*Eighth Workshop Dynamical Systems Applied
to Biology and Natural Sciences DSABNS 2017
Évora, Portugal, January 31st - February 3rd, 2017*

BRANCHING IN FLUIDIC NETWORKS WITH PERMEABLE WALLS: AN EXTENSION OF HESS-MURRAY'S LAW

Vinicius R. Pepe[1], Luiz A. O. Rocha[1] and Antonio F. Miguel[2]*

[1]Department of Mechanical Engineering,
Federal University of Rio Grande do Sul, Porto Alegre, Brazil

[2]Department of Physics, School of Science and Technology,
University of Évora, Évora, Portugal

afm@uevora.pt

The branching of fluidic networks becomes a subject of great interest due to its importance in understanding the behavior of branching networks in biology (cardiovascular and bronchial systems, river basins, the structure of plants and trees, etc..), as well as for the biomimetic design of engineering systems [1, 2]. The successive division of tubes and their hierarchical structure are distinctive features of tree-shaped networks. As the network progresses, tubes become smaller, both in length and diameter with the successive division of tubes. The design of these networks is generally assumed as being described by the Hess-Murray's law [1, 2]. Assuming a HagenPoiseuille flow and applying the principle of minimum work to the total power requirement, Hess [3] and Murray [4] show that that the cube of the diameter of the parent vessel equals the sum of the cubes of the diameters of the daughter vessels. Using the constructal law, Bejan et al. [5] later derived an equation predicting the lengths of branching tubes by minimizing the overall flow resistance over a finite-size space. For laminar flow, they also found that the cube of the length of a parent tube should be equal the sum of the cubes of the lengths of the daughter tubes. Although proposed first to the optimal design of vessels of cardiovascular system, experimental results seem also to support Hess-Murrays law for the bronchial trees of mammals, the leaf veins of plants, etc. [2, 6]. However, this law not always hold well for the diameter of acinar airways and for some pulmonary veins [6, 7]. This paper addresses a fundamental issue of distributing a fluid flow in a network of vessels with permeable walls. A numerical study is presented to investigate the influence of wall permeability on the optimum geometrical relationship governing the ratio of sizes of the tubes in

 ISBN: 978-989-98750-3-6

a branching network. A generalized version of Hess-Murrays law, including the diameters and lengths of vessels, is derived for the design of fluidic networks and hierarchical fluid distribution systems with permeable walls.

References

[1] A. Bejan and S. Lorente. (2008). *Design with Constructal Theory*, Wiley, New Jersey.

[2] A. F. Miguel. (2013). *Quantitative unifying theory of natural design of flow systems: emergence and evolution*, in: Constructal Law and the Unifying Principle of Design, Springer, NY, pp. 21–38.

[3] W.R. Hess. (1917). *Über die periphere Regulierung der Blutzirkulation*, Pflüger's Archiv fr die Gesamte Physiologie des Menschen und der Tiere, **168**, 439–490.

[4] C.D. Murray. (1926). *The physiological principle of minimum work applied to the angle of branching of arteries*, J. Gen. Physiol., **9**, 835–841.

[5] A. Bejan, L. A. O. Rocha and S. Lorente. (2000). *Thermodynamic optimization of geometry: T and Y-shaped constructs of fluid streams*, Int. J. Therm. Sci., **39**, 949–960.

[6] A. F. Miguel. (2016). *Toward an optimal design principle in symmetric and asymmetric tree flow networks*, Journal of Theoretical Biology, **389**, 101–109.

[7] T. F. Sherman. (1981). *On connecting large vessels to small*, J. Gen. Physiol., **78**, 431–453.

 ISBN: 978-989-98750-3-6