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Function of Several Variable

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Function of Several Variables

In 2014 spring semester, I have learnt about the function with one variable so that I want to explore the function with several variables. This essay mainly talks about the function with two variables and their properties.

Definition: f (x,y) Domain: Let D represent domain of f (x,y). The range of F consists of all real numbers f (x,y) Where (x,y) is in D.

Then we solve some problems.

I. Describe the domain of f, and find the indicated function value.

1. *f (u,v)*=uv/(u-2v)

   Domain of f is {(u,v):u≠2v}

   $f(2,3) = \frac{2\times3}{2-2\times3} = \frac{-3}{2}$

   *f (-1,4)= (-1)*4/-1-2*4=4/9*

   *f (0,1) =0*1/0=2*1=0*

2. *f(x,y,z)*=$\sqrt{25-x\text{^}2-y\text{^}2-z\text{^}2}$

   Domain of f is {(x,y,z): x^2+y^2+z^2≤25}

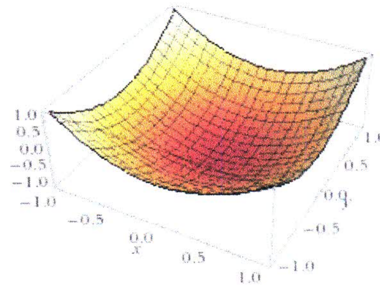
   f(1,-2,2)=$\sqrt{25-1\text{^}2-(-2)\text{^}2-2\text{^}2} = 4$

   f(-3,0,2)=$\sqrt{25-(-3)\text{^}2-0\text{^}2-2\text{^}2} = 2\sqrt{3}$

II. Sketch the graph of f.

1. *f(x,y)=x^+y^2-1*

Step: I use an online tool to graph the following graph which is in www.wolframalpha.com



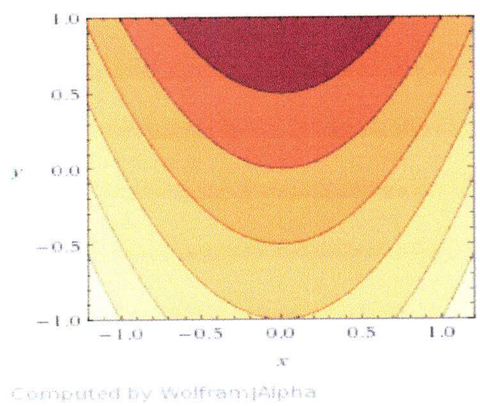
III. Sketch the level curve of f for the given value of K.

1. *f(x,y)=x^2-y k=0*

Step: we should let *x^2-y=K=0*

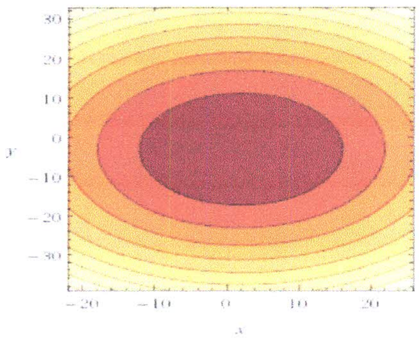
So we get the *y=x^2*

Then the curve through the origin (0,0) is the level curve of f when the *K=0*



2. *f(x,y)=(x-2)^2+(y+3)^2 k= 1,4*

Step: same like the first one.



IV. Find the equation of the level curve of f that contains the point P.

f (x,y)=y arctanx P(1,4)

Solve:

Step 1: Put the P (1,4)into the function f . *4*arctan 1=K then K=* π

Step 2: Then we find the equation of the level curve of f is *y arctanx =*π

V. Then we can find the equation of the level curve of f which with three variables.

f (x,y,z)=x^2+4y^2-z^2 P (2,-1,3)

Step1: *f (2,-1,3)=2^2+4(-1)^2-3^2=-1*

Step2: we find that the equation of the level curve of f is *x^2+4y^2-z^2= -1*

VI. Describe the level surface of f for the given values of K.

1. *f (x,y,z)=x^2+y^2+z^2   K= -1,0,4*

Solve:

For *K= -1 f (x,y,z)=x^2+y^2+z^2= -1* is not right, because *f(x,y,z)=x^2+y^2+z^2* $\geq$*0*. So there is no level surface if *K= -1*.

For *K= 0 f (x,y,z)=x^2+y^2+z^2=0* . We get the solution that *x=0,y=0,z=0.* So the level surface is the origin (0,0,0).

For K= 4 $f(x,y,z)=x^2+y^2+z^2=4$. We get the level surface is with center (0,0,0) and radium 2.

Limits of functions with several variables

Let the **function f of two variables** be defined throughout the interior of a circle with center (a,b), expect possibly at (a,b) itself. The statement Lim $f(x,y)= L$ means that for every $\epsilon > 0$ there is a $\delta > 0$ such tha $(x,y)\to(a,b)$

If $0 < \sqrt{(x-a)^2+(y-b)^2} < \delta$ then $|f(x,y) - L| < \epsilon$ (SWOKOWSKI)

We can use the definition to solve some kinds of problems.

TWO-PATH RULE:

If two different paths to a point P (a,b) produce two different limiting values for f, then Lim $(x,y)\to(a,b)\ f(x,y)$ does not exist.

I. Find the limit.

1. $lim_{(x,y)\to(0,0)}(x^2-2)/(3+xy)$

   Solve: We just need to put the approximate value of x,y into function $(x^2-2)/(3+xy)$.

   $(0^2-2)/(3+0*0)= -2/3$

   So the result is -2/3

2. $lim_{(x,y)\to(0,0)}(x^4-y^4)/(x^2+y^2)$

   Solve: We find that if we put x=0 and y=0 into the $(x^4-y^4)/(x^2+y^2)$, the denominator will be 0, so we cannot do that. However, we find that there is a same factor we can delete it. $(x^4-y^4)/(x^2+y^2)= (x^2+y^2)(x^2-y^2)/ (x^2+y^2)= (x^2-y^2)$.

   Then we can put the x=0 and y=0 into the (x^2-y^2). $(0^2-0^2)=0$

   So the result is 0.

II. Show that the limit does not exist.

1. $\lim_{(x,y)\to(0,0)}(2x^2 - y\text{^}2)/(x^2 + 2y\text{^}2)$

   Solve: We can use the two paths method. We need to find two different paths. If we get two different limits, it is showing that the limit does not exist.

   Let y=0 $\lim_{(x,y)\to(0,0)}(2x^2 - y\text{^}2)/(x^2 + 2y\text{^}2)$ =2

   Let y=x $\lim_{(x,y)\to(0,0)}(2x^2 - y\text{^}2)/(x^2 + 2y\text{^}2)$ = 0

   $0 \neq 2$ So the limit does not exist.

2. $\lim_{(x,y)\to(0,0)}\frac{3x^2y^3}{2y^5-2x^5}$

   Solve:

   Let y=0 $\lim_{(x,y)\to(0,0)}\frac{3x^2y^3}{2y^5-2x^5}$ = 0

   Let y=2x $\lim_{(x,y)\to(0,0)}\frac{3x^2y^3}{2y^5-2x^5}$ = 31

   $0 \neq 31$ So the limit does not exist.

3. $\lim_{(x,y,z)\to(0,0,0)}\frac{xy+yz+xz}{x^2+y^2+z^2}$

   Solve:

   Let x=y=0 $\lim_{(x,y,z)\to(0,0,0)}\frac{xy+yz+xz}{x^2+y^2+z^2}$ =0

   Let x=y=z $\lim_{(x,y,z)\to(0,0,0)}\frac{xy+yz+xz}{x^2+y^2+z^2}$ = 1

   $0 \neq 1$ So the limit does not exist.

Partial derivatives

First Partial Derivative

we defined the derivative f'(x) of a function of one variable as $f'(x)=lim_{h\to0}\, f(x+h) - f(x)/h$

Let f be a function of two variables. **The first partial derivative** of f with respect to x and y are the function f x and f y such that

$fx(x,y) = lim_{h \to 0} f(x+h,y) - f(x,y)/h$ $fy(x,y) = lim_{h \to 0} f(x,y+h) - f(x,y)/h$

Notation of Partial Derivatives

$Z = fx\ (x,y) = \partial f / \partial x = \partial z / \partial x = Zx$

$Z = fy\ (x,y) = \partial f / \partial y = \partial z / \partial y = Zy$

Now we discuss how to use the partial derivatives method to find the derivatives of function with several variables.

I. Find the first partial derivatives of f.

The method: we need to treat other variables as constants.

1. *f (x,y) = 2*x^4*y^3-xy^2+3y+1*

   solve:

   *fx (x,y) = 8*x^3*y^3-y^*

   *fy (x,y) = 6x^4*y^2-2xy+3*

2. *f (r,s)* $=\sqrt{r\hat{}2 + s\hat{}2}$

   solve:

   Firstly, we can rewrite *f (r,s)* $=\sqrt{r\hat{}2 + s\hat{}2}$ *= (r^2+s^2)^(1/2)*

   fr (r,s) = (1/2) (r^2+s^2)^(-1/2)*2r = r/(r^2+s^2)^(1/2)*

   fs (r,s) = (1/2) (r^2+s^2)^(-1/2)*2s = s/(r^2+s^2)^(1/2)*

3. *f (x,y) = xe^y+ysinx*

   solve:

   fx (x,y) = e^y + ycosx

   fy (x,y) = x e^y + sinx

4. $f(x,y,z) = 3x^z + x y^2$

   solve:

   $f_x(x,y,z) = 6xz + y^2$

   $f_y(x,y,z) = 2xy$

   $f_z(x,y,z) = 3x^2$

5. $f(r,s,t) = r^2 e^{(2s)} \cos t$

   $f_r(r,s,t) = 2re^{(2s)} \cos t$

   $f_s(r,s,t) = 2r^2 e^{(2s)} \cos t$

   $f_t(r,s,t) = - r^2 e^{(2s)} \sin t$

Second Partial Derivative

If $W = f(x,y)$, we write

$(\partial^2/\partial x^2) f(x,y) = f_{xx}(x,y) = \partial^2 w/\partial x^2 = W_{xx}$

$(\partial^2/\partial y \partial x) f(x,y) = f_{xy}(x,y) = \partial^2 W/\partial y \partial x = W_{xy}$

Now let me show some questions.

I. Verify that $W_{xy} = W_{yx}$

1. $W = x y^4 - 2 x^2 y^3 + 4 x^2 - 3y$

   $W_x = f_x(x,y) = y^4 - 4 x y^3 + 8x$

   $W_y = f_y(x,y) = 4 x y^3 - 6 x^2 y^2 - 3$

   $W_{xy} = (\partial / \partial y) f_x(x,y) = (\partial / \partial y)(y^4 - 4 x y^3 + 8x) = 4y^3 - 12xy^2$

   $W_{yx} = (\partial / \partial x) f_y(x,y) = (\partial / \partial x)(4 x y^3 - 6 x^2 y^2 - 3) = 4y^3 - 12xy^2$

   So $W_{xy} = W_{yx}$

2. $W = x^3 e^{(-2y)} + y^{(-2)} \cos x$

$Wx = fx(x,y) = 3x^{2} e^{(-2y)} - y^{(-2)} \sin x$

$Wy = fy(x,y) = (-2)x^{3} e^{(-2y)} + (-2)y^{(-3)} \cos x$

$Wxy = (\partial / \partial y)fx(x,y) = (\partial / \partial y)(3x^{2} e^{(-2y)} - y^{(-2)} \sin x)$

$= (-6)x^{2} e^{(-2y)}+2y^{(-3)}\sin x$

$Wyx = (\partial / \partial x)\, fy(x,y) = (\partial / \partial x)((-2)x^{3} e^{(-2y)} + (-2)y^{(-3)} \cos x)$

$= (-6)x^{2} e^{(-2y)}+2y^{(-3)}\sin x$

So $Wxy = Wyx$

II. *If* $W = 3 x^{2} y^{3} z +2 x y^{4} z^{2} - yz$, *Find* $Wxyz$.

Solve:

$Wx = fx(x,y,z) = 6x y^{3} z +2 y^{4} z^{2}$

$Wxy = (\partial / \partial y)fx(x,y,z)$

$= (\partial / \partial y)(6x y^{3} z +2 y^{4} z^{2})$

$= 18x y^{2} z + 8y^{3} z^{2}$

$Wxyz = (\partial^{2} / \partial z\partial y)\, fx(x,y,z) = (\partial / \partial z)\, Wxy$

$= (\partial / \partial z)\, (18x y^{2} z + 8y^{3} z^{2})$

$= 18x y^{2} + 16 y^{3} z$

III. *If* $w = \sin xyz$, *find* $\partial^{3} w / \partial z\, \partial y\, \partial x$.

$\partial^{3} w / \partial z\, \partial y\, \partial x$

$(\partial / \partial z)fz(x,y,z) = xy \cos xyz$

$fzy(x,y,z) = (\partial / \partial y)\, (\partial / \partial z)fz(x,y,z) = (\partial / \partial y)(xy \cos xyz)$

$= x \cos xyz - x^{2} yz\sin xyz$