

THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING  
IN SOLID AND STRUCTURAL MECHANICS

Towards enhanced mechanical braking systems for trains

Thermomechanical capacity of wheel treads at stop braking

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Department of Applied Mechanics  
CHALMERS UNIVERSITY OF TECHNOLOGY

Göteborg, Sweden 2017

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## **ABSTRACT**

Modern trains are equipped with different braking systems, between which the braking effort can be distributed. Among these, tread braking is still the most common system for friction braking. Tread brake systems are cheap and robust. However, extensive usage of tread brakes demands knowledge of operational limits to ensure safety and to decrease life cycle costs (LCC) of the running gear.

In the present work, a state-of-the-art literature survey has been compiled which covers topics related to establishing operational limits such as: brake control and blended braking, braking temperatures, brake block materials, wear and rolling contact fatigue of wheels due to tread braking, and capacity of tread brakes and brake discs.

A methodology to simulate full-scale brake rig tests, including wheel–rail contact, has been further developed. It includes an axisymmetric thermal analysis, a 3D mechanical wheel–rail contact analysis, and a 3D thermomechanical analysis of the braked wheel. The behaviour of ER7 grade railway wheel material is mimicked by use of a plasticity model calibrated against results from cyclic experiments on test specimens. The results from the simulations in terms of predicted fatigue lives show good agreement with full-scale test rig results for three combinations of initial velocity and brake block material.

The developed methodology is employed in parametric studies. These consist of braking load cases characterised by operational parameters such as axle load, maximum vehicle speed, deceleration, block material, and initial wheel temperature. Damage evolution in the wheel tread is studied. A strong influence of high temperatures on rolling contact fatigue formation in the wheel tread was observed. In particular, braking temperatures over 450 °C can result in a very short life up to crack initiation. However, for braking temperatures in the range of 300 – 350 °C, the wheel is found to be more resistant to fatigue due to strain hardening.

Keywords: Railway wheels, tread braking, high temperature fatigue, rolling contact fatigue, full-scale brake rig testing, stop braking, finite element analysis



## PREFACE

The work presented in this thesis is part of the activity in the CHARMEC Centre of Excellence in Railway Mechanics at the Department of Applied Mechanics, Chalmers University of Technology, Gothenburg. The current project aims at an overall cost-effective partitioning of braking power between the parts in the full braking systems. The project is supported financially by CHARMEC's industrial partners. Especially the support from Bombardier Transportation, Faiveley Transport, Lucchini Sweden and SNC-Lavalin is gratefully acknowledged.

The work would not have been possible to accomplish without the contribution of my supervisors. My deepest gratitude goes to my main supervisor Dr Tore Vernersson for his invaluable guidance, endless support, simple explanations and encouragement. I am also deeply grateful to my co-supervisor Professor Roger Lundén for encouraging and inspiring me during my project and for supporting and assisting me in my writing. I would also like to acknowledge Professor Emeritus Bengt Åkesson and Professor Anders Ekberg who helped me to improve my manuscripts.

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Gothenburg, March 2017  
Mandeep Singh Walia

*If you want to shine like a sun, first burn like a sun (Dr A.P.J. Abdul Kalam)*



# THESIS

This thesis consists of an extended summary and the following appended papers:

- Paper A** M. S. Walia. Enhanced mechanical braking systems for modern trains: State-of-the-art survey, Chalmers Applied Mechanics/CHARMEC, Gothenburg, 2017, 33 pp. *Internal report*
- Paper B** A. Esmaeili, M. S. Walia, K. Handa, K. Ikeuchi, M. Ekh, T. Vernersson and J. Ahlström. A methodology to predict thermomechanical cracking of railway wheel treads: from experiments to numerical predictions, Chalmers Applied Mechanics/CHARMEC, Gothenburg, 2017, 29 pp. *To be submitted for international publication*
- Paper C** M. S. Walia, A. Esmaeili, T. Vernersson and R. Lundén. Thermomechanical capacity of wheel treads at stop braking: A parametric study, Chalmers Applied Mechanics/CHARMEC, Gothenburg, 2017, 21 pp. *To be submitted for international publication*

The following paper constitutes an early version of **Paper C** and has not been included in the thesis:

M. S. Walia, A. Esmaeili, K. Handa, T. Vernersson and R. Lundén. Thermal impact on rolling contact fatigue and capacity of railway wheels, *Proceedings of EuroBrake 2016*, Milan, Italy, 2016, 10 pp.





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# Extended Summary

# 1 Introduction

## 1.1 Background and motivation

In the past decades, environmental awareness, new regulations, demands on cost reductions and improved operational performance have led to accelerated new developments in railway operations. Together with the stride towards shorter travel times, higher axle loads and increased traffic volumes with improved safety, this imposes great challenges on the railway industry. In this context, it is highly important to understand the factors that limit the capacity of various systems and components that are in usage. In particular, the current thesis focuses on understanding the role of the tread brake system in inducing wheel damage. In particular, ways to be able to predict damage to avoid removal of trains from operation, operation failures or even derailments. In addition to safety, these issues are major cost drivers.

Braking systems are critical for reliable and safe operation of trains. Modern trains are often equipped with a computer controlled braking system that can flexibly distribute braking power between different braking subsystems. These subsystems can consist of electrodynamic (ED) brakes that act in combination with friction brakes in form of tread brakes and /or disc brakes. Ideally, ED brakes are utilized which allows regenerated energy to be fed back to the main power supply. However, as the efficiency of the ED brake is speed-dependent, additional braking must be performed using friction brakes. Hence the modern use of the friction brakes has changed substantially, from a situation where it takes all of the braking energy and is in very frequent use, to the current situation where it is used only in certain speed ranges. Note, however, that if the ED brake does not have sufficient capacity (e.g., due to malfunctioning, or in a situation of emergency), the friction brakes will have to take a larger part (or all) of the braking effort.

In this thesis, the main focus has been on tread braked wheels with application to stop braking. Tread brakes are widely used on freight, metro and suburban trains. Commonly the wheels are the limiting factor for the braking capacity. The available guidelines/standards [1] relating to wheel design and capacity limits for the wheel are based on “traditional criteria”, e.g., change of gauge and build-up of global residual stresses that essentially relate to drag braking. Studies [2], show that these

guidelines/standards need to be updated to consider also stop braking applications. Recent studies [3] try to highlight the impact of braking temperatures on rolling contact fatigue (RCF). However, the thermal impact on RCF and the thermal capacity of tread braked wheels need to be further investigated. A goal of this study is to perform life predictions and analyse various operational parameters that limit the wheel tread life during different stop braking scenarios.

## 1.2 Research collaborations

The work in this thesis has been carried out within the CHARMEC project SD10 “Enhanced mechanical braking systems for modern trains”. This project is carried out in collaboration with the CHARMEC project MU32 “Modelling of thermo-mechanically loaded rail and wheel steels” at the Department of Applied Mechanics. In MU32, a material plasticity model is developed, which is calibrated against mechanical tests performed on pearlitic steels at elevated temperatures. The developed material model is used in the current project for finite element simulations. In addition, full-scale tests featuring three series of repeated stop braking cases have been performed by the Railway Technical Research Institute (RTRI) in Japan. Results of these tests are here compared to finite element simulations.

## 2 Tread braked wheels – overview

Tread (block) brakes are still one of the most common braking systems on railway vehicles. Block braking is the ordinarily used system on freight wagons and is also common on passenger trains. Often, but not necessarily, block brakes are used in combination with disc brakes and electrodynamic brakes. The tread braking action is carried out by pressing the brake block(s) against the tread of a wheel. In addition, the tread is also in rolling contact with the rail, see Figure 1. The vehicle can be equipped with four different standard block configurations, illustrated in [4]. Furthermore, brake blocks are commonly manufactured from cast iron, organic composite, or sinter materials.

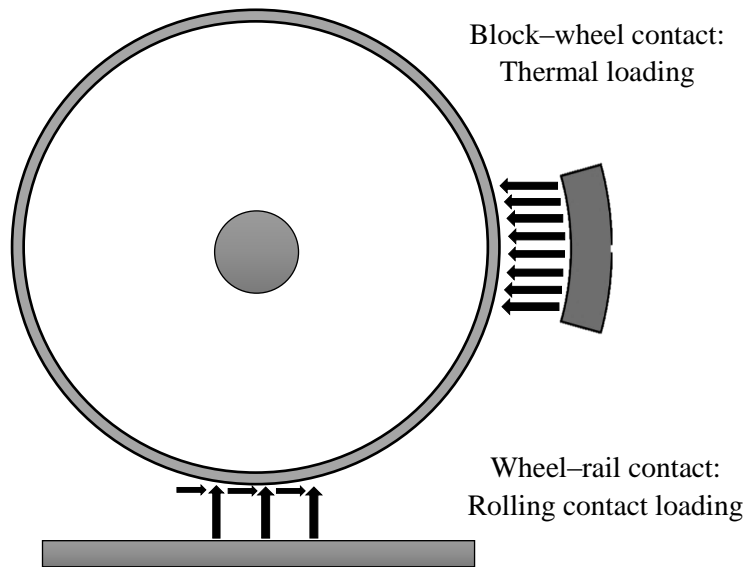


Figure 1: Schematic diagram of wheel, block and rail where the wheel is subjected to combined thermal and rolling contact loading.

## 2.1 Thermal loading

At tread braking, the frictional heat generated at the area of contact is partitioned between block and wheel and further conducted from the hot wheel into the cold rail. Average temperature of the wheel can be obtained by accounting for the heat partitioning between the brake block(s), the wheel and the rail [4]. In a simplified approach, the temperature distribution on the wheel is computed presuming an evenly distributed heat flux on the wheel–block contact area using the heat partitioning model developed in [4, 5] including influence of rail chill [6]. The model, which is extensively used in **Paper B** and **Paper C**, is based on the third-body concept [7], accounting for the temperature jump at the interface between two bodies in contact. In addition to the heat transfer from wheel to rail, dissipation of heat to the surroundings by convection and radiation is accounted for based on the classical formulae [8]

$$q^{\text{conv}} = -h^{\text{conv}}A_s(T - T_\infty) \quad (1)$$

and

$$q^{\text{rad}} = -\varepsilon\sigma A_s(T^4 - T_\infty^4) \quad (2)$$

Here  $T$  is the absolute temperature,  $T_\infty$  a reference temperature (room temperature),  $h^{\text{conv}}$  the convection heat transfer coefficient,  $\sigma$  the Stefan–Boltzmann constant,  $A_s$  the exposed surface area, and  $\varepsilon$  the emissivity. In the analyses, the average heat input at the wheel–block interface is represented by polynomial functions over time and implemented using the Fortran subroutine DFLUX [9] in a 3D FE model in ABAQUS, see Figure 2.

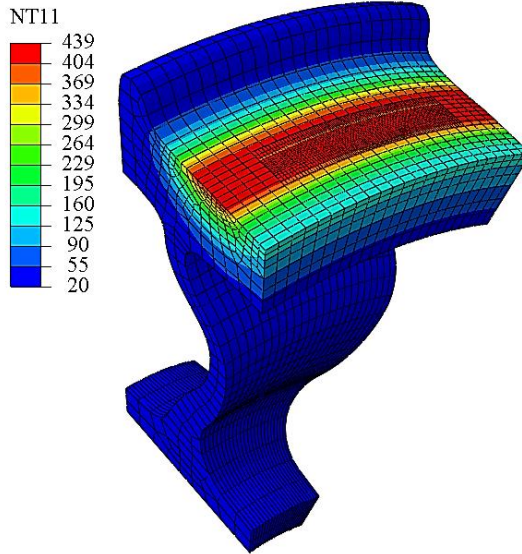


Figure 2: Example of calculated temperature distribution (°C) over the wheel surface in the 3D FE model (for case S160 with contact band of 50 mm in **Paper B**).

## 2.2 Rolling contact loading

A classic approach for evaluating the normal pressure and tangential tractions in the wheel–rail interface is to use Hertz–Carter theory for frictional rolling contact [10, 11]. In the current thesis, the normal pressure distribution is approximated using a so-called superellipsoid, which is a generalization of an ellipsoid, given by

$$p(x, y) = p_0 \left[ \left( 1 - \left( \frac{|x|}{a} \right)^{\frac{2}{e}} - \left( \frac{|y|}{b} \right)^{\frac{2}{e}} \right)^{\frac{n}{2}} \right]^{\frac{e}{n}} \quad (3)$$

where  $e$  and  $n$  are shape controlling exponents,  $a$  and  $b$  are the semi-axes of the contact superellipse of the wheel–rail contact patch. Further,  $x$  and  $y$  are local coordinates from the centre of the contact patch, and  $p_0$  is the peak contact pressure. The exponents  $e$  and  $n$ , the semi-axes  $a$  and  $b$  and the peak contact pressure  $p_0$  control the shape of the superellipsoid. They are determined using an optimization algorithm in Matlab by fitting of the superellipsoid to contact pressure distributions evaluated from a wheel–rail indentation analysis, see **Paper B** and **Paper C**.

Further, the interfacial shear stresses that are required to transmit the wheel–rail tractive forces, are introduced assuming a linear variation of tractions along the rolling direction inside the stick zone. This is motivated by FE results from rolling contact simulations for an elastoplastic material model [12] where calculated stick zone tractions suggest that approximation with a linear function would give a satisfactory representation. The model is thus based on the following assumptions, where only assumption 4 is not in agreement with the classical Carter case (see also Figure 3a),

1. The contact patch is partitioned into a leading zone of stick and a trailing zone of slip.
2. The tractions in the slip zone can be estimated as the coefficient of friction times the contact pressure.
3. The shape of the stick zone is found by mirroring the leading edge around the centre of the stick zone.
4. The tractions in the stick zone are assumed to grow linearly along the rolling direction from the leading edge back to the edge of the slip zone, see Figure 3b.



Following the Carter model, the tractions in the slip zone are calculated as

$$q_{\text{slip}}(x, y) = \mu p(x, y) \tag{4}$$

where  $p(x, y)$  is the normal pressure distribution given by equation (3). The tractions in the stick zone are given by

$$q_{\text{stick}}(x, y) = \mu p_* \frac{x + a_*}{x_* + a_*} \tag{5}$$

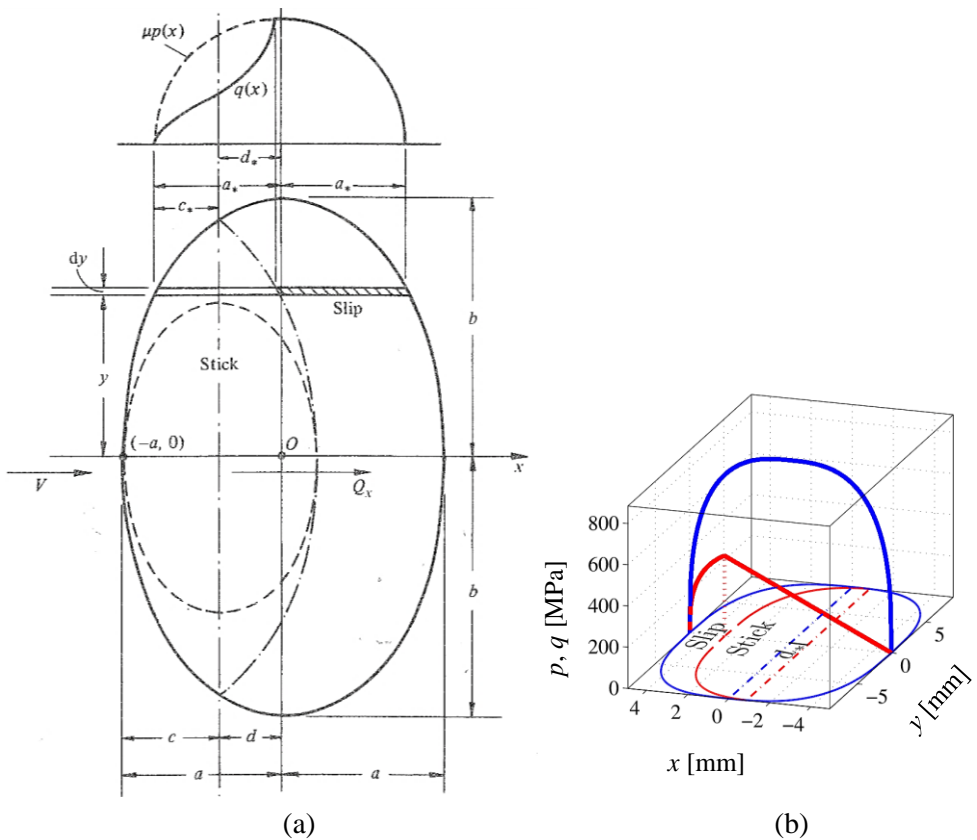


Figure 3: Illustration of stick and slip zones: (a) based on strip theory according to Carter (figure taken from [13]) and (b) pressure and traction distributions used in the present thermomechanical analysis. Slip and stick zones are in opposite directions in the two figures.

Here  $p_*$  is the contact pressure at the boundary between stick and slip zones (at  $x = x_*$ ) given by

$$p_*(y) = p_0 \left[ \left( 1 - \left( \frac{|x_*|}{a} \right)^{\frac{2}{e}} - \left( \frac{|y|}{b} \right)^{\frac{2}{e}} \right)^{\frac{e}{n}} \right]^{\frac{n}{2}} \quad (6)$$

with  $x_* = -a_* + 2c_*$  and  $c_* = a_* - d_*$ . The distance  $d_*$  is determined by the requirement on the resulting total traction load for a given coefficient of friction. The evaluated normal and tangential contact stress distributions are employed using the Fortran subroutines DLOAD and ULTRACLOAD [9], respectively, in a 3D FE model in ABAQUS.

### 2.3 Thermomechanical analysis

During braking, the temperature increases in the wheel and can introduce a change of the global wheel behaviour, i.e., axial rim displacements during and after braking, and also residual stresses after braking [14]. The thermal capacity of tread braked wheels may be limited by this global thermomechanical behaviour [1]. In addition, elevated temperatures may influence the rolling contact fatigue (RCF) damage introduced to the wheel tread [15]. Wheel tread damage caused by RCF calls for premature machining of wheels, which shortens their expected service life [16]. For a detailed study of the impact from combined thermal and mechanical loads, imposed by braking and rolling contact, numerical modelling using the finite element (FE) method is adopted in this thesis.

In **Paper B** and **Paper C**, detailed studies of wheel tread material behaviour when subjected to thermomechanical loading from simultaneous braking and rolling contact loads are carried out. **Paper B** introduces a 3D computational framework that allows for the analysis of the wheel tread material subjected to thermal and mechanical loading when employing an elastoplastic material model. The framework developed in **Paper B** is further used in **Paper C** where different braking load cases have been investigated. The FE modelling in this study involves sequential thermal and mechanical analyses conducted in three steps, see also Figure 4,

1. Heat partitioning between brake block, wheel and rail during stop braking is accounted for in axisymmetric thermal analyses using models developed in [4, 5].
2. Wheel–rail contact is simulated for indentation type of loading using a detailed FE model.
3. Wheel temperature histories, obtained in the first step, and the mechanical rolling contact stresses, obtained in the second step, are applied as loads in a 3D structural analysis.

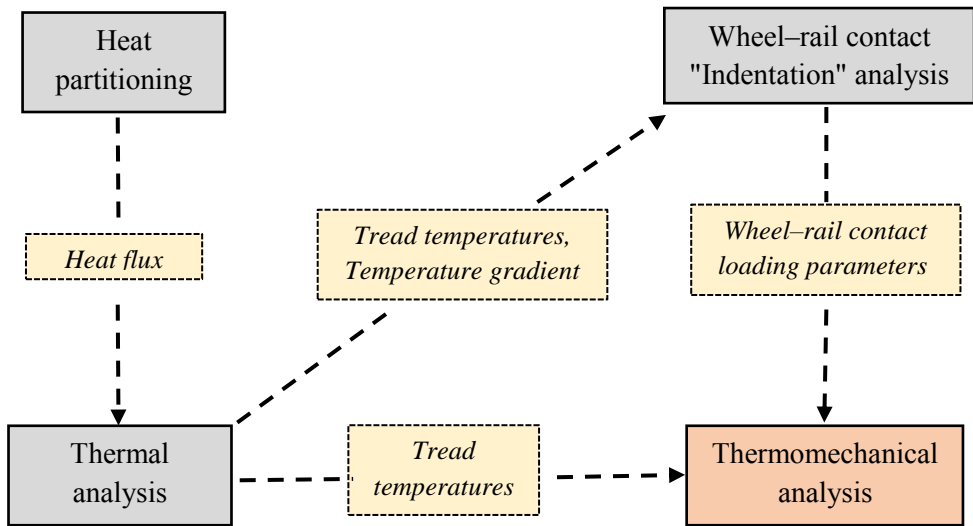


Figure 4: Schematic illustration of the steps followed for sequential thermo-mechanical analysis. Outputs of the analyses are given in italics.

Further, the material response of the wheel is evaluated by studying the evolution of the total effective plastic strain at the most critical region on the wheel tread. In the current studies, the dominating mechanism behind wheel tread fatigue is found to be plastic deformations resulting from the combined thermal and mechanical loading. Two methods for assessing fatigue life are utilized in **Paper B**; one is based on ratchetting failure (RF) and the other is based on low cycle fatigue (LCF) damage [17]. **In Paper B**, for the considered types of stop braking, it is found that the LCF damage mechanism has a low influence on the failure of the material and that RF is

likely to be the dominant damage mechanism. This is in line with the results in [18]. Based on a comparison between experimental and simulated results, an estimate for the critical strain that controls the ratchetting life is obtained. In **Paper C**, the fatigue life is assessed based on an RF analysis with aid of the critical strain value obtained in **Paper B**.

## 3 Summary of Appended Papers

### 3.1 Paper A

The main aim of this literature survey is to briefly provide an overview of modern mechanical braking systems (tread and disc brakes) and of electrodynamic (ED) brakes. For tread braking, the recent studies and developments in brake block types are discussed. Numerical modelling techniques to assess braking temperatures, thermomechanical loading, wear and rolling contact fatigue are highlighted. There are current standards/guidelines regarding thermal capacity of railway wheels at drag braking. Additional efforts are required to establish the limits for wheels subjected to stop braking. For brake discs, thermal limits can be established accounting for the thermomechanical behaviour along with better degradation models. Developments in ED brakes has enhanced their role in braking effort and improved their usage in blended braking. Main objectives and importance of brake blending in modern trains are discussed. In addition, strategies for cost-efficient blended braking for decreased Life Cycle Costs (LCC) are described from earlier case studies. Developments in LCC analysis in the railway industry and available standards are discussed briefly. Based on the requirements of the project SD10, possible laboratory testing and in-field testing techniques are mentioned. These can be used to enhance thermomechanical and wear simulations. Modern instrumentation with low cost, easy usage and high robustness are briefly discussed.

## 3.2 Paper B

Thermal cracking of railway wheel treads is studied by full-scale brake rig tests and finite element simulations. The main goal is to perform thermomechanical rolling contact fatigue life predictions. The wheel tread material is subjected to simultaneous mechanical and thermal loads due to rolling contact and stop braking, respectively. Full-scale tests featuring three series of repeated stop braking cases have been performed in a brake rig featuring a tread braked wheel that is in rolling contact with a so-called rail-wheel. The brake rig test conditions have been simulated numerically. In particular, the effect of “hot bands” on the tread as indicated by the experimental findings is accounted for. Stresses induced by heat from braking, as well as from tractive rolling contact loading on the tread are considered. The mechanical response of the wheel material ER7 is simulated using an elastoplastic material model calibrated against data from cyclic experiments at room temperature and up to 625 °C. Finally, the methodology for predicting fatigue life with respect to ratchetting failure is discussed. FE simulations results for crack initiation on the wheel tread are found to be in good agreement with experimental results.

## 3.3 Paper C

Usage of trains varies considerably and operational parameters may significantly influence the life of the railway wheels. The aim of the study is to find limits for tread braking with respect to the influence of thermal impact on rolling contact fatigue (RCF) of the wheel tread. Braking load cases studied include variations of operational parameters such as axle load, maximum vehicle speed, deceleration, block material and initial wheel temperature. An S-shaped wheel in new condition is studied using 3D FE simulations. Frictional rolling contact stress distributions induced by braking are evaluated from elastoplastic wheel–rail contact simulations. The effects of simultaneous thermal and mechanical loading, are studied in uncoupled thermomechanical analyses. A temperature-dependent elastoplastic model is utilised to account for elevated temperatures. The evolution of wheel tread damage is studied for the various brake load cases. Results from the parametric study indicate that load cases with braking temperatures over 450 °C can result in a very short life up to crack initiation on wheel treads. A strong influence of high temperatures on the rolling contact fatigue of the wheel treads is observed.

## 4 Conclusions and Outlook

The state-of-the-art survey, presented in **Paper A**, gives an overview of the range of topics related to railway braking such as classification of braking systems, brake control, brake blending, mechanical brakes (brake blocks for tread brakes, modelling of braking temperatures, thermomechanical loading, wear, rolling contact fatigue and capacity of both wheels and brake discs), electrodynamic brakes, cost-efficient blending and possible instruments for experimental studies.

**Paper B** presents a methodology to simulate the conditions in full-scale brake rig experiments. The wheel is subjected to both thermal and mechanical loading from frictional sliding contact with a brake block and rolling contact with a rail-wheel. A sophisticated material model is calibrated against data from LCF tests for ER7 steel at different temperatures. The model is able to capture the cyclic behaviour of the material. The results from FE simulations, in terms of ratchetting life, show good agreement with full-scale test rig results for three combinations of initial velocity and brake block material (organic composite and sinter).

The methodology and material model developed in **Paper B** is further used for parametric studies in **Paper C**. Braking load cases involve operational parameters: axle loads, maximum vehicle speed, decelerations, block material and initial wheel temperature. Results clearly indicate a strong influence of high temperatures on rolling contact fatigue of the wheel treads.

Future studies will include field experiments, where the braking effort is distributed between different braking systems. In the first phase, calibration of a simulation model will be carried out using short-term data for measured temperatures and wear of the wheel tread. In the second phase, long-term sampled data from brake control devices will be collected and used for calculation of wheel and block behaviour for longer durations with the aid of the calibrated model. In addition, rig experiments will be performed and non-linear wear rate models will be developed and implemented in numerical simulations. Further, models for estimation of wear and degradation of brake discs and brake pads will be developed and thermomechanical simulations will be performed. The degradation models developed in this project and models from previous projects will be combined into a model where limiting phenomena for usage of braking subsystems. Such a model will be useful for optimisation of brake power partitioning in trains.

## References

1. Railway applications – Wheelsets and bogies - Wheels – Technical approval procedure – Part 1: Forged and rolled wheels. *European Committee for Standardization (CEN)*, Brussels, Belgium, 2011, 50 pp.
2. **Teimourimanesh, S.** Thermal Capacity of Railway Wheels - Temperatures, residual stresses and fatigue damage with special focus on metro applications, *Doctoral thesis, Department of Applied Mechanics, Chalmers University of Technology*, Chalmers University of Technology, Göteborg, 2014.
3. **Caprioli, S.** Thermal impact on rolling contact fatigue of railway wheels, *Doctoral thesis, Department of Applied Mechanics, Chalmers University of Technology*, Göteborg, 2014.
4. **Vernersson, T.** Temperatures at railway tread braking. Part 1: modelling. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 2007, **221**(2), 167-182.
5. **Vernersson, T.** Temperatures at railway tread braking. Part 2: calibration and numerical examples. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 2007, **221**(4), 429-441.
6. **Vernersson, T. and Lundén, R.** Temperatures at railway tread braking. Part 3: wheel and block temperatures and the influence of rail chill. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 2007, **221**(4), 443-454.
7. **Laraqi, N.** Velocity and relative contact size effects on the thermal constriction resistance in sliding solids. *Journal of Heat Transfer, Transactions of the ASME*, 1997, **119**(1), 173-177.
8. **Holman, J.P.**, *Heat transfer*, 8th edition, McGraw-Hill, New York, 1997, 696 pp.
9. *ABAQUS 6.13 documentation*. 2013, Dassault Systèmes: Providence, RI, USA.
10. **Hertz, H.** On the contact of elastic solids. *J. reine und angewandte Mathematik*, 1881, **92**(156-171), 110.
11. **Carter, F.** On the action of a locomotive driving wheel. *Proceedings of Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 1926, The Royal Society, 151-157.
12. **Zhao, X. and Li, Z.** A three-dimensional finite element solution of frictional wheel–rail rolling contact in elasto-plasticity. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, 2015, **229**(1), 86-100.

13. **Johnson, K.L.**, *Contact mechanics*, 9th edition, Cambridge University Press, Cambridge, UK, 1989, 452 pp.
14. **Teimourimanesh, S., Vernersson, T., and Lundén, R.** Thermal capacity of tread-braked railway wheels. Part 1: Modelling. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 2016, **230**(3), 784-797.
15. **Handa, K., Kimura, Y., and Mishima, Y.** Surface cracks initiation on carbon steel railway wheels under concurrent load of continuous rolling contact and cyclic frictional heat. *Wear*, 2010, **268**(1-2), 50-58.
16. **Tunna, J., Sinclair, J., and Perez, J.** A review of wheel wear and rolling contact fatigue. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 2007, **221**(2), 271-289.
17. **Ringsberg, J.W.** Life prediction of rolling contact fatigue crack initiation. *International Journal of Fatigue*, 2001, **23**(7), 575-586.
18. **Vernersson, T., Caprioli, S., Kabo, E., Hansson, H., and Ekberg, A.** Wheel tread damage: a numerical study of railway wheel tread plasticity under thermomechanical loading. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 2010, **224**(5), 435-443.