

## **2017 Research Week Proposal**

**Title** – Generalized Parabolas and their Properties

**Program of Study** – Mathematics

**Presentation Type** – PowerPoint

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**Category** – Basic

**Abstract:** Most students of mathematics learn early on in their careers that a parabola is the set of points equidistant from a given point (called a focus) and a given line (called a directrix). In their paper “Generalized Parabolas,” Daniel Joseph, Gregory Hartman, and Caleb Gibson (2011)<sup>1</sup> extend this idea by generalizing the directrix, and defining a generalized parabola to be the set of points equidistant from a given point and a curve, rather than a line. We undertake to investigate the geometric properties of generalized parabolas as they relate to their respective directrix and focus pairs. We investigate appearances of asymptotes, corners and cusps on generalized parabolas. We also describe generalized parabolas in terms of variable circles in order to further our knowledge of their properties. Using this perspective, we provide a proof for the conditions that lead to an asymptote on a generalized parabola. The variable circle description also provides a method for finding a directrix given a generalized parabola and a focus point. Using methods of calculus and trigonometry, we are able to explain the appearance of sharp corners on a generalized parabola. Considering generalized parabolas as the centers of variable circles has helped us to understand certain characteristics about these curves. The results that we have so far uncovered

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<sup>1</sup> Dan Joseph, Gregory Hartman, Caleb Gibson. “Generalized Parabolas.” The College Mathematics Journal, Vol. 42, No. 4 (September 2011), pp. 275-282

suggest that there are many more characteristics of generalized parabolas that could be understood in a different light using this perspective.