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Abstract:

Motivated by efficiency and equity concerns, public resource managers have increasingly utilized hybrid allocation mechanisms that combine features of commonly used price (e.g., auction) and non-price (e.g., lottery) mechanisms. This study serves as an initial investigation of these hybrid mechanisms, exploring theoretically and experimentally how the opportunity to obtain a homogeneous good in a subsequent lottery affects Nash equilibrium bids in discriminative and uniform price auctions. The lottery imposes an opportunity cost to winning the auction, systematically reducing equilibrium auction bids. In contrast to the uniform price auction, equilibrium bids in the uniform price hybrid mechanism vary with bidder risk preferences. Experimental evidence suggests that the presence of the lottery and risk attitudes (elicited through a preceding experiment) impact auction bids in the directions predicted by theory. Finally, we find that theoretically and experimentally, the subsequent lottery does not compromise the efficiency of the auction in the hybrid mechanisms.

JEL classification: D44; D81; C91; H40

Keywords: allocation mechanisms; auctions; lotteries; risk aversion; experiments

I. Introduction

That the price mechanism is able to achieve an efficient allocation of resources is well known.[1] However, non-price mechanisms commonly relied upon to allocate publicly provided goods including health care, public housing, recreational opportunities, among other goods, fail to achieve efficiency. Some have argued that the use of such non-price mechanisms reflects concerns for fairness (see Aubert [1959], Goodwin [1992], Hofstee [1990]). Taylor et al. [2003] argue a lottery, one such non-price mechanism, is "usually employed to resolve allocation problems in order to reflect a spirit of fairness and equality, since everyone has an equal chance to win, regardless of whatever characteristics or qualities one may possess" (p. 1316).[2] While the efficiency properties of specific mechanisms within the class of non-price mechanisms may vary (see Taylor et al. [2003]), in general the choice between a price and a non-price mechanism implies a tradeoff between efficiency and equity. Historical and continued current use of non-price mechanisms for some publicly provided goods suggests that policy actions may be at least partially driven by some other objective. Even if resource managers would otherwise favor moving to a pure price system to achieve efficiency and/or maximize revenue, they may be reluctant or unable to do so due to political constraints such as public disapproval.[3]

Recently implemented and proposed mechanisms, which we label "hybrid mechanisms," combine features of price and non-price mechanisms. The goods for which hybrid mechanisms have either been used or proposed tend to be (i) publicly provided, and (ii) goods that have historically been allocated through the exclusive use of non-price rationing devices such as

[1] Carlton [1991] states "Microeconomics is concerned with the efficient allocation of goods. It is typical for economists to focus on the price system as the mechanism used to achieve this efficient allocation. An impersonal auction system is often in the back of most economists' minds when they think of efficient resource allocation."
[2] Boyce [1994] challenges the view that the use of lotteries stems primarily from their desirable equity properties. He proposes a model in which a small proportion of proceeds from the allocation are redistributed to the population. He shows that in this setting, a lottery is preferred to an auction.
[3] For example, Kahneman et al. [1986] found that people preferred queues to lotteries to auctions among mechanisms used to allocate concert tickets.

lottery or queue. Revenue generation and increased efficiency are commonly cited motives for moving or proposing a move from a non-price system to a hybrid system.

The mechanism used to allocate rights of passage through the Panama Canal to ships is one example of a hybrid mechanism. Of the approximately 38 ships that pass through the Canal each day, 24 slots are reserved in advance, one slot is distributed in an auction, and the remaining rights of passage are allocated to ships in the queue [Schnexnayder, 2007]. A second example stems from the recent debates surrounding U.S. immigration policy reforms. Current immigration policy distributes immigration visas through non-price mechanisms including lottery, queue, and preference by family status and special skills. There is ongoing debate over how to modify the current system (see Simon [1986, 1987] and Becker [1987] for early discussions), and recent proposals favor a hybrid mechanism over a move to a pure price system. Gross and MacLaury [2001] propose allocating Specialty Worker visas (H-1B), over and above the current allotment, via auction during periods of excess demand. Posner [2005] discusses using hybrid mechanisms for visas that grant permanent resident status.

Two additional examples relate to the allocation of recreational opportunities. First, perhaps in response to declining budgets, several states have used hybrid mechanisms to distribute big game hunting permits, a setting in which there is often a very small number of permits relative to the number of interested parties. Permits, as few as a single permit, are initially offered for sale through a central auction with the remaining permits allocated via a lottery (sometimes with a nominal entry fee in one or both).[4] Rafting permits for the Colorado River in the Grand Canyon are also allocated using a system that effectively combines a price

[4] See Nguyen et al. [2007] for a brief discussion of Maine's experience with moose hunting permit auctions. Some other U.S. states and Canadian provinces rely on non-profit foundations, such as the Foundation for North American Wild Sheep (www.fnaws.org), to hold auctions for as few as a single hunting permit while the local wildlife agency continues to manage allocation of the majority of permits through the lottery.

and non-price mechanism. An individual who wishes to raft this section of the Colorado River has two options. First, she may enter a lottery to obtain a non-commercial rafting permit.[5] Alternatively, she can pay a commercial rafting company to guide her, making use of the company's commercial rafting permit to access the river.

The use of hybrid mechanisms has not been restricted to the allocation of homogeneous goods, such as hunting permits and immigration visas. The Federal Onshore Oil and Gas Leasing Reform Act of 1987 initiated a move from the previous lottery system to a hybrid system in which all available tracts are initially offered for sale through a sealed-bid first-price auction with a reserve bid (Haspel [1990]).[6] Those tracts not sold via auction (because of lack of bids or bids that fail to exceed reserve prices) are then offered for sale through a lottery. More recently, the Alaska Department of Natural Resources [2006] used a similar hybrid mechanism to allocate state lands to Alaska residents. Other applications include the allocation of vendor spaces (e.g., Kobey's Swap Meet [2006] at the San Diego Sports Arena) and undeveloped housing lots (e.g., the "Ocean View Auction and Lottery" sponsored by the Norfolk Redevelopment and Housing Authority in 2003 [PR Newswire, 2003]).

While increasingly implemented in practice, a rigorous examination of the incentive properties associated with hybrid mechanisms has not yet been undertaken. This is a necessary first step before one can begin to develop formal theories to explain why such hybrid mechanisms are employed to allocate goods in some settings while price and non-price mechanisms continued to be used in other settings. This paper provides first insights for a specific hybrid system: a fixed number of non-transferable homogeneous units are available for

[5] Until October 2006, non-commercial rafting permits were allocated via queue. In 2000, the average wait time for a non-commercial applicant was 14 years. See www.gcpba.org.

[6] Haspel [1990] notes that the adoption of the hybrid or two-tier allocation mechanism was seen as a compromise by those who advocated using a price mechanism, but such individuals were "pleasantly surprised" at the outcome of the program.

allocation, of which a portion are allocated through an auction with the remainder allocated via a lottery in which auction winners are precluded from participating. In particular, we build upon the work of Vickrey [1962] and Harris and Raviv [1981] to investigate theoretically how the opportunity to obtain a homogeneous good in a subsequent lottery affects equilibrium auction bidding in uniform price and discriminative auctions. Our model of bidder behavior assumes singleton demands, private values, and homogeneous risk preferences. With risk-neutral agents, we establish that: (a) hybrid mechanism equilibrium bids are simply the pure auction equilibrium bids scaled by the probability of losing the lottery; and (b) the addition of the lottery does not change the familiar revenue equivalence result. With identically risk-averse agents, we are able to show that, with the addition of the lottery: (c) uniform price auction bids are no longer independent of risk preferences; and (d) uniform price and discriminative auctions continue to allocate units to individuals with the highest valuations, i.e., auctions remain efficient.

To complement our theoretical derivations, we conduct a series of laboratory experiments with induced values to test theoretical predictions. The focus of the experiments is on how the conditional probability of winning the lottery and risk attitudes affect auction bidding. To investigate the role of risk attitudes, we include in our econometric analysis a risk measure elicited through a preceding paired lottery-choice procedure (e.g., Holt and Laury [2002]). This approach is a departure from previous empirical auction studies where it is common to use indirect approaches, for example examining risk parameters estimated from structural econometric models, or by testing for departures from risk-neutral bidding.

We find that the subsequent lottery and risk preferences affect the estimated bid function in the directions predicted by theory for experimental uniform price and discriminative hybrid mechanisms. In particular, increasing the conditional probability of winning the lottery (e.g., allocating more units via lottery, less via auction) decreases auction bids; risk aversion (risk seeking) increases (decreases) auction bids. These findings have import for the healthy debate over the role of risk attitudes in experimental auctions (see Kagel [1995]). We find that average revenue from the uniform price and discriminative hybrids are approximately equivalent.

Finally, the experimental results confirm the theory in that, in the hybrid mechanisms, auctioned units continue to go to those with the highest valuations. Although the presence of the lottery in the hybrid mechanisms reduces efficiency relative to the pure auction counterparts, the inefficiency stems only from the lottery component, not from reduced efficiency in the auction component. Hence, even if the public sector must allow lotteries for political reasons, governments would be able to introduce an auction to allocate some units without the fear that the subsequent lottery allocation of similar items will reduce the efficiency properties of the auction.

II. Model

Consider a seller who has S total units (or permits) available for allocation among $N > S$ potential buyers. Bidders demand at most one unit of the good. Let v_i represent the private monetary value of a unit to bidder i where v_i is independently drawn from the distribution characterized by density function g and cumulative distribution function G with support $[0, \bar{v}]$. The allocation proceeds as follows. First, the seller allocates $Q < S$ units via auction. Then, the seller allocates the remaining S-Q units via lottery among the N-Q remaining buyers as auction winners are ineligible for participation in the lottery. Therefore, the probability of winning the subsequent lottery is $\frac{S-Q}{N-Q} \in (0,1)$.

Assume that there are neither entry fees for either the auction or lottery nor a user fee for lottery or auction winners. We consider two common multiple unit auction formats, the uniform price and discriminative auctions, both defined explicitly below. Assuming homogenous risk preferences, let $b_{ij,k} = b_{j,k}(v_i)$ represent the bid function for bidder *i* for mechanism *j* when all bidders have risk preferences *k*.[7] Let $\pi_i(b_{j,k}(v_i)) = v_i$ represent its inverse. For each auction format, we first examine how the opportunity to obtain a unit through the hybrid mechanism affects the equilibrium bid function under risk neutrality and risk aversion. Following this discussion, we examine auction revenues.

A. Hybrid mechanism with uniform price auction

The uniform price auction is the multiple unit counterpart of the second-price auction. We consider a uniform price auction in which all winners pay a price equal to the first rejected bid. Harris and Raviv [1981] prove that the familiar value revelation result for second-price auctions, where it is optimal to bid one's value regardless of risk preferences, extends to the uniform price auction. We show that a related result holds for the uniform price hybrid. However, the addition of a subsequent lottery in this setting serves as an additional (opportunity) cost to winning the auction, which causes bidder *i* to shade her bid relative to her monetary value, v_i. In the uniform price hybrid, bidder *i* submits a bid equal to her benefit of winning the auction (as in the pure uniform price auction) but that benefit is now lower than v_i as a result of the subsequent lottery. For the risk-neutral case, Proposition 1 shows that bidders scale their bids exactly by the probability of losing the lottery.

<u>Proposition 1</u>: When bidders are risk neutral, the Nash equilibrium bid function for bidder *i* who

[7] When we allow for risk aversion, we assume all bidders are identically risk averse. The absence of a subscript following the comma indicates that the result holds whether all bidders are risk neutral or all bidders are identically risk averse.

faces a hybrid allocation mechanism with a uniform price auction (denoted *UH* for uniform price hybrid) is given by

$$b_{UH,N}(v_i) = \left(1 - \frac{S-Q}{N-Q}\right) v_i = \left(1 - \frac{S-Q}{N-Q}\right) b_U(v_i) \tag{1}$$

where $\left(1 - \frac{S-Q}{N-Q}\right)$ represents the probability of losing the lottery and $b_U(\cdot)$ denotes the Nash equilibrium bid in the uniform price auction.

Proof: See appendix.

Whereas equilibrium bids in the uniform price auction are independent of bidders' risk preferences, risk preferences do affect equilibrium bids in the uniform price hybrid.[8] The intuition behind this result is as follows. In a pure uniform price auction setting, where a risk-neutral bidder bids her value, there is no incentive for a risk-averse agent to increase her bid above her value. Doing so affects the probability of winning but does not affect the payoffs upon winning (at least over the range for which she would want to win) because the price is independent of her bid. However, in the uniform price hybrid, risk aversion implies a lower opportunity cost of winning the auction (relative to risk neutrality) and therefore an incentive to bid more. This occurs because increasing the probability of winning the auction, by bidding more, decreases the probability of winning the lottery given that only non-auction winners enter the lottery. Although the equilibrium bid under risk aversion exceeds that under risk neutrality in the uniform price hybrid, the risk averse bidder continues to bid less than she would in the absence of the lottery. Proposition 2 addresses these results.

Proposition 2: $b_{UH,N}(v_i) < b_{UH,A}(v_i) < b_U(v_i) = v_i \forall v_i > 0$ where $b_{UH,A}(v_i)$ ($b_{UH,N}(v_i)$) denotes the

[8] This result is consistent with Nash equilibrium bidding in a second-price auction for risky prizes (see Neilson [1994]).

equilibrium bid function for risk averse (neutral) bidder i facing the uniform price hybrid and $b_U(v_i) = v_i$ represents the equilibrium bid function for bidder i facing the pure uniform price auction. The first inequality holds weakly for weak risk aversion.

Proof:

Consider the second inequality first. For the proof, we make use of the following two expressions, which hold in equilibrium for the uniform price auction and uniform price hybrid respectively:

$$U(v_i - b_U) = U(0) \tag{2}$$

$$U(v_i - b_{UH,A}) = pU(v_i) + (1-p)U(0) \tag{3}$$

where $U'(\cdot) > 0, U''(\cdot) < 0$. Expression (2) implicitly defines the equilibrium bid function for a bidder in a uniform price auction. To see this, let $EU(b_U) \equiv EU(b_U(v_i))$ represent the expected utility from bidding $b_U \equiv b_U(v_i)$ in the uniform price auction:

$$EU(b_U) = \int_0^{b_U} U(v_i - \hat{b})dH(\hat{b}) + \int_{b_U}^{\bar{b}_U} U(0)dH(\hat{b}) = \int_0^{b_U} U(v_i - \hat{b})h(\hat{b})d\hat{b} - \int_{\bar{b}_U}^{b_U} U(0)h(\hat{b})d\hat{b}$$

where $H(\cdot)$ denotes the distribution of the first rejected bid and $\bar{b}_U$ represents the highest bid in the uniform price auction. The optimal bid for an expected utility maximizer solves the following (see for example Neilson [1994], equation (2), p. 139):

$$\frac{d}{db_U} EU(b_U) = 0 \Leftrightarrow U(v_i - b_U)h(b_U) - U(0)h(b_U) = 0,$$

which reduces to expression (2).

Expression (3) implicitly defines the equilibrium bid function for a bidder in the uniform price hybrid. Expression (3) also defines the participation constraint for the uniform price hybrid, which we assume holds for all bidders. That is, there exists a positive bid, denoted $b_{UH,A}$, such that bidder i is indifferent between winning the auction and paying $b_{UH,A}$, and taking part in the lottery. For certain parameter values, the value of $b_{UH,A}$ for which expression (3) holds is negative. Under these conditions, the bidder requires compensation for submitting a non-negative bid in the auction and thus the participation constraint does not hold. To derive expression (3), note that the expected utility from bidding $b_{UH,A}(v_i) \equiv b_{UH,A}$ in the uniform price hybrid is given by:

$$EU(b_{UH,A}) = \int_0^{b_{UH,A}} U(v_i - \hat{b})dH(\hat{b}) + \int_{b_{UH,A}}^{\bar{b}_{UH,A}} [(1-p)U(v_i) + pU(0)]dH(\hat{b})$$
$$= \int_0^{b_{UH,A}} U(v_i - \hat{b})h(\hat{b})d\hat{b} - \int_{\bar{b}_{UH,A}}^{b_{UH,A}} [(1-p)U(v_i) + pU(0)]h(\hat{b})d\hat{b}$$

where $\bar{b}_{UH}$ represents the highest bid in the uniform price hybrid. Expression (3) follows from

$\frac{d}{db_{UH,A}} EU(b_{UH,A}) = 0$ as above. Given (2) and (3), for $p \in (0,1)$,

$U(v_i - b_U) = U(0) < pU(v_i) + (1-p)U(0) = U(v_i - b_{UH,A})$. With U increasing, $v_i - b_U < v_i - b_{UH,A}$, which yields the result.

To prove the first inequality, note that by Jensen's inequality

$$U(v_i - b_{UH,A}) < U(p \cdot v_i + (1-p) \cdot 0) \tag{4}$$

which because U is increasing implies the following:

$$v_i - b_{UH,A} < p \cdot v_i + (1-p) \cdot 0 . \tag{5}$$

The right hand side of equation (5) is equal to $U(v_i - b_{UH,N}) = v_i - b_{UH,N}$. Substitution yields the result that $b_{UH,N}(v_i) < b_{UH,A}(v_i)\ \forall v_i > 0$.

In contrast to the pure uniform price auction, Proposition 2 suggests that the addition of the lottery causes risk-averse bidders in the uniform price hybrid to shade their bids below their values. The lottery in effect changes the auction from one in which each bidder has probabilities of winning certain values, $U(v_i - b_i)$, if she wins the auction or $U(0)$ if not, to one in which each bidder has a chance of winning a certain value, $U(v_i - b_i)$, or winning an uncertain value $pU(v_i) + (1-p)U(0) > U(0)$ for $p \in (0,1)$. Because the lottery makes the alternative to winning the auction more desirable, it reduces the benefits of winning a unit of the good in the auction. As expected, the reduction is less pronounced for risk-averse bidders as they are willing to pay a premium to avoid the risk associated with playing the lottery.

B. *Hybrid mechanism with discriminative auction*

A second auction format available to the seller is the discriminative auction, the multiple unit counterpart of the first-price auction where winners pay prices equal to their respective bids. Harris and Raviv [1981] derive the equilibrium bid function for the discriminative auction. We adapt the Harris and Raviv model to describe the problem faced by a bidder in the hybrid mechanism with a discriminative auction. Here, bidder *i* wins the auction if at least $N - Q$ bidders have values less than $\pi_i(b_{DH}(v_i))$ where the subscript *DH* denotes the discriminative hybrid mechanism. The probability of this event is equal to the probability distribution of the $(N-Q)^{th}$ order statistic for a random sample of size $N-1$ drawn from *G* evaluated at $\pi(b) \equiv \pi_i(b_{DH}(v_i))$ given by:

$$F(\pi(b)) = \frac{(N-1)!}{(N-Q-1)!(Q-1)!} \int_0^{\pi(b)} [G(v)]^{N-Q-1} [1-G(v)]^{Q-1} g(v) dv . \tag{7}$$

If bidder *i* wins the discriminative auction then her payoff is $U(v_i - b_i)$ where

$U'(\cdot) > 0, U''(\cdot) \leq 0$.[9] Otherwise, she has a second opportunity to obtain a unit through the lottery. Therefore, bidder *i* chooses her bid, b_{DH}, to maximize her expected utility equal to:

$$F(\pi(b_{DH}(v_i)))U(v_i - b_{DH}(v_i)) + [1 - F(\pi(b_{DH}(v_i)))]\left\{\frac{S-Q}{N-Q}U(v_i) + \left[1 - \frac{S-Q}{N-Q}\right]U(0)\right\} \quad (8)$$

where $U(0)$ represents utility if the individual does not obtain a unit in either the auction or lottery. As in the discriminative auction, the equilibrium bid function in the discriminative hybrid varies with risk preferences. We begin in Proposition 3 with the assumption of risk neutrality, where, analogous to the uniform price hybrid case, the optimal bidding strategy entails scaling the equilibrium bid in the pure auction setting by the probability of losing the lottery.

Proposition 3: With risk neutral bidders, the Nash equilibrium bid function for bidder *i* who faces a discriminative hybrid mechanism is

$$b_{DH,N}(v_i) = \left(1 - \frac{S-Q}{N-Q}\right)\frac{1}{F(v_i)}\int_0^{v_i} xf(x)dx = \left(1 - \frac{S-Q}{N-Q}\right)b_{D,N}(v_i). \quad (9)$$

Proof: See appendix.

Proposition 4 below shows that a similar result holds under risk aversion: the presence of the lottery causes risk-averse bidders to shade their bids in the discriminative hybrid relative to their equilibrium bid in an analogous discriminative auction. Before proceeding to the proposition, we briefly discuss the intuition behind the proof, which relies on the observation that *i's bid in the uniform price hybrid under risk aversion is the amount she values the item being auctioned given the subsequent lottery*. Bidder *i*, whose value of winning a unit of the good in the pure auction is given by v_i, values winning a unit of the good in the auction component of

[9] We assume $U'' = 0$ and $U'' < 0$ characterize risk neutral and risk averse bidders respectively. Haris and Raviv [1981] also assume $U(0) = 0$. Later in our analysis, we make a similar assumption. However, the assumption is unnecessary at this point.

the hybrid setting at $b_{UH,A}(v_i)$. Therefore, the discriminative hybrid under risk aversion is comparable to a discriminative auction under risk aversion where the distribution of bidder values has changed. As a result, if we replace v_i in $b_{D,A}(v_i)$ with $b_{UH,A}(v_i)$ we obtain an expression that characterizes the equilibrium bid function for the discriminative hybrid under risk aversion, $b_{DH,A}(v_i)$.

<u>Proposition 4</u>: $b_{DH,A}(v_i) < b_{D,A}(v_i)$

<u>Proof</u>:

In the uniform price auction, the Nash equilibrium bid for risk averse bidder *i*, denoted $b_{U,A}(v_i)$, is equal to her value for a unit of the good being auctioned or v_i (Harris and Raviv [1981]). In the uniform price hybrid, the Nash equilibrium bid for bidder *i*, denoted $b_{UH,A}(v_i)$ is equal to her value for a unit of the good being auctioned given the subsequent lottery. Therefore, to obtain an expression that characterizes the equilibrium bid function for the discriminative hybrid under risk aversion, $b_{DH,A}(v_i)$, substitute $b_{UH,A}(v_i)$ for v_i in $b_{D,A}(v_i)$ as follows:

$$b_{DH,A}(v_i) = b_{D,A}(b_{UH,A}(v_i)) \tag{10}$$

where by Proposition 2, $b_{UH,A}(v_i) < v_i$. If $b_{UH,A}(v_i)$ and $b_{D,A}(v_i)$ are increasing in v_i, then

$$b_{DH,A}(v_i) = b_{D,A}(b_{UH,A}(v_i)) < b_{D,A}(v_i).$$

First show $b_{UH,A}(v_i)$ is increasing in v_i. In equilibrium for the uniform price hybrid, the following holds:

$$U(v_i - b_{UH,A}) = pU(v_i) + (1-p)U(0) \tag{11}$$

with $b_{UH,A} > 0$. Implicit differentiation yields:

$$b'_{UH,A}(v_i) = \frac{U'(v_i - b_{UH,A}) - pU'(v_i)}{U'(v_i - b_{UH,A})} \tag{12}$$

which is positive if $U'(v_i - b_{UH,A}) > pU'(v_i)$. Because $v_i - b_{UH,A} \le v_i$ and by strict concavity of the utility function, $U'(v_i - b_{UH,A}) > U'(v_i) > pU'(v_i)$ for $p < 1$.

Now show $b_{D,A}(v_i)$ is increasing in v_i. The first order condition for the equilibrium bid in the discriminative hybrid is (see appendix equation (A6)):

$$b'_{DH,A}(v_i) = \frac{f(v_i)}{F(v_i)} \frac{U(v_i - b_{DH,A}) - [pU(v_i) + (1-p)U(0)]}{U'(v_i - b_{DH,A})}.$$

This expression is positive provided $U(v_i - b_{DH,A}) > [pU(v_i) + (1-p)U(0)] = U(v_i - b_{UH,A})$ where the equality results from equation (11). Because U is strictly increasing, $v_i - b_{DH,A} > v_i - b_{UH,A} \Leftrightarrow b_{UH,A} > b_{DH,A}$. In order to prove the last inequality rewrite $b_{DH,A}(v_i)$ as $b_{D,A}(b_{UH,A}(v_i))$. Then $b_{DH,A}(v_i) = b_{D,A}(b_{UH,A}(v_i)) < b_{UH,A}(v_i)$ provided $b_{D,A}(v_i) < v_i$ which Harris and Raviv [1981] prove.

Finally, we examine how risk aversion affects the equilibrium bid function under the discriminative hybrid mechanism by comparing $b_{DH,N}(v_i)$ with $b_{DH,A}(v_i)$. Recall that in the uniform price hybrid, risk aversion serves to increase equilibrium bids relative to the risk-neutral case. Proposition 5 extends Harris and Raviv's [1981] result that risk aversion increases the equilibrium bid relative to risk neutrality in the discriminative auction to this hybrid setting.

<u>Proposition 5</u>: Assume $U(0) = 0$. Then $b_{DH,A}(v_i) > b_{DH,N}(v_i) \; \forall \; v_i > 0$.

<u>Proof</u>: See Appendix.

Propositions 1 through 5 suggest two general results. First, the addition of the lottery component of the hybrid mechanisms reduces bidders' expected payoffs of winning the auction.

As a result, relative to the respective pure auctions, equilibrium bids in the hybrid mechanisms are lower (given the probability of winning the lottery is strictly less than one and holding risk preferences constant). Second, as in Harris and Raviv's [1981] analysis of discriminative auctions, relative to risk neutrality, risk aversion serves to increase equilibrium bids in both hybrid settings. Therefore, risk aversion and the subsequent lottery have opposite effects on bids in the hybrid mechanisms.

The relationships between equilibrium bids suggest differences in expected auction revenues across mechanisms (and risk preferences). The next subsection addresses these differences, which are important to the seller who may care about the relationships between equilibrium bids to the extent that they inform revenue differences.

C.   Auction revenues

In order to compare the actual and expected revenues for the allocation mechanisms, it is convenient to rank bidders from lowest to highest valuation such that $v_1 \leq v_2 \leq ... \leq v_N$. Our discussion of hybrid mechanisms adapts Cox et al. [1985] who derive revenue expressions for discriminative and uniform price auctions. Consider actual revenues first. In the case of the uniform price hybrid mechanism with risk neutral bidders, actual revenue equals the first bid rejected times the number of units auctioned, *Q*. Therefore, actual revenue, *R*, for the uniform price hybrid mechanism under risk neutrality is:

$$R_{UH,N} = Q\left(1 - \frac{S-Q}{N-Q}\right)v_{N-Q} \tag{13}$$

where $\left(1 - \frac{S-Q}{N-Q}\right)v_{N-Q}$ represents the first bid rejected. Actual revenue in the discriminative hybrid mechanism with risk neutral bidders equals the sum of the highest *Q* bids or

$$R_{DH,N} = \sum_{j=N-Q+1}^{N} b_{DH,N}(v_j) \tag{14}$$

where bids are ranked from lowest to highest, $b_{DH,N}(v_1) \leq b_{DH,N}(v_2) \leq ... \leq b_{DH,N}(v_N)$. Under risk aversion, the following equations describe actual auction revenues in the uniform price and discriminative hybrids respectively:

$$R_{UH,A} = Q \cdot \widetilde{b}_{UH,A} \tag{15}$$

$$R_{DH,A} = \sum_{j=N-Q+1}^{N} b_{DH,A}(v_j) \tag{16}$$

where $\widetilde{b}_{UH,A}$ denotes the first rejected bid in the uniform price hybrid with risk averse bidders.

In terms of expected auction revenues, Harris and Raviv [1981, pp. 1492-1493] prove that with risk neutral bidders expected revenue in the discriminative auction equals expected revenue in the uniform price auction. Proposition 6 extends Harris and Raviv's revenue equivalence result to the hybrid mechanisms discussed here.

Proposition 6: $E(R_{UH,N}) = E(R_{DH,N}) < E(R_{DH,A})$ where $E(R_{UH,N})$ and $E(R_{DH,N})$ represent expected revenues for the uniform price hybrid and discriminative hybrid (both with risk neutral bidders) respectively.

Proof: See appendix.

With risk averse bidders, if the addition of the lottery reduces bids more in the uniform price setting than in the discriminative setting, expected revenues for the discriminative hybrid exceed expected revenues for the uniform price hybrid. This is in line with the established result that a (risk-neutral) seller facing risk averse bidders prefers the discriminative (or first-price) auction to the uniform price (or second-price) auction. Determining whether this condition is satisfied requires specifying a utility function and value distribution.

The experimental design described in section III permits empirical tests of the revenue hypotheses stated as Proposition 6. Tests of additional revenue hypotheses that stem from the theory are stated and proved in the appendix.

III. Experimental Design

The main objective of the experiments is to allow empirical tests of the theoretical propositions put forth on bidding behavior and revenue generation for hybrid allocation mechanisms. In all but the uniform price auction, theory suggests interaction effects between bidding behavior or revenue generation and risk preferences. Thus, how to elicit or otherwise control for participant risk preferences is a central issue. For first-price auctions, one common approach is to assume bidders have utility functions that exhibit constant relative risk aversion (CRRA) and use a structural model to estimate the risk aversion parameter directly from bidding data.[10] Risk preferences can thus serve as an additional degree of freedom when estimating bid functions. If some bidders engage in ad hoc bidding strategies that are spuriously consistent with risk-averse Nash equilibrium bidding, this confounds estimated risk parameters and conclusions about the role of risk aversion drawn from them. We are unaware of appropriate structural methods for the multiple unit auctions we consider (and, likewise, the hybrid allocation mechanisms).

As an alternative, we directly elicit risk preferences from a separate experiment and use this to control for risk attitudes in our analysis. The validity of this procedure pivots on whether risk attitudes are similar across the risk elicitation and allocation mechanism experiments. Existing research suggests that risk preferences can vary across institutions (e.g., Isaac and James

[10] An alternative approach is to induce risk-neutrality (Walker et al. [1990]), but, consistent with the theory, we wish to allow the possibility that experiment participants are risk averse.

[2000], Berg et al. [2005]), and over time within the same institution (James [2007]). Further, James [2007] provides evidence that elicited risk preferences may correlate with confusion. Two important distinctions of the present study are that our analysis does not rely upon a particular utility specification to elicit risk preferences and the procedure used to elicit risk attitudes is reasonably similar to the hybrid mechanisms of interest, e.g., they both involve lotteries.

Risk elicitation is through the lottery-choice procedure of Holt and Laury [2002], with three important modifications.[11] First, to reduce the effects of potential participant confusion, participants make choices between a certain payoff and a lottery, rather than between two lotteries. Second, the first and last decision tasks involve a choice between two certain (different) payoffs. This allows a rudimentary examination of experiment transparency and saliency. Third, as Holt and Laury [2002] find an interaction between payoff levels and risk, and Rabin's [2000] critique casts doubt on whether any utility function (assuming expected utility theory) can explain behavior over a wide range of gambles, the expected payoff from the risk preference experiment is on par with the payoff in the allocation experiment for successful bidders. To equate these expected payoffs we necessarily use a higher experimental $ to US $ exchange rate in the hybrid allocation experiments relative to the pure auction experiments.

Table 1 presents the eleven decision tasks in the risk preference experiment. As a reference point, Table 1 also presents the implied range of the coefficient of relative risk aversion, *r*, for the CRRA utility function $U(x) = x^{1-r} \big/ 1-r$. Specifically, the bound on *r* that corresponds with a particular decision task is for an individual who switches from the certain payoff (Option A) to an uncertain payoff (Option B) at this gamble. Choices are simultaneous

[11] The Holt and Laury [2002] procedure has been widely employed in experimental laboratory settings (see for example Eckel and Wilson [2004] and Lusk and Coble [2005]) and in field experiments (see for example Harrison et al. [2005] and Harrison et al. [2004]).

(without feedback) and one randomly determined decision task determines payoffs. To prevent spillover effects, announcement of the elicitation experiment outcome occurs after the completion of the allocation experiment.

The allocation experiment consists of N = 12 players bidding for a "good" in a series of 20 independent, multiple unit pure auction or hybrid allocations (i.e., decision periods). In a specific session, players face only one of four allocation mechanisms: uniform price auction, discriminative auction, uniform price hybrid mechanism, and discriminative hybrid mechanism. Inclusion of the pure uniform price and discriminative auctions serves to establish a baseline from which to evaluate lottery effects. Prior to each decision period the individual receives an induced value for a unit of the good, determined by a random draw from a uniform distribution on the interval [$0.00, $20.00].[12] The value distribution is common knowledge, value realizations are private information, and there is a new independent draw for each player in each period. Bids are constrained to be within this same range, which is common in the experimental auction literature (e.g., Cox et al. [1984, 1985]). A minimum bid of zero represents a rational institutional constraint as it insures nonnegative revenues. The maximum bid imposes a rationality constraint and partially avoids extreme negative earnings that may stem from excessive overbidding. This is especially important for initial auctions in an experiment with inexperienced bidders.

The quantity for sale in each auction, Q, is announced prior to bidding. Q is either 1, 2, or

[12] Unfortunately there was a programming error in some initial sessions whereby the values were drawn from the distribution U ~ [$5.00, $20.00]. We conjecture that this had little or no effect on bidding behavior. In particular, it would be rather difficult for a bidder to determine that her random draws, and those of other participants, were in fact coming from a slightly different distribution. This is especially true since players with values between $0 and $5, if they indeed were generated, given the number of auctioned units in relation to the number of players, would not be expected to influence observed auction prices. We compared behavior between sessions with and without the correct distribution employed and found no meaningful differences, in both a statistical and a qualitative sense. Throughout our analysis, in calculating optimal bids, we assume agents behaved as if they faced the announced versus the actual value distribution.

3, with each amount having an equal chance of being selected. Exactly one unit is allocated to each of the highest Q bidders. Further, for hybrid auctions, the number of available lottery units, $L \equiv S - Q$, is announced. L is either 1, 2, or 3 and each amount has an equal chance of being selected. Only those who do not win the auction are eligible for the lottery, and all entrants have an equal chance of winning the lottery. The decision period concludes with an announcement to each player of whether they won the auction (or lottery), earnings, all successful auction bids, and price(s) paid by all successful bidders. A history of prior auction outcomes is given prior to each auction.

A. Equilibrium Bid Functions

To characterize theoretical bidding patterns based on the experimental design we derive equilibrium bid functions for the hybrid mechanisms in the risk neutral case and using the CRRA utility function presented above. For the discriminative auction with risk neutral bidders, we rely on the equilibrium bid function derived by Cox et al. [1984]. Cox et al. [1982] derive the equilibrium bid function for the discriminative auction when bidders have heterogeneous CRRA preferences. We adapt their model to the case of homogeneous bidders with CRRA preferences. We solve for the uniform price hybrid bid functions using equation (3). For the discriminative hybrid mechanisms under CRRA, we derive equilibrium bid functions with the technique used to prove Proposition 4. In particular, we note that $b_{DH,CRRA}(v) = b_{D,CRRA}(b_{UH,CRRA}(v))$.

Table 2 presents the equilibrium bid functions for the hybrid mechanisms, and for purposes of comparison the analogous pure auction bid functions. Further, the characteristics of the bid functions are illustrated in Figures 1 and 2 based on our experimental design parameters. Figure 1 shows how risk neutral bid functions vary with respect to the probability of winning the lottery. In particular, for both hybrid mechanisms, Figure 1 presents bid functions for the unit combinations {Q=1, L=1}, {2, 2}, and {3, 3}. This spans the range of experiment parameters, for which the probability of winning the lottery (conditional on being in the lottery) varies from 1/11 (9.1%) to 3/9 (33.3%). As a baseline, the uniform price auction and discriminative auction (Q=1) are also presented. For the uniform price hybrid, bid functions are linear, and increasing the probability of winning the auction serves to decrease the slope of the bid function in relation to value. For the discriminative hybrid, bid functions are linear for Q=1 and concave for Q>1.

Figure 2 shows the effect of risk preferences, using the CRRA utility specification with Q,L=2 for purposes of illustration. In particular, uniform price and discriminative hybrid bid functions are presented under moderate risk aversion (r=0.5), risk neutrality, and moderate risk seeking (r=-0.5). We note that the curvature of the functions is the same as under risk-neutrality. The functions suggest quite pronounced risk effects. For either mechanism, for example, the slope of the bid function decreases by about 0.3 between the moderate risk seeking and moderate risk aversion cases.

B. Participant Pool and Procedures

In the spring and summer of 2006, 144 students recruited from the general population at the University of Tennessee participated in one of 12 experimental sessions conducted at the Experimental Economics Laboratory. There are three sessions (replications) of each mechanism. Approximately 55% of subjects had previously participated in an economics experiment. Seventy-five percent of participants have experience bidding in non-experimental auctions (such as eBay). Average earnings were approximately $20, paid in cash at the conclusion of the session. Sessions lasted approximately one hour.

Decisions are made via laptop computers. The experiments are programmed and conducted with the software z-Tree [Fischbacher, 2007]. The software collects all decisions and makes all relevant earnings calculations. Written instructions are provided to each participant and displayed on-screen. The experiment moderator reads instructions aloud, one screen at a time, and answers any questions prior to proceeding to the next instruction screen. After instructions for the allocation experiment are read, participants are asked to answer three questions (using pencil and paper) to assess their understanding of the auction mechanism. Participants are paid US$0.75 for each correct answer. An experiment coordinator privately checks answers for each individual, and re-explains procedures and fields questions in the case of wrong answers. Following the short quiz, there are three unpaid practice rounds. Questions are encouraged and addressed.

In pilot sessions we observed some considerable overbidding in uniform price auction treatments. With Q small relative to N, the probability of positive earnings in any auction is small. As such, it may be quite difficult to overcome a large loss due to gross overbidding in any period. Even though a participant is likely to learn from such a loss, a few individuals in pilot experiments engaged in relatively perverse bidding behavior after incurring a large loss, likely motivated by the remote opportunity to become solvent. To mitigate such behavior, the first time a participant incurred a large loss (~$5), which occurred for 7 of 144 participants, we refunded the loss. A coordinator discretely conveyed this information to the participant, and stated that this was a one-time measure. This procedure was not disclosed prior to the experiment. Upon the conclusion of the allocation mechanism experiment, a short questionnaire was administered that included among other things an assessment of how well instructions were understood and prior participation in auctions.

IV. Results

A. Risk Preference Elicitation

The last column of Table 1 presents risk elicitation experiment results. Overall, the decision tasks presented in the experiment appear to be well understood by most participants. In particular, of the two decision tasks involving certain payoffs, just 8 of 144 of participants choose the option with the lowest payoff in either of these. Consistent with Holt and Laury [2002] there is a low incidence of apparent preference reversals, as only 19 of 144 switched from Option A to B and then back to A.[13]

Excluding the two choices between certain payoffs, the mean and median number of safe choices (Option A) selected are 5.9 and 6, respectively. Referring to Table 1, this suggests that the representative individual chooses Option A for decision tasks 1 through 7, and then chooses Option B for the remaining tasks. About 16%, 23%, and 61% of participants can be characterized as risk loving, risk neutral and risk averse, respectively. We use the number of safe choices to construct a measure of risk attitude in the analysis that follows.

B. Bidding Behavior

The functional form and included model covariates for the econometric bid model are motivated by the equilibrium bid functions described in III.A. and illustrated in the Figures, with the effects of the lottery measured relative to the corresponding pure auction treatment. For the uniform price auction, the specification is simply the auction bid as a linear function of an intercept and induced value

$$Bid_{ij} = \alpha + \beta * Value_{ij} + \varepsilon_{ij} \tag{17}$$

where ε_{ij} is a random error term corresponding with individual i in allocation period j. The equilibrium bid function (Table 2) is exactly recovered when α=0 and β=1. The probability of

[13] In the analysis that follows, for these individuals with preference reversals, as an approximation we use the total number of safe choices indicated. We note that treating responses in this manner, or omitting them completely, makes no real difference in this analysis.

winning the lottery is included as an interaction term with value, $Value_{ij} * \left(\frac{S-Q}{N-Q} \right)$, which allows the slope of the bid function to vary with this probability. Further, the slope of the estimated bid function varies by risk attitude through inclusion of an interaction between an elicited risk preference measure with induced value, $Value_{ij} * Risk_{ij}$. The risk measure is simply the number of safe choices (excluding choices between certain outcomes) selected in the risk elicitation experiment minus five. This gives the risk measure a value of zero for risk neutrality and larger positive (negative) values for higher degrees of risk aversion (seeking). Further, this is consistent with the theory that suggests no risk effect for risk neutrality, and opposite effects for risk seekers and averters.[14] The intercept varies across uniform price auction and uniform price hybrid treatments as, for instance, under certain conditions uniform price hybrid participants may have negative optimal bids. The estimable bid function for the uniform price hybrid is thus

$$Bid_{ij} = \alpha + \beta * Value_{ij} + \lambda * Value_{ij} * \left(\frac{S-Q}{N-Q} \right) + \gamma * Value_{ij} * Risk_{ij} + \varepsilon_{ij} \qquad (18)$$

The risk neutral equilibrium bid function is exactly recovered with α=0, β=1, λ=-1, and γ=0.

The functional form and included covariates are similar for the case of the discriminative auction and hybrid. The main difference is that, in contrast to the uniform price auction, both the number of auction units and risk preferences affect discriminative auction bidding. Although for Q>1 the equilibrium bid function is nonlinear, the Figures suggest that the curvature is slight and that a linear approximation is reasonable. The econometric specification has the discriminative auction bids as a linear function of an intercept, induced value, an interaction between induced value and the number of units, and an interaction between induced value and the risk measure.

[14] To make our bid and revenue analyses consistent under our theoretical assumption of homogeneous risk preferences, we assume that bidder *i* perceives all other bidders as having risk preferences identical to her own. This is a palatable assumption given that participants have no clear signals on the distribution of group risk preferences.

For the discriminative hybrid, we include two additional interaction terms, a risk-value interaction term and an interaction between induced value and the probability of winning the lottery. That is to say, the estimable bid function for the discriminative hybrid has identical form to equation (18) with the inclusion of a value-auction unit interaction, $\theta * Value_{ij} * Q$.

Recall that auction bids are constrained to be between the minimum and maximum values of the value distribution. A modest fraction of bids (about 10%) are equal to the supports of the distribution, mostly zero bids in the hybrid treatments. We treat these as censored observations in the sense that individuals may justifiably choose to bid below (above) the lower (upper) support in the absence of such constraints. For instance, overbidding is common in experimental uniform price auctions, where the winning bidders do not set the price. Kagel and Levin [1993] discuss one possible explanation for the overbidding—individuals may see it as strategic to bid above their values as doing so increases the probability of winning, and in more cases than not the realized uniform price lies below the over-bidder's valuation. Also, as illustrated in Section II.A., some participants in some hybrid mechanism auctions may have optimal bids that are negative. Given that individuals cannot officially withdraw from the auction, and that bids are constrained to be non-negative, some zero bids may come from individuals who would have bid negative if given the option. We account for the double-censored data, with individuals who participate in 20 auctions, using a random effects Tobit model (the random effect is participant-specific).[15] Data from all treatments are pooled.

[15] We also estimated standard random and fixed effects models that ignore the censoring. Random and fixed effects estimates are similar to one another (Hausman Test: p=0.33), and all estimated slope coefficients have the same sign, and are of similar magnitude and statistical significance when compared with the random effects Tobit. The main difference is that the estimated intercept for the discriminative hybrid is no longer statistically significant in the standard random effects model (these intercepts drop out of the fixed effects model due to perfect collinearity with the fixed effects).

Table 3 presents the estimated bid functions.[16] We report a measure of fit, Pseudo-R^2, of 0.789, calculated as the squared correlation between the actual and the estimated conditional mean for noncensored observations (see Dhrymes [1986]). The two main results, consistent with Propositions 1 through 5, are that the lottery reduces bids and risk aversion increases bids. In particular, the presence of the lottery, as measured by the probability of winning the lottery, decreases the slope of the bid function for both formats. The effects are statistically significant at the 1% level. We note that this effect is robust to inclusion/exclusion of variables related to our risk measure. Further, at the 1% significance level, risk aversion (seeking) increases (decreases) the slope of the bid function for both the uniform price and discriminative hybrid. Overall, there appears to be merit in using an elicited risk preference measure in a subsequent experiment that can be argued as being reasonably close in terms of the nature of the risk.

The slope of the uniform price bid function is not statistically different than one, indeed it equals 0.99, suggesting a near perfect correlation between bids and value. Further, we comfortably fail to reject the null joint hypothesis of unit slope and zero intercept [χ^2(2 d.f.)=0.61, p=0.74]. Many experimental second-price auction studies find evidence of overbidding. However, our results are similar to Garrat et al. [2005], who use participants known to have substantial experience in eBay auctions. Three-fourths of our subjects report having previous auction experience. Evaluated at the mean risk measure (Risk=0.85) and mean number of auction units (Q=1.98), the slope of the discriminative auction bid function is 0.96, and is statistically different from 1 [z=-2.39, p=0.02]. The intercept is equal to -0.57 and is not statistically different from zero. Inconsistent with the theory, there is no statistically significant effect of risk preference on the slope of the discriminative auction bid function. The number of

[16] We initially estimated bid and revenue models that controlled for individual characteristics such as age, previous experiment participation and previous auction participation. These effects are jointly equal to zero and omitted from our final models.

auction units also has no discernable effect. Although the auction-unit effect is peculiar, we have no basis of empirical comparison as we know of no studies that vary the number of auction units within session.

C. Revenue

Our bidding analysis, as well as theory, suggests that we should expect lower revenue from the hybrid allocation mechanisms. Tests of revenue data, however, are quite capable of yielding results that differ from the tests of individual bidding (see Cox, Smith, and Walker [1984]), and so we undertake an analysis of revenue that parallels our bidding analysis. As noted by Tenorio [1993], although revenue hypotheses are stated in *ex ante* terms, empirical tests commonly rely on *ex post* realizations. We follow a similar convention here.

The revenue functions presented in Section II.C have a similar form to equilibrium bid functions. The revenue function for the uniform price auction and hybrid is the number of auction units, Q, multiplied by the bid of the individual with the $(N-Q)^{th}$ highest value, denoted v_{N-Q}. Thus, for the uniform price auction the revenue function is simply $Q \times v_{N-Q}$. As the probability of winning the lottery and risk preferences shift the slope of the bid function in relation to value for all bidders, including the one with the $(N-Q)^{th}$ highest value, it follows that these factors shift the revenue function in relation to $Q \times v_{N-Q}$. The revenue function for the discriminative auction and discriminative hybrid is the sum of the top Q bids. As equilibrium bid functions, for our parameters, are approximately linear functions of value, the revenue function becomes approximately linear in relation to the sum of the top Q values, $\sum_{j=N-Q+1}^{N} v_j$. Similar to how the probability of winning the lottery and risk preferences change the slope of the bid function in relation to value, these factors shift the revenue function relative to $\sum_{j=N-Q+1}^{N} v_j$.

An important consideration is how to include risk in an analysis of revenue, which depends on the risk attitudes of several individuals. Theoretically, for homogeneous risk preferences, those with the highest Q values are auction winners, and the individual with the Q+1 highest value sets the uniform price. As an approximation here, we assume that individuals with the top Q+2 values are important in determining revenue. [17] We include measures of risk by averaging over the individuals with the top Q+2 values. In contrast to session-specific averages, the measures utilized here necessarily vary across the 20 auctions within a session.

Table 4 presents estimated revenue functions. The estimator is a standard random effects model, rather than a random effects Tobit, as there is no obvious censoring issue, e.g., there are no $0 revenue outcomes. The two main observations are that the probability of winning the lottery has a negative and statistically significant effect on revenue for the discriminative hybrid only, and there is no significant relationship between average risk and the revenue function of either hybrid mechanism. Curious about this finding, we re-estimated our bid functions while restricting the sample to those with high values. In doing so, we find that the risk effects do likewise go away for the hybrid mechanisms, but the probability of winning the lottery remains negative and statistically different from zero for both hybrid mechanisms. We fail to reject the null hypothesis that all risk coefficients are jointly equal to zero [$\chi^2(3)=2.25$, $p=0.52$].

The estimated revenue function for the uniform price auction has a slope of 1.04 and an intercept of 0.89. These parameters are jointly statistically different from the theoretical revenue function which has unit slope and zero intercept [$\chi^2(2)=24.45$, $p<0.01$]. The actual difference is

[17] This measure seems reasonable given that, as discussed later, the allocation mechanisms are highly efficient, which means those with the highest values tend to be auction winners.

small in magnitude, as estimated revenue is about 5% higher than predicted revenue. An interesting finding is that the estimated uniform price hybrid revenue function is statistically identical to that from the uniform price auction [$\chi^2(2)=1.92$, $p=0.38$]. Thus, although there are very strong lottery effects on bidding behavior, these effects do not translate into lower revenues.

Turning to the discriminative auction, we find weak evidence that the discriminative auction generates higher revenue than the uniform price auction. Note that the slopes of the two revenue functions are not directly comparable since they are relative to different value measures. Instead, we compare the two by comparing the estimated revenues based on the expected values for $Q \times v_{N-Q}$ and $\sum_{j=N-Q+1}^{N} v_j$ for the value distribution employed in the experiment (i.e., Uniform[0, 20]). Estimated revenue for the discriminative auction is \$0.19 lower for $Q=1$ [$z=-0.21$, $p=0.84$], \$0.46 higher for $Q=2$ [$z=0.53$, $p=0.59$], and \$3.11 higher for $Q=3$ [$z=3.08$, $p<0.01$].[18] Our results are thus approximately consistent with theory under the assumption of risk-neutrality. As mentioned above the lottery serves to decrease discriminative auction revenues. For $Q=2$, for instance, the model suggests that expected revenue decreases by \$0.77 for each additional unit allocated by lottery. Putting this into perspective, the expected difference in revenue based on the theoretical revenue functions under risk neutral bidding is \$2.87.

Although the discriminative auction revealed a slight revenue advantage over uniform price, the advantage disappears in the hybrid setting due to insignificant lottery effects in the uniform price hybrid. Evaluated at the means of the covariates, for seven of the nine Q, L combinations employed in the experiment, revenues from the two hybrid mechanisms are not statistically different. In one of the two cases of statistical difference $\{Q=3, L=1\}$ the

[18] Although the discussion here and in the remainder of this subsection suggests directional (i.e., one-sided) alternative hypotheses, the p-values reported are for two-sided tests.

discriminative hybrid is higher by \$2.27 and in the other $\{Q=2, L=3\}$ the uniform price hybrid is higher by \$1.96. This (approximate) revenue equivalence is consistent with the theory under risk neutral bidding. Although revenue differences for the hybrid mechanisms in the general case of risk aversion depend on the form of the utility function, for CRRA expected revenue is higher for the discriminative hybrid.

D. Efficiency

Under certain assumptions, both the pure auction and the auction *component* of the hybrid mechanisms we examine are theoretically efficient, in the sense that *auction* units accrue to those with the highest valuations. No assumptions about risk preferences are needed for the uniform price auction, but for the other allocation mechanisms efficiency depends upon homogeneous risk preferences with risk neutrality as a special case. The analysis above suggests some heterogeneity in risk preferences, although the revenue analysis suggests that the average winning bidder bids as if risk neutral. We estimate the following efficiencies, defined as the percentage of available economic surplus captured in the auction, which in this setting is simply the sum of the values for winning bidders as a percentage of the sum of values for those with the Q^{th} highest valuations: uniform price auction = 95.69% (robust s.e. = 1.69); discriminative auction = 97.78% (1.33); uniform price hybrid = 95.77% (1.09); discriminative hybrid = 97.52% (0.08). Previous uniform price and discriminative auction experiments yield efficiencies in the neighborhood of 90 to 100%, and our results are in agreement. Comparing like pure auction and hybrid mechanisms, they are within a fraction of a percent. Hence, apparently no behavioral anomalies render the auction mechanism inefficient with the addition of the subsequent lottery.

V. Discussion

Motivated by the increasing use of combined price and non-price instruments for allocating publicly provided goods, this paper provides first insights on their theoretical and empirical properties. In particular, we investigate “hybrid” mechanisms that allocate a portion of available units via auction and the remainder through a random lottery. We consider multiple unit uniform price and discriminative auction formats, for bidders with independent private values, singleton demands, and homogeneous risk preferences.

Theoretically, the lottery serves as an opportunity cost of winning the good through the auction, and thus reduces equilibrium auction bids and revenues relative to the no lottery case. The lottery also introduces an additional source of risk, which serves to increase bids under risk aversion relative to the risk neutral case. Thus, unlike Nash equilibrium bidding in the pure uniform price auction, which is independent of risk attitudes, equilibrium bidding in the uniform price hybrid mechanism varies with risk preferences. Under homogeneous risk preferences, the auction components of the hybrid mechanisms remain efficient.

Results from laboratory experiments with induced values largely support the theoretical propositions put forth. A departure from other auction studies that consider risk preferences, we elicit individual bidder risk preferences from a separate lottery-choice experiment and use these measures in our econometric estimation of bid functions. There appears to be merit in this more direct approach, as the risk measure is statistically significant, and shifts the slope of the empirical bid functions in the anticipated direction, for both hybrid mechanisms. One interesting finding is that the risk measure does not correlate with bids for the pure discriminative auction, and this adds to the debate on whether risk parameters estimated directly from auction data are actually capturing risk preferences.

The auction component of the hybrid mechanism remains highly efficient in our experiments. However, our revenue analysis reveals that auction winners engage in different bidding strategies in the process. In particular, we find that there no longer is a relationship between risk preferences and revenue generation for either hybrid mechanism. For the uniform price hybrid, this disparate bidding behavior from winners is even more pronounced as the lottery has no discernable effect on auction revenues. An extended experimental design coupled with a rich behavioral model may be able to provide valuable insight on observed bidding patterns. This would appear to have import for auction behavior in other situations where auction bidders have an outside but uncertain opportunity to obtain the good, such as when an auction bidder can participate in a subsequent auction for the same good. We conclude the paper with discussion of three other possible avenues for future research.

First, our theoretical results assume homogeneous risk preferences. In light of recent empirical work that suggests important differences across individuals in risk preferences, a useful extension of our work would be to allow for this. Allowing for heterogeneity in risk preferences among bidders renders the auction component of both hybrid mechanisms asymmetric and as such inefficient [Cox et al., 1982; Krishna, 2002]. That is, with heterogeneous risk preferences, the hybrid auction-lottery mechanisms we examine no longer guarantee that auction units go to bidders with the highest values. In terms of the discriminative hybrid, this is an expected result since the discriminative auction is inefficient in general with asymmetric bidders. In contrast, the pure uniform price auction is efficient under heterogeneous risk preferences (i.e., equilibrium bid functions remain independent of risk preferences). However, the addition of the lottery in the uniform price hybrid reverses this result. The intuition is straightforward. Since risk aversion increases equilibrium bids in both hybrid mechanisms, in a model with heterogeneous risk preferences, a risk-averse bidder with a lower value could outbid a risk-neutral bidder with a higher value.[19]

Second, our theoretical analysis and experimental design assume no entry or user fees for the auction or lottery. However, as fees are in place in some of the example mechanisms we cite, an exploration of their effects is a useful extension of our model. Here, we take preliminary steps towards this goal by examining the effects of a user fee for lottery winners on auction participation and equilibrium bids in the uniform price hybrid. Let τ denote a user fee for lottery winners. The participation constraint with this modification becomes:

$$U\left(v_i - b_{UH,A}\right) = pU\left(v_i - \tau\right) + \left(1 - p\right)U\left(0\right). \tag{19}$$

A comparison of equations (3) and (19) suggests that the presence of (or an increase in) the user fee for lottery winners encourages participation in the auction. Similarly, for those values of $b_{UH,A}$ for which the participation constraint holds, implicit differentiation of expression (19) suggests that a higher user fee for lottery winners increases equilibrium bids in the uniform price hybrid:

$$\frac{db_{UH,A}}{d\tau} = \frac{pU'\left(v - \tau\right)}{U'\left(v - b_{UH,A}\right)} > 0 .$$

These results are intuitive as the presence of a user fee for lottery winners decreases the opportunity cost of winning the auction. Similar comparisons allow an exploration of the impacts of other factors, such as risk preferences, entry fees, and changes in the probability of winning the lottery on the participation constraint and on equilibrium bidding behavior. An experimental design that allows bidders to opt out of the auction component of the hybrid mechanism would be a valuable complement to a theoretical exploration of these issues.

[19]We thank an anonymous reviewer for bringing this result to our attention. When we allow for heterogeneous risk preferences among bidders, the efficiency of the hybrid mechanisms depends on the distribution of values and the distribution of risk preferences among bidders. A formal example for the uniform price hybrid under heterogeneous CRRA preferences is developed in the appendix to illustrate the inefficiency of the uniform price hybrid mechanism in this setting.

A final related extension involves the optimal design of a hybrid mechanism. Some important features include the optimal distribution of units between the auction and lottery, the optimal user and/or entry fees, and the optimal order of the auction and lottery components of the mechanism. Analysis of these issues requires an explicit model of the seller's objective. Although revenue maximization is the typical assumption, a revenue-maximizing seller would not choose a hybrid mechanism. Therefore, as noted earlier, the use of such mechanisms suggests the presence of more complex motives on the part of the seller. A political economy model of the mechanism design problem facing the seller would allow an exploration of these issues and represents an important extension to the results we present here.

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Table 1. Parameters and Results of Risk Elicitation Experiment

Decision Task	Option A	Option B	CRRA coefficient of relative risk aversion (*r*)	Proportion of Participants
1	Receive \$3.00	0% chance of \$5.00, 100% chance of \$0.50	–	–
2	Receive \$3.00	10% chance of \$5.00, 90% chance of \$0.50	[-∞, -3.508]	0
3	Receive \$3.00	20% chance of \$5.00, 80% chance of \$0.50	[-3.507, -2.146]	0
4	Receive \$3.00	30% chance of \$5.00, 70% chance of \$0.50	[-2.145, -1.336]	0.014
5	Receive \$3.00	40% chance of \$5.00, 60% chance of \$0.50	[-1.335, -.742]	0.042
6	Receive \$3.00	50% chance of \$5.00, 50% chance of \$0.50	[-.741, -.250]	0.104
7	Receive \$3.00	60% chance of \$5.00, 40% chance of \$0.50	[-.249, .194]	0.229
8	Receive \$3.00	70% chance of \$5.00, 30% chance of \$0.50	[.195, .631]	0.306
9	Receive \$3.00	80% chance of \$5.00, 20% chance of \$0.50	[.632, 1.112]	0.174
10	Receive \$3.00	90% chance of \$5.00, 10% chance of \$0.50	[1.113, 1.758]	0.097
11	Receive \$3.00	100% chance of \$5.00, 0% chance of \$0.50	[1.759, ∞]	0.035

Notes: The risk coefficient corresponds to an individual that switches from the certain payoff (Option A) and the uncertain payoff (Option B) at this task.

Table 2. Equilibrium Bid Functions

	Risk Neutral: $U(x) = x$	CRRA: $U(x) = x^{1-r} / 1-r$
Uniform Price Auction	$b_{U,N} = v_i$	$b_{U,CRRA} = v_i$
Discriminative Auction	$b_{D,N} = \bar{v}\dfrac{N-Q}{N}\dfrac{I_{v_i/\bar{v}}(N-Q+1,Q)}{I_{v_i/\bar{v}}(N-Q,Q)}$	$b_{D,CRRA} = v_i - \bar{v}\left[I_{v_i/\bar{v}}(N-Q,Q)\right]^{\frac{-1}{1-r}} \cdot \int_0^{v_i/\bar{v}} \left[I_Y(N-Q,Q)\right]^{\frac{1}{1-r}} dY$
Uniform Price Hybrid	$b_{UH,N} = \left(1-\dfrac{S-Q}{N-Q}\right)b_{U,N}$	$b_{UH,CRRA} = \left(1-\left(\dfrac{S-Q}{N-Q}\right)^{\frac{1}{1-r}}\right)b_{U,CRRA}$
Discriminative Hybrid	$b_{DH,N} = \left(1-\dfrac{S-Q}{N-Q}\right)b_{D,N}$	$b_{DH,CRRA} = \left(1-\left(\dfrac{S-Q}{N-Q}\right)^{\frac{1}{1-r}}\right)b_{D,CRRA}$

Notes: $I_{v_i/\bar{v}}(N-Q,Q) = \dfrac{\Gamma(N)}{\Gamma(N-Q)\Gamma(Q)}\int_0^{v_i/\bar{v}} y^{N-Q-1}(1-y)^{Q-1}\,dy$ is the incomplete beta function.

Table 3. Estimated Bid Functions (Random Effects Tobit)

Variable	Parameter	Uniform Price	Uniform Price Hybrid	Discriminative	Discriminative Hybrid
Intercept	α	0.373 (0.478)	-0.463 (0.477)	-0.572 (0.456)	-2.672** (0.467)
Value	β	0.993** (0.018)	0.993** (0.018)	0.965** (0.022)	0.965** (0.022)
Value x Q	θ			0.005 (0.007)	0.005 (0.007)
Value x $\left(\frac{S-Q}{N-Q}\right)$	λ		-0.232** (0.074)		-0.299** (0.083)
Value x Risk	γ		0.038** (0.013)	-0.014 (0.012)	0.024** (0.010)
Log-L			-18806.349		
Pseudo-R^2			0.789		
N			2880		

Notes: A participant-specific random effect is assumed. Standard errors in parentheses. *, ** denotes coefficient is statistically different from zero at the 5% and 1% level, respectively.

Table 4. Estimated Revenue Functions (Random Effects Model)

Variable	Uniform Price	Uniform Price Hybrid	Discriminative	Discriminative Hybrid
Intercept	0.890 (0.969)	-0.807 (0.942)	2.047* (0.854)	0.572 (0.838)
$\tilde{v}$	1.039** (0.023)	1.039** (0.023)	0.878** (0.021)	0.878** (0.021)
$\tilde{v}$ x $\left(\frac{S-Q}{N-Q}\right)$		0.078 (0.088)		-0.209* (0.089)
$\tilde{v}$ x Risk (average)		-0.024 (0.018)	-0.006 (0.016)	-0.008 (0.013)
χ^2(10 d.f.)		5407.30		
R^2		0.958		
N		240		

Notes: Standard errors in parentheses. *, ** denotes coefficient is statistically different than zero at the 5% and 1% level, respectively.

For uniform price and discriminative treatments, $\tilde{v} = Q\, v_{N-Q}$ and $\tilde{v} = \sum_{j=N-Q+1}^{N} v_j$, respectively.

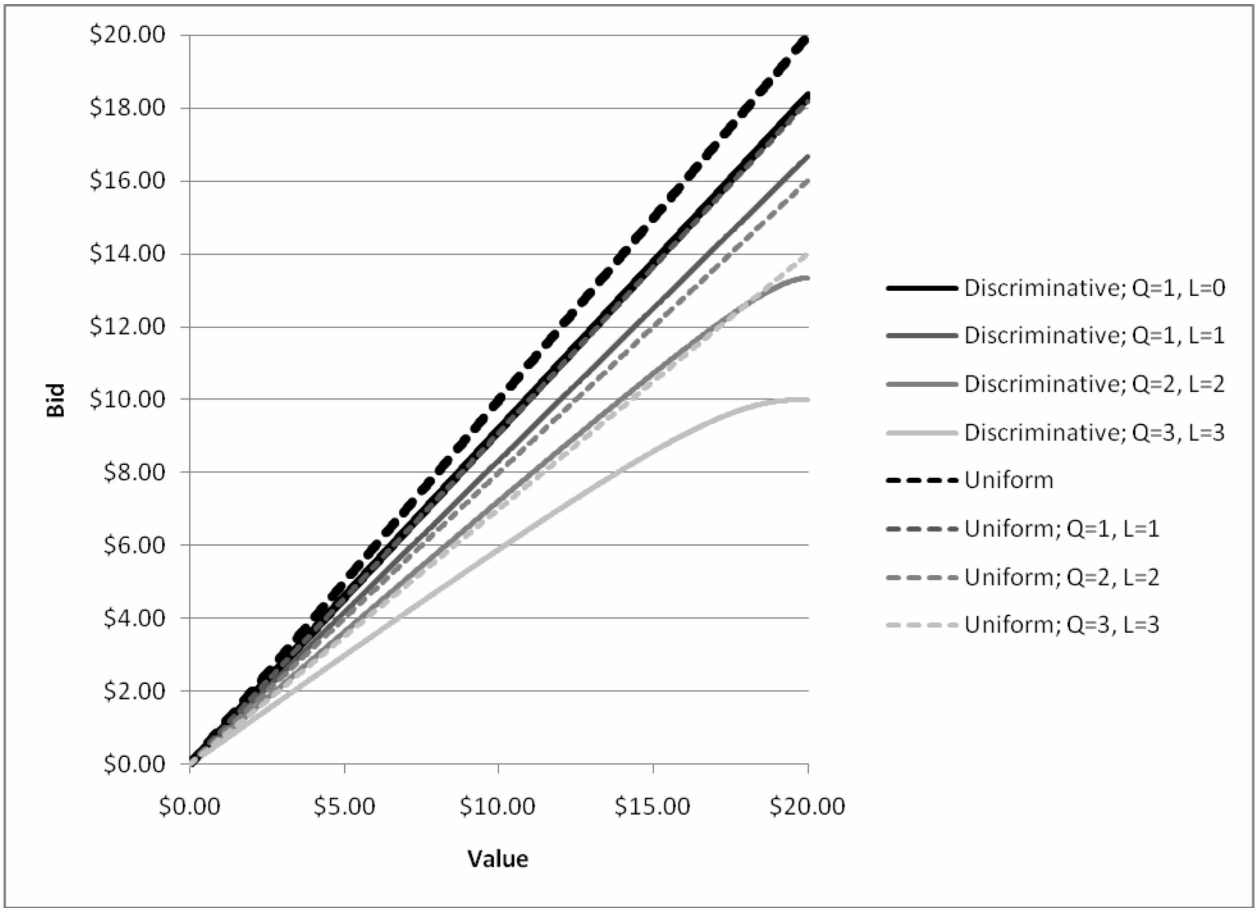

Figure 1. Equilibrium Bid Functions: Lottery Effects

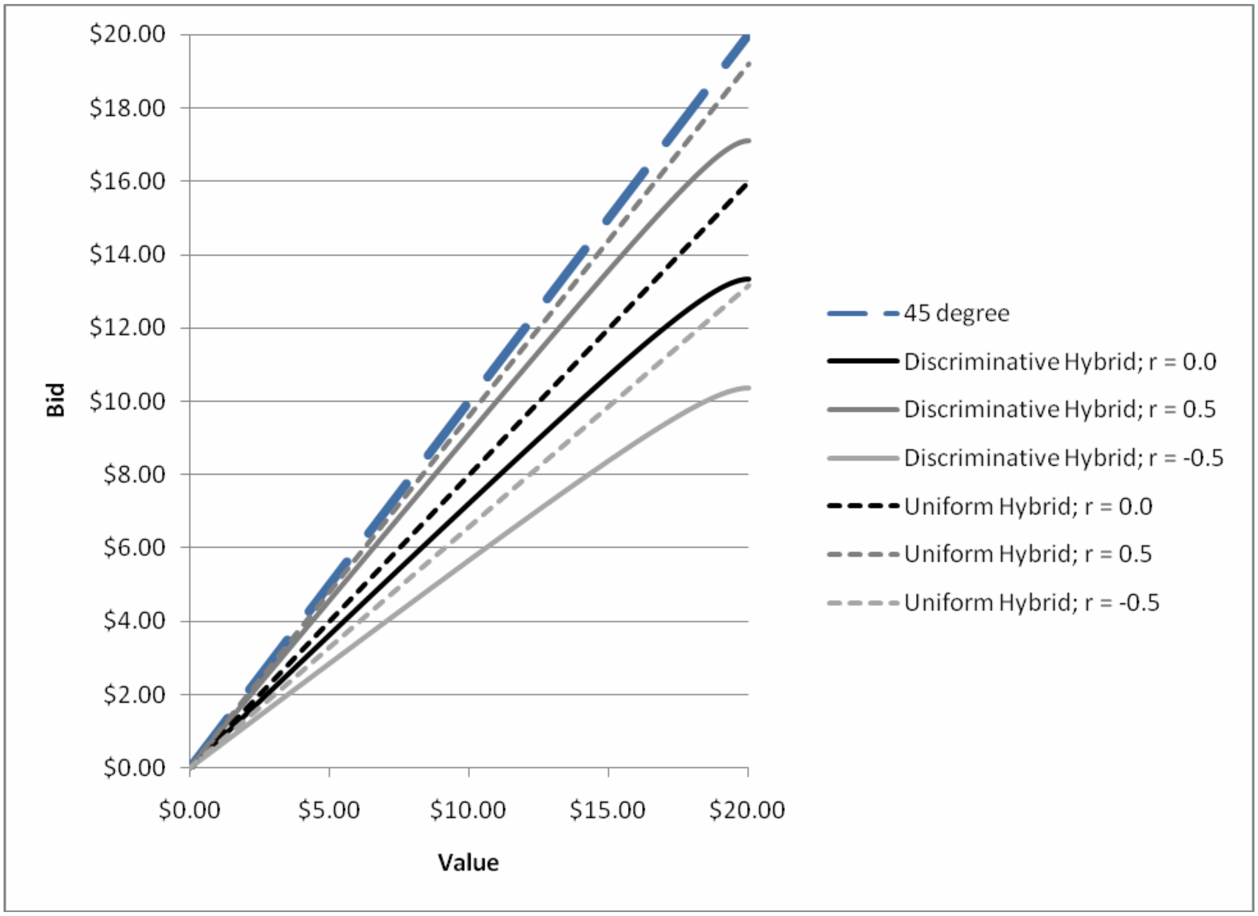

Figure 2. Equilibrium Bid Functions: Risk Effects