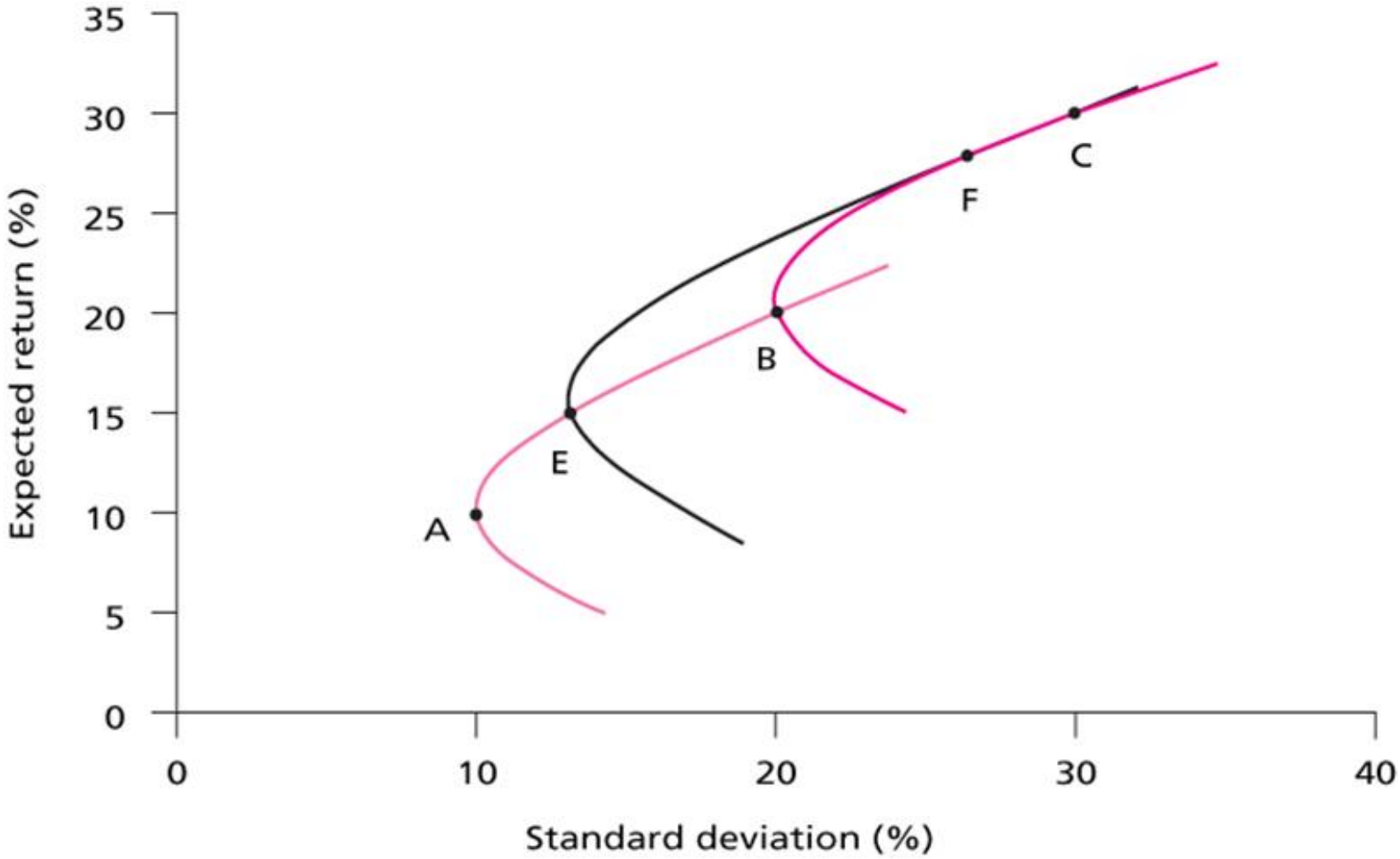
Case 1: Riskless asset and one risky asset

$E(r_1) = 14\%$
$\sigma_1 = 20\%$
$E(r_2) = r_f = 6\%$
$\sigma_2 = 0$
The return of a risky asset is calculated at the net of a safe asset. This is called the *Premium at risk from a riskless asset to a risky asset*

- How to achieve a target expected return $E(r) = 11\%$?

You know that $E(r) = r_f + w[E(r_1) - r_f]$ and $\sigma = w\sigma_1$
$w = \frac{E(r)-r_f}{E(r_1)-r_f} = 62.5\%$
$E(r) = r_f + \frac{E(r_1)-r_f}{\sigma_1}\sigma$
And $\sigma = 12.5\%$

Portfolio with many assets



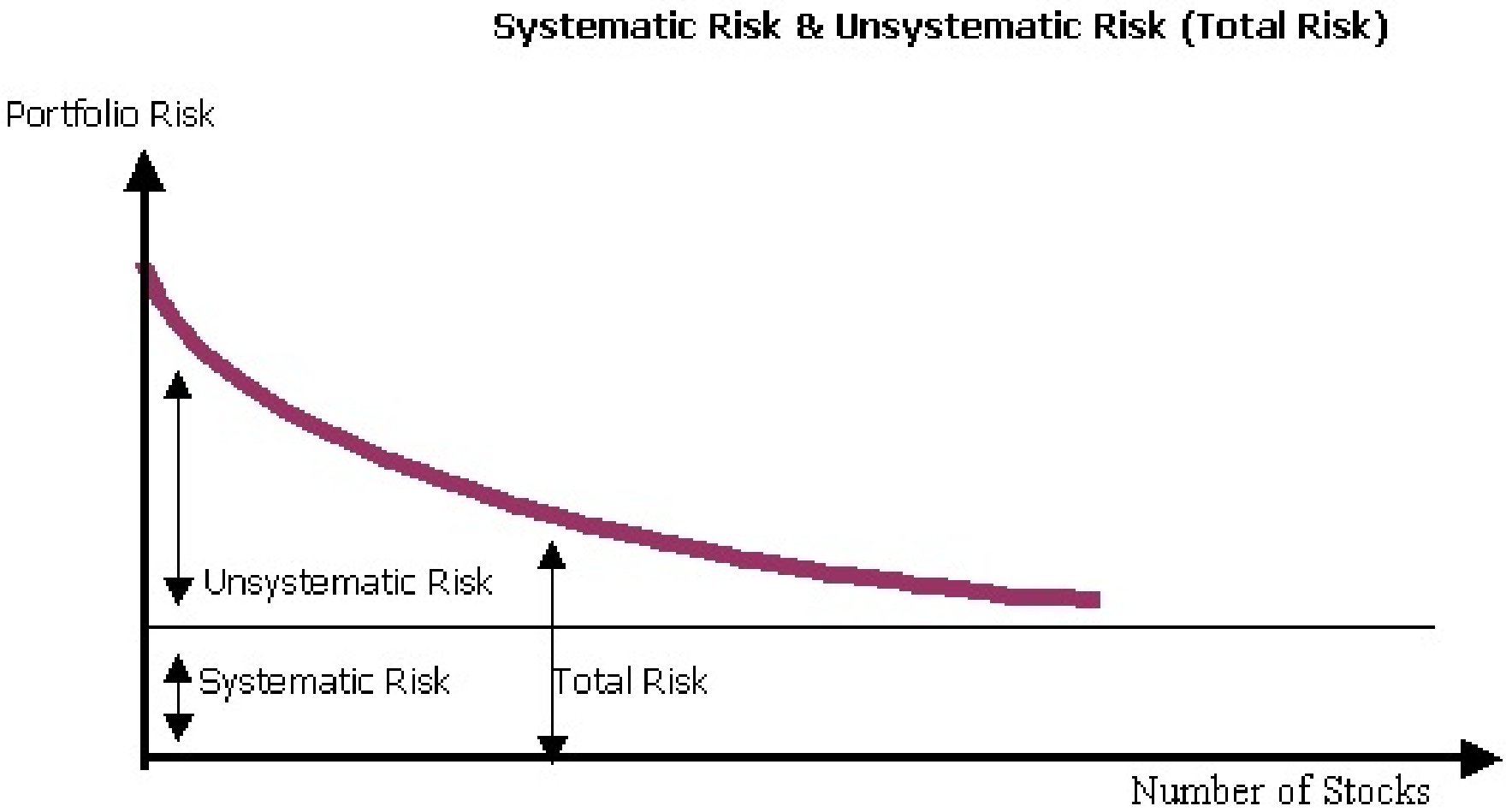
Diversification

Suppose $w_i = \frac{1}{N}, i = 1, ..., n$
The portfolio variance $\sigma = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{N} \frac{1}{N} \sigma_{i,j}^2 =$
$\frac{1}{N^2} \sum_{i=1}^{n} \sigma_i^2 + \frac{1}{N^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{i,j}$
Let $n \longrightarrow \infty$
Let $\frac{1}{N^2 - n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{i,j} = \tilde{\sigma}_{i,j}$ [1] $\Rightarrow$ Systematic risk

- Markowitz contribution II : **Diversification**

[1] Covariance accros different assets, Cov(i,j)

Portfolio with many assets



Proposition

- The variance of a diversified portfolio is irrelevant to the variance of individual assets. It is relevant to the covariance between them and equals the average of all the covariance.
- Only unsystematic risks can be diversified
- Systematic risks cannot be diversified. They can be hedged and transferred only.

Markowitz contribution III : **Distinguishing systematic and unsystematic risks**

Proposition Con't

- There is **systematic risk premium** contained in the **expected return**. **Unsystematic risk premium** cannot be got through transaction in competitive markets.
  $E(R_i)$ = *Only systematic risk premium contained.*
  σ_i = *Both systematic and unsystematic risk volatilities contained*