OIL AND GAS EXTRACTION

Scattering of acoustic waves bubbles of gas in reservoirs

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The article is concerned with questions of acoustic wave scattering by gas bubbles. The results obtained allow determining the acoustic resonant frequency, in which the effect of oil-producing formation wave treatment with gas bubbles is maximum.

From professional literature it is known that for increasing of flow rate of oil wells it is normally to use acoustic methods of action at geopolitical environment of productive layers [1-5]. During the acoustic treatment of productive layers the key parameters are the frequency and amplitude of the pulse action. This technology gets particular relevance in the oil fields, which are in advanced stage of development, which is characterized by the presence of gas bubbles in the fluid of a certain size. This occurs when the pressure in the fluid becomes less than the saturation pressure of gas oil.

The interaction problem of the incident wave with bubbles of gas in the gas-liquid environment are published in works [1-5], sufficiently describing the picture of the amplitudes of the wave field generated in the gas-liquid environment of the resulting oscillations bubbles. However, insufficient attention was given to studies of scattered waves, which are generated by the interaction of the incident wave with bubbles, and evaluated the amplitude of the wave field. We shall consider the determination of the resonance frequency of the acoustic effect on the gas-saturated environment.

Suppose that in a homogeneous environment of the characterized density ρ and speed of sound c is distributed acoustic wave harmonic with frequency ω:

$$p_0(r,t)=p_m e^{ikr-i\omega t}=p_0(r)e^{i\omega t}, \qquad (1)$$

where $p_0(r) = p_m e^{ikr}$ – complex amplitude of pressure; $\kappa = \omega/c$- wave number; **I=V - 1** - complex number.

Later times multiplier $e^{-i\omega t}$ omit as considering the established processes.

If there are obstacles in the environment as gas bubbles to the incident wave (1) is added to the wave, which is called the scattered wave. The amplitude of the pressure is denoted by $p_s(r)$. The sum of pressure $p_0(r) + p_s(r) = p(r)$ determines the acoustic pressure field in the medium and presence of a bubble.

Permeable environment wave in bubble is denoted by $p_n(r)$. We consider the scattering of acoustic waves on bubbles, which size is much smaller than the length of the incident wave kR0

<< 1. So, we will consider scattering of a plane acoustic harmonic waves (1) with frequency ω in the gas bubbles of radius R0, filled with gas with density ρn at the speed of sound CN, located in the liquid.

The amplitude of the pressure p (r) in the environment satisfies the Helmholtz equation [6]:

$$\frac{\partial^2 p}{\partial r^2}+k^2p=0. \qquad (2)$$

As the ambient pressure field ps (r) = p (r) - p0 (r), and the incident wave satisfies the Helmholtz equation, the scattered wave and we have:

$$\frac{\partial^2 p_s}{\partial r^2}+k^2p_s=0. \qquad (3)$$

The scattered wave must satisfy the radiation conditions, that determines the traveling wave that goes from bubbles to infinity. On the border of obstacle (bubble) the boundary conditions should be performed: the equality of pressure and normal velocity components of particles on the surface of the bubble (*r*=*R*$_0$):

$$v_n=\frac{1}{i\omega\rho_n}\frac{\partial p_n}{\partial r} \qquad (4)$$

$$p_0+p_s=p_n; \qquad (5)$$

$$\frac{1}{i\omega\rho}\frac{\partial(p_0+p_s)}{\partial r}=v_n,$$

where p_n - pressure gas (air) on the surface of bubble;

– vibrational displacements of the surface normal speed

Neglecting viscosity and thermal conductivity, we believe that the gas inside the blister describes the linear equation of state [6]:

$$p_n=\rho_n c_n^2. \qquad (6)$$

Equation (6) can be represented through vibrational velocity vB in the form [6]:

$$p_n=-i\frac{3\rho_{n_0}c_n^2}{\omega R_0}v_n=-i\frac{3\kappa_n}{\omega R_0}v_n, \qquad (7)$$

where v_B = *dR/dt* - oscillatory velocity of normal displacement of bubble surface ; R – bubble variable radius, k = p "-0s2p elasticity gas bubbles; rp0 - the average density of the gas.

Field pressure, which ambient with gas bubbles, we search in series the following line:

$$p_s(r,\Theta)=\sum_{n=0}^{\infty}p_m B_n H_n^{(1)}(kr)P_n(\Theta), \qquad (8)$$

where $H_n^{(1)}$*(кг)* - Hankel function of the 1st type; *Pn(@)* - Legendre polynomial; B_n – coefficients, which determined from the boundary conditions of the problem (4), (5).

When kg << 1 the first term can be limited to which n = 0. Then, taking into account that the Legendre polynomial P0 (&) = 1 and asymptotically function H0 (1) (kg) is given by H0 (1) (kg) = —*i** e^{ikr}/kr and the scattered field (8) can be written as

$$p_s(r)=-ip_mB_0\frac{e^{ikr}}{kr}, \qquad (9)$$

where the coefficient B0 corresponds exclusive scattering.

The coefficient B0 is determined from the boundary conditions (4) and (5) that at r = R0 are given by

$$p_m e^{ikR_0} - iB_0 p_m \frac{e^{ikR_0}}{kR_0} = -i\frac{3\rho_{n_0} c_n^2}{\omega R_0} v_n; \qquad (10)$$

$$p_m e^{ikR_0}\left(\frac{k}{\omega\rho} + \frac{B_0}{\omega\rho k R_0^2} - \frac{ikB_0}{\omega\rho k R_0}\right) = v_n. \qquad (11)$$

Expand eikR0 function in a Taylor line in dagrees kR0. Taking into account that kR0 << 1, we shall present the line of only the first two members:

where the squared

$$e^{ikR_0} \approx 1 + ikR_0. \qquad (12)$$

Substituting (12) into the system of equations (10) - (11), we obtain:

$$\begin{aligned} p_m(1+ikR_0)\left(1-\frac{iB_0}{kR_0}\right) &= -i\frac{3\rho_{n_0} c_n^2}{\omega R_0} v_n; \\ p_m(1+ikR_0)\left(\frac{k^2R_0^2 + B_0 - iB_0 kR_0}{\omega\rho k R_0^2}\right) &= v_n. \end{aligned} \qquad (13)$$

By solving the system of equations (13) according to the coefficient B, we find:

$$B_0 = \frac{kR_0 + i(kR_0)^2\left(\frac{\omega_0}{\omega}\right)^2}{i\left(1-\frac{\omega_0^2}{\omega^2}(1-ikR_0)\right)}, \qquad (14)$$

$\omega_0^2 = \frac{3\rho_{n_0} c_n^2}{\rho R_0^2}$ - circular frequency of oscillation of the gas bubble.

After substituting (14) into (9) we obtain the formula for determining the field scattered by the gas bubbles, during the term of the acoustic wave at him (1):

$$p_s(r) = \frac{p_m e^{ikr}}{kr}\left[\frac{kR_0 + i(kR_0)^2\left(\frac{\omega_0}{\omega}\right)^2}{(\omega_0^2/\omega^2 - 1) - ikR_0}\right]. \qquad (15)$$

Note that the coefficient B0 (14) defines the complex amplitude of the scattered wave. As can be seen, the amplitude of the scattered wave has a resonant character. It takes the maximum value in aqueous ω = ω0.

Note: If the expression (14) we neglect the small member (kR0) 2, then we obtain:

$$B_0 = \frac{ikR_0}{\left[\left(\frac{\omega_0^2}{\omega^2} - 1\right) - ikR_0\right]},$$

which coincides with the result of [6], where the members (kR0) 2 neglected and not taken into account the radial velocity componentщо

$v_{r_0} = \frac{\partial p_0}{\partial r}$. in the incident wave

Module relation (15) determines the pressure dependence of the amplitude of the scattered wave on the frequency of the incident wave.

After introducing the dimensionless frequency co =co$_0$/co formula (16) takes the form:

$$p_s = \frac{p_m R_0}{r}\sqrt{\frac{1+(kR_0)^2\overline{\omega}^4}{\left(\overline{\omega}^2-1\right)^2+(kR_0)^2}}. \qquad (17)$$

During the acoustic effect on the gas bubble usually, (kR) 2 << 1, and this profile can be neglected. Then from (17) we obtain:

$$p_s = \frac{p_m R_0}{r}\frac{1}{\left(\overline{\omega}^2-1\right)}, \qquad (18)$$

which coincides with the data [6] at (R0) 2 << 1.

However, formula (17) at the point u = 1 allows to pinpoint the amplitude of the scattered field, as opposed to dependence (18).

So we get the dependence of the amplitude of pressure waves scattered by gas bubbles, during its interaction with the incident acoustic wave (1) of the frequency without affecting co dissipative energy losses due to viscosity and thermal conductivity, are available in a production environment. This dependence can be used to estimate the pressure field emitted by the gas bubbles, when choosing an acoustic modes of action on the environment.

Note that in [5] a general formula was obtained for the pressure wave emitted by the gas bubbles, based on dissipative losses, which are characterized by the parameter p attenuation of the incident acoustic pressure wave and coefficient.

$$\alpha = 1 + \frac{4\eta}{\rho c R_0}$$

(η - dynamic viscosity of the environment).

Table 1 The dependence of the dimensionless amplitude of the scattered wave from the dimensionless frequency ω for different sizes of bubbles

$\overline{\omega}$	0	0,5	1	2	3	4	5
$p_s/\left(\frac{p_m R_0}{r}\right)$, при $R_0 = 1\cdot10^{-4}$ м	1,0	1,333	500,0	0,333	0,125	0,066	0,041
$p_s/\left(\frac{p_m R_0}{r}\right)$, при $R_0 = 1\cdot10^{-5}$ м	1,0	1,333	5000,0	0,333	0,125	0,066	0,041

Table 2 The dependence of the dimensionless amplitude of the scattered wave from a distance for different modes of interaction of acoustic waves with bubble

r/R_0	1	10	100	500	1000	5000
p_s/p_m ($\overline{\omega}=1$)	500	50	5	1	0, 5	0 ,1
p_s/p_m ($\overline{\omega}=2$)	0,333	0,333	0,333	$6{,}66\cdot10^{-4}$	$3{,}33\cdot10^{-4}$	$6{,}66\cdot10^{-5}$

Using this dependence and laying it parameters b = 0, a = 1 (environment without viscosity and dissipative losses), we obtain the formula for the amplitude of the pressure wave emitted by gas bubbles, as:

$$p_s = \frac{p_m R_0}{r}\frac{1}{\left(\frac{\omega_0}{\omega}\right)^2-1}\left(\frac{\omega_0}{\Omega}\right). \qquad (19)$$

For environments without resistance $\omega_0 = \Omega$ **[5]**. Thus, the dependence

(19) coincides with formula (17) obtained for idealized environment. In reality resonant scattering bubbles is significant, but not as much as the theory that ignores the loss of

mechanical energy. Resonant bubbles not only scatter but also absorb the energy of the incident acoustic wave, and due to the large amplitude of doing it effectively.

In Table. 1 it is shown the data for the dimensionless pressure amplitude of the scattered wave for some radii blubble.

where $k = \omega/c = \frac{32000}{1600} = 20$ depending on the dimensionless frequency -.

The data in Table 1 show that the bubble highly responsive to a frequency that to its resonant frequency. At other frequencies the scattered field pressure slightly

dependent on $p_s \Big/ \left(\frac{p_m R_0}{r}\right)$ and does not depend on the size of the bubble.

The Table. 2 shows the dependence of the dimensionless ratio of the amplitude of the incident pressure wave scattered from a distance r/R0 dimensionless ratios for some frequency -. As you can see in Table. 2, the ratio p_s / p_m by - = 1 (resonant mode) is significantly higher than the corresponding value in $\bar{\omega} = 2$. That is, the resonant mode maximizes the ambient gas blister pressure field acting on its surrounding environment.

Summarizing conducted theoretical studies, we can conclude that the best effect of processing fluid from blisters can be achieved at the resonance frequency of the acoustic mode of action of ω, which coincides with the natural frequency of bubble. In this action the effect of peening environment with bubbless associated with the amplitude of the scattered waves is the strongest, which reduces viscosity and connection of fluid (oil) with the solid phase of environmental layer, which is accompanied by improvements to the inflow of fluids downhole and increasing of their flow rate.

This is especially important for depleted oil fields that are in the final stages of their exploitation and which have a bubble mode of fluid flow.