Lateral Boundary Conditions at the staircase-like boundary of ocean models

Eugene Kazantsev, Florian Lemarié, Eric Blayo

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LJK/INRIA, AirSea, Grenoble

PACO project: LEFE-GMMC, LEFE-MANU

Rectangular Box Configuration of the NEMO:

$$30^\circ \times 20^\circ \text{ with } \frac{1}{4}^\circ \text{ resolution and 5 } z \text{ levels.}$$

$$120 \times 80 \times 5 \text{ nodes in } (x, y, z) \text{ coordinates, 64 time steps per day.}$$

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \underbrace{v(\omega + f)}_{\text{Enstr.cons.}} - \frac{\partial (u^2 + v^2)/2}{\partial x} - w\frac{\partial u}{\partial z} - \underbrace{\frac{\partial A_u^h \xi}{\partial x} + \frac{\partial A_u^h \omega}{\partial y}}_{\text{Vort-Div. form.}} + \\
&+ \underbrace{\frac{\partial}{\partial z} A_u^z \frac{\partial u}{\partial z}}_{\text{Implicit}} + \underbrace{g\int_0^z \frac{\partial \rho(x,y,\zeta)}{\partial x} d\zeta}_{\text{Pressure gradient}} + \underbrace{g\frac{\partial (\eta + T_c \phi)}{\partial x}}_{\text{EGW filter}} \\
\xi &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \qquad \omega = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \quad \text{Divergence, Vorticity} \\
w &= \int_H^z \xi(x,y,\zeta) d\zeta; \quad w(x,y,H) = 0 \quad \text{Vertical velocity} \\
A^z &= 1.2 \times 10^{-4} \frac{m^2}{s}; \; A^h = 200\frac{m^2}{s}, \quad f = \frac{4\pi}{86400}\sin(lat) \\
\left.\frac{\partial u}{\partial z}\right|_{w_0} &= \frac{-0.1\frac{N}{m^2}\cos\left(B\pi * \frac{lat - 24^\circ}{44^\circ - 24^\circ}\right)}{hz_1\rho_0} \qquad \begin{matrix} B = 2, & \text{Double gyre} \\ B = 1, & \text{Single gyre} \end{matrix}
\end{aligned}$$

$$(u_\perp, \omega)_{\text{Lateral Boundary}} = 0 \text{ (Impermeability and Free-Slip conditions)}$$

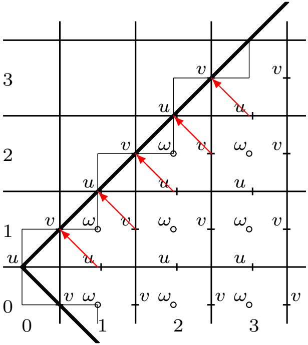

Figure: 45° rotated grid

Impermeability + free-slip boundary conditions:

$$(\vec{V}, \vec{n}) = 0, \quad \frac{\partial(\vec{V}, \vec{\tau})}{\partial \vec{n}} = 0$$

Impermeability: $u|_{bnd} = v|_{bnd}$,

Free-slip:

$$v|_{bnd} = u|_{bnd} = u(x + h/2, y - h/2)$$
$$u|_{bnd} = v|_{bnd} = v(x + h/2, y - h/2)$$

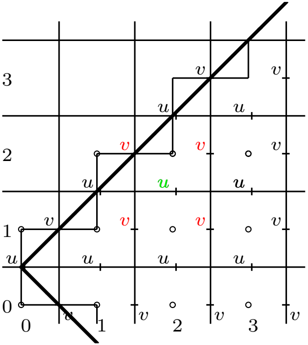

Figure: 45° rotated grid

$$\frac{\partial u}{\partial t} = f\overline{\overline{v}}^{xy}$$
$$\frac{\partial v}{\partial t} = -f\overline{\overline{u}}^{xy}$$
$$u(t) = u(0)\exp(ift)$$

$$\frac{\partial}{\partial t}\begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \\ v_1 \\ v_2 \\ \dots \\ v_n \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & f/4 & f/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & f/4 & f/4 & 0 & 0 \\ & & & \dots & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & f/4 & f/4 \\ -f/4 & -f/4 & 0 & 0 & 0 & 0 & 0 & 0 \\ -f/4 & -f/4 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & \dots & & & & \\ 0 & 0 & -f/4 & -f/4 & 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \\ v_1 \\ v_2 \\ \dots \\ v_n \end{pmatrix}$$

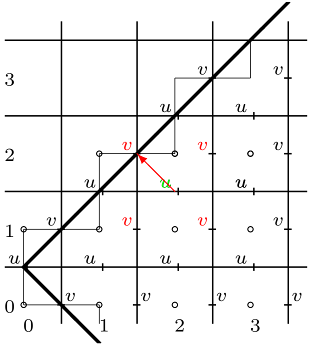

Figure: 45° rotated grid

$$\frac{\partial u}{\partial t} = f\overline{\overline{v}}^{xy}$$

$$\frac{\partial v}{\partial t} = -f\overline{\overline{u}}^{xy}$$

$$u(t) = u(0)e^{+ft/4}$$

$$\frac{\partial}{\partial t}\begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \\ v_1 \\ v_2 \\ \dots \\ v_n \end{pmatrix} = \begin{pmatrix} f/4 & 0 & 0 & 0 & f/4 & f/4 & 0 & 0 \\ 0 & f/4 & 0 & 0 & f/4 & f/4 & 0 & 0 \\ & & & \dots & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & f/4 & f/4 \\ -f/4 & -f/4 & 0 & -f/4 & 0 & 0 & 0 & 0 \\ -f/4 & -f/4 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & \dots & & & & \\ 0 & 0 & -f/4 & -f/4 & 0 & 0 & 0 & -f/4 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \\ v_1 \\ v_2 \\ \dots \\ v_n \end{pmatrix}$$

Optimal Control of the Discrete Derivatives near the Boundary

Coefficients α are allowed to vary in order to find the best fit with requirements of the model and data (see [1, 2]).

$$\left.\frac{\partial v}{\partial x}\right|_{\omega_b} = \frac{\alpha_1 v_{1/2} + \alpha_2 v_{3/2}}{h}$$

$$\left.\frac{\partial u}{\partial y}\right|_{\omega_b} = \frac{\alpha_1 u_{1/2} + \alpha_2 u_{3/2}}{h}$$

$$\left.\frac{\partial v}{\partial x}\right|_{\omega_b} = \frac{\alpha_1 v_{-1/2} + \alpha_2 v_{1/2}}{h}$$

$$\left.\frac{\partial u}{\partial y}\right|_{\omega_b} = \frac{\alpha_1 u_{-1/2} + \alpha_2 u_{1/2}}{h}$$

$$\left.\frac{\partial \omega}{\partial x}\right|_{v} = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\left.\frac{\partial \omega}{\partial y}\right|_{u} = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

with the Initial Guess

$$\alpha_1 = \alpha_2 = 0$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

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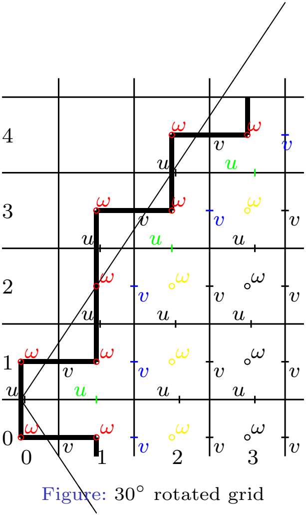


Figure: 30° rotated grid

The model: $x(t) = \mathcal{M}_{0,t}(x(0), \alpha)$ with $x = (u, v, T, S, ssh)^T$

Cost function J

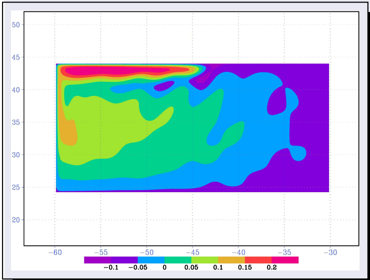
$$J = 10^{-4}(\|x(0) - x_{bgr}\|^2 + \|\alpha - \alpha_{bgr}\|^2) +$$
$$+ \int_{t=0}^{T} t \int\int (u - u_{\text{ref}})^2 + (v - v_{\text{ref}})^2 + (ssh - ssh_{\text{ref}})^2 dxdydt$$

Layout:

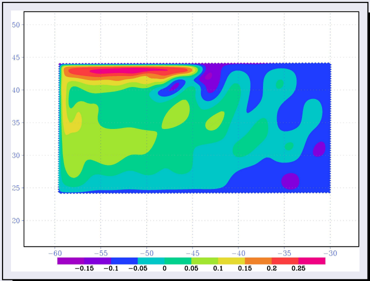
- Joint control of the initial point $x(0)$ (interpolation errors) and the set of α;
- Artificially generated data by the same model on the aligned grid;
- Data Assimilation with the sequence of assimilation windows: 10, 30, 50 days with 30 iterations made in each window;
- Analysis of the solution on the 800 days interval.

- Minimization is performed by M1QN3 (JC Gilbert, C.Lemarechal);
- Adjoint is generated by Tapenade (Ecuador team, INRIA).

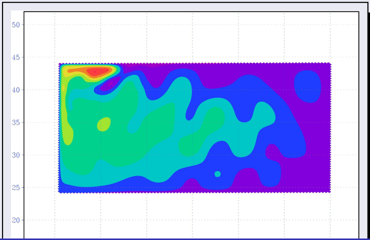
Reference, Optimal and Conventional BC 800 days later



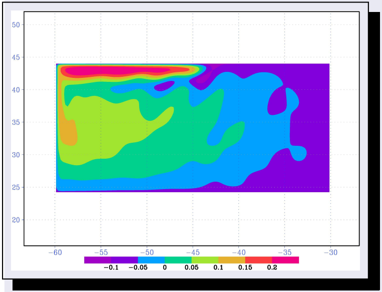
Reference SSH



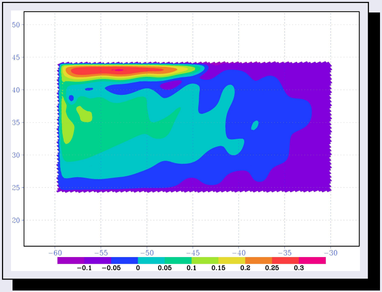
Rotated grid Optimal BC SSH



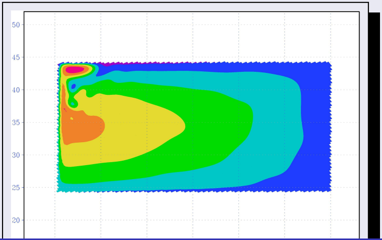
Reference, Optimal and Conventional BC 800 days later



Reference SSH



Rotated grid Optimal BC SSH



$$\left.\frac{\partial v}{\partial x}\right|_{\omega_\circ} = \frac{\alpha_1 v_{-1/2} + \alpha_2 v_{1/2}}{h}$$

$$\left.\frac{\partial u}{\partial y}\right|_{\omega_\circ} = \frac{\alpha_1 u_{-1/2} + \alpha_2 u_{1/2}}{h}$$

$$\left.\frac{\partial v}{\partial x}\right|_{\omega_\circ} = \frac{\alpha_1 v_{1/2} + \alpha_2 v_{3/2}}{h}$$

$$\left.\frac{\partial u}{\partial y}\right|_{\omega_\circ} = \frac{\alpha_1 u_{1/2} + \alpha_2 u_{3/2}}{h}$$

$$\left.\frac{\partial \omega}{\partial x}\right|_{v} = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\left.\frac{\partial \omega}{\partial y}\right|_{u} = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\alpha_1 = 0, \quad \alpha_2 = 0$$

$$\alpha_1 = -1.5, \quad \alpha_2 = 0.5$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

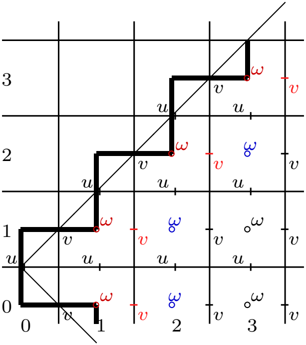


Figure: 45° rotated grid

That means the tangential velocity component is added to the vorticity formula:

$$\omega_\circ = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{u+v}{h}$$

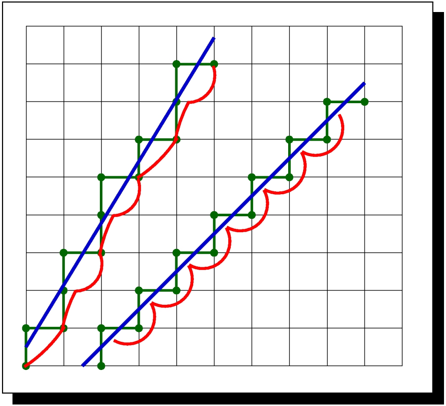
$$= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{\vec{V}.\vec{\tau}}{h/\sqrt{2}}$$

$$\omega_\circ = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{u+v}{h}$$
$$= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{\vec{V}.\vec{\tau}}{h/\sqrt{2}}$$

- Free-slip condition on a curvilinear boundary (see [4]): $\omega|_{bnd} = \frac{\vec{V}.\vec{\tau}}{R}$

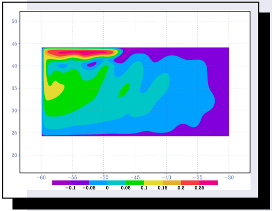
The optimized boundary is supposed to be a curvilinear one with:

- Constant $R_{45^\circ} = -h/\sqrt{2}$ for the 45° rotated grid,
- Variable R_{30° : $-h < R_{30^\circ} < 5h$ for the 30° rotated grid

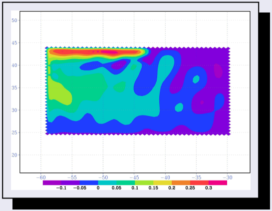


Physical boundary (blue), approximated by the grid (green) and optimized one (red).

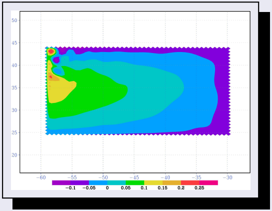
1°/2 resolution



Reference SSH

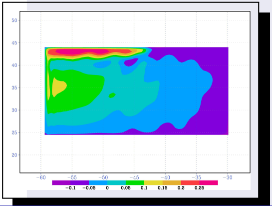


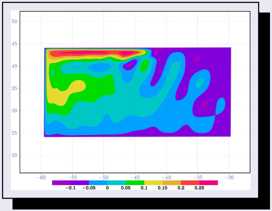
Rotated grid constant $R = -h/\sqrt{2}$ BC SSH

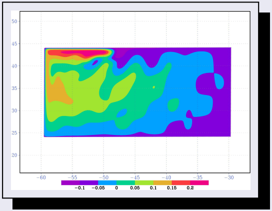


Rotated grid conventional BC SSH

1°/8 resolution

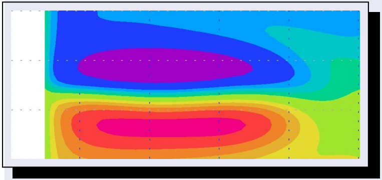




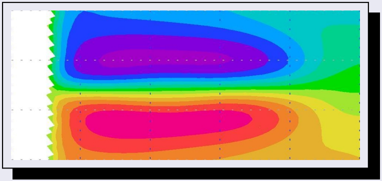


Double Gyre Control: 30° rotation, SSH

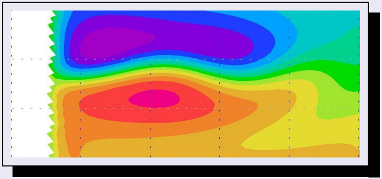
Reference, Optimal and Conventional BC. 10 years average.



Reference SSH



Rotated grid Optimal BC SSH



Rotated grid conventional BC SSH

Boundary Conditions influence is important

- Optimal BCs allows **to correct errors** committed by the discretization,
- The model **is closer** to the reference one with optimal BC,
- Data assimilation allows to get the **optimized position and shape** of the boundary.

BUT

As well as for any adjoint parameter estimation

- The control may violate the model physics;
- The physical meaning of the optimal boundary is difficult to understand;
- The set of α is not unique;
- The problem of identifiability is not addressed yet;
- The problem of stability is not even posed.

References:

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