Evaluating the Effects of Social Interactions on a Distributed Demand Side Management System for Domestic Appliances

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Abstract In presence of time-variable energy tariffs, users will try to schedule the usage of their electrical appliances with the goal of minimising their bill. If the variable price component depends on the peak aggregate demand during each given hour, users will be incentivised to redistribute their consumption during the day, thus lowering the overall peak consumption. The process can be automated by means of an Energy Management System that chooses the best schedule while satisfying the user's constraints on the maximum toler-able delays. In turn, users' thresholds on delay tolerance may slowly change over time. In fact, users may be willing to modify their threshold to match the threshold of their social group, especially if there is evidence that friends with a more flexible approach have paid a lower bill. We provide an algorithmic framework that models the effect of social interactions in a distributed Demand Side Management System and show that such interactions can increase the flexibility of users' schedules and lower the peak power, resulting in a smoother usage of energy throughout the day. Additionally, we provide an alternative description of the model by using Markov Chains and study the corresponding convergence times. We conclude that the users reach a steady state after a limited number of interactions.

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1 Introduction

The application of Demand Side Management (DSM) approaches to the smart grid ecosystem has recently gained popularity and several frameworks aimed at shaping the aggregated power load curve of groups of customers [24] (e.g. to reduce/shift consumption peaks [30]), avoiding outages and improving power quality [12], or maximising the usage of Renewable Energy Sources (RESes) [14] have been proposed.

In DSM, different strategies may be adopted to motivate users to alter their energy usage patterns. Historically, stakeholders have focused on price-based policies: dynamic pricing schemes exhibiting hourly variations and reflecting the costs incurred by the smart grid system to satisfy the customers' demand have proven to be effective when the objective is the minimisation of the

users' bill [8,13]. To this aim, customers may opt for co-ordinated optimization schemes to avoid the drawbacks of uncoordinated shifts in their energy usage schedules (e.g. excessive consumption peaks during low-price periods). Coordinated solutions include centralized and distributed DSM frameworks: the former typically maximize a shared utility function [6], whereas in the latter each consumer locally defines her energy plan according to her personal preferences (e.g. bill minimisation or comfort maximization).

However, several recent studies investigated the effectiveness of non-monetary strategies in shaping and reducing consumption by stimulating long-term changes in beliefs and norms [11,4,23]. Indeed, consumers do not live in isolation: they can interact with each others and with public institutions, and therefore being influenced in their own attitudes, preferences and possible actions. This approach is consistent with the sociological paradigm of "Homo Socialis" [16], according to which agents' actions are not only determined by the desire to maximize their utility but also driven by shared norms, roles, and relationships. Therefore, social norms are nowadays an important topic of research not only in contemporary sociology, economics or game theory, but also in environmental sciences [1,19].

The main objective of our work is to take into account both the role of price policies and the influence mechanisms of norms on energy consumption behaviours. To this aim, we propose a distributed game-theoretic DSM framework which relies on an agent-based modeling approach. The framework includes a model of the social structures and interactions which define the reciprocal influences of socially-connected agents. These models make it possible to study the impact of social interactions on the user tolerance to delaying the starting time of their appliances, with or without knowledge of the electricity bill and delay tolerance levels of other users. In turn, the impact of delay tolerance on the electricity bill is modeled by using a load scheduling game for distributed demand side management, where rational agents run a distributed protocol to minimise the users' bills. The main contributions of this paper are:

- a description of the interaction mechanisms aimed at modeling the influence of the society on individual delay tolerance preferences;
- an evaluation of how social interactions modify individual delay tolerance preferences, which, in turn, affects the aggregated energy consumption curve obtained at the end of the execution of the load scheduling game;
- a Markov-chain model of the interaction mechanisms and the evaluation of the time necessary to converge to the steady state.

The remainder of the paper is structured as follows: Section 2 provides a short overview of the related literature, whereas Section 3 describes the structure of the agent-based model. An analytical model based on Markov chains which captures the evolution of the agent-based model is proposed in Section 4. The performance assessment of our proposed model and the discussion of the experimental results are provided in Section 5. Conclusions are drawn in the final Section.

2 Related Work

A substantial body of literature addresses the problem of Demand Response (DR) in smart grids (see [29] for a complete overview). Most of the proposed frameworks assume fully automated DR mechanisms which directly control the user's electrical loads and operate independently, without need of a direct involvement of the user. Alternatively, Agent-Based Models (ABMs) have raised interest in the smart grid research community as a promising approach to capture the complex interdependencies in the behaviour of electricity users who take direct actions on their (controllable) electrical appliances. Several recent studies propose frameworks incorporating multi-agent systems to collaboratively detect and react to grid outages [26], to improve the voltage profile by means of active/reactive power load control [2], or to develop energy market control strategies [9].

Some ABMs tackle socio-behavioural aspects of the interactions among users and between users and utilities: Worm et al. [31] propose a two-layered framework including a short-term choice model which captures the effects of energy price variations on the users' consumption patterns, depending on their comfort needs and on the presence of local RESes (e.g. solar panels), and a long-term behavioural model which defines how interactions within the social network may alter users' attitudes towards comfort requirements, energy efficiency, usage of RESes, and price policies. In this work we also study the effects of social interactions on a user-defined delay tolerance threshold, considering users' personal price-delay tolerance trade-off, but we depart from the aforementioned work by explicitly taking into account influence and imitation mechanisms within the proposed ABM, analogously to what has been done e.g. in [3,28] for residential water consumption.

Ramchurn et al. [27] describe a decentralised DSM framework which allows autonomous software agents installed at the customers' premises to collaboratively schedule the usage of domestic controllable appliances with the aim of minimising peaks in the aggregated

consumption within a neighbourhood, assuming the usage of dynamic pricing. The framework includes an adaptive mechanism which models the learning process adopted by the users to modify the deferral time of their controllable loads based on predicted market prices for the next day. Our proposed solution is also aimed at peak shaving and adopts a similar learning approach to update the users' delay tolerance threshold.

In the distributed DSM systems proposed by Barbato *et al.* [7] and Chavali *et al.* [10], residential user agents are modelled as rational entities who solve a Mixed Integer Linear Program (MILP) to minimise their energy bill. Under this assumption (i.e., each user applies a best response strategy), the users can be considered as players in a non-cooperative game theoretical framework: it has been proved in [removed citation]that such game is a generalised ordinal potential game which converge in a few steps to a pure Nash Equilibrium. In this paper we adopt the same assumptions and build upon the theoretical results therein discussed. However, in this paper the MILP formulation therein provided has been modified to take into account delay tolerance thresholds based on the users' attitudes.

Among the ABMs investigating influence and imitation mechanisms between agents, Helbing *et al.* [18] propose a model of norm formation in scenarios where agents exhibit incompatible preferences, and where rewards or punishment mechanisms are adopted to encourage conformity to the behaviour of others. In [20], Klein *et al.* introduce a computational model for habit contagion and change in a social network, in which cognitive processes are combined with interaction mechanisms. In this paper, we adopt similar models to capture imitation mechanisms between users.

3 The ABM Framework

We consider a two-stage interaction model.

In the first stage, which happens daily, user publish information about their current energy consumption behaviour. The parameters that we consider in this paper are the users' tolerance to delaying the starting time of their appliances and the daily bill. This stage may leverage a gamification platform such as the one described in [15], which could connect either real-life friends or simply people having similar habits. Users observe the information published by their social connections and modify their own tolerance. The platform may influence users behavior by filtering information, for example by advertising information about users that are saving on their bill. The output of this stage is a numerical parameter representing the delay tolerance

Table 1: Table of Symbols

Notation	Description
$\mathcal{U}, \mathcal{T}$	set of users and set of time slots within the optimisation horizon
$\mathcal{A}_u = \mathcal{A}_u^F \cup \mathcal{A}_u^S$	set of appliances of user $u \in \mathcal{U}$, including non-shiftable ($\mathcal{A}_u^F$) and shiftable appliances ($\mathcal{A}_u^S$)
$\mathcal{I} = \{\mathcal{I}_u\}_{u \in \mathcal{U}}$	set of strategies profiles $\mathcal{I}_u$ of users $u \in \mathcal{U}$
$\mathcal{P} = \{P_u\}_{u \in \mathcal{U}}$	set of utility functions P_u of users $u \in \mathcal{U}$
$\mathcal{J}\mathcal{U}$	set of iterations of the load scheduling game played by the users in $\mathcal{U}$
N_{au}, c_{au}^t	load profile duration of appliance $a \in \mathcal{A}_u$ owned by user $u \in \mathcal{U}$
c_{au}^t	comfort profile of appliance $a \in \mathcal{A}_u$ owned by user $u \in \mathcal{U}$ during each time slot $t \in \mathcal{T}$
γ_{au}	minimum delay tolerance level of user $u \in \mathcal{U}$ for appliance $a \in \mathcal{A}_u^S$
l_{an}^u	power consumption of appliance $a \in \mathcal{A}_u$ owned by user $u \in \mathcal{U}$ during slot $n \in \mathcal{N}_{au} = \{1, 2, \ldots, N_{au}\}$
b_t	electricity price during time slot $t \in \mathcal{T}$
s, c^{Anc}	slope of the energy cost function and cost of ancillary services
T	duration of a time slot
π_u	contractual limit of user $u \in \mathcal{U}$ on maximum energy consumption per slot
y_{ut}^j	energy consumption of user $u \in \mathcal{U}$ during slot $t \in \mathcal{T}$ at game iteration $j \in \mathcal{J}\mathcal{U}$
p_{ut}^j	aggregated energy consumption of users in $\mathcal{U} \setminus \{u\}$ during slot $t \in \mathcal{T}$ at game iteration $j \in \mathcal{J}\mathcal{U}$
x_{at}^j	binary variable set to 1 if the start time of appliance a of user u is scheduled at time $t \in \mathcal{T}$ at game iteration $j \in \mathcal{J}\mathcal{U}$, 0 otherwise

threshold of each user. This stage may either involve the end-user or may be automated.

In the second stage, a EMS schedules the deferrable appliances over the following day taking as input from each user the desired starting time and the delay tolerance. This stage is fully automated and leverages an EMS coordinator that circulates consumption information among the EMS of the various users until a satisfactory schedule is found.

In the rest of the Section, we first describe the automated load scheduling stage and then the social interaction stage. Table 1 lists all the symbols used in this Section.

3.1 The Load Scheduling Algorithm

We consider a set of residential users, $\mathcal{U}$, who allocate their power demand over a 24-hour time period divided into a set, $\mathcal{T}$, of time slots of duration T. Each user $u \in \mathcal{U}$ owns a set of non-interruptible electric appli-

ances, $\mathcal{A}_u$, that must run once a day. The load profile of each appliance $a \in \mathcal{A}_u$ lasts N_{au} time slots and its value in the nth time slot is given by l_{an}^u (with $n \in \mathcal{N}_{au} = \{1, 2.., N_{au}\}$). For the sake of easiness, we assume that l_{an}^u is constant for the whole duration of the nth time slot. Each appliance $a \in \mathcal{A}_u$ is also associated to a set of parameters $c_{au}^t \in [0, 1]$ which define the comfort level perceived by user u in starting appliance a at time slot $t \in \mathcal{T}$. The rationale behind the definition of such comfort level is the following: each user decides a preferred time slot for the starting time of her appliances. However, in case of deferrable appliances, she can tolerate to delay the starting time up to a certain number of time slots. Intuitively, the higher the delay is, the less comfortable such schedule is perceived by the user. It follows that the less pronounced the preference of user u for starting appliance a at time slot t is, the lower c_{au}^t is. In the extreme case of $c_{au}^t = 0$, user u will never turn on appliance a at slot t. In addition, user u specifies a threshold $\gamma_{au} \in (0, 1]$ indicating the minimum acceptable delay tolerance level for appliance a, which defines the degree of flexibility of the user in scheduling her appliances: the lower γ_{au} is, the more tolerant to delaying their starting time the user will be.

Each user $u \in \mathcal{U}$ may own two different types of appliances. *Non-shiftable* appliances (e.g., lights, TV), represented by the subset $\mathcal{A}_u^F \subseteq \mathcal{A}_u$, have a predefined starting time that cannot be shifted. Such constraint is imposed by assuming that there exists exactly one time slot $\bar{t}_{au} \in \mathcal{T}$ such that the condition expressed by Eq. (1) is satisfied

$$c_{au}^t = \begin{cases} 1 & \text{if } t = \bar{t}_{au} \\ 0 & \text{else} \end{cases} \tag{1}$$

which guarantees that non-shiftable devices have only one allowed starting time and that the system is forced to start them at time $\bar{t}_{au}$. Note that in this study we consider interruptible appliances in the same class as non-shiftable ones, since state-of-the-art algorithms for the automatic control of such type of appliances are typically designed to have no impact on the comfort level perceived by the users. Conversely, *shiftable* appliances (e.g., washing machine, dishwasher), represented by the subset $\mathcal{A}_u^S \subseteq \mathcal{A}_u$, are controllable devices and their starting time is an output of a scheduling algorithm. For these appliances, c_{au}^t may assume non-zero values in multiple slots, providing that there exists at least one time slot t such that $c_{au}^t = 1$ (i.e., t is u's preferred starting time for appliance a).

As depicted in Figure 1, the scheduling strategy i_u of player u is described by a set of binary decision variables

defined as per Eq. (2):

$$x_{au}^t = \begin{cases} 1 & \text{if appliance } a \text{ of user } u \text{ is started at time } t \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The set of all strategies of u is denoted by $\mathcal{I}_u$.

We say that the strategy i_u is *feasible* if it satisfies the following constraints:

$$\sum_{a \in \mathcal{A}_u^F} c_{au}^t x_{au}^t \geq |\mathcal{A}_u^F| \tag{3}$$

$$\sum_{t \in \mathcal{T}} c_{au}^t x_{au}^t \geq \gamma_{au} \quad \forall a \in \mathcal{A}_u^S \tag{4}$$

$$\sum_{t \in \mathcal{T}} x_{au}^t = 1 \quad \forall a \in \mathcal{A}_u \tag{5}$$

Constraints (3)-(4) ensure that the starting time of every appliance a provides a comfort level higher than the acceptability threshold γ_{au}. Constraints (5) impose that each appliance is executed exactly once per day.[1]

The pair $(\{c_{au}^t \mid t \in \mathcal{T}, a \in \mathcal{A}_u\}, \{\gamma_{au} \mid a \in \mathcal{A}_u\})$ is called the *comfort characteristic* of user u. Moreover, a comfort characteristic $C_u := (\{c_{au}^t \mid t \in \mathcal{T}, a \in \mathcal{A}_u\}, \{\gamma_{au} \mid a \in \mathcal{A}_u\})$ of user u is said to be *consistent* with the contractual limit[2] π_u on the amount of purchasable energy per time slot if there exists a strategy i_u such that:

$$\sum_{a \in \mathcal{A}_u} \sum_{\substack{n \in \mathcal{N}_{au}: \\ n \leq t}} l_{an}^u \cdot x_{au}^{t-n+1} \leq \pi_u \quad \forall t \in \mathcal{T} \tag{6}$$

Constraints (6) determine the overall consumption of the appliances in each time slot and bound the amount of purchasable energy in order not to exceed the contractual limit, π_u. Such consumption depends on the scheduling strategy: the energy required by each device a in every time slot t is equal to the energy consumption indicated by the nth sample (with $n \in \mathcal{N}_{au} = \{1, 2.., N_{au}\}$) of the load profile, l_{an}^u, executed at time t. Note that the energy amount indicated by the nth sample of the appliance load profile is consumed during slot t only in case the appliance is started at time $t - n + 1$, thus if $x_{au}^{t-n+1} = 1$.

The non empty set of feasible scheduling strategies consistent with π_u is denoted by $\mathcal{I}_u^C$. We also say that $\mathcal{I}_u^C$ is determined by (C_u, π_u).

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[1] Such a condition can be easily generalised to include an upper bound on the number of usages of an appliance.

[2] Contracts typically impose a limit Π on the peak power consumption, which can be straightforwardly converted in an energy consumption limit per time slot $\pi_u = \Pi T$, where T is the time slot duration.

We model the price of electricity at time $t \in \mathcal{T}$, $b_t(\cdot)$ as an increasing function of the total energy demand of the group of users $\mathcal{U}$ at time t [25]. Under this assumption, prices will increase during peak consumption periods. Therefore, an energy utility may impose such a price function with the goal of inducing peak shaving. For this reason, due to the conflicting goals of the users, the load scheduling problem cannot be solved with a centralised approach. Thus, we adopt the distributed game-theoretic framework proposed in [removed citation], which models the problem as a game $\mathcal{G} = \{\mathcal{U}, \mathcal{I}, \mathcal{P}\}$, defined by: i) the *players* representing the users in the set $\mathcal{U}$, each one associated to a comfort characteristic C_u and a contractual limit π_u on the purchasable energy per time slot; ii) the *strategy* set $\mathcal{I} \triangleq \prod_{u \in \mathcal{U}} \mathcal{I}_u^{C_u}$, where $\mathcal{I}_u^{C_u}$ indicates the strategy set of player u corresponding to her feasible load schedules determined by (C_u, π_u) (we assume here that such set is not empty); iii) the *payoff function* set $\mathcal{P} \triangleq \{P_u\}_{u \in \mathcal{U}}$, where P_u is the payoff function of user u, which coincides with her daily electricity bill.

The payoff function of each player, P_u, is defined as a function of $\mathcal{I}$ as per Eq. (7):

$$P_u(\mathcal{I}) = \sum_{t \in \mathcal{T}} y_{ut} b_t(y_t) \tag{7}$$

where

$$y_{ut} = \sum_{a \in \mathcal{A}_u} \sum_{n \in \mathcal{N}_{au} : n \leq t} l_{an}^u \cdot x_{au}^{t-n+1} \tag{8}$$

is the energy demand of user u at time t and $b_t(y_t)$ is the price of electricity at time t defined as a linear function of $y_t = \sum_{u \in \mathcal{U}} y_{ut}$, which represents the total electricity demand of the players at time t. In [removed citation], it has been proved that such function is a regular pricing function. It thence follows that $\mathcal{G}$ is a generalised ordinal potential game, with $P(\mathcal{I})$ being the potential function. Potential games admit at least one pure Nash Equilibrium which can be obtained by applying the Finite Improvement Property (FIP). Such propriety guarantees that any sequence of asynchronous improvement steps is finite and converges to a pure Nash equilibrium. In particular, a succession of best response updates converges to a pure equilibrium [21]. As proposed in [removed citation], we assume that the best response method is implemented in an iterative way as follows. Users in $\mathcal{U}$ are listed in a predefined order. The first user initiates the algorithm by choosing his/her optimal load schedule assuming flat tariffs. Then, as depicted in Figure 2, the user communicates her scheduled energy profile to the next user in the list, who executes the same operations but considering the hourly energy prices calculated from the expected
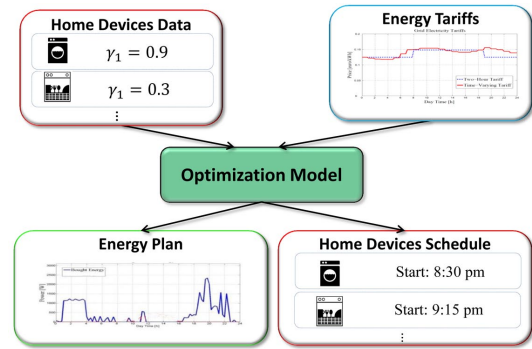

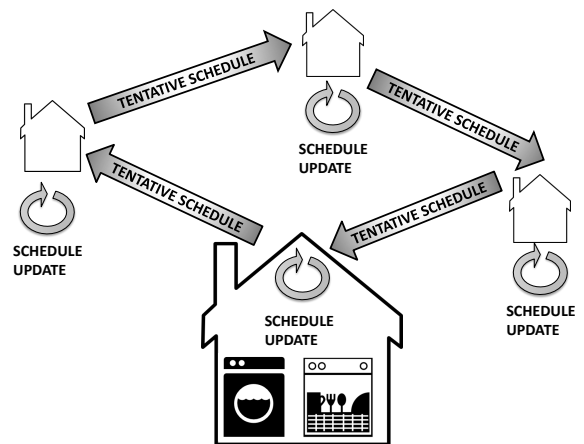Fig. 1: Inputs and outputs of the adopted load scheduling algorithm

Fig. 2: Iterative implementation of the best-response mechanism

hourly load obtained by summing the schedules of all the users in the list. At every iteration energy prices are updated and, as a consequence, other users can decide to modify their schedules. The procedure stops when none of the users alters her schedule in an iteration, meaning that convergence is reached.

In order to find the optimal schedule, each user solves the following Mixed Integer Non-linear Programming Model that minimises the objective function expressed by Formula (9):

$$\min \sum_{t \in \mathcal{T}} y_{ut} \cdot b_t \tag{9}$$

subject to constraints (3)-(6), where $b_t = b^{Anc} + s(y_{ut} + p_{ut})$, being p_{ut} the total energy demand of the players of the set $\mathcal{U} \setminus \{u\}$ received by user u at the current game iteration, b^{Anc} the cost of ancillary services (e.g., electricity transport, distribution and dispatching, frequency regulation, power balance) and s the slope of the cost function, respectively.

3.2 The Agent-based Model

We consider a time span of $\mathcal{L}$ days. During each day, users socially interact and influence one another's delay tolerance thresholds γ_{au}. The social interaction is mediated by a platform that collects each user's delay tolerance thresholds and the daily energy bill and distributes that information to the user's friends or to other similar users according to some filtering rules. For example a user might be willing to compare only to similar users that pay a lower bill. As a consequence of the social interaction, user u may modify his/her own values of γ_{au} to be used in the next execution of the load scheduling game (i.e., during the next optimisation time horizon). This modification can be the outcome of a user's rational decision, or it can be fully automated.

Users may not interact with their friends on regular daily basis (e.g. they may decide to communicate their delay tolerance thresholds only occasionally). Therefore, we characterised each user with a parameter p_u^ℓ which is set to 1 if user u is willing to compare (and possibly revise) her delay tolerance thresholds on day $\ell \in \mathcal{L}$, to 0 otherwise.

Similarly to the approach proposed in [9], in order to capture the users' capability to make rational decisions about whom to imitate (and up to which extent), for each appliance $a \in \mathcal{A}_u^S$ we consider a two-dimensional attitude space[3] (see Figure 3) where each user locates herself and her neighbours based on the delay tolerance threshold γ_{au}^ℓ of the current day ℓ and normalised daily energy bill per appliance B_{au}^ℓ defined by means of Eq. (10):

$$B_{au}^\ell = \frac{\sum_{t \in \mathcal{T}} b_t(y_{ut}) \sum_{n \in \mathcal{N}_{au}:\, n \le t} l_{an}^u \cdot x_{au}^{t-n+1}}{\sum_{t \in \mathcal{T}} b_t(y_{ut}) \cdot y_{ut}} \quad (10)$$

Based on her position, user u defines an area of interest A_u^ℓ (see shaded area in Figure 3) representing acceptable bill-delay threshold pairs. The criteria for the definition of such area depend on the personal attitude of the user (e.g., a user with a strong hedonistic attitude would be willing to imitate users with a higher delay tolerance threshold than hers, though their bill is -even significantly- higher than hers, whereas she would never imitate users with a lower delay tolerance threshold, even if their daily expense is lower than hers) and may be revised at each game execution.

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[3] This assumption also enables the users to redefine the sets $\mathcal{A}_u^F, \mathcal{A}_u^S$ before each game execution, e.g. in case some appliances are not regularly used on daily basis.
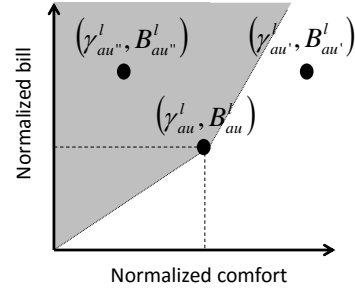

Fig. 3: Users' attitude space

The interaction protocol executed by user u at day $\ell \in \mathcal{L}$ proceeds as follows. Let $R[u]$ be the list of u's social neighbours. User u defines a boolean parameter $\eta_{u'}$ computed as per Eq. (11):

$$\eta_{u'} = \begin{cases} 1 & \text{if } (\gamma_{au}^\ell, B_{au}^\ell) \in A_u^\ell \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

For each neighbour $u' \in R[u]$, user u updates the delay tolerance threshold γ_{au}^ℓ of each appliance $a: a \in \mathcal{A}_u^S \wedge a \in \mathcal{A}_{u'}^S$ by means of Eq. (12):

$$\gamma_{au}^{\ell+1} = \begin{cases} \gamma_{au}^\ell + \dfrac{1}{|R[u]|} \Gamma(u) & \text{if (♣) holds} \\ \gamma_{au}^\ell & \text{otherwise} \end{cases} \quad (12)$$

where

$$\Gamma(u) = \sum_{\substack{u' \in R[u]: \\ \eta_{u'} \wedge p_u^\ell p_{u'}^\ell = 1}} h_a(u, u')(\gamma_{au'}^\ell - \gamma_{au}^\ell) \quad (13)$$

and $h_a(u, u')$ is the similarity between the comfort profiles of users u and u' with respect to appliance a. In this paper we will define similarity as half the number of time slots where c_{au}^t and $c_{au'}^t$ are both non-zero. The condition (♣) is given by Eq. (14):

$$\gamma_{au}^\ell + \frac{1}{|R[u]|} \sum_{\substack{u' \in R[u]: \\ \eta_{u'}=1 \wedge p_u^\ell p_{u'}^\ell=1}} h_a(u, u')(\gamma_{au'}^\ell - \gamma_{au}^\ell) > 0 \quad (14)$$

User u will then use the updated delay tolerance thresholds $\gamma_{au}^{\ell+1}$ in the next day for the execution of the load scheduling game.

4 Modeling the System Evolution with Markov Chains

The agent-based model described in Section 3.2 can be analysed with the help of a discrete-time Markov Chain.

We make following assumptions:

1. The user delay tolerance threshold can only take values that are integer multiples of a fixed step $\Delta\gamma$. The total number of possible tolerance threshold levels is $1/(\Delta\gamma) + 1$.
2. The agent state is represented by its tolerance thresholds at any given time for each appliance.
3. Each day is divided in iterations. At each iteration τ a single agent interacts with another user and modifies its delay tolerance threshold. This is slightly different than the model in Section 3.2, which considers that the effects of multiple interactions are applied at the same time. This assumption will make the Markov matrix more sparse and, thus, more manageable. On the other hand, the system will evolve more slowly.

Without loss of generality we consider that each user has a single appliance. Since the thresholds for the various appliances evolve independently, the full state of each user can be described by a set of identical Markov Chains evolving independently.

The full state at iteration τ for appliance a is thus given by the tuple: $(\gamma_{a1}^\tau, \gamma_{a2}^\tau, \ldots, \gamma_{a|\mathcal{U}|}^\tau)$. Thus, the total number of states is $|\mathcal{U}|^{1/\Delta\gamma+1}$, which grows exponentially fast as $\Delta\gamma$ becomes small. Therefore the model can be used only with a small number of levels.

At each time slot, an agent u uniformly at random interacts with another user u' uniformly at random and changes its delay threshold as per Eq. (15):

$$\gamma_{au}^{\tau+1} = \gamma_{au}^\tau +$$
$$\begin{cases} \lfloor \frac{\delta_{u,u'}}{\Delta\gamma} \rfloor \Delta\gamma & \text{with probability } \frac{\delta_{u,u'}}{\Delta\gamma} - \lfloor \frac{\delta}{\Delta\gamma} \rfloor \\ \lceil \frac{\delta_{u,u'}}{\Delta\gamma} \rceil \Delta\gamma & \text{with probability } \lceil \frac{\delta_{u,u'}}{\Delta\gamma} \rceil - \frac{\delta_{u,u'}}{\Delta\gamma} \end{cases} \quad (15)$$

with

$$\delta_{u,u'} = h_a(u,u')(\gamma_{au'}^\tau - \gamma_{au}^\tau) \quad (16)$$

Thus, the transition probability is given by Formula (17):

$$\Pr\left[(\ldots, \gamma_{au}^\tau + k\Delta\gamma, \ldots, \gamma_{au'}^{\tau+1}, \ldots)|\right.$$
$$\left.(\ldots, \gamma_{au}^\tau, \ldots, \gamma_{au'}^\tau, \ldots)\right] = \frac{1}{|\mathcal{U}|(|\mathcal{U}|-1)} \times$$
$$\left[\left\| \left\{ u' : k = \frac{\delta_{u,u'}}{\Delta\gamma} \right\} \right\| + \sum_{u' : k = \lfloor \frac{\delta_{u,u'}}{\Delta\gamma} \rfloor} \frac{\delta_{u,u'}}{\Delta\gamma} - \lfloor \frac{\delta}{\Delta\gamma} \rfloor + \right.$$
$$\left. \sum_{u' : k = \lceil \frac{\delta_{u,u'}}{\Delta\gamma} \rceil} \lceil \frac{\delta_{u,u'}}{\Delta\gamma} \rceil - \frac{\delta_{u,u'}}{\Delta\gamma} \right] \quad (17)$$

for all integer k such that $0 \le \gamma_{au}^\tau + k\Delta\gamma \le 1$, while it is equal to zero in all other cases.
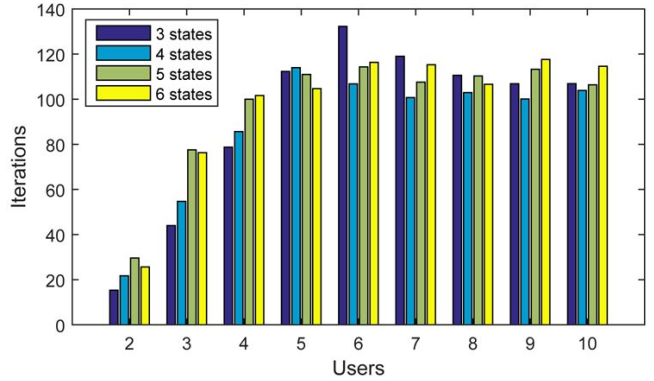

Fig. 4: Number of iterations to a steady state, depending on the number of users and states per users.

We used Equation (17) to model an homogeneous system with $h_a(u,u') = 1/2$ for all u, u' and study the number of time slots to reach steady state.

Figure 4 shows the average number of interactions after which the state probabilities change over time by less than 1%. The results are averaged over 100 Monte Carlo simulations with random initial conditions. The Figure shows that the number of discrete threshold levels has a limited impact on the convergence time, making it possible to study this important parameter with large discretisation steps, resulting in smaller, more manageable chains.

Additionally, we note that, for a very small number of users, the interactions to reach steady state grow as the number of users grows. Instead, for more than 5 users, the number of social interactions to reach equilibrium does not depend on the size of the social community. Consequently, we can say that, for a large network, the number of users has a limited impact on the convergence time, whereas the important parameter is the frequency at which users interact. Frequent interactions, with a large set of neighbours results in faster convergence.

5 Performance Assessment

5.1 Description of the Scenarios

In our tests, the 24-hour time horizon is represented by a set $\mathcal{T}$ of 24 time slots of $T = 1$ hour each. We consider $|\mathcal{L}| = 30$ consecutive executions of the load scheduling game presented in Section 3 and simulate social interactions at the end of each execution. The parameters of the electricity tariff, b_t, are defined based on the real-time pricing currently used in Italy. Specifically, $b^{Anc} = 0.5 \, \text{€}/\text{MWh}$ and $s = 2.3 \times 10^{-2} \, \text{€}/(\text{MWh})^2$.

We consider a scenario with $\mathcal{U} = 50$ users. The set $R[u]$ of each user's neighbours is computed based on the topology of a random scale-free network graph generated according to the Barabasi-Albert model with mean degree d, which is a popular generative model for based social networks and online communities [5]. If not differently stated, we assume $d = 6$.

Each user u has a contractual limit, π_u, of $3\,\mathrm{kWh}$ and owns 4 shiftable appliances (i.e., $\mathcal{A}_u^S = \{$ washing machine, dishwasher, boiler, recharge of robotic vacuum cleaner$\}$) and 7 non-shiftable ones (i.e., $\mathcal{A}_u^F = \{$ refrigerator, purifier, lights, microwave oven, oven, TV, iron$\}$). The operation of shiftable appliances is assumed to be controllable and fully automatised (i.e. by means of a home energy management system such as the one described in [17]). The energy consumption patterns of each appliance have been extracted from a real dataset [22]. On average, the energy consumption due to deferrable appliances accounts for 55% of the total daily consumption. For each appliance, the comfort curve c_{au}^t assumes a right-angled triangular shape of 4 slots duration randomly placed within the 24-hours scheduling horizon, with values $[1, 0.75, 0.5, 0.25]$ (i.e., we assume that the preferred starting time is the slot t such that $c_{au}^t = 1$ and that users' satisfaction decreases linearly with delay). The initial values of the appliance delay tolerance thresholds γ_{au}^1 are randomly chosen with uniform distribution in the range $[0, 0.75]$. If not differently stated, we assume that the probability of setting $p_u^\ell = 1$ is 85%.

In order to evaluate the performance of the proposed interaction mechanism we consider four scenarios: the first one assumes that the area of interest A_u^ℓ of user u is defined as:

$$A_u^\ell = \{(\gamma_{au'}^\ell, B_{au'}^\ell) \colon B_{au'}^\ell < B_{au}^\ell\}$$

(i.e., users imitate neighbours whose daily bill is lower than theirs); in the second one we define A_u^ℓ as:

$$A_u^\ell = \{(\gamma_{au'}^\ell, B_{au'}^\ell) \colon \gamma_{au'}^\ell < \gamma_{au}^\ell\}$$

(i.e., users imitate neighbours who impose lower delay tolerance thresholds than theirs), whereas in the third we set A_u^ℓ as:

$$A_u^\ell = \{(\gamma_{au'}^\ell, B_{au'}^\ell) \colon \gamma_{au'}^\ell < \gamma_{au}^\ell \wedge B_{au'}^\ell < B_{au}^\ell\}$$

(i.e., users imitate neighbours who have both lower daily bill and delay tolerance thresholds). Finally, in the fourth one users imitate all their neighbours, regardless to their bills and threshold values. The daily results obtained in the four scenarios are compared to the ones obtained during the first day (i.e. for $\ell = 1$), when no social interactions among the users have occurred. For

the assessment, the following metrics are measured: *individual bill*, i.e. the electricity bill of each user $u \in \mathcal{U}$, and *peak demand*, i.e. the peak of the aggregated energy demand of the group of users $\mathcal{U}$ defined as $\max_t y_t$.

5.2 Numerical Results

Average results obtained over 50 instances of each scenario are reported in Figure 5, which shows that in the first three scenarios the imitation of virtuous behaviours lead to non negligible decreases of the individual daily bills and of the aggregate peak energy consumption. Bill reductions are more consistent in scenario 2, i.e. when users imitate neighbours with lower delay tolerance thresholds regardless to their bill (0.5% reduction versus 0.3% in scenario 1 and 0.4% in scenario 3).
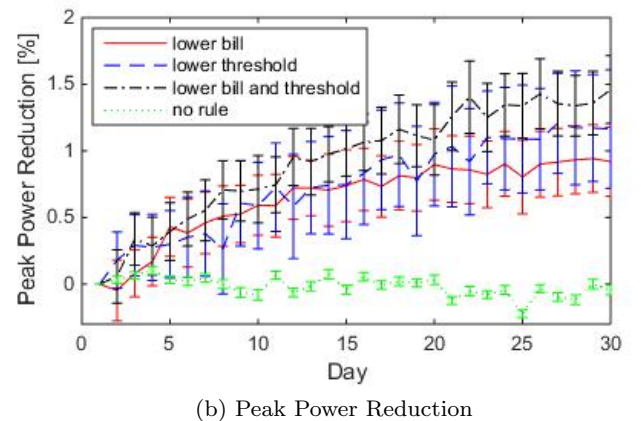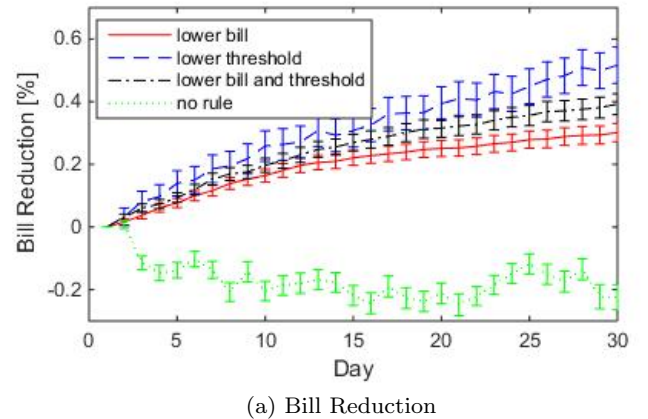

(a) Bill Reduction

(b) Peak Power Reduction

Fig. 5: Percentual reduction of daily bill and peak energy consumption depending on the definition of the users' area of interest A_u^ℓ, with $|\mathcal{U}| = 50$. 95% confidence intervals are plotted.

This is due to the fact that achieving a low bill does not necessarily imply a low delay tolerance threshold: users can indeed achieve low bills if their preferred appliance usage periods are very different from the ones of the other users (e.g. they span night hours), which would lead to lower values of the aggregate power consumption and, consequently, to lower energy prices. It follows that imitating users with low bills does not always lead to an increase of the flexibility of the individual schedules. Conversely, the results obtained in the fourth scenario show that imitating all the neighbours regardless to their experienced bills and tolerance thresholds causes an opposite trend, i.e. an increase of the average bill.

In terms of aggregate peak reduction, simulation results make it possible to conclude that Scenario 1 leads to the least peak power reduction, which settles at about 0.9% less than the peak power obtained with no social interaction. Better performance is obtained in second and third scenarios, which provide similar peak power reductions, whereas the fourth scenario does not lead to noticeable benefits.

It is worth noting that imitating the behaviour of other users disregarding the delay tolerance threshold and the bill does not lead to any advantages and could also result in an increase of the bill and of the peak power. This is shown by the lower lines in Figures 5 (a) and (b). It is therefore necessary that any social interaction is mediated by some filtering mechanism which could be implemented by the interaction platform.

Figure 6 shows the evolution of the delay tolerance threshold γ_{au}^{ℓ} of the washing machine for the whole population of users in a representative instance of the two scenarios. As depicted in Figure 6a, in the first scenario the imitation of the neighbours with lower bills leads to a homogenisation of the thresholds.

Moreover, the average value of γ_{au}^{ℓ} tends to decrease, due to the fact that people experiencing lower bills are more likely to have chosen low delay tolerance thresholds. It follows that imitating them leads to more elasticity in the scheduling patterns. However, some users never alter their delay tolerance threshold: this happens when none of their neighbours ever experiences lower bill than theirs. Conversely, in scenario 2 the benefit of imitating neighbours with lower delay tolerance threshold clearly emerges: in this case, γ_{au}^{ℓ} never increases with time w.r.t. its initial value and remains constant only in case the delay tolerance threshold of a given user is always lower than the one of all her neighbours. Results obtained in scenario 3 (resp. 4) shows trends analogous to Scenario 2 (resp. 1) and are thus not reported for the sake of conciseness.

Table 2 shows the the aggregate peak power consumption and total bill of the group of users under three different assumptions: (1) the users do not tolerate any delay in the starting time of their shiftable appliances, which thus behave as non-shiftable ones; (2) the users have variable tolerance thresholds but do not socially interact with one another and therefore their thresholds do not change over time; (3) users imitate neighbours that experience a bill lower than their own (former Scenario 1). The savings from (1) to (2) are in line with previous literature on Demand Side Management (e.g. [7]); the additional saving from (2) to (3) is due to the social interaction, when only the users with a better bill are imitated.

We now further refine our assessment focusing on scenario 1 (i.e. the imitation of users experiencing lower bills). Fig. 7 reports bill and peak power savings depending on the level of participation of the users to the social interactions. Intuitively, the more the users are willing to interact and revise their strategies based on the comparison of their bill to those of their neighbours, the higher are the obtained savings. Fig. 7 shows that increasing the level of user activity from 50% to 100%
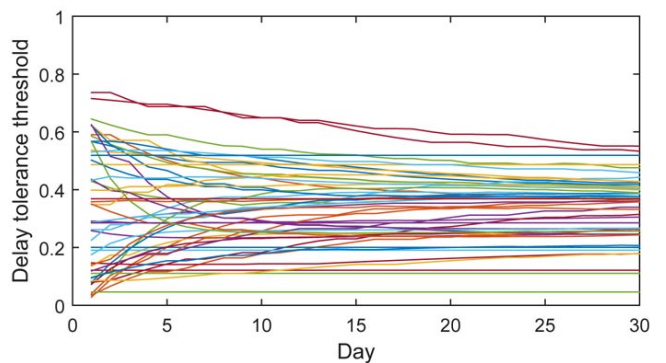
(a) Scenario 1
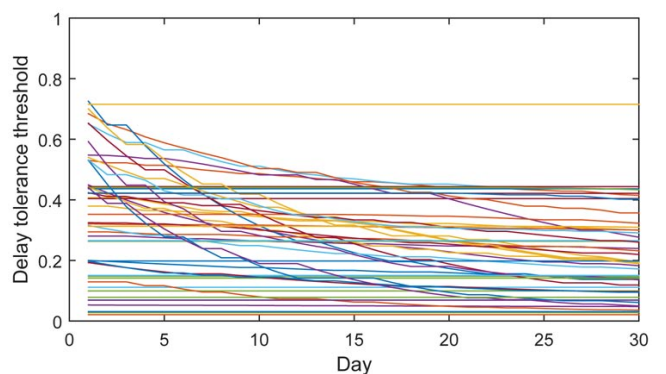
(b) Scenario 2

Fig. 6: Trend of the whashing machine delay tolerance threshold γ_{au}^{ℓ} over time depending on the definition of the users' area of interest A_u^{ℓ}.

Table 2: Comparison of the aggregate peak power and aggregate bill after 30 days.

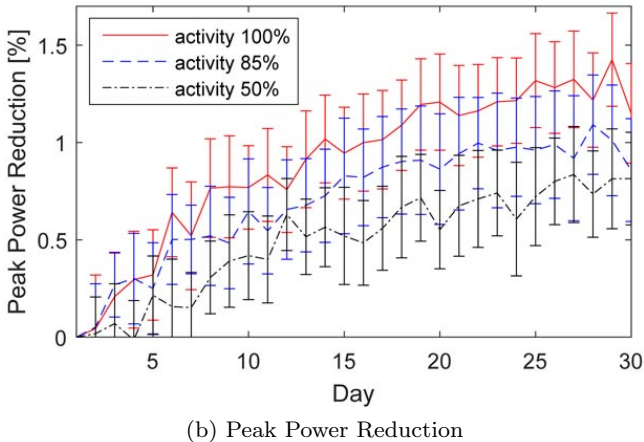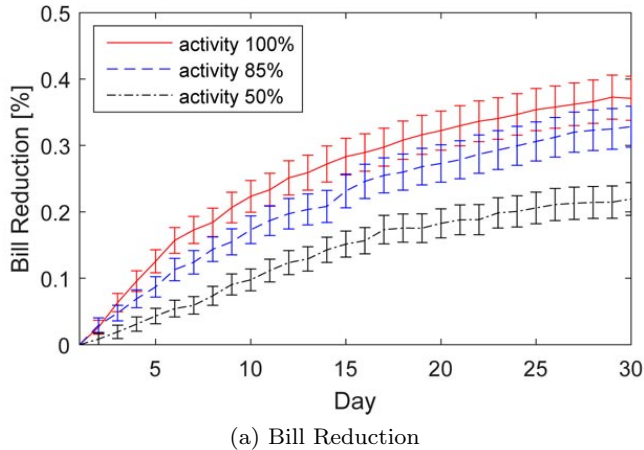| Strategy	Peak Power (kW)	Bill (€)
No delay tolerated	26.7	180
Random delay tolerance, no social interaction	20.8	169
Social interaction with users with lower bill	20.6	168

(a) Bill Reduction

(b) Peak Power Reduction

Fig. 7: Percentual reduction of daily bill and peak energy consumption for different values of the parameter p_u^ℓ. 95% confidence intervals are plotted.

doubles both the bill reduction and the peak power reduction achievable in one month.

Moreover, we evaluate the impact of the mean degree of connectivity d of the social network: Fig. 8 shows that increasing d from 3 (which corresponds to an user interacting with around 4% of the other users) to 6 (i.e. each user interacts on average with 10% of the other users) almost triplicates the bill reduction and doubles the peak power reduction. Further increasing d did not lead to noticeable additional savings.

(a) Bill Reduction
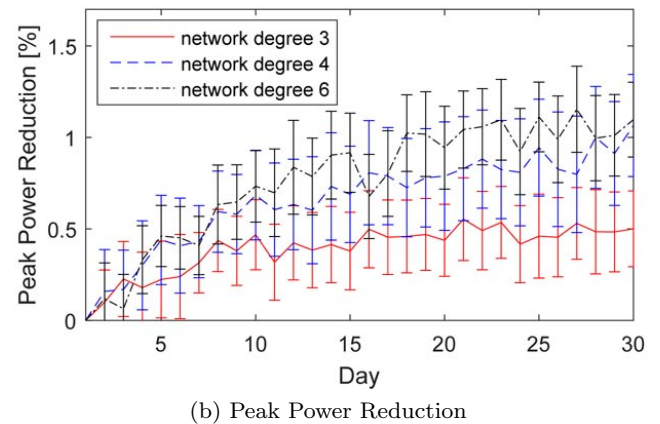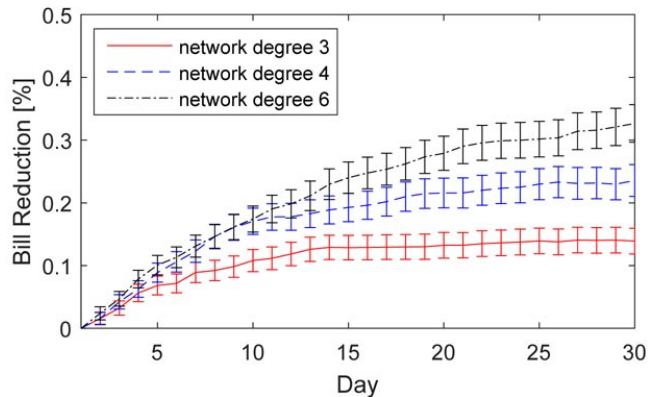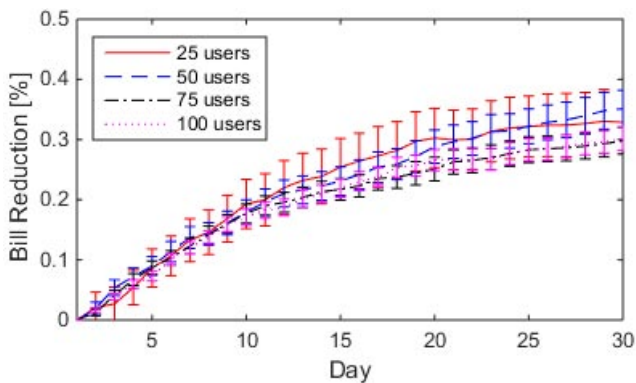

(b) Peak Power Reduction

Fig. 8: Percentual reduction of daily bill and peak energy consumption depending on the average connectivity degree of the social network. 95% confidence intervals are plotted.

Finally, we investigate the impact of the total number of users participating to the DSM protocol. Interestingly, as shown in Fig. 9 increasing the cardinality of the set of users does not lead to significant variations in the average bill reduction, whereas the peak power reduction is highest for very small groups (e.g. 25 users). In fact, when there are fewer users the impact of each user on the peak power is larger and it is sufficient that a single user lowers his/her delay tolerance threshold to have a beneficial effect on the peak power. As the number of users grows, it is necessary that many users improve their tolerance for having a significant effect.

6 Conclusion

In this paper, we study the impact that social interaction has on the peak power demand of a group of users who own shiftable domestic appliances. One of the problems with demand side management is that user flexibility varies over time, depending on observed savings and social pressure. In this study, we assume that

(a) Bill Reduction
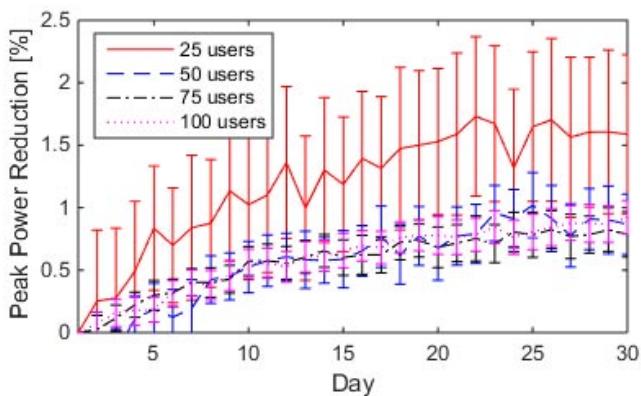
(b) Peak Power Reduction

Fig. 9: Percentual reduction of daily bill and peak energy consumption depending on the total number of users. 95% confidence intervals are plotted.

users are willing to vary their delay tolerance thresholds over time by matching the ones of the users of their social group. We show with simulations that social interaction cannot, by itself, drive the users to shave their aggregate power consumption. Instead, if the users imitate other users with lower bill or with greater delay tolerance, the overall peak demand lowers, with beneficial results on the energy management costs.

The models and the findings of this paper can be used by utilities to study how much user awareness of other users' behaviour can impact on the demand. We also show that a limited number of interactions with a relatively small set of neighbours is sufficient to achieve a steady state condition.

7 Acknowledgments

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