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Use of Response Surfaces in the Design of a Simple Step Stress Accelerated Test Plan

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Industrial Engineering

by

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Abstract

In designing accelerated testing plans, cost is a factor that is missing in much of the literature. This paper explicits considers cost by developing an optimization model with the objective to minimize costs for a simple step stress accelerated test plan. Two methodologies are employed. One is an optimization approach in which an attempt is made to quantify the behavior of a series-parallel hardware system over all stages of testing using a response surface, and then an optimization model is used to determine the settings for stresses and failure mode modifications for all stages of testing prior to the start of testing. The second methodology or sequential stage approach is to generate a response surface using data from a completed test stage to determine the settings of stresses and failure mode modifications for the next stage. Then this process is repeated for all stages of testing.

When validating the results of the optimization model through simulation, the model overestimated costs. Assuming the simulated optimal settings are the true value of cost, the sequential approach produced suboptimal results. This is because each stage of testing results in narrowing the search parameters of a solution. However, it was found that the sequential stage approach had similar costs to that of the optimization model. Although the optimization model has a better solution, it requires much more data initially whereas the sequential stage approach does not require information about the system for all stages prior to testing. If information of the system's behavior is known for all stages prior to testing, then the optimization approach is more advantageous, yet most cases have limited information so the sequential stage approach should be utilized.

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The completion of this thesis would not be possible without my advisor Dr. Ed Pohl. I am grateful for his guidance and knowledge in this subject matter, his encouragement in times of stress, and the numerous opportunities he has afforded me during the time we worked together. I would also like to thank my family and friends who helped me endure the many periods of frustration and confusion. Thank you all for kind words, tough love, and empathy. Lastly, I would like to specially thank my friend Mary who advised and inspired me to restart my academic endeavors and seek something I could be passionate about. I could not have done this alone. Thank you all.

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Chapter 1

Introduction

1.1 Background

The rise of reliability engineering dates back to the World War II era. Prior to World War II, reliability was used in the study of the fatigue life of materials by Wallodie Weibull and was even used in the insurance industry as the probability of a human's survival [8]. During World War II, complex electronic systems were introduced to the military and high failure rates were observed in vacuum tubes, which were used in radios and radar [19]. Since then, the military has taken an interest in the study of reliability to increase equipment reliability and prevent frequent maintenance [3]. Some examples of this are: Aeronautical Radio, Inc. (ARINC), which was an organization developed by airlines to investigate defective vacuum tubes initially and have since focused on reliability issues in the military; the Rome Air Development Center (RADC), which was developed to study reliability issues in the Air Force; and the Advisory Group on the Reliability of Electronic Equipment (AGREE), which was developed by the Department of Defense, that published a report prompting the military to establish quality and reliability requirements for components from suppliers, testing requirements for equipment, and the collection of data from testing for use in diagnosing problems [19].

A majority of the research during this time period assumed an exponential distribution for failure times, which was developed by Epstein and Sobel [10], and whose density is given as

$$f(x;\theta) = \frac{1}{\theta}e^{-x/\theta},\ \theta > 0,\ x \geq 0, \tag{1.1}$$

where x is a measure of time and θ is average life of the equipment or component. However, the Weibull distribution would also receive significant attention in research [19]. The Weibull distribution is given as

$$F(x) = 1 - e^{-\phi(x)},\ \phi(x) > 0 \tag{1.2}$$

where $\phi(x)$ is a positive, non-decreasing function [3]. Many of the documents developed during this time period contributed to or became standards for the military like Military Handbook 781, regarding the methods and requirements for environmental testing, the Military Handbook 217, which catalogs the failure rates of military components, and the Military Handbook 189, which provide guidelines for improving the reliability of military equipment [19].

Reliability growth testing is a method of improving reliability over time through engineering redesign and a test-analyze-fix approach. Typically, testing is conducted for a fixed amount of time or until a failure occurs in the system. Then failure data is analyzed, and changes are implemented to reduce the number of failure modes in the system. However, in many cases, the testing time is significantly less than the expected life of a component [8]. Thus, there is need for accelerated life testing, which is subjecting tested units to elevated levels of environmental or operational stresses like humidity, vibration, voltage, and temperature in order to estimate the life distribution of the product or component under normal operating conditions [21].

The two most common types of accelerated life tests (ALT) are: constant stress accelerated life tests and step stress accelerated life tests. In constant stress ALT, multiple copies of a unit are tested at a constant level of high stress until failure, and in step stress ALT, multiple copies of a unit are subjected to a high level of stress, x_1, and then increased to another stress level x_2 until all units fail. Many accelerated life tests only consider two stresses, which are called simple stress accelerated life tests. Step stress ALTs can be categorized into two different types of tests: time-step and failure-step. In time-step stress accelerated testing, units are tested at x_1 until a predetermined time, τ, and then all units still functioning are elevated to x_2 until all units fail. In failure-step stress accelerated testing, units are tested at x_1 until a predetermined proportion, p_1, have failed, and then the remaining functioning units are elevated to x_2 until the remaining units fail. Failure-step stress ALT is not common because this requires continuous monitoring, which is an inconvenience [20].

There has been a renewed interest in the field of accelerated reliability growth testing (ARGT) because there is a demand for high reliability requirements and long lifetimes for products, and product testing under normal conditions would be time and cost consuming [16]. Therefore, ARGT is necessary, and planning is essential to the success of ARGT [12]. Design of experiment (DOE) methodologies have been integrated into ARGT to provide time and cost-efficient strategies while keeping the effects of noise variables low [23]. The purpose of DOE is to minimize the number of experiments (combinations of stresses) to conduct while predicting the expected lifetime of the product or component under normal operating conditions [24].

1.2 Motivation

There has been an abundance of work conducted in planning accelerated life tests; however, a majority of the work does not consider cost modeling within its framework [22]. This is an aspect of the problem that has real world application as cost can be a driving force for projects or an enormous limiting factor. The focus of this paper is to develop an accelerated testing plan employing a response surface that models the relationship between cost, testing time, and reliability, and then using the response surface to find the most efficient plan to minimize costs while meeting a target system reliability for a series-parallel hardware system. Although there is no evidence that response surfaces have been used to plan accelerated reliability growth testing, it is believed that this approach will help to minimize costs by means of optimization. Rather than developing an experimental design to explore the surface, it is assumed that a model fit to the response surface can generate a plan with minimal costs.

Chapter 2

Literature Review

In the following sections, a description of the systematic literature review process is given along with an overview of accelerated life testing, design of experiment techniques, and the incorporation of design of experiments in accelerated reliability growth testing.

2.1 Research Method

A systematic literature review inspired by [25] was conducted using the Compendex, EBSCO, and Proquest databases. The purpose for using a systematic literature review was not only to find a comprehensive list of articles regarding the subject matter but also to ensure that the material used for the work is of high quality. The search criteria are summarized below:

Combinations of the search terms were used in each of the databases for a thorough search. Many references were eliminated by use of the criteria given in Table 1. Additionally, several papers regarding software systems and accelerated degradation testing were excluded by narrowing the focus of the research. Many sources were added from doing a reverse reference search in relevant articles.

Table 2.1: Search Criteria for Literature Review

Criteria	Description
Search Terms	Accelerated Testing Design of Experiments Reliability Growth
Databases	Compendex EBSCO ProQuest
Exclusion Criteria	Duplicate Papers Papers written in languages other than English Papers with incomplete information (title, author, publisher, year, etc.)

2.2 Accelerated Testing Models

Many ARGT models are extensions of existing reliability growth models by employing accelerating stresses and relating those stresses to the life of the product or component. The two most common types of stress loading are constant and step-stress. Constant stress models run units at a constant level of stress until failures and modify the level between test phases. Step stress models apply a constant stress to units until a specified time and then increase the stress for another specified period of time. This is repeated until all units fail. Other methods of accelerated stress loading are progressive, which is continuously increasing the stress level; cyclic, which is subjecting a unit to repeating patterns of high levels of stress; and random, which is subjecting the unit to randomly changing levels of stress. Although, these methods of stress loading exist, they are not typically used in research and will not be discussed further. The most prevalent failure inducing mechanisms are temperature, humidity, power cycling, and vibration. The following sections describe several common constant stress and step stress models.

2.2.1 Constant Stress Models

In constant stress models, a constant level of stress is applied to units under testing until those units fail. The level of stress can be adjusted after a stage of testing is complete. The advantages of constant stress models are that it is easy to hold units at a constant stress, and there are proven data analysis methods and empirical evidence to support these models [21]. An assumption for ARGT is that failure modes operate in the same fashion during normal operating use as the accelerated environment. ARGT yields failures more quickly than standard testing, and it also assumes that no new failure modes are present [8]. The relationship between failure times is assumed to be linear and is given as

$$t_n = \mathrm{A}(t_s), \tag{2.1}$$

where t_n is the time to failure under normal stress, t_s is the time to failure under an accelerated stress, and A is an acceleration factor. The failure or hazard rate under normal conditions is given as

$$\lambda_n(t) = \frac{1}{\mathrm{A}}\lambda_s\left(\frac{t}{\mathrm{A}}\right), \tag{2.2}$$

where λ_s is the hazard rate under acceleration. Some common life distributions are the exponential, Weibull, normal, and lognormal distributions, which are discussed in detail in [21] and are expressed as linear transformations of the accelerated conditions in [8].

When temperature is the failure mechanism, the Arrhenius model is the most widely used. The Arrhenius model, based on chemical reaction rates, presents the time to failure as

$$\theta = ae^{\left(\frac{b}{T}\right)}, \tag{2.3}$$

where a and b are constants dependent on product specifications and test methods, and T is the temperature in Kelvin. The acceleration factor for θ_1 at temperature T_1 and θ_2 at T_2 is given as

$$\mathrm{AF} = \frac{\theta_1}{\theta_2} = e^{\left[B\left(\frac{1}{T_1}-\frac{1}{T_2}\right)\right]}. \tag{2.4}$$

Alternate forms of this model are given for temperatures with exponential, Weibull, and lognormal distributions in [21].

Another common model in ARGT is the inverse power law model, usually referred to as the power law model. This model is applicable to a single accelerated stress, V, and given as

$$\theta(V) = aV^b, \tag{2.5}$$

where a and b are constants dependent on product specifications and test methods. The power can also be expressed as

$$\theta(V) = a\left(\frac{V_0}{V}\right)^b, \tag{2.6}$$

where a and b have the same definition, and V_0 is a standard level of stress. Alternate forms of model (2.5) are given for stresses that follow the exponential, Weibull, and lognormal distributions in [21].

An alternative and similar model to the Arrhenius model, which is based on quantum mechanics, is the Eyring model. This is given as

$$\theta = \left(\frac{a}{T}\right) e^{\left(\frac{b}{T}\right)}, \tag{2.7}$$

where parameters have the same definition as model (2.3). This model is extended for cases when temperature and other stresses are applied. The time to failure for the Generalized Eyring Model is given as

$$\theta = \left(\frac{a}{T}\right) e^{\left(\frac{b}{T}\right)} e^{\left(V\left[c+\left(\frac{d}{T}\right)\right]\right)}, \tag{2.8}$$

where a, b, c, and d are constants dependent on product specifications and test methods, T is temperature in Kelvin, and V is another stress. Model (2.8) can be expanded to include more than two stresses [15]. The use of multiple stresses in ARGT provides a better estimation of the life of a product than single stress testing, and using lower levels of multiple stresses can reduce the chance of atypical failure modes under normal operating conditions [6].

Feinberg [12] proposed a ARGT constant stress model with the following assumptions: (i) an acceleration factor, A, exists and can be estimated; (ii) time is linearly compressed by A; (iii) both compressed and uncompressed time periods can yield the same reliability growth. The mean time to failure (MTTF) under an acceleration applied at the beginning of testing, $\theta^A(t)$, is given as

$$\theta^A(t) = \frac{\theta_1^u}{1-\alpha}\left(\frac{t}{t_1^u}\right)^{\alpha} A^{\alpha}, \quad t_1^s < t < t_f^s, \tag{2.9}$$

where θ_1^u is the initial failure for an unstressed test, t_1^u is the initial unstressed test time, t_f^s is the end time of the stressed test, and α is the growth rate. Accelerated testing increases the effect of reliability growth by a factor of A^{α}. model is extended in which different acceleration factors are used for different phases of testing are considered and given as

$$\theta_i^A(t) = \frac{\theta_1^u}{1-\alpha}\left(\frac{t}{t_1^u}\right)^\alpha A_i^\alpha, \quad t_i^s < t < t_f^{s+1}, \tag{2.10}$$

where A_i is the ith acceleration factor in the ith stage of testing. Another extension of model (2.9) considers an acceleration factor that is applied after the first failure during an unstressed test, which is given as

$$\theta^A(t) = \frac{\theta_1^u}{1-\alpha}\left(\frac{t}{t_1}\right)^\alpha \left(1-\frac{t_1}{t_f^s}\right)^\alpha, \quad t_1 > t_s. \tag{2.11}$$

Krasich [16] develops a methodology for ARGT. Both environmental and operational stresses are considered, and Krasich notes that a key to successful ARGT is knowledge of how the environment and operations of the product will affect its life. The overall product reliability is given as

$$R(t_0) = R_U(t_0)\prod_i R_{ei}(t_0)\prod_j R_{0j}(t_0), \tag{2.12}$$

where R_e is the reliability of the product under environmental stresses, R_o is the reliability of the product under operational stresses, R_u is the reliability of the product under interaction of stresses, and i and j are indexes for the respective number of environmental and operational stresses. Thermal cycling, thermal exposure, humidity, vibration, and operational cycling are considered as acceleration factors.

2.2.2 Step Stress Models

In step stress models, units under testing are subjected to a stress level for a fixed amount of time, and then the stress level is increased if the unit does not fail. This is repeated until all units fail. An assumption of step stress ARGT is that increases in stress linearly affect the time to failure. The advantage of step-stress testing is that failures can appear quicker than constant stress testing. However, failures induced at high level stresses in later stages of testing may not be reflective of failures under normal operating conditions. Additionally, step stress tests need another model to approximate the life under constant stress as most products are run constantly. Like constant stress models, a distribution (i.e., exponential, Weibull, lognormal, etc.) is usually assumed for the time to failure. Thus, existing constant stress models with minor adjustments are applicable to step stress ARGT [21].

Feinberg [12] proposed a step stress model that is given as

$$M(t^{ss}) = \frac{M_1}{1-\alpha}\left(\frac{A_{\text{eff}}t^{ss}N}{t_1}\right)^{\alpha},\ t^{ss} > t_1, \tag{2.13}$$

where A_{eff} is the effective step-stress acceleration factor and the sum of all acceleration factors, and t^{ss} is the hold time or time units are tested at a constant stress level.

Step stress models require an additional model that relates the stress to conditions of normal use by assessing the cumulative effect of stresses that vary over time. Such a model is called a cumulative damage or cumulative exposure model. Miner's Rule [21] is a commonly used cumulative damage model for fatigue. It is given as

$$\sum_{i=1}^{k}\frac{n_i}{N_i} = C, \tag{2.14}$$

where k is the number of stresses, n_i is the number of cycles accumulated at stress i, N_i is the average number of cycles to fail at the ith stress, and C is the amount of life consumed. When $C = 1$, failure occurs.

Miller and Nelson [20] consider a simple step stress ALT in which units are tested at one stress for a length of time, τ_1, and then the stress is changed. By minimizing the asymptotic variance of the maximum likelihood estimation (MLE) of the mean time to failure, optimum times for τ_1 in a time-step SSALT and a optimum proportion, p_1, for failure-step SSALT are developed. The assumptions of the model are that the life distribution of units are exponential when stress is constant, and the characteristic life, θ, is a log-linear function of the stress x given as

$$\theta(x) = e^{a+bx}, \tag{2.15}$$

where a and b are parameters of the log linear function between the stress and characteristic life. Bai, Kim and Lee [2] extend this work by incorporating both Type I censoring, which is terminating the test at a predetermined time, and Type II censoring, which is terminating the test after a predetermined number of failures.

In [1], a simple step stress ALT with a Weibull distribution and Type I censoring is considered because the Weibull distribution allows for more flexibility. The assumptions are that the life distributions of units are Weibull when stress is constant rather than exponential, stress is log-linearly related to the scale parameter of the Weibull distribution, given as $\theta(x) = exp(a + bx)$, the shape parameter of the Weibull distribution is constant and independent of the stress, and the test is terminated at a predetermined time. Fard and Li [11] develop an optimal hold time by minimizing the asymptotic variance of the MLE of reliability. Yuan and Liu [27] assume that the model's parameters (a, b, and β) are not known and apply a Bayesian approach to the problem. A joint prior distribution is developed for the unknown parameters, and the asymptotic variance of the MLE of the time to change stresses that corresponds to the closest approximation of the lifetime distribution under normal operating conditions.

In [9] an optimum low stress level and hold time for a simple step stress ALT are developed by minimizing the asymptotic variance of the reliability estimate at normal conditions. The assumptions of the simple step stress ALT are that there is only one stress with two different accelerated levels and rather then assuming a distribution, failure rate is modeled using Cox's proportional hazards (PH) model, which is a multiple regression approach for reliability estimation that assumes stresses are multiplicative instead of additive. The PH model is given as

$$\lambda(t : z) = \lambda_0(t)e^{\boldsymbol{\beta}\mathbf{z}}, \tag{2.16}$$

where $\boldsymbol{z}$ is a vector of stresses, $\boldsymbol{\beta}$ is a vector of regression coefficients, and $\lambda(t) = a + bt$ is the baseline failure rate with parameters a and b. The ratio of hazard rates between at different stresses is assumed to be constant. Assuming there are initial parameters a, b, and β from engineering judgments, an optimum plan for the low stress level and hold time was developed considering censoring.

In [26] van Dorp et al. apply a Bayesian approach to a step stress accelerated life test. The motivation for this model is that many models assume there is one stress is the primary failure inducing mechanism, non-Bayesian approaches require large sample sizes because prior information cannot be incorporated, and that given there are several different stresses, it is difficult to specify a singular time transformation function or acceleration function. The assumptions of this model are: (i) the test will be conducted on a few highly reliability units, (ii) the failure rates of the units are exponential distributed, which increase as a function of the stress levels, (iii) there exists some prior information on the failure rates at normal and accelerated operating conditions, (iv) testing is identical for all units, which starts at normal operating stress and increases to a higher level after a fixed period of time, and (v) failures can only be observed at discrete time intervals, in which they are removed before testing is resumed.

The multivariate ordered Dirichlet distribution is considered as the prior distribution because incorporating engineering judgments is facilitated and the quality of the information can be quantified. The ordered Dirichlet distribution is given as

$$\Pi\{(u_0,\ldots,u_m)\} = \frac{\Gamma(\beta)}{\prod_{j=0}^{m+1}\Gamma(\beta\alpha_j)} \prod_{j=0}^{m+1}\left(u_{j-1}-u_j\right)^{\beta\alpha_j-1}, \tag{2.17}$$

where $\beta > 0$ and $\alpha_j > 0$ are prior parameters, $j = 1,\ldots,m$, $u_j = e^{-c\lambda_j}$ is the transformation of the failure rate λ_j, and $\sum_{j=0}^{m+1}\alpha_j = 1$. When deriving the likelihood function, it is assumed that stress levels are not changed instantaneously. Rather stresses increase linearly until the next level, which is known as the ramping phenomenon. The likelihood given the number of initial testing units, n, and the likelihood of the number of failures s' is expressed as

$$L(u'|n,s') = \prod_{i=0}^{m}\left[(u_i)^{\frac{2L_i-\rho_i}{2c}}(u_{i-1})^{\frac{\rho_i}{2c}}\right]^{n_i-s_i}\left[1-(u_i)^{\frac{2L_i-\rho_i}{2c}}(u_{i-1})^{\frac{\rho_i}{2c}}\right]^{s_i}, \tag{2.18}$$

where L_i is the length of time a unit is tested at stress level i, ρ_i is the length of time to ramp from stress level i to stress level $i+1$, c is a constant in the failure rate transformation, s_i is the total number of failures in the time interval (t_i, t_{i+1}), and n_i is the number of units remaining in the test. The posterior distribution, which is a product of the prior and likelihood function given as

$$\prod_{i=0}^{m}\left[(u_i)^{\frac{2L_i-\rho_i}{2c}}(u_{i-1})^{\frac{\rho_i}{2c}}\right]^{n_i-s_i}\left[1-(u_i)^{\frac{2L_i-\rho_i}{2c}}(u_{i-1})^{\frac{\rho_i}{2c}}\right]^{s_i}\prod_{j=0}^{m+1}\left(u_{j-1}-u_j\right)^{\beta\alpha_j-1}. \tag{2.19}$$

2.3 Design of Experiment Techniques

Complete l^k factorial designs have k factors with l levels. Typically factorial designs are restricted to two levels: high ('+') and low ('-'). These designs provide the most information about the experiment because it considers all combinations of levels of factors as well as the interactions of the factors. However, complete factorial designs can become large even at two levels of each factors [4]. An experiment with eight factors would require $2^5 = 32$ runs. In the case of ALT, each run may take several hundred hours, which would be practically infeasible to conduct 32 runs. Thus, more efficient methods of conducting experiments is needed.

Fractional factorial designs can reduce the number of runs significantly by choosing a subset or fraction of the complete factorial design. A 2^{5-2} factorial design would reduce the number of runs by $2^{-2} = 1/4$ to eight runs. Although this provides an advantage of reducing the number of runs significantly, the disadvantage is that many of the effects are hidden or confounded by the main effect factors that the experimenter deems the most important. This assumes that many of the confounded effects are not significant and do not affect the response. Fractional factorial designs are particularly useful if it can be estimated which main effects and interactions are significant so that the remaining effects can be confounded [4].

Two efficient methods, Latin square design and split-plot design, consider blocking, which is the reduction of variation by noise factors. Latin square design considers two blocking factors and reduces the number of runs by n, where n is the number of levels in treatments and blocks. An experiment with three treatment levels and three levels for each block would normally require

27 runs (i.e., running every combination of treatment level and block levels); however, the Latin square design reduces it to 9 runs by only running each treatment level with every pair combination of block levels, which can be seen in Figure 2.3 where A, B, and C are the treatment levels. The disadvantages of Latin square design are that there must be two blocking factors and the levels in the treatment and blocks must be the same and interaction effects are not discernible in this design [4].

		Block 1		
		1	2	3
Block 2	1	A	B	C
	2	C	A	B
	3	B	C	A

Figure 2.1: Latin Square Design

Split-plot designs consider multiple factors and are useful when one of the factors is not easily manipulated. In a split-plot design, each block, referred to as a whole plot, is divided into subplots. An example is given in [4], where four different types of coating are used to treat steel bars in different positions in a furnace. In the example, the furnace temperature is difficult to manipulate and adjusting it for each run would result in large variances. The experiment can be found in Figure 2.3. The advantages of this design are that variance is reduced because all main effects and interactions between the heat and position have the same source of error and it provides convenience for scenarios where blocks may already exist.

	Heat (Whole Plots), C°					
	360	370	380	380	370	360
Coating-Position (Subplots)	C2-73	C1-65	C3-147	C4-153	C4-150	C1-33
	C3-83	C3-87	C1-155	C3-90	C1-140	C4-54
	C1-67	C4-86	C2-127	C2-100	C3-121	C2-8
	C4-89	C2-91	C4-212	C1-108	C2-142	C3-46

Figure 2.2: Split-Plot Design [4]

Response surface methodology (RSM) is designed to develop a relationship between a response and a number of predictor variables, which can be expressed as

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + \ldots + b_kx_k + \varepsilon, \tag{2.20}$$

where $\hat{y}$ is an estimation of the response, $b_i,\ i = 0,\ldots,k$ are constant coefficients, $x_i,\ i = 1,\ldots,k$ are the predictor variables, and ε is an error term. A technique of RSM is the method of steepest ascent, in which the experimenter starts at a region near the normal operating conditions of the system and moves toward the optimal response in a minimal amount of steps or runs. Using the contour plot resulting from equation (2.20) at the initial region, the experimenter moves toward the optimum region. This is continued until there is no increase in the response from equation (2.20), and the experimenter is in the optimum region [4]. The line moving perpendicularly from one contour plot to the next is termed the path of steepest ascent, and is also where the rate of increase of the response is a maximum. Once in the optimum region, a more sophisticated model is required like a second order model given as

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + \ldots + b_kx_k + b_{11}x_1^2 + b_{22}x_2^2 + \ldots + b_{kk}x_k^2 + b_{12}x_1x_2 + \ldots + b_{k-1,k}x_{k-1}x_k + \varepsilon. \tag{2.21}$$

There are several optimal statistical criteria used to evaluate the experimental design. The most widely cited in the literature is the D-optimal design. In the D-optimal design, the volume of the confidence region is minimized, which maximizes the determinant of the information matrix of the design. In the G-optimal design, the maximum variance of responses are minimized. Although the D-optimal design requires a known underlying model, it is useful when other classical models are not applicable.

2.4 Design of Experiment Approaches to ARGT

Design of experiments (DOE) is a suitable approach to ARGT because it can evaluate multiple stresses in a single experiment, the interactions of those stresses, and allow the user to configure the design characteristics of the product to remain reliable under stress [17]. There are a number of advantages to combining DOE with ARGT. The primary advantage is that both methods reduce the amount of time and cost of testing. The purpose of ARGT is to reduce the amount of testing time while DOE provides the vast amounts of information from a minimum number of tests. Advantages of incorporating DOE with ARGT are that no new failure modes are introduced when combining accelerated stresses with normal operating stresses can be verified and appropriate levels of stress for acceleration can be identified [15]. Additionally, there are methods like test sequence randomization and blocking that can reduce variance and the effect of extraneous factors [23]. The remainder of the section is dedicated to describing several methodologies of combining DOE approaches with ARGT.

McKinney [18] develops a methodology for system-level ARGT that addresses the following deficiencies: assumption of a life distribution, assumption of a stress relationship function, and extrapolation outside of the test data. The assumptions for his methodology are: (i) the factors being studied are quantitative, meaning they can be described as points on a scale; (ii) the interactions are negligible; (iii) the factors can be equally spaced from one level to the next; (iv) the errors are independent and normally distributed with mean zero and common variance; (v) the design limits of the test article can be determined or approximated; (vi) multiple, identical units are available for test; and (vii) the test stresses can be applied simultaneously.

There are three phases to this methodology: planning, design, and analysis. In the planning phase, performance measures (i.e., time to failure), stresses (i.e., temperature, humidity, vibration, etc.), and level of stresses (maximum operation levels to maximum design limits) are determined. In the design phase, McKinney suggests the use of a one-third replicate of a three-factorial test with three levels, requiring nine test cells. Furthermore, he estimates the minimum number of units to be tested in each cell and gives guidelines for testing time. In the analysis phase, if a cell has at least one failure, then the data is used for the MTTF. However, if there is not a failure for a particular cell, then a 0.5 χ^2 confidence limit with two degrees of freedom is used to calculate the MTTF. Traditional analysis of variance (ANOVA) and regression analysis are used to evaluate the natural logarithms of the MTTFs. The null hypothesis (H_0) is that none of the factors has an effect on the MTTF. By using an experimental design, no assumptions regarding the life distribution or stress relationship were necessary, and extrapolation was not an issue because the levels of stress being tested overlapped with the operational levels.

Dietrich and Mazzuchi [7] raise concerns with McKinney's methodology. The primary issue is the use of standard analysis procedures: needing multiple components to satisfy the normality assumption when using a logarithmic transformation causes non-normality and calculating a pseudo-MTTF using a 0.5 $\chi^2_{(2)}$ confidence limit. Additionally, atypical failure modes were not considered for combined stresses.

Hakim-Mashhadi [15] analyzes two different distributions in the context of a DOE methodology: the power-Weibull and power-lognormal distribution. He first assumes that the product's life distribution follows the Weibull distribution under current stress, x, and the scale parameter, θ, is a function of the stress, which can be expressed linearly as

$$\ln\theta(x) = a - b(\ln x), \tag{2.22}$$

where a and b are parameters of the product and test method. An example is presented for light bulbs where the expected life of light bulbs are determined from accelerating levels of voltage and estimating the values of $\ln\theta$ for each level of stress through regression analysis. The expected life of light bulbs is also determined using the power-lognormal distribution as the underlying assumption for product's life distribution. A demonstration of incorporating fractional factorial designs and accelerated testing is given. Hakim-Mashhadi notes several considerations when designing an experiment with accelerated life testing: the dependent measure, acceleration factors, levels of the acceleration factors, interactions of factors, the number of runs, and the replications of the experiment.

Clark et al. [5] present a methodology that does not assume design limits of a product to be known. Rather in this methodology the design limits are determined through a technique called destructive evaluation. In this technique, a N-factor, 2-level incomplete factorial design is used for

N+1 experiments, in which each stress is started at its maximum design level and incremented until failure. In the last experiment, all failures are incremented in a manner such that all failures should occur simultaneously. However, failure typically occurs earlier due to the interactions among stresses. The maximum stress level (MSL) is obtained from taking the lower tail of the probability density function for the design limits in order to avoid non-stress failure modes.

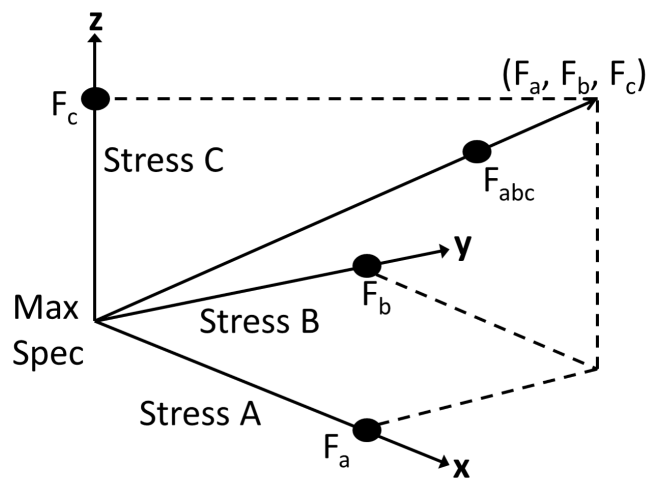


Figure 2.3: Destructive Evaluation of a Three-Stress Accelerated Life Test [5]

Accuracy and acceleration should be considered as objectives when designing the ALT. When trying to achieve greater acceleration, choose stress profiles close to the MSL. Additionally, testing more units at higher stress levels in a constant stress test or increasing the length of steps in a step stress test would have to the same effect. When trying to achieve greater accuracy, choose stress profiles closer the maximum design limits, testing more units at lower stress levels in a constant test, or decrease the length of steps as stress level increases in a step stress test. Using an appropriate model, the failure rate can be predicted, and an s-confidence can be obtained by setting the number of test units, the length of the test, and stress levels. This approach was applied to commercial off-the-shelf single-board electronics, and assumes that all units were identical, that units should be of low-cost and high-volume to keep costs minimal, and that the length of experiments should be kept as short as possible to ensure that failures occur due to stress rather than reliability issues.

Thomas and Gaines [24] address the difficulties in balancing a statistically designed experiment and incorporating engineering judgments because statistically designed experiments provide the most insight when little is known about the system under testing and incorporating prior information is difficult especially when much is known about some aspects of the system while little else is known about the rest of the system. Engineering judgment is vital to initialize the design of the ALT, and then using statistical criteria improvements can be made. The methodology laid forth is dependent on engineering knowledge about some performance measure of the combination of factors. In this example, relative severity was used; however, a number of different measures can be considered.

First, a full factorial design is considered and a severity rating is assigned to each combination of stress levels. Next the severity rating of each of the main factors was calculated. The main effect with the highest severity was selected and split so that the remaining main effects are conditioned on a high level of that stress. The main effect with the next highest severity is the next split and the remaining main effects are conditioned on the high level of that stress. This continues until all main effects have been selected. The construction of this hierarchal tree is such that the large horizontal distances indicate a large importance. Experiments with large importance are kept and others are eliminated reducing the complete factorial design to an engineering design, a set of five conditional tests.

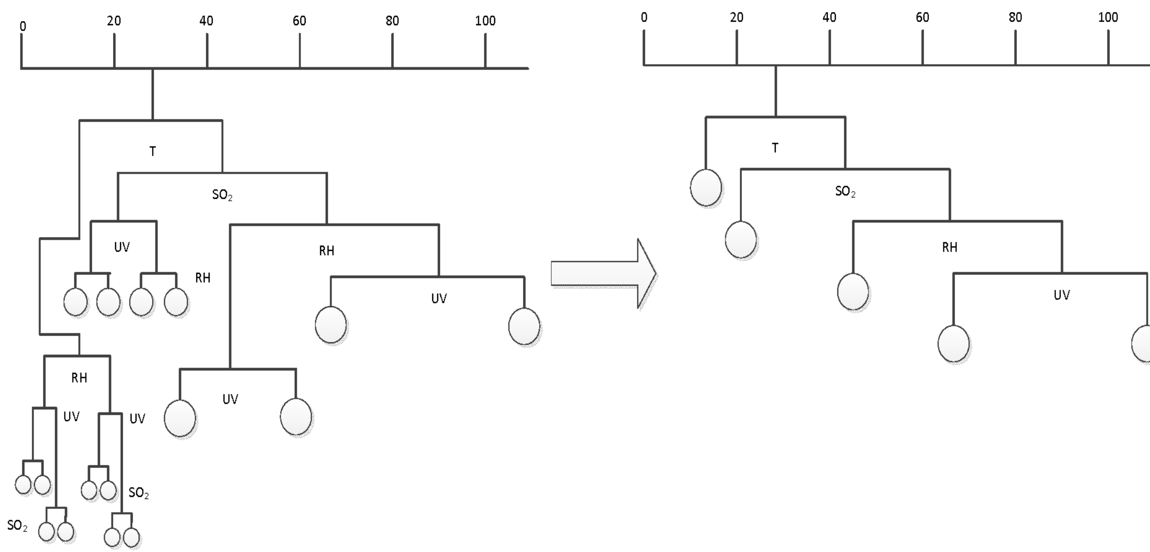


Figure 2.4: Conversion of Factorial Test Design to Engineering Design [24]

The five tests are used to generate a polynomial response surface. The elimination of tests create an issue because several effects are confounded. Thus, fractional factorial design should be considered for supplemental testing. The use of extrapolation is necessary in order to describe the system under normal operating conditions, and the use of extrapolation is allowable by using multiple levels of stresses. It is recommended that at least five stresses be used assuming that the same failure mode is the primary failure mode in all five levels.

Dietrich and Mazzuchi [7] continue the work from van Dorp et al. [26] by observing that the testing order is not significant. Rather the prior estimates of the failure rates and the rank order of the failure rates are the only necessary information to conduct the test at each level. This allows the incorporation of DOE methodologies, which Dietrich and Mazzuchi use by randomizing the test order and by combining different stresses. Somerville et al. [23] extend the work of the randomly ordered life testing design. It is shown that when ordering the stress levels in a strictly increasing distribution, the conditional reliability or probability that a unit will survive test stage i, u_i, can be expressed as

$$u_i = \frac{R(\tau_{i+1})}{R(\tau_i)} = e^{-\lambda_i(\tau_{i+1}-\tau_i)}, \tag{2.23}$$

where τ_i is the time at the end of test $i-1$, $R(t)$ is the reliability of a unit from time 0 to time t, and λ_i is the average failure rate at stress i. Then each item at each test stage is considered a Bernoulli trial resulting in a joint conditional sampling probability density function or likelihood function for the ALT, which is given as

$$f(\underline{s}|\underline{u}) = \prod_{i=1}^{m} \binom{n_i}{s_i} (1-u_i)^{s_i} (u_i)^{n_i - s_i},\ n_i = \sum_{j=1}^{k} n_{ij},\ s_i = \sum_{j=1}^{k} s_{ij} \tag{2.24}$$

where n_i is the number of units at the start of test stage i and s_i is the number of failed units at test stage i.

Guo and Pan [14] develop a D-optimal design for two-stress two-level ALT with both censored and uncensored data, which minimizes the uncertainty of estimation of the model's parameters. The assumptions are that failure times are lognormally distributed and the only the location parameter, β, is affected by stress. The log-failure time is given as

$$Y_{i,j} = \ln t_{i,j} = \beta_0 + \beta_1 X_i, 1 + \beta_2 X_{i,2} + \beta_{12} X_{i,1} X_{i,2} + \sigma\varepsilon_{i,j}, \tag{2.25}$$

where $t_{i,j}$ is the failure time for the ith run and the jth unit, $\boldsymbol{\beta}$ is a vector quantifying the effect of stress on failure times, σ is the variance, and $\varepsilon_{i,j}$ is random error. For uncensored failure data, the optimal design is found at the boundary of the feasible region by maximizing the determinant of the Fisher information matrix. For censored failure data, the optimal design is dependent on β and σ. These parameters can be estimated from engineering judgment or existing data analysis.

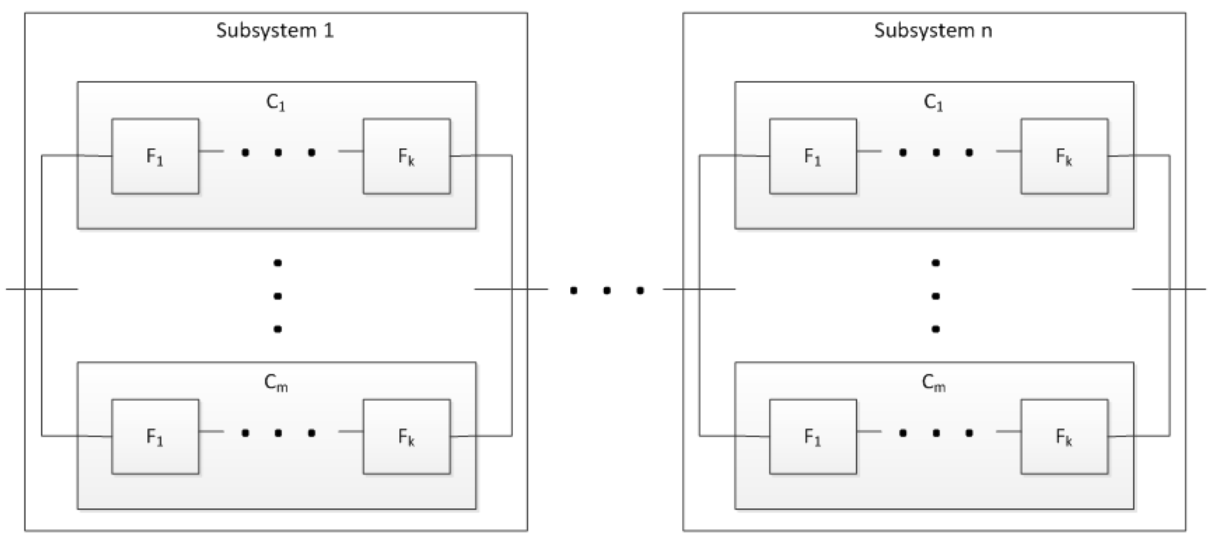
Ginebra and Sen [13] employ a minimax approach to experiment design for ALT. Both the lognormal and Weibull distributions are considered. Minimax designs seek to minimize the maximum of the optimality function over a feasible region of interest, P. A search over the entire region is too difficult so a subset is used for a rectangular region bounded by the upper and lower limits of the probabilities of failure for the design and highest stress. Two design sets are investigated: a set of designs with k equally spaced stress levels with equal units tested at each stress, and a set of designs with k equally spaced stress levels with a number of allocated units proportional to the level of stress. The minimax approach requires the best performance from the smallest number of failures at the design and highest stresses, and it was found that the recommended design use the locally optimal designs for the lowest feasible probabilities of failure for the design and highest stresses.

Chapter 3

Problem Formulation and Methodology

3.1 System Description

A series-parallel hardware system is considered. This is a common system in which multiple subsystems modeled as a parallel system are joined in series. The system is composed of n subsystems each with m_i components, where $i = 1, \ldots, n$, and each component m_i has k failure modes, where $k = 1, \ldots$. Multiple failure modes within a component are modeled as components in series. A general example of the systems under consideration can be found in Figure 3.1.



The assumptions of the model are:

- one failure in the series configuration causes system failure;
- each failure mode is Weibull distributed;
- the scale (η) and shape (β) parameters can vary between components;
- the shape parameter is constant and independent of stress;

- the scale parameter is a log linear function of transformed stress, temperature (T) and voltage (V), given as

$$\eta = \frac{a}{T} e^{\left(\frac{b}{T} + V\left[c + \frac{d}{T}\right]\right)}; \tag{3.1}$$

- the parameters β, a, b, c, and d are assumed to be known or derivable;
- there are multiple failure modes for each different component in the system;
- the cause of each failure can be identified with certainty;
- modifications of the system are instantaneous and perfect;
- modifications can remove the failure mode completely or increase the performance of the component;
- failure modes can experience improvement more than once;
- and modifications are identical for all copies of the system.

The measure of interest is the system reliability, given as

$$R_{\text{sys}} = \prod_{i=1}^{n} \left[1 - \prod_{j=1}^{m_i} (1 - R_j) \right]. \tag{3.2}$$

3.2 Testing Procedure

A simple step stress accelerated life test is used with two levels of two different stresses: temperature and voltage. There are two steps in each stage of testing. In the first step, N identical copies of the system are tested at stresses x_1 and y_1 for a fixed amount of time (hold time), τ. At the end of the first step, $t = \tau$, failed units are removed, and at least one of the stresses are increased, such that $x_1 \leq x_2$ and $y_1 \leq y_2$. The remainder of the units are tested until they all fail. The levels of stresses in testing are between the operating stress or design stress and the maximum load stress, $x_D \leq x_1 < x_2 \leq x_M$ and $y_D \leq y_1 < y_2 \leq y_M$, such that exposing the system to these high levels of stress should not yield failure modes that are uncharacteristic of normal use. At the end of each stage of testing, repairs and modifications are made with a probability of reducing the effect of each failure mode. Modifications are identical for each copy of the system under testing. Testing is concluded when one of the following criteria are met: (i) system reliability meets the target reliability, (ii) the total testing time allocation is exhausted, or (iii) fixes become too costly (e.g., system redesign).

The goal of this problem is to identify the minimum cost of an accelerated testing plan given specific constraints on time and reliability and the ability to manipulate factors during testing. The constraints of this problem are that a desired reliability must be met and testing must not exceed the total testing time allocation for the program. The factors that can be manipulated are the levels of stresses during each step of each stage of testing and the level of improvement made to each failure mode. It is assumed that the number of failure modes in each component is known. It is also assumed that the experimenter knows how the system behaves. This can normally be obtained from engineering judgment. An optimization model can be used to minimize costs, which is given as

$$\begin{aligned} \min \; C = \sum_{i=1}^{S} \left[\sum_{j=1}^{2} c_{ij}^{T}(t_j) + \sum_{k=1}^{F} c_i^{I}(z_{ki}) \right] \\ \text{s.t.} \; R_{\text{sys}} \geq R_{\text{goal}} \\ \sum_{i=1}^{S} t_i \leq T \quad , \end{aligned} \tag{3.3}$$

where C is the total cost of testing, $c_{ij}^{T}(t)$ is a time-dependent cost function of testing of step j in stage i, $c_i^{I}(z_{ki})$ is the improvement cost of making modifications to failure mode z_{ki} in stage i for $k = 1, \ldots, F$, F is the total number of failure modes, S is the total number of stages, t_i is the total testing time of stage i, T is the total testing time allocation. It is assumed that $c_{i1}^{T} < c_{i2}^{T}$ and $c_{ij}^{T} < c_{(i+1)j}^{T}$. This is a reasonable assumption because the cost of testing at higher levels of stress would accrue higher energy costs. Additionally, costs would increase in later stages of testing because the system is closer to being released into the field. It is also assumed that making improvements and modifications to the system will increase in later stages of testing, $c_i^I < c_{i+1}^I$.

3.3 Test Environment

For each failure mode, estimates were made for the scale and shape parameters. Units were placed on test at a high level of temperature and voltage. At time τ, failed units were removed from the test, and the remainder of the units are tested until they fail. Testing time was recorded along with the failure mode that caused the failure. At the end of each stage of testing, modifications are made to each component, where failure modes of the component can either be (i) completely removed, (ii) improved, which would increase the performance of the component, or (iii) not affected. However, making modifications to the system could introduce new failure modes. For example, if a microchip with two failure modes is replaced on a circuit board with a higher quality chip. One failure mode could be completely eliminated, and the other failure mode could be improved, reducing the rate of occurrence of that failure mode. However, replacing the chip could have also introduced a new failure mode. New failure modes are not exclusive to making improvements to the system but can also be revealed through more extensive testing.

Reliability is a function of the failure modes, $\{R_{\text{sys}} \,:\, f(z_{ki})\ i = 1,\ldots,S;\ k = 1,\ldots, F\}$, where $0 \leq z_{ki} \leq 1$ indicates the level of improvement of the failure mode in stage i. A value of 0 means there is no fix to the failure mode and a value of 1 means the failure mode is completely removed.

As failure modes are removed and components are improved, the reliability of the system should be improving. Test duration is a function of the stresses and reliability, $\{t_i \,:\, f(x_j, y_j, R_i)\ i = 1,\ldots,S;\ j = 1,2\}$, where x_j, y_j are the levels of stresses x and y in the jth step of testing. As the levels of stress increase, test duration decreases. However, reliability should be extending the length of tests because the system's performance is improving.

To calculate testing cost, costs were estimated for each level of stress with increasingly higher costs for higher levels of stress. Testing cost is a function of stresses and test duration, $\{c_{ij}^T \,:\, f(x_j, y_j, t_i)\ i = 1,\ldots,S;\ j = 1,2\}$. It is assumed that higher levels of stress and longer tests stages would consume higher levels of energy, thereby incurring higher levels of costs. To calculate improvement costs, costs were estimated for making improvements to each failure mode. It was assumed that if a failure mode remained unchanged it would not incur a cost but completely removing a failure mode would be more expensive than simply improving its performance. Improvement costs are a function of failure modes, $\{c_i^I(z_{ki}) i = 1,\ldots,S;\ k = 1,\ldots,F;\, 0 \leq z_k \leq 1\}$. Additionally, It is assumed that both testing and improvement costs would increase from stage i to stage $i+1$. This is a reasonable assumption because the costs would increase in later stages of testing because the system is closer to being released into the field.

3.4 Methodology

A major assumption of the problem is that some knowledge of the system's behavior is available to the user. In most cases engineering judgment can be used to estimate the behavior of a system, or there could be previous sets of data that can be used as estimations for system performance. However, in instances where neither are available, simulation can be employed.

The main approach is to optimize a test design plan in which all settings are determined prior to testing. The steps to the optimization approach are:

- Obtain data for system performance in all stages
- Use data to construct response surface
- Fit model to response surface
- Use optimization model to determine optimal settings
- Test system
- Compare results to the model

Another approach is a sequential stage approach in which analysis is completed after each stage of testing to determine the optimal settings for the next stage of testing. The steps to this sequential approach are:

- Test system
- Construct response surface from test data
- Fit model to response surface
- Determine optimal settings for next stage
- Repeat until all stages have been completed

The main difference in the two approaches is that the optimization approach attempts to establish test settings prior any testing based on previously collected data whereas the sequential approach uses the previous stage testing data to establish test settings for the next stage of testing. Another difference is the amount of data being analyzed. In the optimization approach the entire response surface is evaluated for all three stages of testing, and although more analysis is conducted during testing, the sequential approach only evaluates portions of the response surface.

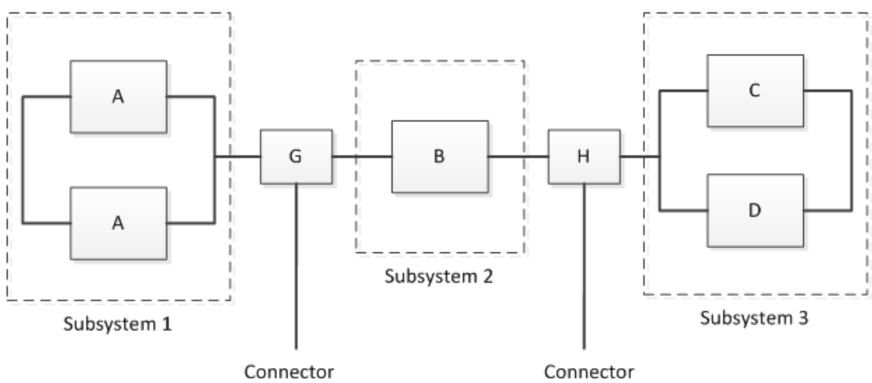
Chapter 4

Results

4.1 Test Setup

A three stage accelerated simple step stress was considered for a series-parallel system. The system consists of three subsystems: one subsystem with two of the same components in parallel, another subsystem is a single component, and the third subsystem has two different components in parallel. The three subsystems are connected in series, and the connections are modeled as components with failure modes to emulate the failure modes that emerge from connecting subsystems. A diagram of the system is represented below.

Figure 4.1: Diagram of series-parallel system



Each component was assumed to have two different known failure modes initially, and one additional failure mode introduced in each subsequent testing stage. It was also assumed that each component had different failure modes such that no failure mode overlapped between components. Failure modes are denoted as z_{ki}, where z is the component, i is the stage of testing, and k is an index of the failure mode (e.g., component A has two failure modes in Stage 1: A_{11}, A_{21}). Each failure mode was assumed to have Weibull distributed failure times with estimated values for the shape and scale parameter. The scale parameter was modeled using the Generalized Eyring model with the constant parameters, a, b, c, and d, estimated by the Method of Maximum Likelihood using Reliasoft Weibull++. The improvement for all failure modes if they were modified but not completely removed was 0.7.

A simulation model was developed using Visual Basic in Microsoft Excel. The simulation model was used to generate random failure times of the system. The data was collected used to construct an underlying model for a response surface. The next section described in detail the steps to construct the response surface.

4.2 Constructing the Response Surface

Four testing stages of data were collected; however, only three stages were used in the construction of the response surface. This is because no improvements are made in the first stage and because subsystems were tested independently. Testing in the first stage, which will be referred to as Stage 0, is used to determine the initial failure modes and establish a baseline reliability.

Each subsystem was tested independently, and the connection was assumed to be perfect for this stage of testing, which would establish an upper bound for the total system's reliability. However, this is not necessarily a realistic assumption because there can be failure modes which emerge from joining subsystems. Therefore, failure modes are introduced in the subsequent testing stage to model failures due to connecting subsystems. The regression for testing time in Stage 0 is

$$t_0 = -(-0.000306x_1 + 0.000449x_2 - 0.000267y_1 + 0.000391y_2)^{-5}, \quad (4.1)$$

where x_j and y_j, $j = 1,2$, are the temperature and voltage stresses, respectively. The Minitab results can be found in Figure A.1 in Appendix A. The residuals followed all assumptions for normality, and the normal probability plot for the residuals were checked for normality with a p-value of 0.13. The graphs can be found in Figures A.2 and A.3 in Appendix A.

The regression for testing cost is

$$c_0^T = -(-0.007378 - 0.000022x_1 + 0.000017x_2 - 0.000016y_1 + 0.000013y_2)^{-2}. \tag{4.2}$$

The system reliability was the same for all test trials of varying stresses, which was approximately 0.113, so no regression was necessary. This is expected because reliability is a function of the modifications of the failure modes, and no improvements were made in the first stage of testing. Furthermore, no analysis was necessary for improvement costs.

In the second stage of testing or Stage 1, the subsystems were connected together, and the full system is simulated. It was assumed that failure modes would be introduced where the subsystems were connected. A new component with failure modes was added in series between the subsystems to model the connections. The Minitab results for testing time, reliability, testing cost, and improvement cost can be found in Appendix A along with their residual plots and the associated normal probability plot. The regression for improvement cost is

$$\begin{aligned} c_1^I =& 4265\mathrm{A}_{11} + 5110\mathrm{A}_{21} + 4885\mathrm{B}_{11} + 6397\mathrm{B}_{21} + 7445\mathrm{C}_{11} + 3425\mathrm{C}_{21} \\ & + 3405\mathrm{D}_{11} + 6046\mathrm{D}_{21} + 2512\mathrm{G}_{11} + 3915\mathrm{G}_{21} + 4095\mathrm{H}_{11} + 6649\mathrm{H}_{21} \end{aligned} \tag{4.3}$$

This was a perfect fit because all the costs for each failure mode were estimated and not based on simulated information, it was not necessary to a residual analysis for improvement cost. The regression for reliability is given as

$$\begin{aligned} R_1 =& (0.333 + 0.070\mathrm{A}_{11} + 0.069\mathrm{A}_{21} + 0.063\mathrm{B}_{11} + 0.062\mathrm{B}_{21} + 0.038\mathrm{C}_{11} + 0.038\mathrm{C}_{21} \\ & + 0.033\mathrm{D}_{11} + 0.032\mathrm{D}_{21} + 0.051\mathrm{G}_{11} + 0.052\mathrm{G}_{21} + 0.058\mathrm{H}_{11} + 0.058\mathrm{H}_{21})^{1/0.281}. \end{aligned} \tag{4.4}$$

The regression for test time is given as

$$
\begin{aligned}
t_1 = -\,(&-0.02491 - 0.000115x_1 + 0.000001x_2 - 0.000088y_1 + 0.000001y_2 \\
&+ 0.003190\mathrm{A}_{11} + 0.003067\mathrm{A}_{21} + 0.001382\mathrm{B}_{11} + 0.001491\mathrm{B}_{21} \\
&+ 0.001451\mathrm{C}_{11} + 0.001415\mathrm{C}_{21} + 0.001025\mathrm{D}_{11} + 0.001109\mathrm{D}_{21} \\
&+ 0.001016\mathrm{G}_{11} + 0.001089\mathrm{G}_{21} + 0.001341\mathrm{H}_{11} + 0.001243\mathrm{H}_{21})^{1/-0.248}.
\end{aligned} \tag{4.5}
$$

The regression for test cost is given as

$$
\begin{aligned}
c_1^T = -\,(&-0.272 - 0.000105x_1 + 0.000086x_2 - 0.000062y_1 + 0.000072y_2 \\
&+ 0.004713\mathrm{A}_{11} + 0.00472\mathrm{A}_{21} + 0.006368\mathrm{B}_{11} + 0.006365\mathrm{B}_{21} \\
&+ 0.00181\mathrm{C}_{11} + 0.002071\mathrm{C}_{21} + 0.001752\mathrm{D}_{11} + 0.001531\mathrm{D}_{21} \\
&+ 0.004781\mathrm{G}_{11} + 0.004866\mathrm{G}_{21} + 0.006047\mathrm{H}_{11} \\
&+ 0.005993\mathrm{H}_{21})^{1/-0.137}.
\end{aligned} \tag{4.6}
$$

The complete regression analysis, residual plots, and normal probability plots for residuals can be found in Appendix A.

In Stage 2, the number of failure modes was increased to model introduced failure modes, which may emerge from making improvements to existing and known failure modes or failure modes that may result from longer testing times. Furthermore, if more failure modes are not introduced in subsequent stages of testing, then it would be pragmatic to make all fixes after the first stage of testing because costs increase in subsequent stages of testing. The regressions for testing time, reliability, testing cost, and improvement cost are similar to those for Stage 1 with the exception of an additional failure mode per component. Improvement cost was not evaluated for the regression because the R^2 value will be 100% because all costs were estimated. The regression for improvement cost is given as

$$
\begin{aligned}
c_2^I =&21232\mathrm{A}_{12} + 25548\mathrm{A}_{22} + 21323\mathrm{A}_{32} + 24424\mathrm{B}_{12} \\
&+ 31983\mathrm{B}_{22} + 24424\mathrm{B}_{32} + 37226\mathrm{C}_{12} + 17126\mathrm{C}_{22} \\
&+ 37726\mathrm{C}_{32} + 17026\mathrm{D}_{12} + 30232\mathrm{D}_{22} + 17026\mathrm{D}_{32} \\
&+ 12560\mathrm{G}_{12} + 19574\mathrm{G}_{22} + 12560\mathrm{G}_{32} + 20476\mathrm{H}_{12} \\
&+ 33246\mathrm{H}_{22} + 20476\mathrm{H}_{32}.
\end{aligned} \tag{4.7}
$$

The regression for reliability is given as

$$
\begin{aligned}
R_2 =&(0.711 + 0.0113\mathrm{A}_{12} + 0.0117\mathrm{A}_{22} + 0.0087\mathrm{A}_{32} \\
&+ 0.0076\mathrm{B}_{12} + 0.0056\mathrm{B}_{22} + 0.0101\mathrm{B}_{32} + 0.0064\mathrm{C}_{12} \\
&+ 0.0050\mathrm{C}_{22} + 0.0077\mathrm{C}_{32} + 0.0049\mathrm{D}_{12} + 0.0049\mathrm{D}_{22} \\
&+ 0.0051\mathrm{D}_{32} + 0.0054\mathrm{G}_{12} + 0.0057\mathrm{G}_{22} + 0.0056\mathrm{G}_{32} \\
&+ 0.0064\mathrm{H}_{12} + 0.0068\mathrm{H}_{22} + 0.0067\mathrm{H}_{32})^{1/0.1973}.
\end{aligned} \tag{4.8}
$$

The regression for test time is given as

$$
\begin{aligned}
t_2 =&-(-0.5970 - 0.000163x_1 + 0.000003x_2 - 0.000126y_1 + 0.000001y_2 \\
&+ 0.001501\mathrm{A}_{12} + 0.001329\mathrm{A}_{22} + 0.002102\mathrm{A}_{32} + 0.001279\mathrm{B}_{12} \\
&+ 0.000905\mathrm{B}_{22} + 0.001815\mathrm{B}_{32} + 0.001447\mathrm{C}_{12} - 0.00330\mathrm{C}_{22} \\
&+ 0.002515\mathrm{C}_{32} + 0.000361\mathrm{D}_{12} + 0.000963\mathrm{D}_{22} + 0.000828\mathrm{D}_{32} \\
&+ 0.000913\mathrm{G}_{12} - 0.000010\mathrm{G}_{22} + 0.001299\mathrm{G}_{32} + 0.001081\mathrm{H}_{12} \\
&+ 0.001651\mathrm{H}_{22} + 0.001571\mathrm{H}_{32})^{1/-0.0385}.
\end{aligned} \tag{4.9}
$$

The regression for test cost is given as

$$
\begin{aligned}
c_2^T =& -(-0.566-0.000089x_1+0.000072x_2-0.000055y_1+0.000058y_2\\
&+0.001354\mathrm{A}_{12}+0.001047\mathrm{A}_{22}+0.001472\mathrm{A}_{32}+0.003439\mathrm{B}_{12}\\
&+0.002230\mathrm{B}_{22}+0.004949\mathrm{B}_{32}+0.001013\mathrm{C}_{12}-0.000203\mathrm{C}_{22}\\
&+0.002204\mathrm{C}_{32}+0.000353\mathrm{D}_{12}+0.000987\mathrm{D}_{22}+0.000839\mathrm{D}_{32}\\
&+0.002287\mathrm{G}_{12}+0.000022\mathrm{G}_{22}+0.003692\mathrm{G}_{32}+0.003421\mathrm{H}_{12}\\
&+0.004684\mathrm{H}_{22}+0.004608\mathrm{H}_{32})^{1/-0.0518}.
\end{aligned}
\tag{4.10}
$$

The complete regressions can be found in Appendix A along with their residual plots and the associated normal probability plots for residuals.

Stage 3 is similar to the previous two stages of testing with an increased number of failure modes. Improvement cost was not evaluated for the regression because the R^2 value will be 100%. The regression for improvement cost is given as

$$
\begin{aligned}
c_3^I =& 21232\mathrm{A}_{13}+25548\mathrm{A}_{23}+21323\mathrm{A}_{33}+25548\mathrm{A}_{43}\\
&+24424\mathrm{B}_{13}+31983\mathrm{B}_{23}+24424\mathrm{B}_{33}+31983\mathrm{B}_{43}\\
&+37226\mathrm{C}_{13}+17126\mathrm{C}_{23}+37726\mathrm{C}_{33}+17126\mathrm{C}_{43}\\
&+17026\mathrm{D}_{13}+30232\mathrm{D}_{23}+17026\mathrm{D}_{33}+30232\mathrm{D}_{43}\\
&+12560\mathrm{G}_{13}+19574\mathrm{G}_{23}+12560\mathrm{G}_{33}+19574\mathrm{G}_{43}\\
&+20476\mathrm{H}_{13}+33246\mathrm{H}_{23}+20476\mathrm{H}_{33}+33246\mathrm{H}_{43}.
\end{aligned}
\tag{4.11}
$$

The regression for reliability is given as

$$\begin{aligned} R_3 =&(0.601 - 0.0041\mathrm{A}_{13} - 0.0034\mathrm{A}_{23} - 0.0034\mathrm{A}_{33} - 0.0048\mathrm{A}_{43} \\ &- 0.0012\mathrm{B}_{13} - 0.0141\mathrm{B}_{23} - 0.0116\mathrm{B}_{33} - 0.0137\mathrm{B}_{43} \\ &- 0.0086\mathrm{C}_{13} - 0.0095\mathrm{C}_{23} - 0.0001\mathrm{C}_{33} - 0.0011\mathrm{C}_{43} \\ &- 0.0088\mathrm{D}_{13} - 0.0087\mathrm{D}_{23} - 0.0094\mathrm{D}_{33} - 0.0083\mathrm{D}_{43} \\ &- 0.0174\mathrm{G}_{13} - 0.0136\mathrm{G}_{23} - 0.0175\mathrm{G}_{33} - 0.0234\mathrm{G}_{43} \\ &- 0.0164\mathrm{H}_{13} - 0.0122\mathrm{H}_{23} - 0.0168\mathrm{H}_{33} - 0.0014\mathrm{H}_{43})^{-1/5}. \end{aligned} \tag{4.12}$$

The regression for test time is given as

$$\begin{aligned} t_3 =& -(-0.0671 - 0.000163x_1 + 0.000001x_2 - 0.000124y_1 + 0.000001y_2 \\ &+ 0.000572\mathrm{A}_{13} + 0.00056\mathrm{A}_{23} + 0.00006\mathrm{A}_{33} + 0.002932\mathrm{A}_{43} \\ &+ 0.000002\mathrm{B}_{13} + 0.000781\mathrm{B}_{23} - 0.000014\mathrm{B}_{33} + 0.001918\mathrm{B}_{43} \\ &+ 0.00128\mathrm{C}_{13} - 0.000304\mathrm{C}_{23} + 0.00228\mathrm{C}_{33} + 0.002132\mathrm{C}_{43} \\ &+ 0.000373\mathrm{D}_{13} + 0.000999\mathrm{D}_{23} + 0.000439\mathrm{D}_{33} + 0.001007\mathrm{D}_{43} \\ &+ 0.000062\mathrm{G}_{13} + 0.000043\mathrm{G}_{23} + 0.000787\mathrm{G}_{33} + 0.001368\mathrm{G}_{43} \\ &+ 0.000025\mathrm{H}_{13} + 0.001588\mathrm{H}_{23} + 0.000045\mathrm{H}_{33} + 0.001529\mathrm{H}_{43})^{1/-0.1881}. \end{aligned} \tag{4.13}$$

The regression for test cost is given as

$$
\begin{aligned}
c_3^T = &-(-0.5333 - 0.000084x_1 + 0.000681x_2 - 0.000052y_1 + 0.000052y_2 \\
&+ 0.000507\mathrm{A}_{13} + 0.000349\mathrm{A}_{23} + 0.000014\mathrm{A}_{33} + 0.002255\mathrm{A}_{43} \\
&+ 0.000028\mathrm{B}_{13} + 0.001854\mathrm{B}_{23} - 0.000034\mathrm{B}_{33} + 0.004848\mathrm{B}_{43} \\
&+ 0.001169\mathrm{C}_{13} - 0.000245\mathrm{C}_{23} + 0.001889\mathrm{C}_{33} + 0.001858\mathrm{C}_{43} \\
&+ 0.000464\mathrm{D}_{13} + 0.001137\mathrm{D}_{23} + 0.000443\mathrm{D}_{33} + 0.000046\mathrm{D}_{43} \\
&+ 0.000046\mathrm{G}_{13} + 0.000017\mathrm{G}_{23} + 0.002128\mathrm{G}_{33} + 0.003571\mathrm{G}_{43} \\
&- 0.000031\mathrm{H}_{13} + 0.004467\mathrm{H}_{23} + 0.000056\mathrm{H}_{33} + 0.004366\mathrm{H}_{43})^{1/-0.1881}.
\end{aligned} \tag{4.14}
$$

The complete regressions can be found in Appendix A along with their residual plots and the associated normal probability plots for residuals.

The regressions for stages 1-3 were used to construct the response surface with total cost being $C_i = c_i^T + c_i^I$. The response of interest is total cost, and the independent variables are reliability and testing time. The response surface can be seen in Figures 4.2 and 4.3. Two figures are given to provide different perspectives.

Figure 4.2: Response Surface of System Performance

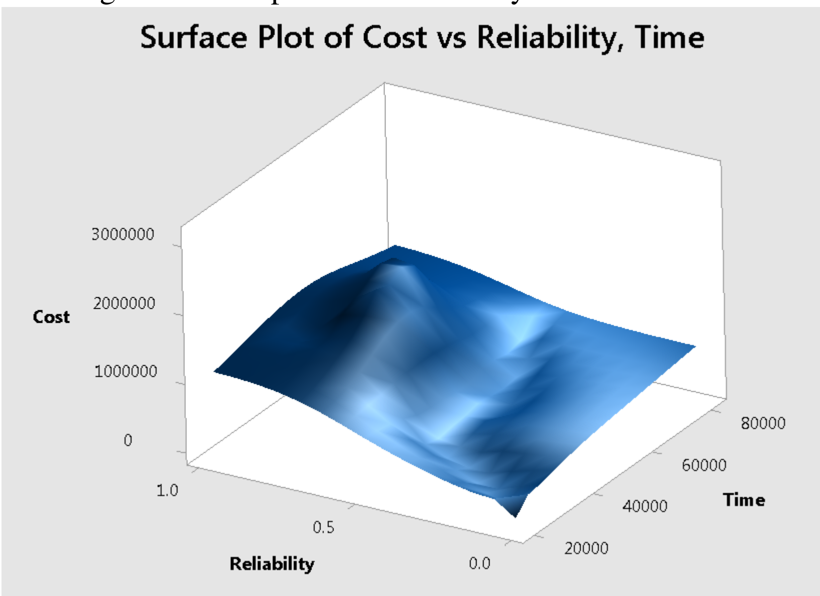
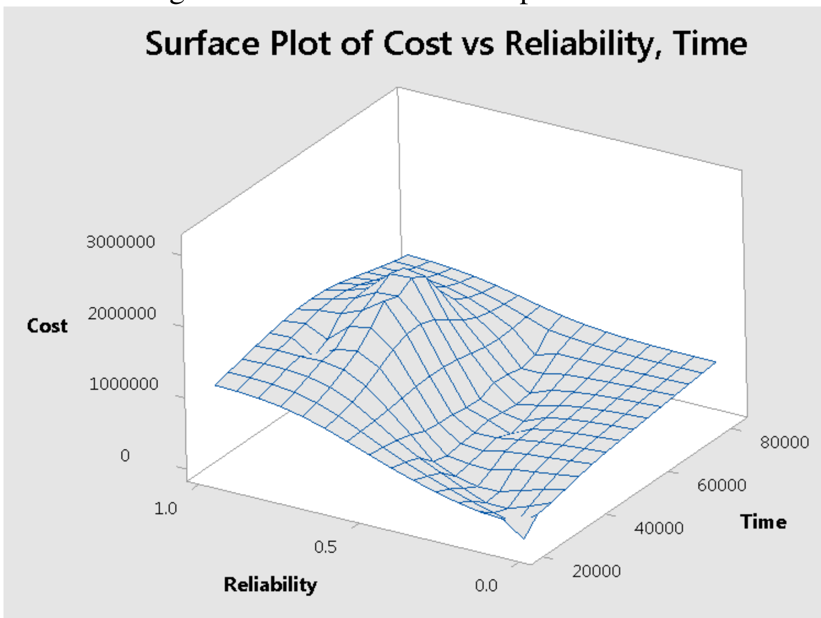


Figure 4.3: Wireframe of Response Surface



The response surface seems like a reasonable fit because total cost increases with longer testing times and higher reliabilities along with later stages of testing. Furthermore, high system reliability is only achievable in the last stage of testing because additional failure modes are not uncovered in the earlier stages.

The response surface was fitted with a quadratic model in R to estimate the cost of each test stage from stresses and failure mode fixes. The resulting model is given as

$$C = -702500 + 0.1917t + 1577000R - 0.00001219t^2 + 1667000R^2 + 5.360tR, \tag{4.15}$$

where t is the function for test time and R is the function for reliability. The function for time is given as

$$t = -13719x_1 + 13592x_2 - 2648y_1 + 1853y_2 + 11599R. \tag{4.16}$$

The reliability function is given as

$$\begin{aligned} R =& 0.3459 + 0.01465\mathrm{A}_1 + 0.01515\mathrm{A}_2 + 0.0234\mathrm{A}_3 + 0.07672\mathrm{A}_4 \\ & + 0.007418\mathrm{B}_1 + 0.004301\mathrm{B}_2 + 0.0277\mathrm{B}_3 + 0.07086\mathrm{B}_4 \\ & + 0.006036\mathrm{C}_1 + 0.004035\mathrm{C}_2 + 0.02345\mathrm{C}_3 + 0.07266\mathrm{C}_4 \\ & + 0.003925\mathrm{D}_1 + 0.004159\mathrm{D}_2 + 0.01695\mathrm{D}_3 + 0.07433\mathrm{D}_4 \\ & + 0.002362\mathrm{G}_1 + 0.004053\mathrm{G}_2 + 0.001456\mathrm{G}_3 + 0.06438\mathrm{G}_4 \\ & + 0.0004075\mathrm{H}_1 + 0.005368\mathrm{H}_2 + 0.01609\mathrm{H}_3 + 0.07893\mathrm{H}_4 \end{aligned} \tag{4.17}$$

The results and analysis of the cost, test time, and reliability functions are given in Appendix A.

4.3 Numerical Example

A simple problem to consider is the series-parallel system shown in Figure 4.1 is under design. The objective is to minimize costs while making modifications to the system to achieve a system reliability of 0.9 at the end of testing. An additional constraint is that the system must be released within the year or approximately within 500,000 minutes of test time. The approach to this problem is to use an optimization model to determine the settings for stresses and failure mode modifications for all three stages prior to the start of testing. Then simulation can be used to compare the results of the optimization model to data from the simulation model.

Following the structure of model (3.3), the optimization model is given as

$$\begin{aligned} \min \ & C = \sum_{i=1}^{3} c_i \\ \text{s.t.} \ & R_{\text{sys}} \geq 0.9 \\ & t_i \leq 500,000 \end{aligned} \tag{4.18}$$

Because it is assumed that the number of failure modes is known at each stage, constraints can be added to indicate which failure modes are not addressed. For stage 1, it is known there are two failure modes per component. Therefore adding a constraints $z_{31} = 0$ and $z_{41} = 0$ for all components z would ensure that those failure modes are not observed or modified in the first stage. Furthermore, constraints can be added to indicate which failure modes have been improved. An example is that the second failure mode of the A component is perfectly fixed for stage 1. The constraint $\mathrm{A}_{22} = 1$ and $\mathrm{A}_{23} = 1$ would be added for stages 2 and 3 to show that a component cannot regress in its level of improvement.

The optimization model is given as

$$
\begin{aligned}
\min\ & C = \sum_{i=1}^{3} c_i^T + c_i^I \\
\text{s.t.}\ & R_3 \geq 0.9 \\
& \sum_{i=1}^{3} t_i \leq 500000 \\
& z_{31} = 0 \ \forall \ z_{31} \\
& z_{41} = 0 \ \forall \ z_{41} \\
& z_{42} = 0 \ \forall \ z_{42} \\
& z_{k(i+1)} \geq z_{ki} \ \forall \ z_{ki} \\
& 300 \leq x_1 < x_2 \leq 400 \\
& 120 \leq y_1 < y_2 \leq 240 \\
& 0 \leq z_{ki} \leq 1 \ \forall \ z_{ki}
\end{aligned}
\tag{4.19}
$$

4.3.1 Solution

A nonlinear optimization package was used to solve for optimality in R. The optimal solution found is given in Table 4.1 The complete settings for optimality for the accelerated testing plan can be found in Table 4.2. This solution seems reasonable, and all constraints are satisfied. The stresses increase from stage to stage to accommodate the longer test times from higher reliability.

Table 4.1: Optimal solution

	Stage 1	Stage 2	Stage 3	Total
Reliability	0.386	0.463	0.900	
Test Time (min)	28,070	31,819	119,772	179,661
Cost	214,530.08	468,033.51	2,638,966.58	3,321,530.17

However, the time for stage 3 seems very high because the the test times for the other stages were much lower.

For Stage 1, it was best to completely remove failure modes: B_1, B_2, C_1, and H_1 in Stage 1 and partially fix: A_1, A_2, C_2, and D_1. For Stage 2, it was best to completely remove failure modes: D_3 and H_3 and partially fix: A_3, B_3, and C_3. For Stage 3, it was best to completely remove failure modes: C_4, D_4, G_4, and H_4 and partially fix: A_4.

An alternative approach was considered in which the reliability was maximized in each stage. This is a more intuitive approach because it seems reasonable to make the most improvements in the earlier stages when costs are lower and then making minimal improvements in the last stage because costs would be higher. The solution for maximizing reliability is found in Table 4.3.

The complete settings for the maximum reliability approach is given in Table B.1 in Appendix B. In each stage, all known failure modes are completely removed after testing. In Stage 1, the failure modes: A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2, G_1, G_2, H_1, and H_2 are completely removed. In Stage 2, the failure modes: A_3, B_3, C_3, D_3, G_3, and H_3 are completely removed. In Stage 3, all failure modes are completely fixed with the exception of H_4.

Like the optimal solution, the stresses increase from stage to stage. As expected, the reliability is higher in the earlier stages. However, there is about an 13% increase in total cost and a 14% increase in total test time. This is appropriate because the higher reliabilities in each stage would cause test times to be longer. Additionally, the optimal solution had a reliability that was closer to the desired reliability, which would reduce costs, whereas the maximum reliability approach had a higher end reliability.

Rather than attempting to determine optimal settings at the beginning of testing, a more realistic approach may be to conduct analysis prior to each subsequent stage of testing. After the first

Table 4.2: Optimal solution settings

	Stage 1	Stage 2	Stage 3
x_1	325	350	375
x_2	350	375	400
y_1	150	120	180
y_2	180	150	240
A_1	0.7	0.7	0.7
A_2	0.7	0.7	0.7
A_3	0	0.7	0.7
A_4	0	0	0.7
B_1	1	1	1
B_2	1	1	1
B_3	0	0.7	0.7
B_4	0	0	0.7
C_1	1	1	1
C_2	0.7	0.7	0.7
C_3	0	0.7	0.7
C_4	0	0	1
D_1	0.7	0.7	0.7
D_2	0	0	0
D_3	0	1	1
D_4	0	1	1
G_1	0	0	0
G_2	0	0	0
G_3	0	0	0
G_4	0	0	1
H_1	1	1	1
H_2	0	0	0
H_3	0	1	1
H_4	0	0	1

Table 4.3: Maximum Reliability Solution

	Stage 1	Stage 2	Stage 3	Total
Reliability	0.418	0.540	0.901	
Test Time (min)	28,436	56,703	119,784	204,923
Cost ($)	312,229.75	799,288.30	2,644,650.22	3,756,168.27

Figure 4.4: Sequential Approach Stage 1

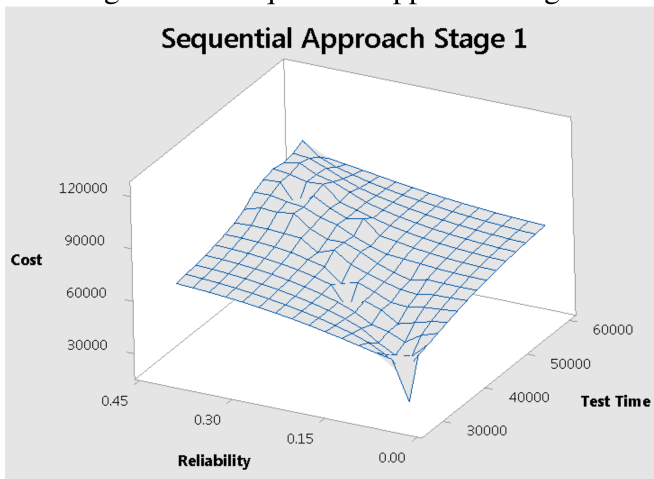
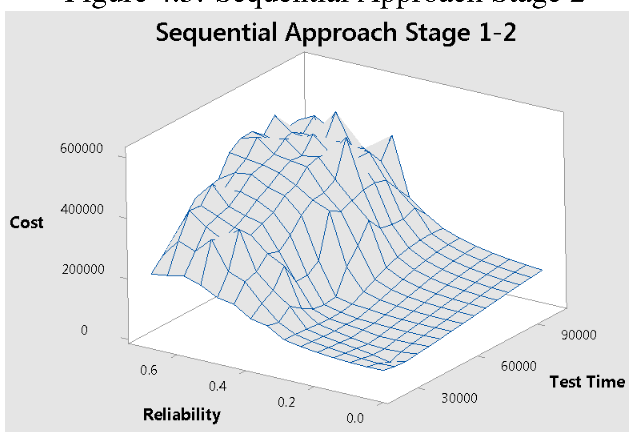


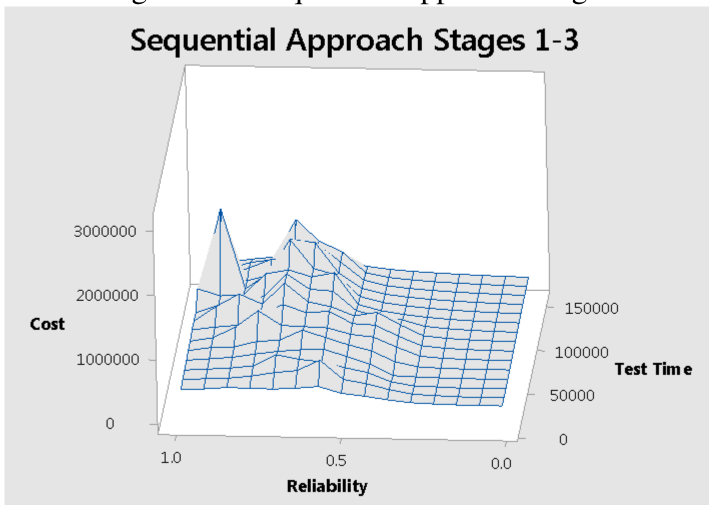
Figure 4.5: Sequential Approach Stage 2



stage of testing, the data can be used to construct and fit a response surface. Then settings can be determined for the next stage of testing. Then in each subsequent testing stage, the data is combined with all the previous testing data to construct another response surface. This is repeated until the last test stage. It is important to note that optimization will not work in earlier stages of testing unless it is specifically known what levels of reliability are desired after each testing stage. In this instance, after each stage of testing, a response surface was generated, and a search was conducted for the lowest cost in the areas with the highest reliability.

The response surfaces of Stage 1, 2, and 3 can be found in Figures 4.4, 4.5, and 4.6, respectively.

Figure 4.6: Sequential Approach Stage 3



The results of the sequential stage analysis approach is given in Table 4.4. The settings for the

Table 4.4: Sequential Approach Solution

	Stage 1	Stage 2	Stage 3	Total
Reliability	0.354	0.629	0.900	
Test Time (min)	36,950	44,634	61471	143,055
Cost ($)	127,164.14	473,646.81	2,642,466.90	3,243,277.85

sequential approach are found in Table B.2. After Stage 1, it was determined that all failure modes should be completely removed with the exception of A_1, A_2, and C_1. After introducing new failure modes, the number of modifications to be considered are greatly reduced because those failure modes that are completely removed cannot be changed anymore. For Stage 2, it was determined that A_1 and A_2 should now be completely removed; A_3 and B_3 are to remain unaffected; C_3 and D_3 are to be improved; and G_3 and H_3 are to be completely removed. In Stage 3, the best settings are to completely remove the remaining failure modes with the exception of A_3, C_1, and D_3, which will be improved and H_4, which will remain unaffected in the system.

It is important to note that an optimization model was not used in the first two stages because a desired reliability was not known. Furthermore, an optimization model was not developed for Stage 3. Rather all possibilities were completely enumerated using simulation, and the lowest cost settings with reliability closest to 0.9 were selected. This is because the fixes in failure

Table 4.5: Comparison of Simulated Results of Optimization Model and Sequential Approach

Stage 1

	Opt Model	Opt Sim	Sequential
Reliability Average	0.3860	0.3083	0.3548
Reliability Std Dev		0.0148	0.0187
Test Time Average	28,070	33,596	36,951
Test Time Std Dev		2,425	3,237
Total Cost Average	214,530.08	110,445.34	127,164.14
Total Cost Std Dev		8,655.85	6,473.99

Stage 2

	Opt Model	Opt Sim	Sequential
Reliability Average	0.4630	0.4875	0.6295
Reliability Std Dev		0.0269	0.0170
Test Time Average	31,819	57,373	44,634
Test Time Std Dev		5,503	7,043
Total Cost Average	468,033.51	528,036.42	473,646.81
Total Cost Std Dev		12,552.03	30,174.57

Stage 3

	Opt Model	Opt Sim	Sequential
Reliability Average	0.9000	0.9069	0.9002
Reliability Std Dev		0.0293	0.0173
Test Time Average	119,772	58,909	61,471
Test Time Std Dev		9,165	16,867
Total Cost Average	2,638,966.58	2,213,321.43	2,642,465.90
Total Cost Std Dev		17,183.92	41,734.20

modes greatly reduces the number of possibilities from stage to stage as completely removed failure modes cannot change.

4.3.2 Results Validation

The settings for both the optimization model and sequential approach were run through the simulation with 1000 replications. A comparison of the results can be found in Table 4.5.

The use of a response surface for all three stages prior to testing fairly estimated the behavior of reliability when simulating the optimization model settings and the sequential approach as well as total cost in the later stages of testing. It did a poor job at estimating testing time in any of the stages as it was over in stages 1 and 3 and under in stage 2. One possibility for the poor estimation of testing time is that there is high variance in testing time. However, there is also high variability in total cost because testing time is a factor in the cost calculation. The sequential approach had higher variance for total cost and test time in all three stages. It is noteworthy that the sequential approach had more similarities to the optimization model than just simulating using the optimal settings. This can be observed in the reliability and total cost in stage 3. This indicates that the optimization model may not have been a good fit for the data. Rather, the sequential approach fits the data better because of the iterative analysis and model fitting.

There are similarities in the response surfaces for stage 1 in both of these scenarios. However, subsequent stages show signs of difference. In the optimization model, the curves are much smoother because all possibilities are considered whereas the sequential approach looks at a subset of the data. This is because the amount of possibilities are reduced when the settings for the next stage of testing are determined. An advantage of the sequential approach is that analysis of data is focused on only a subset of the all possibilities, which reduces the amount of time needed to search for a solution. However, the sequential approach does require analysis after each stage of testing whereas the optimization model only requires analysis prior to testing. Although, the optimization model approach does require less analysis during testing, the model in this instance was not necessarily a good fit because the sequential approach had more similar results to the optimization model than the simulation based on the optimal settings.

4.4 Alternative Scenarios

Additional scenarios were considered to observe how changes in the shape parameters of components, β, and fix effectiveness factor would affect the results. For the shape parameter, the values were manipulated to either be relatively close together (Tight β) or manipulated to be far apart (Loose β) while all other parameters remained the same from the original problem. For the FEF scenario, the FEF was varied between components (e.g., 3 components have FEF of 0.9 and 3 components have FEF of 0.6) and within components (e.g., 2 failure modes of the same compo-

Table 4.6: Comparison of Scenarios

Stage 1

	Tight β	Loose β	Intercomponent	Intracomponent
Reliability Average	0.3433	0.3681	0.3710	0.3642
Reliability Std Dev	0.0260	0.0148	0.0129	0.0108
Test Time Average	47,418	34,250	51,874	59,262
Test Time Std Dev	3,723	1,279	5,339	6,316
Total Cost Average	169,721.83	121,762.31	156,178.34	155,133.51
Total Cost Std Dev	7,446.20	2,558.90	5,812.22	5,614.62

Stage 2

	Tight β	Loose β	Intercomponent	Intracomponent
Reliability Average	0.5467	0.6431	0.5664	0.4045
Reliability Std Dev	0.0325	0.0305	0.0115	0.0172
Test Time Average	67,026	45,163	64,989	65,330
Test Time Std Dev	11,330	6,210	14,685	9,289
Total Cost Average	529,997.59	441,802.29	547,060.49	445880.85
Total Cost Std Dev	45,319.22	24839.10	78,319.22	37,156.98

Stage 3

	Tight β	Loose β	Intercomponent	Intracomponent
Reliability Average	0.9084	0.9010	0.9024	0.9003
Reliability Std Dev	0.0473	0.0145	0.0199	0.0183
Test Time Average	63,095	61,112	85,806	76,582
Test Time Std Dev	25,407	17,668	18,648	18,124
Total Cost Average	2,353,521.25	2,556,116.46	2,485,388.35	2,271,892.10
Total Cost Std Dev	50,814.36	35,336.96	37,296.17	27,186.51

nent have FEF of 0.9 and the other 2 failure modes have FEF of 0.6) while all other parameters remained unchanged from the original problem. The FEF variation between components will be referenced as Intercomponent, and the FEF variation within components will be referenced as Intracomponent. The results of these variations can be found in Table 4.6, and the settings for each scenario can be found in Appendix B.

The tight β scenario demonstrated higher variance for reliability, test time, and total cost for all three stages when compared to both the loose β scenario and the optimization and sequential approaches. It also had higher averages for test time in all three stages and higher averages for total cost in all stages except for stage 3 when compared to loose β. In the tight β variant, there were a three failure modes that did not have any fixes whereas the loose β scenario had modifications made to all failure modes.

There was not a significant difference between the intercomponent and intracomponent scenarios. However, there was more variance for both scenarios for test time and total cost in all three stages when compared to the optimization and sequential approaches. This is expected because the fix effectiveness factors for the failure modes were varied rather than remaining constant.

Chapter 5

Conclusion

5.1 Conclusion

From a review of the literature, there are few instances when cost is considered in the framework of an accelerated testing framework. In this study, cost is explicitly considered as the objective was to minimize the total cost of the test given a desired reliability at the end of testing and a time constraint. Response surfaces were used as estimators of the system's behavior, and two approaches were considered. In one approach, a response surface was constructed for all three stages, and an optimization model was used to determine the settings for temperature and failure mode fixes prior to any testing. The other approach used response surfaces to analyze a testing stage to determine the best settings for the subsequent stage of testing.

The approach of optimizing all settings for all stages prior to testing performed decently in later stages in estimating both reliability and total cost. However, it was poor in estimating testing time for any testing stage. One possible reason for this is that simulation was used where failure times are randomly generated as evidenced by high variance. It would be interesting to evaluate this methodology would be using a non-simulated data set.

The sequential approach where response surfaces are generated after each test stage seems like a much more pragmatic approach and had similar end results to that of the optimization method. However, there is much more computation involved because a complete set of analysis is required after each stage of testing whereas analysis is only required once before testing using the optimization model. Furthermore, the variance was higher than that of the optimization method for test time and for total cost in later stages. Another disadvantage of the sequential approach was that an optimization model was not used in this instance because desired reliabilities were not known for each stage and it was less efficient construct a optimization model. One avenue of exploration for this could be the use of dual response surface optimization where one objective is to maximize reliability and the other objective is to minimize costs. Another avenue could be the use of experimental design to explore the response surface.

The largest deviation from fit was the test time as the optimization model overestimated test time in stages 1 and 3 and underestimated in stage 2. As previously stated, this may be due to the stochasticity of generating failure times with simulation. However, it may indicate a better fitting model is necessary for test time. In this study, linear regressions were used and the normality tests were generally close to 0.05. It is possible that nonlinear regression may have been more appropriate to estimate test time.

5.2 Future Work

One area where this problem can be further investigated is instead of assuming a constant value for the fix effectiveness factor, the fix effectiveness factor can be a random variable with a probability of being fixed or remaining in the system. Additionally, the problem can be expanded to consider censored data.

It was previously mentioned that a dual response surface optimization or design of experiments approach could be incorporated to the sequential stage approach.

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Appendix A

Response Surface Results

Figure A.1: Results for Stage 0 Time Regression

Regression Analysis: Time versus Temp1, Temp2, Volt1, Volt2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	8	4.18022	0.522527	261.80	0.000
Temp1	1	0.01999	0.019989	10.01	0.002
Temp2	1	0.04306	0.043057	21.57	0.000
Volt1	1	0.02190	0.021897	10.97	0.001
Volt2	1	0.04718	0.047177	23.64	0.000

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0446759	77.96%	77.66%	77.55%

Regression Equation

Time^-0.2 = -0.000306 Temp1 + 0.000449 Temp2 - 0.000267 Volt1 + 0.000391 Volt2

Figure A.2: Residuals for Stage 0 Time

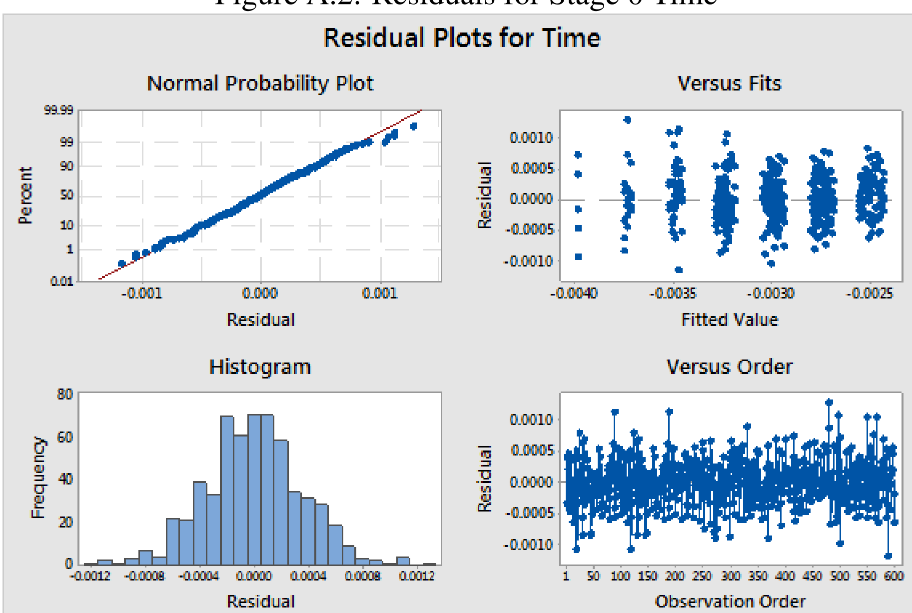

Figure A.3: Normal Probability Plot for Stage 0 Time

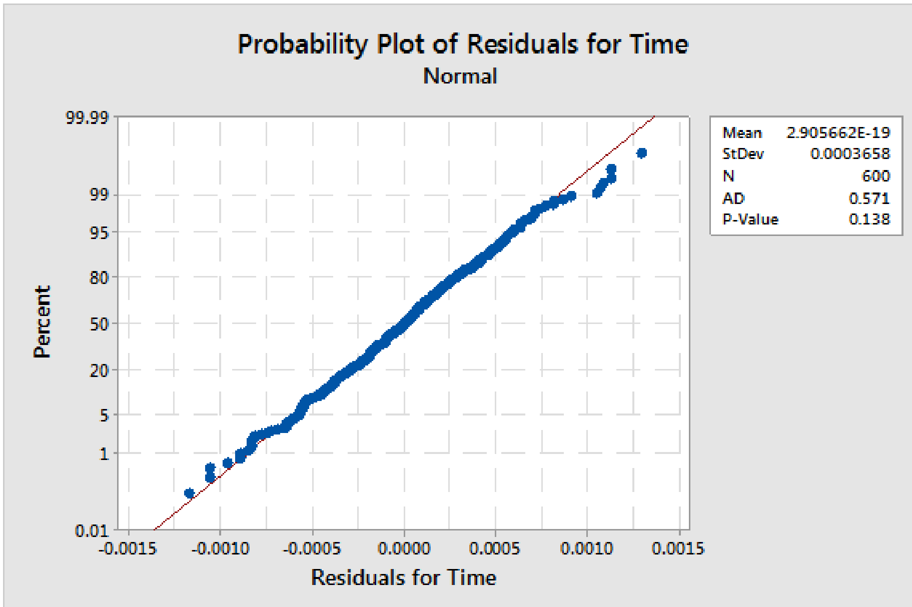

Figure A.4: Results for Stage 0 Cost Regression

Regression Analysis: Cost versus Temp1, Temp2, Volt1, Volt2

Method

Box-Cox transformation	
Rounded λ	0.844626
Estimated λ	0.844626
95% CI for λ	(0.755126, 0.944126)

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	4	8786029669	2196507417	6217.62	0.000
Temp1	1	88094115	88094115	249.37	0.000
Temp2	1	189073744	189073744	535.21	0.000
Volt1	1	70432162	70432162	199.37	0.000
Volt2	1	90134204	90134204	255.14	0.000

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
594.366	97.66%	97.64%	97.63%

Regression Equation

Cost = -(-0.007378 - 0.000022 Temp1 + 0.000017 Temp2 - 0.000016 Volt1
+ 0.000013 Volt2)^(-2)

Figure A.5: Residuals for Stage 0 Cost

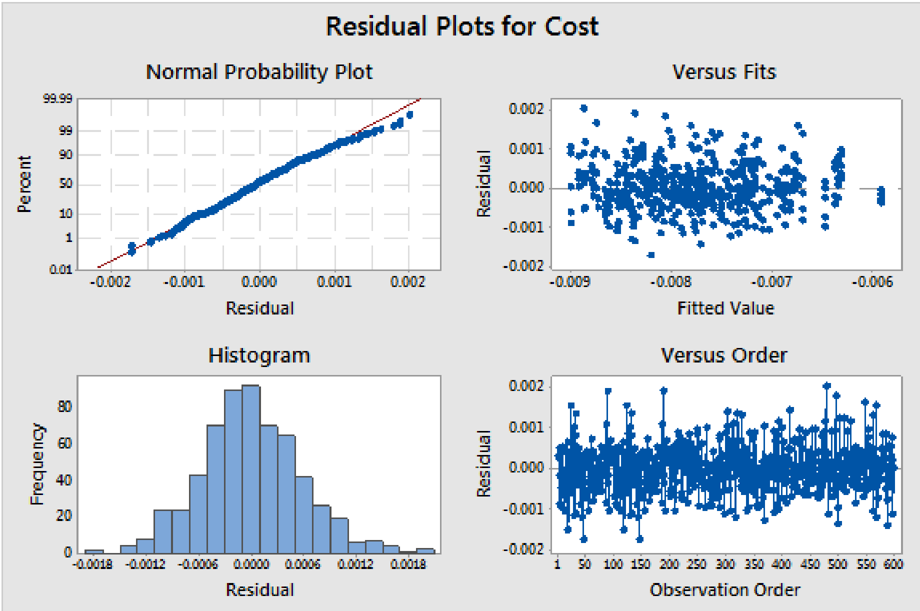

Figure A.6: Normal Probability Plot for Stage 0 Cost

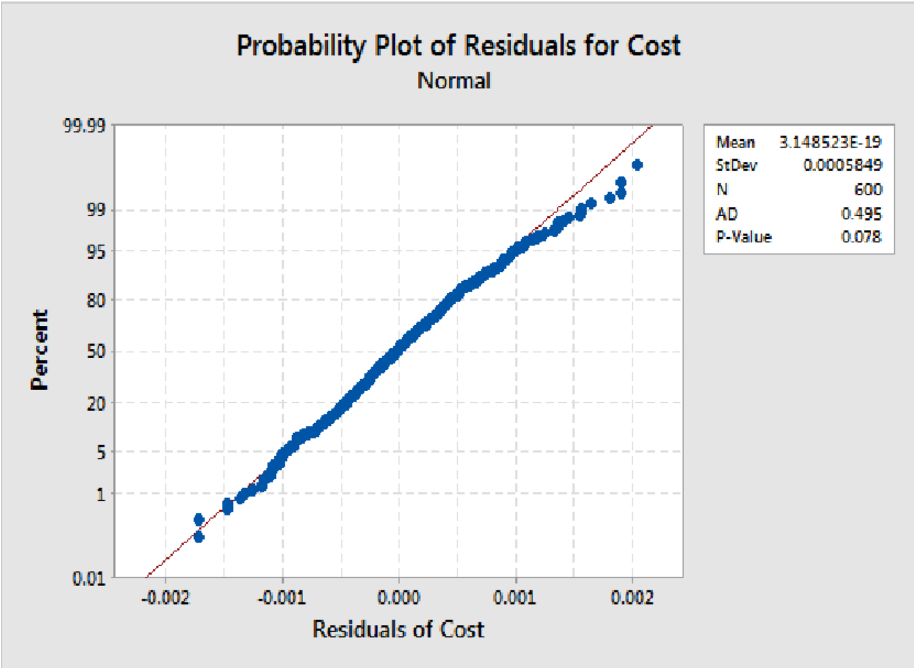

Figure A.7: Results for Stage 1 Improvement Cost Regression

Regression Analysis: Improve Cost versus A1, A2, B1, B2, G, H, D1, D2, G1, G2, H1, H2

Method

Categorical predictor coding (1, 0)

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	24	8.36844E+11	34868510323	*	*
A1	2	55936458758	27968229379	*	*
A2	2	64694735142	32347367571	*	*
B1	2	66470982378	33235491189	*	*
B2	2	1.25344E+11	62672101781	*	*
C1	2	98163120313	49081560156	*	*
C2	2	41291535671	20645767836	*	*
D1	2	38100790565	19050395283	*	*
D2	2	98409215955	49204607978	*	*
G1	2	21050073813	10525036906	*	*
G2	2	58735725832	29367862916	*	*
H1	2	48540881002	24270440501	*	*
H2	2	1.23081E+11	61540263972	*	*
Error	19933	0	0		
Lack-of-Fit	14123	0	0	*	*
Pure Error	5810	0	0		
Total	19957	8.36844E+11			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0	100.00%	100.00%	100.00%

Regression Equation

Improve Cost = 4265 A1 + 5110 A2 + 4885 B1 + 6397 B2 + 7445 C1 + 3425 C2 + 3405 D1 + 6046 D2 + 2512 G1 + 3915 G2 + 4095 H1 + 6649 H2

Figure A.8: Results for Stage 1 Reliability

Regression Analysis: Reliability versus A1, A2, B1, B2, G, H, D1, D2, G1, G2, H1, H2

Method

Categorical predictor coding (1, 0)

Box-Cox transformation	
Rounded λ	0.28124
Estimated λ	0.28124
95% CI for λ	(0.274740, 0.287740)

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	24	111.268	4.63619	18562.14	0.000
A1	2	18.068	9.03419	36170.65	0.000
A2	2	17.756	8.87804	35545.49	0.000
B1	2	10.279	5.13968	20578.00	0.000
B2	2	12.700	6.34978	25422.95	0.000
C1	2	3.122	1.56117	6250.53	0.000
C2	2	5.318	2.65883	10645.28	0.000
D1	2	3.801	1.90028	7608.23	0.000
D2	2	3.180	1.58982	6365.26	0.000
G1	2	9.637	4.81856	19292.31	0.000
G2	2	9.730	4.86483	19477.57	0.000
H1	2	9.670	4.83481	19357.37	0.000
H2	2	8.455	4.22744	16925.64	0.000
Error	19933	4.979	0.00025		
Lack-of-Fit	14123	3.838	0.00027	1.38	0.000
Pure Error	5810	1.141	0.00020		
Total	19957	116.247			

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.0158040	95.72%	95.71%	95.71%

Regression Equation

Reliability^0.28124 = 0.332690 + 0.069524 A1 + 0.068863 A2 + 0.063463 B1 + 0.062393 B2 + 0.038109 C1 + 0.037819 C2 + 0.033138 D1 + 0.032675 D2 + 0.051397 G1 + 0.051633 G2 + 0.057679 H1 + 0.057556 H2

Figure A.9: Residuals for Stage 1 Reliability

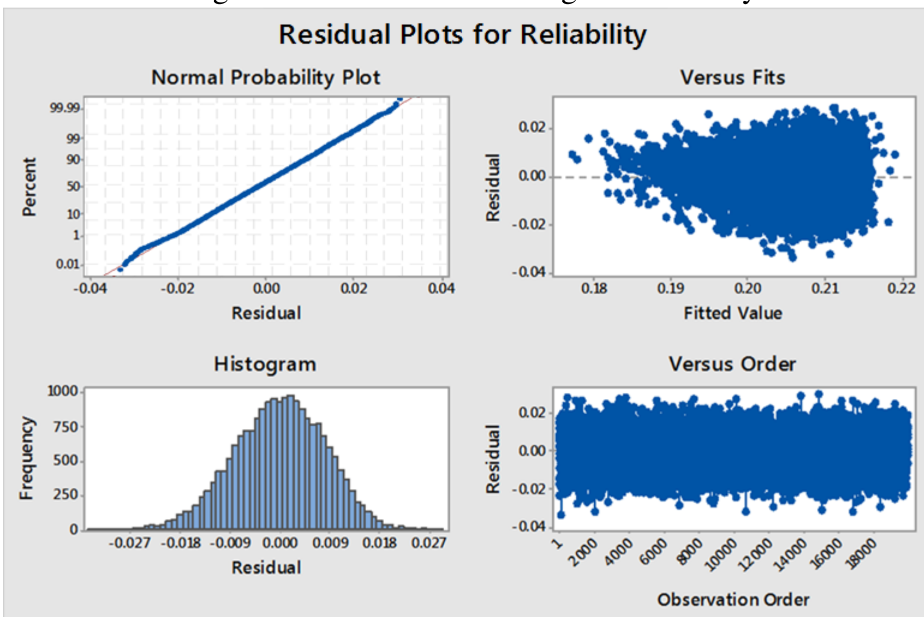

Figure A.10: Normal Probability Plot for Stage 1 Reliability

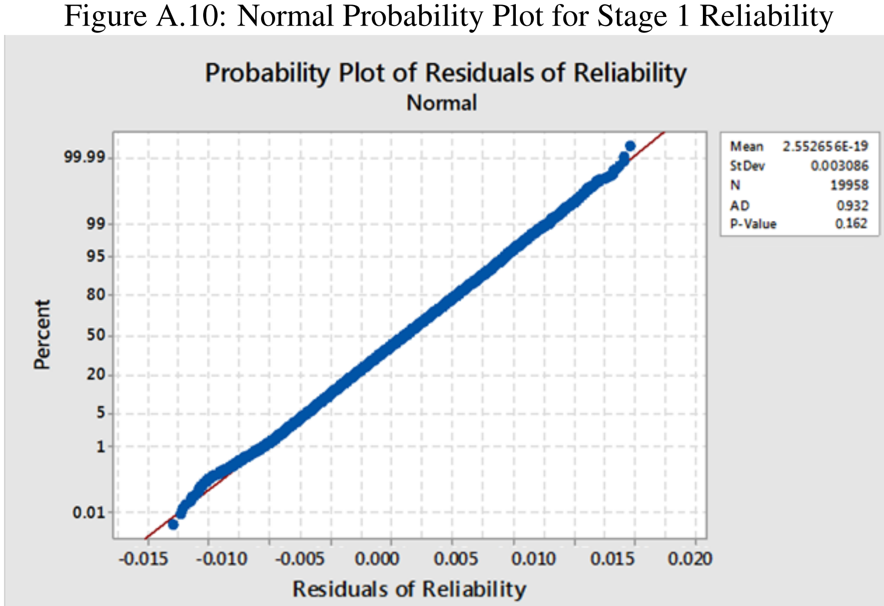

Figure A.11: Results for Stage 1 Test Time

Regression Analysis: Test Duration versus Temp1, Temp2, Volt1, Volt2, A1, A2, B1, B2, ...

Method

Categorical predictor coding (1, 0)

Box-Cox transformation	
Rounded λ	-0.247953
Estimated λ	-0.247953
95% CI for λ	(-0.275453, -0.220453)

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	28	0.399914	0.014283	1497.20	0.000
Temp1	1	0.122529	0.122529	12844.28	0.000
Temp2	1	0.000001	0.000001	0.09	0.760
Volt1	1	0.103882	0.103882	10889.62	0.000
Volt2	1	0.000004	0.000004	0.42	0.518
A1	2	0.030760	0.015380	1612.23	0.000
A2	2	0.024645	0.012322	1291.72	0.000
B1	2	0.004874	0.002437	255.46	0.000
B2	2	0.006787	0.003394	355.75	0.000
D1	2	0.003812	0.001906	199.81	0.000
D2	2	0.007010	0.003505	367.43	0.000
E1	2	0.003270	0.001635	171.40	0.000
E2	2	0.003061	0.001530	160.42	0.000
G1	2	0.003496	0.001748	183.26	0.000
G2	2	0.004275	0.002138	224.08	0.000
H1	2	0.005191	0.002595	272.05	0.000
H2	2	0.003915	0.001957	205.17	0.000
Error	19929	0.190114	0.000010		
Lack-of-Fit	19763	0.188909	0.000010	1.32	0.009
Pure Error	166	0.001205	0.000007		
Total	19957	0.590028			

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.0030886	77.78%	77.73%	77.68%

Regression Equation

-Test Duration^-0.247953 = -0.024910 - 0.000115 Temp1 + 0.000001 Temp2 - 0.000088 Volt1 + 0.000001 Volt2 + 0.003190 A1 + 0.003067 A2 + 0.001382 B1 + 0.001491 B2 + 0.001451 G + 0.001415 H + 0.001025 D1 + 0.001109 D2 + 0.001016 G1 + 0.001089 G2 + 0.001341 H1 + 0.001243 H2

Figure A.12: Residuals for Stage 1 Test Time

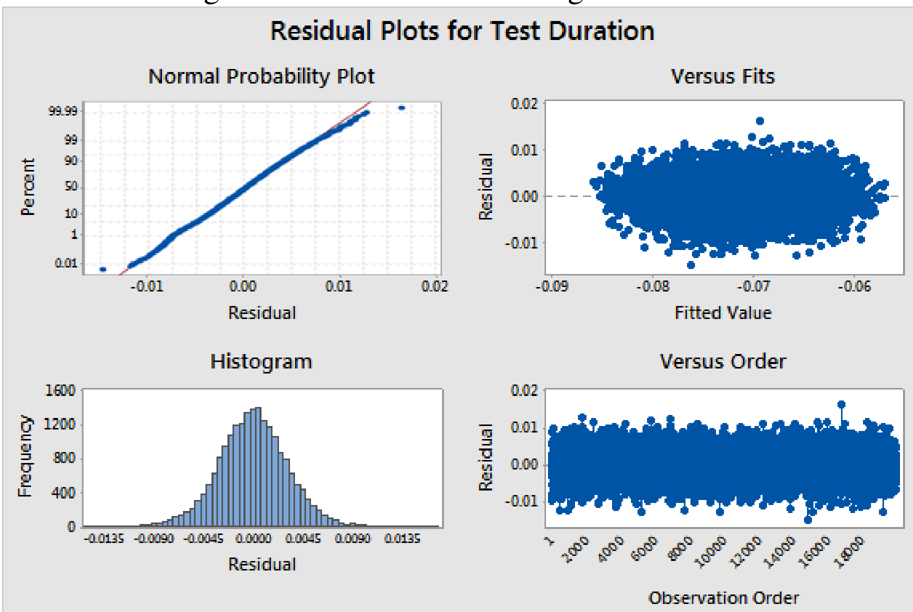

Figure A.13: Normal Probability Plot for Stage 1 Test Time

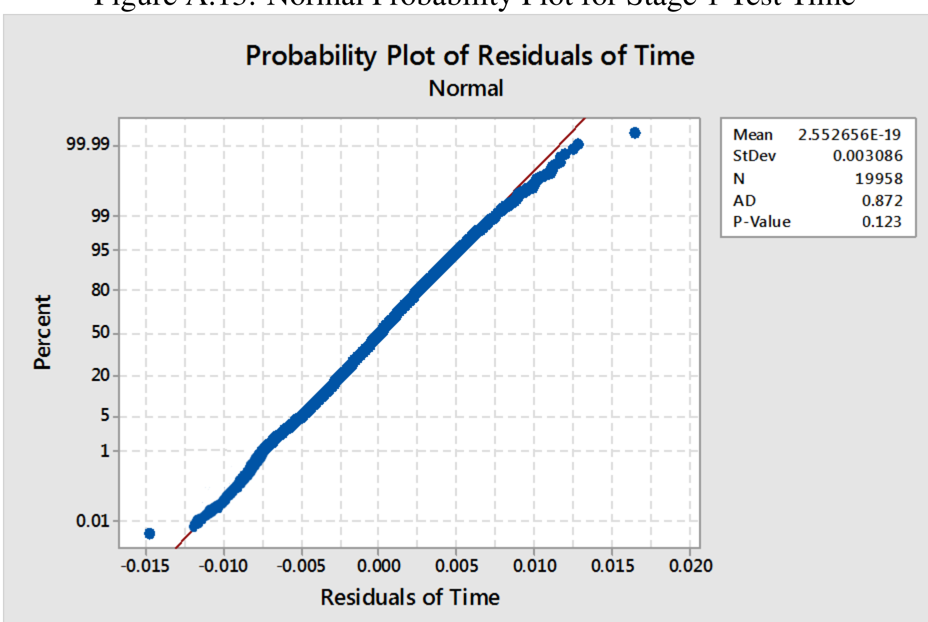

Figure A.14: Results for Stage 1 Test Cost

Regression Analysis: Test Cost versus Temp1, Temp2, Volt1, Volt2, A1, A2, B1, B2, ...

Method

Categorical predictor coding (1, 0)

Box-Cox transformation	
Rounded λ	-0.136984
Estimated λ	-0.136984
95% CI for λ	(-0.170484, -0.104484)

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	28	0.94052	0.033590	766.63	0.000
Temp1	1	0.10309	0.103094	2352.93	0.000
Temp2	1	0.06970	0.069697	1590.69	0.000
Volt1	1	0.05252	0.052518	1198.62	0.000
Volt2	1	0.06948	0.069485	1585.85	0.000
A1	2	0.06798	0.033991	775.78	0.000
A2	2	0.05862	0.029311	668.96	0.000
B1	2	0.10119	0.050596	1154.75	0.000
B2	2	0.12310	0.061549	1404.73	0.000
C1	2	0.00638	0.003190	72.79	0.000
C2	2	0.01499	0.007496	171.07	0.000
E1	2	0.00979	0.004894	111.71	0.000
E2	2	0.00607	0.003035	69.28	0.000
G1	2	0.07659	0.038293	873.96	0.000
G2	2	0.08538	0.042691	974.35	0.000
H1	2	0.10529	0.052647	1201.56	0.000
H2	2	0.09083	0.045413	1036.45	0.000
Error	19929	0.87319	0.000044		
Lack-of-Fit	19763	0.86626	0.000044	1.05	0.346
Pure Error	166	0.00693	0.000042		
Total	19957	1.81371			

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.0066193	71.86%	71.79%	71.71%

Regression Equation

-Test Cost^-0.136984 = -0.271644 - 0.000105 Temp1 + 0.000086 Temp2 - 0.000062 Volt1 + 0.000072 Volt2 + 0.004713 A1 + 0.004720 A2 + 0.006368 B1 + 0.006365 B2 + 0.001810 C1 + 0.002071 C2 + 0.001752 D1 + 0.001531 D2 + 0.004781 G1 + 0.004866 G2 + 0.006047 H1 + 0.005993 H2

Figure A.15: Residuals for Stage 1 Test Cost

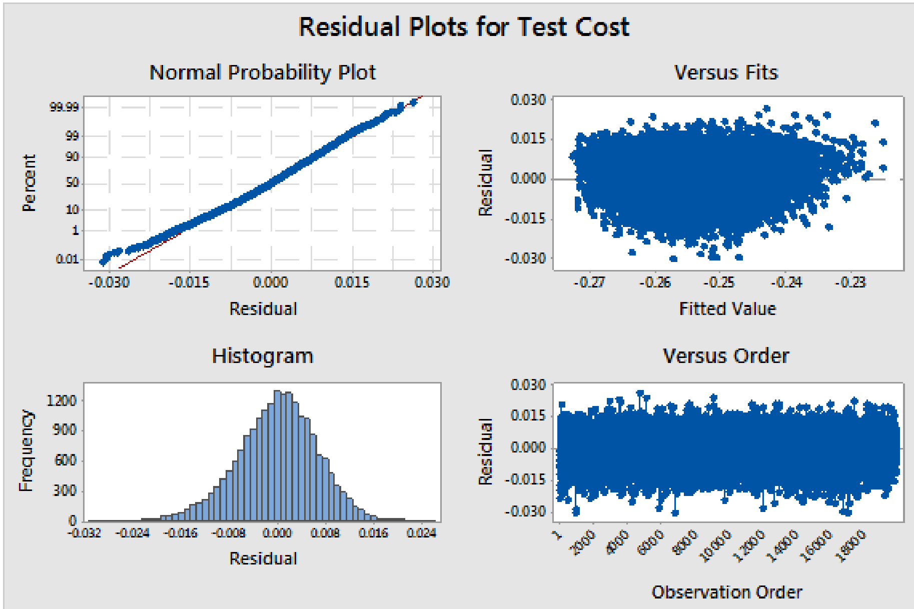

Figure A.16: Normal Probability Plot for Stage 1 Test Cost

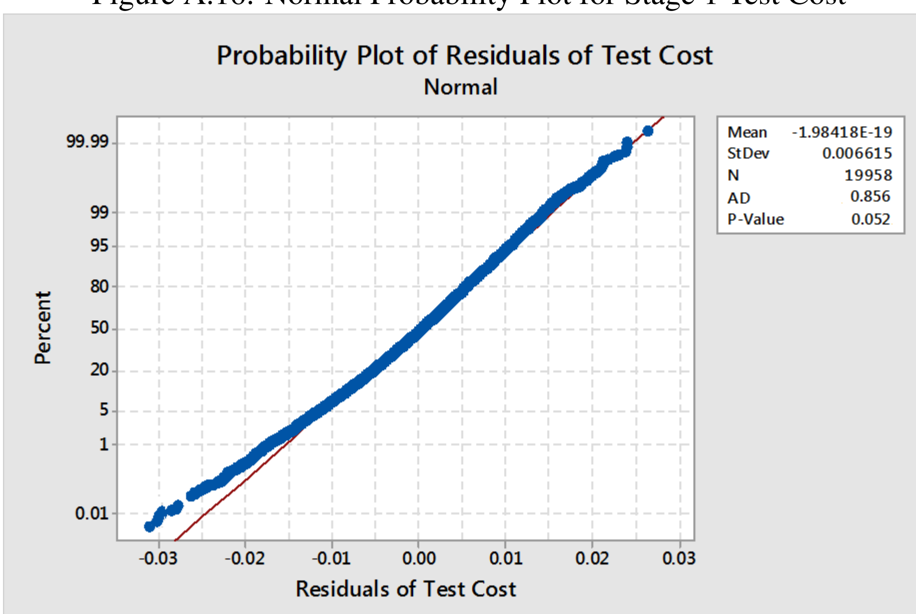

Figure A.17: Results for Stage 2 Reliability

Regression Analysis: Reliability versus A1, A2, A3, B1, B2, B3, C1, C2, ...

Method

Categorical predictor coding (1, 0)

Box-Cox transformation	
Rounded λ	0.197336
Estimated λ	0.197336
95% CI for λ	(0.182836, 0.211836)

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	36	6.9740	0.193721	982.74	0.000
A1	2	1.1553	0.577625	2930.27	0.000
A2	2	1.2465	0.623250	3161.72	0.000
A3	2	0.6344	0.317212	1609.20	0.000
B1	2	0.3663	0.183139	929.06	0.000
B2	2	0.2359	0.117927	598.24	0.000
B3	2	0.1677	0.083871	425.48	0.000
C1	2	0.2081	0.104064	527.91	0.000
C2	2	0.2242	0.112109	568.73	0.000
C3	2	0.1145	0.057249	290.42	0.000
D1	2	0.2090	0.104501	530.13	0.000
D2	2	0.1892	0.094576	479.78	0.000
D3	2	0.2276	0.113776	577.18	0.000
G1	2	0.2618	0.130923	664.17	0.000
G2	2	0.2870	0.143503	727.98	0.000
G3	2	0.2702	0.135096	685.34	0.000
H1	2	0.2962	0.148119	751.40	0.000
H2	2	0.2847	0.142348	722.13	0.000
H3	2	0.3303	0.165154	837.82	0.000
Error	49758	9.8085	0.000197		
Lack-of-Fit	48540	9.5204	0.000196	0.83	1.000
Pure Error	1218	0.2881	0.000237		
Total	49794	16.7824			

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.0140401	61.56%	61.51%	61.47%

Regression Equation

Reliability^0.197336 = 0.711077 + 0.011337 A1 + 0.011696 A2 + 0.008677 A3
+ 0.007630 B1 + 0.005638 B2 + 0.010053 B3 + 0.006378 C1 + 0.005022 C2
+ 0.007657 C3 + 0.004878 D1 + 0.004943 D2 + 0.005125 D3 + 0.005368 G1
+ 0.005650 G2 + 0.005585 G3 + 0.006356 H1 + 0.006808 H2 + 0.006691 H3

Figure A.18: Residuals for Stage 2 Reliability

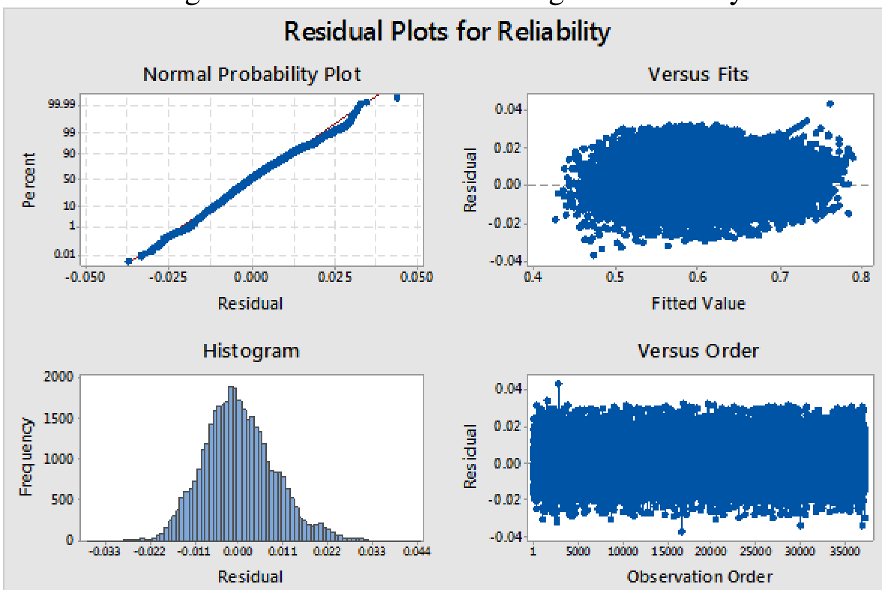

Figure A.19: Normal Probability Plot for Stage 2 Reliability

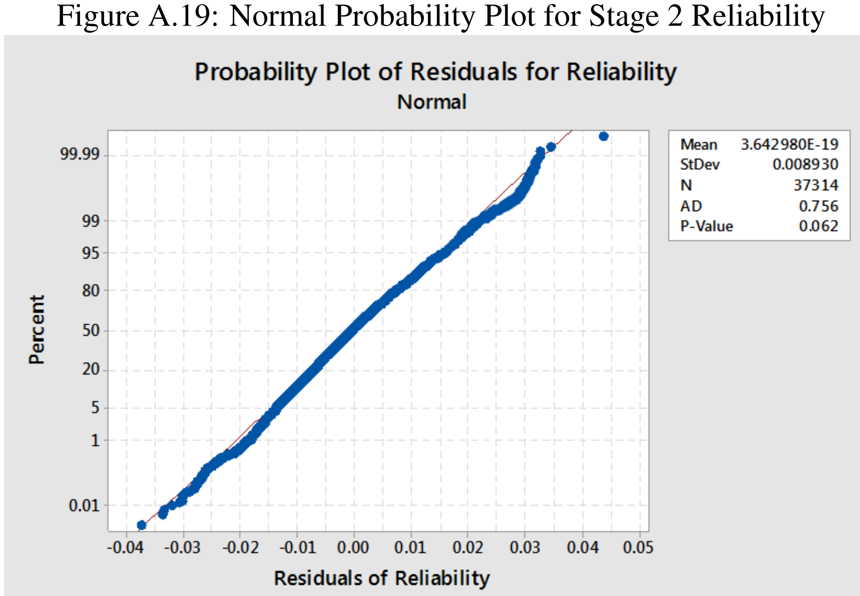

Figure A.20: Results for Stage 2 Test Time

Regression Analysis: Test Time versus Temp1, Temp2, Volt1, Volt2, A1, A2, A3, B1, ...

Method

Categorical predictor coding (1, 0)

Box-Cox transformation	
Rounded λ	-0.0385078
Estimated λ	-0.0385078
95% CI for λ	(-0.0580078, -0.0190078)

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	40	1.71439	0.042860	1517.24	0.000
Temp1	1	0.61874	0.618738	21903.32	0.000
Temp2	1	0.00027	0.000267	9.45	0.002
Volt1	1	0.53755	0.537547	19029.18	0.000
Volt2	1	0.00006	0.000063	4.22	0.036
A1	2	0.02037	0.010186	360.58	0.000
A2	2	0.01239	0.006196	219.33	0.000
A3	2	0.03603	0.018015	637.74	0.000
B1	2	0.01014	0.005070	179.49	0.000
B2	2	0.00556	0.002781	98.46	0.000
B3	2	0.00598	0.002991	105.89	0.000
C1	2	0.00977	0.004883	172.85	0.000
C2	2	0.00095	0.000473	16.74	0.000
C3	2	0.01279	0.006394	226.36	0.000
D1	2	0.00101	0.000505	17.88	0.000
D2	2	0.00586	0.002928	103.66	0.000
D3	2	0.00542	0.002709	95.89	0.000
G1	2	0.00697	0.003484	123.32	0.000
G2	2	0.00015	0.000074	2.63	0.052
G3	2	0.01404	0.007018	248.43	0.000
H1	2	0.00855	0.004273	151.25	0.000
H2	2	0.01661	0.008304	293.95	0.000
H3	2	0.01819	0.009096	321.99	0.000
Error	49754	1.40548	0.000028		
Lack-of-Fit	49736	1.40493	0.000028	0.92	0.643
Pure Error	18	0.00055	0.000031		
Total	49794	3.11987			

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.0053149	54.95%	54.91%	54.88%

Regression Equation

-Test Time^-0.0385078 = -0.596976 - 0.000163 Temp1 + 0.000003 Temp2 - 0.000126 Volt1 + 0.000001 Volt2 + 0.001501 A1 + 0.001329 A2 + 0.002102 A3 + 0.001279 B1 + 0.000905 B2 + 0.001815 B3 + 0.001447 C1 - 0.000330 C2 + 0.002515 C3 + 0.000361 D1 + 0.000963 D2 + 0.000828 D3 + 0.000913 G1 - 0.000010 G2 + 0.001299 G3 + 0.001081 H1 + 0.001651 H2 + 0.001571 H3

Figure A.21: Residuals for Stage 2 Test Time

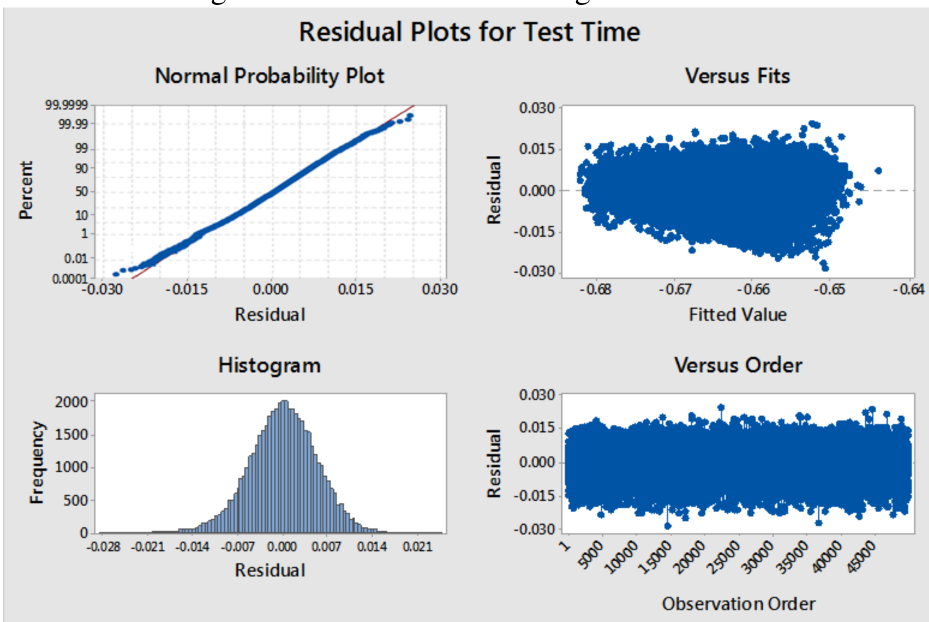

Figure A.22: Normal Probability Plot for Stage 2 Test Time

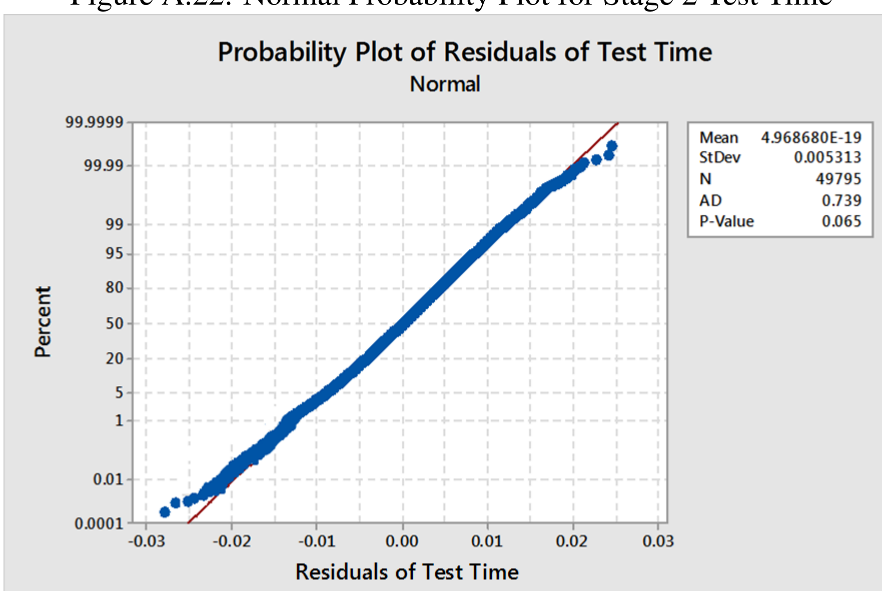

Figure A.23: Results for Stage 2 Test Cost

Regression Analysis: Test Cost versus Temp1, Temp2, Volt1, Volt2, A1, A2, A3, B1, ...

```
Method

Categorical predictor coding  (1, 0)

Box-Cox transformation
Rounded λ                     -0.0510781
Estimated λ                   -0.0510781
95% CI for λ                  (-0.0745781, -0.0275781)


Analysis of Variance for Transformed Response

Source            DF   Adj SS    Adj MS  F-Value  P-Value
RDgrDssion        40  1.13807  0.028452   820.49    0.000
  Temp1            1  0.18325  0.183247  5284.50    0.000
  Temp2            1  0.12108  0.121078  3491.66    0.000
  Volt1            1  0.10214  0.102140  2945.52    0.000
  Volt2            1  0.11346  0.113464  3272.10    0.000
  A1               2  0.01642  0.008208   236.72    0.000
  A2               2  0.00802  0.004012   115.69    0.000
  A3               2  0.01761  0.008806   253.94    0.000
  B1               2  0.07334  0.036671  1057.53    0.000
  B2               2  0.03366  0.016830   485.35    0.000
  B3               2  0.04343  0.021714   626.19    0.000
  C1               2  0.00455  0.002273    65.55    0.000
  C2               2  0.00044  0.000219     6.31    0.002
  C3               2  0.01098  0.005492   158.37    0.000
  D1               2  0.00106  0.000528    15.22    0.000
  D2               2  0.00668  0.003338    96.25    0.000
  D3               2  0.00586  0.002931    84.53    0.000
  G1               2  0.04378  0.021892   631.32    0.000
  G2               2  0.00047  0.000233     6.95    0.009
  G3               2  0.11379  0.056893  1640.69    0.000
  H1               2  0.08559  0.042793  1234.08    0.000
  H2               2  0.13398  0.066991  1931.88    0.000
  H3               2  0.15647  0.078237  2256.21    0.000
Error          49754  1.72529  0.000035
  Lack-of-Fit  49736  1.72472  0.000035     1.11    0.425
  Pure Error      18  0.00056  0.000031
Total          49794  2.86335


Model Summary for Transformed Response

        S    R-sq  R-sq(adj)  R-sq(pred)
0.0058887  69.75%     69.70%      69.65%


Regression Equation

-Test Cost^-0.0510781 = -0.566023 - 0.000089 Temp1 + 0.000072 Temp2 -
 0.000055 Volt1 + 0.000058 Volt2 + 0.001354 A1 + 0.001047 A2 + 0.001472 A3
+ 0.003439 B1 + 0.002230 B2 + 0.004949 B3 + 0.001013 C1 - 0.000203 C2
+ 0.002204 C3 + 0.000353 D1 + 0.000987 D2 + 0.000839 D3 + 0.002287 G1
+ 0.000022 G2 + 0.003693 G3 + 0.003421 H1 + 0.004684 H2 + 0.004608 H3
```

Figure A.24: Residuals for Stage 2 Test Cost

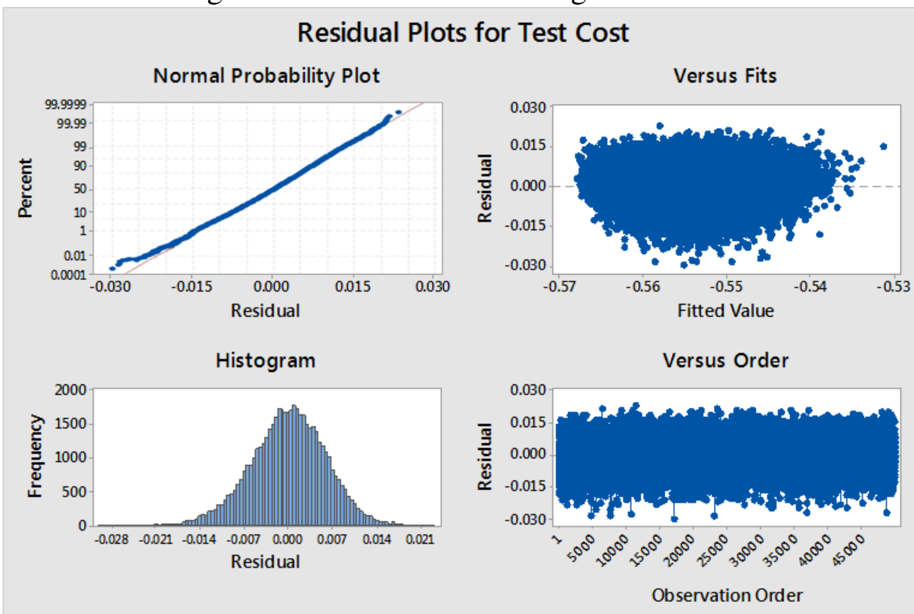

Figure A.25: Normal Probability Plot for Stage 2 Test Cost

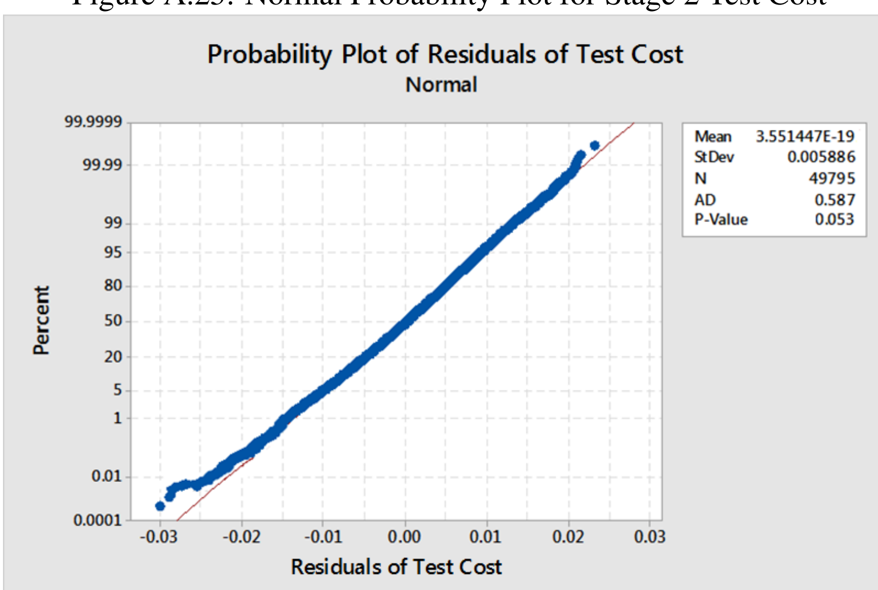

Figure A.26: Results for Stage 3 Reliability

Regression Analysis: Reliability versus A1, A2, A3, A4, B1, B2, B3, B4, ...

Method

Categorical predictor coding (1, 0)

Box-Cox transformation

Rounded λ	-2.79389
Estimated λ	-2.79389
95% CI for λ	(-2.86339, -2.72439)

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	48	11.0496	0.230200	717.55	0.000
A1	2	1.6054	0.802684	2502.03	0.000
A2	2	1.5785	0.789231	2460.10	0.000
A3	2	1.8268	0.913392	2847.12	0.000
A4	2	1.0918	0.545889	1701.58	0.000
B1	2	0.0032	0.001260	5.31	0.005
B2	2	0.0035	0.001760	5.48	0.004
B3	2	0.0080	0.004020	12.53	0.000
B4	2	0.0085	0.004241	13.22	0.000
C1	2	0.2113	0.105630	329.26	0.000
C2	2	0.4267	0.213356	665.05	0.000
C3	2	0.0694	0.034711	108.20	0.000
C4	2	0.2297	0.114825	357.92	0.000
D1	2	0.2236	0.111814	348.53	0.000
D2	2	0.1451	0.072543	226.12	0.000
D3	2	0.1962	0.098077	305.71	0.000
D4	2	0.1569	0.078455	244.55	0.000
G1	2	0.0120	0.006001	18.70	0.000
G2	2	0.0048	0.002388	7.44	0.001
G3	2	0.0104	0.005217	16.26	0.000
G4	2	1.9067	0.953355	2971.69	0.000
H1	2	0.0291	0.014549	45.35	0.000
H2	2	0.0053	0.002651	8.26	0.000
H3	2	0.0365	0.018237	56.85	0.000
H4	2	1.2058	0.602908	1879.31	0.000
Error	99951	22.0656	0.000321		
Lack-of-Fit	99898	22.0560	0.000321	1.79	0.004
Pure Error	53	0.0095	0.000180		
Total	99999	43.1152			

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.0179112	51.63%	51.59%	51.56%

Regression Equation

(Reliability^-5) = 0.600574 - 0.00405 A1 - 0.00337 A2 - 0.0034 A3 - 0.00477 A4
- 0.001175 B1 - 0.01414 B2 - 0.01161 B3 - 0.01369 B4 - 0.00857 C1 - 0.00947 C2
- 0.00011 C3 - 0.011 C4 - 0.00883 D1 - 0.00868 D2 - 0.00936 D3 - 0.00826 D4
- 0.01742 G1 - 0.01359 G2 - 0.01745 G3 - 0.02347 G4 - 0.01643 H1 - 0.01217 H2
- 0.01677 H3 - 0.00135 H4

Figure A.27: Residuals for Stage 3 Reliability

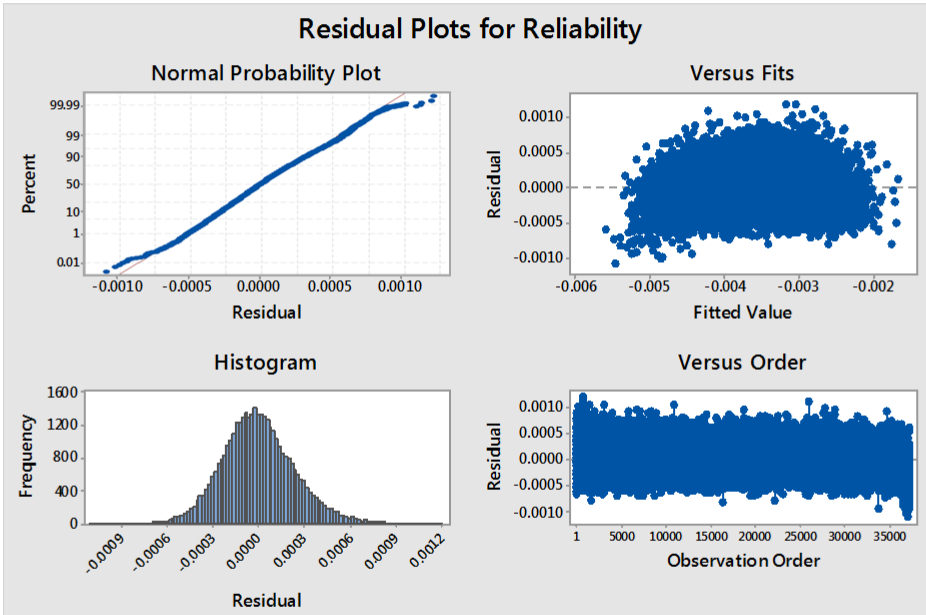

Figure A.28: Normal Probability Plot for Stage 3 Reliability

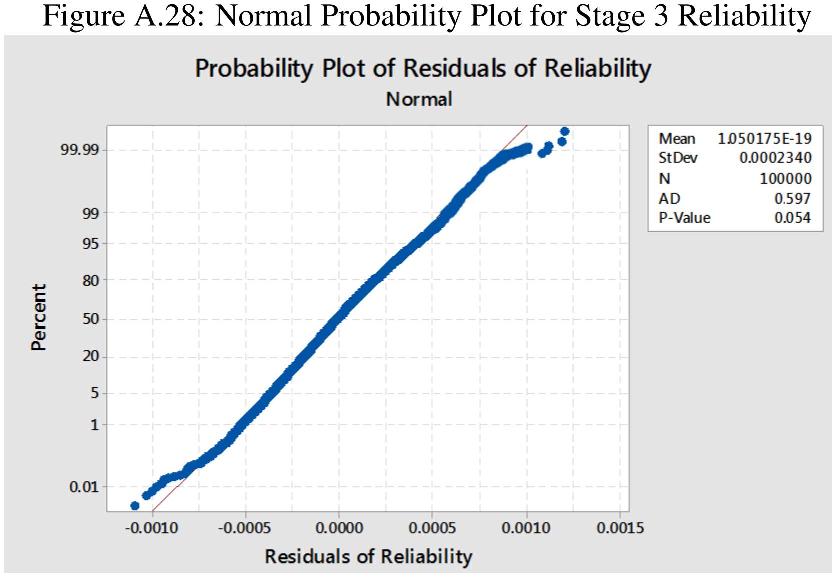

Figure A.29: Results for Stage 3 Test Time

Regression Analysis: Test Time versus Temp1, Temp2, Volt1, Volt2, A1, A2, A3, A4, ...

Method

Categorical predictor coding (1, 0)

Box-Cox transformation	
Rounded λ	-0.188104
Estimated λ	-0.188104
95% CI for λ	(-0.201604, -0.173604)

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	52	3.49243	0.06716	4278.23	0.000
Temp1	1	1.24055	1.24055	79022.68	0.000
Temp2	1	0.00032	0.00018	8.93	0.007
Volt1	1	1.04331	1.04331	66459.09	0.000
Volt2	1	0.00012	0.00012	7.54	0.006
A1	2	0.00511	0.00255	162.72	0.000
A2	2	0.00431	0.00215	137.17	0.000
A3	2	0.00364	0.00021	10.70	0.002
A4	2	0.13092	0.06546	4169.72	0.000
B1	2	0.00522	0.00269	163.83	0.000
B2	2	0.00623	0.00328	294.05	0.000
B3	2	0.00045	0.00014	7.12	0.006
B4	2	0.05637	0.02818	1795.36	0.000
C1	2	0.01611	0.00806	513.20	0.000
C2	2	0.00148	0.00074	47.22	0.000
C3	2	0.02064	0.01032	657.44	0.000
C4	2	0.07935	0.03968	2527.31	0.000
D1	2	0.00222	0.00111	70.73	0.000
D2	2	0.01293	0.00647	411.91	0.000
D3	2	0.00309	0.00155	98.55	0.000
D4	2	0.01303	0.00652	415.07	0.000
G1	2	0.00010	0.00005	3.34	0.036
G2	2	0.00026	0.00018	8.88	0.001
G3	2	0.01048	0.00524	333.65	0.000
G4	2	0.03381	0.01690	1076.69	0.000
H1	2	0.00018	0.00011	5.30	0.004
H2	2	0.03137	0.01568	999.13	0.000
H3	2	0.00014	0.00006	3.12	0.030
H4	2	0.03172	0.01586	1010.17	0.000
Error	99947	1.56903	0.00002		
Total	99999	5.06146			

Regression Equation

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.0039621	69.00%	68.98%	68.97%

(Test Time^-0.188104) = -0.067051 - 0.000163 Temp1 + 0.000001 Temp2 -
0.000124 Volt1 + 0.000001 Volt2 + 0.000572 A1 + 0.00056 A2 + 0.00006 A3
+ 0.002932 A4 + 0.000002 B1 + 0.000781 B2 - 0.000014 B3 + 0.001918 B4
+ 0.00128 C1 - 0.000304 C2 + 0.00228 C3 + 0.002132 C4 + 0.000373 D1
+ 0.000999 D2 + 0.000439 D3 + 0.001007 D4 + 0.000062 G1 + 0.000043 G2
+ 0.000787 G3 + 0.001368 G4 + 0.000025 H1 + 0.001588 H2 + 0.000045 H3
+ 0.001529 H4

Figure A.30: Residuals for Stage 3 Test Time

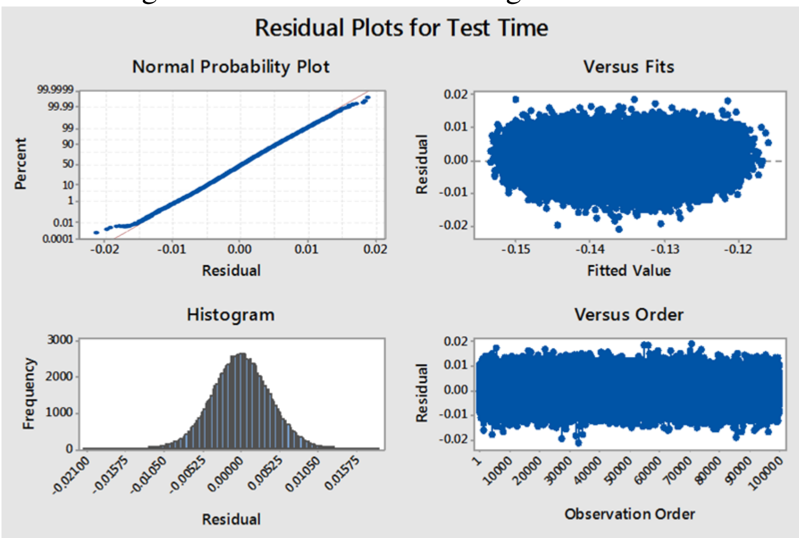

Figure A.31: Normal Probability Plot for Stage 3 Test Time

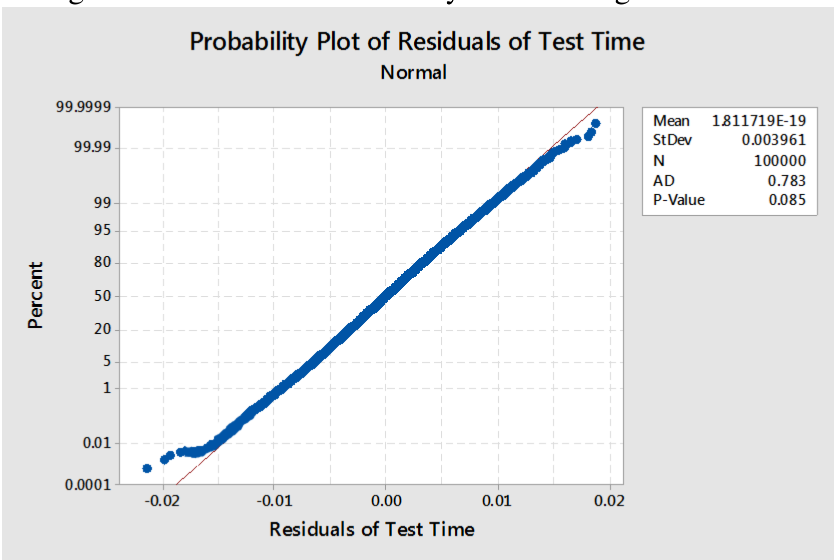

Figure A.32: Results for Stage 3 Test Cost

Regression Analysis: Test Cost versus Temp1, Temp2, Volt1, Volt2, A1, A2, A3, A4, ...

Method

Categorical predictor coding (1, 0)

Box-Cox transformation	
Rounded λ	-0.0486824
Estimated λ	-0.0486824
95% CI for λ	(-0.0651824, -0.0321824)

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	52	2.06884	0.039785	1980.12	0.000
Temp1	1	0.33451	0.334514	16648.70	0.000
Temp2	1	0.21497	0.214971	10699.08	0.000
Volt1	1	0.18448	0.184483	9181.68	0.000
Volt2	1	0.21636	0.216361	10768.29	0.000
A1	2	0.00393	0.001966	97.86	0.000
A2	2	0.00189	0.000946	47.10	0.000
A3	2	0.00001	0.000004	0.19	0.823
A4	2	0.08110	0.040551	2018.21	0.000
B1	2	0.00002	0.000009	0.45	0.639
B2	2	0.05242	0.026209	1304.43	0.000
B3	2	0.00002	0.000008	0.38	0.687
B4	2	0.35684	0.178422	8880.07	0.000
C1	2	0.01288	0.006441	320.59	0.000
C2	2	0.00097	0.000483	24.05	0.000
C3	2	0.01439	0.007195	358.08	0.000
C4	2	0.06048	0.030239	1505.01	0.000
D1	2	0.00348	0.001738	86.49	0.000
D2	2	0.01768	0.008838	439.86	0.000
D3	2	0.00309	0.001547	76.98	0.000
D4	2	0.01668	0.008339	415.04	0.000
G1	2	0.00008	0.000042	2.10	0.122
G2	2	0.00001	0.000005	0.27	0.763
G3	2	0.07610	0.038050	1893.73	0.000
G4	2	0.23008	0.115039	5725.50	0.000
H1	2	0.00003	0.000013	0.64	0.525
H2	2	0.24790	0.123948	6168.89	0.000
H3	2	0.00008	0.000041	2.06	0.127
H4	2	0.23821	0.119107	5927.95	0.000
Error	99947	2.00818	0.000020		
Total	99999	4.07703			

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.0044825	50.74%	50.72%	50.69%

Regression Equation

```
(Test Cost^-0.0486824) = -0.533335 - 0.000084 Temp1 + 0.0000681 Temp2 -
 0.000052 Volt1 + 0.000052 Volt2 + 0.000507 A1 + 0.000349 A2 + 0.000014 A3
+ 0.002255 A4 + 0.000028 B1 + 0.001854 B2 - 0.000034 B3 + 0.004848 B4
+ 0.001169 C1 - 0.000245 C2 + 0.001889 C3 + 0.001858 C4 + 0.000464 D1
+ 0.001137 D2 + 0.000443 D3 + 0.001134 D4 + 0.000046 G1 + 0.000017 G2
+ 0.002128 G3 + 0.003571 G4 - 0.000031 H1 + 0.004467 H2 + 0.000056 H3
+ 0.004366 H4
```

Figure A.33: Residuals for Stage 3 Test Cost

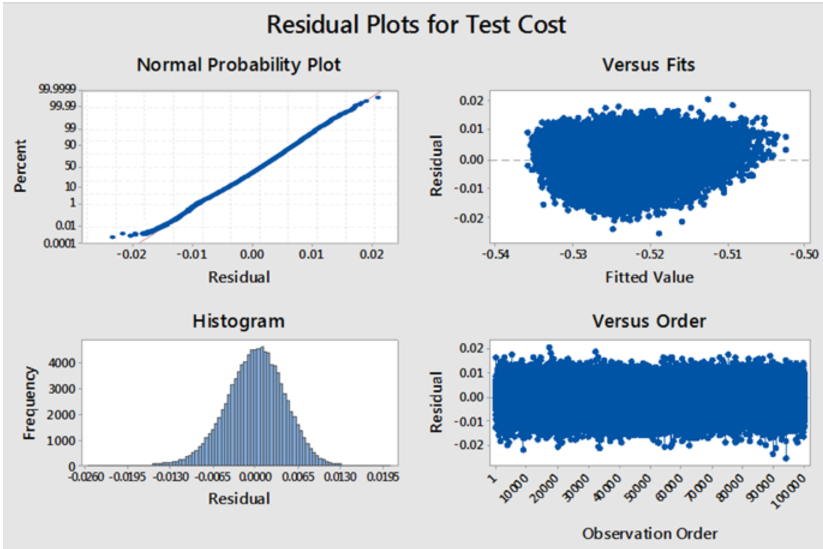

Figure A.34: Normal Probability Plot for Stage 3 Test Cost

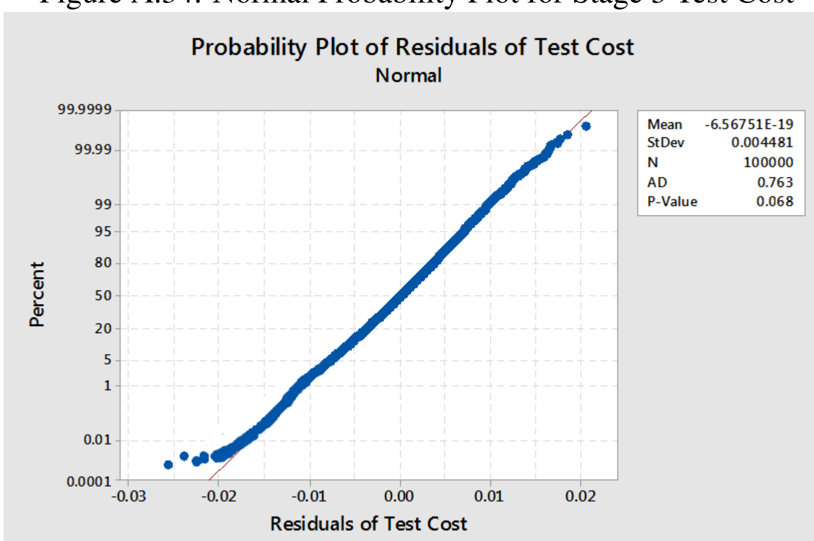

Figure A.35: Regression of Reliability for Entire Response Surface

```
Reliability<-lm(Rel~A1+A2+A3+A4+B1+B2+B3+B4+D1+D2+D3+D4+E1+E2+E3+E4+C11+C12+C13+C14+C21+C22+C23+C24)
summary(Reliability)

Call:
lm(formula = Rel ~ A1 + A2 + A3 + A4 + B1 + B2 + B3 + B4 + D1 +
    D2 + D3 + D4 + E1 + E2 + E3 + E4 + C11 + C12 + C13 + C14 +
    C21 + C22 + C23 + C24)

Residuals:
     Min       1Q   Median       3Q      Max
-0.32233 -0.04360 -0.01337  0.03133  0.54692

Coefficients:
             Estimate Std. Error  t value Pr(>|t|)
(Intercept) 3.459e-01  1.864e-04 1855.658   <2e-16 ***
A1          1.465e-02  7.864e-05  186.301   <2e-16 ***
A2          1.515e-02  7.867e-05  192.529   <2e-16 ***
A3          2.346e-02  7.860e-05  298.529   <2e-16 ***
A4          7.672e-02  1.023e-04  749.812   <2e-16 ***
B1          7.418e-03  7.865e-05   94.311   <2e-16 ***
B2          4.301e-03  7.866e-05   54.679   <2e-16 ***
B3          2.227e-02  7.859e-05  283.422   <2e-16 ***
B4          7.086e-02  1.023e-04  692.598   <2e-16 ***
D1          6.036e-03  7.866e-05   76.738   <2e-16 ***
D2          4.035e-03  7.864e-05   51.305   <2e-16 ***
D3          2.345e-02  7.860e-05  298.313   <2e-16 ***
D4          7.266e-02  1.023e-04  710.068   <2e-16 ***
E1          3.925e-03  7.864e-05   49.906   <2e-16 ***
E2          4.159e-03  7.864e-05   52.884   <2e-16 ***
E3          1.695e-02  7.860e-05  215.633   <2e-16 ***
E4          7.433e-02  1.024e-04  726.212   <2e-16 ***
C11         2.362e-03  7.864e-05   30.039   <2e-16 ***
C12         4.053e-03  7.864e-05   51.539   <2e-16 ***
C13         1.456e-02  7.860e-05  185.279   <2e-16 ***
C14         6.438e-02  1.023e-04  629.042   <2e-16 ***
C21         4.075e-03  7.867e-05   51.805   <2e-16 ***
C22         5.368e-03  7.865e-05   68.253   <2e-16 ***
C23         1.609e-02  7.857e-05  204.785   <2e-16 ***
C24         7.893e-02  1.023e-04  771.413   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Gen
Residual standard error: 0.08296 on 6675219 degrees of freedom
Multiple R-squared:  0.7931,    Adjusted R-squared:  0.7931
F-statistic: 5.331e+05 on 48 and 6675219 DF,  p-value: < 2.2e-16
```

Figure A.36: Regression of Test Time for Entire Response Surface

```
> TTime=lm(Time~Temp1+Temp2+Volt1+Volt2+Rel+0)
> summary(TTime)

Call:
lm(formula = Time ~ Temp1 + Temp2 + Volt1 + Volt2 + Rel + 0)

Residuals:
   Min     1Q Median     3Q    Max
-61955  -5915  -1416   5006  91841

Coefficients:
Estimate Std. Error t value Pr(>|t|)
Temp1 -13719.461      2.174  6310.7   <2e-16 ***
Temp2  13592.207      2.038  6669.4   <2e-16 ***
Volt1  -2648.317      1.164 -2275.2   <2e-16 ***
Volt2   1853.281      1.078  1719.2   <2e-16 ***
Rel    11598.989     27.504   421.7   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 12850 on 6675265 degrees of freedom
Multiple R-squared:  0.9725,    Adjusted R-squared:  0.9725
F-statistic: 7.864e+07 on 3 and 6675265 DF,  p-value: < 2.2e-16
```

Figure A.37: Regression of Total Cost for Entire Response Surface

```
Time2<-Time^2
Rel2<-Rel^2
TotalCost<-lm(Cost~Time+Rel+Time2+Rel2+Time*Rel)
summary(TotalCost)

Call:
lm(formula = Cost ~ Time + Rel + Time2 + Rel2 + Time * Rel)

Residuals:
     Min       1Q   Median       3Q      Max
-2528795  -141833   -40301   128896  1581053

Coefficients:
              Estimate Std. Error  t value Pr(>|t|)
(Intercept) -7.025e+05  2.046e+03 -343.339  < 2e-16 ***
Time         1.917e-01  2.701e-02    7.096 1.28e-12 ***
Rel          1.577e+06  5.951e+03  265.039  < 2e-16 ***
Time2       -1.219e-05  1.328e-07  -91.821  < 2e-16 ***
Rel2         1.667e+06  5.100e+03  326.739  < 2e-16 ***
Time:Rel     5.360e+00  1.969e-02  272.151  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 298400 on 6675262 degrees of freedom
Multiple R-squared:  0.8444,     Adjusted R-squared:  0.8444
F-statistic: 7.245e+06 on 5 and 6675262 DF,  p-value: < 2.2e-16
```

Appendix B

Variable Settings

Table B.1: Maximum Reliability Settings

	Stage 1	Stage 2	Stage 3
x_1	325	350	375
x_2	350	375	400
y_1	150	120	180
y_2	180	150	240
A_1	1	1	1
A_2	1	1	1
A_3	0	1	1
A_4	0	0	1
B_1	1	1	1
B_2	1	1	1
B_3	0	1	1
B_4	0	0	1
C_1	1	1	1
C_2	1	1	1
C_3	0	1	1
C_4	0	0	1
D_1	1	1	1
D_2	1	1	1
D_3	0	1	1
D_4	0	0	1
G_1	1	1	1
G_2	1	1	1
G_3	0	1	1
G_4	0	0	1
H_1	1	1	1
H_2	1	1	1
H_3	0	1	1
H_4	0	0	0

Table B.2: Sequential Approach Settings

	Stage 1	Stage 2	Stage 3
x_1	325	350	300
x_2	350	400	325
y_1	210	180	150
y_2	240	210	240
A_1	0.7	1	1
A_2	0.7	1	1
A_3	0	0	0.7
A_4	0	0	1
B_1	1	1	1
B_2	1	1	1
B_3	0	0	1
B_4	0	0	1
C_1	0.7	0.7	0.7
C_2	1	1	1
C_3	0	0.7	0.7
C_4	0	0	1
D_1	1	1	1
D_2	1	1	1
D_3	0	0.7	0.7
D_4	0	0	1
G_1	1	1	1
G_2	1	1	1
G_3	0	1	1
G_4	0	0	1
H_1	1	1	1
H_2	1	1	1
H_3	0	1	1
H_4	0	0	0

Table B.3: Tight β Settings

	Stage 1	Stage 2	Stage 3
x_1	325	325	300
x_2	375	350	350
y_1	120	210	120
y_2	210	240	240
A_1	1	1	1
A_2	1	1	1
A_3	0	0	0
A_4	0	0	1
B_1	1	1	1
B_2	1	1	1
B_3	0	1	1
B_4	0	0	1
C_1	1	1	1
C_2	0	0	0
C_3	0	0.7	0.7
C_4	0	0	0.7
D_1	0.7	1	1
D_2	1	1	1
D_3	0	0	0.7
D_4	0	0	0.7
G_1	1	1	1
G_2	1	1	1
G_3	0	0.7	1
G_4	0	0	1
H_1	1	1	1
H_2	1	1	1
H_3	0	0	1
H_4	0	0	0

Table B.4: Loose β Settings

	Stage 1	Stage 2	Stage 3
x_1	325	300	325
x_2	350	375	400
y_1	210	150	180
y_2	240	180	240
A_1	0.7	1	1
A_2	0.7	0.7	1
A_3	0	0	0.7
A_4	0	0	1
B_1	1	1	1
B_2	1	1	1
B_3	0	0.7	1
B_4	0	0	1
C_1	0.7	0.7	0.7
C_2	1	1	1
C_3	0	0.7	0.7
C_4	0	0	0
D_1	1	1	1
D_2	1	1	1
D_3	0	0	0.7
D_4	0	0	1
G_1	1	1	1
G_2	1	1	1
G_3	0	1	1
G_4	0	0	1
H_1	1	1	1
H_2	1	1	1
H_3	0	1	1
H_4	0	0	1

Table B.5: Intercomponent Settings

	Stage 1	Stage 2	Stage 3
x_1	325	300	325
x_2	350	375	375
y_1	210	150	120
y_2	240	240	150
A_1	0.9	0.9	0.9
A_2	0.9	1	1
A_3	0	0	0.9
A_4	0	0	0.9
B_1	0.6	0.6	0.6
B_2	1	1	1
B_3	0	0.6	1
B_4	0	0	0.6
C_1	0.9	0.9	0.9
C_2	1	1	1
C_3	0	0.9	1
C_4	0	0	0.9
D_1	1	1	1
D_2	1	1	1
D_3	0	1	1
D_4	0	0	0
G_1	1	1	1
G_2	1	1	1
G_3	0	0	0.9
G_4	0	0	0.9
H_1	1	1	1
H_2	1	1	1
H_3	0	0.6	1
H_4	0	0	1

Table B.6: Intracomponent Settings

	Stage 1	Stage 2	Stage 3
x_1	325	350	300
x_2	350	400	325
y_1	210	180	150
y_2	240	210	240
A_1	0.9	0.9	0.9
A_2	0.6	1	1
A_3	0	0	0.9
A_4	0	0	0.6
B_1	0.9	0.9	0.9
B_2	0.9	0.9	1
B_3	0	0.6	1
B_4	0	0	0.9
C_1	1	1	1
C_2	0.6	0.6	1
C_3	0	0.6	0.6
C_4	0	0	0.9
D_1	0.9	0.9	1
D_2	1	1	1
D_3	0	0	0
D_4	0	0	0
G_1	1	1	1
G_2	1	1	1
G_3	0	0.9	1
G_4	0	0	1
H_1	0.9	1	1
H_2	1	1	1
H_3	0	0	1
H_4	0	0	1