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Rubrique

Sensitivity of the electrocardiographic forward problem to the heart potential measuement noise and conductivity uncertainties

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ABSTRACT. In this work we are interested in quantifying the conductivity and epicardial potential boundary data uncertainties for the forward problem of electrocardiography (ECG). Indeed these input data are very important for the computation of the torso potential and consequently for the computation of the ECG. We use a stochastic approach for two dimensional torso geometry. We attribute probability density functions for the various source of randomness, and apply stochastic finite elements based on generalized polynomial chaos method. This work is the first step in order to quantify the uncertainties in inverse problem, which the goal is to complete the epicardial data. The efficiency of this approach to solve the forward ECG problem and the usability to quantify the effect of organs conductivity and epicardial boundary data uncertainties in the torso are demonstrated through a number of numerical simulations on a 2D computational mesh of the torso geometry.

KEYWORDS : electrocardiographic forward problem, stochastic finite elements, polynomial chaos, uncertainty quantification, stochastic processes, stochastic Galerkin

1. Introduction

Many studies have been performed on the forward problem of electrocardiography, in order to create more accurate methods allowing to find the electrical potential on the heart surface. However the data required by the mathematical electrocardiographic model, is in practice subject to uncertainties due to measurement errors or modeling assumptions and the resulting lack of knowledge. Therefore the idea of uncertainties quantification has attracted much interest in the last few years [1, 4]. The goal is to propagate information on the uncertainty of input data to the solution of a PDE [5]. Moreover the electrical potential in the torso depends on some physical parameters and on the geometry of the patient. In this work we are interested in studying the effect of the conductivity uncertainties, and also epicardial boundary data, in the ECG forward problem solved via stochastic finite element method (SFEM). For this aim we consider a stochastic approach in which the parameters of the model will be viewed as having statistical distributions, then as result the solutions of the stochastic system obtained have statistical characteristics, and we can determine the mean and the standard deviation of the electrical potential in the torso.

2. Stochastic forward problem of electrocardiography

2.1. Function spaces and notation

We give in the following a short overview of the notations, and definition of the stochastic Sobolev space used throughout this paper. Let D be the spacial domain. Ω is sample space that belongs to a probability space (Ω, A, P), A denotes the σ-algebra of subsets of Ω, and let P be the probability measure. Following the theory of Wiener [7] , as well as Xiu and Karniadakis [5] , we can represent any general second-order random process $X(\omega)$, $\omega \in \Omega$, in terms of a collection of finite number of random variables. We represent this random process by a vector $\boldsymbol{\xi} = \boldsymbol{\xi}(\omega) = (\xi_1(\omega), ..., \xi_N(\omega)) \in \mathbb{R}^N$, where N is the dimension of the approximated stochastic space. We assume that each random variable is independent, its image space is given by $\Theta_i \equiv \xi_i(\Omega) \subset \mathbb{R}$. Each random variable is characterised by a probability density function (PDF) $\rho_i : \Theta_i \longrightarrow \mathbb{R}^+$, for $i = 1, \ldots, N$. Then, we define the joint PDF of the random vector $\boldsymbol{\xi}$

$$\rho(\boldsymbol{\xi}) = \prod_{i=1}^{N} \rho_i(\xi_i) \qquad \forall \boldsymbol{\xi} \in \Theta,$$

where the support of ρ is $\Theta = \prod_{i=1}^{N} \Theta_i$. The probability measure on Θ is $\rho(\boldsymbol{\xi}) d\boldsymbol{\xi}$. As commented in [5], this allows us to conduct numerical formulations in the finite dimensional (N-dimensional) random space Θ.

In this paper we treat a stochastic problem of electrocardiography, and we suppose that the conductivity parameter and the epicardial boundary data two different are independent source of uncertainties two different and independent source of uncertainties, which will be represented by two random process. For the conductivity parameter random process we define the probability space (respectively the vector of random variables, PDF, the PDF support) with (Ω_0, A_0, P_0), (respectively $\boldsymbol{\xi_0}, \rho_0, \Theta_0$) and with (Ω_1, A_1, P_1), (respectively $\boldsymbol{\xi_1}, \rho_1, \Theta_1$) for the epicardial data random process.

Let us denote $\Theta = \Theta_0 \times \Theta_1$ and $L^2(\Theta) = L^2(\Theta_0) \times L^2(\Theta_1)$ the space of random variables X with finite second moments :

$$\mathbb{E}[X^2(\boldsymbol{\xi_0}, \boldsymbol{\xi_1})] = \int_{\Theta_1} \left(\int_{\Theta_0} X^2(\boldsymbol{\xi_0}, \boldsymbol{\xi_1}) \rho(\boldsymbol{\xi_0}) d\boldsymbol{\xi_0} \right) \rho(\boldsymbol{\xi_1}) d\boldsymbol{\xi_1} < +\infty,$$

where $\mathbb{E}[.]$ denotes the mathematical expectation operator. This space is a Hilbert space with respect to the inner product :

$$\langle X, Y \rangle_{L^2(\Theta)} = \mathbb{E}[XY] = \int_{\Theta_1} \left(\int_{\Theta_0} XY(\boldsymbol{\xi_0}, \boldsymbol{\xi_1}) \rho(\boldsymbol{\xi_0}) d\boldsymbol{\xi_0} \right) \rho(\boldsymbol{\xi_1}) d\boldsymbol{\xi_1}$$

Additionally, we consider a spatial domain D and we define the tensor product Hilbert space $H^1(D) \otimes L^2(\Theta)$ of second-order random fields as :

$$L^2(D) \otimes L^2(\Theta) = \left\{ u : D \otimes \Theta \longrightarrow \mathbb{R}, \int_{\Theta_1} \left(\int_{\Theta_0} \int_D |u(x, \boldsymbol{\xi_0}, \boldsymbol{\xi_1})|^2 \, dx \rho(\boldsymbol{\xi_0}) d\boldsymbol{\xi_0} \right) \rho(\boldsymbol{\xi_1}) d\boldsymbol{\xi_1} \right\}$$

Analogously, the tensor product spaces $H^1(D) \otimes L^2(\Theta)$ and $H_0^1(D) \otimes L^2(\Theta)$ can be defined.

2.2. Stochastic formulation of the forward problem

Under our assumption the conductivity uncertainties and epicardial boundary data uncertainties do not interact, and they are supposed to be independent each other, consequently we represent the stochastic forward solution of the Laplace equation as random field depending to the both kinds of uncertainties. For the space domain we use a 2D computational mesh of the torso geometry (see Figure 1)

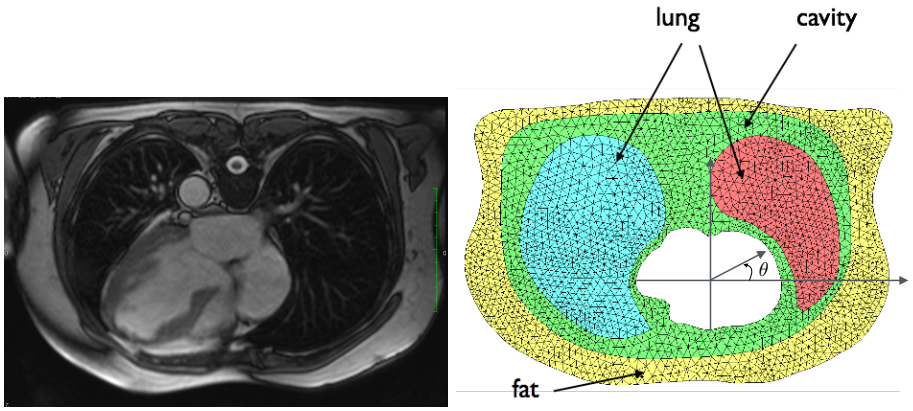

Figure 1 – MRI 2D slice of the torso (left), 2D computational mesh of the torso geometry showing the different regions of the torso considered in this study : fat, lungs and torso cavity, (right). The angle θ is the second polar coordinate.

Since we suppose that the conductivity parameter (σ) depends on the space (x) and on the stochastic variable (ξ_0), and the boundary epicardial data (f) depends on the space (x) and on a stochastic variable (ξ_1). Thus, the solution of the Laplace equation will depend on space and the both stochastic variables $u(x, \xi_0, \xi_1)$. The stochastic forward problem of electrocardiography can be written as follows

$$\begin{cases} \bigtriangledown.(\sigma(x,\xi_0) \bigtriangledown u(x,\xi_0,\xi_1)) = 0 \quad \text{in } D \times \Omega, \\ u(x,\xi_0,\xi_1) = f(x,\xi_1) \quad \text{on } \Gamma_{\text{i}} \times \Omega, \\ \sigma(x,\xi_0)\frac{\partial u(x,\xi_0,\xi_1)}{\partial n} = 0 \quad \text{on } \Gamma_{\text{c}} \times \Omega, \end{cases} \tag{1}$$

Where, Γ_{i} and Γ_{c} are the epicardial and torso boundaries respectively.
The weak formulation of SPDEs is based on an extension of the deterministic theory [3], test function become random fields and an integration over stochastic space is done with respect to the corresponding measure. Thus, the weak form involves expectations of the weak problem formulation in the physical space. Then, denoting by u_f the extension of f to the whole domain, we look for $\tilde{u} \in H_0^1(D) \otimes L^2(\Theta)$, where $\tilde{u} = u - u_f$ is the weak solution of (1), if for all $v \in H_0^1(D) \otimes L^2(\Theta)$, we have :

$$\mathbb{E}\left[\int_D \sigma(x,\xi_1)\nabla\tilde{u}(x,\xi_0,\xi_1).\nabla v(x,\xi_0,\xi_1)dx\right] + \mathbb{E}\left[\int_D \sigma(x,\xi_1)\nabla u_f(x,\xi_0).\nabla v(x,\xi_0,\xi_1)dx\right] = 0. \tag{2}$$

3. Descretization of the stochastic forward problem

A stochastic process $X(\xi)$ of a parameter or a variable X is represented by weighted sum of orthogonal polynomials $\{\Psi_i(\xi)\}$ denoting the generalized chaos polynomial. More details about the different choices of PDFs could be found in [5].
We have

$$X(\xi) = \sum_{i=0}^{p} \hat{X}_i \Psi_i(\xi),$$

where $\hat{X}_i$ are the projections of the random process on the stochastic basis $\{\Psi_i(\xi)\}_{i=1}^{p}$ with respect to the joint PDF ρ.

$$\hat{X}_i = \int_\Omega X(\xi)\Psi_i(\xi)d\rho = \langle X(\xi).\Psi_i(\xi)\rangle_\rho.$$

In order to solve the equation (2) we use the stochastic Galerkin (SG) method to compute the approximate solutions. To develop this method, we denote $Y_\sigma^p \subset L2(\Theta_0)$ and $Y_{u_f}^p \subset L2(\Theta_1)$ the stochastic approximation spaces, and we have $Y_\sigma^p \times Y_{u_f}^q \subset L2(\Theta)$.
In our case we suppose that the conductivity parameter varie uniformly like in [4, 2] and we use the Legendre chaos polynomials which are more suitable for uniform probability density, in other hand we assigned Gaussian probability density to the epicardial boundary data, the corresponding stochastic orthogonal basis to Gaussian random field is Hermite chaos polynomials [5].

$$Y_\sigma^P = span\{\ L_0, ...,\ L_p\}.$$
$$Y_{u_f}^P = span\{H_0, ..., H_p\}.$$

In this study we have targeted to evaluate in the same time, two different source of uncertainties on the electrical potential, then σ, u_f and u are now expressed in the Galerkin space $Y_\sigma^p \times Y_{u_f}^q$ as follows :

$$\sigma(x,\xi_0) = \sum_{l=1}^{r} \hat{\sigma}_l(x)L_l(\xi_0). \tag{3}$$

$$u_f(x,\xi_1) = \sum_{k=1}^{q} (\tilde{u}_f)_k(x) H_k(\xi_1). \tag{4}$$

$$u(x,\xi_0,\xi_1) = \sum_{i=1}^{p}\sum_{j=1}^{q} \hat{u}_{ij}(x) L_i(\xi_0) H_j(\xi_1) \tag{5}$$

By substituting (4),(3),(5) into the stochastic diffusion equation (1) and by projecting the result on the polynomial basis $\{L_m(\xi_0)H_n(\xi_1)\}_{m,n=1}^{(p,q)}$:
For $m = 1, ..., q$ et $n = 1, ..., p$,

$$\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{l=1}^{r} D_{jn} C_{iml} \nabla.(\hat{\sigma}_l(x)\nabla)\hat{u}_{ij}(x)) = 0 \qquad \text{in } D,$$

$$\begin{aligned}
\hat{u}_{11}(x) &= (\tilde{u}_f)_1(x) && \text{on } \Gamma_{\mathrm{i}}, \forall i = 1, ...p, \\
\hat{u}_{12}(x) &= (\tilde{u_f})_2(x) && \text{on } \Gamma_{\mathrm{i}} \forall i, = 1, ...p, \\
\hat{u}_{ij}(x) &= 0 && \text{on } \Gamma_{\mathrm{i}} \forall i, = 2, ...p, j = 3, ...q, \\
\hat{\sigma}_l(x)\frac{\partial \hat{u}_{ij}(x)}{\partial n} &= 0 && \text{on } \Gamma_{\mathrm{c}} \forall i = 1, ...p, j = 1, ...q,
\end{aligned} \tag{6}$$

Where $C_{iml} = \mathbb{E}[L_i(\xi_0), L_m(\xi_0), L_l(\xi_0)]$ et $D_{jn} = \mathbb{E}[H_j(\xi_1), H_n(\xi_1)]$.
For the spatial domain, we define a subspace $V_h \subset H_0^1(D)$ of standard Lagrange finite element functions on a triangulation of the domain D.

$$V_h := span\,\{\phi_1, \phi_2, ..., \phi_{Nx}\}$$

Obviously this ordering induces the following block structure of the linear system of equations :

$$\begin{bmatrix}
A^{(1,1;1,1)} & A^{(1,1;1,2)} & \dots & A^{(1,1;1,q)} & A^{(1,1;2,1)} & \dots & A^{(1,1;p,q)} \\
A^{(1,2;1,1)} & A^{(1,2;1,2)} & \dots & A^{(1,2;1,q)} & A^{(1,2;2,1)} & \dots & A^{(1,2;p,q)} \\
\vdots & \vdots & \dots & \vdots & \vdots & \dots & \vdots \\
A^{(1,q;1,1)} & A^{(1,q;1,2)} & \dots & A^{(1,q;1,q)} & A^{(1,q;2,1)} & \dots & A^{(1,q;p,q)} \\
A^{(2,1;1,1)} & A^{(2,1;1,2)} & \dots & A^{(2,1;1,q)} & A^{(2,1;2,1)} & \dots & A^{(2,1;p,q)} \\
\vdots & \vdots & \dots & \vdots & \vdots & \dots & \vdots \\
A^{(p,q;1,1)} & A^{(p,q;1,2)} & \dots & A^{(p,q;1,q)} & A^{(p,q;2,1)} & \dots & A^{(p,q;p,q)}
\end{bmatrix}
\begin{bmatrix}
\hat{U}_{11} \\ \hat{U}_{12} \\ \vdots \\ \hat{U}_{1q} \\ \hat{U}_{21} \\ \vdots \\ \hat{U}_{pq}
\end{bmatrix}
=
\begin{bmatrix}
B^{11} \\ B^{12} \\ \vdots \\ B^{1q} \\ B^{21} \\ \vdots \\ B^{pq}
\end{bmatrix}$$

where every matrix $A^{(i,j;m,n)} \in \mathbb{R}^{Nx} \times \mathbb{R}^{Nx}$ is a linear combination of finite element stiffness matrices

$$A^{(i,j;m,n)} = D_{j,n} \sum_{l=1}^{r} C_{iml} K_l \quad \forall i, m = 1, ...p; j, n = 1, ...q, \tag{7}$$

$$K_l = [K_l]_{h,t} = (\sigma_l \nabla \phi_h . \nabla \phi_t) \quad \forall l = 1, ...r, \tag{8}$$

h denotes the degrees of freedom of the nodes of the mesh in which the electrical potential values is unknown.
Similarly, every vector $B^{ij} \in R^{Nx}$ is a linear combination of finite element load vectors :

$$B^{ij} = \sum_{l=1}^{r} C_{iml} f_l \quad \forall i = 1, ...p, j = 1, ...q, \tag{9}$$

$$f_l = \sum_{x_h \in \Gamma_i} \hat{u}_{ij} \left(\sigma_l \nabla \phi_h . \nabla \phi_t\right) \quad \forall l = 1, ...r, \tag{10}$$

with h denoting the degrees of freedom of the (known) Dirichlet boundary conditions of the solution.

4. Results

In this section we conduct the numerical simulation obtained in order to show the influence of the conductivity variabilities and the epicardial potential data uncertainties on the electrical potential in the torso. For instance we suppose that the electrical potential in the heart boundary is equal to U_{ex}.

$$U_{ex} = sin(y).$$

Since we assume that the uncertainty of the conductivity value follows a uniform probability density, as probability density functions $\rho_0$we use the Legendre polynomials defined on the interval $\Omega = [-1, 1]$. We also suppose that the true conductivity uncertainty interval is centered by σ_T, the true conductivity see Table 1. In other hand U_{ex} will represent the mean of the Gaussian random field representing the epicardial boundary data uncertainty, we denote its stdev by (ν).

organ category	conductivity (σ_T :S/m)
lungs	0.096
torso cavity	0.200
fat	0.045

Tableau 1 – Conductivity values corresponding to the organs that are considered in the model.

In the following we present four cases, in the first case we only study the effect of epicardial boundary data uncertainties where we gradually increase the stdev ν from zero to 50%. In the second (respectively, third, fourth) case we add the effect of fat (respectively, cavity, lung) with $\pm 50\%$ of uncertainties. Figure 2 summarize the obtained results for all cases. First we see that the forward solution after adding epicardial boundary data uncertainties is more sensitive to the torso cavity and lung conductivities than it is for fat. This result is in line with the numerical results obtained in [4, 2]. Second we remark that the influence of organs conductivity uncertainties disappear when $\nu \geq 10^{-1}$ and all curves take the same values as the case with only epicardial boundary data uncertainty. Figure 3 displays an example for obtained results with respect to lung with $\pm 50\%$ of conductivity uncertainties and epicardial boundary data uncertainty with different values of ν. Figure 3(a) shows the mean value of $u(x, \xi_0, \xi_1)$. Figure3 (b) (respectively Figure 3(c), Figure3(d), and Figure3(e)) shows $u(x, \xi_0, \xi_1)$ stdev with respect to $\pm 50\%$ lung uncertainties and $\nu = 0.03$ (respectively $\nu = 0.05, \nu = 0.1, \nu = 0.5$), finally Figure3(f) represents the case supposing that there is no conductivity uncertainties.

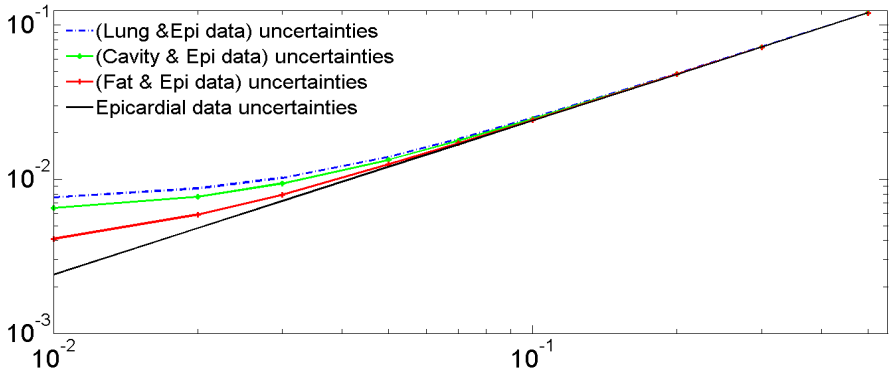

Figure 2 – The effects of $\pm 50\%$ uncertainty to each organ conductivity from it's reference conductivity, and different levels of uncertainty on the the epicardial boundary data. X-axis denote the different stdev value (ν) of the Gaussian epicardial data boundary field. Y-axis the mean square of the stdev value of $u(x, \xi_0, \xi_1)$.

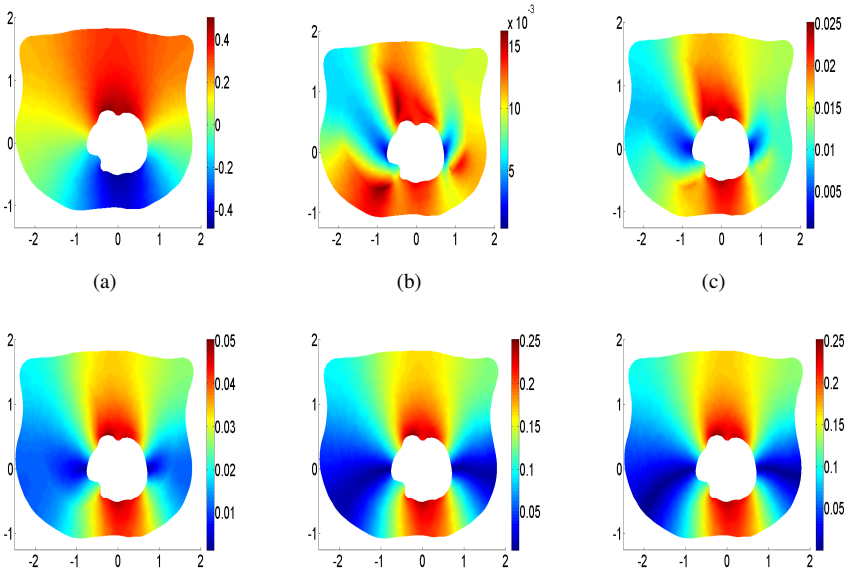

Figure 3 – Mean value of the SFE panel (a). Standard deviation of the SFE solution for $\pm 50\%$ of uncerainty for lung and epirdial data uncertainty for $\nu = 0.03$ panel(b) (respectively $\nu = 0.05$ panel(c), $\nu = 0.1$ panel(d) , $\nu = 0.5$ panel(e)). Panel(f) shows the Standard deviation of the SFE solution with only epirdial data uncertainty for $\nu = 0.5$

5. Conclusion :

This work is a novel approach studying the sensitivity of forward problem of electrocardiography, with respect to different sources of uncertainty using chaos polynomial and SFE method. The obtained results allow to classify the influence of each parameter. We conclude that epicardial potential boundary data uncertainty have a strong effect on forward problem solution errors, compared to the organs conductivity, which at some level of boundary data uncertainty becomes insignificant. This finding suggests that the precise determination of the epicardial boundary data is very important. In a next work we will solve the inverse problem following the formulation presented in [1], using stochastic approach developed in this work, and we will study the uncertainties in the case of the inverse problem.

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