# Hierarchical Model Predictive/Sliding Mode Control of Nonlinear Constrained Uncertain Systems

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Abstract– This paper presents an overview of some hierarchical control schemes composed by a high level Model Predictive Control (MPC) and a low level Sliding Mode Control (SMC). The latter is realized by using the so-called Integral Sliding Mode (ISM) control approach and is meant to reject the matched disturbances affecting the plant, thus providing a system with reduced uncertainty for the MPC design. Both continuous and discrete-time solutions are discussed in the paper. Moreover, assuming the presence of a network in the control loop, a networked version of the control scheme is presented. It is a model-based event-triggered MPC/ISM control scheme aimed at minimizing the packets transmission. The input-to-state (practical) stability properties of the proposed approaches are also addressed in the paper.

*Keywords:* Predictive control, sliding mode control, robust control, uncertain dynamic systems, networks.

## 1. INTRODUCTION

Recent research in the Model Predictive Control (MPC) field has paid attention to the presence of unavoidable modeling uncertainties or external disturbances affecting the plants, so as to motivate the introduction of robust control strategies [Maciejowski, 2002; Rawlings and Mayne, 2009; Magni et al., 2009; Mayne, 2014]. Two main approaches have been followed: the first one, called open-loop nominal approach, introduces tightened constraints in order to guarantee robust constraint satisfaction and recursive feasibility [Limon Marruedo et al., 2002; Lazar and Heemels, 2009; Pin et al., 2009], while the second one is based on the solution of a closed-loop min-max optimization problem that explicitly takes into account model uncertainty [Scokaert and Mayne, 1998; Fontes and Magni, 2003; Bemporad et al., 2003; Magni et al., 2003; Limon Marruedo et al., 2006]. In the open-loop nominal approach the real constraints are shrunk in order to guarantee that the original constraints are fulfilled by the real system for any possible uncertainty realization. However, this method results in being too conservative in view of the open-loop off-line approximation of the worst possible effect of the disturbance necessary to compute the tightened constraints. In the closed-loop min-max approach the cost function is minimized for the worst possible uncertainty, while satisfying the constraints for any possible perturbation. The drawback of this approach is the very high computational burden since it either requires the solution to difficult on-line min–max optimization problems, see e.g. [Magni et al., 2003], or the off-line computation of polytopic robust positive invariant sets, see [Mayne et al., 2006]. In addition, both the approaches are conservative, mainly because they (implicitly or explicitly) rely on worst-case techniques. If the uncertainties or the state and control disturbances are characterized as stochastic processes, the conservativeness of a deterministic worst case approach can be significantly reduced by the development of stochastic MPC algorithms with probabilistic state and/or input constraints (see [Farina et al., 2015] and the references therein reported).

A different approach to reduce the conservativeness inherent in any robust MPC control algorithm, that merges MPC and Sliding Mode Control (SMC), has been investigated in recent years.

SMC is a widely appreciated control methodology particularly effective in case of systems which are affected by a wide class of uncertain terms due to modeling uncertainties and external disturbances. According to the sliding modes control theory, the system states are forced to reach a prespecified sliding manifold in finite time so that the desired dynamics is assigned to the controlled system [Utkin, 1992; Edwards and Spurgen, 1998; Utkin et al., 1999]. When the so-called sliding variable is steered to zero and the system state belongs to the sliding manifold, then a sliding mode is enforced. The sliding manifold is made attractive

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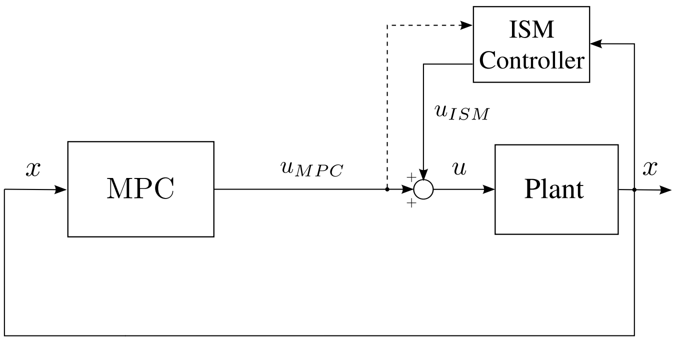


Figure 1. The hierarchical MPC/ISM control scheme for nonlinear constrained uncertain systems.

via a discontinuous control law. After a transient phase, called "reaching phase", during which the insensitiveness of the system with respect to the uncertain terms is not guaranteed, the so-called "sliding phase" starts and the controlled system proves to be robust versus a specific class of uncertainties, the so-called "matched uncertainties", i.e., uncertain terms which act on the same channel of the control variable. The main drawback of SMC is the so-called chattering phenomenon, i.e., oscillations of the controlled variable due to the discontinuity of the control input [Boiko et al., 2007; Levant, 2010]. The most effective solution to alleviate this phenomenon is the generation of higher order sliding modes (HOSM), which are able to reduce the chattering effect while maintaining good stability properties [Bartolini et al., 1998, 2000; Levant, 2003; Dinuzzo and Ferrara, 2009; Ferrara et al., 2014b].

In order to improve the robustness feature of conventional SMC, the so-called Integral Sliding Mode (ISM) has been introduced [Utkin and Shi, 1996]. ISM, by removing the reaching phase, ensures the generation of an ideal sliding mode and the robustness of the controlled system since the initial time instant. Note that, higher order ISM algorithms have also been presented [Levant and Alelishvili, 2007; Goggia et al., 2014; Ferrara and Incremona, 2015].

MPC and SMC was first used in a combined scheme in [Muske et al., 2007; Garcia-Gabin et al., 2009]. In particular, the MPC has been used to update the parameters of the sliding manifold. A different idea to combine MPC and SMC was proposed in [Rubagotti et al., 2011a; Ferrara et al., 2013; Raimondo et al., 2014; Ferrara et al., 2014a], where a hierarchical control scheme composed by a high level MPC and a low level ISM is proposed (see Figure 1). The ISM component has the aim to reduce the difference between the dynamics of the nominal closed-loop system and the actual evolution of the state. Thanks to the presence of the ISM controller at the low level, the MPC can be designed on a system with reduced disturbances. The main idea is first introduced for continuous-time control systems, since the ISM is generally designed to generate a continuous-time control action [Rubagotti et al., 2011a]. In particular, it can be proved that, if only matched uncertainties affect the system, the predictive control law can be designed on the nominal model of the plant, since ISM completely rejects the matched uncertainties. Otherwise, if also unmatched uncertainties are present, it has been proved that their effect is not amplified by the ISM control, so that a robust MPC needs to be applied to cover only the residual uncertainties. In particular in [Rubagotti et al., 2011a] a continuous-time model with sampled data has been used to design a robust MPC with tightened constraints. According to [Magni and Scattolini, 2004], the optimization is performed in discrete-time with respect to a piecewise constant control signal. Regional Input-to-State Stability (ISS), developed for discrete-time systems in [Magni et al., 2006], and Input-to-State practical Stability (ISpS) are proved. The proposed scheme is only conceptual. In fact, since a finite non-negligible sampling time is used in any real application, the sliding manifold is not exactly reached but the chattering occurs making the sliding variable ultimately bounded within a boundary layer of the sliding manifold.

In order to provide a practical solution, a multirate discrete-time scheme is required [Raimondo et al., 2014]. In particular two different finite sampling times are used for the ISM (shorter) and for the MPC (longer). Hence also a discrete ISM is required, which consists in finding the control value capable of ideally steering the state to the sliding manifold in one sampling time. As a consequence, the control law is not defined on a discontinuous domain anymore, thus leading to a reduction of the chattering effect. The main drawback is that the discrete ISM needs the knowledge of the disturbance acting at the same time instant of the control action. Since it is impossible to get it, the matched disturbance is never completely rejected, and differently from the continuous-time scheme, a robust MPC is always necessary even if only matched uncertainties are present. Another step in the direction of a more effective application of the proposed hierarchical architecture is the possibility to introduce a bound also on the ISM control input. The equivalent disturbance visible by the MPC is computed as a function of the maximal input allocated to SMC, and a dynamic input allocation between SMC and MPC is provided. The dynamic allocation allows the adaptation of input partitioning according to the state conditions. If the state is far from the origin, ISM is applied with the minimum actuation range in order to ensure the MPC feasibility. Otherwise, if the state is closer to the origin and MPC requires less control amplitude, more amplitude is allocated to the ISM in order to provide a stronger disturbance rejection. ISpS has been proved also for this formulation [Raimondo et al., 2014]. A practical example of the MPC/ISM control strategy has been presented in [Ferrara et al., 2013], where a robust hierarchical multi-loop control scheme aimed at solving motion control problems for robot manipulators has been proposed.

A different practical situation that has been considered is the presence of a network between the two layers. The goal in this case, because of delays and packet loss induced by the presence of the network in the control loop [Tabuada, 2007; Gupta and Chow, 2010; Garcia and Antsaklis, 2013], is to avoid packets transmission if not necessary, both in the direct path (from the controller to the plant) and in the feedback path (from the sensor to the controller). To achieve this goal a model-based event-triggered MPC/ISM control scheme for nonlinear constrained continuous-time uncertain systems is discussed [Ferrara et al., 2014a]. The key elements of the proposed control scheme are a model-based controller and a smart actuator/sensor, both containing a copy of the nominal model of the plant. The sensor intelligence is provided by a suitable triggering condition, which enables to determine when it is necessary to transmit the measured state and to update the state of the nominal model. Furthermore, an asynchronous packetized version of a quasi-infinite horizon MPC with tightened constraints is designed, such that the optimization problem is solved only when a triggering event occurs. The ISpS of the controlled system is proved also in this case.

## 2. PROBLEM FORMULATION

Let first introduce the continuous-time model

$$\dot{x}(t) = \mathcal{F}(x(t), u(t), d(t)), \ t \geq 0 \tag{1}$$

where $t \in \mathbb{R}_{\geq 0}$, $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the control variable, and $d \in \mathbb{R}^q$ is the disturbance term. Given system (1), which is assumed to be forward complete, assume also that $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ denotes the nominal model, being

$$f(\hat{x}(t), u(t)) \triangleq h(\hat{x}(t)) + Bu(t) \tag{2}$$

with $\hat{x} \in \mathbb{R}^n$ being the state of the nominal model, $h : \mathbb{R}^n \to \mathbb{R}^n$ with $h(0) = 0$, and $B \in \mathbb{R}^{n\times 1}$. Then, the plant system can be expressed as

$$\dot{x}(t) = h(x(t)) + Bu(t) + w(t), \ t \geq 0 \tag{3}$$

where $w \triangleq \mathcal{F}(x(t), u(t), d(t)) - f(\hat{x}(t), u(t))$ denotes the additive uncertainty.

Then consider the discrete-time form of system (3)

$$x_{t+1} = h_d(x_t) + B_d u_t + w_t, \, t \geq 0 \tag{4}$$

where $t \in \mathbb{Z}_{\geq 0}$, $h_d$ and $B_d$ are the corresponding of $h$ and $B$ in the discrete-time form, respectively. The nominal evolution of (4) at time $t + 1$ is expressed as

$$\hat{x}_{t+1} = h_d(\hat{x}_t) + B_d u_t, \, t \geq 0 \tag{5}$$

Systems (3) and (4) are such that the state and the control variables are restricted to fulfill the following constraints

$$x \in \mathcal{X} \tag{6}$$

$$u \in \bar{\mathcal{U}} \tag{7}$$

where $\mathcal{X}$ and $\bar{\mathcal{U}}$ are compact sets containing the origin as an interior point. Moreover, the uncertainty $w$ is such that

$$w \in \mathcal{W} \tag{8}$$

where $\mathcal{W}$ is a compact set containing the origin, with $\mathcal{W}^{sup} = \sup_{w\in\mathcal{W}}\{|w|\}$ known, and $|\cdot|$ being the Euclidean norm. This last property can be obtained starting from system (1), assuming that the disturbance $d \in \mathcal{D}$, where $\mathcal{D}$ is a compact set containing the origin, and $\mathcal{D}^{sup} = \sup_{d\in\mathcal{D}}\{|d|\}$ is known.

The control objective consists in designing robust control schemes that guarantee ISpS and that state and control variables comply with constraints (6) and (7), respectively, despite of the presence of the uncertain terms.

## 3. THE HIERARCHICAL ARCHITECTURE

The control architecture considered in this paper is the combination of two components: a high level controller based on MPC, and a low level ISM controller (see Figure 1). The overall control law can be expressed as follows

$$u = u_{MPC} + u_{ISM} \tag{9}$$

The ISM approach is used to produce a control action aimed to reduce the difference between the nominal predicted dynamics of the closed-loop system and the actual one. In this way, the MPC, which has the role to stabilize the system and to optimize the performance, can be designed on a system with a reduced uncertainty.

### *3.1 Integral Sliding Mode Control*

ISM control enables to generate an ideal sliding mode of the controlled system on a particular sliding manifold starting from the initial time instant $t_0$ [Utkin and Shi, 1996]. This allows one to reduce to a minimum or eliminate the so-called reaching phase, improving the robustness issues of the controlled system. Considering $u_{MPC}$ as stabilizing controller designed relying on the plant with reduced uncertainties (how the uncertainties can be reduced with respect to the original ones will be clarified in the following), the discontinuous control action to compensate the matched uncertainties affecting the system is based on the definition of a suitable manifold. This sliding manifold, named "integral sliding manifold", is defined in a general form as

$$\Sigma(t) = s(x(t)) - \varphi(t) = 0 \tag{10}$$

where $\Sigma$ is the auxiliary sliding variable, $s$ is the actual sliding variable, chosen, for instance, as a linear combination of the states, i.e.,

$$s = Gx \tag{11}$$

with $G \in \mathbb{R}^{1\times n}$, and the integral term $\varphi$ is given by

$$\varphi(t) = s(x(t_0)) + \int_{t_0}^{t} \frac{\partial s}{\partial x}\dot{x}(\zeta)d\zeta \tag{12}$$

with the initial condition $\varphi(t_0) = s(x(t_0))$. By virtue of the choice of $\varphi(t)$ and $\varphi(t_0)$, the controlled system is in sliding mode on the manifold $\Sigma(t) = 0$ since the initial time instant. Then, the ISM control action is defined as

$$u_{ISM} = -U_{max}\,\mathrm{sgn}(\Sigma) \tag{13}$$

where $U_{max} > 0$ is suitably chosen in order to dominate the matched uncertainties and enforce the sliding mode on the sliding manifold $\Sigma = 0$ [Utkin and Shi, 1996].

Assume that the uncertain term $w$ can be written as

$$w(t) = Bw_m(x(t)) + B^{\perp}w_u(x(t)) \tag{14}$$

where $B^{\perp} \in \mathbb{R}^{n\times(n-1)}$, $w_m$ represents the "matched" uncertainty, which in practical applications is due to unavoidable unmodeled dynamics, parameter uncertainties and disturbances, while $w_u$ is the so-called "unmatched" uncertainty. It is possible to prove that the equivalent control, that is the control obtained by posing the first time derivative of the auxiliary sliding manifold equal to zero, i.e., $\dot{\Sigma} = 0$, is such that $u_{ISM_{eq}} = -w_m$, so as to have a rejection of the matched uncertainty in sliding mode from $t_0$. Moreover it is possible to show that the choice of the sliding manifold such that $G = B^T$ allows one to maintain the unmatched uncertainty without amplifying them [Rubagotti et al., 2011b].

### *3.2 Model Predictive Control*

As for the MPC component, since the ISM control is able to reject or reduce to a minimum the matched uncertainty, it can be designed relying on a system with reduced disturbances. A robust MPC is required in order to cope with the residual uncertainty. Note that, if the matched uncertainty is completely rejected and no other disturbances affect the system, a robust control action is not necessary, avoiding more computational burden. For the sake of simplicity, in the following only the continuous-time formulation of the algorithm is considered, without lost of generality. The aim is to solve a Finite-Horizon Optimal Control Problem (FHOCP) which consists in minimizing, at any sampling time instant $t_k$, a suitably defined cost function with respect to the control sequence $\mathbf{u}_{[t_k,t_{k+N-1}]}$, with $N \geq 1$ being the prediction horizon. To make the control action robust, tightened constraints that can be described via a Pontryagin difference of sets as

$$\mathcal{X}_{kT+\tau} = \mathcal{X} \sim \mathcal{B}_{kT+\tau} \tag{15}$$

$$= \{x \in \mathbb{R}^n : x + z \in \mathcal{X}, \forall\, z \in \mathcal{B}_{kT+\tau}\}$$

are included. The set $\mathcal{B}_{kT+\tau}$ will be suitably specified for each case in the following. Then, given two positive definite matrices $Q$ and $R$, the cost function to be minimized is

$$J(x(t_k), \mathbf{u}_{[t_k,t_{k+N-1}]}, N) = \\ = \int_{t_k}^{t_{k+N}} [x^T(\tau)Qx(\tau) + u^T(\tau)Ru(\tau)]d\tau + \\ + x^T(t_{k+N})\Pi x(t_{k+N}) \tag{16}$$

where $\Pi$ is the terminal state weight so as to ensure the stability of the controlled system. The minimization has to be accomplished subjected to the state dynamics, the constraints on the state variables

$$x(t) \in \mathcal{X}_{t-t_k} \tag{17}$$

and the constraint $\bar{u}_{MPC}$ on the control variable $u(t)$, which can be determined considering that a quantity equal to $U_{max}$ allocated for the ISM component (see (13)) must be subtracted to the control bound in (7), i.e.,

$$\bar{u}_{MPC} = \bar{u} - U_{max} \tag{18}$$

Finally, a terminal constraint $x(t_{k+N}) \in \mathcal{X}_f$, with

$$\mathcal{X}_f \triangleq \{x \mid x^T \Pi x \leq \rho_f\}, \quad \mathcal{X}_f \subseteq \mathcal{X} \tag{19}$$

where $\rho_f$ is a positive constant, is considered. The terminal penalty and the terminal constraint are chosen as in [Rubagotti et al., 2011a], in order to guarantee stability properties. Then, according to the "Receding Horizon" (RH) strategy, the applied control input is given by the following piecewise-constant signal

$$u_{MPC}(t) = \kappa_{MPC}(x(t_k)), \quad t \in [t_k, t_{k+1}) \tag{20}$$

where

$$\kappa_{MPC}(x(t_k)) \triangleq u^o(t_k) \tag{21}$$

with $u^o(t_k)$ being the first value of the optimal control sequence obtained by solving the FHOCP at $t_k$.

## 4. ROBUST MPC/ISM IN CONTINUOUS-TIME FOR SAMPLED-DATA NONLINEAR SYSTEMS

The first conceptual control scheme was developed for continuous-time sampled data nonlinear systems, since the ISM is generally designed to generate a continuous-time control action [Rubagotti et al., 2011a]. In [Utkin and Shi, 1996] it is shown that the ISM control completely rejects the matched uncertainties so that the system composed by the plant and the ISM can be rewritten as

$$\dot{x}(t) = h(x(t)) + Bu(t) + B^{\perp} w_u(x(t)) \tag{22}$$

where only the unmatched uncertainty is present, that, thanks to a suitable choice of the sliding manifold, is not affected by the ISM control action [Rubagotti et al., 2011b]. The MPC is then computed on the basis of the model relying on system (22), according to the FHOCP described in the previous section. In order to take into account the presence of the residual uncertainty, the set $\mathcal{B}_{kT+\tau}$ in (15) is defined as

$$\mathcal{B}_{kT+\tau} \triangleq z \in \mathbb{R}^n : |z| \leq \gamma \left( \tau + T\mathcal{L}_\tau \frac{\mathcal{L}_T^k - 1}{\mathcal{L}_T - 1} \right) \tag{23}$$

where $\tau \in [0, T]$, $\mathcal{L}_\tau \triangleq \mathcal{L}(\tau)$ being a positive continuous function defined in $[0, T]$ such that $\mathcal{L}_0 = 1$, and $\gamma$ is the upperbound of the residual uncertain terms, after the rejection operated by the ISM control. Assuming that the residual uncertainty is small enough, relying on [Magni et al., 2006], the ISpS of the closed-loop system (3) has been proved [Rubagotti et al., 2011a].

## 5. ROBUST MULTIRATE MPC/ISM FOR DISCRETE-TIME NONLINEAR SYSTEMS

The continuous-time scheme is only conceptual, in fact, since a finite non negligible sampling time is used in any real application, the SMC cannot guarantee the complete rejection of the matched uncertainty term. Moreover, due to the fact that the sampling time is not negligible, the so-called chattering phenomenon can occur. A possible solution is to compute at each sampling time the value of the control variable capable of steering the state to the sliding manifold in one sampling time (see [Bartolini et al., 1995] for a discussion on discrete-time sliding mode). To achieve this goal, in this work, the sliding manifold is chosen as the difference between the predicted and the actual evolution of the plant, i.e.,

$$s_{tM+i} = C(x_{tM+i} - \hat{x}_{tM+i|tM}), \ i = 1, \ldots, M \tag{24}$$

where $C \in \mathbb{R}^{1 \times n}$. Then, the value of $s$ in one step is given by

$$s_{tM+i+1} = C(h_d(x_{tM+i}) - h_d(\hat{x}_{tM+i|tM} + B_d u_{ISM}) \\ + d_{tM+i}) \tag{25}$$

so that the ISM control component results in being

$$u_{ISM} = -(CB)^{-1} C(h_d(x_{tM+i}) - h_d(\hat{x}_{tM+i|tM}) + d_{tM+i}) \tag{26}$$

Note that the disturbance estimation is necessary to compute the control law. An easy choice is to take the value of the disturbance at the previous time instant, defining $\hat{d}_t = d_{t-1}$. In view of this approximation also the matched disturbance is not completely rejected but an upperbound on the residual disturbance can be computed. This is the reason why a robust MPC approach needs to be suitably coupled with the adopted SMC strategy. These aspects are hereafter briefly addressed.

In this hierarchical multirate control approach, the sliding mode component, which is light and effective from a computational point of view, is computed with a shorter sampling time. The MPC component acts with an execution rate limited due to the complexity of the constrained optimization problem solved at each time instant. The structure of the control law is the same in (9). The high level controller takes advantage of the disturbance reduction operated by the sliding mode component. Given the state constraints (6), the tightened set in this case is the following

$$\mathcal{B}^{\nu-k}_{\mathcal{W}^{sup}} \triangleq z \in \mathbb{R}^n : |z| \leq \sum_{i=0}^{\nu-k-1} \lambda^{oi}(\gamma) \tag{27}$$

with $\nu \geq k$. Note that the cost function for the FHOCP is

$$\begin{aligned} &J(x(t_k), \mathbf{u}_{k,k+N-1}, N) = \\ &= \sum_{\nu=k}^{k+N-1} x^T_{\nu|k} Q x_{\nu|k} + u^T_{\nu}(\tau) R u_{\nu} + \\ &+ x^T_{k+N|k} \Pi x_{k+N|k} \end{aligned} \tag{28}$$

subjected to the state dynamics, the constraints on the state variables in (15), and the constraint $\bar{u}_{MPC}$ on the control variable $u(t)$.

Another step in the direction of a more effective application of the proposed hierarchical architecture is the possibility to introduce an input bound also on the ISM control. In this case the equivalent disturbances visible by the MPC can be computed as a function of the maximal input allocated to SMC [Raimondo et al., 2014]. The choice of the upperbound for the ISM control and consequently for the MPC (see (18)) can be done dynamically. For example in [Raimondo et al., 2014] the following strategy has been proposed: if the state is far from the origin, since the SMC cannot guarantee the robustness of the controlled system [Utkin, 1992], it is generated with the minimum actuation range in order to provide a large domain of attraction; if the state is closer to the origin more input is allocated to the SMC in order to provide a stronger disturbance rejection. Also for this scheme ISpS has been proved in [Raimondo et al., 2014].

## 6. MODEL-BASED EVENT-TRIGGERED MPC/ISM

Another practical situation that has been considered is the presence of a network between the two layers. The goal in this case, because of delays and packet loss induced by the presence of the network in the control loop is to reduce the packet transmission over the network. To achieve this goal a model-based event-triggered MPC/ISM control scheme for nonlinear constrained continuous-time uncertain systems is discussed in [Ferrara et al., 2014a]. As illustrated in Figure 2, the control scheme includes three key blocks, the model-based controller, the ISM controller and the smart actuator/sensor. The main novelty of this approach is the fact that MPC is called asynchronously only when necessary. Hence, it is necessary to develop an asynchronous MPC that provides input suggestions for an unknown possibly infinite horizon, a smart sensor that decides when it is necessary to provide a new measure to the MPC and a smart actuator that is able to generate a synchronous input suggestion for the plant on the basis of the asynchronous MPC suggestions. The model-based controller contains the MPC algorithm that is called when a new instance of the plant state is transmitted over the network. The ISM controller has the task of rejecting the matched uncertainties at any sampling time instant on the basis of the actual state. Note that, in Figure 2, the notation $x/\varnothing$ means that the actual state is sent or not through the network. The overall control law $u(t)$ is chosen as in (9).

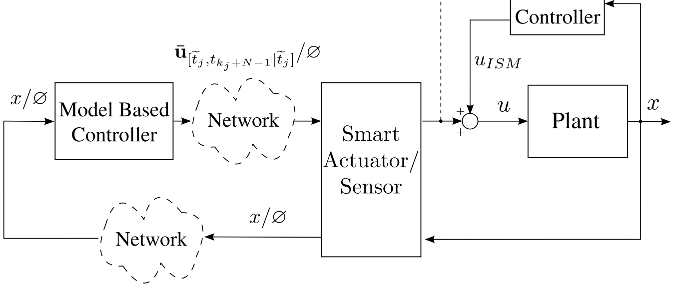


Figure 2. The model-based event-triggered MPC/ISM control scheme.

### 6.1 The Smart Actuator/Sensor

The smart sensor contains a copy of the nominal model of the plant, which provides the computed state $\hat{x}$ to the triggering condition block. This block, relying on the measured state $x$, computes the state error $e(t) = \hat{x}(t) - x(t)$, and verifies the following "triggering condition",

$$|e| \leq \max\{\varepsilon_1 |x|, \varepsilon_2\} \tag{29}$$

where $0 < \varepsilon_2 < \varepsilon_1 < 1$. If $|e| > \max\{\varepsilon_1 |x|, \varepsilon_2\}$, the actual state is sent to the controller and the state of the nominal model is updated. Note that even if in practice the triggering condition cannot be verified in continuous-time, a sampling time shorter than the one used in the MPC will be adopted so that the MPC is called in an asynchronous way.

As previously mentioned, the smart actuator provides asynchronous signals starting from the solution of the asynchronous optimization problem performed by the MPC. In particular, it will use the element of the finite optimal vector for a finite time, while later it will compute the control action based on the auxiliary control law. Specifically, the auxiliary control $\kappa(x(t))$, $t \geq t_{k_j+N}$, is applied after the end of the prediction horizon till the triggering condition is again violated and the MPC is recalled with the updated state.

### 6.2 The Asynchronous Model Predictive Control

By virtue of the rejection of the matched uncertainties produced by the ISM control (13), the MPC component can be developed relying on the system with reduced uncertainties (22). In order to cope with the residual uncertainty following the idea behind the control algorithm presented in the previous sections, the set $\mathcal{B}_{kT+\tau}$ is defined as

$$\mathcal{B}_{kT+\tau} \triangleq z \in \mathbb{R}^n : |z| \leq \gamma \cdot \\ \cdot \begin{cases} \left(\tau + T\mathcal{L}_{\tau} \frac{\mathcal{L}_T^k - 1}{\mathcal{L}_T - 1}\right), & k \leq N \\ \left(\tau + T\mathcal{L}_{\kappa_{f_\tau}} \left(\mathcal{L}^{k-N}_{\kappa_{f_T}} \frac{\mathcal{L}_T^N - 1}{\mathcal{L}_T - 1} + \frac{\mathcal{L}^{k-N}_{\kappa_{f_T}} - 1}{\mathcal{L}_{\kappa_{f_T}} - 1}\right)\right), & k > N \end{cases} \tag{30}$$

with $\tau \in [0, T]$. This definition of the tightened set guarantees that, if the nominal state evolution belongs to $\mathcal{X}_{kT+\tau}$ in (15), then the perturbed trajectory of the system fulfills (6).

Letting $\widetilde{t}_j$ the asynchronous triggering instant and $t_{k_j}$ the first sampling instant just after $\widetilde{t}_j$, considering the FHCOP problem, the associated finite horizon piecewise-constant control signal $\mathbf{u}_{[\widetilde{t}_j, t_{k_j+N}|\widetilde{t}_j)}$ is such that

$$\mathbf{u}_{[\widetilde{t}_j, t_{k_j}|\widetilde{t}_j)}(t|\widetilde{t}_j) = u_0(\widetilde{t}_j)$$

for all $t \in [\widetilde{t}_j, t_{k_j})$ and

$$\mathbf{u}_{[t_{k_j+1}, t_{k_j+N}|\widetilde{t}_j)}(t|\widetilde{t}_j) = u_i(\widetilde{t}_j)$$

for all $t \in [t_{k_j+i}, t_{k_j+i+1})$ and all $i \in [1, \ldots, N-1]$. Note that, because the FHOCP is not solved at any sampling time as usual but at any asynchronous triggering time, the first value of the vector $\bar{\mathbf{u}}_{[\widetilde{t}_j, t_{k_j+N-1}|\widetilde{t}_j]}$ is applied only from the triggering time $\widetilde{t}_j$ to $t_{k_j}$. Analogously to the previous described scheme, also in this case, assuming to have residual uncertainties small enough, the ISpS of the closed loop system (3) can be proved.

## 7. ILLUSTRATIVE EXAMPLE

As illustrative example, the model-based event-triggered MPC/ISM control strategy is performed in simulation on a cart moving on a plane.

The plant in question is described by the following equations

$$\begin{cases} \dot{x}_1(t) = x_2(t) + w_u \\ \dot{x}_2(t) = \frac{1}{M}\left(-k_0 x_1(t) - h_0 x_2(t) + u(t) + w_m(t)\right) \end{cases} \quad (31)$$

where the control variable $u$ is the force applied to the cart. Moreover, $M$ =1 kg is the mass of the cart, which is assumed to be known, $k_0 = 0.33\mathrm{N\,m^{-1}}$ is the stiffness of the spring, $h_0 = 1.1\mathrm{N\,s\,m^{-1}}$ is the damping factor, while the matched uncertain term is $w_m = W_m \sin(x_2)$ with $W_m$=1 N. Note that $w_m$ is unknown for the controller. In (31), signal $w_u$ is the unmatched uncertain disturbance, which is generated as a random noise, such that $|w_u| \leq 0.02\,\mathrm{m\,s^{-1}}$. Furthermore, the nominal model of the plant is expressed as follows

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) \\ \dot{\hat{x}}_2(t) = \frac{1}{M}\left(-k_0 \hat{x}_1(t) - h_0 \hat{x}_2(t) + u(t)\right) \end{cases} \quad (32)$$

with initial condition $x(0) = \hat{x}(0) = [-2.2,\ 1.7]^T$.

To perform the simulation tests, the Euler solver is used with a numerical integration step $\tau_i$ equal to 0.0005 s, while the MPC sampling time is chosen as $T$=0.2 s. The prediction horizon of the FHOCP is $N$=3, while the quantities $Q$ and $R$ in (16) are chosen as $Q = \mathbb{I}_2$, and $R$=1, respectively. The resulting auxiliary control law and the matrix $\Pi$ are equal to

$$\kappa_f(x(\widetilde{t}_j)) = Kx(\widetilde{t}_j), \quad K = [0.6413,\ 0.7306] \quad (33)$$

and

$$\Pi = \begin{bmatrix} 8.7647 & 3.6217 \\ 3.6217 & 4.6226 \end{bmatrix} \quad (34)$$

Moreover, the MPC has been tuned to satisfy the stability condition. The considered control and state constraints are set to $|u| \leq 2\,\mathrm{N}$, $|x_1|$, $|\hat{x}_1| \leq 3\,\mathrm{m}$, $|x_2|$, and $|\hat{x}_2| \leq 3\,\mathrm{m\,s^{-1}}$. The relative degree of the system is $r$=1, since the sliding variable is selected as $s = m_1 x_1 + x_2$, with $m_1$=1. Moreover, the transient trajectory $\varphi$ is chosen as in (12). The ISM control parameter in (13) has the amplitude $U_{max}$ =1. The triggering condition in (29) is specified by choosing $\varepsilon_1$=0.5 and $\varepsilon_2$=0.05.

In order to evaluate the closed-loop performance, we have considered three different indexes: i) the number of updates of the actual state, denoted with $n_{up}$; ii) the root mean square (RMS) value of the plant state, $x_{RMS}$; iii) the RMS value of the auxiliary sliding variable, i.e. $\Sigma_{RMS}$. These indexes are determined as

$$n_{up} = \sum_{i=0}^{n_s} f_{up}(\tau_i), \quad x_{RMS} = \sqrt{\frac{1}{n_s}\sum_{i=1}^{n_s}\sum_{j=1}^{n} x_{ji}^2}$$

$$\Sigma_{RMS} = \sqrt{\frac{1}{n_s}\sum_{i=1}^{n_s} \Sigma_i^2}$$

where $f_{up}(\cdot)$ is a flag equal to 1 when the actual state is transmitted over the network, equal to zero otherwise, and $n_s$ is the number of integration steps during the simulation; $x_{ji}$, and $\Sigma_i$ are the $j$-th component of the state vector, and the auxiliary sliding variable at the $i$-th integration step, respectively. Figure 3 shows the time evolution of the state variables of the plant and of the nominal model, which are both steered to a vicinity of zero, depending, as before, on the amplitude of the unmatched uncertain term. Figure 4 illustrates the control variable $u(t)$. In Figure 5, the relative threshold defined in (29), and the flags values are reported. As expected, both the states and the input respect the pre-specified constraints. Figure 6 shows the auxiliary sliding variable $\Sigma$ (black line) together with the sliding variable $s$ (green

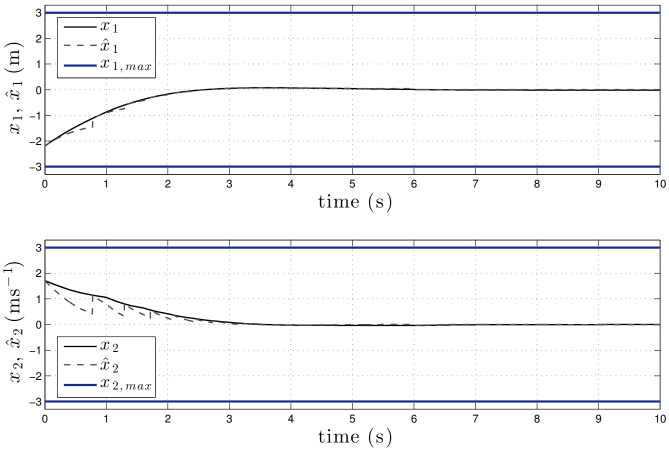


Figure 3. Time evolution of the state variables of the plant ($x_1$: top, $x_2$: bottom, solid black line), and of the model ($\hat{x}_1$: top, $\hat{x}_2$: bottom, dashed gray line).

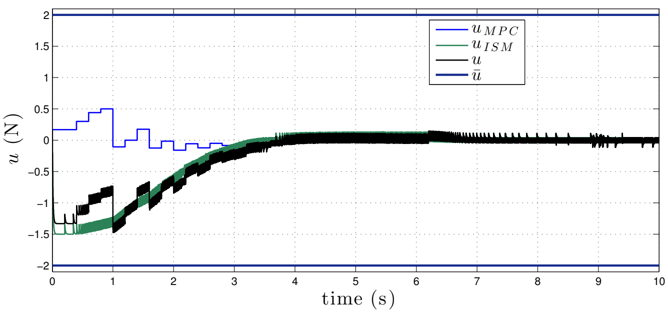


Figure 4. Time evolution of the control variable $u(t)$ (solid black line) along with the MPC (solid blue line) and ISM (solid green line) components.

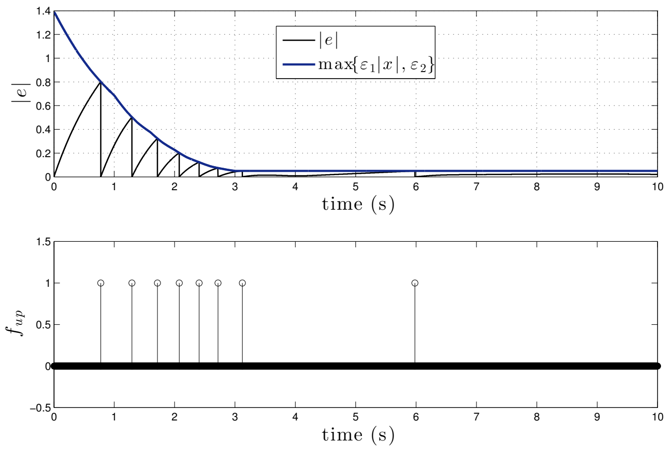


Figure 5. Time evolution of the state error $e$, and updates of the actual state when the event triggering condition is active.

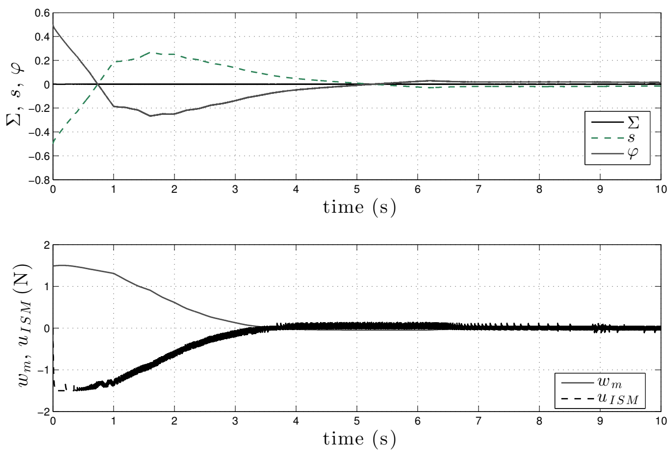


Figure 6. Time evolution of the auxiliary sliding variable $\Sigma$ (black line), of the sliding variable $s$ (green line) and of the transient function $\varphi$ (gray line), and the ISM component $u_{ISM}$ with respect to the matched uncertainty $w_m$.

line) and the transient function $\varphi$ (gray line). Moreover, the effect of the ISM component $u_{ISM}$ is illustrated with the corresponding rejected matched uncertainty term $w_m$. All the results are summarized in Table 1. One can notice

Table 1. Performance indices.

| | $n_{up}$ | $x_{RMS}$ | $\Sigma_{RMS}$ |
|---|---|---|---|
| on | 8 | 0.0707 | 0.0019 |
| off | 51 | 0.0705 | 0.0019 |

that the RMS value of the state and the RMS value of the auxiliary sliding variable are evidently small even when the triggering mechanism is activated. Finally, the number of updates is evidently reduced with respect to the case in which the state is always transmitted over the network, thus implying advantages in terms of packet loss, delays and jitter.

## 8. CONCLUSIONS

In this paper, recent results on the combined use of MPC and SMC have been discussed. The proposed solutions allow one to reduce the computation burden typical of the constrained optimization problem in presence of uncertainty terms. The introduction of an ISM component presents the advantage to completely reject or reduce to a minimum the matched uncertainties of the plant, so that the MPC component can be designed on a system with reduced disturbances. This is beneficial in terms of feasibility and stability properties of the MPC algorithm, which, in presence of large disturbances, could not guarantee feasible solutions while satisfying input and control constraints. Three different robust control schemes, recently published in the literature, have been presented. The first one is a robust MPC/ISM control scheme for nonlinear continuous-time uncertain systems. The second scheme is the corresponding multirate discrete-time version. Finally, the third scheme is a model-based event-triggered MPC/ISM control scheme for continuous-time systems.

## REFERENCES

Bartolini, G., Ferrara, A., and Usai, E. (1998). Chattering avoidance by second-order sliding mode control. *IEEE Trans. Automat. Control*, 43(2), 241–246. doi:10.1109/9.661074.

Bartolini, G., Ferrara, A., Usai, E., and Utkin, V. (2000). On multi-input chattering-free second-order sliding mode control. *IEEE Trans. Automat. Control*, 45(9), 1711–1717. doi:10.1109/9.880629.

Bartolini, G., Ferrara, A., and Utkin, V. (1995). Adaptive sliding mode control in discrete-time systems. *Automatica*, 31(5), 769 – 773. doi:http://dx.doi.org/10.1016/0005-1098(94)00154-B.

Bemporad, A., Borrelli, F., and Morari, M. (2003). Min-max control of constrained uncertain discrete-time linear systems. *IEEE Trans. Automat. Control*, 48(9), 1600–1606. doi:10.1109/TAC.2003.816984.

Boiko, I., Fridman, L., Pisano, A., and Usai, E. (2007). Analysis of chattering in systems with second-order sliding modes. *IEEE Trans. Automat. Control*, 52(11), 2085–2102. doi:10.1109/TAC.2007.908319.

Dinuzzo, F. and Ferrara, A. (2009). Higher order sliding mode controllers with optimal reaching. *IEEE Trans. Automat. Control*, 54(9), 2126 –2136. doi:10.1109/TAC.2009.2026940.

Edwards, C. and Spurgen, S.K. (1998). *Sliding Mode Control: Theory and Applications*. Taylor and Francis, London, UK.

Farina, M., Giulioni, L., Magni, L., and Scattolini, R. (2015). An approach to output-feedback MPC of stochastic linear discrete-time systems. *Automatica*, 55(0), 140 – 149. doi:http://dx.doi.org/10.1016/j.automatica.2015.02.039.

Ferrara, A., Incremona, G.P., and Magni, L. (2013). A robust MPC/ISM hierarchical multi-loop control scheme for robot manipulators. In *Proc. 52th IEEE Conf. Decision Control*. Florence, Italy, to appear.

Ferrara, A. and Incremona, G.P. (2015). Design of an integral suboptimal second order sliding mode controller for the robust motion control of robot manipulators. In *IEEE Trans. Control Syst. Technol.* doi:10.1109/TCST.2015.2420624.

Ferrara, A., Incremona, G., and Magni, L. (2014a). Model-based event-triggered robust MPC/ISM. In *Proc. Eu-*

ropean Control Conf.*, 2931–2936. Strasbourg. doi:10.1109/ECC.2014.6862523.

Ferrara, A., Incremona, G.P., and Rubagotti, M. (2014b). Third order Sliding Mode Control with box state constraints. In *Proc. 53th IEEE Conf. Decision Control*, 4727–4732.

Fontes, F.A.C.C. and Magni, L. (2003). Min-max model predictive control of nonlinear systems using discontinuous feedbacks. *IEEE Trans. Automat. Control*, 48(10), 1750–1755. doi:10.1109/TAC.2003.817915.

Garcia, E. and Antsaklis, P. (2013). Model-based event-triggered control for systems with quantization and time-varying network delays. *IEEE Trans. Automat. Control*, 58(2), 422–434. doi:10.1109/TAC.2012.2211411.

Garcia-Gabin, W., Zambrano, D., and Camacho, E.F. (2009). Sliding mode predictive control of a solar air conditioning plant. *Control Engineering Practice*, 17(6), 652 – 663. doi:http://dx.doi.org/10.1016/j.conengprac.2008.10.015.

Goggia, T., Sorniotti, A., De Novellis, L., Ferrara, A., Gruber, P., Theunissen, J., Steenbeke, D., Knauder, B., and Zehetner, J. (2014). Integral sliding mode for the torque-vectoring control of fully electric vehicles: Theoretical design and experimental assessment. *IEEE Trans. vehicular Tech.*, PP(99), 1–1. doi:10.1109/TVT.2014.2339401.

Gupta, R. and Chow, M.Y. (2010). Networked control system: Overview and research trends. *IEEE Trans. Ind. Electron.*, 57(7), 2527–2535. doi:10.1109/TIE.2009.2035462.

Lazar, M. and Heemels, W. (2009). Predictive control of hybrid systems: Input-to-state stability results for sub-optimal solutions. *Automatica*, 45(1), 180 – 185. doi:http://dx.doi.org/10.1016/j.automatica.2008.06.007.

Levant, A. (2003). Higher-order sliding modes, differentiation and output-feedback control. *Int. J. Control*, 76(9-10), 924–941. doi:10.1080/0020717031000099029.

Levant, A. (2010). Chattering analysis. *IEEE Trans. Automat. Control*, 55(6), 1380–1389. doi:10.1109/TAC.2010.2041973.

Levant, A. and Alelishvili, L. (2007). Integral high-order sliding modes. *IEEE Trans. Automat. Control*, 52(7), 1278 –1282. doi:10.1109/TAC.2007.900830.

Limon Marruedo, D., Alamo, T., and Camacho, E. (2002). Input-to-state stable MPC for constrained discrete-time nonlinear systems with bounded additive uncertainties. In *Proc. 41st IEEE Conf. Decision Control*, volume 4, 4619–4624 vol.4. Las Vegas, NV, USA. doi:10.1109/CDC.2002.1185106.

Limon Marruedo, D., Alamo, T., Salas, F., and Camacho, E. (2006). Input to state stability of min-max MPC controllers for nonlinear systems with bounded uncertainties. *Automatica*, 42(5), 797–803.

Maciejowski, J.M. (2002). *Predictive Control with Constraints*. Prentice Hall, Essex, England.

Magni, L., Raimondo, D., and Allgöwer, F. (2009). *Nonlinear Model Predictive Control: Towards New Challenging Applications*. Lecture Notes in Control and Information Sciences. Springer.

Magni, L., Raimondo, D., and Scattolini, R. (2006). Regional input-to-state stability for nonlinear model predictive control. *IEEE Trans. Automat. Control*, 51(9), 1548–1553. doi:10.1109/TAC.2006.880808.

Magni, L. and Scattolini, R. (2004). Model predictive control of continuous-time nonlinear systems with piecewise constant control. *IEEE Trans. Automat. Control*, 49(6), 900–906. doi:10.1109/TAC.2004.829595.

Magni, L., De Nicolao, G., Scattolini, R., and Allgower, F. (2003). Robust model predictive control for nonlinear discrete-time systems. *International Journal of Robust and Nonlinear Control*, 13(3-4), 229–246.

Mayne, D., Rakovifa, S., Findeisen, R., and Allgower, F. (2006). Robust output feedback model predictive control of constrained linear systems. *Automatica*, 42(7), 1217 – 1222. doi:http://dx.doi.org/10.1016/j.automatica.2006.03.005.

Mayne, D.Q. (2014). Model predictive control: Recent developments and future promise. *Automatica*, 50(12), 2967–2986.

Muske, K., Ashrafiuon, H., and Nikkhah, M. (2007). A predictive and sliding mode cascade controller. In *American Control Conf.*, 4540–4545. doi:10.1109/ACC.2007.4282633.

Pin, G., Raimondo, D., Magni, L., and Parisini, T. (2009). Robust model predictive control of nonlinear systems with bounded and state-dependent uncertainties. *IEEE Trans. Automat. Control*, 54(7), 1681–1687. doi:10.1109/TAC.2009.2020641.

Raimondo, D.M., Rubagotti, M., Jones, C.N., Magni, L., Ferrara, A., and Morari, M. (2014). Multirate sliding mode disturbance compensation for model predictive control. *Int. J. Robust and Nonlinear Control*. doi:10.1002/rnc.3244.

Rawlings, J. and Mayne, D. (2009). *Model Predictive Control: Theory and Design*. Nob Hill Pub, Llc.

Rubagotti, M., Raimondo, D., Ferrara, A., and Magni, L. (2011a). Robust model predictive control with integral sliding mode in continuous-time sampled-data nonlinear systems. *IEEE Trans. Automat. Control*, 56(3), 556–570. doi:10.1109/TAC.2010.2074590.

Rubagotti, M., Estrada, A., Castanos, F., Ferrara, A., and Fridman, L.M. (2011b). Integral Sliding Mode Control for Nonlinear Systems With Matched and Unmatched Perturbations. *IEEE Trans. Automat. Control*, 56(11), 2699–2704.

Scokaert, P.O.M. and Mayne, D. (1998). Min-max feedback model predictive control for constrained linear systems. *IEEE Trans. Automat. Control*, 43(8), 1136–1142. doi:10.1109/9.704989.

Tabuada, P. (2007). Event-triggered real-time scheduling of stabilizing control tasks. *Automatic Control, IEEE Transactions on*, 52(9), 1680–1685. doi:10.1109/TAC.2007.904277.

Utkin, V.I. (1992). *Sliding Modes in Optimization and Control Problems*. Springer Verlag, New York.

Utkin, V.I., Guldner, J., and Shi, J. (1999). *Sliding Model Control in Electromechanical Systems*. Taylor and Francis, London, UK.

Utkin, V.I. and Shi, J. (1996). Integral sliding mode in systems operating under uncertainty conditions. In *Proc. 35th IEEE Conf. Decision Control*, volume 4, 4591–4596. Kobe, Japan. doi:10.1109/CDC.1996.577594.