#### QATAR UNIVERSITY

#### COLLEGE OF ENGINEERING

#### ON THE DEGREES OF FREEDOM OF THE RELAY X-CHANNEL

BY

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#### Abstract

Interference is a principal source of capacity limitations in today's multi-access multi-user wireless systems. Despite the fact that the capacity of interference channels is still an unsolved problem, the research community has already established a substantial work towards this goal. In effort to provide alternative attainable expressions for performance limits in interference channels, the concept of the Degrees of Freedom (DoF) has been introduced. DoF describes network capacity in terms of the number of maximum possible simultaneous interference-free streams.

X-channel is defined where there are two transmitters, two receivers and each transmitter has an independent message for each receiver. Interference channel, broadcast channel and the multiple access channel are special cases of the X-channel. In this thesis, we further investigate the effect of a relay on the DoF of a single input single output (SISO) X-channel with no channel state information at transmitters (CSIT). In contrast to previous work, which has focused on two antennas at the relay to achieve the optimal  $\frac{4}{3}$  DoF, we focus on the case of a single antenna half duplex relay. We show that with a single antenna relay and delayed output feedback, the upper bound of  $\frac{4}{3}$  DoF for the X-channel is achievable and we provide the achievability scheme.

We revisit the previously studied case of single antenna relay in the more practical setting of alternating CSIT. We show that the optimal  $\frac{4}{3}$  DoF achievability does not mandate full CSIT availability. For the case of partial alternating CSIT availability at the relay transmitters, we propose a scheme that can achieve the optimal  $\frac{4}{3}$  DoF and we deduce the minimum CSIT availability for the proposed scheme to achieve optimality.

# Contents

	List	of Tab	les	vi
	List	of Figu	ires	vii
	Ack	nowledg	gements	viii
	Ded	ication		ix
L	Intr	$\mathbf{coduct}$	ion	1
	1.1	Motiv	ation	2
	1.2	Objec	tives	3
	1.3	Contri	ibutions	3
	1.4	Thesis	s organization	4
2	Bac	kgroui	nd	5
	2.1	Main	concepts	5
		2.1.1	X, interference, multiple access (MAC) and broadcast	
			channels	5
		2.1.2	Degrees of freedom (DoF)	8
		2.1.3	Interference alignment	9
		2.1.4	Channel state information (CSI)	10
		2.1.5	Alternating CSIT	11
		2.1.6	Wireless relays	11
	2.2	Releva	ant literature	12

3	Blind X-Channel Guided by a Single Antenna Relay with Par-			
	tial Delayed Output Feedback		16	
	3.1	System Model	17	
	3.2	Achievable Scheme	18	
	3.3	Results	22	
4 Blind X-Channel Guided by a Single Antenna Relay with Al-				
	ternating CSIT		23	
	4.1	Proposed alternating CSIT scheme	25	
	4.2	Results	32	
5	Con	clusions and Future Work	35	
Bi	Bibliography 3			

# List of Tables

4.1	CSIT permutations	s which achieve $\frac{4}{3}$ DoF		32
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# List of Figures

2.1	The X-channel	6
2.2	The interference channel	6
2.3	The broadcast channel	7
2.4	The MAC channel	8
2.5	Interference alignment	10
3.1	System model composed of X-channel with a relay	17
3.2	Delayed output feedback achievable scheme	19
4.1	CSIT states for $\frac{4}{3}$ DoF Achievable Scheme	25
4.2	Alternating CSI achievable scheme	27
4.3	The DoF achievability region.	34

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# Dedication

This thesis is dedicated to my beloved family and my dear husband Momen.

# Chapter 1

# Introduction

Channel capacity is considered as a theoretical limit used to study the performance of communication systems. Since the publication of his seminal work on the mathematical theory of communication, the work of Claud Shannon on channel capacity started a new era of research (which was later referred to as Information Theory) and technological advancement in the field of telecommunications. The capacity of wireless channels has attracted a lot of researchers in the past few decades. Recent work focuses on asymptotic capacity characterizations of wireless networks in order to study their performance limits. The capacity of a channel is defined by Shannon as the maximum rate of information that can be reliably transmitted through a channel [23].

It was widely believed that in order to have a reliable communication using a noisy channel with small error probability, data rate needed to be reduced. Contrary to this widely spread belief, Shannon showed that using more intelligent coding of information one can send high data rate with as small error of probability as desired. However, if one attempts to communicate above the channel capacity which is the maximum reliable communication rate, then it is not possible to achieve a vanishing error probability. Despite the extensive work in the field of information theory, the exact channel capacity of many com-

munication systems configurations involving interference – studied in a subfield of information theory referred to as Network Information Theory – is still unknown. In order to approximate the capacity of a communication network, the Degrees of Freedom approximation is used as an asymptotic way to represent the capacity scaling with transmitted signal power.

A principal source of performance limitation of wireless channel (measured using channel capacity) is caused by interference [20]. Interference can be thought of in its simplest form as receiving undesired messages at the receiver simultaneously with the desired message. In order to combat interference, two main techniques are used; interference cancellation and interference alignment. Interference cancellation [1] is performed at the receiver with the objective of eliminating the undesired messages. Interference alignment designs the signals at the transmitter so that they cast overlapping shadows at the receivers where they form interference (undesired) while remaining separable from the interference at the receivers where they are anticipated (desired) [9]. Hence, interference alignment forces all interfering signals to be aligned at the receiver while maximizing the number of interference free signalling dimensions in order to characterize the degrees of freedom of the channel.

#### 1.1 Motivation

Wireless communication continues to show a very high impact in our daily life. Achieving high data rates and having a reliable communication is limited by some factors including fading, interference and path loss. A promising technique to overcome this problem, is to deploy relaying nodes between transmitter and receiver. These nodes are called relays, which can improve the communication link and extend its coverage to improve its throughput. The relay receives the transmitted message, processes it and relays the processed output to the

receiver. This relaying strategy can be beneficial when the transmitter-receiver pair are experiencing a large obstruction that blocks the communication link between them or separated by a large distance. It can also be beneficial in the cases where the direct link between the transmitter and the receiver is weak, in which case the relay can cooperate with the transmitter to increase the signal reliability at the receiver. This last scenario is called cooperative relaying.

#### 1.2 Objectives

The main objective of the thesis is to provide further insights and contributions to the problem of finding the capacity of wireless interference limited networks utilizing the degrees of freedom capacity scaling methods. In particular, we look at the influence of a single antenna relay in a SISO (Single Input Single Output) X-channel with blind sources having no CSIT (Channel State Information at Transmitters). We study the case where all nodes and the relay are having a single antenna. Moreover, we study some scenarios of blind transmitters X-channel guided by a single antenna relay under the practical case of alternating CSIT at the relay.

#### 1.3 Contributions

The main contributions in this thesis can be summarized as follows:

- We introduce a scheme for blind X-channel guided by a relay incorporating partial delayed output feedback, which achieves the upper bound of  $\frac{4}{3}$  DoF for the SISO X-channel.
- We address a practical gap in DoF achievability in blind X-channel guided by a relay, where it was shown that a single antenna relay can achieve the upper bound of  $\frac{4}{3}$  DoF if perfect CSIT is available, while the same upper

bound can only be achieved under delayed CSIT using a 2-antenna relay. We show that a single antenna relay can still be used to guid the blind X-channel to achieve the upper bound under the assumption of alternating (partial) CSIT.

• We further state and proof a theorem that puts the limits on the partial availability (alternation) of CSIT that enables the achievement of the upper bound.

#### 1.4 Thesis organization

This thesis studies relay aided X-channels under different scenarios of channel state information availability. The thesis is organized as follows. Chapter 2 presents a background information of the basics used throughout the thesis followed by the motivating literature survey that exposes the gap of knowledge addressed in this thesis. Chapter 3 introduces the system model of a single antenna relay aided X-channel and summarizes current upper bounds and achievability schemes on this system under different scenarios followed by our proposed achievable scheme for the single antenna relay aided X-channel with delayed output feedback. Chapter 4 provides our proposed extension of the results on relay aided X-channel with single antenna relay for the case of alternating channel state information and provides a theorem that states the limits on CSIT alternation patterns. Chapter 5 gives a summary of the thesis findings as well as discusses the directions for future work.

# Chapter 2

# Background

#### 2.1 Main concepts

In this part, we go over the main concepts relevant to our work in order to facilitate a self contained structure of the thesis.

# 2.1.1 X, interference, multiple access (MAC) and broadcast channels

An X-channel is established where there are two transmitters, two receivers and each transmitter has an independent message for each receiver as shown in Figure 2.1. Interference channel, broadcast channel and the multiple access channel are special cases of the X-channel [10].

The interference channel is shown in Figure 2.2. There is strictly a one-to-one correspondence between transmitters and receivers. Each transmitter wants to deliver a message to its corresponding receiver and each receiver is only interested in the message from its corresponding transmitter while the channels interfere with each other. The interference channel is a subset of the X-channel. This is true only if each transmitter sends information to one corresponding receiver instead of sending a message to each receiver.

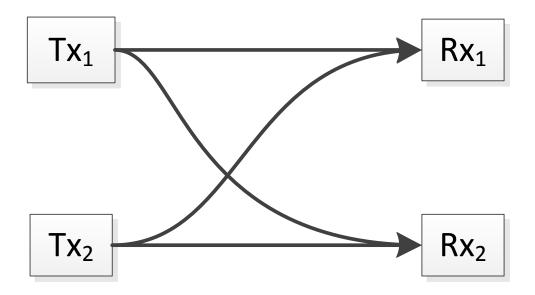


Figure 2.1: The X-channel.

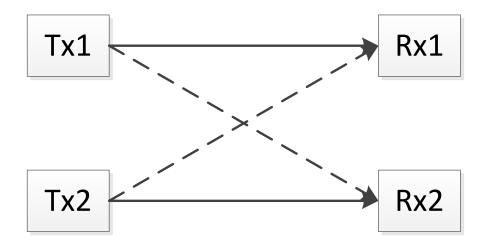


Figure 2.2: The interference channel.

Figure 2.3 shows the broadcast channel. A broadcast channel in its simple scenario is a channel with one sender and multiple receivers. The transmitter sends a common message to all the receivers. Broadcast channel is considered as a subset of the x-channel if one of the transmitters is omitted and the remaining transmitter sends a common broadcast message to all receivers.

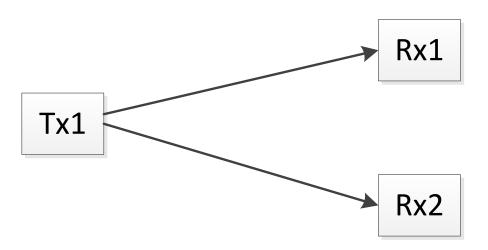


Figure 2.3: The broadcast channel.

Finally, the multiple access (MAC) channel in Figure 2.4 is a channel where transmitters send messages to a single receiver. The receiver is interested in all received messages. In the x-channel, if we consider that there is only one receiver that receives all messages transmitted from the two transmitters then it is a multiple access channel. Now it is clear that interference, broadcast and multiple access channels are special cases of the X-channel.

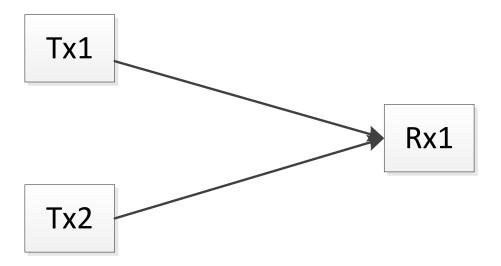


Figure 2.4: The MAC channel.

#### 2.1.2 Degrees of freedom (DoF)

Degrees of freedom (DoF) is an important metric in the estimation of capacity of wireless networks. It is widely used to characterize how capacity scales with total transmit power (i.e, signal to noise ratio, SNR) as power approaches infinity. This is defined as in the expression [2].

$$D = \lim_{P \to \infty} \frac{C(P)}{\log(P)},\tag{2.1}$$

where C is the capacity, D is degrees of freedom and P is the power.

DoF is clearly a capacity approximation that is accurate at high SNR and it was firstly named as the pre-log factor which may be expressed as

$$C(P) = D\log(P) + o(\log(P)). \tag{2.2}$$

where  $o(\log(P))$  is a function such that

$$\lim_{p \to \infty} \frac{f(P)}{\log(P)} = 0. \tag{2.3}$$

It is also known as the multiplexing gain since it measures the number of signals multiplexed over the air. DoF may also be interpreted as the number of signaling dimensions, where one signal dimension corresponds to interference-free transmission stream under additive white Gaussian noise (AWGN) with SNR increasing proportionally with P as P approaches infinity [11].

It is worth noting that in practice, interference-free dimensions are created using the interference alignment technique. Over multiple dimensions, signals are encoded so that they are aligned in the space observed by the receiver. Consequently, the interfering signals are aligned under a subspace at each receiver, keeping the desired signal separable in another orthogonal subspace [6].

#### 2.1.3 Interference alignment

Interference is known to limit the performance of the wireless channel capacity. In order to manage interference, it can be canceled (decoded then subtracted), treated as noise or aligned in limited dimensions that are orthogonal to the signal dimension. Interference is decoded if it is stronger than the desired signal. It is considered as noise if it is weaker in comparison to the desired signal. Interference alignment is used when the interfering (undesired) signal is in a comparable strength to the desired signal [3]. Interference alignment in its simplest form is a linear precoding technique. It encodes the signals to maximize the interference-free subspace for the desired signals and forces all the interfering signals into an orthogonal subspace with the lowest possible dimension (see Fig. 2.5). The number of non-interfering signals is known as the multiplexing gain or DoF. Since interference alignment is performed at the

transmitter (precoding) and intended for the space of the receiver, knowledge of channel state information at the transmitters (CSIT) is a major requirement to calculate the interference alignment precoders [3].

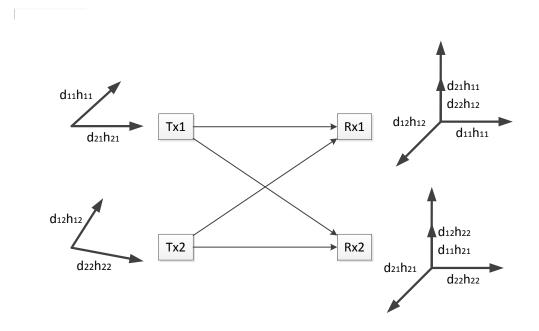


Figure 2.5: Interference alignment.

#### 2.1.4 Channel state information (CSI)

Channel state information (CSI) provides the properties of the communication channel to the communicating parties. It describes how the signal is affected as it propagates from transmitters to receivers. CSI is usually estimated by receivers and fedback to transmitters in order to provide a reliable communication; hence, transmitters and receivers may encounter different CSI availability. CSI is usually available at the receiver (CSIR) but at transmitter (CSIT) it can have different states, namely, perfect, delayed or no CSI. CSIT can be also imperfect, affected by the quality of the feedback link due to decoding errors on the feedback link or quantization errors due to limited capacity of the feedback channel. In analyzing the fundamental system performance it is common to

assume perfect CSI as it leads to the upper limits of the communication system. However, practical considerations requires looking at imperfect CSIT scenarios, in particular the case where CSIT at the transmitter is changing during the communication session (aka alternating CSIT).

#### 2.1.5 Alternating CSIT

The idea of alternating CSIT assumes that the availability of channel states is varying over time (per channel use) instead of a fixed state as perfect, delayed or not available during the whole communication session (total number of channel uses). Alternating CSIT is more towards practical systems and the nature of wireless networks. Under the assumption of alternating CSIT, we sum all the channel uses with a specific channel state and divide it by the total number of channel uses to calculate the ratio of CSIT availability of that specific state. The ratio of channels with different channel states gives a good information about how the alternating CSI is distributed among all channels. Assuming that  $\lambda_i$ , where  $i \in \{p, d, n\}$  is the CSIT availability type,  $p \triangleq \text{perfect}$ ,  $d \triangleq \text{delayed}$  and  $n \triangleq \text{not}$  available, we can write

$$\lambda_i = \frac{\sum u_i}{\sum u},\tag{2.4}$$

where u stands for channel use and  $u_i$  is channel use with CSIT availability of type i.

#### 2.1.6 Wireless relays

Relays are used to extend the coverage of a wireless network and improve its throughput. A Relay is deployed as a node between transmitter and receiver to overcome the communication problem. Relays can operate in two modes: full and half duplex. Half duplex relays can forward a message from source to destination over two channel uses, one for receiving the message from the source and one for forwarding the message to the destination. Full duplex relays transmit and receive data to be forwarded in one channel use. Despite the current research activities in full duplex wireless system, most of today's practical systems operate in half duplex mode due to non-trivial implementation problems of sending and receiving within the same channel use. The implementation of full duplex relays is quite demanding in practice due to the significant level of interference.

#### 2.2 Relevant literature

Wireless communication continues to attract many researchers and the need to develop signalling schemes with high spectrum efficiency and capacity is ever demanding. A key bottleneck limiting the capacity of wireless networks is interference among all other limiting factors. In order to overcome this limitation and provide a high spectral efficiency, interference alignment was proposed as a way of addressing these issues. Multiple interference alignment techniques have been proposed and an overview of those techniques is given in [11].

Interference alignment in its original form requires perfect and instantaneous CSIT to achieve the maximum DoF for different wireless network models [10, 3]. The work in [10] provided an achievable scheme for the MIMO X-channel to achieve  $\frac{4}{3}M$  DoF using a simple zero forcing technique with perfect CSIT. In addition, the achievability scheme in [3] with perfect CSIT is done based on interference alignment to achieve  $\frac{k}{2}$  DoF for the k-user interference channel. Recent work showed that imperfect CSIT can still be useful in order to facilitate interference alignment and provide a DoF gain greater than one [17, 18]. Contrary to the widely spread belief that the delayed CSIT is not

useful, authors in [17], interestingly, considered a scenario of a delayed CSIT, where CSIT of the previous channel use becomes available to the transmitter in the current channel use (time slot). Their work showed that the delayed CSIT can provide a DoF gain for many wireless network models such as X-channel, broadcast channel and interference channel [18, 25, 28, 29, 21].

The X-channel consists of two transmitters and two receivers, where each transmitter has an independent message for each receiver [10]. It was shown in [10] that for a MIMO (Multiple Input-Multiple Output) X-channel with each node equipped with M antennas, the maximum achievable DoF is  $\frac{4}{3}M$ . A zero forcing technique was used to achieve the maximum multiplexing gain. Similarly, for the SISO (Single Input Single Output) X-channel,  $\frac{4}{3}$  is the maximum achievable DoF. However, for the delayed CSIT case, it is shown in [18] that  $\frac{8}{7}$  DoF is achievable using a retrospective interference alignment scheme with all the nodes equipped with a single antenna. Later, in [8],  $\frac{6}{5}$  DoF was shown to be achievable which as an updated result for the work done in [18] with the same settings. Recently, in [14], it was proven that  $\frac{6}{5}$  is the upper bound for the X-channel with single antenna nodes in the case of delayed CSIT. However, when there is no CSIT, the maximum achievable DoF is one [10].

The relay aided X-channel was studied by many researchers with different scenarios [13, 32, 16, 15, 7, 19, 33]. In [19], the relay is used to randomize the static channel coefficients and achieve the optimal DoF for the quasi-static X-channel. In addition, the two-way relay in MIMO X-channel was studied in [32, 16, 15] where each work was an extension of the previous one by changing the system specifications. In [12], two interference alignment schemes are proposed for the  $M \times 2$  X-channel, one with half duplex relay and the other one with full duplex relay achieving the maximum DoF and the relay operates in the amplify and forward mode.

However, in [4] the relay is shown to be unable to provide any DoF gain for a fully connected interference and X-channel when all nodes are equipped with perfect CSIT and the channels are frequency selective. The work in [27] considered a relay aided SISO X-channel to facilitate interference alignment to achieve the maximum DoF without CSIT. An extension of the work done in [27] is considered, where the achievable scheme is done for the k-user X-channel with blind transmitters and a half duplex relay with multiple antennas or multi half duplex relays [26]. The work in [33] focused on the two user X-channel where each user is equipped with M antennas. It provides an achievable scheme using interference alignment for the two M-antenna users. The scheme achieves  $\frac{4}{3}M$  DoF with half duplex relay equipped with M antennas.

The idea of alternating CSIT was studied by many researchers as a representation of practical systems behavior. It allows the channels to have different CSIT availability types across different channel uses instead of a fixed perfect, delayed or no CSIT type across all channel uses. An interesting research work is proposed in [24]. The model allows the CSIT to vary over time (channel uses) which is more towards the practical systems and the nature of wireless networks. For this model, a two-user MISO broadcast channel is considered where one of the users has perfect CSIT and the other has no CSIT. The interesting result is that for this system without alternating CSIT, the optimal DoF is 1 for the constant CSIT setting, while with the alternating CSIT,  $\frac{3}{2}$  DoF is achievable. In [5], the alternating CSIT is considered for two user broadcast channel in the presence of topological and practical feedback settings. Recent work in [22] studied the DoF of the k-user MISO broadcast channel through utilizing the synergistic benefits of alternating CSIT. Also, the research work presented in [30] shows that alternating CSIT has a synergistic benefits, and  $\frac{4}{3}$  DoF is achievable for the two-user SISO X-channel. The work is extended in [31] to show the benefit of alternating CSIT with certain distribution of the CSIT availability types. An achievable scheme is presented to achieve  $\frac{5}{4}$  DoF, which is between the maximum DoF for the X-channel of  $\frac{4}{3}$  and the  $\frac{6}{5}$  maximum DoF for the delayed CSIT case.

This thesis work is inspired by the work done in [27]. The work in [27] shows that  $\frac{4}{3}$  DoF is achievable, which is the upper bound for SISO X-channel. In contrast to the work in [27] which assumes two antenna and perfect CSIT at the relay, Two cases are studied, the first case utilizes a single antenna relay and perfect CSIT at the relay. The second case considers a relay with 2-antennas and delayed CSIT at the relay. Both cases achieve  $\frac{4}{3}$  DoF. The achievable scheme is based on joint beam-forming. The relay is a half-duplex relay. We consider in this thesis the same system with single antenna relay and partial delayed output feedback instead of perfect CSIT at the relay. We further extend the work done in [27] to the case of alternating CSIT to address the practical gap between full perfect CSIT and completely delayed CSIT (requiring 2-antennas) by providing the achievability scheme and stating and proving the minimum requirements on the CSIT availability ratios,  $\lambda_i$ , needed to achieve the upper bound on the total DoF of  $\frac{4}{3}$  considering the alternating CSIT scenario.

It must be noted that both considered scenarios in this thesis are closer to practical aspects of real-life systems since they both avoid the use of perfect genie-aided CSIT.

# Chapter 3

# Blind X-Channel Guided by a Single Antenna Relay with Partial Delayed Output Feedback

We revisit the problem of relay aided SISO X-channel with single antenna relay. In our proposed setting we replace the impractical requirement of perfect CSIT availability with delayed output feedback, where perfect CSIT means it is instantaneous with no errors. An achievable scheme of a relay aided SISO X channel is presented here which is capable of achieving the  $\frac{4}{3}$  DoF upper bound under the assumption of delayed output feedback. We consider the typical assumption of receivers having global CSI in all channel uses while the transmitters are blind having no CSIT at all times. In [18], it is summarized that the delayed feedback can happen in one or more the following three different settings:

1. Delayed CSIT: where the channel coefficients are fedback to the transmitters.

- 2. Delayed output feedback: where the received signals are fedback to transmitters without the explicit channel states.
- 3. Delayed Shannon feedback: where the received signals along with the channel states are fedback to the transmitters.

In all three cases, the delayed feedback means that no instantaneous information is available for transmitters through the feedback channel; instead, a version of the feedback, delayed by one channel use, is available at the transmitters.

#### 3.1 System Model

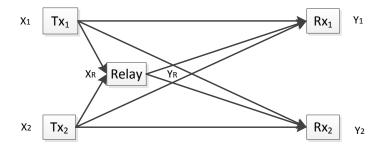


Figure 3.1: System model composed of X-channel with a relay.

A SISO X-channel aided by a single antenna decode and forward half-duplex relay is considered as shown in figure 3.1. The channel consists of two transmitters,  $Tx_1$  and  $Tx_2$ , and two receivers,  $Rx_1$  and  $Rx_2$ , where each transmitter has an independent message for each receiver and all nodes are equipped with a single antenna. The received signals during the different channel uses are as follows:

In the first channel use when the relay works as a receiver, we have

$$Y_1(t) = h_{11}(t)X_1(t) + h_{12}(t)X_2 + Z_1(t), (3.1)$$

$$Y_2(t) = h_{21}(t)X_1(t) + h_{22}(t)X_2 + Z_2(t), \tag{3.2}$$

$$Y_R(t) = h_{R1}(t)X_1(t) + h_{R2}(t)X_2 + Z_R(t).$$
(3.3)

In the second channel use when the relay works as a transmitter, we have

$$Y_1(t) = h_{11}(t)X_1(t) + h_{12}(t)X_2 + h_{1R}(t)X_R(t) + Z_1(t), \tag{3.4}$$

$$Y_2(t) = h_{21}(t)X_1(t) + h_{22}(t)X_2 + h_{2R}(t)X_R(t) + Z_2(t),$$
(3.5)

where  $X_i(t)$  is the transmitted message from transmitter i or the relay at time slot (channel use) t and is subjected to power constraint  $E[\|X(t)\|^2] \leq P$ ,  $h_{ij}(t)$  is the channel coefficient from transmitter j to receiver i at channel use number t and all channel coefficients are drawn independently from a continuous distribution for each channel use and  $Z_i(t) \in \mathcal{C}^{N_i}, i \in 1, 2, R$  is the AWGN with unit variance for the receivers  $Rx_1, Rx_2$  and the relay, respectively. We assume all receivers will have perfect global CSI during all t channel uses and no CSIT availability at transmitters. The relay is assumed to have a delayed output feedback from the receivers.

#### 3.2 Achievable Scheme

We consider a case where a single antenna relay has a delayed output feedback and no CSIT. We propose a scheme that can achieve the  $\frac{4}{3}$  DoF for the X-channel. The achievable scheme is shown in Figure 3.2.

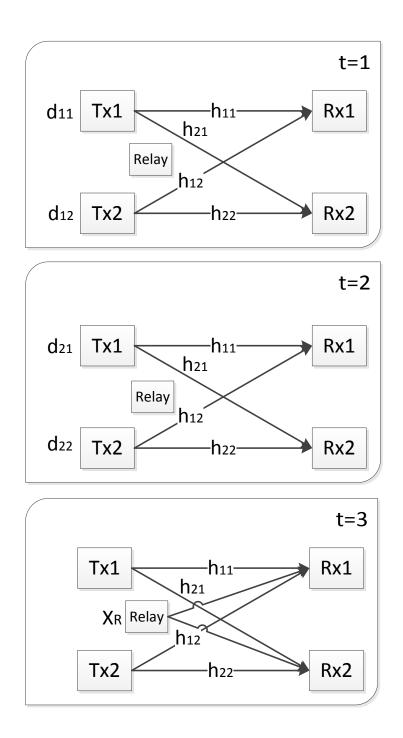


Figure 3.2: Delayed output feedback achievable scheme

We consider sending four independent messages over three channel uses. Let  $Tx_1$  transmit the data symbol  $d_{11}$  and  $Tx_2$  transmit  $d_{12}$  to  $Rx_1$ . For  $Rx_2$ , the data symbol  $d_{21}$  is transmitted from  $Tx_1$  and  $d_{22}$  is transmitted from  $Tx_2$ , where  $d_{ij}(t)$  is the data symbol sent from transmitter j to receiver i during channel use (time slot) number t.

In the first channel use, i.e., t = 1,  $Tx_1$  and  $Tx_2$  send  $d_{11}$  and  $d_{12}$  to  $Rx_1$  respectively.

The received signals at  $Rx_1$  and  $Rx_2$  and the relay are:

$$Y_1(1) = h_{11}(1)d_{11}(1) + h_{12}(1)d_{12}, (3.6)$$

$$Y_2(1) = h_{21}(1)d_{11}(1) + h_{22}(1)d_{12}, (3.7)$$

$$Y_R(1) = h_{R1}(1)d_{11}(1) + h_{R2}(1)d_{12}. (3.8)$$

In the second channel use, i.e., t = 2,  $Tx_1$  and  $Tx_2$  will send  $d_{21}$  and  $d_{22}$  to  $Rx_2$ .

The received signals at  $Rx_1$ ,  $Rx_2$  and the relay are:

$$Y_1(2) = h_{11}(2)d_{21}(2) + h_{12}(2)d_{22}, (3.9)$$

$$Y_2(2) = h_{21}(2)d_{21}(2) + h_{22}(2)d_{22}, (3.10)$$

$$Y_R(2) = h_{R1}(2)d_{21}(2) + h_{R2}(2)d_{22}. (3.11)$$

In the previous two channel uses, the relay was silent and only receiving messages. It is a half-duplex relay and it can not receive and transmit at the same time slot (channel use). Now from the delayed output feedback, relay would have  $Y_2(1)$  by the start of channel use t = 2, and by start of channel use t = 3 relay would have  $Y_1(2)$ , which can be used to provide each receiver with a second linear equation. In addition, it will cancel the interference at the other receiver while the transmitters will be silent. Hence, the relay can send

at t = 3 the following,

$$X_R(3) = Y_1(2) + Y_2(1) (3.12)$$

and the received signals at  $Rx_1$  and  $Rx_2$  are:

$$Y_1(3) = h_{1R}(3)X_R(3) = h_{1R}(3)(Y_1(2) + Y_2(1)), \tag{3.13}$$

$$Y_2(3) = h_{2R}(3)X_R(3) = h_{2R}(3)(Y_1(2) + Y_2(1)).$$
(3.14)

After the third channel use,  $Rx_1$  will subtract  $Y_1(2)$  from  $Y_1(3)$  to get the second linear equation. The first linear equation for  $Rx_1$  was received in the first channel use. Hence,  $Rx_1$  has enough equations to solve for its desired messages as follows,

$$Y_1(3) - h_{1R}(3)Y_1(2)$$

$$= h_{1R}(3)Y_1(2) + h_{1R}(3)Y_2(1) - h_{1R}(3)Y_1(2)$$

$$= h_{1R}(3)Y_2(1)$$

For  $Rx_2$ , the second equation is formed in the same manner by subtracting  $Y_2(1)$  from  $Y_2(3)$ , where the first equation has been received in the second channel use as follows,

$$Y_2(3) - h_{2R}(3)Y_2(1)$$

$$= h_{2R}(3)Y_1(2) + h_{2R}(3)Y_2(1) - h_{2R}(3)Y_2(1)$$

$$= h_{2R}(3)Y_1(2)$$

At the end of this transmission,  $\frac{4}{3}$  DoF is achieved by sending two messages for each receiver in a total of three channel uses and utilizing the delayed output feedback at the relay.

#### 3.3 Results

We have proposed an achievable scheme for a relay aided X-channel with output feedback. For the X-channel aided by a single antenna relay and delayed output feedback,  $\frac{4}{3}$  DoF is achievable using our proposed scheme. The delayed output feedback info is available for selected receivers in the first two channel uses. In the first channel use,  $Rx_2$  sends its output feedback to the relay, while in the second channel use  $Rx_1$  provides the output feedback. The achievable scheme can also work vice versa between receivers by giving attention to follow the scheme in the third channel use. In other words each receiver should provide a delayed output feedback only once at the first two channel uses. The two receivers can not provide simultaneous delayed output feedback in the same channel use. Accordingly, only partial delayed output feedback is needed (only two of them is explicitly needed). Since,  $\frac{4}{3}$  DoF is the upper bound for the X-channel, the proposed scheme with partial delayed output feedback was able to achieve the upper bound with a single antenna relay and no CSIT at all transmitters and relay. In the presented scheme we have addressed the practical aspects of delayed CSI.

# Chapter 4

# Blind X-Channel Guided by a Single Antenna Relay with Alternating CSIT

Alternating CSI is a situation where the CSI is considered to have different states of availability across the different channels and across the different channel uses instead of a one fixed state of availability. The possible CSI availability states, i, are: perfect, i = p, delayed, i = d and no CSI, i = n. The fraction of channels associated with CSI availability state, i, is denoted by  $\lambda_i$ . The distribution of  $\lambda_i$  for the different CSI availability states is denoted by the 3-tuple  $\Lambda = (\lambda_p, \lambda_d, \lambda_n)$ , where  $\sum_{i \in \{p,d,n\}} \lambda_i = 1$ . In order to calculate the fraction of each CSI availability state, one needs to sum the channels with that specific CSI availability state in all channel uses and divide it by the total channels to get the CSI state fraction. The total number of channels is the number of channel uses multiplied by the total number of channels fedback in one channel use.

It is well known that  $\frac{4}{3}$  DoF is achievable for the SISO X-channel with perfect CSIT and that X-channel with perfect CSIT cannot be further enhanced by

adding a relay or assuming output feedback. Hence, this is the ultimate upper bound for the SISO X-channel. The initial achievable scheme for the X-channel was derived assuming a perfect CSIT in all channel uses [10]. Later work considered the idea of alternating CSIT where it was shown that perfect CSIT does not need to be available all the time to achieve the upper bound [27, 30]. In other words, there is redundancy in the CSIT availability, which is not fully utilized in several proposed achievable schemes. To further motivate our work in this Chapter, consider the example of calculating the CSIT distribution,  $\Lambda$ , for the achievable scheme in Figure 4.1 proposed in [27]. There are four feedback channels in every channel use, so the total number of feedback channels is twelve. For perfect CSIT, the sum of perfect channels is four from the third channel use. Then for the delayed CSIT, there are two delayed channels at t=1 from  $Rx_2$  and two delayed channels at t=2 from  $Rx_1$ . Hence, the state distribution is  $\Lambda = (4/12, 4/12, 4/12)$ . Following the same method used in [27, 30], we can study the alternating CSIT availability distribution for the SISO X-channel which gives the following results, respectively:

- 1.  $\frac{4}{3}$  DoF is achievable with the scheme given in [27] for SISO X-channel with blind transmitters guided by a relay with alternating CSIT of distribution  $\Lambda = (4/18, 4/18, 10/18)$  instead of their studied fully perfect CSIT at relay.
- 2.  $\frac{4}{3}$  DoF with alternating CSIT for the SISO X-channel using the scheme given in [30] is achievable for  $\Lambda = (4/12, 4/12, 4/12)$ .

Remark 1. From the previous results of CSIT distributions it can be observed that in order to achieve the optimal DoF in both cases, there should be four perfect CSIT availability states whether in the same channel use across the different channels or alternating.

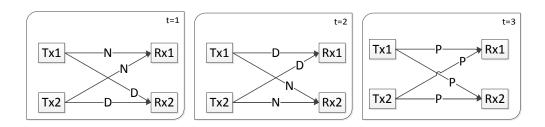


Figure 4.1: CSIT states for  $\frac{4}{3}$  DoF Achievable Scheme

Remark 2. In the case of alternating CSIT [30],  $\frac{5}{4}$  DoF is achievable having only two perfect CSIT and the rest are a mix of delayed and no CSIT, which further demonstrates that if the perfect CSIT availability is less than four, the optimal DoF can not be achieved, however, a sub-optimal DoF is achievable.

Remark 3. In the case of the SISO X-channel with relay [27], the number of feedback channels from receivers to relay are the four channels from transmitters and two channels from the relay.

#### 4.1 Proposed alternating CSIT scheme

Consider the SISO X-channel with the system model described in Fig. 4.1 with alternating CSIT pattern at the relay given by  $\Lambda = (nd, dn, pp)$  over three channel uses, where the first element nd means no CSIT from  $Rx_1$  and delayed CSIT from  $Rx_2$  in the first channel use, the second element dn indicates delayed CSIT from  $Rx_2$  and no CSIT from  $Rx_1$  in the second channel use and the last element pp represents perfect CSIT from both receivers. Our proposed scheme is composed of two phases spanning three channel uses. The first phase, which spans two channel uses is dedicated to simultaneously sending data directly from both transmitters to receivers. In the second phase, which has only one

time use, the relay in addition to one of the transmitters are sending data. The scheme is shown in Fig. 4.2.

• **Phase one:** In this phase, the two transmitters send all data symbols in two channel uses, i.e.,  $d_{11}$ ,  $d_{12}$ ,  $d_{21}$ ,  $d_{22}$ . In the first channel use  $d_{11}$  and  $d_{12}$  are sent and they are intended for  $Rx_1$ , while it is received as interference for  $Rx_2$ . As a result the received signals are given by

$$Y_1(1) = h_{11}(1)d_{11}(1) + h_{12}(1)d_{12}, (4.1)$$

$$Y_2(1) = h_{21}(1)d_{11}(1) + h_{22}(1)d_{12}, (4.2)$$

$$Y_R(1) = h_{R1}(1)d_{11}(1) + h_{R2}(1)d_{12}. (4.3)$$

Where  $h_{ij}(t)$  is the channel gain between transmitter  $Tx_j$  and receiver  $Rx_i$  during channel use t and  $d_{ij}$  is the data transmitted from transmitter  $Tx_j$  and intended for receiver  $Rx_i$ .

For the second channel use,  $d_{21}$ ,  $d_{22}$  are sent and the received signals are given by

$$Y_1(2) = h_{11}(2)d_{21}(2) + h_{12}(2)d_{22}, (4.4)$$

$$Y_2(2) = h_{21}(2)d_{21}(2) + h_{22}(2)d_{22}, (4.5)$$

$$Y_R(2) = h_{R1}(2)d_{21}(2) + h_{R2}(2)d_{22}. (4.6)$$

• Phase two: From Phase one, each receiver has one desired equation and one interference. The objective of Phase two is that by its end each receiver should have one additional equation to decode its intended messages, without producing extra interference to the destinations. In this phase,  $Tx_1$  transmit  $X_1(3) = d_{11} + d_{21}$ . Simultaneously, the Relay combines a message from the previously received symbols and transmit

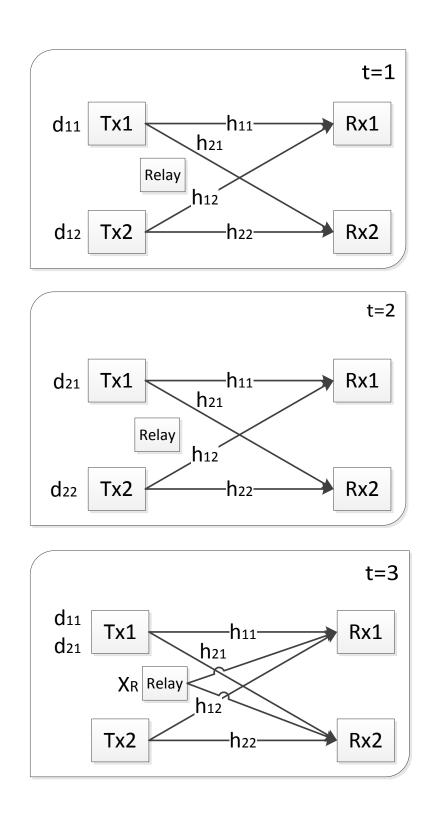


Figure 4.2: Alternating CSI achievable scheme

 $X_R(3) = \alpha Y_R(1) + \beta Y_R(2)$ . The values of  $\alpha$  and  $\beta$  should be designed in a way to align interference (similar to the scheme in [27]). In the following we will inspect the received signals in this phase and deduce the values of  $\alpha$  and  $\beta$  to be calculated by the relay using its CSIT knowledge. For the third channel use, the received signals are as follows:

For  $Rx_1$  we receive

$$Y_{1}(3) = h_{11}(3)X_{1}(3) + h_{1R}X_{R}(3)$$

$$= h_{11}(3)d_{11} + h_{11}(3)d_{21} + \alpha h_{1R}(3)h_{R1}(1)d_{11} + \alpha h_{1R}(3)h_{R2}(1)d_{12}$$

$$+ \beta h_{1R}(3)h_{R1}(2)d_{21} + \beta h_{1R}(3)h_{R2}(2)d_{22}$$

$$= [h_{11}(3) + \alpha h_{1R}(3)h_{R1}(1)]d_{11} + \alpha h_{1R}(3)h_{R2}(1)d_{12}$$

$$+ [h_{11}(3) + \beta h_{1R}(3)h_{R1}(2)]d_{21} + \beta h_{1R}(3)h_{R2}(2)d_{22}. \tag{4.7}$$

Similarly, for  $Rx_2$  we have

$$Y_2(3) = [h_{21}(3) + \alpha h_{2R}(3)h_{R1}(1)]d_{11} + \alpha h_{2R}(3)h_{R2}(1)d_{12}$$

$$+[h_{21}(3) + \beta h_{2R}(3)h_{R1}(2)]d_{21} + \beta h_{2R}(3)h_{R2}(2)d_{22}.$$
(4.8)

If vectors aligned in one dimension to align interference as follows,

$$\frac{h_{11}(3) + \beta h_{1R}(3)h_{R1}(2)}{\beta h_{1R}(3)h_{R2}(2)} = \frac{h_{11}(2)}{h_{12}(2)}$$
(4.9)

$$\frac{h_{21}(3) + \alpha h_{2R}(3)h_{R1}(1)}{\alpha h_{2R}(3)h_{R2}(1)} = \frac{h_{21}(1)}{h_{22}(1)},\tag{4.10}$$

then  $Rx_1$  can subtract the following from  $Y_1(3)$ 

$$Y_1(2) \cdot \frac{\beta h_{1R}(3) h_{R2}(2)}{h_{12}(2)} \tag{4.11}$$

and  $Rx_2$  can subtract the following from  $Y_2(3)$ 

$$Y_2(1) \cdot \frac{\alpha h_{2R}(3) h_{R2}(1)}{h_{22}(1)} \tag{4.12}$$

in order to produce an equation for decoding. This can be done by choosing  $\alpha$  and  $\beta$  as follows

$$\alpha = \frac{h_{21}(3)h_{22}(1)}{h_{21}(1)h_{2R}(3)h_{R2}(1) - h_{22}(1)h_{2R}(3)h_{R1}(1)}$$
(4.13)

$$\beta = \frac{h_{11}(3)h_{12}(2)}{h_{11}(2)h_{1R}(3)h_{R2}(2) - h_{12}(2)h_{1R}(3)h_{R1}(2)}. (4.14)$$

The scheme is following an alternating CSI pattern of (nd,dn,pp), so  $\alpha$  and  $\beta$  can be computed at the relay. The CSI of  $Rx_2$  is delayed in the first channel use and for  $Rx_1$  the CSI is delayed in the second channel use. While for the third channel use both receivers have a perfect CSI. This combination of CSI status will make the relay able to compute the two symbols  $\alpha$  and  $\beta$  with the relay having perfect CSI only in the third channel use. The symbols can be computed at the receivers also since they are assumed to have perfect CSI. After that the receivers can subtract the interference and obtain

$$Y_1'(3) = Y_1(3) - Y_1(2) \cdot \frac{\beta h_{1R}(3) h_{R2}(2)}{h_{12}(2)}$$
$$= (h_{11}(3) + \alpha h_{1R}(3) h_{R1}(1)) d_{11} + \alpha h_{1R}(3) h_{R2}(1) d_{12}$$
(4.15)

$$Y_2'(3) = Y_2(3) - Y_2(1) \cdot \frac{\beta h_{2R}(3) h_{R2}(1)}{h_{22}(1)}$$
$$= (h_{21}(3) + \beta h_{2R}(3) h_{R1}(2)) d_{21} + \beta h_{2R}(3) h_{R2}(2) d_{22}. \tag{4.16}$$

Now we have two linearly independent equations for each destination,  $Y'_1(3)$  and  $Y_1(1)$ ,  $Y'_2(3)$  and  $Y_2(2)$ . Each destination can recover their intended messages

and  $\frac{4}{3}$  degrees of freedom is achieved. The next section is a theorem for the prescribed achievable scheme for the SISO X-channel with alternating CSI.

**Theorem 1.** For the two users SISO X-channel in time varying and frequency selective setting with blind Txs and single antenna relay, the upper bound on the total degrees of freedom is achievable iff the following requirements on the CSI alternating pattern are satisfied at the relay:

- Requirement 1: The ratio of unknown CSI for channels to  $Rx_1$  and  $Rx_2$  are allowed to be  $\lambda_n \leq \frac{1}{3}$
- Requirement 2: The CSI of the channels to  $Rx_1$  and  $Rx_2$  should be perfectly known in the third channel use(CSI for Rx1 and Rx2)).
- Requirement 3: At each channel use, at least one transmitter should have some CSI (perfect or delayed). Two transmitters should not be simultaneously with unknown CSI.
- Requirement 4: A receiver can't have no CSI in consecutive channel uses.

**Note:** The relay has global CSI and all CSI data are fed-back from receivers to relay. All CSI mentioned above is read at the relay since transmitters are blind.

*Proof.* We start with the necessity proof by a proof of each requirement in the theorem as follows:

• Requirement 1: If the achievable scheme is done following the CSI pattern (nd,nn,pp), then in phase two the relay will not be able to compute the symbols  $\alpha$  and  $\beta$  in equations 4.13 and 4.14 since the delayed CSI for  $Rx_2$  is not available. Taking into account the case where the

scheme order of execution is switched, the same argument applies to the case (nn,nd,pp). This proofs requirement 1 in the theorem.

- Requirement 2: For the CSI pattern (nd,dn,xx), where xx stands for any channel state other than pp, if the last channel use has a state different than perfect CSI, then the step in 4.13 and 4.14 will not be done correctly since perfect information of CSI is needed at the relay for the symbols  $\alpha$  and  $\beta$  to align the interference and solve for the intended messages.
- Requirement 3: From requirement 2 we can have simultaneous no CSI only in the first two channel uses. Consider the CSI pattern (nn,dn,pp), where the first channel use has no CSI so. Accordingly, in phase two the relay will not have the information of both receivers' channels. Referring to equations 4.19 and 4.20, the relay requires information of both channels in order for the scheme to be able to compute the symbols. Hence, the scheme will not work for this pattern. The same follows for the pattern (nd,nn,pp), but the channels are not available for  $Rx_1$  instead of  $Rx_2$ . The same applies for the CSI patterns (nn,nd,pp) and (dn,nn,pp) but exchanging the receivers.
- Requirement 4: Consider the CSI pattern (nd,nn,pp), where  $Rx_1$  has no CSI in consecutive channel uses, in order to compute the symbols  $\alpha$  and  $\beta$  in equations 4.13 and 4.14, the delayed CSI is needed for  $Rx_2$  in the first channel use and for  $Rx_1$  in the second channel use. The same argument can be applied to show that the scheme will not work for the three other possible permutations of CSI alternation patterns (dn,nn,pp),(nn,dn,pp) and (nn,nd,pp).

The sufficiency proof follows from the achievable scheme described in section 4.1.

## 4.2 Results

Table 4.1: CSIT permutations which achieve  $\frac{4}{3}$  DoF.

Λ	Pattern	$(\lambda_p, \lambda_d, \lambda_n)$
$\Lambda_1$	(dn, nd, pp)	$(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$
$\Lambda_2$	(nd, dn, pp)	$(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$
$\Lambda_3$	(dd, dn, pp)	$(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$
$\Lambda_4$	(dd, nd, pp)	$(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$
$\Lambda_5$	(nd, dd, pp)	$\begin{pmatrix} \left(\frac{1}{3}, \frac{1}{2}, \frac{1}{6}\right) \\ \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \end{pmatrix}$
$\Lambda_6$	(dn, dd, pp)	$(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$
$\Lambda_7$	(dd, dd, pp)	$(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$ $(\frac{1}{3}, \frac{2}{3}, 0)$
$\Lambda_8$	(dd, pd, pp)	$(\frac{1}{2}, \frac{1}{2}, 0)$
$\Lambda_9$	(dd, dp, pp)	$(\frac{1}{2}, \frac{1}{2}, 0)$
$\Lambda_{10}$	(pd, dp, pp)	$(\frac{2}{3}, \frac{1}{3}, 0)$
$\Lambda_{11}$	(pd, pd, pp)	$(\frac{2}{3}, \frac{1}{3}, 0)$
$\Lambda_{12}$	(dp, pd, pp)	$(\frac{2}{3}, \frac{1}{3}, 0)$
$\Lambda_{13}$	(dp, dp, pp)	$(\frac{2}{3}, \frac{1}{3}, 0)$
$\Lambda_{14}$	(pp, pd, pp)	$(\frac{5}{6}, \frac{1}{6}, 0)$
$\Lambda_{15}$	(pp, dp, pp)	$(\frac{5}{6}, \frac{1}{6}, 0)$
$\Lambda_{16}$	(dp, pp, pp)	$(\frac{5}{6}, \frac{1}{6}, 0)$
$\Lambda_{17}$	(pd, pp, pp)	$(\frac{5}{6}, \frac{1}{6}, 0)$
$\Lambda_{18}$	(pd, dn, pp)	$\begin{pmatrix} \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right) \\ \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \end{pmatrix}$
$\Lambda_{19}$	(pd, nd, pp)	$(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$
$\Lambda_{20}$	(dp, dn, pp)	$(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$
$\Lambda_{21}$	(dp, nd, pp)	$(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$
$\Lambda_{22}$	(pp, pp, pp)	(1,0,0)

As a result, for the previously studied relay aided X-channel we addressed the idea of alternating CSIT which is towards the practical systems. The optimal  $\frac{4}{3}$  DoF is achieved by a scheme that doesn't mandate full CSIT availability. However, partial alternating CSIT availability at the relay transmitters is used. In comparison to the work done in [27], where they achieve  $\frac{4}{3}$  DoF with full perfect in case of single antenna then our result is more practical. While for the delayed CSI case they achieve  $\frac{4}{3}$  with 2-antenna relay but here we achieve it under the assumption of fractional perfect with single antenna relay. Also we deduce the minimum CSIT availability for the proposed scheme. A theorem followed by its proof where discussed to achieve the optimality of the proposed

scheme. Table 4.1 lists all possible CSI distributions that achieves  $\frac{4}{3}$  as per theorem. In the presented scheme we have addressed the practical aspects of alternating CSI.

In the following we will try to determine the region over channel state availability ratios, defined by the 3-tuple  $\Lambda(lambda_p, \lambda_d, \lambda_n)$ , over which the maximum DoF of 4/3 is achievable using our scheme under alternating CSIT. It is worth noting that it suffices to define the region using the 2-tuple  $(\lambda_p, \lambda_d)$  since  $\lambda_n = 1 - (\lambda_p + \lambda_d)$ .

It must be noted that by definition

$$\lambda_p + \lambda_d \le 1. \tag{4.17}$$

From Requirement 1 in Theorem 1 it can be deduced that

$$\lambda_p + \lambda_d \ge \frac{2}{3}.\tag{4.18}$$

Requirement 2 in Theorem 1 indicates that at least two channel uses out of the total 6 uses will have perfect CSIT, i = p; hence,

$$\lambda_p \ge \frac{1}{3}.\tag{4.19}$$

Requirement 3 in Theorem 1 mandates that

$$\lambda_p + \lambda_d \ge \frac{1}{2},\tag{4.20}$$

which is a redundant constraint compared to 4.18; hence, it will be ignored.

Accordingly, combining 4.17, 4.18 and 4.19, we get the region shown in Fig. 4.3, which includes all the points (the actual DoF achievability points) listed in Table 4.1.

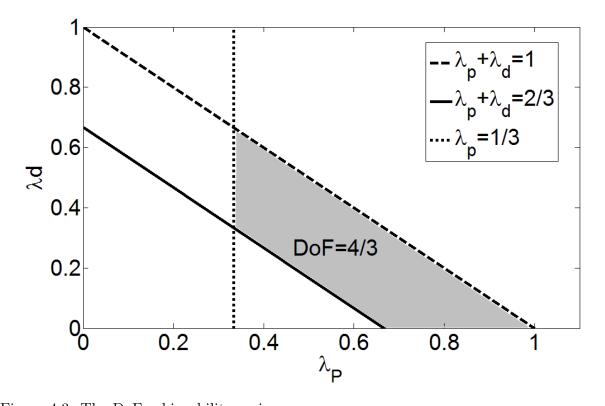


Figure 4.3: The DoF achievability region.

## Chapter 5

## Conclusions and Future Work

In this work, we show the advantage of aiding the X-Channel by a single antenna relay with no CSIT. We have shown that the relay has a positive impact in the X-channel when no CSIT is available. We have shown that  $\frac{4}{3}$  DoF is achievable with single antenna relay and delayed output feedback instead of perfect CSI at the relay. The delayed output feedback achieved  $\frac{4}{3}$  DoF while the X-channel with delayed CSIT achieved a maximum  $\frac{6}{5}$  DoF. This matched the optimal DoF for the X-channel with perfect CSIT and no relay. Hence, the result we obtained is optimal and is better than the DoF for the X-channel with blind transmitters where the DoF is one. The single antenna relay here was helpful for the X-channel with blind transmitters.

In addition, we examined the idea of alternating CSI. We have shown that the optimal  $\frac{4}{3}$  DoF is achievable with partially alternating CSI for the X-channel with single antenna relay. The partial alternating CSI is considered at the relay transmitters to achieve the optimal degrees of freedom. We showed that we don't need the full CSI information and an alternating part of it can work efficiently. We have studied the minimum CSIT availability with a proved theorem to attain optimality. The theorem has 4 requirements that is mainly concerned about specific CSIT availability  $\lambda$ . The alternating CSI helps to use

the available CSI in an efficient way better than the fixed CSI towards practical systems, where some of CSI information could be unavailable at all times.

The practical side of all DoF studies is about the design of the achievable schemes which can be applied to real systems, however, achievable schemes when applied to real systems may not be able to achieve the upper bound due to practical imperfections.

Future work includes extending the work to cover the relay aided MIMO X-channel with delayed output feedback and delayed CSI. Furthermore, examine the impact of the relay on other channel models in terms of achievable schemes and alternating channel state information. Also looking for the idea of alternating CSI in other channel models and examines its impact in the achievable degrees of freedom. Alternating CSI could be studied to consider practical systems specifications. In addition, the work can be extended to cover the X-channel aided by multiple relays considering blind transmitters and delayed CSI at the relay side.

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