PRIVATE EQUITY: ITS ROLE IN PORTFOLIO OPTIMIZATION

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Abstract

Alternative investments have increasingly been used to complement a traditional portfolio of stocks and bonds. Among them, Private Equity is found to be able to provide diversification benefits and higher expected returns. This study uses the traditional mean-variance portfolio optimization process with several inputs: “equilibrium” returns for the traditional assets as a neutral starting point generated by the Black-Litterman model; and a range of expected returns of private equity fund types. We find that private equity funds in earlier stages are more suitable for investors seeking higher expected returns and with higher levels of risk appetite, while private equity in later stages are more suitable for investors with lower risk appetite, seeking for more modest levels of returns. In both cases, it is notable that the portfolio gains efficiency after the inclusion of private equity. The diversification benefits from low correlations are also observed.

Keywords: Private Equity; Asset Allocation; Portfolio Optimization; Mean-Variance

Dedication

We dedicate this research to our families, especially to Jenny Vo and Juliana Hazime, for all the support and patience during our long hours of work.

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Introduction

In the past decade, private equity funds have increasingly becoming relevant to investment portfolios. Many institutions and private investors have been allocating more to private equity funds, which has fueled its massive growth of both assets-under-management and average percentage allocations.

The motivation for the study comes from several peer-reviewed articles discussing the potential benefits of including private equity in the optimization problem of a traditional portfolio of stocks or stocks and bonds. In particular, most recent papers have demonstrated that when used properly, private equity funds can help investors reap benefits and diversify risks. Nonetheless, due to their nature of low market transparency and illiquidity, private equity funds remain a favourable choice mainly to pension funds, endowments, and investors with longer horizons.

Our study aims to extend the data using a private equity funds database and see if the viability and appropriateness of using private equity have changed in recent years and also with regards to the 2008 financial crisis. We perform similar analyses with peer-reviewed articles but with a few innovations. First, we include a bond index to replicate bonds as an asset class in the investor' portfolio, while previous papers mainly focus on a portfolio of just public equity and private equity. Second, we employ the Black-Litterman model in joint usage with traditional Markowitz mean-variance approach to improve the portfolio optimization process.

Our conclusions show that there are potential risk diversification benefits arising from the low correlations between private equity funds and traditional asset classes such as stocks and bonds. We also find that private equity funds of earlier stages, while offering higher expected returns and some diversification benefits, are more suitable for investors who demand such high return levels and have the risk capacity and tolerance for them. Private equity funds of later stages will benefit investors who have more moderate expectations on returns with lower levels of risk tolerance.

1: Types of Private Equity Funds

Private equity (PE) funds are different from mutual funds and hedge funds in the sense that they invest in illiquid, private companies (Metrick & Yasuda, 2011). PE funds are also obliged to return money to investors within a specified timeframe and thus have finite life, typically 10 years, and a fixed fund size that is pre-determined (2011). In this paper, we examine the three main types of PE, namely Venture Capital (VC), Buyout (BO), and Mezzanine (ME).

Venture Capital is a type of investment usually made in rapidly growing companies that need a lot of capital, and is provided by both individual and organizations/funds. In exchange for the capital provided, investors typically demand a percentage ownership of the company ranging between 25% and 55% (as opposed to 'pure' private equity where ownership is typically 100%). Investors also demand a high return over a smaller horizon (3-7 years) (Venture Capital, 2011). After this period, the equity is either listed on a public stock exchange or sold back to the original company. Compared to traditional sources of financing such as bank loans or supplier credit, VC is more difficult for small business to get due to the inherent riskiness of it (2011).

Buyout refers to a private equity firm/fund (may even include the target firm's upper management) using debt to take a publicly-held firm private. This practice is normally referred to as Leveraged Buyout since the debt used in the acquisition does not need to cover all the capital required (hence leveraged). After the firm is taken private, a restructuring of ownership and capitalization structure occurs, and the investor makes a return when the firm is eventually sold (Leveraged Buyouts, 2016). In a Buyout, existing shareholders 'win' as they normally are paid a premium over market price, and the party taking control gets access to cash flows and control of the once public company. On the other hand, existing creditors 'lose' due to increased risks associated with new debt that finances the Buyout. This also reduces the market value of outstanding bonds and places uncertainty in future debt repayments (2016).

Mezzanine financing refers to introducing subordinate debt into a target firm with existing debt, which comes with a warrant to convert itself into equity in the borrower's firm with a predetermined price per share if the debt is not repaid in time or in full. Lenders can receive equity right away or as a default payment in the aforementioned scenario, and the motivation for lenders arises from their interest in the firm mixed with demands for security of payment and reducing uncertainties. Mezzanine financing still gives the original owners control of the company (provided that the firm continues to do well), but comes with normally a higher interest rate than traditional or senior debts (Mezzanine Financing, 2016).

2: Literature Review

The private equity funds industry, as reported by Metrick & Yasuda (2011), underwent massive growth in the past decade, from $100 billion in 1994 (Fenn & Liang, 1998) to $2.4 trillion assets under management as of June 2015 (Preqin, 2016). For private equity investment value, generally half to two-thirds are accounted for by buyouts (BO), and for the number of investment deals, the majority is classified under venture capital (VC). One of the main drivers behind this growth was institutional investors' allocation to private equity, which rose from 3%/2% on average in 1997 to 12%/6% in 2007 for large foundations/endowments (Metrick & Yasuda, 2011).

With the growth of private equity funds and increasing rate of adoption, the body of theoretical and empirical research has expanded in an attempt to provide reasons for the attractiveness of alternative investments. These focus on the realm of portfolio management, which explores the use of hedge funds, commodities, real estate, private equity, and more. The literature on asset allocation with alternative investments comprises of describing the effects of adding one or more alternative investment asset class(es) to a traditional mixed-asset portfolio of stocks and bonds. Karavas (2000) found that adding managed futures, hedge funds, and traditional alternative investments to a classic stocks and bonds portfolios with a 10-20% minimum allocation provide significant benefits. Jaggi et al. (2011) showed that adding active hedge fund strategies focusing on the same asset class comprised in traditional long-only portfolios improves total & risk-adjusted performance. Jaggi's research is related to French (2005)'s research where it is found that portfolios with hedge funds has efficient frontiers that dominate those with only equities and bonds. Emmrich & McGroarty (2013) extended Jaffe (1989)'s research who found evidence in favour of allocating a small percentage to gold. The authors found that in the 80's and 90's such move is unwise, but from the 2000's and especially since the financial crisis of 2007, including gold in a traditional portfolio provides superior risk-adjusted returns. In certain markets such as China, Hammoudeh et al. found that (2014) the low and positive correlations between the stock markets and the commodity markets have positive implications for the use of this alternative asset class in portfolio strategies to diversify and reduce risks.

However, Zhu et al. (2004) finds that private equity should not be considered as an alternative to public equity, but rather as a complement to it, and recommends to invest in private equity through a fund of funds to reduce risks. In addition, Huang & Zhong (2013) shows that REITSs and treasury inflation-protected securities (TIPS) (commonly classified under traditional investments) and commodities provide positive diversification benefits to portfolios. Commodities, while classified as an alternative investment asset class, have seen inconclusive evidence for their benefits. For instance, Gorton & Rouwenhorst (2004) conducted a study using an equally weighted index of monthly returns of commodity futures from July 1959 to December 2004 and found that it offered diversification benefits to equity and bond portfolios. Conover et al. (2010) used a sample period of 36 years to prove that with a minimum commodity allocation of more than 5% in the investment portfolio, investors can reap substantial benefits in terms of return and diversification of risks. On the other hand, Erb & Harvey (2006) found that portfolios of commodities futures has an average compound excess return has historically been close to zero. Erb et al. also found that these portfolios exhibit some excess return if they (1) periodically rebalance or (2) overweight commodity futures with high returns, and the authors warned against extrapolating past performance into the future. Hedi Arouri & Nguyen (2010) found positive effects of including oil asset into a diversified portfolio of stocks from the Stoxx 600. And Daskalaki & Skiadopoulos (2011), through a process of portfolio optimization, found that adding commodities to a traditional portfolio is only beneficial to non-mean-variance investors, and such benefits do not hold out-of-sample (except for the 2005-2008 commodity boom period).

Within the scope of this paper, we begin by exploring the notion that successful private equity investments can provide good returns and enhance portfolio diversification. This notion is corroborated by Lamm & Ghaleb-Harter (2001) and by Milner & Vos (2003), who proved that private equity has low correlations with listed equity and (2) adding private equity investments enhance the performance of portfolios largely weighted in listed equities. Some original thoughts that gave rise to the initial growth of PE include Lerner (2000)'s assertion that large institutional investors such as pension funds and university endowments want illiquid investments with longer horizons such as private equity in their portfolios. But at the same time, it is worthwhile to note that because of these characteristics of low market transparency and illiquidity, there has been reluctance in considering PE as a viable investment and/or asset class (Schmidt, 2006).

In support of the benefits of utilizing PE in a portfolio, Schmidt (2006) used data from 642 U.S. PE investments from Venture Capital, Buyout, and Mezzanine (amongst other types), showed that characteristic portfolios and mixed-PE types portfolios during the late 1990s outperformed their stocks-only counterparts. Schmidt also showed that mixed-asset portfolios with PE weights between 3% to 65% (unconstrained, and depending on the type of PE investment) offer higher performance ratios and lower standard deviations of returns. Garay (2009) found that PE gives investors the opportunity to achieve high long-term returns than traditional vehicles based on two reasons: (1) the marketplace of private companies are vast and with it comes more opportunities and (2) investors can actively influence the target firms to drive excess return (Szado, 2008 as quoted in Garay, 2009). From a portfolio perspective, Garay also noted that PE provides a certain degree of desirable diversification (2009). Aigner et al. (2012), using the study's first-order autoregressive Markov-switching model (ARMS) to capture the characteristics of stocks, bonds and listed private equity (LPE). He found that investors, even very risk-averse ones, can benefit from adding LPE to a portfolio of stocks and bonds due to the high diversification effects. The authors favoured LPE due to their enhanced risk-return profile and higher liquidity compared to their underlying direct PE investments. They also found that the ARMS model outperformed the standard geometric Brownian motion model portfolio for the same three asset classes, even during the financial crisis.

However, Bond et al. (2007) discovered that PE (and hedge funds) offer no better diversification of risk and offering better returns than real estate (as an alternative investment asset class) if each was being considered for inclusion in a traditional portfolio. Bond et al. found this by using portfolios made from a sample of 8 core assets and 4 alternative assets divided into the bull period from August 1990 – December 1999 and the bear period from January 2000 – July 2006 in the United Kingdom market. Fischer & Lind-Braucher (2010) explored how traditional and alternative investments including PE performed during the dot-com bubble of 2000 and the financial crisis of 2008 using a sample from 1999 to 2009 and constrained PE to 40% of mixed-asset portfolio. The authors found that, using different return estimators and risk measures, globally-diversified portfolios with mixed assets provide better cushions, but the optimizers' results returned very little (3%-4% under one out of four models) to none allocation to PE.

Another important point to consider for review is the nature of alternative asset classes' investment and portfolio returns. Due to the nature of PE and/or hedge funds, such data is sensitive and usually not made publicly or proprietarily available by fund managers (Milner & Vos, 2003; Fenn & Liang, 1998; Wright & Robbie, 1998). Encouragingly, the interest in obtaining and analyzing these data has led to more standardized performance measurements as well as databases seeking to gather them (Milner et al., 2003). However, there exists some potential biases in the process of data gathering, such as delisting bias, survivorship bias, and backfill bias. According to Jorion & Schwarz (2013), "survivorship bias" occurs when funds are delisted from the database of current, "live" funds, mainly due to poor performance. The author also explained "backfill bias" as adding funds in their incubation periods with good performance to the databases (2013). For instance, the data provider may add a new fund in 2016 then "backfill" the data for earlier periods. This creates a bias due to one of the main inclusion criteria is the correlation with past performance (2013). Schneeweis et al. (2011) also discussed arising issues from these biases such as the lack of a widely accepted set of definitions of returns or firm characteristics. Scheeweis et al. voiced common potential solutions, however, such as the removal of the first two years of reported return, the exclusion of small assets-under-management funds, or the merging of multiple provider databases (2011).

In approaching this project, we also considered the approach of Lamm & Ghaleb-Harter (2001)'s paper on hedge funds and US taxable portfolios. In this paper, the authors used standard mean-variance analysis for portfolios that include hedge funds as an asset class but with an important modification. The authors assumed pre-tax returns from engineered sources (from DB Asset Management) and used hedge funds as a variable in the portfolio optimization process. The engineered vectors of returns and subsequently the portfolio allocations came from the use of the Black-Litterman model (Lamm & Ghaleb-Harter, 2001). This approach is similar to this project's and will be discussed in greater length below.

3: Data

The data for this research was obtained from two sources: the VentureXpert database and Bloomberg L.P. VentureXpert is a database that provides comprehensive, real-time data on thousands of private equity funds and is operated by Thomson Venture Economics, a private equity and venture capital research division of Thomson Financial Services. Bloomberg data was obtained from a Bloomberg Terminal, operated by Bloomberg Professional Services, which provides a platform for real-time data, news and analytics on a wide variety of markets and financial products.

VentureXpert uses cashflow schedules and financial reports to calculate internal rate of return (IRR). These returns are time-weighted, and are calculated in three ways: average rate of return, capital-weighted rate of return, and pooled rate of return. The first uses arithmetic means from sample IRRs while the second weigh the returns by the size of cashflows (but ignores scale and timing). In this paper, we utilize the third method, pooled IRR, in our returns reporting and calculation due to its ability to capture both the scale of the investment and the timing of the cashflows from various funds of the same category. “Pooled” means treating the funds as a single “fund” by aggregating their cashflows and use them to calculate a rate of return. This method follows closely how investors use an investment-weighted methodology to measure return, rather than aggregating each fund’s return which may skew results (Private Equity Glossary, n.d.).

The VentureXpert database provides quarterly fund performance data that we used in this paper. Since PE firms agree to share performance data only if return data is not attributed to any fund, the database provide these data on an aggregated basis for fund categories. These fund categories include Venture Capital Early Stage (V1), Venture Capital Late/Expansion (V2), Venture Capital Multi-Stage (V3), Venture Capital All (V4), Buyout, Mezzanine. We obtained from VentureXpert the pooled IRR on a quarterly basis from 2001 to 2015.

One limitation for using performance data from VentureXpert is the risk of backfill bias and survivorship bias, as the number of funds in each category has typically increased over time due to the database's expansion. Therefore, returns are re-calculated to account for the change in number of funds.

We also obtained quarterly price and return data from Bloomberg for the following major indices: MSCI USA, MSCI ex USA, and Bloomberg Barclays Global Aggregate Total Return Index (Unhedged USD). The bond index (hereby referred to as Bonds, one of the assets being studied), is a preferred measure of global investment grade debt from 24 local currency markets. It includes treasury, government-related, corporate and securitized fixed-rate bonds from both developed and emerging markets issuers. Data obtained was quarterly prices from 2001 to 2015. All data were obtained in USD currency as calculated by Bloomberg to avoid any local currency conversion effects and to make a more robust comparison.

We obtained the 2016 bonds' market capitalizations and weights for the relevant regions from the Bank of International Settlements (Bank of International Settlements, n.d.) for use with our Black-Litterman model (to be discussed below). For the equities' market capitalizations and weights, we obtained the data from Bloomberg as of the end of quarter four, 2015.

4: Methodology

To determine the role of PE in a traditional portfolio of stocks and bonds, we used the data explained above to first calculate historical returns and volatility of the different PE funds and indices. We then calculated the correlations between the PE funds and the indices. Finally, we used Black-Litterman model to engineer the returns of our traditional assets and create a reasonable starting point for portfolio optimizations to achieve efficient asset allocations of PE.

4.1 Returns, Volatility, and Correlation

For the quarterly prices of the major indices, their returns were calculated and subsequently converted to compounded annual returns. For the quarterly pooled IRR of the various PE funds, their returns were also converted to compounded annual returns to form the basis for comparison and optimization.

Our initial steps involve calculating the correlations between individual PE funds' returns and that of each of the major indices as explained above. If the returns of these PE funds possess low levels of correlation to the equity and bond indices, there exists a case for potential benefits from portfolio diversification strategies with the inclusion of PE funds. We started the process by calculating covariances to provide a view of co-movement and dependency between the funds and the indices. The covariance between asset "i" and "j" is calculated as follows:

$$COV_{ij} = \frac{\sum(i - \bar{\imath})(j - \bar{\jmath})}{\mathrm{n}}$$

Where $\bar{\imath}$ and $\bar{\jmath}$ are average returns of asset i and j respectively, over n observations.

Positive and negative covariances explain the co-movement or opposite-movement of assets in a portfolio. This measure, however, does not explain the degree of dependency i.e. how "strong" is the relationship between such assets. We calculated the Pearson correlation coefficients to address this limitation, which is calculated as follows:

$$\rho_{ij} = \frac{COV_{ij}}{\sigma_i \sigma_j}$$

Where σ_i and σ_j are the standard deviations of asset i and j respectively.

4.2 The Black-Litterman model

In this paper, we are using the Black-Litterman model for portfolio optimization. With regards to returns and expected returns calculations, using this model, according to Lee (2000), can help mitigate the problem of estimation error-maximization by "spreading the errors throughout the vector of expected returns" (Idzorek, 2005). Since the one of most critical inputs in Markowitz' mean-variance optimization is the vector of expected returns, it has to be reasonable as a starting point for optimization, and is not subjected to causing wide variations in the outputs. Best and Grauer (1991), however, found that a small change in one of the assets' returns can cause huge allocation changes in the portfolio. Therefore, Black and Litterman (1992), He and Litterman (1999), and Litterman (2003) explored different methods and forecasts to counteract this model. The Black-Litterman model, in its final iteration, uses "equilibrium" returns as a neutral starting point. Because the model uses market capitalization weights, this method is only used to estimate the returns of the traditional assets. They are derived using reserve optimization using the formula below:

$$\Pi = \lambda \sum w_{mkt}$$

Where Π is the implied excess "equilibrium" return vector (Nx1);

λ is a proportionality constant based on the formulas in Black;

$\sum$ is the covariance matrix of excess returns (NxN);

w_{mkt} is the market capitalization weights of all assets (Nx1).

Using the given formula, we engineered the implied expected returns for the 3 assets in the paper: the MSCI USA, MSCI ex USA, and Bonds. Given the market capitalizations obtained, we formed a Nx1 vector for w_{mkt}. We picked λ to be 2 as the commonly used risk aversion parameter. The results are outlined in Table 1, first three rows. These engineered returns were subsequently used in portfolio optimization processes.

4.3 Portfolio Optimization

To set the inputs for the optimization process, we individually add the different types of private equity funds (V1 to V4, Buyout, and Mezzanine) to the equilibrium portfolio of 2 equity assets (MSCI USA and MSCI ex USA) and 1 bond asset (Bonds). The equilibrium portfolio is expected to clear the market, and 2 equity assets were chosen because the US equity market is much larger than the others to be included in just one equity asset. In light of the Black-Litterman model, we express our views of possible outcomes by using 3 levels of expected returns: underperforming, comparable to, and outperforming equity. These returns are set using historical mean return plus/minus two standard deviations, so we can say with 95% confidence that such returns fall within these levels. The intuition is to produce optimal portfolios that behave reasonably and do not require investors to express their views for all remaining assets.

We used Markowitz' mean-variance optimization approach to generate a frontier of optimal portfolios for the assumed range of returns. The optimal portfolios represent the highest expected return for the given amount of risk (McClure, 2010). We use a set of various portfolios to create an 'Efficient Frontier' to represent the best possible combination of expected returns for every level of possible risk.

The return of a portfolio is calculated as follows:

$$E[r_p] = \sum_i w_i E(r_i)$$

Where:

$$\sum_i w_i = 1$$

The variance of the Return on a portfolio is:

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij}$$

Following Markowitz (1952), and Best and Grauer (1991), we formulate the optimization process as a parametric quadratic programming problem. We subject the optimization only to a budget general constraint. The procedure is given by:

$$\max \{\lambda \mu' w - \frac{1}{2} w' \sum w \mid Aw \leq b\}$$

Where λ is the investor's risk tolerance parameter;

μ is a vector of unities plus expected rates of return;

$\sum$ is the covariance matrix of excess returns (NxN);

A is a matrix of constraints (MxN);

b is a M-vector.

We then used the optimization results to plot efficient frontiers.

5: Results and Discussions

The results of standard deviations and returns are discussed, along with our observations of correlations and the efficient frontiers obtained from portfolio optimization processes. We assume a portfolio allocation of 5% or more represent a sizable position and will be considered. Any allocation less than 5% will be deemed insignificant.

5.1 Historical Returns and Standard Deviations

Table 1: Historical Returns and Standard Deviations

	Return	Risk
MSCI USA	0.093	0.182
MSCI ex USA	0.055	0.202
Bonds	0.059	0.066
V1	0.280	0.740
V2	0.167	0.267
V3	0.150	0.298
V4	0.225	0.555
Buyout	0.142	0.131
Mezzanine	0.106	0.082

In the table above, V3 stands for Venture-Capital Multi-Stage, which is a representation of the whole asset class. V4 represents V1 and V2, which is an approximate ‘average’ of venture capital in its early and late stages. The return/risk profiles, as detailed in Table 1 above, are consistent with the fund types’ definitions.

5.2 Expected Returns and Standard Deviations

Table 2: Expected Returns and Standard Deviations

	Return	Risk
MSCI USA	0.119*	0.182
MSCI ex USA	0.121*	0.202
Bonds	0.012*	0.066
V1	0.13 to 0.43**	0.740
V2	0.11 to 0.22**	0.267
V3	0.09 to 0.21**	0.298
V4	0.11 to 0.34**	0.555
Buyout	0.12 to 0.17**	0.131
Mezzanine	0.09 to 0.12**	0.082

* Engineered using Black-Litterman model
** Expressed views

Risks (standard deviations) were calculated based on historical annual returns. As noted, the engineered implied returns and standard deviations for the equity and bond assets behave as expected. The MSCI ex USA, which includes emerging and frontier markets, gives a higher return and higher risk. Bonds provide much lower return with much lower risks. The private equity funds types have returns that vary from the low end of 8% to the high end of 14%, with V4 having the highest risk (56%) and Mezzanine with the lowest risk (8.2%).

5.3 Correlations

Table 3: Correlation Matrix

	MSCI USA	MSCI ex USA	Bonds	V1	V2	V3	V4	Buyout	Mezzanine
MSCI USA	1.000								
MSCI ex USA	0.720	1.000							
Bonds	0.165	0.128	1.000						
V1	0.330	0.310	-0.304	1.000					
V2	0.565	0.534	-0.192	0.897	1.000				
V3	0.478	0.431	-0.344	0.945	0.944	1.000			
V4	0.355	0.340	-0.323	0.997	0.917	0.963	1.000		
Buyout	0.458	0.697	0.016	0.165	0.368	0.367	0.207	1.000	
Mezzanine	0.64	0.61	0.00	0.55	0.72	0.67	0.58	0.61	1.00

The private equity funds types from V1 to V4 are highly correlated since they are of different stages, which we expect. However, their correlations with equity, both the MSCI USA and MSCI ex USA, are not as low as we initially expected. Evidently, the highest correlation is 0.697 of V2 and the lowest of 0.31 of V1. We suspect that the sub-prime mortgage crisis of 2008 has caused the correlations to be higher ever since, but more research should be done to warrant such claim.

We expect the correlations between V3 with V1 and V2 to be lower, and that between V3 and Buyout and Mezzanine to be higher, but this is not the case. V3 stands for all private equity fund types, so one of the reasons that we suspect to be behind such correlations is due to price-weighted returns as calculated by VentureXpert. We believe the cause cannot be market capitalization-weighted due to the nature of Buyout and Mezzanine. Unfortunately, this is something that we cannot verify as no information is given by the database.

Nonetheless, with correlations of 0.3 to 0.5 of some of these funds, there are benefits of diversification by including them in a traditional portfolio. The correlations with bonds, as expected, are negative for most of the PE funds, further adding to the diversification benefits. It's worth noting that Buyout is a very viable choice for portfolio inclusion due to its low correlations with the other assets and with the other fund types as well. This is expected due to the nature of Buyout as explained above.

5.4 Efficient Frontiers

The efficient frontiers for V1 have negligible weights (<5%) to this private equity fund type at 13.2% expected return. This is due to the extreme volatility of 74% with returns lower than both of the equities. Theoretically, V1 is Venture Capital – Early Stage so the risk/return profile matches the fund's description. For instance, annual returns vary from +365% to -42%. Due to this, investors should demand a much higher rate of return before allocation of V1 in a traditional portfolio is warranted. Evidently, at 28% expected return, we observe higher allocations to V1 at 8% desired portfolio return and going up from there. At 42.8% expected return, V1 is strongly favoured and allocations are evident throughout the range of desired returns.

V2 is inefficient at 11.4% expected return i.e. it does not lie on the efficient frontier. However, the optimizer allocates significant weights (varying from 10% to 5%) to V2 at lower levels of desired portfolio returns, up to 10% (where V2 drops to 5%). We believe that this is due to V2's low correlation that the allocated weights are meant to be for diversification purposes. At 16.7% and 22% expected returns, similar to V1, V2 is more favoured and allocation is a viable option.

We can observe some similarities for V4 since it is a 'combination' of V1 and V2. The similarities more closely resemble that of V1 than V2 due to the higher correlation between them.

Depending on the risk tolerance and capacity, V4 might be a more reasonable middle-ground and more inclusive private equity fund type to be included as an asset class.

As explained in the Correlations section above, we expect different results for V3 from the plotted frontiers. At 9% expected return, the efficient frontiers do not trace through V3 due to its high standard deviation yet low return. Compared to V1, V2, and V4, V3 appear to be more favoured at its mean return level and +2 standard deviations upper bound (21%) expected return, with a minimum position of 10% in both scenarios. If the correlations between V3 and the traditional asset classes in the portfolio behave as expected (lower, due to the inclusion of Buyout and Mezzanine), we may see higher allocations to V3 at all three expected returns' views.

In the case of Buyout, we observe allocations to this private equity fund type at all three levels of expected returns. This signifies the feasibility of having Buyout as an additional asset class in a traditional portfolio of stocks & bonds. Due to the characteristics of Buyout (its risk/return profile and correlations with the other asset classes), the optimizer allocates a minimum of 20% at all three levels expected returns. There is also stronger evidence of higher allocations to Buyout than to equity the higher Buyout's expected return is. Adding Buyout fund to a traditional portfolio fits a wide range of investors' views on expected returns, bolstering its benefits.

Lastly, for Mezzanine, more significant benefits are observed, compared to Buyout. We have allocations to Mezzanine at all three levels of expected returns. As Mezzanine is close to a firm going for an Initial Public Offering, there is less risk and higher return for this type of private equity. The optimizer allocates a minimum of 39% in this type of private equity at all three levels of expected returns, which is a very significant position. Similar to Buyout, but to a greater extent, adding Mezzanine fund to a traditional portfolio can shift the efficient frontiers leftwards, implying lower levels of standard deviations for the portfolio.

6: Limitations

One of the main limitations in the analysis belongs to how much the Black-Litterman model used for engineered returns can deliver. The model does not give the best possible portfolios but only the best portfolios given investors' views on different asset classes. The model also assumes that views are independent of one another, which means that if the investor has views on separate assets, there are no mathematical solution for optimization. This paper did not explore that, but potential research topics that investigate the optimal allocations of a traditional stocks & bonds portfolio with more than one alternative asset class (stocks, bonds, venture capital, and commodities, for instance) may experience this issue.

The private equity database used, VentureXpert, suffers from backfill bias, which affects the quality and quantity of inputs used in this paper. The database also lacks descriptions for the different types of private equity and lacks information on selection criteria for these funds. This limitation was also experienced by Milner & Vos (2003) who used an earlier version of VentureXpert. These limitations may affect investors' ability to express valid views on the risk and performance of private equity fund types, which are instrumental to the Black-Litterman approach used in this paper.

Lastly, the real-life applicability of the results in our research may be limited. The private equity funds listed in our data may not be investable in real life and if they are, there might be limited information about them. As of 2014, there is a very limited number of investable private equity funds' indices and ETFs that are traded publicly. Our research is also based on data of pooled funds that suggest investors investing in broad funds and/or types instead of individual private equity funds that may not offer the aforementioned diversification benefits.

7: Conclusion

The addition of private equity funds to a traditional portfolio of stocks & bonds has the potential to improve the risk/return profile of the overall portfolio. The suitability of this type of investment to specific investors on the other hand, depends mainly on two factors: the investor's ability to express accurate views and his risk appetite. Private equity funds of earlier stages which are riskier appear to be more suitable for investors who demand or expect higher levels of returns. Private equity funds of later stages will benefit investors who have lower risk tolerance and more moderate expectations on returns. For all different types of private equity funds, however, correlations between such funds and traditional asset classes can be considered low, which supports the literature on diversification benefits of this asset class.

Through using the Black-Litterman model and Markowitz's mean-variance approach to optimization, the paper has shown that, in general, for private equity funds of later stages in the financing cycle and for investors with optimistic views on the performance of private equity as an asset class, private equity can improve the risk-return profile of their portfolios, compared to having just equities and bonds.

Appendices

Appendix A: Historical Returns and Standard Deviations

Year	MSCI USA	MSCI ex USA	Bonds	V1	V2	V3	V4	Buyout	Mezzanine
1991	27.17%	11.55%	16.04%	21.46%	28.17%	18.60%	20.76%	4.89%	0.42%
1992	4.16%	-13.04%	5.80%	13.04%	16.38%	13.18%	13.40%	6.17%	10.28%
1993	7.02%	32.35%	11.08%	18.19%	34.26%	13.43%	17.20%	14.69%	19.63%
1994	-0.86%	4.83%	0.23%	14.75%	12.33%	19.59%	16.97%	22.11%	6.99%
1995	34.74%	7.81%	19.66%	58.65%	33.83%	26.24%	40.07%	14.81%	19.08%
1996	21.36%	4.66%	4.91%	48.57%	21.60%	27.80%	36.46%	23.13%	21.90%
1997	31.73%	0.18%	3.79%	38.84%	25.26%	25.23%	31.71%	9.95%	13.18%
1998	28.79%	12.36%	13.71%	42.86%	17.72%	11.33%	29.12%	12.93%	18.55%
1999	20.86%	28.80%	-5.17%	365.85%	116.42%	134.75%	273.01%	18.31%	24.52%
2000	-13.56%	-16.34%	3.18%	27.54%	10.63%	5.79%	21.60%	2.55%	7.12%
2001	-13.23%	-20.98%	1.57%	-41.97%	-31.49%	-34.80%	-39.31%	-7.39%	-11.29%
2002	-23.97%	-16.53%	16.53%	-34.98%	-23.97%	-30.49%	-32.73%	4.65%	-1.01%
2003	26.78%	37.50%	12.51%	-7.99%	12.44%	-1.35%	-4.13%	15.72%	15.09%
2004	8.80%	18.26%	9.27%	11.86%	1.52%	22.79%	13.58%	32.91%	8.94%
2005	3.80%	13.89%	-4.49%	2.88%	8.83%	13.44%	6.38%	26.19%	13.31%
2006	13.18%	23.84%	6.64%	14.77%	24.57%	19.77%	17.24%	40.98%	16.36%
2007	4.09%	14.07%	9.48%	14.01%	21.74%	14.38%	14.93%	38.71%	13.65%
2008	-38.58%	-47.07%	4.79%	-16.83%	-17.08%	-16.66%	-16.78%	-19.33%	-3.14%
2009	24.20%	37.43%	6.93%	1.98%	11.20%	2.04%	2.93%	16.23%	6.80%
2010	13.18%	8.42%	5.54%	13.53%	27.32%	11.58%	14.29%	21.01%	9.15%
2011	-0.11%	-16.13%	5.64%	13.77%	12.13%	10.39%	12.43%	12.07%	13.09%
2012	13.52%	13.35%	4.32%	8.70%	2.29%	5.97%	7.10%	13.98%	12.82%
2013	29.85%	12.26%	-2.60%	27.30%	33.12%	27.43%	27.91%	15.80%	12.97%
2014	11.10%	-6.29%	0.59%	27.76%	9.02%	22.14%	24.10%	7.19%	10.42%
2015	-0.76%	-7.98%	-3.15%	15.38%	9.09%	12.14%	13.79%	5.95%	5.21%
μ	9.33%	5.49%	5.87%	28.00%	16.69%	14.99%	22.48%	14.17%	10.56%
σ	18.19%	20.15%	6.59%	74.00%	26.70%	29.77%	55.52%	13.13%	8.19%

Appendix B: Portfolio Allocations

Table 4: Portfolio Allocations for V1

V1: Early Stage with expected return of 13.2%					
MSCI USA	**MSCI ex USA**	**Bonds**	**V1**	**Return**	**Risk**
0.00	0.02	0.95	0.03	0.02	0.06
0.08	0.06	0.83	0.02	0.03	0.06
0.17	0.10	0.71	0.02	0.04	0.07
0.25	0.14	0.59	0.01	0.06	0.09
0.33	0.19	0.47	0.00	0.07	0.10
0.42	0.23	0.36	0.00	0.08	0.12
0.50	0.27	0.24	0.00	0.09	0.14
0.57	0.31	0.12	0.00	0.11	0.16
0.65	0.35	0.00	0.00	0.12	0.18
0.00	0.00	0.00	1.00	0.13	0.74
V1: Early Stage with expected return of 28%					
MSCI USA	**MSCI ex USA**	**Bonds**	**V1**	**Return**	**Risk**
0.00	0.02	0.95	0.03	0.02	0.06
0.16	0.10	0.70	0.04	0.05	0.08
0.33	0.18	0.45	0.05	0.08	0.11
0.49	0.26	0.19	0.06	0.11	0.15
0.57	0.32	0.00	0.11	0.14	0.20
0.41	0.30	0.00	0.29	0.17	0.28
0.25	0.28	0.00	0.46	0.19	0.39
0.09	0.27	0.00	0.64	0.22	0.50
0.00	0.18	0.00	0.82	0.25	0.62
0.00	0.00	0.00	1.00	0.28	0.74
V1: Early Stage with expected return of 42.8%					
MSCI USA	**MSCI ex USA**	**Bonds**	**V1**	**Return**	**Risk**
0.00	0.02	0.95	0.03	0.03	0.06
0.17	0.10	0.66	0.08	0.07	0.09
0.33	0.18	0.37	0.12	0.12	0.15
0.50	0.26	0.08	0.16	0.16	0.21
0.46	0.26	0.00	0.28	0.21	0.28
0.36	0.22	0.00	0.42	0.25	0.36
0.25	0.19	0.00	0.57	0.29	0.45
0.14	0.15	0.00	0.71	0.34	0.55
0.03	0.12	0.00	0.86	0.38	0.64
0.00	0.00	0.00	1.00	0.43	0.74

Table 5: Portfolio Allocations for V2

V2: Late/Expansion Stage with expected return of 11.4%					
MSCI USA	**MSCI ex USA**	**Bonds**	**V2**	**Return**	**Risk**
0.00	0.00	0.91	0.09	0.02	0.06
0.06	0.04	0.80	0.10	0.03	0.06
0.13	0.08	0.70	0.09	0.04	0.07
0.20	0.11	0.60	0.08	0.05	0.08
0.28	0.15	0.50	0.07	0.07	0.10
0.35	0.19	0.39	0.07	0.08	0.11
0.42	0.23	0.29	0.06	0.09	0.13
0.50	0.27	0.19	0.05	0.10	0.14
0.57	0.30	0.09	0.04	0.11	0.16
0.00	1.00	0.00	0.00	0.12	0.20
V2: Late/Expansion Stage with expected return of 16.7%					
MSCI USA	**MSCI ex USA**	**Bonds**	**V2**	**Return**	**Risk**
0.00	0.00	0.91	0.09	0.03	0.06
0.04	0.03	0.78	0.15	0.04	0.07
0.11	0.06	0.65	0.18	0.06	0.08
0.17	0.09	0.52	0.22	0.07	0.10
0.24	0.12	0.39	0.25	0.09	0.12
0.30	0.15	0.26	0.28	0.10	0.14
0.37	0.18	0.13	0.32	0.12	0.16
0.43	0.21	0.00	0.35	0.14	0.19
0.20	0.12	0.00	0.67	0.15	0.22
0.00	0.00	0.00	1.00	0.17	0.27
V2: Late/Expansion Stage with expected return of 22%					
MSCI USA	**MSCI ex USA**	**Bonds**	**V2**	**Return**	**Risk**
0.00	0.00	0.91	0.09	0.03	0.06
0.00	0.01	0.80	0.19	0.05	0.07
0.04	0.02	0.68	0.27	0.07	0.08
0.07	0.03	0.55	0.34	0.09	0.10
0.11	0.04	0.43	0.42	0.12	0.13
0.14	0.06	0.31	0.50	0.14	0.15
0.17	0.07	0.18	0.57	0.16	0.18
0.21	0.08	0.06	0.65	0.18	0.21
0.15	0.05	0.00	0.79	0.20	0.24
0.00	0.00	0.00	1.00	0.22	0.27

Table 6: Portfolio Allocations for V3

V3: Multi - Stage with expected return of 9%					
MSCI USA	**MSCI ex USA**	**Bonds**	**V3**	**Return**	**Risk**
0.00	0.00	0.90	0.10	0.02	0.06
0.06	0.05	0.79	0.10	0.03	0.06
0.14	0.09	0.69	0.08	0.04	0.07
0.21	0.13	0.59	0.06	0.05	0.08
0.29	0.17	0.50	0.05	0.07	0.10
0.37	0.21	0.40	0.03	0.08	0.11
0.45	0.24	0.30	0.01	0.09	0.13
0.52	0.28	0.20	0.00	0.10	0.14
0.59	0.32	0.09	0.00	0.11	0.16
0.00	1.00	0.00	0.00	0.12	0.20
V3: Multi - Stage with expected return of 15%					
MSCI USA	**MSCI ex USA**	**Bonds**	**V3**	**Return**	**Risk**
0.00	0.00	0.90	0.10	0.03	0.06
0.04	0.04	0.78	0.14	0.04	0.06
0.12	0.08	0.65	0.15	0.05	0.08
0.20	0.12	0.53	0.16	0.07	0.09
0.27	0.16	0.40	0.17	0.08	0.11
0.35	0.20	0.28	0.18	0.10	0.13
0.42	0.24	0.15	0.19	0.11	0.15
0.50	0.28	0.03	0.20	0.12	0.17
0.20	0.25	0.00	0.55	0.14	0.21
0.00	0.00	0.00	1.00	0.15	0.30
V3: Multi - Stage with expected return of 20.9%					
MSCI USA	**MSCI ex USA**	**Bonds**	**V3**	**Return**	**Risk**
0.00	0.00	0.90	0.10	0.03	0.06
0.01	0.03	0.78	0.18	0.05	0.06
0.07	0.06	0.64	0.23	0.07	0.08
0.13	0.09	0.49	0.28	0.09	0.11
0.19	0.12	0.35	0.33	0.11	0.13
0.25	0.16	0.21	0.38	0.13	0.16
0.31	0.19	0.07	0.43	0.15	0.19
0.26	0.18	0.00	0.56	0.17	0.22
0.10	0.12	0.00	0.78	0.19	0.25
0.00	0.00	0.00	1.00	0.21	0.30

Table 7: Portfolio Allocations for V4

V4: Venture All with expected return of 11.4%					
MSCI USA	**MSCI ex USA**	**Bonds**	**V4**	**Return**	**Risk**
0.00	0.00	0.95	0.05	0.02	0.06
0.07	0.05	0.84	0.04	0.03	0.06
0.15	0.09	0.74	0.03	0.04	0.07
0.22	0.13	0.63	0.02	0.05	0.08
0.30	0.17	0.52	0.01	0.06	0.10
0.37	0.21	0.42	0.01	0.07	0.11
0.45	0.24	0.31	0.00	0.09	0.13
0.52	0.28	0.20	0.00	0.10	0.14
0.59	0.32	0.09	0.00	0.11	0.16
0.00	1.00	0.00	0.00	0.12	0.20
V4: Venture All with expected return of 22.5%					
MSCI USA	**MSCI ex USA**	**Bonds**	**V4**	**Return**	**Risk**
0.00	0.00	0.95	0.05	0.02	0.06
0.12	0.07	0.75	0.06	0.05	0.07
0.25	0.14	0.55	0.07	0.07	0.10
0.38	0.20	0.35	0.08	0.09	0.13
0.50	0.26	0.15	0.09	0.11	0.16
0.55	0.30	0.00	0.15	0.14	0.20
0.37	0.26	0.00	0.36	0.16	0.26
0.20	0.23	0.00	0.57	0.18	0.35
0.02	0.19	0.00	0.79	0.20	0.45
0.00	0.00	0.00	1.00	0.22	0.56
V4: Venture All with expected return of 33.6%					
MSCI USA	**MSCI ex USA**	**Bonds**	**V4**	**Return**	**Risk**
0.00	0.00	0.95	0.05	0.03	0.06
0.12	0.07	0.72	0.09	0.06	0.08
0.24	0.13	0.49	0.14	0.10	0.12
0.37	0.18	0.26	0.18	0.13	0.16
0.49	0.24	0.03	0.23	0.16	0.21
0.42	0.21	0.00	0.37	0.20	0.27
0.31	0.16	0.00	0.53	0.23	0.33
0.20	0.12	0.00	0.68	0.27	0.40
0.09	0.07	0.00	0.84	0.30	0.48
0.00	0.00	0.00	1.00	0.34	0.56

Table 8: Portfolio Allocations for Buyout

		Buyout with expected return of 11.5%			
MSCI USA	**MSCI ex USA**	**Bonds**	**Buyout**	**Return**	**Risk**
0.00	0.00	0.80	0.20	0.03	0.06
0.02	0.00	0.71	0.27	0.04	0.06
0.05	0.00	0.61	0.34	0.05	0.06
0.08	0.00	0.52	0.40	0.06	0.07
0.11	0.00	0.43	0.47	0.07	0.08
0.14	0.00	0.33	0.53	0.08	0.09
0.16	0.00	0.24	0.60	0.09	0.10
0.19	0.00	0.14	0.66	0.10	0.11
0.22	0.00	0.05	0.73	0.11	0.12
0.00	1.00	0.00	0.00	0.12	0.20
		Buyout with expected return of 14.2%			
MSCI USA	**MSCI ex USA**	**Bonds**	**Buyout**	**Return**	**Risk**
0.00	0.00	0.80	0.20	0.04	0.06
0.01	0.00	0.71	0.28	0.05	0.06
0.02	0.00	0.62	0.36	0.06	0.06
0.04	0.00	0.53	0.44	0.07	0.07
0.05	0.00	0.44	0.51	0.08	0.08
0.06	0.00	0.35	0.59	0.10	0.09
0.08	0.00	0.25	0.67	0.11	0.10
0.09	0.00	0.16	0.74	0.12	0.11
0.11	0.00	0.07	0.82	0.13	0.12
0.00	0.00	0.00	1.00	0.14	0.13
		Buyout with expected return of 16.8%			
MSCI USA	**MSCI ex USA**	**Bonds**	**Buyout**	**Return**	**Risk**
0.00	0.00	0.80	0.20	0.04	0.06
0.00	0.00	0.71	0.29	0.06	0.06
0.00	0.00	0.62	0.38	0.07	0.06
0.01	0.00	0.53	0.46	0.08	0.07
0.01	0.00	0.44	0.55	0.10	0.08
0.01	0.00	0.35	0.63	0.11	0.09
0.02	0.00	0.26	0.72	0.13	0.10
0.02	0.00	0.17	0.81	0.14	0.11
0.03	0.00	0.08	0.89	0.15	0.12
0.00	0.00	0.00	1.00	0.17	0.13

Table 9: Portfolio Allocations for Mezzanine

Mezzanine with expected return of 8.9%					
MSCI USA	**MSCI ex USA**	**Bonds**	**Mezzanine**	**Return**	**Risk**
0.00	0.00	0.61	0.39	0.04	0.05
0.00	0.00	0.49	0.51	0.05	0.05
0.00	0.00	0.38	0.62	0.06	0.06
0.00	0.00	0.27	0.73	0.07	0.06
0.00	0.00	0.16	0.84	0.08	0.07
0.00	0.00	0.04	0.96	0.09	0.08
0.12	0.06	0.00	0.82	0.09	0.09
0.29	0.17	0.00	0.53	0.10	0.12
0.46	0.29	0.00	0.25	0.11	0.15
0.00	1.00	0.00	0.00	0.12	0.20
Mezzanine with expected return of 10.6%					
MSCI USA	**MSCI ex USA**	**Bonds**	**Mezzanine**	**Return**	**Risk**
0.00	0.00	0.61	0.39	0.05	0.05
0.00	0.00	0.52	0.48	0.06	0.05
0.00	0.00	0.44	0.56	0.06	0.05
0.00	0.00	0.35	0.65	0.07	0.06
0.00	0.00	0.27	0.73	0.08	0.06
0.00	0.00	0.18	0.82	0.09	0.07
0.00	0.00	0.09	0.91	0.10	0.07
0.00	0.00	0.01	0.99	0.10	0.08
0.27	0.24	0.00	0.49	0.11	0.12
0.00	1.00	0.00	0.00	0.12	0.20
Mezzanine with expected return of 12.2%					
MSCI USA	**MSCI ex USA**	**Bonds**	**Mezzanine**	**Return**	**Risk**
0.00	0.00	0.61	0.39	0.06	0.05
0.00	0.00	0.54	0.46	0.06	0.05
0.00	0.00	0.47	0.53	0.07	0.05
0.00	0.00	0.40	0.60	0.08	0.06
0.00	0.00	0.34	0.66	0.09	0.06
0.00	0.00	0.27	0.73	0.09	0.06
0.00	0.00	0.20	0.80	0.10	0.07
0.00	0.00	0.13	0.87	0.11	0.07
0.00	0.00	0.07	0.93	0.11	0.08
0.00	0.00	0.00	1.00	0.12	0.08

Appendix C: Efficient Frontiers

Figure 1: Efficient Frontier for V1 (13% Expected Return)

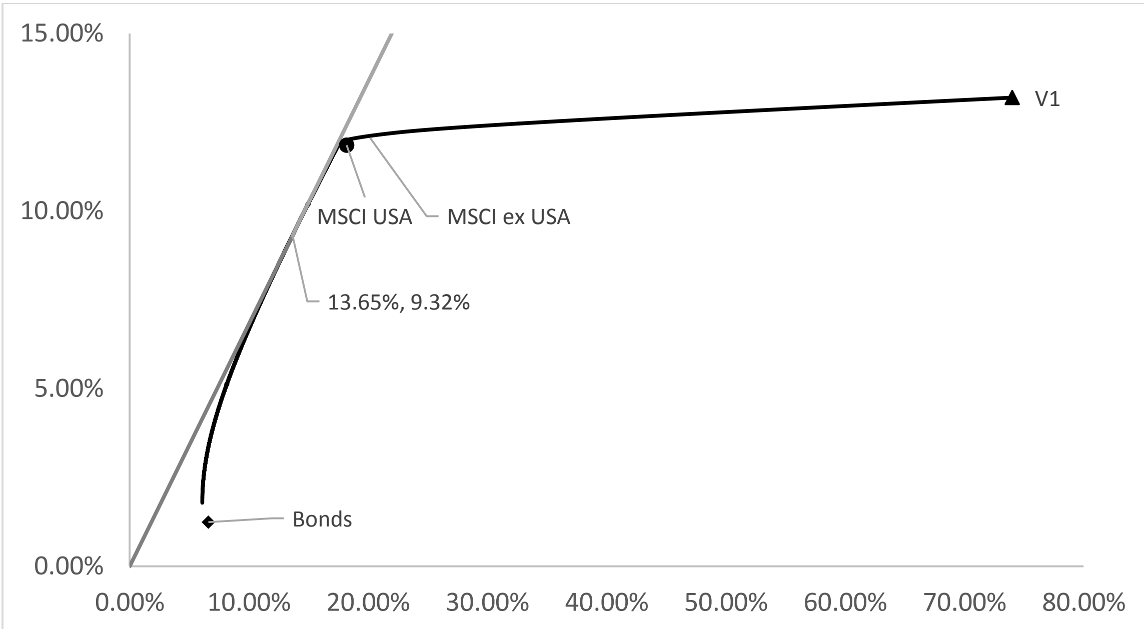


Figure 2: Efficient Frontier for V1 (28% Expected Return)

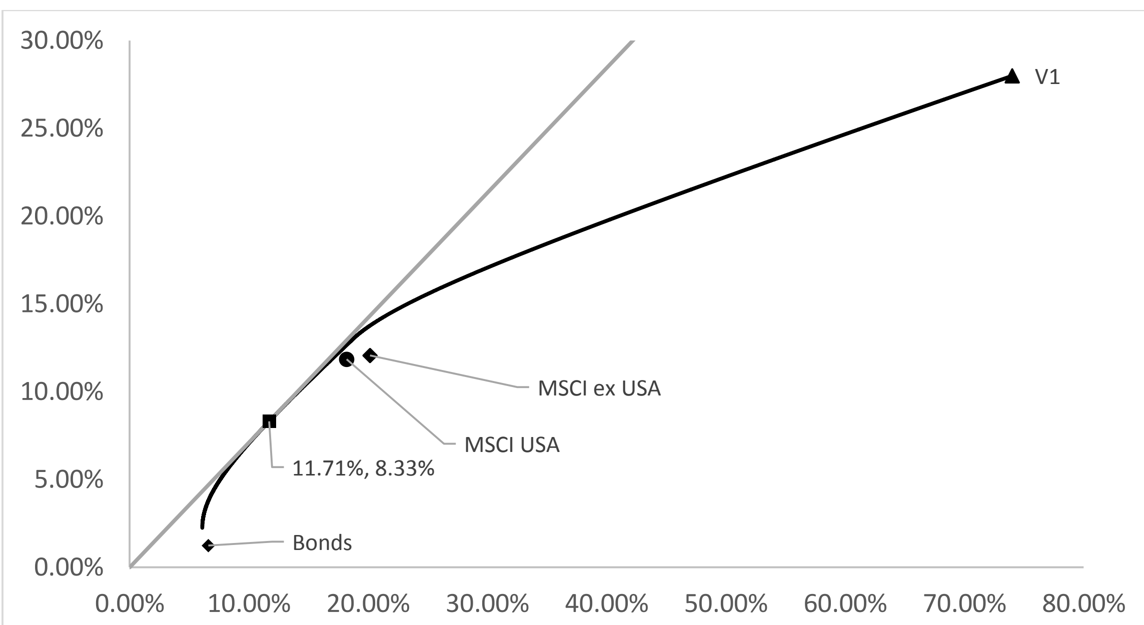


Figure 3: Efficient Frontier for V1 (42% Expected Return)

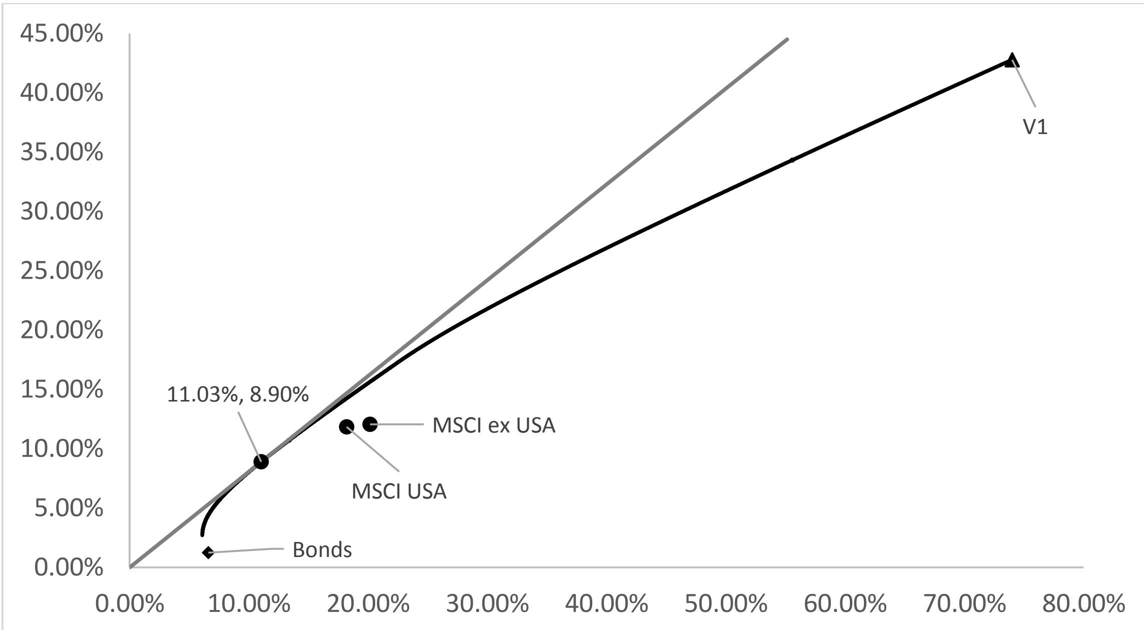


Figure 4: Efficient Frontier for V2 (11% Expected Return)

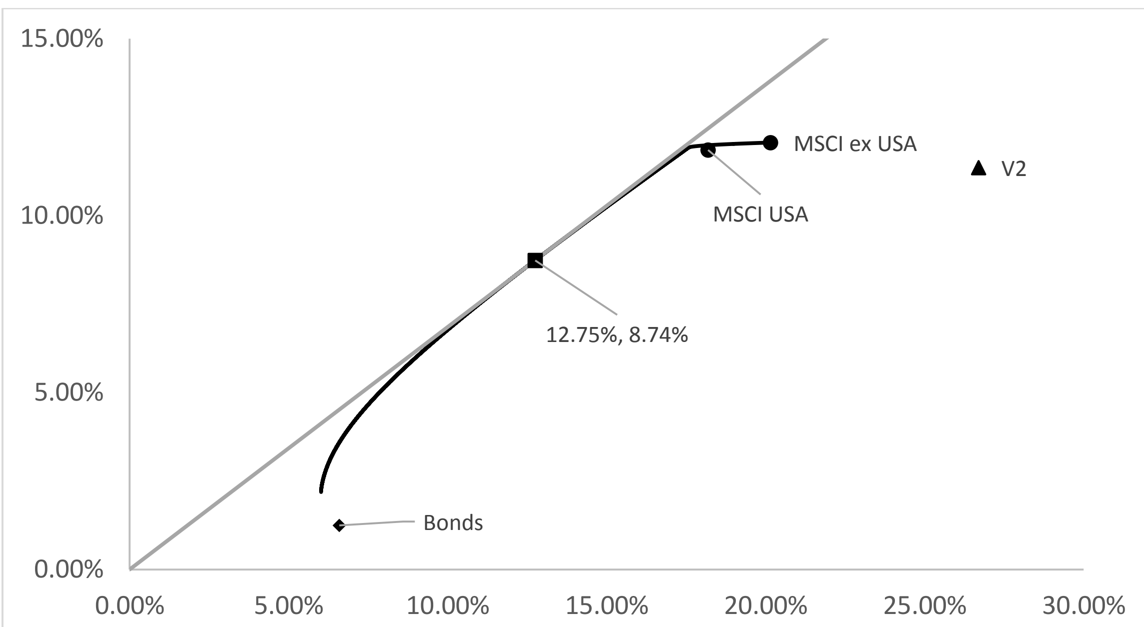


Figure 5: Efficient Frontier for V2 (16% Expected Return)

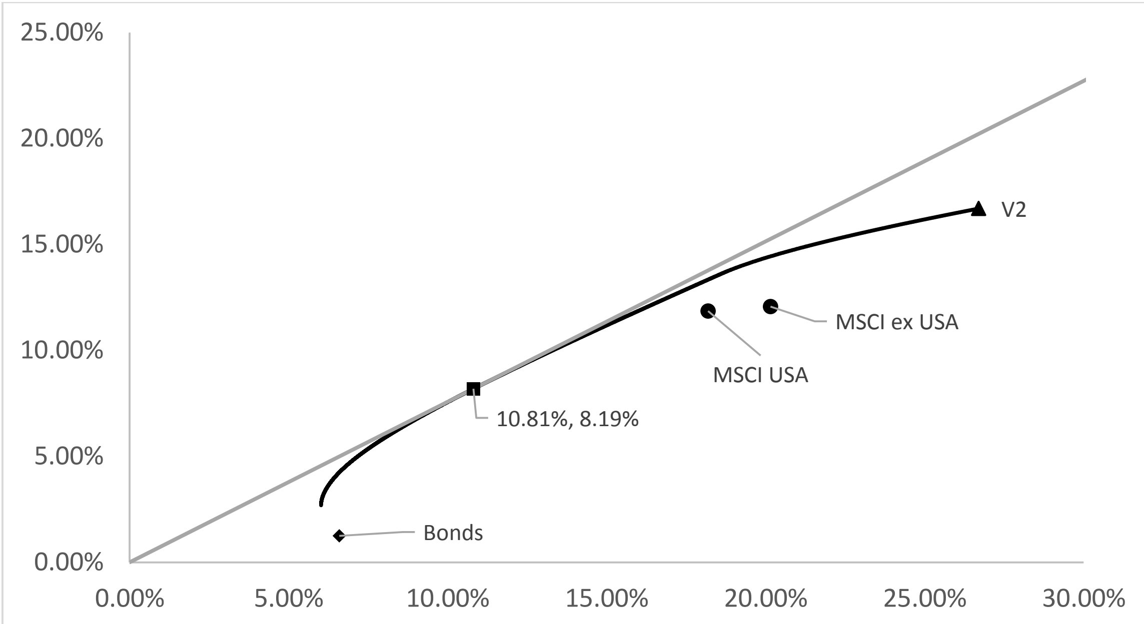


Figure 6: Efficient Frontier for V2 (22% Expected Return)

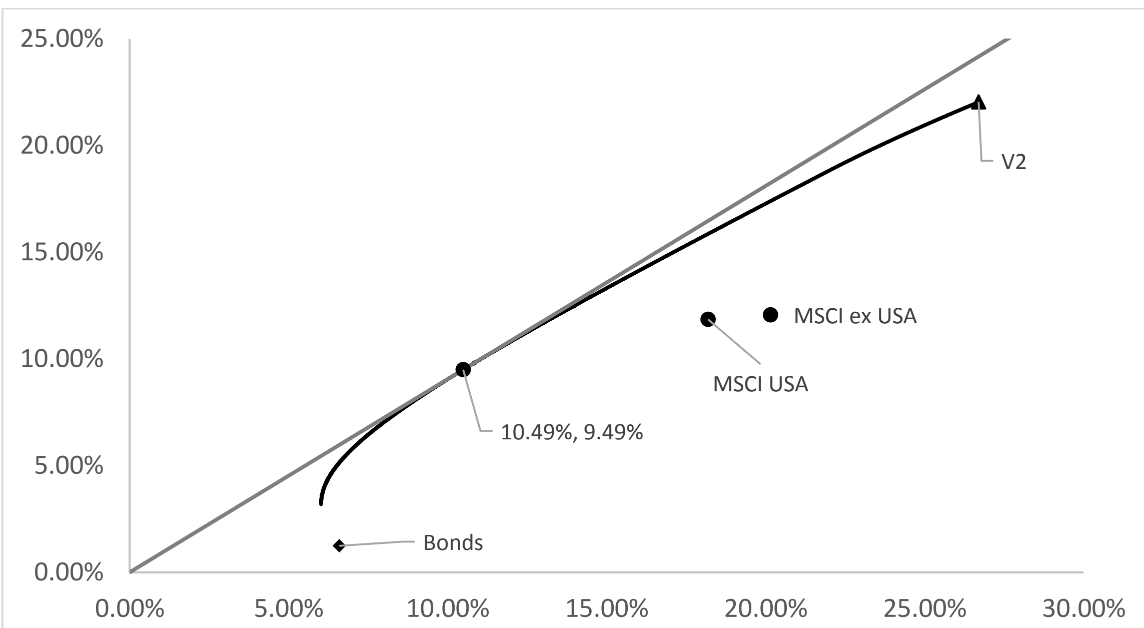


Figure 7: Efficient Frontier for V3 (9% Expected Return)

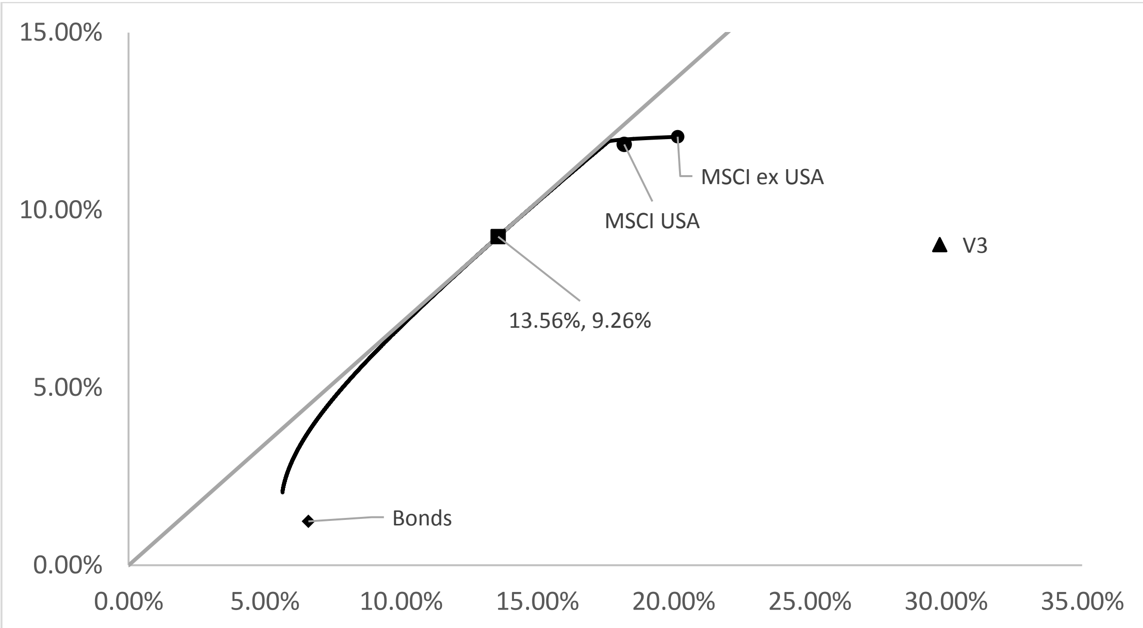


Figure 8: Efficient Frontier for V3 (15% Expected Return)

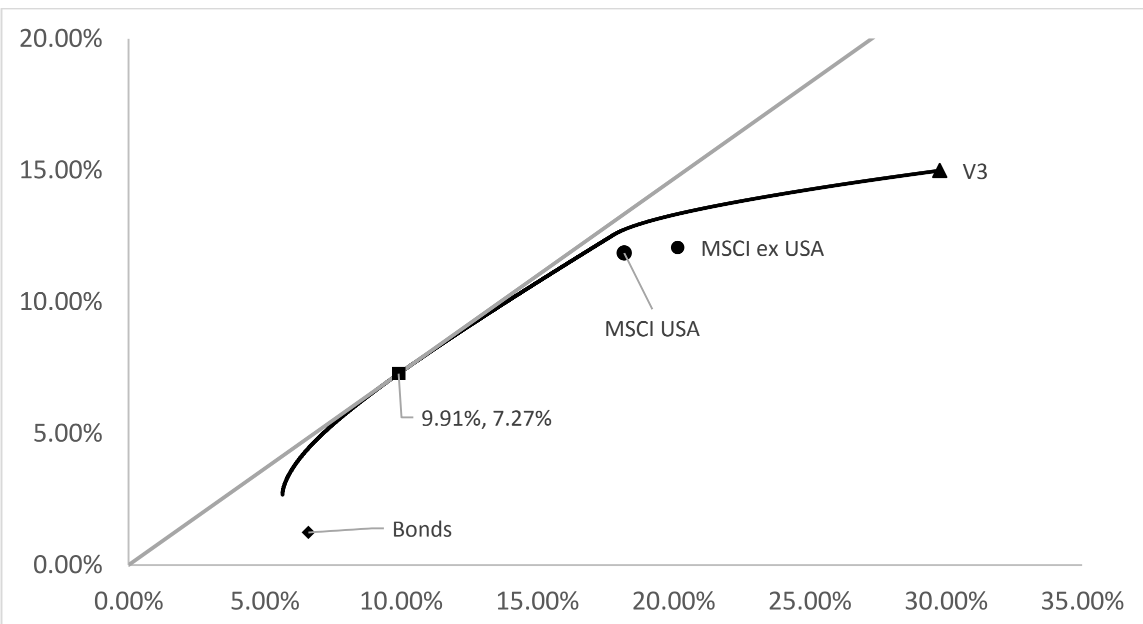


Figure 9: Efficient Frontier for V3 (21% Expected Return)

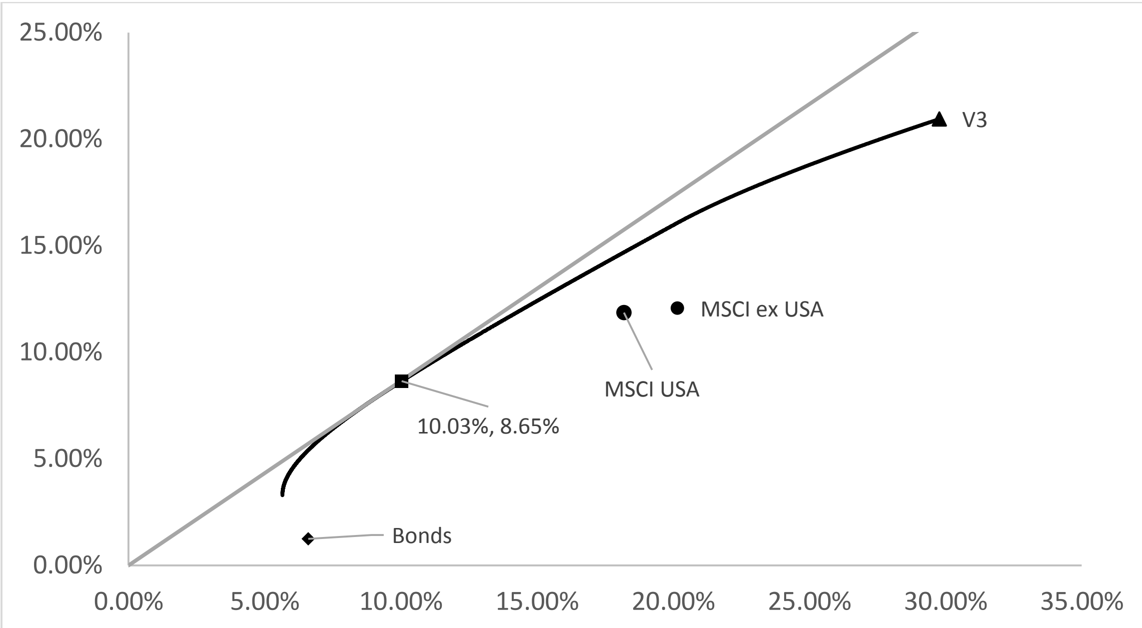


Figure 10: Efficient Frontier for V4 (11% Expected Return)

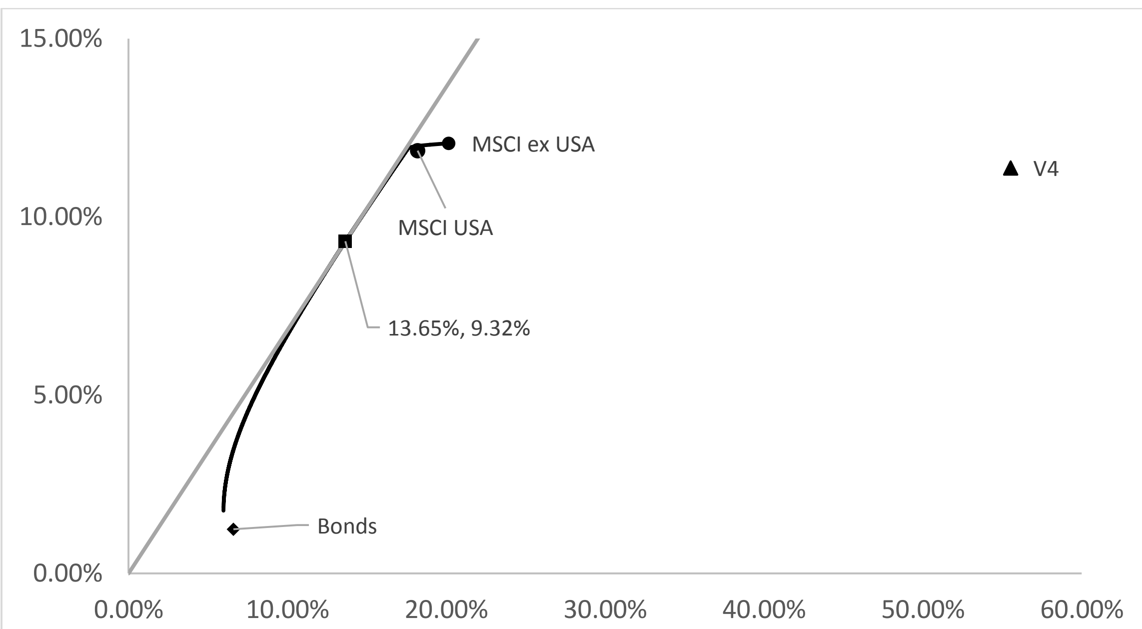


Figure 11: Efficient Frontier for V4 (22% Expected Return)

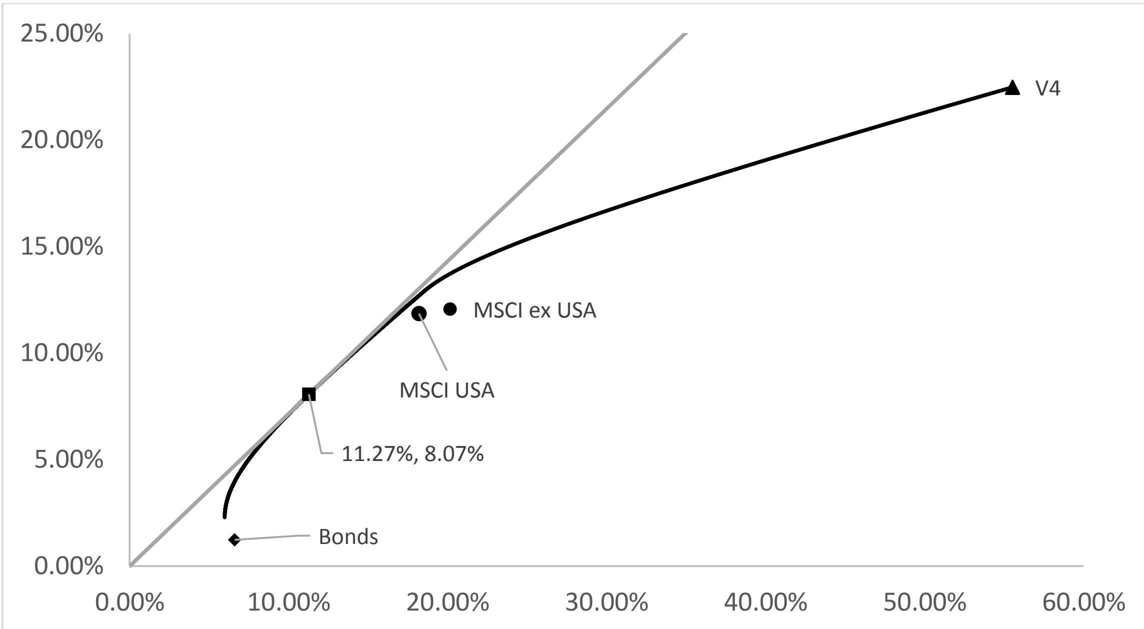


Figure 12: Efficient Frontier for V4 (34% Expected Return)

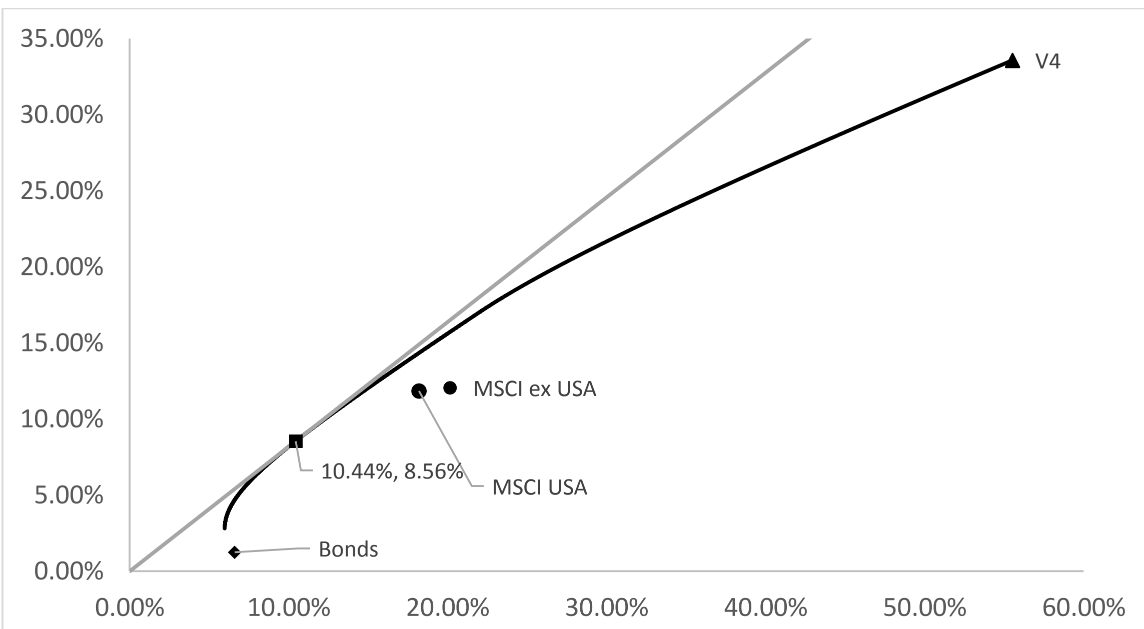


Figure 13: Efficient Frontier for Buyout (12% Expected Return)

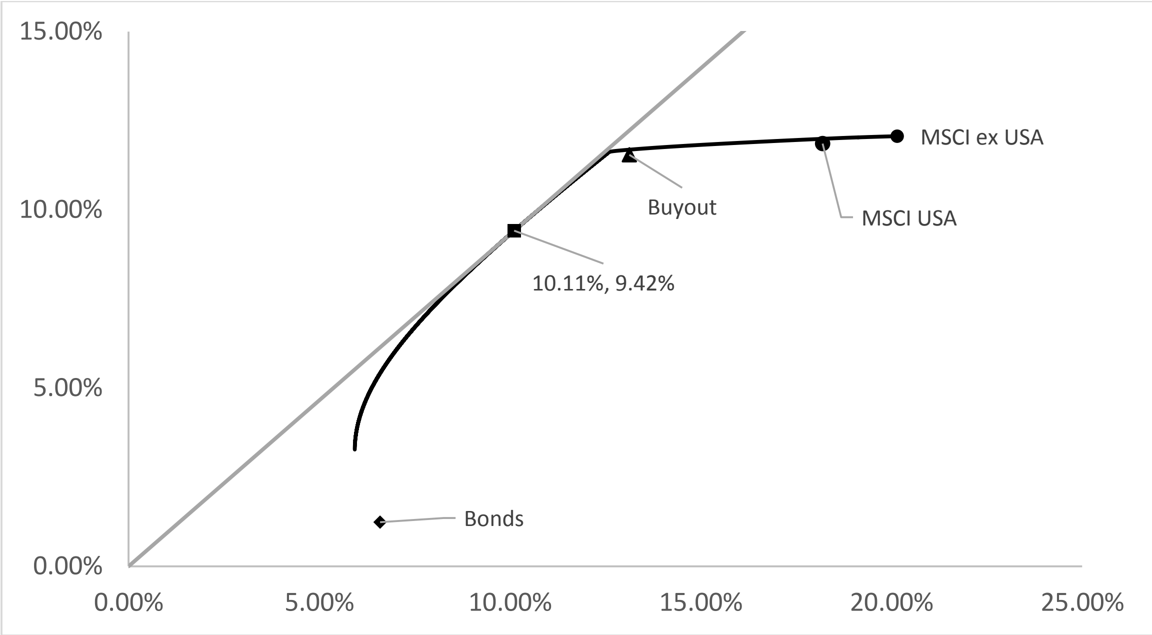


Figure 14: Efficient Frontier for Buyout (14% Expected Return)

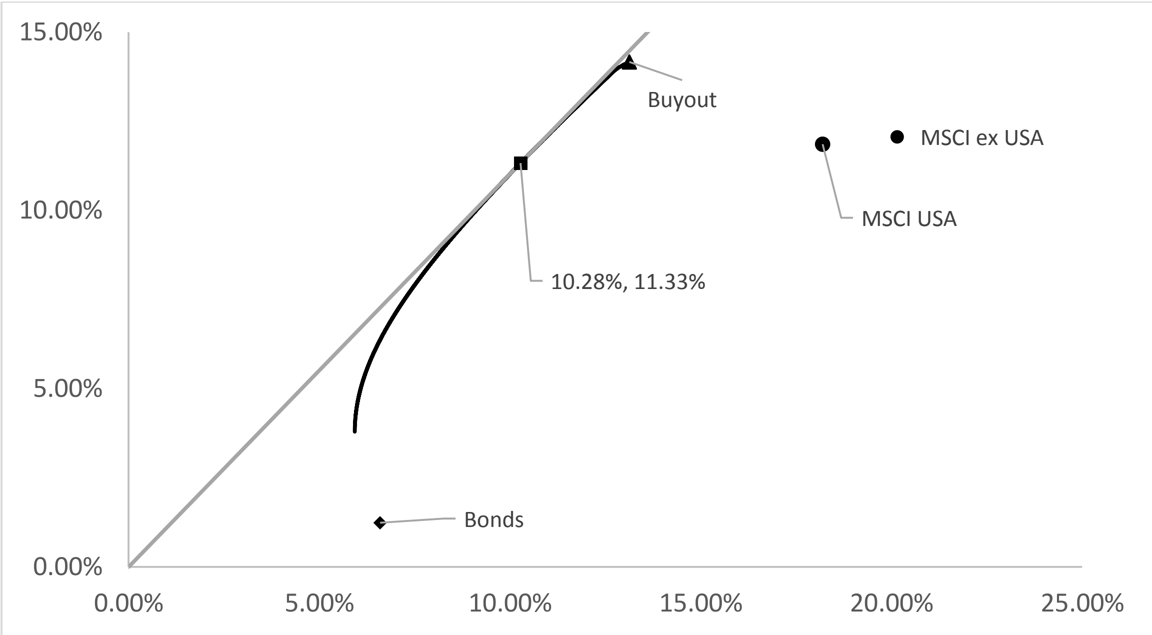


Figure 15: Efficient Frontier for Buyout (17% Expected Return)

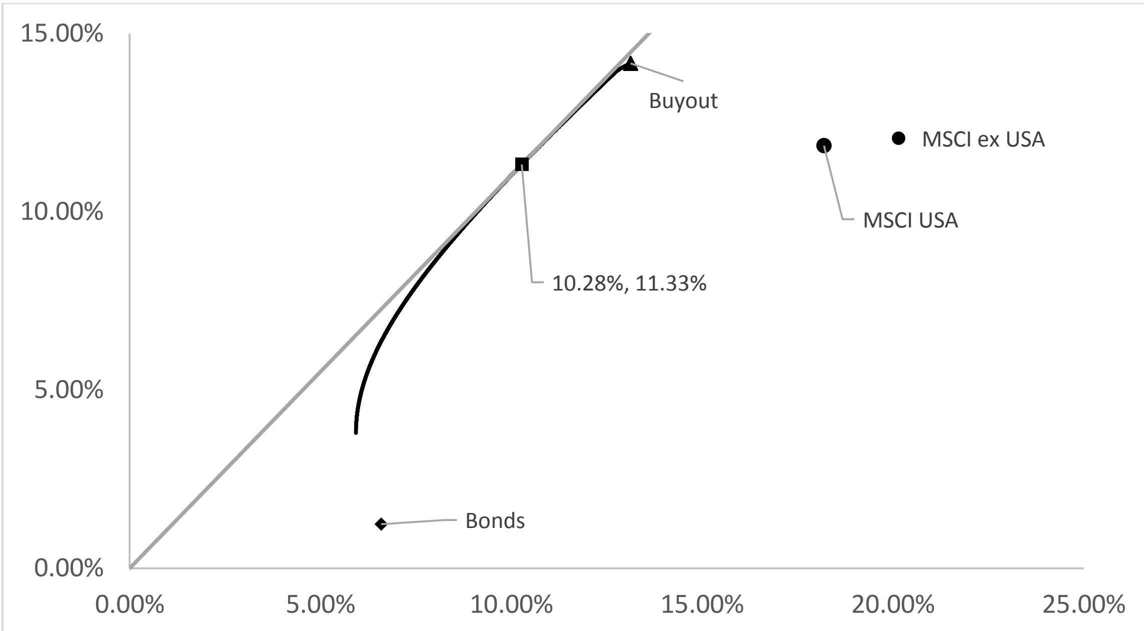


Figure 16: Efficient Frontier for Mezzanine (9% Expected Return)

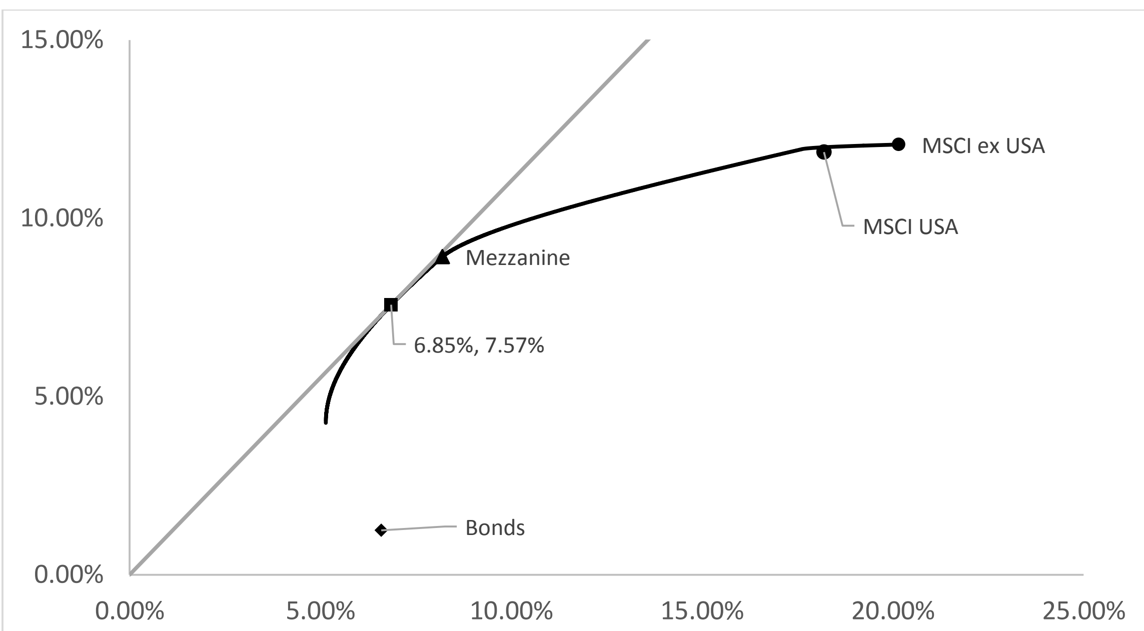


Figure 17: Efficient Frontier for Mezzanine (11% Expected Return)

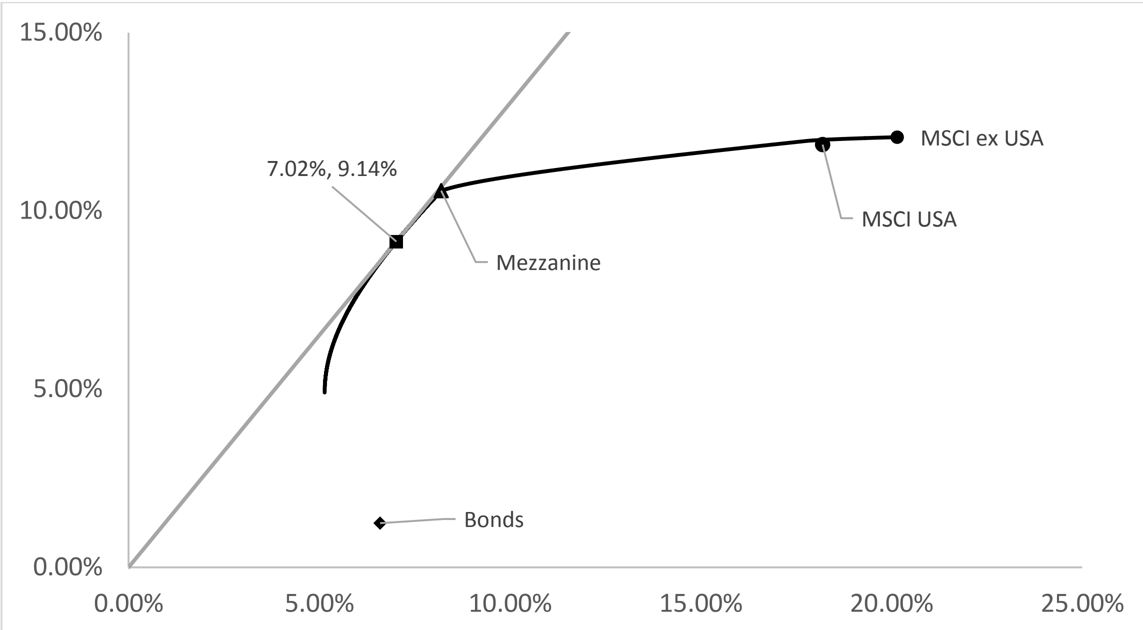
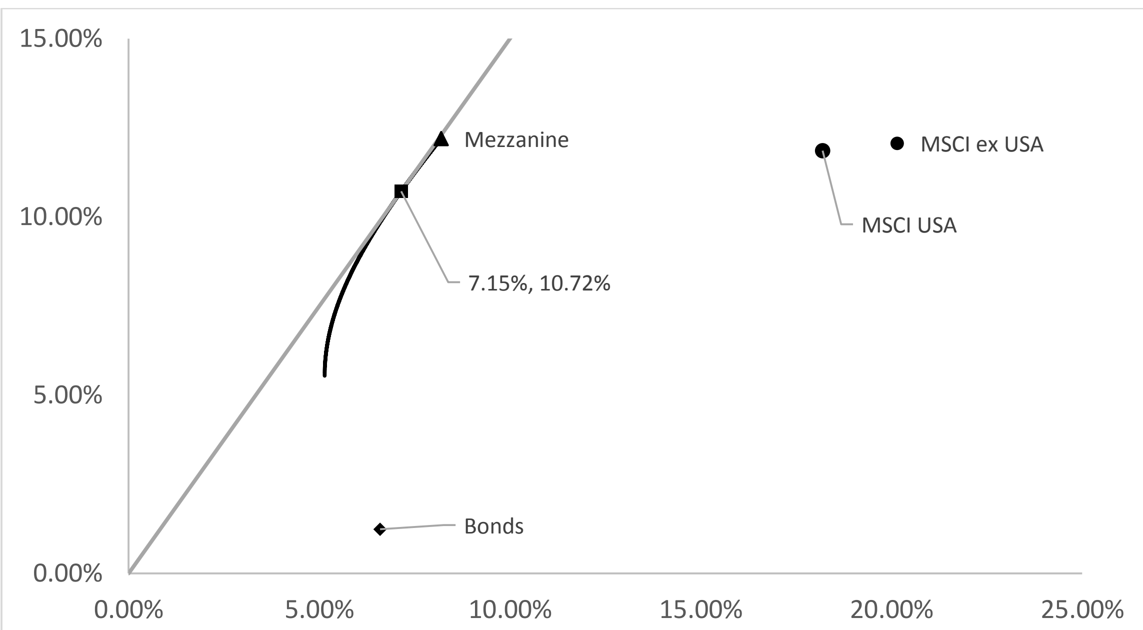


Figure 18: Efficient Frontier for Mezzanine (12% Expected Return)



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