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New developments in frontier models for objective assessments

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To my family

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The realization of this thesis has been possible thanks to some key teachings, collaborations and suggestions.

Firstly, my most sincere thanks goes to my supervisor prof. Bernardo Maggi, University of Rome La Sapienza, who introduced me to the production efficiency theory in my degree course transmitting his passion for the topic. He has followed my research for the past 6 years and has continuously encouraged me whilst leaving me free to pursue my own ideas an find my own way forwards. He has provided solid foundations for my understanding of the literature allowing me to propose new methods.

I am particularly grateful to my friend, colleague and main coauthor Francesco Vidoli who is and has been a guide for me. He encouraged me to start this educational path, has always believed in me and has taught me how to write a paper, how to use some statistical tools and LaTeX (he is a source of knowledge and ideas!) and even how to set up my own research sometimes pretending not to know in order to make me more independent. He also encouraged me to overcome my shyness and to speak in public, making me more self-confident every day. All his advice has been very precious.

Special thanks should be given to my parents for having always encouraged me to study and to follow my dreams, to my sister, my brother and my friends for their unconditional support at all times and last but not least my husband Daniele who has always supported my lifestyle choices and who is very patient with his wife's love of studying.

MOTIVATIONS AND MAIN OBJECTIVES

"Creativity is intelligence having fun"
Albert Einstein

This dissertation is the result of some innovative proposals, in the wide framework of production efficiency frontier models, that have the common goal of reducing subjective choices of the researcher by using, as far as possible, objective methods.

In particular, the first proposal links the economic efficiency theory to the spatial econometrics with the aim of taking into account - in the efficiency evaluation of a productive unit - the neighborhood effects in a global way avoiding the subjective selection of a set of variables identifying territorial effects. The method called *Spatial Stochastic Frontier Analysis* (SSFA) has been published in Fusco and Vidoli (2013) for the production efficiency analysis and generalized in this thesis to be able to also analyze the cost efficiency.

The second proposal, instead aims to introduce enhancements in the methods using frontier techniques to aggregate simple indicators in a composite indicator. Subjectivity is avoided in the identification of the set of aggregation weights necessary for constructing the composite indicator, in the definition of a preference structure among simple indicators and in the extreme values and outliers influence removal. The two methods proposed, called respectively *Directional Benefit of the Doubt* (D-BoD) and *Robust Directional Benefit of the Doubt* (RD-BoD), have been published in Fusco (2015) and Vidoli, Fusco and Mazziotta (2015).

The dissertation consists of four parts: the first one introduces the foundations of the economic efficiency analysis and gives key economic concepts and definitions needed for a proper understanding of the following parts, focusing both on parametric and on nonparametric methods for cross-sectional and panel data and for mono-output and multi-output production processes; the second one discusses the fundamentals of the spatial econometrics, on the main connection proposals with the efficiency theory and shows in detail the SSFA method and the related R package called `SSFA` implemented to allow other researchers to use it; in the third part the concept of composite indicator and the required steps for its construction are discussed and D-BoD and RD-BoD are shown, moreover the related R package `Compind` is presented; all proposed methods have been tested both on simulated data and on real data and the results are shown in the fourth part. In the last part, two innovative applications, respectively on the estimation of non performing loans of commercial banks (Fusco and Maggi, 2016) and on the estimation of the local governments' expenditure needs (Vidoli and Fusco, 2017) by using the efficiency and spatial theories, are also included .

Scientific acknowledgments

I wish to thank my co-authors Francesco Vidoli (Fusco and Vidoli, 2013; Vidoli, Fusco and Mazziotta, 2015; Vidoli, Cardillo, Fusco and Canello, 2016; Vidoli and Fusco, 2017), Prof. Bernardo Maggi (Fusco and Maggi, 2016), Prof. Claudio Mazziotta (Vidoli, Fusco and Mazziotta, 2015) and Prof. Jacopo Canello (Vidoli, Cardillo, Fusco and Canello, 2016) for the fruitful collaboration.

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LIST OF ABBREVATIONS AND SYMBOLS

Abbrevations

ACF Autocorrelation function

AE Allocative efficiency

BLUE Best Linear Unbiased Estimators

BLUP Best Linear Unbiased Prediction

BoD Benefit of Doubt

BoD-PVC Benefit of Doubt Penalized by Coefficient of Variation

CD Cobb-Douglas

CE Cost efficiency

CI Composite Indicator

COLS Corrected Ordinary Least Squares

CRS Constant Returns to Scale

CCR-DEA Data Envelopment Analysis with Constant Returns to Scale

D-BoD Directional Benefit of Doubt

DEA Data Envelopment Analysis

DFA Deterministic Frontier Analysis

DGP Data Generating Process

DRS Decreasing Returns to Scale

EM Expectation-maximization

FDH Free Disposal Hull

FGLS Feasible Generalized Least Squares

FFF Fourier Flexible Functional form

GLS Generalised Least Squares

GWR Geographically Weighted Regression

IRS Increasing Returns to Scale

LAs Local Authorities

LISA Local indicators of spatial association

LWR Local Weighted Regression

MCMC Markov chain Monte Carlo

ML Maximum Likelihood

MLE Maximum Likelihood Estimation

MOLS Modified Ordinary Least Squares

MPVC Method of Penalties for Coefficient of Variation

NPL Non Performing Loan

OLS Ordinary Least Squares

Order-m Robust Nonparametric Efficiency Analysis - Order-m

PCA Principal Component Analysis

PACF Partial autocorrelation function

R-BoD Robust Benefit of Doubt

RD-BoD Robust directional Benefit of Doubt

RE Revenue efficiency

RMSE Root Mean Squared Error

SAC Spatial AutoCorrelation model

SAC-mixed Mixed-regressive-spatial autoregressive model with a spatial autoregressive disturbance

SAR Spatial AutoRegressive model

SEM Spatial Error Model

SFA Stochastic Frontier Analysis

SSFA Spatial Stochastic Frontier Analysis

StoNED Stochastic Nonparametric Envelopment of Data

StoNEZD Stochastic Nonparametric Envelopment of Data with z-variables

TE Technical efficiency

TL Translog

2SLS Two Stage Least Squares

VRS Variable Returns to Scale

Symbols

$i = 1, ..., N$	Producer or decision making unit
$\mathbf{x} = (x_1, ..., x_P)$	Input vector
$\mathbb{R}_+^P$	Input space
$\mathbf{y} = (y_1, ..., y_Q)$	Output vector
$\mathbb{R}_+^Q$	Output space
$\chi = \{(X_i, Y_i), i = 1, ..., n\}$	Random sample
$\Psi(\mathbf{x}, \mathbf{y})$	Technology set or Production set
$L(\mathbf{y})$	Input set
$P(\mathbf{x})$	Output set
$f(\mathbf{x})$	Production frontier
θ	Input-oriented measure of technical efficiency
ϕ	Output-oriented measure of technical efficiency
$\mathbf{w} = (w_1, ..., w_P)$	Input price vector
$\mathbf{C} = (C_1, ..., C_N)$	Cost vector
$c(\mathbf{y}, \mathbf{w})$	Cost frontier
$\mathbf{p} = (p_1, ..., p_Q)$	Output price vector
$r(\mathbf{x}, \mathbf{p})$	Revenue frontier
$D_I(\mathbf{x}, \mathbf{y})$	Input distance function
λ	Input distance at page (15) or Lagrange multiplier at page (20) or spatial autoregressive coefficient at page (53)
$D_O(\mathbf{x}, \mathbf{y})$	Output distance function
μ	Output distance
$\overrightarrow{D}_T(\mathbf{x}, \mathbf{y}; \mathbf{g})$	General directional distance function
$\mathbf{g}$	Directional vector
g_x	Input direction
g_y	Output direction
β	Parameter vector
$\mathbf{u} = (u_1, ..., u_N)$	Inefficiency vector
$\mathbf{v} = (v_1, ..., v_N)$	Noise vector
$\epsilon = (\epsilon_1, ..., \epsilon_N)$	Compound error vector
$N(\mu, \sigma)$	Normal distribution
$N^+(\mu, \sigma)$	Half-Normal distribution
W	Spatial weights matrix
d_{ij}	i and j distance
ρ	Error spatial lag
$\mathbb{E}$	Expected value
$\mathbb{V}$	Variance

Part I

Efficiency analysis

CHAPTER 1

THE MEASUREMENT OF EFFICIENCY

The efficient use of physical (*e.g.* labor, energy, capital, etc.) and monetary resources, or an efficient production process, are the two main objectives of any production unit to be competitive on the market and to maximize the profits.

Since 1950, beginning with Koopmans (1951), Debreu (1951) and Shephard (1953) a wide literature has developed in econometrics, management sciences, operations research and mathematical statistics areas with the aim to implement new tools for analyzing productivity and efficiency of firms (please see Shephard (1970) for a comprehensive presentation of the underlying economic theory).

This chapter introduces to the foundations of the economic efficiency analysis and gives key economic concepts and definitions needed for a proper understanding of the following chapters.

1.1 Production frontiers and technical efficiency

1.1.1 Production frontier

Consider the case of a production unit i using a positive vector of P inputs, denoted by $\mathbf{x} = (x_1, ..., x_P)$, $\mathbf{x} \in \mathbb{R}_+^P$ to produce a positive vector of Q outputs, denoted by $\mathbf{y} = (y_1, ..., y_Q)$, $\mathbf{y} \in \mathbb{R}_+^Q$.

To determine if an input set can produce an output set knowledge on technology, that is the set of feasible production activities of i, is needed.
The general way to characterize the technology of i is the *technology set* $\Psi(\mathbf{x}, \mathbf{y})$, which represents the all combinations of input and output such that $\mathbf{y}$ can be produced with the input vector $\mathbf{x}$, *i.e.* $\Psi = \{(\mathbf{x}, \mathbf{y}) : \mathbf{x}\ \textit{can produce}\ \mathbf{y}\}$.

The production technology is assumed to satisfy the following properties (Chambers, 1988):

Ψ1: $(\mathbf{x}, \mathbf{0}) \in \Psi$ and $(\mathbf{0}, \mathbf{y}) \in \Psi \Rightarrow \mathbf{y} = \mathbf{0}$

Ψ2: Ψ is a closed set

Ψ3: Ψ is bounded for each $\mathbf{x} \in \mathbb{R}_+^P$

Ψ4: $(\mathbf{x}, \mathbf{y}) \in \Psi \Rightarrow (\lambda\mathbf{x}, \mathbf{y}) \in \Psi$ for $\lambda \geq 1$

Ψ5: $(\mathbf{x}, \mathbf{y}) \in \Psi \Rightarrow (\mathbf{x}, \lambda\mathbf{y}) \in \Psi$ for $\lambda \in [0, 1]$

Ψ6: $(\mathbf{x}, \mathbf{y}) \in \Psi \Rightarrow (\mathbf{x}', \mathbf{y}') \in \Psi\ \forall\ (-\mathbf{x}', \mathbf{y}') \leq (-\mathbf{x}, \mathbf{y})$

Ψ7: Ψ is a convex set for $\mathbf{x} \in \mathbb{R}_+^P$

These properties state that any non-negative level of input can produce at least a zero level of output and the production of a positive output is impossible without at least one input (there is no *free lunch*); guarantee the existence of at least one input and one output efficient vector; ensure that finite inputs cannot produce infinite outputs; guarantee a feasible production that is any input-output, where the input quantity is larger and the output one is smaller, is also in the technology set (*free* *disposability*); sometimes it is required that any input-output combinations have to lie in the set of possible production (*convexity*).

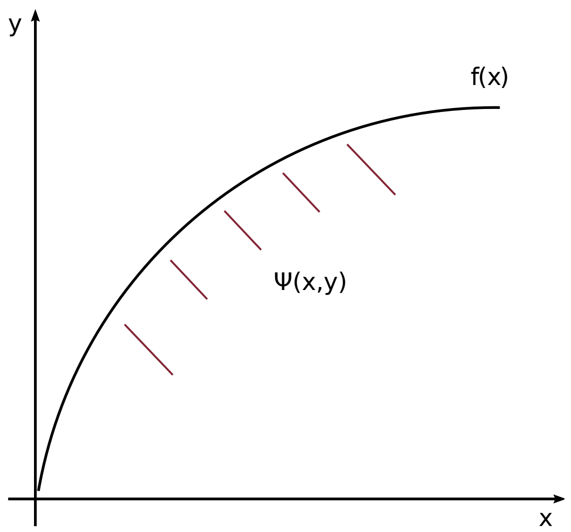
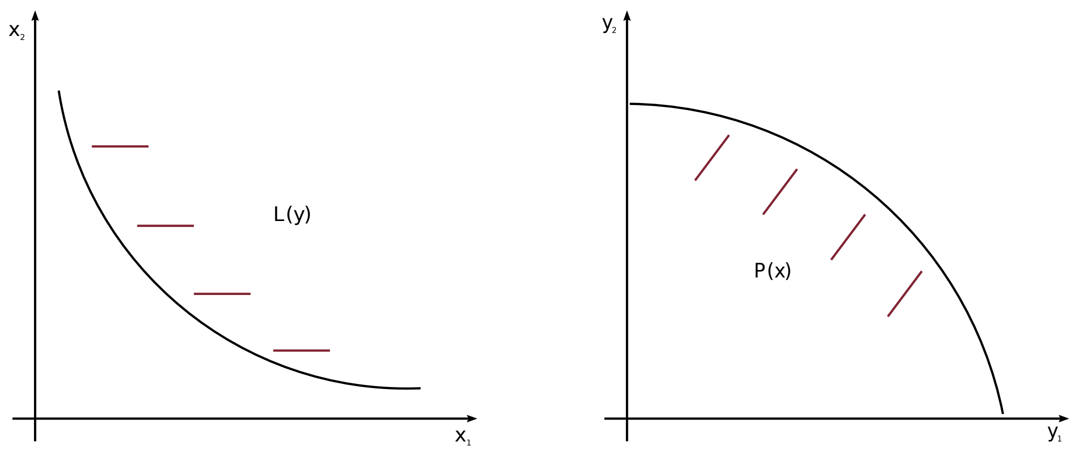


Figure 1.1: The Graph of Production Technology (P=1, Q=1)

In Figure 1.1 $\Psi(\mathbf{x},\mathbf{y})$ is the region beneath the curve $f(\mathbf{x})$, called *production frontier*, that represents the maximum output obtainable by the given input vector or the minimum input usage required to produce any given output vector, *i.e.*:

$$f(\mathbf{x}) = \max\{\mathbf{y} : \mathbf{y} \in P(\mathbf{x})\} = \max\{\mathbf{y} : \mathbf{x} \in L(\mathbf{y})\} \tag{1.1}$$

where $L(\mathbf{y})$ and $P(\mathbf{x})$ are respectively the *input* and the *output set* of production technology (see Figure 1.2).



(a) $L(\mathbf{y})$ (b) $P(\mathbf{x})$

Figure 1.2: The input and output sets of production technology ($P = 2$, $Q = 2$)

Note that $f(\mathbf{x})$ is defined in terms of $\Psi(\mathbf{x},\mathbf{y})$ and so it is assumed to satisfy the subsequent properties:

f1: $f(\mathbf{0}) = \mathbf{0}$

f2: f is upper semi-continuous on $\mathbb{R}_+^P$

f3: $f(\mathbf{x}) > \mathbf{0} \Rightarrow f(\lambda\mathbf{x}) \to +\infty$ as $\lambda \to +\infty$

f4: $f(\lambda\mathbf{x}) \geq f(\mathbf{x}), \lambda \geq 1$ for $\mathbf{x} \in \mathbb{R}_+^P$

f5: $\mathbf{x}' \geq \mathbf{x} \Rightarrow f(\mathbf{x}') \geq f(\mathbf{x})$

f6: f is quasi-concave on $\mathbb{R}_+^P$

1.1.2 Technical efficiency

One of the main objectives of a firm is to minimize inputs used to produce a given output set or to obtain the maximum output from a given input set.

Koopmans (1951) gave a general definition of *efficient subset* of Ψ: $(\mathbf{x},\mathbf{y})$ is technical efficient in Ψ if and only if it cannot be dominated by some $(\mathbf{x}',\mathbf{y}') \in \Psi$.

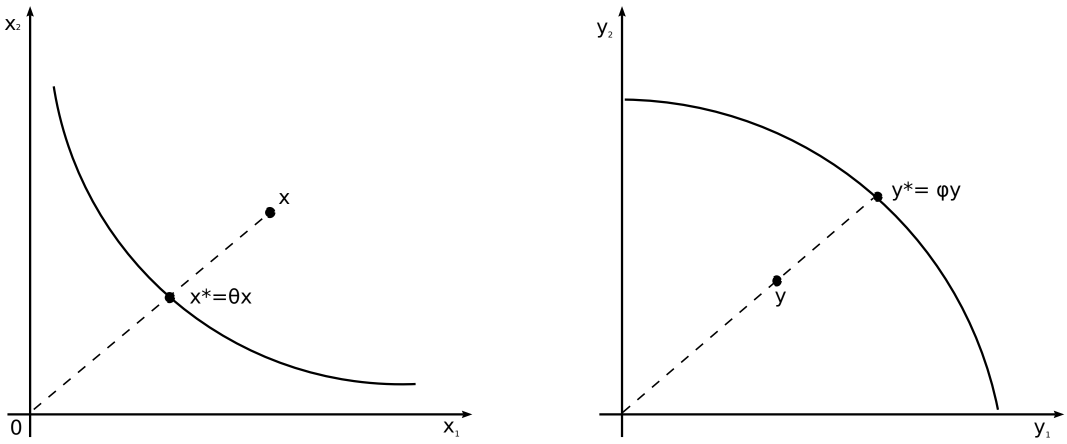
This definition can be split in:

- *input-oriented efficiency* if output quantity is fixed and no more reduction of inputs is possible;
- *output-oriented* if input quantity is fixed and no more increment of outputs is possible.

Debreu-Farrell Efficiency

Debreu (1951) and Farrell (1957) proposed two measures of Technical efficiency (TE) where the basic idea is to seek if it is possible to reduce the input quantities without changing the output quantities produced and vice versa:

- *input-oriented measure of TE*: $TE_I(\mathbf{x},\mathbf{y}) = min\{\theta : \theta x \in L(\mathbf{y})\}$.
- *output-oriented measure of TE*: $TE_O(\mathbf{x},\mathbf{y}) = [max\{\phi : \phi y \in P(\mathbf{x})\}]^{-1}$



(a) TE_I (b) TE_O

Figure 1.3: Input and output efficiency ($P = 2$, $Q = 2$)

More simply, in a single-input and single-output case, TE is measured as the ratio between the observed input $\mathbf{xA}$ and the minimum input $\mathbf{xA}^*$ (observed output $\mathbf{yA}$ and the maximum output $\mathbf{yA}^*$), under the assumption of fixed output (fixed input) *i.e.* in the Figure 1.4 for a generic firm A:

$$TE_I = \frac{\mathbf{xA}}{\mathbf{xA}^*} \geq 1 \quad and \quad TE_O = \frac{\mathbf{yA}}{\mathbf{yA}^*} \leq 1 \tag{1.2}$$

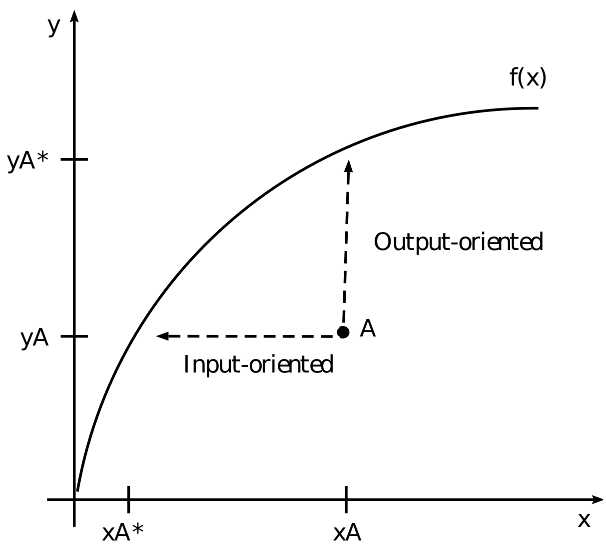


Figure 1.4: Input-oriented and output-oriented technical efficiency ($P = 1$, $Q = 1$)

1.2 Cost frontiers and cost efficiency

Now, consider also that firms are subjected to a strictly positive vector of input prices given by $\mathbf{w} = (w_1, ..., w_P) \in \mathbb{R}_+^P$ and that they attempt to minimize the cost of producing the output vector $\mathbf{y}$ given by $\mathbf{w}^T\mathbf{x} = \sum_p \mathbf{w}_p \mathbf{x}_p$.

It is possible to derive a *cost frontier* that represents the minimum expenditure required to produce any output given input prices *i.e.*:

$$c(\mathbf{y}, \mathbf{w}) = \min_x \left\{ \mathbf{w}^T\mathbf{x} : \mathbf{x} \in L(\mathbf{y}) \right\} \tag{1.3}$$

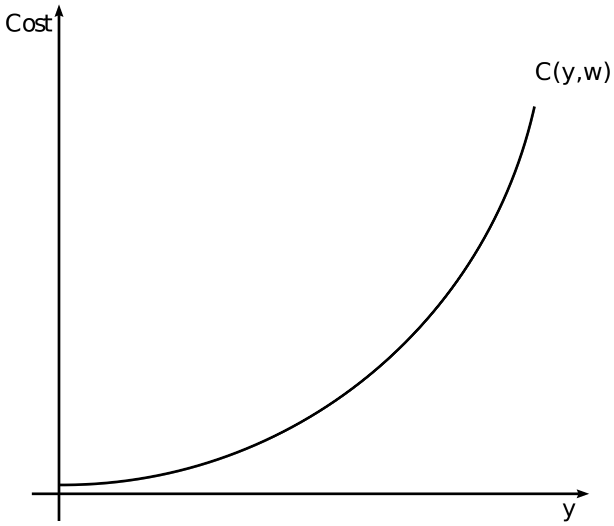


Figure 1.5: Cost frontier ($Q = 1$)

Note that $c(\mathbf{y}, \mathbf{w})$ is defined in terms of $L(\mathbf{y})$ and so it is assumed to satisfy the subsequent properties:

c1: $c(\mathbf{0}, \mathbf{w}) = \mathbf{0}$ and $c(\mathbf{y}, \mathbf{w}) > \mathbf{0}$ for $\mathbf{y} \geq \mathbf{0}$

c2: $c(\mathbf{y}, \lambda\mathbf{w}) = \lambda c(\mathbf{y}, \mathbf{w})$ for $\lambda > 0$

c3: $c(\mathbf{y}, \mathbf{w}') \geq c(\mathbf{y}, \mathbf{w})$ for $\mathbf{w}' \geq \mathbf{w}$

c4: c is concave in $\mathbf{w}$

c5: c is continuous in $\mathbf{w}$

c6: $c(\lambda\mathbf{y}, \mathbf{w}) \leq c(\mathbf{y}, \mathbf{w})$ for $0 \leq \lambda \leq 1$

c7: c is lower semi-continuous in $\mathbf{y}$

c8: $c(\mathbf{y}', \mathbf{w}) \leq c(\mathbf{y}, \mathbf{w})$ for $0 \leq \mathbf{y}' \leq \mathbf{y}$

c9: If Ψ is convex, then c is a convex function in $\mathbf{y}$

1.2.1 Cost efficiency

Cost efficiency (CE) can be measured as the ratio between the minimum cost and the observed cost under the assumption of fixed output and input prices *i.e.* $CE(\mathbf{x},\mathbf{y},\mathbf{w}) = c(\mathbf{y},\mathbf{w})/\mathbf{w}^T\mathbf{x}$ and for a generic firm A (Figure 1.6):

$$CE = \frac{\mathbf{wA}^T\mathbf{xA}^*}{\mathbf{wA}^T\mathbf{xA}} \leq 1 \tag{1.4}$$

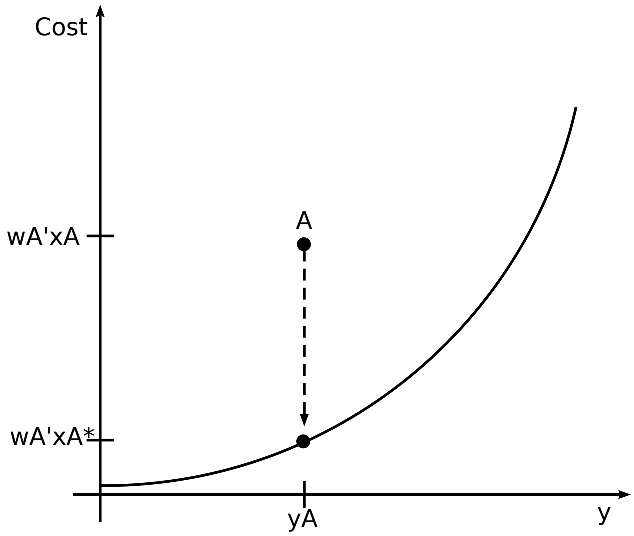


Figure 1.6: Cost efficiency ($Q = 1$)

Moreover, it can be noted that the inefficiency in term of costs could depend from two types of bad management of inputs: *(i)* an inappropriate quantity of resources to produce a fixed quantity of outputs *i.e.* a *technical inputs inefficiency* (Chapter 1 at page 6); *(ii)* an inappropriate mix of resources. Figure 1.7 shows that the firm could reach the full technical efficiency (*isoquant curve*) by reducing the inputs quantity from x to x', but by adding the *isocost curve*, that is the curve of all combinations of inputs which cost the same total amount, the cost efficiency is reached at quantity x^* *i.e.* with a different allocation of the inputs x_1 and x_2 namely *input allocative efficiency* (AE).

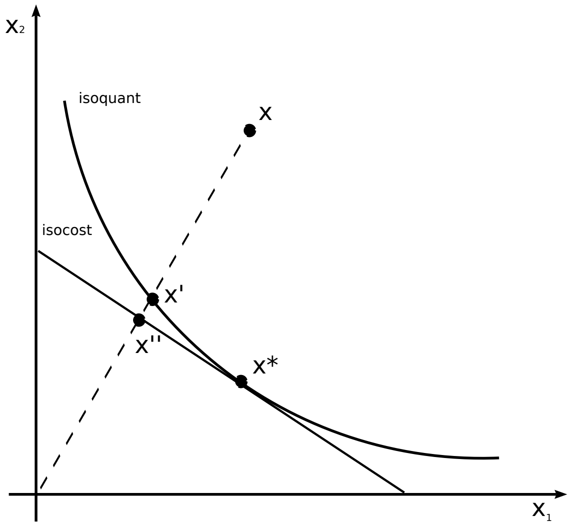


Figure 1.7: Decomposition of cost efficiency

Therefore, the cost efficiency can be decomposed in:

$$CE = \frac{\mathbf{w}^T\mathbf{x}^*}{\mathbf{w}^T\mathbf{x}} = \frac{\mathbf{w}^T\mathbf{x}^*}{\mathbf{w}^T\mathbf{x}'}\frac{\mathbf{w}^T\mathbf{x}'}{\mathbf{w}^T\mathbf{x}} = AE \cdot TE \tag{1.5}$$

1.3 Revenue functions

A firm can maximize profits by minimizing costs or maximizing revenues, and so in this section it is assumed that producers are subjected to a strictly positive vector of output prices given by $\mathbf{p} = (p_1, ..., p_Q) \in \mathbb{R}_+^Q$ and that they attempt to maximize the revenue of producing the output vector $\mathbf{y}$, given by $\mathbf{p}^T\mathbf{y} = \sum_q \mathbf{p}_q\mathbf{y}_q$, obtainable from the input vector $\mathbf{x}$ at their disposal.

It is possible to derive a *revenue frontier* that represents the maximum revenue obtainable from any input given the output prices *i.e.*:

$$r(\mathbf{x},\mathbf{p}) = \max_y \left\{\mathbf{p}^T\mathbf{y} : \mathbf{y} \in P(\mathbf{x})\right\} \tag{1.6}$$

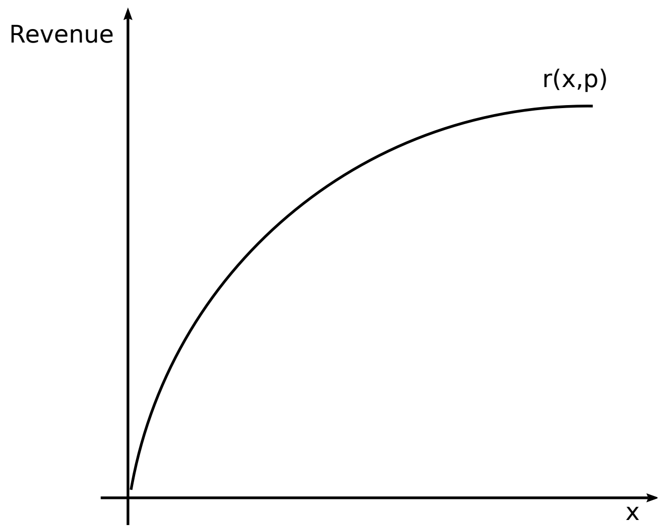


Figure 1.8: Revenue frontier $(P = 1)$

Note that $r(\mathbf{x}, \mathbf{p})$ is defined in terms of $P(\mathbf{x})$ and so it is assumed to satisfy the subsequent properties:

r1: $r(\mathbf{0}, \mathbf{p}) = \mathbf{0}$ and $r(\mathbf{x}, \mathbf{p}) > \mathbf{0}$ for $\mathbf{x} \geq \mathbf{0}$

r2: $r(\mathbf{x}, \lambda\mathbf{p}) = \lambda r(\mathbf{x}, \mathbf{p})$ for $\lambda > 0$

r3: $r(\mathbf{x}, \mathbf{p}') \geq r(\mathbf{x}, \mathbf{p})$ for $\mathbf{p}' \geq \mathbf{p}$

r4: r is convex in $\mathbf{p}$

r5: r is continuous in $\mathbf{p}$

r6: $r(\lambda\mathbf{x}, \mathbf{p}) \geq r(\mathbf{x}, \mathbf{p})$ for $\lambda \geq 1$

r7: r is upper semi-continuous in $\mathbf{x}$

r8: $r(\mathbf{x}', \mathbf{p}) \geq r(\mathbf{x}, \mathbf{p})$ for $x' \geq x$

r9: If Ψ is convex, then $r(\mathbf{x}, \mathbf{p})$ is a concave function in $\mathbf{x}$

1.3.1 Revenue efficiency

Revenue efficiency (RE) can be measured as the ratio between the actual revenue and the maximum revenue under the assumption of fixed input and output prices *i.e.* $RE(\mathbf{x}, \mathbf{y}, \mathbf{p}) = \mathbf{p}^T\mathbf{y}/r(\mathbf{x}, \mathbf{p})$ and for a generic firm A (Figure 1.9):

$$RE = \frac{\mathbf{pA}^T\mathbf{pA}}{\mathbf{pA}^T\mathbf{pA}^*} \leq 1 \tag{1.7}$$

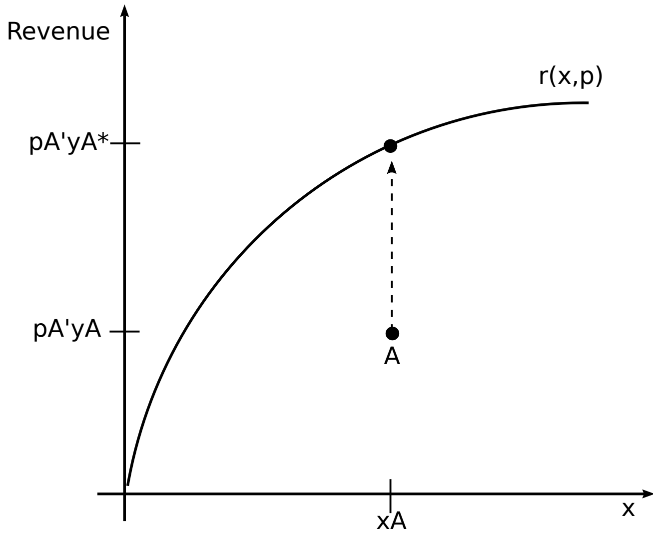


Figure 1.9: Revenue efficiency $(P = 1)$

CHAPTER 2

SHADOW PRICE APPROACH, DUALITY AND DISTANCE FUNCTIONS

Standard production functions, generally, present the implicit strong hypothesis that producers use multiple inputs to obtain a single-output. However, sometimes, the case of a multi-input/multi-output production process exists also and a representation of this setting has been provided by the so-called *Shephard distance functions* (Shephard, 1970).

Moreover, in some cases the direct estimation of a cost or a revenue frontier may not be practical or appropriate: it would not be practical when input or output prices do not differ among firms or it might not be appropriate when there is a systematic difference in cost-minimizing behavior in an industry, for example when political, union or regulatory factors cause shadow prices to deviate from market prices in a systematic way. In this situation, the duality between cost and production functions breaks down, and the resulting bias in the cost frontier estimates makes the cost efficiency calculation and decomposition biased as well (object of analysis of the

Chapter 12).
Distance functions, instead, do not require price information, they are robust to systematic deviations from cost-minimizing behavior, and they do not suffer from simultaneous equations bias when firms are cost minimizers or shadow cost minimizers (Coelli, 2000).

Therefore, distance functions through the duality theorems, explained in the following sections, allow to estimate *shadow-pricing models* when prices are not observable, are endogenous and/or are difficult to estimate in a consistent way or, finally, if the goal is to estimate the monetary effects of the production of an undesirable output (object of analysis of the Chapter 13).

For these reasons distance functions are regularly utilized in theoretical and empirical economics papers both in a parametric and in a non-parametric way. Färe et al. (1985) and Färe et al. (1994), for example, used linear programming methods to construct non-parametric distance functions for the measurement of technical efficiency and productivity growth in multi-output firms, or Lovell, Richardson, P. Travers and Wood (1994), Grosskopf et al. (1997), Coelli and Perelman (1999) and Coelli and Perelman (2000) proposed to estimate parametric distance functions using econometric methods.

The following sections give an introduction to input and output distance functions and on their duality with cost and revenue frontiers.

2.1 Cost frontiers and input distance functions

An input distance function defined as $D_I(\mathbf{x},\mathbf{y}) = max\{\lambda > 1 : \mathbf{x}/\lambda \in L(\mathbf{y})\}$ indicates the largest proportional contraction in the observed input vector $\mathbf{x}$ such that the contracted vector $(\mathbf{x}/\lambda)$ is still an element of the original input set $L(\mathbf{y})$.

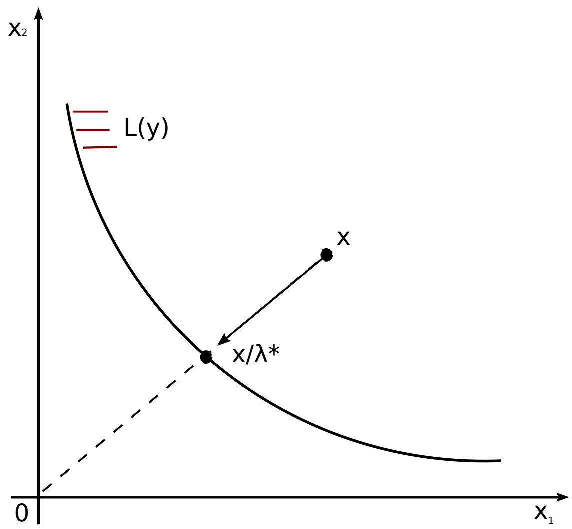


Figure 2.1: Input distance function $(P = 2)$

Färe et al. (1985) and Färe et al. (1994) proved that in the particular case of single-output, distance functions are closely linked with the *Farrell efficiency* measure, inasmuch, they provide radial measures of the distance from an output-input bundle to the boundary of production technology. More specifically the input distance function is the reciprocal of the Farrell efficiency, *i.e.* $D_I(\mathbf{x},\mathbf{y}) = [TE_I(\mathbf{x},\mathbf{y})]^{-1} = [min\{\theta : \theta x \in L(\mathbf{y})\}]^{-1}$.

More simply, taking up the graph (a) in Figure 1.3a, which depicts the input technical efficiency, $D_I(\mathbf{x},\mathbf{y})$ can be represented as in Figure 2.2 and it can be written:

$$D_I(\mathbf{x},\mathbf{y}) = \frac{0A}{0B} \quad \text{and} \quad TE_I = \frac{0B}{0A} = \frac{1}{D_I(\mathbf{x},\mathbf{y})}$$

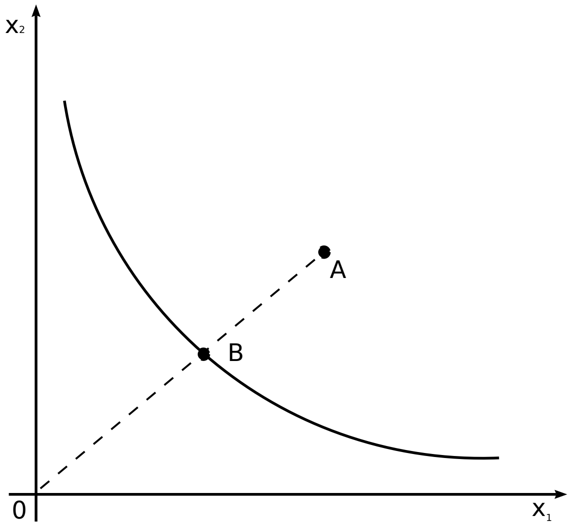


Figure 2.2: Input distance function vs Farrell input efficiency ($P = 2$)

Its properties can be summarized as:

D_I1: $D_I(0,\mathbf{y}) = 0$ and $D_I(\mathbf{x},0) = +\infty$

D_I2: $D_I(\mathbf{x},\mathbf{y})$ is an upper semi-continuous function

D_I3: $D_I(\lambda\mathbf{x},\mathbf{y}) = \lambda D_I(\mathbf{x},\mathbf{y})$ for $\lambda > 0$

D_I4: $D_I(\mathbf{x},\lambda\mathbf{y}) \leq D_I(\mathbf{x},\mathbf{y})\ for\ \lambda \geq 1$

D_I5: $D_I(\lambda\mathbf{x},\mathbf{y}) \geq D_I(\mathbf{x},\mathbf{y})\ for\ \lambda \geq 1$

D_I6: $D_I(\mathbf{x}',\mathbf{y}) \geq D_I(\mathbf{x},\mathbf{y})\ for\ \mathbf{x}' \geq \mathbf{x}\ and\ D_I(\mathbf{x},\mathbf{y}') \leq D_I(\mathbf{x},\mathbf{y})\ for\ \mathbf{y}' \geq \mathbf{y}$

D_I7: If $L(\mathbf{y})$ is a convex set for $\mathbf{y}$ then $D_I(\mathbf{x},\mathbf{y})$ is a concave function in $\mathbf{x}$

These assumptions ensure the distance function to be non-negative, non-decreasing in $\mathbf{x}$ and non-increasing in $\mathbf{y}$, to be linearly homogeneous in $\mathbf{x}$ and that if $\mathbf{x}$ belongs to the input set $L(\mathbf{y})$ then $D_I(\mathbf{x},\mathbf{y}) \geq 1$ and that the distance is equal to 1 only if $\mathbf{x}$ is on the frontier.

Taking advance of the envelope theorem (see Shephard, 1970 or Färe, 1988), the cost frontier can be derived from equation (2.1):

$$c(\mathbf{y},\mathbf{w}) = \underset{x}{min}\left\{\mathbf{w}^T\mathbf{x} : \mathbf{x} \in L(\mathbf{y})\right\} = \underset{x}{min}\left\{\mathbf{w}^T\mathbf{x} : D_I(\mathbf{x},\mathbf{y}) \geq 1\right\} \tag{2.1}$$

In fact, if $c(\mathbf{y},\mathbf{w}) = \underset{x}{min}\left\{\mathbf{w}^T\mathbf{x} : D_I(\mathbf{x},\mathbf{y}) \geq 1\right\}$ satisfies conditions {c1-c5,c7,c8} (Chapter 1 at page 8), then $c(\mathbf{y},\mathbf{w})$ is *dual* to $D_I(\mathbf{x},\mathbf{y})$ in the sense that $D_I(\mathbf{x},\mathbf{y}) = \underset{w}{min}\left\{\mathbf{w}^T\mathbf{x} : c(\mathbf{y},\mathbf{w}) \geq 1\right\}$ satisfies properties $\{D_I1 - D_I3, D_I6, D_I7\}$.

Therefore, the cost function can be derived from the input distance function by minimizing with respect to inputs, and the input distance function is obtainable from the cost function by minimizing with respect to input prices. This duality between the input distance function and the cost function can be used to obtain the absolute shadow prices of inputs (Shephard, 1970 , Färe and Primont, 1995).

Assuming that the cost and the distance functions are both differentiable, a *Lagrange* problem can be set up to minimize the cost (see *e.g.* Rodríguez-Álvarez et al., 2007):

$$\min_{x} \Lambda = \mathbf{w}^T\mathbf{x} + \lambda(1 - D_I(\mathbf{x},\mathbf{y})) \tag{2.2}$$

and first order conditions with respect to inputs yield the relationship:

$$\mathbf{w} = \lambda \nabla_x D_I(\mathbf{x},\mathbf{y}) \tag{2.3}$$

where ∇ is the gradient operator.
At the optimum, in force of the homogeneity of degree 1 of $D_I(\mathbf{x},\mathbf{y})$ (see Jacobsen, 1972), the Lagrange multiplier equals the cost function, *i.e.*, $\lambda = c(\mathbf{y},\mathbf{w})$. Thus, equation (2.3) may be written in terms of the following system of equations:

$$\mathbf{w} = c(\mathbf{y},\mathbf{w})\nabla_x D_I(\mathbf{x},\mathbf{y}) \tag{2.4}$$

Now, by using the second part of the Shephard's duality theorem, equation (2.4) can be written as:

$$D_I(\mathbf{x},\mathbf{y}) = \mathbf{w}^*(\mathbf{x},\mathbf{y})\mathbf{x} \tag{2.5}$$

where $\mathbf{w}^*(\mathbf{x},\mathbf{y})$ represents the input price vector that minimizes the cost.

Applying Shephard's dual lemma to expression (2.5), the shadow price formula can be written as:

$$\nabla_x D_I(\mathbf{x},\mathbf{y}) = \mathbf{w}^*(\mathbf{x},\mathbf{y}) \tag{2.6}$$

Consequently, by combining (2.6) with (2.4):

$$\mathbf{w} = c(\mathbf{y},\mathbf{w})\mathbf{w}^*(\mathbf{x},\mathbf{y}) \tag{2.7}$$

The main difficulty that arises in order to obtain absolute shadow prices from expression (2.7) relies on the dependence of the cost function $c(\mathbf{y},\mathbf{w})$ on $\mathbf{w}$, that is precisely the vector of shadow prices sought.

Therefore, in order to obtain $c(\mathbf{y},\mathbf{w})$ the Hailua and Veemanb (2000) shadow prices ratio is used:

$$c_{pp'} = \frac{\mathbf{w}_p^*}{\mathbf{w}_{p'}^*} = \frac{\partial D_I(\mathbf{x},\mathbf{y})/\partial \mathbf{x}_p}{\partial D_I(\mathbf{x},\mathbf{y})/\partial \mathbf{x}_{p'}} \tag{2.8}$$

It represents the relative shadow price of $\mathbf{x}_p$ with respect $\mathbf{x}_{p'}$ and $\mathbf{w}_p^*$ and $\mathbf{w}_{p'}^*$ are the shadow prices of the inputs $\mathbf{x}_p$ and $\mathbf{x}_{p'}$, respectively.

Assuming, at last, that *"The absolute shadow price* $\mathbf{w}_p$ *of an input p, equals its observed market price* $\mathbf{w}_p^o$*"*, the absolute shadow price of $\mathbf{x}_{p'}$ is given by:

$$\mathbf{w}_{p'} = \mathbf{w}_p^o \frac{\partial D_I(\mathbf{x},\mathbf{y})/\partial \mathbf{x}_{p'}}{\partial D_I(\mathbf{x},\mathbf{y})/\partial \mathbf{x}_p} \tag{2.9}$$

Finally, the AE can be obtained comparing the shadow cost function with the cost function calculated by the means of the actual prices, formally:

$$AE = \frac{c(\mathbf{y},\mathbf{w'_p})}{c(\mathbf{y},\mathbf{w^o_p})} \tag{2.10}$$

2.2 Revenue frontiers and output distance functions

The output distance defined as $D_O(\mathbf{x},\mathbf{y}) = min\{\mu \in [0,1] : \mathbf{y}/\mu \in P(\mathbf{x})\}$ indicates the smallest proportional reduction in the observed output vector $\mathbf{y}$ provided that the expanded vector $(\mathbf{y}/\mu)$ is still an element of the original output set $P(\mathbf{x})$ (Grosskopf et al., 1995).

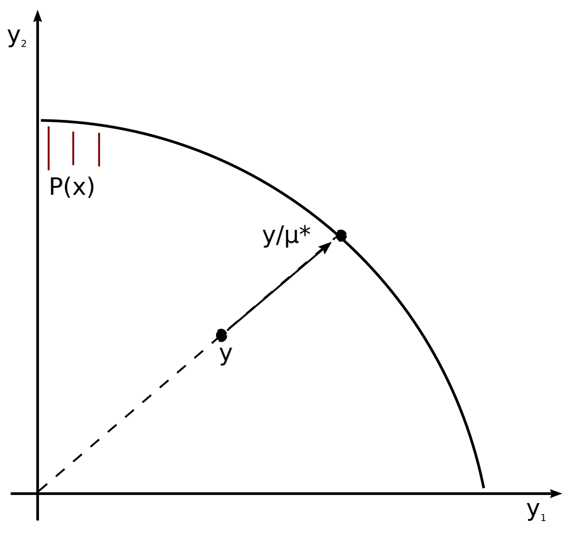


Figure 2.3: Output distance function ($Q = 2$)

Färe et al. (1985) and Färe et al. (1994) proved that, in the particular case of single-input, output distance function equal the Farrell efficiency *i.e.* $D_O(\mathbf{x},\mathbf{y}) = TE_O(\mathbf{x},\mathbf{y}) = [max\{\phi : \phi y \in P(\mathbf{x})\}]^{-1}$.

Simplifying, taking up the graph (b) in Figure 1.3b, which depicts the output technical efficiency, $D_O(\mathbf{x},\mathbf{y})$ can be represented as in Figure 2.4 and it can be written as:

$$D_O(\mathbf{x},\mathbf{y}) = \frac{0A}{0B} \quad \text{and} \quad TE_O = \frac{0A}{0B} = D_O(\mathbf{x},\mathbf{y})$$

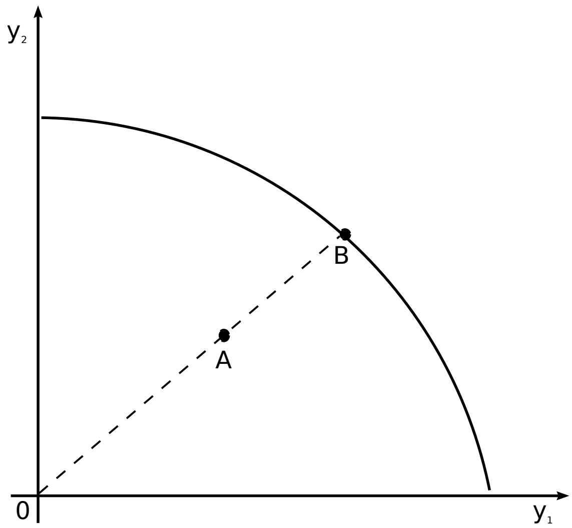


Figure 2.4: Output distance function vs Farrell output efficiency ($Q = 2$)

The properties can be summarized as:

D_O1: $D_O(\mathbf{x},0) = 0$ and $D_O(0,\mathbf{y}) = +\infty$

D_O2: $D_O(\mathbf{x},\mathbf{y})$ is a lower semi-continuous function

D_O3: $D_O(\mathbf{x},\lambda\mathbf{y}) = \lambda D_O(\mathbf{x},\mathbf{y})\ for\ \lambda > 0$

D_O4: $D_O(\lambda\mathbf{x},\mathbf{y}) \leq D_O(\mathbf{x},\mathbf{y}) for\ \lambda \geq 1$

D_O5: $D_O(\mathbf{x},\lambda\mathbf{y}) \leq D_O(\mathbf{x},\mathbf{y})\ for\ 0 \leq \lambda \leq 1$

D_O6: $D_O(\mathbf{x}',\mathbf{y}) \leq D_O(\mathbf{x},\mathbf{y})\ for\ \mathbf{x}' \geq \mathbf{x}\ and\ D_O(\mathbf{x},\mathbf{y}') \leq D_O(\mathbf{x},\mathbf{y})\ for\ \mathbf{y}' \leq \mathbf{y}$

D_O7: If $P(\mathbf{x})$ is a convex set for $\mathbf{x}$ then $D_O(\mathbf{x},\mathbf{y})$ is a convex function in y

Taking advantage of the envelope theorem (see Shephard, 1970 or Färe, 1988), the revenue frontier can be derived from equation (2.11):

$$r(\mathbf{x},\mathbf{p}) = \underset{y}{max}\left\{\mathbf{p}^T\mathbf{y} : \mathbf{y} \in P(\mathbf{x})\right\} = \underset{y}{max}\left\{\mathbf{p}^T\mathbf{y} : D_O(\mathbf{x},\mathbf{y}) \leq 1\right\} \tag{2.11}$$

In fact, if $r(\mathbf{x},\mathbf{p}) = \underset{y}{max}\left\{\mathbf{p}^T\mathbf{y} : D_O(\mathbf{x},\mathbf{y}) \leq 1\right\}$ satisfies conditions {r1-r5,r7,r8} (Chapter 1 page 11), then $r(\mathbf{x},\mathbf{p})$ is *dual* to $D_O(\mathbf{x},\mathbf{y})$ in the sense that $D_O(\mathbf{x},\mathbf{y}) = \underset{p}{max}\left\{\mathbf{p}^T\mathbf{y} : r(\mathbf{x},\mathbf{p}) \leq 1\right\}$ satisfies properties $\{D_O1 - D_O3, D_O6, D_O7\}$.

Therefore, the revenue function can be derived from the output distance function by maximizing with respect to outputs, and the output distance function is obtainable from the revenue function by maximizing with respect to output prices. This duality between the output distance function and the revenue function can be used to obtain the absolute shadow prices of outputs (Shephard, 1970 , Färe and Primont, 1995).

Assuming that the revenue and distance functions are both differentiable, a *Lagrange* problem can be set up to maximize revenue:

$$\max_y \Lambda = \mathbf{p}^T\mathbf{y} + \lambda(D_O(\mathbf{x},\mathbf{y}) - 1) \tag{2.12}$$

and first order conditions with respect to outputs yield the relationship (Färe and Primont, 1995):

$$\mathbf{p} = -\lambda \nabla_y D_O(\mathbf{x},\mathbf{y}) \tag{2.13}$$

At the optimum, in force of the homogeneity of degree 1 of $D_O(\mathbf{x},\mathbf{y})$ (see Jacobsen, 1972), the negative of the Lagrange multiplier equals the revenue function, *i.e.*, $-\lambda = \Lambda = r(\mathbf{x},\mathbf{p})$. Thus, equation (2.13) may be written in terms of the following system of equations:

$$\mathbf{p} = r(\mathbf{x},\mathbf{p}) \nabla_y D_O(\mathbf{x},\mathbf{y}) \tag{2.14}$$

Now by means of the second part of the duality theorem (2.11), it is obtained that:

$$D_O(\mathbf{x},\mathbf{y}) = \mathbf{p}^*(\mathbf{x},\mathbf{y})\mathbf{y} \tag{2.15}$$

where $p^*(\mathbf{x},\mathbf{y})$ represents the output price-vector that maximises revenue.

Applying Shephard's dual lemma to expression (2.15), yields:

$$\nabla_y D_O(\mathbf{x},\mathbf{y}) = \mathbf{p}^*(\mathbf{x},\mathbf{y}) \tag{2.16}$$

expression that combined with equation (2.14), leads to:

$$\mathbf{p} = r(\mathbf{x},\mathbf{p})\mathbf{p}^*(\mathbf{x},\mathbf{y}) \tag{2.17}$$

where, $\mathbf{p}^*(\mathbf{x},\mathbf{y})$ is obtained from the gradient of the distance function, and represents revenue-deflated output prices. The main difficulty that arises in order to obtain absolute shadow prices from expression (2.17) relies on the dependence of the revenue function $r(\mathbf{x},\mathbf{p})$ on $\mathbf{p}$, that is precisely the vector of shadow prices sought.

Therefore, in order to obtain $r(\mathbf{x},\mathbf{p})$ it is assumed that *"The observed price of an output q, $\mathbf{p}_q^o$, equals its absolute shadow price $\mathbf{p}_q$"*, and thus the maximum revenue is obtained as:

$$r = \frac{\mathbf{p}_q^o}{\mathbf{p}_q^*(\mathbf{x},\mathbf{y})} \tag{2.18}$$

expression that can be used to calculate the absolute shadow prices of the remaining outputs from its deflated shadow prices $\mathbf{p}^*$. The absolute shadow price for output $\mathbf{y}_{q'}$, denoted by $\mathbf{p}_{q'}$, is given by (Färe, 1993):

$$\mathbf{p}_{q'} = r \cdot \mathbf{p}_{q'}^*(\mathbf{x},\mathbf{y}) = r \cdot \frac{\partial D_O(\mathbf{x},\mathbf{y})}{\partial \mathbf{y}_{q'}} = \mathbf{p}_q^o \cdot \frac{\partial D_O(\mathbf{x},\mathbf{y})/\partial \mathbf{y}_{q'}}{\partial D_O(\mathbf{x},\mathbf{y})/\partial \mathbf{y}_q} \tag{2.19}$$

2.3 Directional efficiency measures

In both *Farrell* and *Shephard* efficiency measures, seen in the previous chapters, the producer has to choose whether to reduce inputs by fixing outputs or to increment outputs by fixing inputs, moreover, in a multi-input or multi-output case, all inputs or all outputs are reduced or expanded by a same multiplicative factor.

Sometimes, it would be useful to be able to decide how intensively to reduce or increase the various inputs and outputs and for this reason in recent years alternative efficiency measures have been proposed like the *directional distance functions* (Chambers et al., 1998). Please see Bogetoft and Otto (2011) for a detailed discussion.

The purpose of directional distance functions is to determine improvements in a given direction $\mathbf{g} \in \mathbb{R}_+^P \times \mathbb{R}_+^Q$ and to define an *excess* function e such as:

$$\overrightarrow{D}_T(\mathbf{x},\mathbf{y};\mathbf{g}) = sup\{e : (x - eg_x, y + eg_y) \in \Psi\} \tag{2.20}$$

where $\mathbf{g} = (g_x, g_y)$ is the directional vector, *i.e.* this function is defined by simultaneously contracting inputs and expanding outputs.

The function satisfies the following properties:

D1: $\overrightarrow{D}_T(\mathbf{x},\mathbf{y};\mathbf{g}) \geq 0 \Leftrightarrow \mathbf{x} \in \Psi$ (*representation*)

D2: $\overrightarrow{D}_T(\mathbf{x}-\alpha\mathbf{g},\mathbf{y}+\alpha\mathbf{g};\mathbf{g}) = \overrightarrow{D}_T(\mathbf{x},\mathbf{y};\mathbf{g}) - \alpha$ for $\alpha \in \mathbb{R}_+$ (*translation*)

D3: $\overrightarrow{D}_T(\mathbf{x},\mathbf{y};\mathbf{g})$ is an additive measure, hence it allows negative values of $\mathbf{x}$ and/or $\mathbf{y}$

From a geometric point of view directional distance functions project the point (x,y) onto the production frontier $f(\mathbf{x})$ in the direction $\mathbf{g} = (g_x, g_y)$ as shown in Figure 2.5.

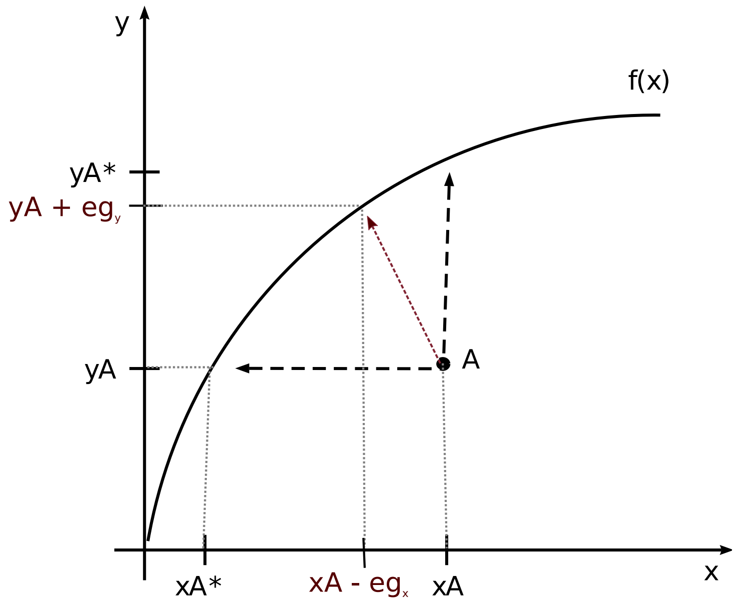


Figure 2.5: Input-oriented and output-oriented directional efficiency ($P = 1$, $Q = 1$)

There are some special cases of $\overrightarrow{D}_T(\mathbf{x},\mathbf{y};\mathbf{g})$ that are interesting:

- $g_y = 0 \Rightarrow \overrightarrow{D}_T(\mathbf{x},\mathbf{y};g_x,0) = \overrightarrow{D}_I(\mathbf{x},\mathbf{y};g_x)$ (Directional input distance function)
- $g_x = 0 \Rightarrow \overrightarrow{D}_T(\mathbf{x},\mathbf{y};0,g_y) = \overrightarrow{D}_O(\mathbf{x},\mathbf{y};g_y)$ (Directional output distance function)
- $g_x = 0$ *and* $g_y = y \Rightarrow \overrightarrow{D}_T(\mathbf{x},\mathbf{y};0,y) = (1/D_O(\mathbf{x},\mathbf{y})) - 1$ (Relation with Shephard's output distance function)
- $g_x = x$ *and* $g_y = 0 \Rightarrow \overrightarrow{D}_T(\mathbf{x},\mathbf{y};x,0) = 1 - 1/D_I(\mathbf{x},\mathbf{y})$ (Relation with Shephard's input distance function)

Long debates have pursued in the economic literature on how the directions may be chosen, please see Färe et al. (2008) for a discussion.

Chapter 3

FRONTIER MODELS ESTIMATION TECHNIQUES

In Chapters 1 and 2 the theory of production technology, technical and economic efficiency has been discussed; in empirical studies both the frontier and the efficiency of a given firm are unknown and must be estimated from a sample of production units for which data, on input and output quantities, are available.

In literature, many different approaches have been proposed and vary from the most popular fully *parametric*, where a functional form of the frontier and a statistical distribution of the efficiency are assumed and the parameters are estimated by statistical and econometric methods, to the most popular fully *nonparametric* techniques, where the frontier is identified only by some specific properties and the relative efficiency is calculated through linear programming methods. However, in recent years, several interesting middle methods have been developed: *semi-nonparametric* and *semi-parametric methods*. Table 3.1 reports a synthetic taxonomy of the most used approaches.

Table 3.1: Estimation techniques by production technology

Production Technology	*Estimation techniques*
Nonparametric	**DEA**, Farrell (1957);Charnes et al. (1978)
	FDH, Deprins et al. (1984); Grosskopf (1996)
	Order-m, Cazals et al. (2002); Daraio and Simar (2007)
Semi-nonparametric	**StoNED** , Kuosmanen and Kortelainen (2012)
	StoNEZD, Johnson and Kuosmanen (2011)
Semi-parametric	Park and Simar (1994), Park et al. (2007)
Parametric	**SFA**, Aigner et al. (1977); Meeusen and van den Broeck (1977)
	DFA, Aigner and Chu (1968)

From a personal point of view, as can be seen from the chapters dedicated to empirical applications, no single approach is best but in some cases it is more convenient to use a parametric approach while in others a nonparametric approach is preferred.

In the following sections some parametric and nonparametric frontier approaches will be presented focusing more on the methods that are used in subsequent chapters.

3.1 Parametric approach

The frontier parametric models are classified according to two main criteria: *(i)* the presence of noise in the estimation procedure, in *deterministic* models it is assumed that all observations (x, y) belong to the production set Ψ *i.e.* $Pr((x,y) \in \Psi) = 1$, while *stochastic* models allow for the presence of the noise in the data and so for some unit i $(x_i, y_i) \notin \Psi$; *(ii)* the type of data analyzed *i.e.* cross-section or panel data.

In an econometric framework the existence of a well-defined production (or cost) structure characterized by smooth, continuous, continuously differentiable and quasi-concave production (or cost) functions is taken as given.

Therefore, by hypothesizing for simplicity the case of N producers that produce in one year (cross-section data) a single output y by using P inputs $\mathbf{x}$, the parametric single-output *production frontier* can be written as:

$$y_i \leq f(\mathbf{x_i};\beta) \tag{3.1}$$

where $f(\mathbf{x_i};\beta)$ is a well-defined smooth, continuous, continuously differentiable, quasi-concave production function and β are the technology parameters to be estimated.

Moreover, it is assumed that prices are imposed by the market and so input prices $\mathbf{w_i}$ are treated as exogenous, the parametric *cost frontier* is:

$$C_i \geq c(\mathbf{y_i}, \mathbf{w_i};\beta) \tag{3.2}$$

where C_i is the observed expenditure incurred by producer i.

The main parametric approaches *pros* are:

- statistical inference can be easily carried out being based on econometric models and so on probabilistic structures;
- the presence of parameter to estimate and so results are easy to interpret in terms of sensitivity of the production function to particular inputs or of cost function to particular outputs and input prices and so on.

Instead, the major *criticisms* to parametric approaches regard:

- the restrictive assumptions about particular functional forms for the frontier and about the distribution of the two elements of the stochastic parts of the model, *i.e.* inefficiency and noise, from which derives, in my opinion, the most applicative problem called "Wrong skewness problem";
- the greater difficulty in estimating multi-input/multi-output cases.

3.1.1 Deterministic Frontier Analysis (DFA)

As hinted above, in a deterministic model all observations (x, y) are assumed to belong to the production set Ψ and this means that in the data there are no random shocks and that the distance to the frontier is pure inefficiency *i.e.*:

$$y_i = f(\mathbf{x_i};\beta) \cdot TE_i \tag{3.3}$$

and TE is measured as:

$$TE(\mathbf{x_i}, y_i) = \frac{y_i}{f(\mathbf{x_i};\beta)} \leq 1 \tag{3.4}$$

The production model is usually rewritten as (see Jondrow et al., 1982 and Battese and Coelli, 1992):

$$y_i = f(\mathbf{x_i}; \beta) \cdot exp\{-u_i\} \tag{3.5}$$

that in logarithmic terms becomes:

$$ln(y_i) = ln[f(\mathbf{x_i}; \beta)] - u_i \tag{3.6}$$

where $u_i \geq 0$ is a measure of the *technical inefficiency* and guarantees that $y_i \leq f(\mathbf{x_i}; \beta)$ and $TE(\mathbf{x_i}, y_i) \leq 1$.

In the same way the cost frontier can be written as:

$$ln(C_i) = ln[c(\mathbf{y_i}, \mathbf{w_i}; \beta)] + u_i \tag{3.7}$$

where $u_i \geq 0$ is a measure of the *cost inefficiency* and guarantees that $C_i \geq c(\mathbf{y_i}, \mathbf{w_i}; \beta)$ and $CE(\mathbf{y_i}, \mathbf{w_i}) = \frac{c(\mathbf{y_i}, \mathbf{w_i}; \beta)}{C_i} \leq 1$.

Note that from a mathematical point of view the production and the cost frontier differ only in the "u_i" sign, therefore, to simplify presentation in the following sections it will refer only to the production frontier case (easily reportable to the cost frontier issue).

In literature, three major methods have been proposed to obtain the deterministic frontier.

Mathematical programming

Aigner and Chu (1968) reformulated the equation (3.6) in a *linear* and a *quadratic* mathematical programming models where parameters are calculated as those βs for which the sum of the deviations (or squared deviations), of the observed output of each producer from the maximum feasible output, is minimized. The principal drawback of the proposed model is that parameters are *calculated* rather than *estimated* and so statistical inference on the results is complicated.

Corrected Ordinary Least Squares (COLS)

Winsten (1957) suggested to estimate the model in equation (3.6) in two stages. In the first stage the slope parameters can be consistently estimated by Ordinary Least Squares (OLS) and in the second step the biased OLS $\widehat{\beta}_0$ intercept is *corrected* by shifting the estimated production function upward until all residuals except one, on which the function is placed, are negative *i.e.*:

$$\widehat{\beta}_0^* = \widehat{\beta}_0 + \max_i \{\widehat{u}_i\} \tag{3.8}$$

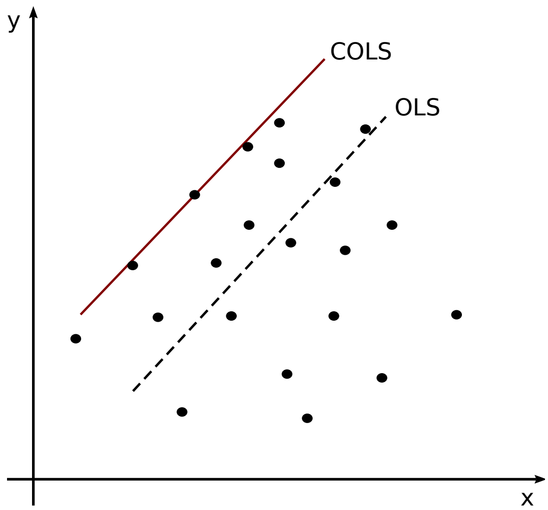


Figure 3.1: COLS (Corrected Ordinary Least Squares)

and TE is measured as follows:

$$TE_i = exp\{\widehat{u}_i - \max_i(\widehat{u}_i)\}. \tag{3.9}$$

An important drawback of COLS is the strong sensitivity to outliers, in fact, as shown in Figure 3.2 the frontier is shifted on the highest point defined "*fully efficient*" and the relative efficiency of all other points is underestimated.

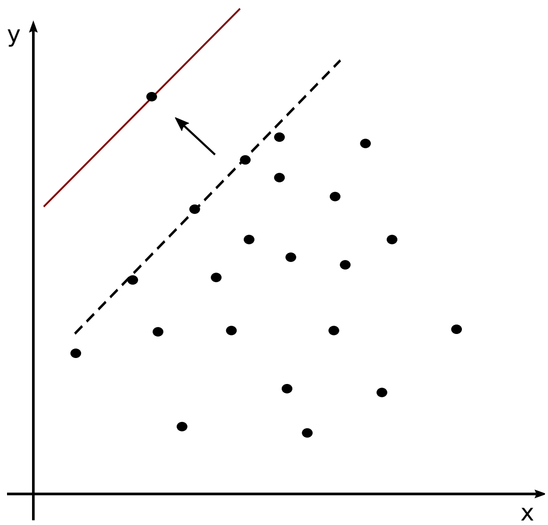


Figure 3.2: COLS (Corrected Ordinary Least Squares) in presence of outliers

Modified Ordinary Least Squares (MOLS)

Afriat (1972) and Richmond (1974) proposed a variation of COLS that hypothesizes a one-side distribution of OLS residuals, such as halfnormal or exponential, and a shift of the frontier equal to the mean of the assumed distribution *i.e.*:

$$\widehat{\beta}_0^* = \widehat{\beta}_0 + E(\widehat{u}_i) \tag{3.10}$$

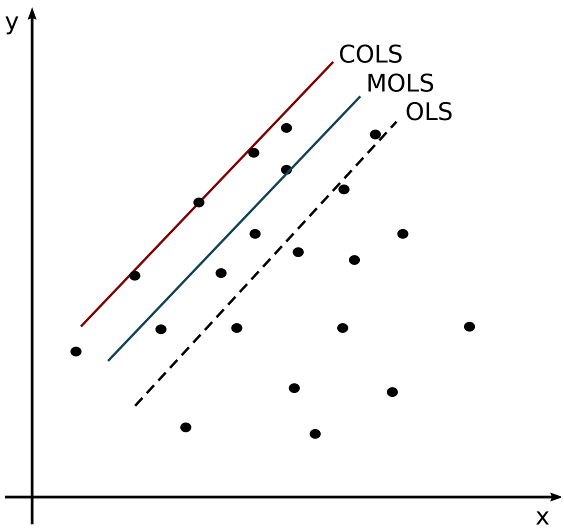


Figure 3.3: MOLS (Modified Ordinary Least Squares)

Afterward, TE is measured as below:

$$TE_i = exp\{\widehat{u}_i - E(\widehat{u}_i)\}. \tag{3.11}$$

Note that to obtain TE, it is assumed $\widehat{u}_i \geq 0$, *i.e.* residuals must have a "*Negative skewness*".

3.1.2 Stochastic Frontier Analysis (SFA)

Stochastic frontier models (SFA) introduced in the same period by Aigner et al. (1977) and Meeusen and van den Broeck (1977) allow the possibility of random shocks (bad weather, machinery breakdown,...), not controllable by producers, in the production process. In deterministic models these types of events cause a translation of the frontier leading to an increment of the inefficiency for the unfortunate producer.

Starting from the deterministic frontier in equation (3.6) and introducing a stochastic term v_i SFA can be specified as follows:

$$ln(y_i) = ln[f(\mathbf{x}_i;\beta)] + v_i - u_i \tag{3.12}$$

where:

- v_i is a two-side noise term;
- u_i is the nonnegative technical inefficiency term;
- v_i and u_i are assumed to be independent of each other and of the regressors.

Figure 3.4 shows a comparison among SFA and the deterministic models discussed in the previous section.

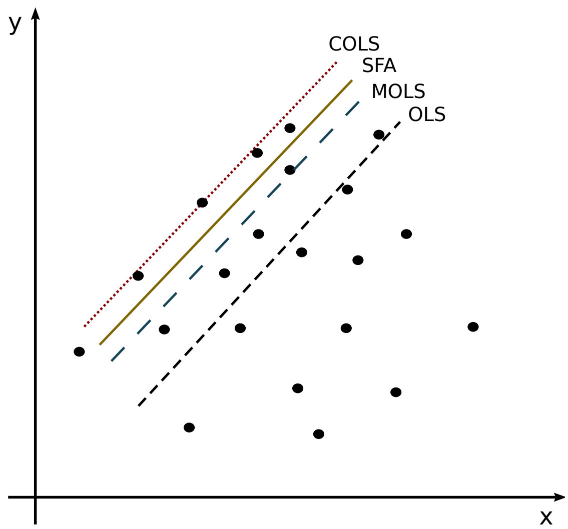


Figure 3.4: Comparison between stochastic and deterministic frontier models

In the same way the stochastic cost frontier can be defined as:

$$ln(C_i) = ln[c(\mathbf{y_i}, \mathbf{w_i};\beta)] + v_i + u_i \tag{3.13}$$

Distributional assumptions

In order to estimate the technical (or cost) efficiency of each producer some distributional assumptions, on the compound error $\varepsilon_i = v_i - u_i$ (or $\varepsilon_i = v_i + u_i$ in the cost case), are required. One of the most used distributions proposed in literature is the *Normal-Half Normal*[1] (Aigner et al., 1977) based on the idea that the modal value of inefficiency term is zero:

[1]Please see Kumbhakar and Lovell (2000) or Greene (2008) for the other distributions for u_i.

- $v_i \sim \; iid \; N(0, \sigma_v^2)$
- $u_i \sim \; iid \; N^+(0, \sigma_u^2)$

The aftermath on the likelihood function of this distributional assumption will be discussed in detail as it will be modified in the innovative part of the thesis.

The mathematical general formulation (usable both for production and cost functions) of the Half normal density function (depicted in Figure 3.5a) of u_i is[2]:

$$f(u) = \frac{2}{\sqrt{2\pi}\sigma_u} \cdot exp\left\{-\frac{u^2}{2\sigma_u^2}\right\} \tag{3.14}$$

and the normal density function of v_i is:

$$f(v) = \frac{1}{\sqrt{2\pi}\sigma_v} \cdot exp\left\{-\frac{v^2}{2\sigma_v^2}\right\} \tag{3.15}$$

In order to derive the density function of the compound error term ε and taking advantage of the independence assumption (see page 29), the joint density function of u_i and v_i is the product of the single density functions, *i.e.*:

$$f(u,v) = \frac{2}{2\pi\sigma_u\sigma_v} \cdot exp\left\{-\frac{u^2}{2\sigma_u^2} - \frac{v^2}{2\sigma_v^2}\right\} \tag{3.16}$$

Then, the general form of the joint density function for u_i and ε_i, since $\varepsilon_i = v_i - s \cdot u_i$ (where $s = 1$ for production functions and $s = -1$ for cost functions), is given by:

$$f(u,\varepsilon) = \frac{2}{2\pi\sigma_u\sigma_v} \cdot exp\left\{-\frac{u^2}{2\sigma_u^2} - \frac{(\varepsilon + su)^2}{2\sigma_v^2}\right\} \tag{3.17}$$

Whereupon, to obtain the marginal density of ε_i (depicted in Figure 3.5b), u_i is integrated out of the joint density:

$$\begin{aligned} f(\varepsilon) &= \int_0^\infty f(u,\varepsilon)\; du \\ &= \frac{2}{\sqrt{2\pi(\sigma_u^2 + \sigma_v^2)}} \cdot \left[1 - \Phi\left(\frac{s\varepsilon(\sigma_u/\sigma_v)}{\sqrt{\sigma_u^2 + \sigma_v^2}}\right)\right] \cdot exp\left\{-\frac{\varepsilon^2}{2(\sigma_u^2 + \sigma_v^2)}\right\} \end{aligned} \tag{3.18}$$

where $\Phi(.)$ is the cumulative distribution function of the standard Normal distribution.

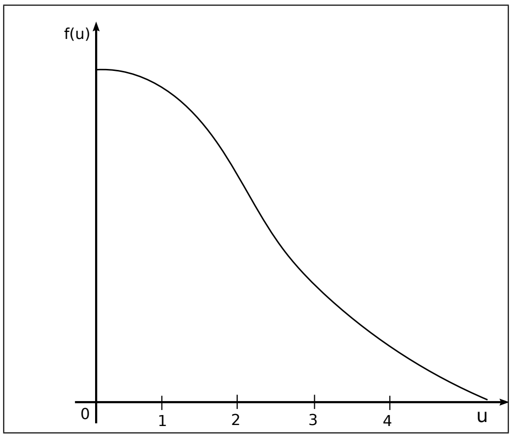
A convenient parameterization that also produces a useful interpretation, is to put $\sigma = \sqrt{(\sigma_u^2 + \sigma_u^2)}$ and $\lambda = \sigma_u/\sigma_v$, in this way the total variance of ε (σ^2) can

[2]For simplicity's sake the equations refer to each company i but the i subscript is dropped.

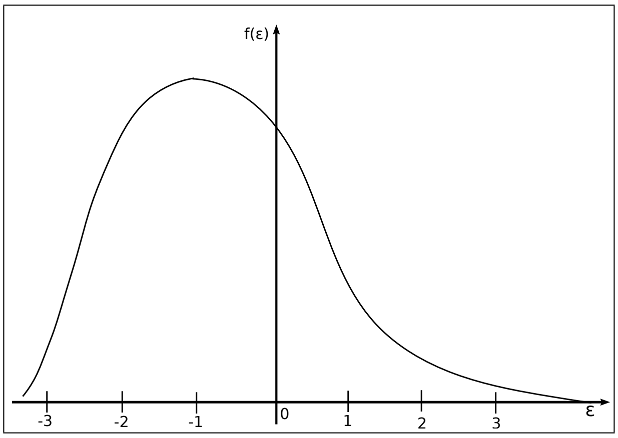
be calculated as the sum of the variances due to the inefficiency (σ_u^2) and to the noise term (σ_v^2) and the parameter (λ) is an immediate suggestion of the amount of inefficiency with respect to the noise[3]. So the (3.18) can be reformulated as:

$$\begin{aligned} f(\varepsilon) &= \frac{2}{\sqrt{2\pi}\sigma} \cdot \left[1 - \Phi\left(\frac{s\varepsilon\lambda}{\sigma}\right)\right] \cdot exp\left\{-\frac{\varepsilon^2}{2\sigma^2}\right\} \\ &= \frac{2}{\sigma} \cdot \phi\left(\frac{\varepsilon}{\sigma}\right) \cdot \Phi\left(-\frac{s\varepsilon\lambda}{\sigma}\right) \end{aligned} \tag{3.19}$$

where $\phi(.)$ is the density function of the standard normal distribution.



(a) Half normal distribution of u_i



(b) Normal- Half normal distribution of ε_i

Figure 3.5: Normal - Half normal distribution model

It is important to note that:

- if $\lambda \rightarrow +\infty$ the distance to the frontier is all due to inefficiency and so deterministic frontier results return;
- if $\lambda \rightarrow 0$ the distance to the frontier is all due to noise, there is no inefficiency in the disturbance, and the model can be efficiently estimated by OLS.

Finally, the log likelihood function to be maximized (or minimized in the case of cost), to obtain maximum likelihood estimates of all parameters, is given by:

$$ln(L) = \sum_{i=1}^{N} \left\{\frac{1}{2} ln\left(\frac{2}{\pi}\right) - ln(\sigma) + ln\left[\Phi\left(-\frac{s\varepsilon_i\lambda}{\sigma}\right)\right] - \frac{\varepsilon_i^2}{2\sigma^2}\right\} \tag{3.20}$$

[3]Some papers use this alternative parameterization $\gamma = \sigma_u^2/\sigma^2$ where the parameter γ measures the proportion of the total variance of ε due to the inefficiency term (see Battese and Corra, 1977; Battese and Coelli, 1992; Coelli, 1991; Greene, 2000, chapter 28).

The last step is the estimation of the inefficiency or efficiency of each firm. In literature the main two proposals are:

- Jondrow et al. (1982) : $E(u_i|\varepsilon_i) = \mu_{*i} + \sigma_* \left\{ \frac{\phi(-\mu_{*i}/\sigma_*)}{\Phi(\mu_{*i}/\sigma_*)} \right\}$
- Battese and Coelli (1988) : $E\left\{exp(-su_i)|\varepsilon_i\right\} = \left\{ \frac{1-\Phi(s\sigma_* - \mu_{*i}/\sigma_*)}{1-\Phi(-\mu_{*i}/\sigma_*)} \right\} exp\left(-s\mu_{*i} + \frac{1}{2}\sigma_*\right)$

where $\mu_{*i} = -s\varepsilon_i\sigma_u^2/\sigma^2$ and $\sigma_* = \sigma_u\sigma_v/\sigma$. Note that $exp\left\{-E(u_i|\varepsilon_i)\right\} \neq E\left\{exp(-su_i)|\varepsilon_i\right\}$.

3.1.3 A brief introduction to panel data frontier models

Cross-sectional frontier models suffer, as pointed out by Schmidt and Sickles (1984), from some difficulties related to the strong assumptions on the compound error term. Panel data, if available, allows to overcome these limitations thanks to the classical panel data estimation techniques that require less distributional assumptions: in fact, in many techniques it is not necessaries to assume the independence of the efficiency term from the regressors or the technical efficiency distribution to be skewed (by avoiding one of the main critical issues of frontier models). Moreover, in panel data models, the efficiency of each producer can be estimated consistently, unlike cross-sectional models, as the time periods $T \rightarrow +\infty$.

In panel data models the production frontier (but also the cost one by changing the sign of u_{it}) can be written as:

$$ln(y_{it}) = ln[f(\mathbf{x}_{it};\beta)] + v_{it} - u_{it} \tag{3.21}$$

where $i = 1,...,I$ indexes producers and $t = 1,...,T$ indexes time periods.

A part of the literature, going back to classical panel data models, assumes that efficiency only varies across producers but is constant through time, *i.e.*:

$$ln(y_{it}) = ln[f(\mathbf{x}_{it};\beta)] + v_{it} - u_i \tag{3.22}$$

Aiming at a more fluid reading of the application in Chapter 13 and to avoid making the discussion heavy going only one of the possible techniques is reported (please see Kumbhakar and Lovell, 2000 or Greene (2008) for a detailed discussion) that is the *"Distribution-Free Approach"* (also named "Free efficiency Method" - Berger, 1993). This approach assumes a stable efficiency of each producer over time, whereas random errors will average out to zero in the end (Berger et al., 1993), *i.e.*:

$$\widehat{u}_i = \max_i(\widehat{u}_i^*) - \widehat{u}_i^* \tag{3.23}$$

where $\widehat{u}_i^* = \sum_t \varepsilon_{it}/\mathrm{T}$.

In this type of models β coefficients can be estimated with a Feasible Generalized Least Squares (FGLS).

3.1.4 Functional forms

Another important choice in the estimation of parametric frontiers, in addition to the u_i distribution, is the functional form of $f(\mathbf{x};\beta)$ (or $c(\mathbf{y_i},\mathbf{w_i};\beta)$). The functional form, in fact, modifies the shape of the isoquants but also the relationship among the regressors. Functional forms should have two characteristics: on the one hand, they should own the properties of the production function (or cost function) discussed in Chapter 1 at page 6 (page 8 for the cost) *i.e.* homogeneity, homotheticity, convexity/concavity etc.; on the other hand, their analytical form should allow empirical analysis and/or the use of statistical and econometric techniques.

In microeconomics, the functional forms are generally divided into two main categories: *rigid* functional forms and *flexible* functional forms. The criterion for distinguishing is the substitution elasticity between the variables that define the domain: if for any pair (x_i, x_j) with $i \neq j$ substitution elasticity is *constant*, then we talk about rigidity. In the opposite case, namely substitution elasticity is different for i and j, we talk about flexibility (see Griffin et al., 1987 for a discussion on the selection of functional forms in production function analysis).

Cobb-Douglas (CD)

The form specified in equation (3.6) and (3.7) is the most used in applications literature of stochastic frontier and econometric inefficiency estimation and it is called *Cobb-Douglas*. The fame of Cobb-Douglas production function is due to the fact that it has universally smooth and convex isoquants and an immediate economic interpretation of results and so of the production technology of the firm. Moreover, it is linearizable with logarithms and so suitable for econometric analysis based on the OLS regression models.

Consider the case of two inputs that is *labor* L and *capital* K to produce the output y the Cobb-Douglas form is (Cobb, 1928):

$$y = AL^{\beta}K^{\alpha} \tag{3.24}$$

β and α give directly the *elasticities* of the output with respect the labor and the capital and their sum provides information about the *returns to scale*:

- if $\alpha + \beta = 1$ the production function has *constant* returns to scale, meaning that an increment of the usage of capital K and labor L of a constant factor generates an increment of the produced output y of the same factor;
- if $\alpha + \beta < 1$ the production function has *decreasing* returns to scale, an increment of the usage of capital K and labor L of a constant factor generates an increment less than proportional of the produced output y;
- if $\alpha + \beta > 1$ the production function has *increasing* returns to scale, an increment of the usage of capital K and labor L of a constant factor generates an increment more than proportional of the produced output y.

The logarithmic general Cobb-Douglas form of a production function is given by:

$$f(\mathbf{x})_{CD} = \alpha_0 + \sum_{p=1}^{P} \beta_p \cdot ln(x_p) + \varepsilon \tag{3.25}$$

Cobb-Douglas is a *rigid* functional form that assumes a constant substitution elasticity among any variables equal to 1, so in the literature some flexible functional forms, that impose fewer restrictions and represent a valid approximation of rigid forms, have been proposed.

Translog (TL)

Translog functional form, with respect to the Cobb-Douglas, has both linear and quadratic terms with the ability of using more than two factor inputs. This function can be approximated by a second order Taylor series (Christensen et al., 1971, 1973) in terms of logarithms and it is characterized by partial substitution elasticities that assume different values. Analytically, a Translog with p inputs can be written as follows:

$$f(\mathbf{x})_{TL} = \alpha_0 + \sum_{p=1}^{P} \beta_p \cdot ln(x_p) + \frac{1}{2} \sum_{p=1}^{P} \sum_{k=1}^{P} \beta_{pk} \cdot ln(x_p) \cdot ln(x_k) + \varepsilon \tag{3.26}$$

Note: If transformed data are used, that is inputs are measured relative to their means, Translog elasticities at means would simply be β_p.

Although Translog allows less restrictions, it is more difficult to interpret than Cobb-Douglas, it also requires the estimation of many parameters and can suffer from curvature violations. Moreover, it derives from the Taylor series approximation that imposes the choice of an arbitrary local point and could generates a large estimation error (White, 1980) in a regression setting.

Fourier Flexible Functional form (FFF)

Fourier Flexible Functional form (FFF) originally proposed by Gallant (1981) is capable of representing a multivariate function when the true functional form is unknown (Dym and McKean, 1972). In fact, a linear combination of sine and cosine functions can exactly approximate every multivariate function with some desirable properties, thanks to the fact that sine and cosine functions are mutually orthogonal and able to cover the entire space of definition of the variables considered.

FFF avoids the limitation of the Taylor series approximation but like other series expansion it has an approximation problem. In fact, to accurately represent a function an infinite number of trigonometric terms, whose coefficients can be estimated only with an infinite number of observations, may be necessary. Gallant (1981) suggested that with a finite number of observations, a second order polynomial in the explanatory variables generates an acceptable error of approximation; this finding is also confirmed by Eastwood and Gallant (1991) suggesting that a number of coefficients equal to the number of observations raised to the 2/3 power can produce consistent and unbiased estimates.

If the explanatory variables are expressed in natural logarithm the FFF is the series expansion of a TL function:

$$\begin{aligned} f(\mathbf{x})_{FFF} = \alpha_0 &+ \sum_{p=1}^{P} \beta_p \cdot ln(x_p) + \frac{1}{2}\sum_{p=1}^{P}\sum_{k=1}^{P} \beta_{pk} \cdot ln(x_p) \cdot ln(x_k) \\ &+ \sum_{p=1}^{P} \delta_p \cdot sin(z_p) + \sum_{p=1}^{P} \lambda_p \cdot cos(z_p) + \sum_{p=1}^{P}\sum_{k=1}^{P} \delta_{pk} \cdot sin(z_p + z_k) \\ &+ \sum_{p=1}^{P}\sum_{k=1}^{P} \lambda_{pk} \cdot cos(z_p + z_k) + \sum_{p=1}^{P}\sum_{k=1}^{P}\sum_{l=1}^{P} \delta_{pkl} \cdot sin(z_p + z_k + z_l) \\ &+ \sum_{p=1}^{P}\sum_{k=1}^{P}\sum_{l=1}^{P} \lambda_{pkl} \cdot cos(z_p + z_k + z_l) + \varepsilon \end{aligned} \tag{3.27}$$

In order to compare the different functional forms, consider a complex form to be estimated where the true function is:

$$Y = \begin{cases} \left(\frac{x}{e}\right)^2 & x \in [0, e) \\ ln(x) & x \in \left[e, e^2\right) \\ 0.25 \cdot cos(x-e)^2 + 1.75 & x \in \left[e^2, e^2 + \pi\right) \\ ln(x - 2\pi) & x \in \left[e^2, 25\right) \end{cases} \tag{3.28}$$

Figure 3.6 shows the estimated regression curves of the three functional form discussed above.

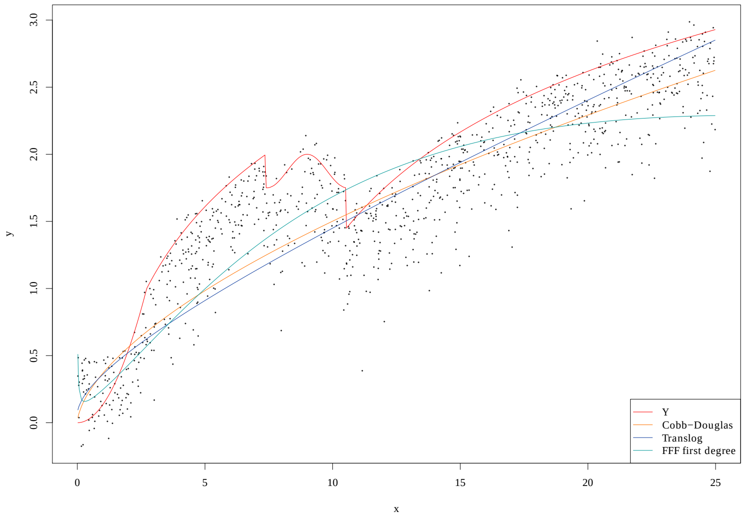


Figure 3.6: Comparison among functional forms

It can be noted how the flexibility of the function increases from the Cobb-Douglas to the FFF of first degree specification: only the FFF begins to catch the congestion point of the true function.

Finally, Figure 3.7 compares the FFF up to third degree that fits the data trends very well.

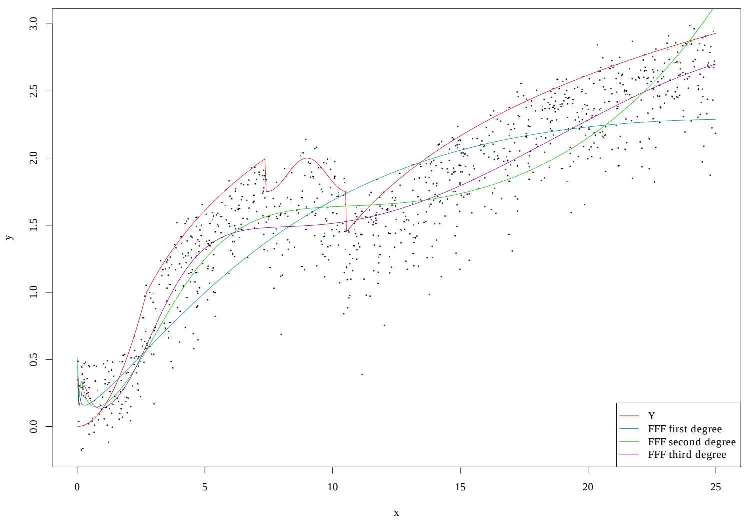


Figure 3.7: Comparison among FFF degrees

3.2 Nonparametric approach

Nonparametric frontier approaches are based on the *envelopment* idea that is the identification of an attainable set (a best-practice frontier), enveloping the cloud of data points, formed by the most efficient units in the sample called "*Benchmarks*". Then, a *relative* efficiency of all other firms is found by comparison with the benchmarks.

In literature, the pioneer was Farrell (1957) developing a first empirical work to identify an efficient frontier of the production possibilities and the resulting efficiency scores. Subsequently, were Charnes et al. (1978) and Banker et al. (1984) to operationalize Farrell's idea by using linear programming techniques. Whereupon, an entire research field has been developed of which only some aspects will be seen aimed at understanding the innovative part of the thesis.

The main *pros* for nonparametric approaches are:

- the very few assumptions required for the identification of the frontier;
- the easy evaluation of multi-input/multi-output cases;
- the possibility to directly compare the individual firm with its peers or combination of peers, in as much as, the frontier derives from the data itself.

Instead, the major *criticisms* to nonparametric approaches regard:

- the deterministic nature of the analysis;
- the difficulty of economic interpretations in term of sensitivity of production to particular inputs;
- the "curse of dimensionality" problem that implies the consideration of large sample sizes to get reliable results.

3.2.1 Free disposal hull (FDH)

Deprins et al. (1984) proposed the Free disposal hull estimator (FDH), that relying only on the *Free disposability* assumption for Ψ (see the production technology property $\Psi 1$ Chapter 1 at page 4), and so it envelopes all the cloud of data points by connecting all positive orthants in the inputs with the negative orthants in the outputs with the vertex at the observed data points (as shown in Figure 3.8), *i.e.*:

$$\Psi_{FDH} = \{(x,y) \in \mathbb{R}_+^{P+Q} | y \leq Y_i, x \geq X_i, (X_i,Y_i) \in \chi\} \tag{3.29}$$

where χ is a random sample $\chi = \{(X_i, Y_i), i = 1, ..., n\}$.

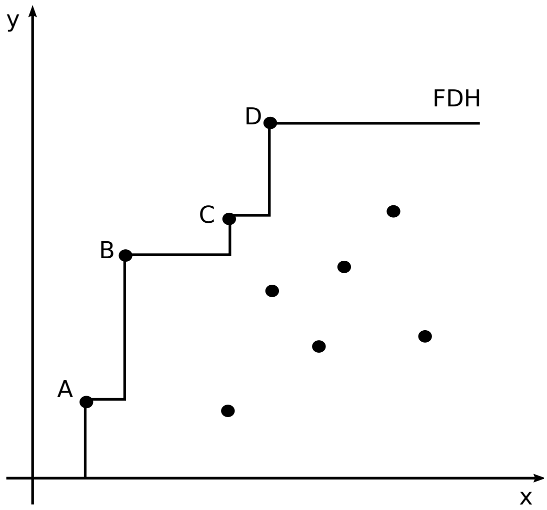


Figure 3.8: FDH (Free disposal hull)

Then, starting from the definitions 1.1.2, for example, the FDH input technical efficiency estimator for a unit (x, y), relative to the boundary of the convex hull of χ_n, is given by:

$$TE_{I_{FDH}} = min\{\theta : (\theta x, y) \in \Psi_{FDH}(\chi)\} \tag{3.30}$$

and so also all the other efficiency measures. In practice, the FDH estimator is computed, using only sorting algorithms, by a simple vector comparison procedure that amounts to a complete enumeration algorithm proposed in Tulkens (1993) and its asymptotic properties have been established in Park et al. (2000) and Daouia et al. (2010).

3.2.2 Data envelopment analysis (DEA)

The Data envelopment analysis (DEA) introduced by Farrell (1957) and popularized by Charnes et al. (1978), that operationalized DEA as a linear programming estimator, adds to the *Free disposability* assumption also the *Convexity* of Ψ (see the production technology property Ψ7 Chapter 1 at page 4), *i.e.*:

$$\begin{aligned}\Psi_{DEA} = \{(x, y) \in \mathbb{R}^{P+Q} | y \leq \sum_{i=1}^{n} \gamma_i Y_i, x \geq \sum_{i=1}^{n} \gamma_i X_i,\ for\ (\gamma_1, ..., \gamma_n) \\ s.t. \sum_{i=1}^{n} \gamma_i = 1; \gamma_i \geq 0; i = 1, ..., n\}\end{aligned} \tag{3.31}$$

Ψ_{DEA} is thus the smallest free disposal convex set covering all the data.

The equality constrained $\sum_{i=1}^{n} \gamma_i = 1$ produces a *Variable Returns to Scale (VRS)* characterization of the DEA (see Banker et al., 1984).

Figure 3.9 shows the differences between Ψ_{DEA} and Ψ_{FDH}, note that if the point c is fully efficient for the FDH it is less efficient for the DEA.

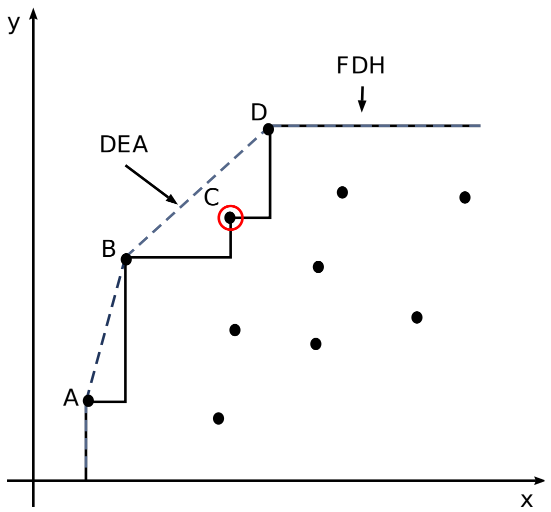
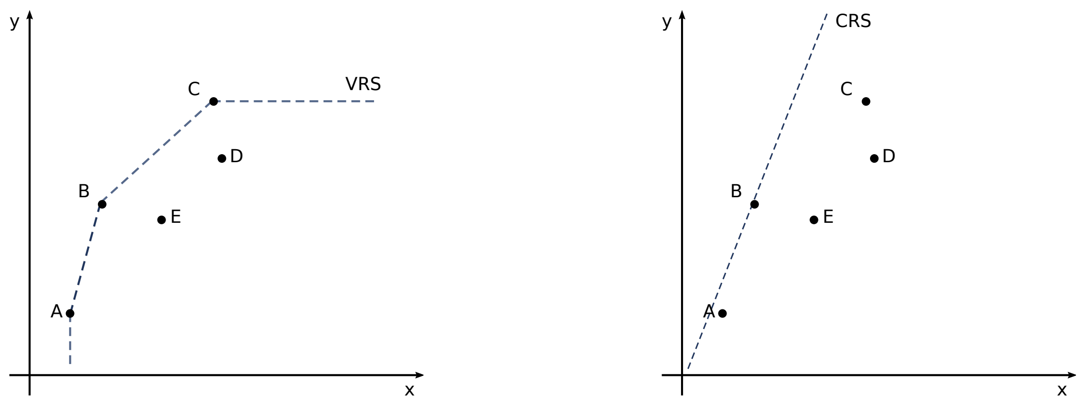


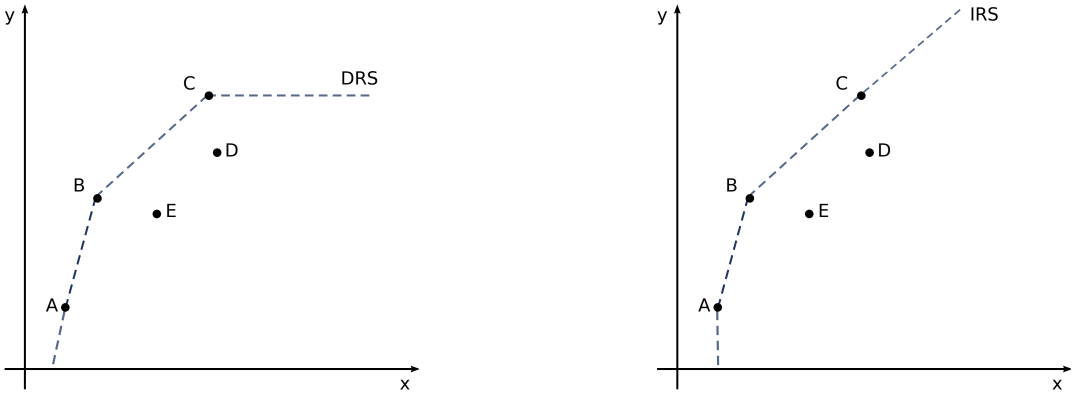
Figure 3.9: Comparison between DEA and FDH production technology

The Ψ_{DEA} also allows for *Constant Returns to Scale (CRS)* (also named CCR-DEA model - Charnes et al., 1978) if the equality constrained $\sum_{i=1}^{n} \gamma_i = 1$ is dropped, *Decreasing Returns to Scale (DRS)* if the equality constrained is changed in $\sum_{i=1}^{n} \gamma_i \leq 1$ and *Increasing Returns to Scale (IRS)* if the equality constrained is modified in $\sum_{i=1}^{n} \gamma_i \geq 1$. See Figure 3.10 for a graphical comparison.



(a) DEA VRS

(b) DEA CRS



(c) DEA DRS

(d) DEA IRS

Figure 3.10: DEA technology sets under different assumptions

Its asymptotic properties have been established in Kneip et al. (2008).

3.2.3 Order-m partial frontiers

Both the FDH and the DEA estimators fully envelope observations in the sample and so their main disadvantage is the sensitivity to outliers or extreme data points.Cazals et al. (2002), with the aim to overcome this drawback, proposed a more robust nonparametric estimator of the frontier (a "partial frontier") based on the concept of expected minimum input function of order m and so called Order-m.

The underlying idea is to identify a partial frontier well near the upper (or lower) boundary, in such a way to be sensitive to the magnitude of the extreme valuable observations but, simultaneously, resistant to their influence in case they are suspicious.

This method mitigates the outliers effect introducing a distribution of probability for the units in the technology set Ψ. In a probabilistic approach the production process, that generates observations in χ_n, can be defined through the joint distribution of the random vector (X,Y) on $\mathbb{R}_+^P \times \mathbb{R}_+^Q$, *i.e.* (Daraio and Simar, 2005):

$$H_{XY}(x,y) = Prob(X \leq x, Y \geq y) \tag{3.32}$$

This probability function can be decomposed as follows:

$$H_{XY}(x,y) = Prob(X \leq x | Y \geq y) Prob(Y \geq y) = F_{X|Y}(x|y) S_Y(y) \tag{3.33}$$

where it is supposed that the survivor function of Y exists *i.e.* $S_Y(y) > 0$ and that the support of the conditional distribution of X $F_{X|Y}(.|y)$ can be viewed as the attainable set of input values X for a unit working at the output level y.

For a given level of y, consider m i.i.d. random variables X_i generated by the conditional distribution of X that is $F_X(x|y)$, the random set of order m can be defined as follows:

$$\widetilde{\Psi}_m(y) = \{(x,y') \in \mathbb{R}_+^{P+Q} | X_i \leq x \text{ for some } 1 \leq i \leq m, y' \geq y\}. \tag{3.34}$$

Next, the benchmark is not identified looking at the lower bound of this support but as the expected minimal value of inputs for m units randomly drawn according to $F_{X|Y}(.|y)$ (*i.e.* units producing at least the output level y):

$$\varphi_m = \mathbb{E}\left[min(X_1,...,X_m|Y \geq y)\right] = \int_0^\infty \left[1 - F_{X|Y}(x|y)\right]^m dx \tag{3.35}$$

For finite m, this is clearly less extreme than the full frontier as depicted in Figure 3.11.

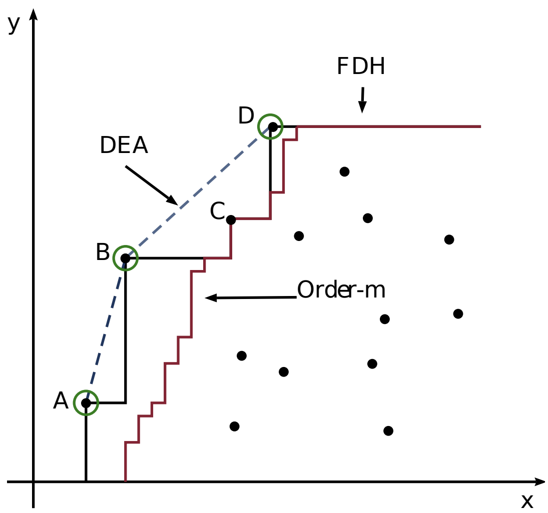


Figure 3.11: Comparison among DEA, FDH and Order-m production technology

Finally, input-oriented efficiency score defined for all y such that $S_Y(y) > 0$, is given by:

$$\widetilde{TE}_{I_m} = min\{\theta : (\theta x, y) \in \Psi_m\} = min\{\theta : H(\theta x, y) > 0\} \tag{3.36}$$

$\widetilde{TE}_{I_m}$ is computed by the following formula:

$$\widetilde{TE}_{I_m} = \min_{i=1,...,m} \left\{ \max_{j=1,...,P} \left(\frac{X_i^j}{x^j} \right) \right\} \tag{3.37}$$

$\widetilde{TE}_{I_m}$ is a random variable since the X_i are random variables generated by $F_X(x|y)$. All results discussed above can be simply reformulated to the output-oriented case.

Part II

Spatial Stochastic Frontiers models

Chapter 4

SPATIAL AUTOCORRELATION ANALYSIS

In traditional regression models, to obtain Best Linear Unbiased Estimators (BLUE) of the parameters, must be valid the so-called Gauss-Markov assumptions on the residuals (ε):

- Zero mean: $\mathbb{E}[\varepsilon] = 0$;
- Homoschedasticity : $\mathbb{E}[\varepsilon^2] = \sigma^2$;

- No serial correlation : $\mathbb{E}[\varepsilon_i \varepsilon_j] = 0, i \neq j$;
- Normality: $\varepsilon \sim N(0, \sigma^2)$.

Absence of serial correlation hypothesis is not valid in presence of *Spatial autocorrelation*. Different definitions of *Spatial autocorrelation* have been proposed in literature starting from 1985. Upton and Fingleton (1985) said that spatial autocorrelation exists if the map presents an organized pattern like, for example, subfigures (a) and (c) in Figure 4.1. In other words, a mapped pattern that significantly deviates from a map where each value of the analysed variable is assigned randomly (see subfigure (b) in Figure 4.1), presents a spatial autocorrelation.

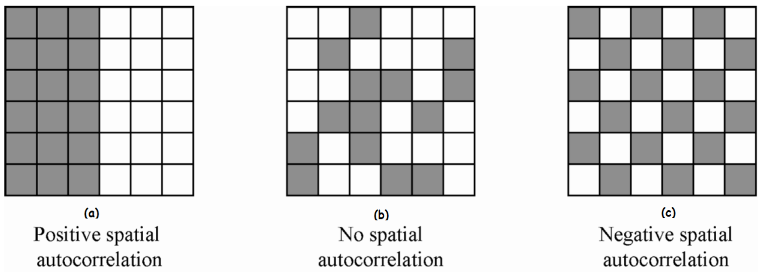


Figure 4.1: Spatial autocorrelation patterns

A conceptual generalization has been proposed by Cliff and Ord (1973) that proposed the comparison among the distributions of some variables in a region or a set of points to test the presence of spatial autocorrelation.
Griffith (1987) and Goodchild (1987) asserted that, in order to evaluate if significant deviations from the random distribution exist it is necessary to calculate an index of comparison. In particular, spatial autocorrelation indexes have to measure "*the degree to which objects or activities at some place on the earth's surface are similar to others objects or activities located nearby*" reflecting Tobler's first law of geography: "*everything is related to everything else, but near things are more related than distant things*".
Moreover, a key definition has been given by Sokal and Oden (1978) that introduced the concept of "*dependence*" *i.e.* spatial autocorrelation exists when the observed value of a variable at one locality is significantly dependent on the values of the same variable at neighboring localities. This dependence is defined by Anselin and Bera (1998) as *"the coincidence of value similarity with locational similarity. In other words, high or low values for a random variable tend to cluster in space (positive spatial autocorrelation) or locations tend to be surrounded by neighbors with very dissimilar values (negative spatial autocorrelation). Of the two types of spatial*

autocorrelation, positive autocorrelation is by far the more intuitive. Negative spatial autocorrelation implies a checkerboard pattern of values and does not always have a meaningful substantive interpretation."
Therefore, positive spatial autocorrelation means that high values tend to be located near high values, medium values near medium values, and low values near low values as shown in Figure 4.2.

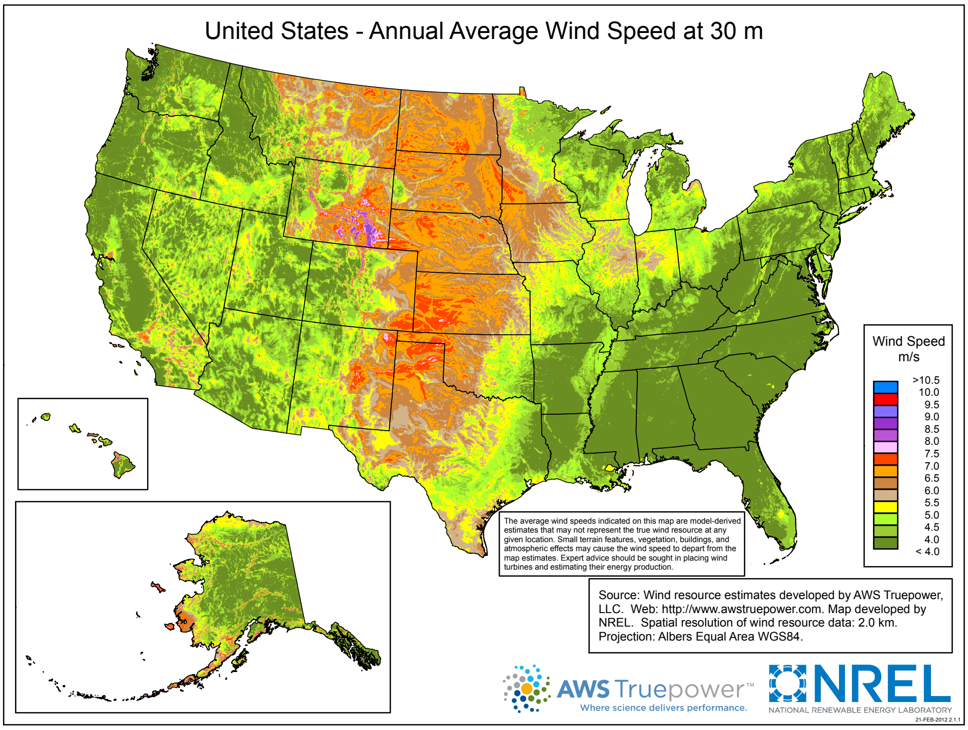


Figure 4.2: Positive spatial autocorrelation - Annual Average Wind Speed at 30 m (United States)

In SFA specification the firm-level efficiency is estimated from the residuals, assuming that all producers in the sample are independent: however, this assumption rules out the possibility to account for spatial effects in the theoretical model. The limitations of this approach were already known in early contributions, considering Farrell (1957) highlighted the importance of incorporating the correlation between technical efficiency and variables representing location, temperature and rainfall. In his analysis, focused on the efficiency patterns in US agricultural firms, he argued that *"the apparent differences in efficiency [...] reflect factors like climate, location and fertility that have not been included in the analysis, as well as genuine differences in efficiency"*.
When spatial effects are significant, the traditional techniques used to estimate the

SFA (MLE or its variants) parameters generate biased results: if the disturbances are spatially correlated, the assumption of a spherical error covariance matrix is violated, leading to biased and inconsistent estimators (LeSage, 1997). In order to overcome these issues, a number of recent contributions in frontier analysis have proposed alternative specifications aiming to incorporate spatial effects in the baseline models. This part of the literature follows the approach used by spatial econometrics, a specific branch of econometrics that deals with spatial interaction (spatial autocorrelation) and spatial structure (spatial heterogeneity) in both cross-sectional and panel data (Paelinck and Klaassen, 1979; Anselin, 1988).

4.1 Definition of the neighborhood

To assess spatial autocorrelation, a very critical and criticized step is the definition of the concepts of "*neighborhood*" and "*nearest neighbor*" (see Anselin, 1988).

Consider again a sample of I units of which a variable x is observed, the set of *neighbors* for a spatial unit i, denoted with $N(i)$ can be defined in various ways (see LeSage and Pace, 2009 and Arbia, 2010 for a detailed discussion):

Critical cut-off distance neighborhood: two sites are said to be neighbors if $0 \leq d \leq d^*$, with d an appropriate distance chosen, and d^* representing the critical cut-off.

Nearest neighbor: two sites are said to be neighbors if $d_{ij} = Min(d_{ij}), \forall i, j$.

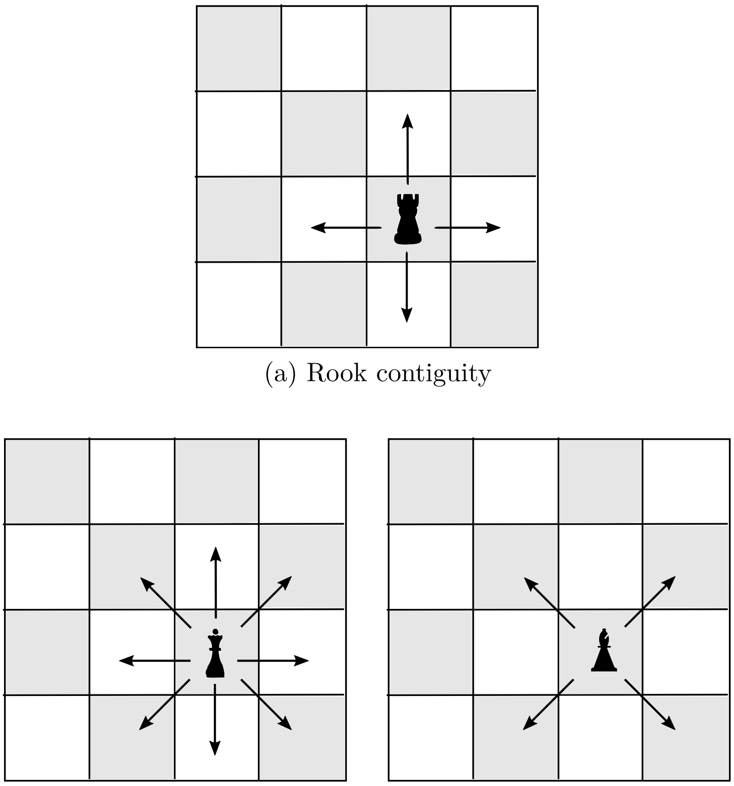
In the literature, different definitions of *distance* d have been proposed: *(i)* the Euclidean distance between centroids of polygons, *(ii)* the Hausdorff distance (Hausdorff, 1914), *(iii)* social distances (Doreian, 1980), *(iv)* economic distances (Case et al., 1993), *(v)* distances measured in terms of the empirically observed flows (Murdoch et al., 1997) or *(vi)* distances measured in terms of trade-based interaction (Aten, 1996, 1997).

Contiguity-based neighborhood: is based on the mere adjacency between two polygons. Two polygons are said to be neighbors if they share a common boundary:

Rook contiguity: two regions are neighbors if they share a common side (Figure 4.3a);

Queen contiguity: two regions are neighbors if they share a common side or vertex (Figure 4.3b);

Bishop: two regions are neighbors if they share a common vertex (Figure 4.3c).



(a) Rook contiguity

(b) Queen contiguity

(c) Bishop contiguity

Figure 4.3: Contiguity neighborhood in a regular lattice

These neighbor links are presented in a $n \times n$ weight matrix **W** whose elements are:

$$w_i j = \begin{cases} 1 & if \ j \in N(i) \\ 0 & otherwise \end{cases} \tag{4.1}$$

For ease of interpretation, it is common practice to normalize **W** such that the elements of each row sum to unity. Since **W** is nonnegative, this ensures that all weights are in [0,1], and it has the effect that the weighting operation can be interpreted as an averaging of neighboring values (Elhorst, 2014).

Finally, another important notion helpful to the subsequent topics is the definition of *spatial lag*: a spatial lag is a variable that averages the neighboring values of a location.

4.2 Detecting spatial autocorrelation

The existence of spatial autocorrelated residuals, *i.e.* of *clusterized data* (see Figure 4.4), can be evaluated with some indicators of spatial association.

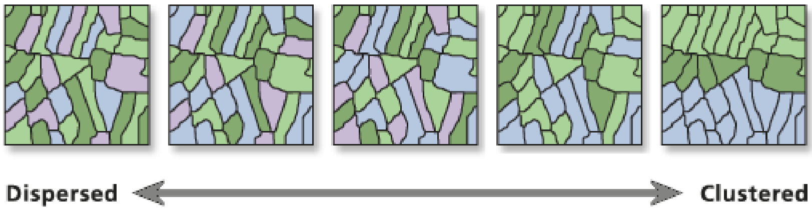


Figure 4.4: Spatial clusters

4.2.1 Moran's I

Moran's I (Moran, 1950) tests the global spatial autocorrelation (of the overall clustering of the data) through the calculation of the cross-products of the variable x deviations from its mean for n observations at locations i, j as:

$$I = \frac{n}{S_0} \frac{\sum_i \sum_j w_{ij}(x_i - \overline{x})(x_j - \overline{x})}{\sum_i (x_i - \overline{x})^2} \tag{4.2}$$

where $\overline{x}$ is the mean of the x variable, w_{ij} are the elements of the weight matrix, and S_0 is the sum of the elements of the weight matrix: $S_0 = \sum_i \sum_j w_{ij}$.

Moran's I is similar but not equivalent to a correlation coefficient. It varies from -1 to +1. In absence of autocorrelation and regardless of the specified weight matrix, the expectation of Moran's I statistic is $[-1/(n-1)]$, which tends to zero as the sample size increases. For a row-standardized spatial weight matrix, the normalizing factor S_0 equals n (since each row sums to 1), and the statistic simplifies to a ratio of a spatial cross product to a variance. A Moran's I coefficient larger than $[-1/(n-1)]$ indicates positive spatial autocorrelation, and a Moran's I less than $[-1/(n-1)]$ indicates negative spatial autocorrelation (see Figure 4.5 for a graphical representation).

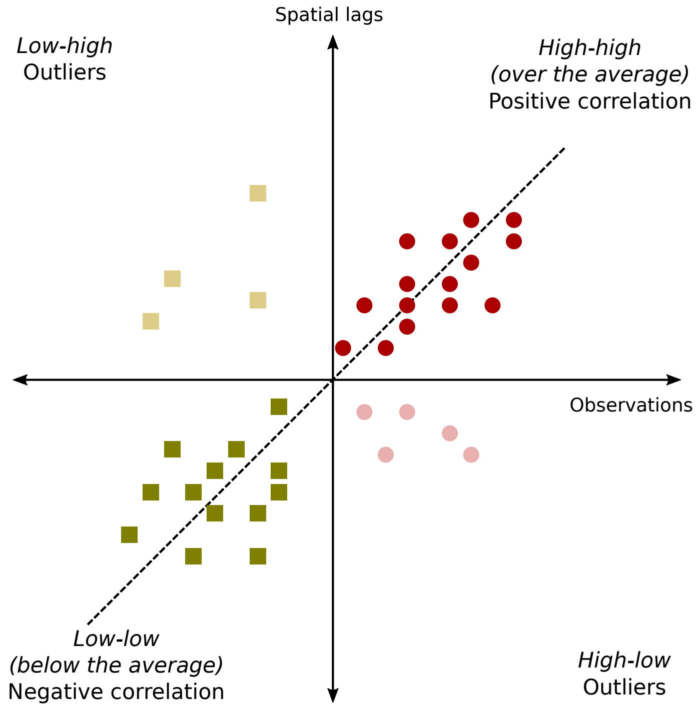


Figure 4.5: Morans'I scatterplot

4.2.2 Geary's C

Geary's C statistic (Geary, 1954) is based on the deviations in responses among the x values of each observation with another one:

$$C = \frac{(n-1)\sum_i \sum_j w_{ij}(x_i - x_j)^2}{2\sum_i \sum_j w_{ij} \sum_i (x_i - \overline{x})^2} \tag{4.3}$$

Geary's C ranges from 0 (maximal positive autocorrelation) to a positive value (high negative autocorrelation) and it assumes value 1 in absence of autocorrelation (Sokal and Oden, 1978). So, if the value of Geary's C is less than 1, it indicates positive spatial autocorrelation.

Moran's I is a more global measure and it is sensitive to extreme values of x, whereas Geary's C is more sensitive to differences in small neighborhoods. In general, Moran's I and Geary's C results give similar conclusions. However, Moran's I is preferred in most cases since Cliff and Ord (1975, 1981), have shown that Moran's I is consistently more powerful than Geary's C.

Local indicators of spatial association (LISA)

Moran's I and Geary's C both measure the *global* spatial autocorrelation that could result null, in mean, hiding the existence of clusters at local level.

A measurement of the *local* spatial autocorrelation is given by the so-called Local indicators of spatial association (LISA - Anselin, 1995) that calculate n Local Moran's I (I_i) and evaluate the statistical significance for each I_i.
LISA measures can be deduced from the Moran's I statistics defined in equation (4.2) as:

$$I_i = \frac{(x_i - \overline{x})}{\sum_i (x_i - \overline{x})^2/n} \sum_j w_{ij}(x_i - x_j) \tag{4.4}$$

and $I = \sum_i I_i/n$.

4.3 Spatial autoregressive models

In this section, a non-exhaustive review of methods for estimating regression models in presence of autocorrelated residuals will be presented.

The classical formulation of a linear regression model, in a matrix formulation, is:

$$y = \mathbf{X}\beta + \varepsilon \tag{4.5}$$

where $\mathbf{X}$ is the covariates matrix and $\varepsilon \sim N(0, \sigma^2\mathbf{I})$.

In a time-series context, the OLS estimator remains consistent even when a lagged dependent variable is present, as long as the error term does not show serial correlation; in a spatial context, instead, this rule does not hold, irrespective of the properties of the error term.

In other terms, ignoring spatial autocorrelation may have serious effects on statistical inference that is on efficiency, consistency, hypothesis testing and, finally, on prediction step. More in particular, Pace and LeSage (2010) wrote that: "*unlike the standard least-squares result for the case of omitted variables, the presence of spatial dependence magnifies conventional omitted variables bias in OLS estimates*" showing that "*using spatial econometric model specifications containing spatial lags of both the dependent and explanatory variables produces estimates whose bias matches the conventional omitted variables case*".

Also for these reasons, in recent years (see *e.g.* Case et al., 1993; Cohen and

Morrison, 2004) economics literature has seen an increasing number of theoretical and applied econometric studies involving spatial issues.

Anselin (2003) proposed a taxonomy of the spatial externalities models for cross-sectional data. This taxonomy depends on the way in which spatial dependence is incorporated in the regression specification. It can be included:

- as an additional covariate in the form of a spatially lagged dependent variable ($\mathbf{W}y$) if the focus of interest is to evaluate if y in place i is affected by the values of the dependent variable in nearby place j;

- as additional covariates in the form of spatially lagged explanatory variables ($\mathbf{WX}$) if the focus of interest is to test if y in place i is affected by the independent variables in both place i and j;

- in the error structure ($\mathbf{W}\varepsilon$) if the focus of interest is to test if the error terms across different spatial units are correlated.

It is important to note that the elements of the weights matrix are nonstochastic and exogenous to the model.

4.3.1 Spatial AutoRegressive model (SAR)

Assuming that the outcome y in a location is affected by the y in all nearby locations (by the way of $\mathbf{W}$), the equation (4.5) can be rewritten as a (first order) *Spatial AutoRegressive model* (SAR or Spatial lag) formally (Anselin, 1988; LeSage and Pace, 2009; Arbia, 2014):

$$y = \lambda \mathbf{W} y + \mathbf{X}\beta + \varepsilon \tag{4.6}$$

where $\lambda < |1|$ is a spatial autoregressive coefficient and $\varepsilon \sim N(0, \sigma_\varepsilon^2 \mathbf{I})$.

Spatial lag term $\mathbf{W}y$ is correlated with the disturbances, even when the latter are independent and identically distributed. In fact, if the reduced form of the (4.6) is considered:

$$y = (\mathbf{I} - \lambda \mathbf{W})^{-1}\mathbf{X}\beta + (\mathbf{I} - \lambda \mathbf{W})^{-1}\varepsilon \tag{4.7}$$

Consequently, the spatial lag term must be treated as an endogenous variable and proper estimation methods must account for this endogeneity (OLS will be biased and inconsistent due to the simultaneity bias) such as Maximum Likelihood (ML) or the Two Stage Least Squares (2SLS).

4.3.2 Spatial Error Model (SEM)

Assuming, instead, that a random shock in a specific location does not only affect y in this location, but it is transmitted to all other locations by the multiplier $\rho \in [0,1]$, the equation (4.5) can be rewritten as a *Spatial Error Model* (SEM), formally (Anselin, 1988; LeSage and Pace, 2009; Arbia, 2014):

$$y = \mathbf{X}\beta + \varepsilon, \varepsilon = \rho\mathbf{W}\varepsilon + \tilde{\varepsilon} \tag{4.8}$$

where $\rho < |1|$ is a spatial autoregressive coefficient and $\tilde{\varepsilon} \sim N(0, \sigma^2_{\tilde{\varepsilon}}\mathbf{I})$.

In this case the reduced form of ε is:

$$\varepsilon = (\mathbf{I} - \rho\mathbf{W})^{-1}\tilde{\varepsilon} \tag{4.9}$$

The SEM is a special case of regression with a non-spherical error term, in which the off-diagonal elements of the covariance matrix express the structure of spatial dependence. Consequently, OLS remains unbiased:

$$\mathbb{E}[\varepsilon] = (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbb{E}[\tilde{\varepsilon}] = 0 \tag{4.10}$$

since $\mathbb{E}[\tilde{\varepsilon}] = 0$.

However, classical estimators for standard errors will be biased, in fact the error variance-covariance matrix becomes:

$$\begin{aligned} \mathbb{E}[\varepsilon\varepsilon^T] &= (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbb{E}[\tilde{\varepsilon}\tilde{\varepsilon}'](\mathbf{I} - \rho\mathbf{W}^T)^{-1} \\ &= \sigma^2_{\tilde{\varepsilon}}(\mathbf{I} - \rho\mathbf{W})^{-1}(\mathbf{I} - \rho\mathbf{W}^T)^{-1} \\ &= \sigma^2_{\tilde{\varepsilon}}\Omega \end{aligned} \tag{4.11}$$

Therefore, the MLE described in Anselin (1988) involves the maximization of the following likelihood function that will be discussed in detail as it will be modified in the innovative part of the thesis:

$$\begin{aligned} L(\beta, \sigma^2_{\tilde{\varepsilon}}, \rho) =& (2\pi)^{-\frac{n}{2}}(\sigma^2_{\tilde{\varepsilon}})^{-\frac{n}{2}} \left| (\mathbf{I} - \rho\mathbf{W})^{-1} \left[(\mathbf{I} - \rho\mathbf{W})^{-1} \right]^T \right|^{-\frac{1}{2}} \\ & \cdot exp \left\{ -\frac{1}{2\sigma^2_{\tilde{\varepsilon}}}(y - \mathbf{X}\beta)^T \left[(\mathbf{I} - \rho\mathbf{W})^{-1}(\mathbf{I} - \rho\mathbf{W^T})^{-1} \right]^{-1} (y - \mathbf{X}\beta) \right\} \end{aligned} \tag{4.12}$$

where:

$$\left|(\mathbf{I}-\rho\mathbf{W})^{-1}\left[(\mathbf{I}-\rho\mathbf{W})^{-1}\right]^{T}\right| = \left|(\mathbf{I}-\rho\mathbf{W})^{-1}\right| \cdot \left|\left[(\mathbf{I}-\rho\mathbf{W})^{-1}\right]^{T}\right| =$$
$$|(\mathbf{I}-\rho\mathbf{W})|^{-1} \cdot |(\mathbf{I}-\rho\mathbf{W})|^{-1} = |(\mathbf{I}-\rho\mathbf{W})|^{-2}$$

So, the log likelihood to be estimated is (please see Lee (2004) for a discussion on the asymptotic properties of the above ML estimator):

$$\begin{aligned} ln\left[L(\beta,\sigma^2_{\tilde{\varepsilon}},\rho)\right] = &-\frac{n}{2}ln(2\pi)-\frac{n}{2}ln(\sigma^2_{\tilde{\varepsilon}})+ln|(\mathbf{I}-\rho\mathbf{W})|- \\ &\frac{1}{2\sigma^2_{\tilde{\varepsilon}}}(y-\mathbf{X}\beta)^T\left[(\mathbf{I}-\rho\mathbf{W})^{-1}(\mathbf{I}-\rho\mathbf{W}^T)^{-1}\right]^{-1}(y-\mathbf{X}\beta) \end{aligned} \tag{4.13}$$

Finally, using Ord (1975), in order to enormously simplify the computation, $ln|(\mathbf{I}-\rho\mathbf{W})|$ can be decomposed as $\sum_{i=1}^{n} ln\left[(1-\rho\psi_i)\right]$ where ψ_i represents the $i-th$ eigenvalue of the weight matrix $\mathbf{W}$.

4.3.3 Spatial AutoCorrelation model (SAC)

Finally, the *Spatial AutoCorrelation model* (SAC, Kelejian and Prucha, 1998) is the logical union of the two estimators shown before.

$$y = \lambda\mathbf{W}y+\mathbf{X}\beta+\varepsilon, \varepsilon = \rho\mathbf{W}\varepsilon+\tilde{\varepsilon} \tag{4.14}$$

or also including the indirect effects on $\mathbf{X}$ the more general model named SAC-mixed is obtained:

$$y = \lambda\mathbf{W}y+\mathbf{X}\beta+\lambda\mathbf{W}\mathbf{X}+\varepsilon, \varepsilon = \rho\mathbf{W}\varepsilon+\tilde{\varepsilon} \tag{4.15}$$

The less general models described above can be derived from the SAC-mixed:

- if $\lambda \neq 0$ and $\rho = 0 \rightarrow$ Spatial AutoRegressive Model
- if $\lambda = 0$ and $\rho \neq 0 \rightarrow$ Spatial Error Model

More specifically, SAC and SAC-mixed models allow to take into account both the impact of the neighbors on the outcome y - decomposing the direct effects from indirect ones - and to make possible the control of the bias due to the omitted variables in the estimation.

4.3.4 Geographically Weighted Regression (GWR)

Models discussed in sections 4.3.1, 4.3.2 and 4.3.3 assume that the relationship among variables is a *stationary* mechanism over the geographical units, *i.e.* they estimate a single average parameter β.

However, it could be a reasonable thought that in many cases the relationship between the dependent variable y and the covariates $\mathbf{X}$ is not constant in the whole area but varies across the geographical units, *i.e.* the mechanism is *non-stationary*.

Geographically Weighted Regression (GWR) introduced by Brunsdon et al. (1996) takes into account these different relations allowing to estimate a β for each unit, *i.e.* the regression model is:

$$y_i = X_i\beta_i + \varepsilon_i \tag{4.16}$$

where:

- y_i is the $i-th$ observation of the dependent variable y at location i;
- X_i is the $i-th$ row of the regressors matrix $\mathbf{X}$;
- β_i is the vector of the β coefficients for each unit at location i;
- ε_i is the random term.

In particular, GWR performs a series of weighted least squares regressions on subsets of the data, where the influence of an observation i decreases with the Euclidean distance to a regression point j. These distance-dependent weights are determined by a kernel function and the range of the input data is set up according to a specific bandwidth in order to carry on a *Local Weighted Regression* (LWR, Fotheringham et al., 2002) for each spatial subset (Figure 4.6).

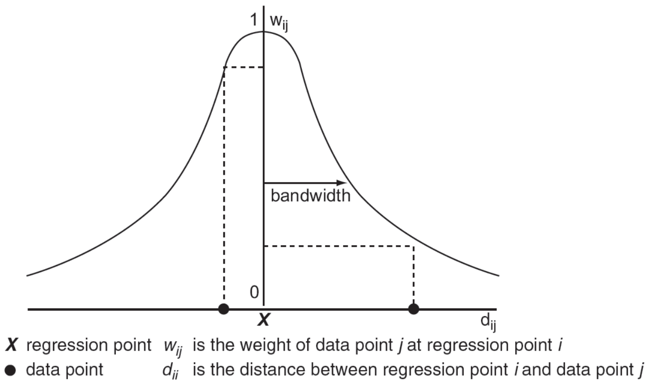


Figure 4.6: GWR kernel function, source: Fotheringham et al. (2002)

Chapter 5

SPATIAL STOCHASTIC FRONTIER ANALYSIS

In previous chapters a general overview of both the frontier analysis and the spatial dependence has been shown in order to introduce key topics for understanding the issue of this innovative chapter.

In particular, as seen in Section 4.3 spatial data positively auto-correlated violates the independence assumption of the classical OLS regression and so also the SFA model[4]; this violation does not permit the assessment of statistical inference because SFA errors can no longer be assumed to have zero covariances with each other.

[4] In terms of homoscedasticity of u, *e.g.* see Jondrow et al. (1982).

Subsequently, given the need to consider for spatial dependence also in frontier analysis, some models have been developed; they can be divided into two major fields distinguishing those that explain inefficiency/efficiency in terms of exogenous determinants analysing the *heterogeneity* from those that consider the *spatial dependence* by including in the model a spatial autoregressive specification.

5.1 Spatial efficiency literature

As far as the first stream is concerned, some authors have proposed to analyze heterogeneity by including contextual factors as regressors or to modelling the inefficiency term. In particular, Lavado and Barrios (2010) used contextual factors for modelling the inefficiency part of a stochastic frontier model by embedding a SAR in the deterministic part and a general linear mixed model into the efficiency equation. Hughes et al. (2011), in a panel-type model with the aim of evaluating a climate-adjusted production frontier for the Australian broad acre cropping industry, considered specific spatial effects in the stochastic production frontier by adding climate effects as dependent variables. Jeleskovic and Schwanebeck (2012) proposed a two step deterministic estimation model to differentiate heterogeneity and inefficiency in world healthcare systems: *(i)* in the first step different fixed effects panel spatial models have been estimated; *(ii)* in the second step the obtained inefficiency has been regressed (also with various fixed effects panel spatial models) as dependent variable onto country specific variables that identify the heterogeneity; finally, Brehm (2013) proposed a correction of the SFA error term for panel data by introducing spatially correlated factors variables that affect the production process.

In the second set of analysis, others proposals consider spatial dependence by including a spatial lag into the dependent variable or into the covariates. More specifically, Affuso (2010) included spatial lag on the dependent variable reformulating the stochastic frontier density function; Glass et al. (2013), Glass et al. (2014) and Glass et al. (2016) introduced the concept of efficiency spillover, extending the non-spatial Cornwell et al. (1990) model to the case of spatial autoregressive dependence; Adetutu et al. (2015) proposed a local spatial stochastic frontier model that accounts for spatial interaction by allowing spatial lags on the inputs and on the exogenous variables to shift the production frontier technology; Han et al. (2013) proposed a method for investigating spillovers effects in panel data by maintaining the Schmidt and Sickles (1984) hypothesis of time-invariant inefficiency, but allowing global spatial dependence through the introduction of a spatial lag on the dependent variable.

Finally, other papers proposed to consider spatial dependence by including a spatial lag on the inefficiency term. Druska and Horrace (2004) extended the Kelejian and Prucha (1999) specification for cross-sectional data based on a standard fixed effects model by assuming an autoregressive specification of the error term and estimating inefficiency with the Generalized Moments Method; Schmidt et al. (2009) used a Bayesian approach to include latent spatial effects, that explain geographical variation of firms' outputs and inefficiency, dependent on a parameter that captures the unobserved spatial characteristics; *e.g.* Areal et al. (2012) suggested, with the aim of measuring the overall effect of spatial factors that affect the production, to include a spatial lag directly into inefficiency allowing the splitting of the inefficiency (u) into a spatial component and into a specific term for every firm through a Bayesian procedure:

$$\begin{aligned} &y = \mathbf{X}\beta + \varepsilon - z \\ &\text{where} \\ &\epsilon \sim N(0, \sigma_\varepsilon^2 \mathbf{I}), z = \rho \mathbf{W} z + \tilde{z} \end{aligned} \tag{5.1}$$

where $\mathbf{W}$ is the connectivity matrix that includes the relative spatial information (see Section 4.1), ρ is the spatial lag parameter ($\rho \in [0,1]$), z and $\tilde{z}$ are the latent variables, representing the inefficiency, whose distributional form is unknown.

Rewriting the specification of z as $z = (\mathbf{I} - \rho\mathbf{W})^{-1}\tilde{z}$ into equation (5.1), Areal et al. (2012) proposed to consider:

$$y = \mathbf{X}\beta + \varepsilon - (\mathbf{I} - \rho\mathbf{W})^{-1}\tilde{z} \tag{5.2}$$

Pavlyuk (2010) proposed to include spatial lags on the overall standard SFA model (see also recent enhancements, Pavlyuk, 2012, 2013).

Following the approach implemented by Areal et al. (2012) in equation (5.2), Fusco and Vidoli (2013) have proposed to measure the global effect of spatial factors by including a spatial lag only in the inefficiency term of a SFA, not using a Bayesian procedure but by reformulating the SFA density function with a SEM.

Therefore, spatial dependence refers to how much the level of technical inefficiency of firm i depends on the levels set by other firms $j = 1, ..., n$, under the assumption that part of the firm i inefficiency depends on the neighbor firm j's performances ($j \neq i$).

5.2 Spatial Stochastic Frontier Analysis (SSFA)

Fusco, E. and Vidoli, F. (2013). ***Spatial stochastic frontier models: controlling spatial global and local heterogeneity.*** **International Review of Applied Economics, 27(5):679–694,** http://dx.doi.org/10.1080/02692171.2013.804493.

Cited in:

Kinfu, Y. and Sawhney, M. (2015). *Inefficiency, heterogeneity and spillover effects in maternal care in India: a spatial stochastic frontier analysis.* BMC Health Services Research, 15, 118.

Vidoli, F. and Canello, J. (2016). *Controlling for spatial heterogeneity in nonparametric efficiency models: An empirical proposal*, European Journal of Operational Research, Volume 249, Issue 2, 1 March 2016, Pages 771-783.

Villano, R. and Fleming, E. and Moss, J. (2016). *Spatial Econometric Analysis: Potential Contribution to the Economic Analysis of Smallholder Development*, Chapter in Causal Inference in Econometrics, Volume 622 of the series Studies in Computational Intelligence, pp 29-55, Springer International Publishing.

Ruiz Fuensanta, M. and Hernández Sancho, F. and Soler i Marco, V., (2015). *In vino veritas: competitive factors in wine-producing industrial districts.* nvestigaciones regionales - Journal of regional research, Asociación Española de Ciencia Regional, issue 32, pages 149-164.

Ramajo, J. and Hewings, G. J.D. (2016). *Modeling Regional Productive Performance Using a Spatial Stochastic Frontier Approach: New Evidence for Europe* (1995-2007) (February 29, 2016).

Starting from equation (3.12), the Normal / Half-Normal cross-section general form (homoskedastic case) of the Spatial Stochastic Frontier model (SSFA) proposed in Fusco and Vidoli (2013) can be respectively rewritten, in a matrix formulation, as:

$$\begin{aligned} y &= \mathbf{X}\beta + v - s \cdot u \\ &= \mathbf{X}\beta + v - s \cdot (\mathbf{I} - \rho \mathbf{W})^{-1} \tilde{u} \end{aligned} \tag{5.3}$$

where:

- $v \sim iid\ N(0, \sigma_v^2 \mathbf{I})$;
- $u \sim\ N^+(0, (\mathbf{I} - \rho\mathbf{W})^{-1}(\mathbf{I} - \rho\mathbf{W}^T)^{-1}\sigma_{\widetilde{u}}^2 \mathbf{I})$;
- u and v are independently distributed of each other, and of the regressors;
- $\widetilde{u} \sim N(0, \sigma_{\widetilde{u}}^2 \mathbf{I})$;
- $s = 1$ for production functions and $s = -1$ for cost functions.

Starting from equations (3.14) and (3.15) at page 30 the density functions for each firm[5] of $(1 - \rho \sum_i w_{i.})^{-1} \widetilde{u}_i \geq 0$ and v_i, noting $(1 - \rho \sum_i w_{i.})$ as $\delta(\rho)$ can be written as:

$$f([\delta(\rho)]^{-1}\widetilde{u}) = \frac{2}{\sqrt{2\pi}[\delta(\rho)]^{-1}\sigma_{\widetilde{u}}} \cdot exp\left\{-\frac{[\delta(\rho)]^{-2}\widetilde{u}^2}{2[\delta(\rho)]^{-2}\sigma_{\widetilde{u}}^2}\right\} \tag{5.4a}$$

$$f(v) = \frac{1}{\sqrt{2\pi}\sigma_v} \cdot exp\left\{-\frac{v^2}{2\sigma_v^2}\right\} \tag{5.4b}$$

Given the independence assumption, the joint density function of $[\delta(\rho)]^{-1}\widetilde{u}$ and v is the product of equations (5.4a) and (5.4b):

$$f([\delta(\rho)]^{-1}\widetilde{u}, v) = \frac{1}{\pi[\delta(\rho)]^{-1}\sigma_{\widetilde{u}}\sigma_v} \cdot exp\left\{-\frac{v^2}{2\sigma_v^2} - \frac{[\delta(\rho)]^{-2}\widetilde{u}^2}{2[\delta(\rho)]^{-2}\sigma_{\widetilde{u}}^2}\right\} \tag{5.5}$$

Since $\varepsilon = v - s \cdot [\delta(\rho)]^{-1}\widetilde{u}$, the joint density function for $[\delta(\rho)]^{-1}\widetilde{u}$ and ε becomes:

$$f([\delta(\rho)]^{-1}\widetilde{u}, \varepsilon) = \frac{1}{\pi[\delta(\rho)]^{-1}\sigma_{\widetilde{u}}\sigma_v} \cdot exp\left\{-\frac{(\varepsilon + s[\delta(\rho)]^{-1}\widetilde{u})^2}{2\sigma_v^2} - \frac{[\delta(\rho)]^{-2}\widetilde{u}^2}{2[\delta(\rho)]^{-2}\sigma_{\widetilde{u}}^2}\right\} \tag{5.6}$$

The marginal density function of ε is obtained by integrating $[\delta(\rho)]^{-1}\widetilde{u}$ out of

[5] For simplicity's sake and to make the notation more consistent with the SFA literature, the model is rewritten for each observation i and the subscript i is dropped in the main variables.

$f([\delta(\rho)]^{-1}\widetilde{u},\varepsilon)$, which yelds:

$$\begin{aligned} f(\varepsilon) &= \int_0^{\infty} f([\delta(\rho)]^{-1}\widetilde{u},\varepsilon)\ du \\ &= \frac{2}{\sigma}\cdot\phi\left(\frac{\varepsilon}{\sigma}\right)\cdot\Phi\left(-\frac{s\lambda\varepsilon}{\sigma}\right) \\ \text{where}& \\ \sigma &= \sqrt{\sigma_v^2+[\delta(\rho)]^{-2}\sigma_{\widetilde{u}}^2} \\ \lambda &= \frac{[\delta(\rho)]^{-1}\sigma_{\widetilde{u}}}{\sigma_v} \end{aligned} \tag{5.7}$$

and $\phi(\cdot)$ e $\Phi(\cdot)$ are the standard normal density and distribution functions, respectively.

The marginal density function $f(\varepsilon)$ is asymmetrically distributed with mean and variance:

$$\begin{aligned} \mathbb{E}(\varepsilon) &= -E([\delta(\rho)]^{-1}\widetilde{u}) = -[\delta(\rho)]^{-1}\sigma_{\widetilde{u}}\sqrt{\frac{2}{\pi}} \\ \mathbb{V}(\varepsilon) &= \sigma_v^2+\frac{\pi-2}{\pi}[\delta(\rho)]^{-2}\sigma_{\widetilde{u}}^2 \end{aligned} \tag{5.8}$$

The log-likelihood function for a sample of n producers is given by:

$$ln(L) = \sum_{i=1}^{N}\left\{\frac{1}{2}ln\left(\frac{2}{\pi}\right)-ln(\sigma)+ln\left[\Phi\left(-\frac{s\lambda\varepsilon_i}{\sigma}\right)\right]-\frac{\varepsilon_i^2}{2\sigma^2}\right\} \tag{5.9}$$

Finally, in order to obtain estimates of the technical efficiency of each producer, following Jondrow et al. (1982), the conditional distribution of $[\delta(\rho)]^{-1}\widetilde{u}$ given ε is calculated as:

$$\begin{aligned} f([\delta(\rho)]^{-1}\widetilde{u}|\varepsilon) &= \frac{f([\delta(\rho)]^{-1}\widetilde{u},\varepsilon)}{f(\varepsilon)} \\ &= \frac{1}{\sqrt{2\pi}\sigma_*}exp\left\{-\frac{[\delta(\rho)]^{-1}\widetilde{u}-\mu_*]^2}{2\sigma_*^2}\right\}\bigg/\left[1-\Phi\left(-\frac{\mu_*}{\sigma_*}\right)\right] \\ \text{where}& \\ \mu_* &= \frac{-s\varepsilon[\delta(\rho)]^{-2}\sigma_{\widetilde{u}}^2}{\sigma^2} \\ \sigma_*^2 &= \frac{[\delta(\rho)]^{-2}\sigma_{\widetilde{u}}^2\sigma_v^2}{\sigma^2} \end{aligned} \tag{5.10}$$

Since $f([\delta(\rho)]^{-1}\widetilde{u}|\varepsilon)$ is distributed as $N^{+}(\mu_{*},\sigma_{*}^{2})$, the point estimator for $[\delta(\rho)]^{-1}\widetilde{u}_i$ can be obtained following Jondrow et al. (1982) or Battese and Coelli (1988)'s proposals. Choosing the second one as suggested in Kumbhakar and Lovell (2000) TE is calculated as below:

$$\begin{aligned} TE_i &= \mathbb{E}\left(exp\left\{-[\delta(\rho)]^{-1}\widetilde{u}_i\right\}|\varepsilon_i\right) \\ &= \left[\frac{1-\Phi(s\sigma_{*}-\mu_{*i}/\sigma_{*})}{1-\Phi(-\mu_{*i}/\sigma_{*})}\right] exp\left\{-s\mu_{*i}+\frac{1}{2}\sigma_{*}^{2}\right\} \end{aligned} \tag{5.11}$$

Therefore, SSFA avoids the subjective choice of the exogenous determinants allowing the evaluation of the *conjoint* effect of a multitude of unknown determinants and focuses the analysis of the spatial dependence only on the inefficiency term. So, it can be considered a prerequisite for subsequent analysis that identifies some determinants of the inefficiency.

5.3 Simulations

To test the properties and accuracy of the SSFA estimator, two simulations have been conducted, the first tests the SSFA ability to sterilize the spatial correlation by using the Data Generating Process (DGP) construction criteria proposed by Banker and Natarajan (2008) and adding a strong spatial correlation in the inefficiency term and the second tests the ability of the SSFA to estimate the parameters of the model by implementing a MCMC where the strength of the spatial correlation is varied.

5.3.1 Banker simulated data

The DGP proposed in Banker and Natarajan (2008) and also used in Johnson and Kuosmanen (2011) allows the construction of a continuous, monotoning, increasing and concave function over the relevant range of input used in the simulation. To test the SSFA ability to sterilize the spatial correlation among the units in a frontier model, the main difference with Banker and Natarajan (2008)'s formulation is due to the spatial correlation added into inefficiency term through a spatial lag parameter and a contiguity matrix.

The true production frontier originally proposed is a third-order polynomial of a single input variable x:

$$y_i = (x_i^3 - 12x_i^2 + 48x_i - 37),\ i = 1, \ldots, n \tag{5.12}$$

The 107 Italian provinces contiguity matrix (year 2008) is used to construct $\mathbf{W}$ and for each unit an input $x \sim Unif[1,4]$, a noise term $v \sim N(0, 0.5^2)$ and an inefficiency term that depends either on a ρ equal to 0.8 are extracted, either to a variance equal to 0.56, and either to a parameter $\gamma \in [0,5]$ that enhances differences in inefficiency between northern and southern provinces, giving lower value to the northern provinces and higher value to the southern ones[6].

More clearly, u may be expressed as:

$$u_i = (1 - \rho \sum_i w_{i.})^{-1} \tilde{u}_i \cdot \gamma \text{ where } \tilde{u}_i \sim N(0, 0.75^2) \tag{5.13}$$

Then, adding the noise term v and the inefficiency term u, the equation (5.12) becomes:

$$y_i = (x_i^3 - 12x_i^2 + 48x_i - 37) \cdot exp(-u_i + v_i),\ i = 1, \ldots, n \tag{5.14}$$

The simulated data represented in Figure 5.1 shows how the presence of a strong spatial auto-correlation (Moran's I index is equal to 0.46), a characteristic of many economic phenomena, is particularly dangerous because it is not easy to diagnose.

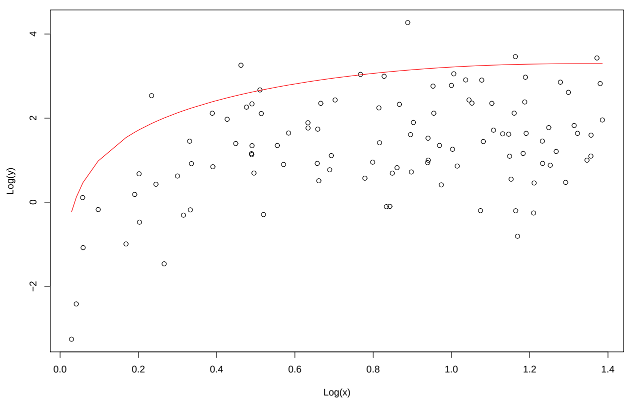


Figure 5.1: Simulated data and the true frontier

Starting from the simulated data, SFA analysis is conducted with the aim of estimating the production efficiency of each unit; results - first column of Table 5.1 - hide the presence of a high spatial correlation in the residuals as shown in the Moran plot in Figure 5.2, making the SFA estimates biased and thereby requiring the use of a model which takes into account and sterilizes the spatial correlation between units.

[6] Equation (5.13) allows the introduction of a strong spatial correlation both local, by the way of γ, and global, by the way of ρ, in the data.

Table 5.1: Simulation results by method. Simulated data.

	SFA	SSFA
Intercept	1.1858**	3.4448
β	1.2734***	41.6332***
σ_u	1.3192	0.5959
σ_v	0.7793*	0.4743*
Moran's I	0.4571***	-0.189
ρ	-	0.7784

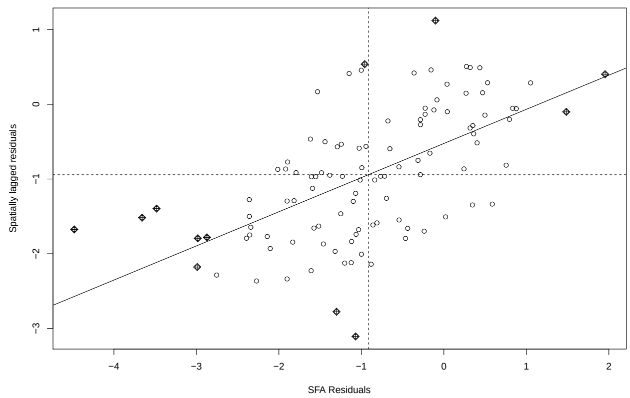
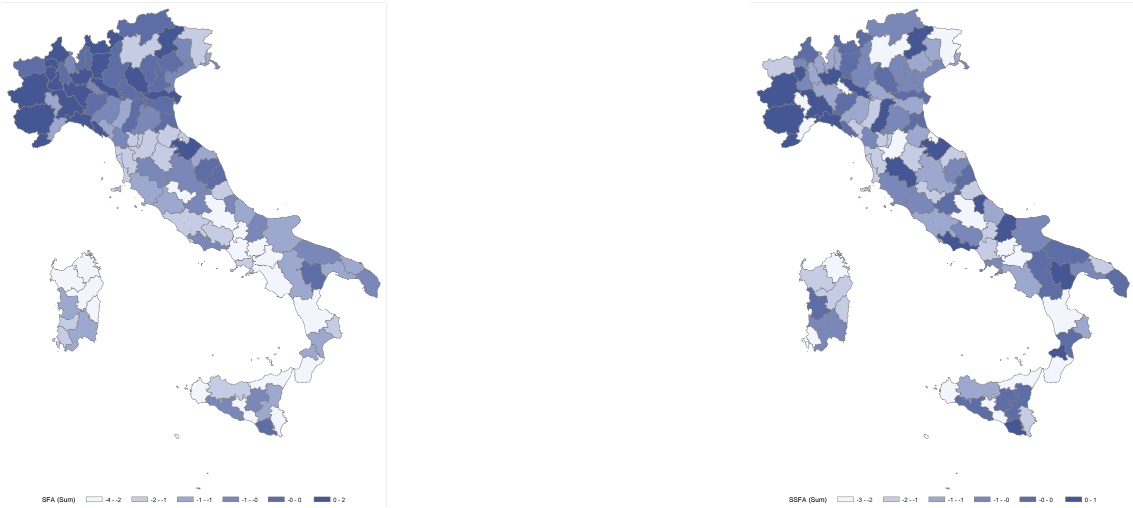


Figure 5.2: Moran plot - SFA residuals

In particular, in the first step, equation (5.14) is estimated both by a SEM and by OLS; the difference in the variance of the errors between the two models, $(\sigma_{\widetilde{u}})$ is placed as the initial value in the next optimization phase of ML.

Table 5.1 shows three results: *(i)* the strong differences in the intercept and the parameter β, *(ii)* the smaller value of σ_u and σ_v, proving that the SSFA model fits the true function better on the frontier and *(iii)* the correct estimation of the spatial lag ρ.

The difference among the residuals estimated is clearly visible in Figure 5.3, where it is shown that the SSFA model residuals are no longer dependent on the territory and the major inefficiency assigned to the southern provinces is fully neutralized.



(a) SFA

(b) SSFA

Figure 5.3: Spatial residual distribution by method

5.3.2 MCMC simulation

In this section a MCMC simulation is implemented with the aim to test the goodness of estimation and the finite sample performance of the SSFA method.

The SSFA model DGP used to simulate 1,000 samples of 100, 200 and 400 observations is:

$$\begin{aligned} y &= 10 + 0.8 \cdot x - u + v \\ u &= (\mathbf{I} - \rho \mathbf{W})^{-1} \tilde{u} \end{aligned} \tag{5.15}$$

where x is extracted from a $N(60,5)$, v from a $N(0,0.1)$, $\tilde{u}$ from a $N(0,3)$ and ρ varies in $(0.1,...,0.9)$ with 0.1 increments. Finally, $\mathbf{W}$ is a sparse matrix as suggested by LeSage and Pace (2009), for its low computational costs, where the average number of neighbors for each observation, identified through the nearest neighbor method, will be equal to 6[7].

[7] *"If W contains all non-zero elements, it would require enormous amounts of memory to store this matrix for problems involving large samples such as the US Census tracts where $n > 60,000$. Fortunately, W is usually sparse, meaning it contains a large proportion of zeros. For example, if one relies on contiguous regions or some number m of nearest neighboring regions to form W, the spatial weight matrix will only contain mn non-zeros as opposed to n^2 non-zeros for a dense matrix. The proportion of non-zeros becomes m/n which falls with n. Contiguity weight matrices have an average of six neighbors per row (for spatially random sets of points on a plane). As an example, using the 3,111 US counties representing the lower 48 states plus the district of Columbia, there are 9,678,321 elements in the $3,111 \times 3,111$ matrix W, but only $3,111 \times 6 = 18,666$ would be non-zero, or 0.1929 percent of the entries. In addition, calculating matrix-vector products such as Wy and WX take much less time for sparse matrices. In both cases, sparse matrices require linear in n operations ($O(n)$) while a dense W would require quadratic in n operations ($O(n^2)$) (LeSage*

According to the data generated from rules and the DGP above, β and ρ are estimated using the SSFA. The SSFA goodness of estimation is evaluated by calculating the *Root Mean Squared Error* (RMSE) of parameters and the results are reported in Table 5.2 and in Figure 5.4.

Table 5.2: RMSE of SSFA parameters

Parameters	*True value*	*RMSE*		
		$n = 100$	$n = 200$	$n = 400$
β	0.8	0.007	0.004	0.002
	0.1	0.050	0.032	0.019
	0.2	0.052	0.029	0.018
	0.3	0.047	0.027	0.016
	0.4	0.041	0.023	0.013
ρ	0.5	0.039	0.022	0.013
	0.6	0.034	0.019	0.012
	0.7	0.028	0.016	0.011
	0.8	0.023	0.013	0.009
	0.9	0.024	0.010	0.005

Some important findings are summarized as follows. The estimation results of the parameters show very low RMSE values, already for samples of 100 units, both for β and for the various ρ parameters[8].

Moreover, Figure 5.4 reports the kernel densities of the repeated estimated ρ parameters, showing that the distributions are very close to the true values and that this accuracy rises with increasing spatial dependence in the error term and with increasing sample size as already indicated by RMSE values.

Below some diagnostics of the MCMC simulation for $n = 400$ are provided to test the stationarity of the MCMC output.
Figure 5.5 displays the trace plots of the simulation showing that the center of the chains appears to be around the ρ true values, with very small fluctuations.

Instead, Figures 5.6 and 5.7 show the *autocorrelation function* (ACF) and the *partial autocorrelation function* (PACF) for ρ that test if the estimated ρ at iteration k is correlated with past versions of itself at lags 1,2,..., in the first case in a global way and in the second case by imposing a partial autocorrelation equal to 0 beyond that iteration.

and Pace, 2009).

[8] Note that only one value for β is reported as it is almost constant at varying rho.

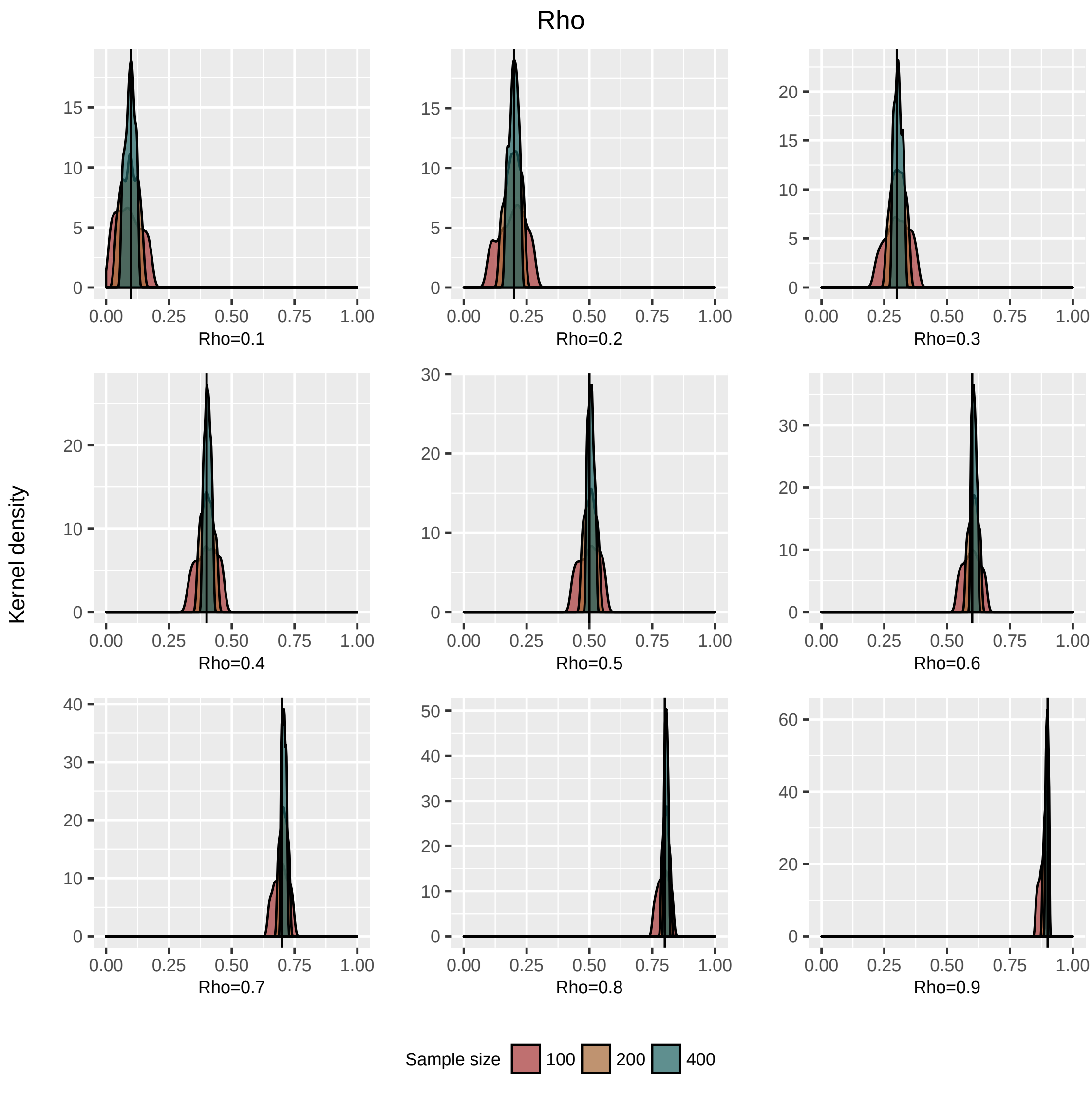


Figure 5.4: Rho MCMC distribution results

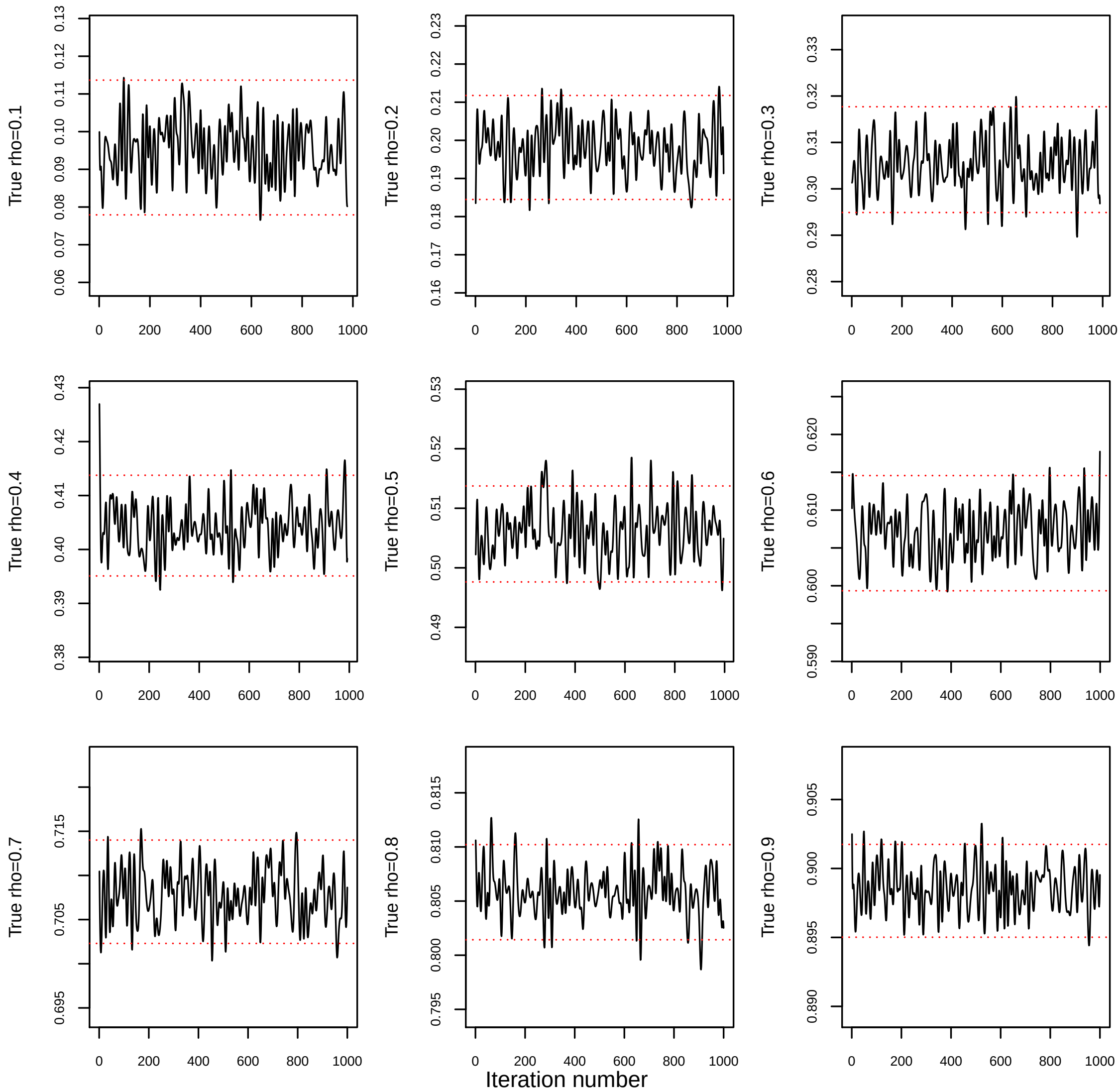


Figure 5.5: Trace plots of ρ parameters ($n = 400$)

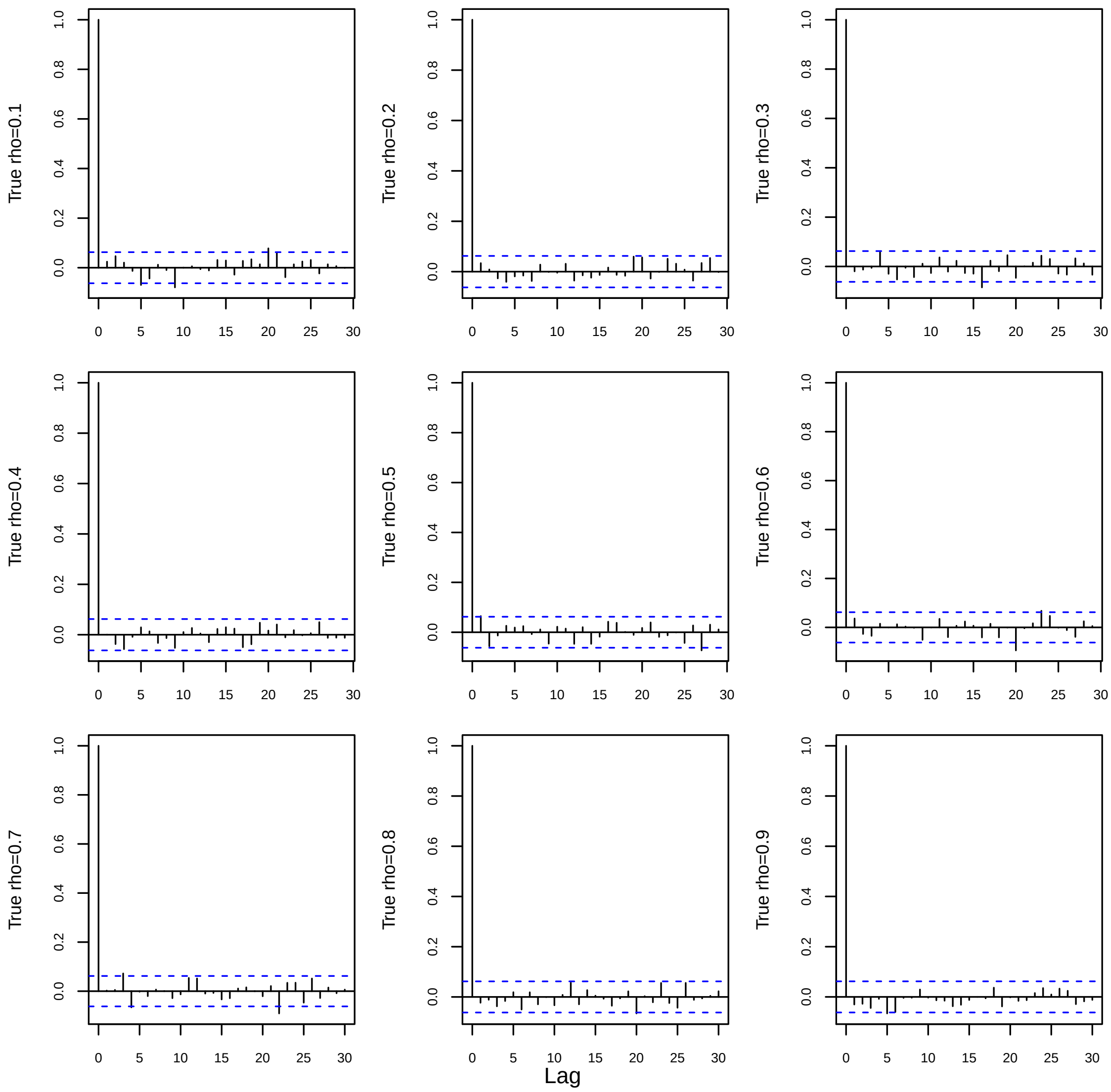


Figure 5.6: ACF plots of ρ parameters ($n = 400$)

In both cases good results are obtained strengthening the stationarity hypothesis of the chain and this indicates that the chain could have reached the right distribution. These results provide simulation evidence that β and ρ estimated by the SSFA are almost surely very near to the true values.

Finally, a comparison with the classical SFA model is provided both graphically in terms of fitted values (Figure 5.8), and in terms of speed of the algorithm(Table 5.3). Note that the algorithm is the Nelder and Mead (1965) with the following constraints on the parameters: $\rho \in [0, 1]$, $\sigma_u^2 \geq 0$ and $\sigma_v^2 \geq 0$.

In particular, Figure 5.8 shows that the SSFA formulation hides an interesting property: the identification of the error part imputable to the spatial proximities and *different for each unit* allows to change (positively or negatively) the intercept

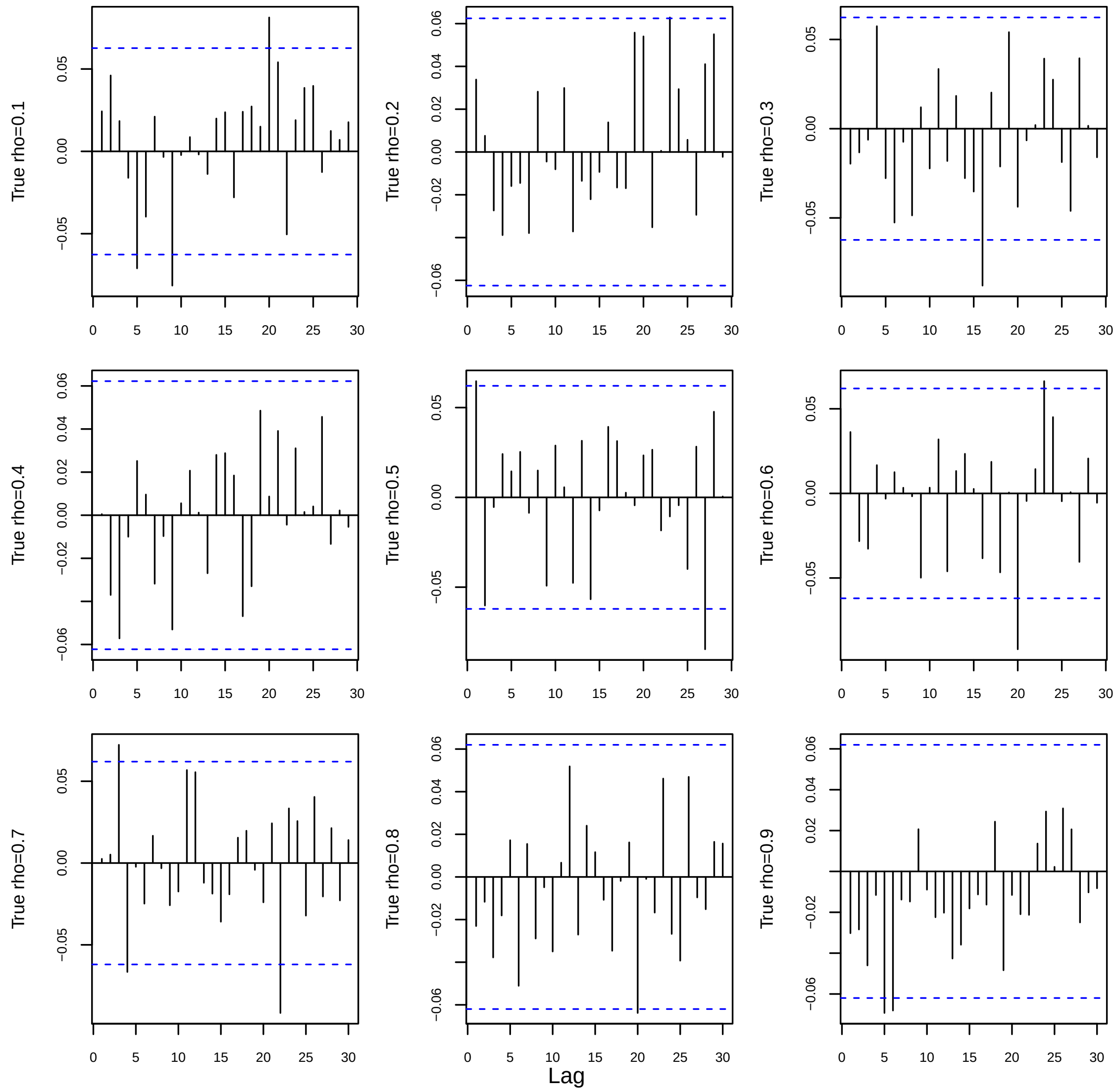


Figure 5.7: PACF plots of ρ parameters ($n = 400$)

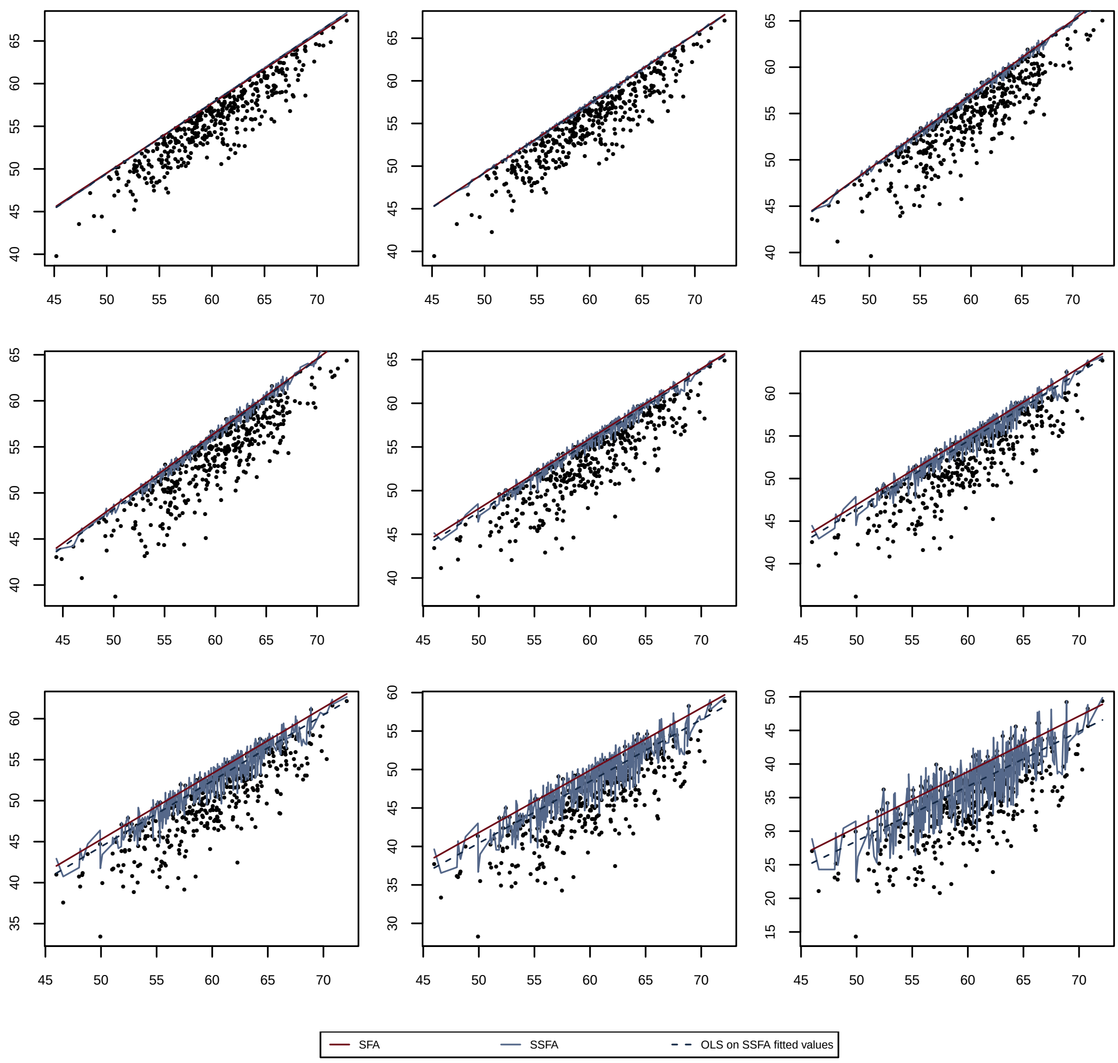


Figure 5.8: SFA and SSFA fitted values, simulated data

of the model by impacting both on the specific estimation of each unit (light blue continuous line) both on the average slope (dark blue dashed line); therefore, in the SSFA framework the intercept is to be intended only as the "medium level" cleansed by the individual spatial effect.

Table 5.3: Comparison of SFA, SSFA and SEM convergence speed

Sample size	*Speed (seconds)*		
	SFA	SSFA	SEM
$n = 100$	0.0335	0.4556	0.2745
$n = 200$	0.0419	0.7547	0.3355
$n = 400$	0.0690	3.2925	0.7745

Finally, Table 5.3 compares the speed of convergence (in seconds) of SFA, SSFA and SEM models showing a not too heavy computational effort of the SSFA with respect the SFA for a single iteration, since the SSFA requires a SEM to find the initial values of β and ρ and has to do matrix operations with $n \times n$ matrices ($\mathbf{W}$).

5.3.3 Spatial weight sensitivity analysis

The main criticism of spatial regression models is the sensibility of the estimations to alternative specifications used for the spatial weight matrix **W** (see LeSage and Pace, 2014 for a detailed survey). Arbia and Fingleton (2008) highlighted that *"[...] critics of spatial econometrics almost always in our experience home in on the arbitrary nature of the weights matrix, asking "how is it defined and why is it precisely like that when it could easily have been like this, what does it mean, and are not the results obtained conditional on somewhat arbitrary decisions taken about its structure?". Some future research on the robustness of outcomes to variations in assumptions about the weight matrix structure would be helpful in allaying such criticisms [...]."*

Accordingly, in this section SSFA results will be tested by varying **W** specification in the same DGP of section 5.3.2 with a sample of 400 units. The test is carried out using the *k-nearest neighborhood criteria* by increasing the average number of neighbors for each observation from 6 to a maximum of 100 given the sample size of 400.

The goal of efficiency analysis is the correct estimation of β coefficients and of the specific efficiency for each producer. For this reason the results are evaluated, on the stability of the estimated β coefficients and on the mean of estimated efficiency in the sample, to different specifications of $\mathbf{W}$. For coherence with the simulation in section 5.3.2 the results are tested for different values of ρ as shown in Tables 5.4 and 5.5.

Table 5.4: Sensitivity analysis of the β coefficients with different ρ and number of neighbors in W $(n = 400)$

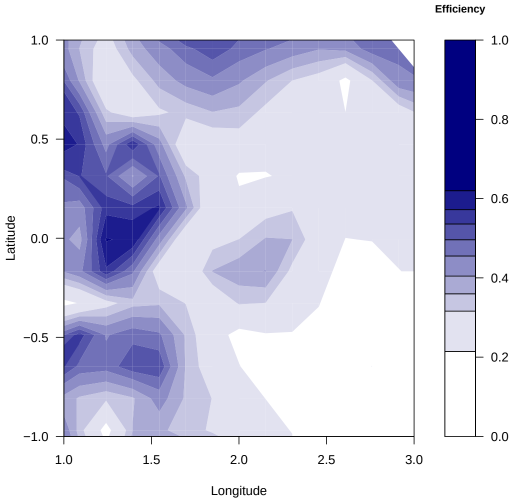
ρ	*SSFA #Neighbors*			
	6	20	30	100
0.4	0.797	0.797	0.798	0.801
0.6	0.795	0.795	0.797	0.800
0.8	0.793	0.793	0.795	0.802

The β coefficients are robust both to growing of the number of neighbors and of the ρ as desirable.

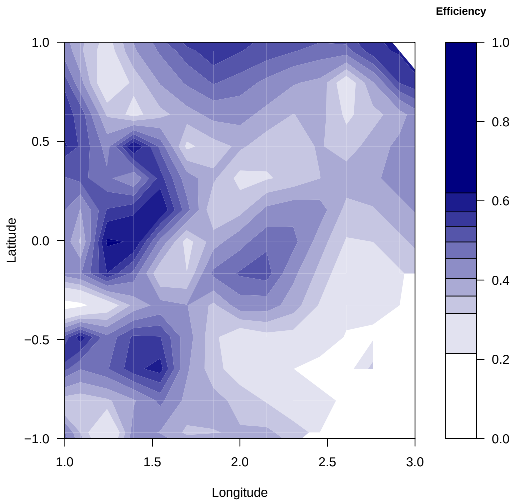
Table 5.5: Sensitivity analysis of the estimated efficiency with different ρ and number of neighbors in W $(n = 400)$

ρ	*SFA*	*SSFA #Neighbors*			
		6	20	30	100
0.4	0.4381	0.4439	0.4422	0.4421	0.4389
0.6	0.4358	0.4488	0.4444	0.4415	0.4355
0.8	0.4373	0.4625	0.4474	0.4361	0.4360

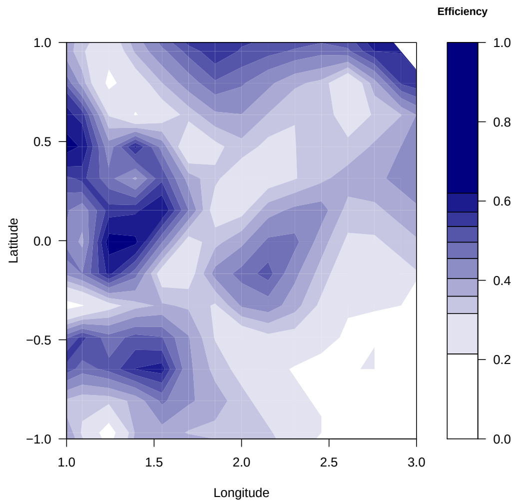
The estimated efficiencies, as shown in Table 5.5, decrease in mean by increasing the number of neighbors and tend to the SFA ones, particularly for high values of ρ, as expected. The result is more evident at local level, in fact, if the number of neighbors tend to N the SSFA model does not split correctly the error term and residuals remain spatial autocorrelated as shown in Figures 5.9 for a $rho = 0.8$.



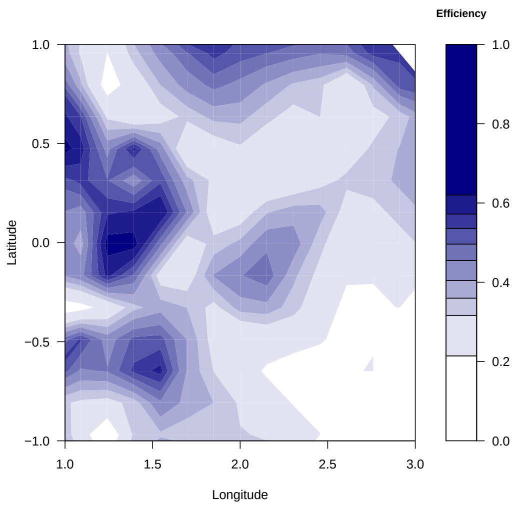
(a) SFA



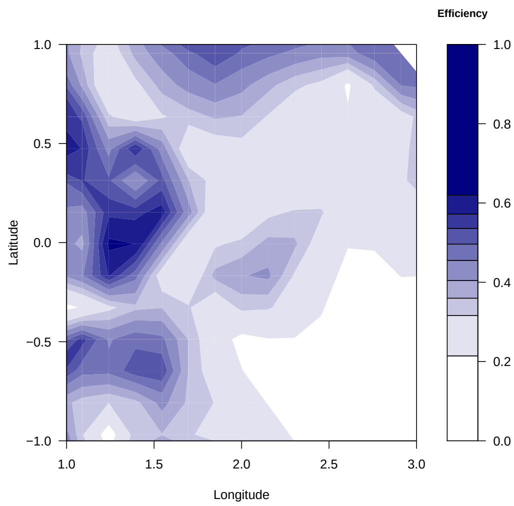
(b) 6 Neighbors



(c) 20 Neighbors



(d) 30 Neighbors



(e) 100 Neighbors

Figure 5.9: W sensitivity analysis ($rho = 0.8$)

5.4 Strengths, drawbacks and possible enhancements

In conclusion, SSFA is an innovative method, suitable to estimate both production and cost frontiers, that presents a lot of advantages with respect to SFA and SEM models and could be defined as a more general method that allows to:

- take into account the global *spatial dependence* of the model that SFA neglects with clear improvements on statistical inference;
- take into account the presence of production (cost) *inefficiency* in the model that SEM neglects leaving in the error term a systematic deterministic component with subsequents bias estimated β parameters

Moreover, SSFA, even if the own algorithm contains a SEM estimation to find initial values of parameters and has to do matrix operations of order $n \times n$, does not require high times of convergence with respect to SFA. So, it is better, in my opinion, to use directly the SSFA for estimating efficiency: in case of absence of spatial autocorrelation SFA is obtained anyway but in case of any degree of spatial autocorrelation (even if very small) in the residuals the firm specific efficiency estimated is more correct.
The major critical points of SSFA method derived from those of the SFA (described in section 3.1 at page 24) like the *wrong skewness* issue that generates convergence problems also for the SSFA and that will be studied soon.
From a technical point of view, only the "Half-normal" distributional case of the inefficiency term has been improved both in the SSFA formulation and in the R package and so "exponential", "truncated normal" or others distributions will be implemented soon.

Moreover, another possible improvement of the convergence speed could be the use of other types of algorithms (Nelder and Mead (1965) is robust but relatively slow) like *e.g.* the Expectation-maximization (EM) algorithm for the MLE estimation for its simplicity, ease of implementation and its memory requirements tend to be modest compared to other methods even in very large sample size problems. Also, the EM algorithm is numerically very stable.

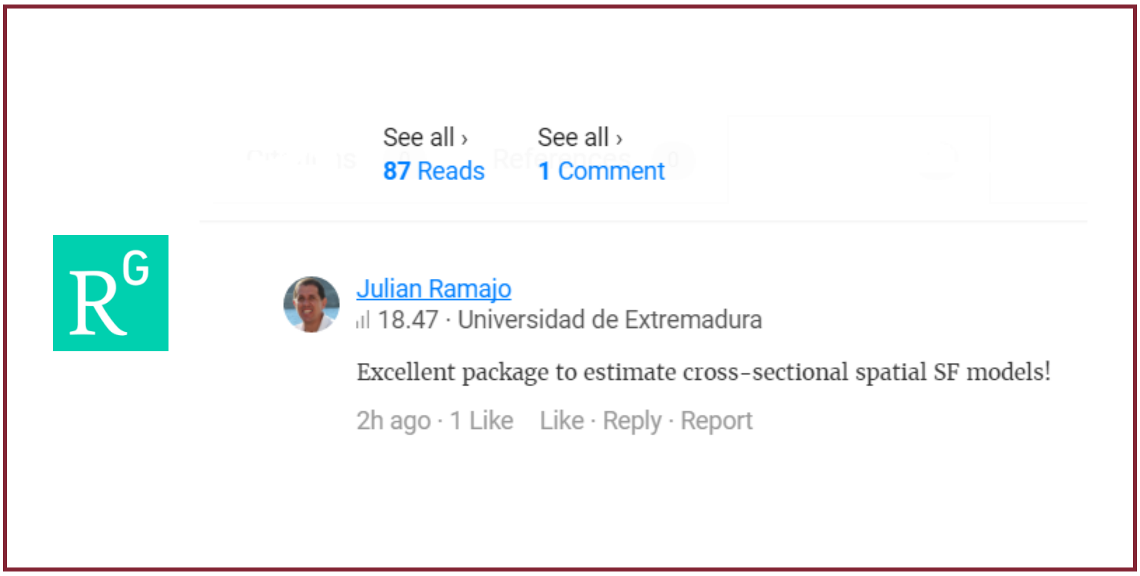
Finally, possible extensions of the SSFA method could also include the use of panel data to obtain more robust measures and to analyze the persistence over time of phenomena that occur in specific spatial clusters.

Chapter 6

SSFA R PACKAGE

6.1 Introduction

To implement the SSFA model proposed in equation (5.3) the code written in R by Straub and Torsten (2011) has been suitably modified and a new package, named **ssfa**, that allows also to make comparison with respect the SFA model has been created.



Below are reported the "*Reference manual*" and the vignette entitled"*Spatial Stochastic frontier models: Instructions for use*" published on CRAN.

6.2 Reference manual

Package 'ssfa'

June 10, 2015

Type Package

Title Spatial Stochastic Frontier Analysis

Version 1.1

Date 2015-06-09

Author Elisa Fusco, Francesco Vidoli

Maintainer Elisa Fusco <fusco_elisa@libero.it>

Description Spatial Stochastic Frontier Analysis (SSFA) is an original method for controlling the spatial heterogeneity in Stochastic Frontier Analysis (SFA) models, for cross-sectional data, by splitting the inefficiency term into three terms: the first one related to spatial peculiarities of the territory in which each single unit operates, the second one related to the specific production features and the third one representing the error term.

Depends Matrix, maxLik, spdep, sp

License GPL-3

Suggests R.rsp

VignetteBuilder R.rsp

NeedsCompilation no

Repository CRAN

Date/Publication 2015-06-10 01:02:51

R topics documented:

`ssfa-package` *Spatial Stochastic Frontier models*

Description

The package implements the Spatial Stochastic Frontier model for cross-sectional data introduced by Fusco and Vidoli (2013). The method controls spatial heterogeneity in SFA models by splitting the inefficiency term into three parts: the first one related to spatial peculiarities of the territory in which each single unit operates, the second one related to the specific production features and the third one representing the error term.

Details

Package:	ssfa
Type:	Package
Version:	1.1
Date:	2015-06-09
License:	GPL-3

Author(s)

Elisa Fusco, Francesco Vidoli

Maintainer: Elisa Fusco <fusco_elisa@libero.it>

References

Fusco, E. and Vidoli, F. (2013). *Spatial stochastic frontier models: controlling spatial global and local heterogeneity*, International Review of Applied Economics, 27(5) 679-694.

`eff.ssfa` *SSFA efficiency*

Description

This function returns the technical efficiency of each producer (without local spatial effects) calculated by the Battese and Coelli (1988) formulation modified by using an autoregressive specification in the inefficiency term u.

Usage

```
eff.ssfa(object, ...)
```

Arguments

`object`	an object of class `ssfa`.
`...`	further arguments for methods.

Value

Technical efficiency of each producer (without local spatial effects).

References

Battese, G. E., and T. J. Coelli (1988). *Prediction of Firm-level Technical Efficiencies with a Generalized Frontier Production Function and Panel Data.* Journal of Econometrics 38(3): 387-399.

Fusco, E. and Vidoli, F. (2013). *Spatial stochastic frontier models: controlling spatial global and local heterogeneity*, International Review of Applied Economics, 27(5) 679-694.

Kumbhakar, S. C., and C. A. K. Lovell (2000). *Stochastic Frontier Analysis*, Cambridge University Press.

See Also

`u.ssfa`

Examples

```
library(ssfa)
data(SSFA_example_data)
data(Italian_W)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data, data_w=Italian_W,
             form = "production", par_rho=TRUE)
eff <- eff.ssfa(ssfa)
```

`fitted.ssfa` *SSFA fitted values*

Description

This function returns the fitted values of the original data used to estimate the SSFA model.

Usage

```
## S3 method for class  ssfa
fitted(object, ...)
```

Arguments

`object`	an object of class `ssfa`.
`...`	further arguments for methods.

Examples

```
library(ssfa)
data(SSFA_example_data)
data(Italian_W)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data, data_w=Italian_W,
             form = "production", par_rho=TRUE)
fitted.ssfa(ssfa)
```

`Italian_W` *Italian provinces spatial weights matrix example*

Description

This is an example dataset that contains the 107 Italian provinces contiguity matrix (year 2008).

Usage

```
data(Italian_W)
```

Format

A data frame with 107 x 107 row-standardized distances between observations (Italian provinces).

References

http://www.istat.it/it/archivio/104317#confini.

Examples

```
data(Italian_W)
```

`L_hNV`	*SFA half-normal log likelihood function*

Description

This function is used to estimate the parameters of the classical SFA model where half-normal distribution of inefficiency term is assumed.

Usage

```
L_hNV(p, y = y, X = X, sc = sc)
```

Arguments

`p`	a vector with the parameters to be estimated.
`y`	the dependent variable.
`X`	the model matrix.
`sc`	specifies the form of the frontier model (-1 = cost, 1 = production).

Value

Value of the SFA log likelihood function.

`L_hNV_rho`	*SSFA half-normal log likelihood function*

Description

This function is used to estimate the parameters of the SSFA model where half-normal distribution of inefficiency term is assumed.

Usage

```
L_hNV_rho(p, y = y, X = X, sc = sc, w = w, sigmau2_sar = sigmau2_sar)
```

Arguments

`p`	a vector with the parameters to be estimated.
`y`	the dependent variable.
`X`	the model matrix.
`sc`	specifies the form of the frontier model (-1 = cost, 1 = production).
`w`	the spatial weight matrix.
`sigmau2_sar`	is the variance of the spatial correlated part of the inefficiency term estimated into `ssfa.fit` function.

Value

Value of the SSFA log likelihood function.

Note

Please note that `sigmau2_sar` is not a free parameter because it is estimated into the `ssfa.fit` function.

See Also

`ssfa`

`plot_fitted` *SSFA plot*

Description

This function allows to plot the data and the fitted values obtained by SSFA model.

Usage

```
plot_fitted(x, y, object, xlab, ylab, main, ...)
```

Arguments

`x`	the x coordinates of points in the plot.
`y`	the y coordinates of points in the plot.
`object`	an object of class `ssfa`.
`xlab`	a title for the x axis.
`ylab`	a title for the y axis.
`main`	an overall title for the plot.
`...`	arguments to be passed to methods, such as graphical parameters (see par).

See Also

plot

Examples

```
library(ssfa)
data(SSFA_example_data)
data(Italian_W)

#### SFA and SSFA comparison
sfa <- ssfa(log_y ~ log_x, data = SSFA_example_data, data_w=Italian_W,
             form = "production", par_rho=FALSE)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data, data_w=Italian_W,
 form = "production", par_rho=TRUE)

sfa_fitted <- fitted.ssfa(sfa)
plot_fitted(SSFA_example_data$log_x, SSFA_example_data$log_y, ssfa)
lines(sort(SSFA_example_data$log_x), sfa_fitted[order(SSFA_example_data$log_x)],col="red")
```

`plot_moran` *SSFA residuals Moran plot*

Description

This function allows to plot the residuals of the object against their spatially lagged values, augmented by reporting the summary of influence measures for the linear relationship between the data and the lag.

Usage

```
plot_moran(x, main, xlab, ylab, labels, listw, ...)
```

Arguments

`x`	an object of class `ssfa`.
`main`	an overall title for the plot.
`xlab`	a label for the x axis.
`ylab`	a label for the y axis.
`labels`	character labels for points with high influence measures, if set to FALSE, no labels are plotted for points with large influence.
`listw`	a listw object from `nb2listw` (see nb2listw).
`...`	arguments to be passed to methods, such as graphical parameters (see par).

References

Anselin, L. (1995). *Local indicators of spatial association, Geographical Analysis*, 27, 93-115.

Anselin, L. (1996). *The Moran scatterplot as an ESDA tool to assess local instability in spatial association*. pp. 111-125 in M. M. Fischer, H. J. Scholten and D. Unwin (eds) Spatial analytical perspectives on GIS, London, Taylor and Francis.

See Also

moran.plot

Examples

```
library(ssfa)
data(SSFA_example_data)
data(Italian_W)

#### SFA and SSFA comparison ###
sfa <- ssfa(log_y ~ log_x, data = SSFA_example_data, data_w=Italian_W,
             form = "production", par_rho=FALSE)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data, data_w=Italian_W,
             form = "production", par_rho=TRUE)

moran.test(residuals.ssfa(sfa), sfa$list_w)
moran.test(residuals.ssfa(ssfa), ssfa$list_w)

plot_moran(sfa, listw=sfa$list_w)
plot_moran(ssfa, listw=ssfa$list_w)
```

residuals.ssfa *SSFA residuals*

Description

This function returns the residuals of the fitted SSFA model.

Usage

```
## S3 method for class  ssfa
residuals(object, ...)
```

Arguments

object	an object of class ssfa.
...	further arguments for methods.

Examples

```
library(ssfa)
data(SSFA_example_data)
data(Italian_W)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data,
             data_w=Italian_W, form = "production", par_rho=TRUE)
residuals.ssfa(ssfa)
```

`ssfa` *Spatial stochastic frontier estimation*

Description

This function estimates the Spatial Stochastic Frontier model introduced by Fusco and Vidoli (2013) in the following form:

$$log(y_i) = log(f(x_i; \beta_i)) + v_i - u_i$$

$$u_i = \rho \sum_i w_{i.} u_i + \widetilde{u}_i$$

where y_i are the outputs, x_i the inputs, v_i the stochastic noise, u_i the inefficiency term, rho the spatial lag, $w_{i.}$ a standardized row of the spatial weights matrix and $\widetilde{u}_i$ the stochastic noise of the inefficiency term.

Usage

```
ssfa(formula, data = NULL, data_w = NULL, intercept = TRUE, pars = NULL, par_rho = TRUE,
                 form = "cost")
```

Arguments

`formula`	an object of class `formula` (or one that can be coerced to that class): a symbolic description of the model to be fitted.
`data`	an optional data frame containing the variables in the model.
`data_w`	a data frame containing the spatial weight matrix.
`intercept`	logical. If true the model includes intercept.
`pars`	initial values for the parameters to be estimated.
`par_rho`	logical. If true the function estimates the Spatial Stochastic Frontier (SSFA) otherwise the classical Stochastic Frontier (SFA).
`form`	specifies the form of the frontier model as "cost" or "production".

Value

`ssfa` returns the following objects of class `ssfa`:

`y`	the dependent variable.
`x`	the covariates.
`X`	the model matrix.
`coef`	the estimated coefficients.
`sc`	the form of the frontier model estimated (-1 = cost, 1 = production).
`hess`	a symmetric matrix giving an estimate of the Hessian at the solution found.
`logLik`	the value of the log likelihood function.

ols	the linear model for the LR-test.
sigmau2	the estimation of sigmau2 (only if par_rho=FALSE): value of inefficiency variance.
sigmau2_dmu	the estimation of sigmau2_dmu (only if par_rho=TRUE): value of the part of the inefficiency variance due to DMU's specificities.
sigmau2_sar	the estimation of sigmau2_sar: value of the part of the inefficiency variance due to the spatial correlation.
sigmav2	the estimation of sigmav2: value of the stochastic error variance.
sigma2	the estimation of sigma2: value of the total variance.
rho	the estimation of the spatial lag parameter rho.
fun	the distribution of the inefficiency term u.
list_w	a listw object from `nb2listw` (See nb2listw).

Note

NOTE 1: In this version the distribution of the inefficiency term u is only "half-normal".

NOTE 2: The method used to maximize the log likelihood function is the Newton-Raphson. Please see the R function `maxNR` of the `maxLik` package for details (Henningsen and Toomet (2011)).

NOTE 3: Please note that the classical SFA inefficiency variance `sigmau2`, in the SSFA, is decomposed into `sigmau2_dmu` and `sigmau2_sar`, respectively the part of inefficiency variance due to DMU's specificities and to the spatial dependence, *i.e.* `sigmau2 = sigmau2_dmu + sigmau2_sar` and consequently the total variance is given by `sigma2 = sigmau2_dmu + sigmau2_sar + sigmav2`.

Author(s)

Fusco E. and Vidoli F.

References

Battese, G. E., and T. J. Coelli (1995). *A Model for Technical Inefficiency Effects in a Stochastic Frontier Production Function for Panel Data.* Empirical Economics 20(2): 325-332.

Fusco, E. and Vidoli, F. (2013). *Spatial stochastic frontier models: controlling spatial global and local heterogeneity*, International Review of Applied Economics, 27(5) 679-694.

Kumbhakar, S. C., and C. A. K. Lovell (2000). *Stochastic Frontier Analysis*, Cambridge University Press.

Henningsen, A. and Toomet, O. (2011). *maxLik: A package for maximum likelihood estimation in R*. Computational Statistics 26(3), 443-458.

Examples

```
library(ssfa)
data(SSFA_example_data)
data(Italian_W)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data,
```

```
            data_w=Italian_W, form = "production", par_rho=TRUE)

### SSFA total variance decomposition
sigma2 = ssfa$sigmau2_dmu + ssfa$sigmau2_sar + ssfa$sigmav2
sigma2
ssfa$sigma2
```

`SSFA_example_data` *Example dataset*

Description

The dataset contains the simulated data used by Fusco and Vidoli (2013) to test the model. Data Generating Process (DGP) follows the construction criteria proposed by Banker and Natarajan (2008), also used by Johnson and Kuosmanen (2011), with the addition of a strong spatial correlation in the inefficiency term through a spatial lag parameter and a contiguity matrix (107 Italian provinces contiguity matrix, year 2008).

Usage

```
data(SSFA_example_data)
```

Format

A data frame with 107 observations (Italian provinces) and 2 variables:

`DMU` the Decision Making Unit name.

`log_x` the input vector (already in logarithmic form).

`log_y` the output vector (already in logarithmic form).

References

Banker, R., and R. Natarajan (2008). *Evaluating Contextual Variables Affecting Productivity using Data Envelopment Analysis*. Operations Research 56 (1): 48-58.

Johnson, A., and T. Kuosmanen (2011). *One-stage Estimation of the Effects of Operational Conditions and Practices on Productive Performance: Asymptotically Normal and Efficient, Root-n Consistent StoNEZD Method*. Journal of Productivity Analysis 36:219-230.

Examples

```
data(SSFA_example_data)
```

`summary` *SSFA summaries*

Description

The function `print.ssfa` is used to display the values of SFA and SSFA estimated coefficients. In particular:

- for SFA the function displays the `Intercept`, the regressors beta coefficients, the inefficiency variance `sigmau2`, the stochastic error variance `sigmav2` and the total variance `sigma2`;
- for SSFA the function displays, in addition, the decomposition of the inefficiency variance into `sigmau2_dmu` and `sigmau2_sar`, respectively the part of inefficiency variance due to DMU's specificities and to the spatial dependence, and finally, the spatial lag parameter `rho`.

The function `summary.ssfa` is used to display the summary results of SFA and SSFA. In particular:

- for SFA the summary shows the estimation of SFA coefficients (`Intercept`, beta coefficients, `sigmau2` and `sigmav2`) and others useful information as the total variance `sigma2`, the inefficiency parameter `Lambda (sigmau/sigmav)`, the Moran `I` statistic, the mean of efficiency, the LR-test and the AIC values;
- for SSFA the summary shows, in addition, the decomposition of the inefficiency variance into `sigmau2_dmu` and `sigmau2_sar` and the spatial lag parameter `rho`.

Usage

```
## S3 method for class  ssfa
print(x, ...)
## S3 method for class  ssfa
summary(object, ...)
```

Arguments

`x`	an object of class `ssfa`.
`object`	an object of class `ssfa`.
`...`	further arguments for methods.

Note

Please note that the classical SFA inefficiency variance `sigmau2`, in the SSFA, is decomposed into `sigmau2_dmu` and `sigmau2_sar`, respectively the part of inefficiency variance due to DMU's specificities and to the spatial dependence, *i.e.* `sigmau2 = sigmau2_dmu + sigmau2_sar` and consequently the total variance is given by `sigma2 = sigmau2_dmu + sigmau2_sar + sigmav2`.

References

Anselin, L. (1995). *Local indicators of spatial association, Geographical Analysis*, 27, 93-115.

Fusco, E. and Vidoli, F. (2013). *Spatial stochastic frontier models: controlling spatial global and local heterogeneity*, International Review of Applied Economics, 27(5) 679-694.

Kumbhakar, S. C., and C. A. K. Lovell (2000). *Stochastic Frontier Analysis*, Cambridge University Press.

Examples

```
library(ssfa)
data(SSFA_example_data)
data(Italian_W)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data,
             data_w=Italian_W, form = "production", par_rho=TRUE)

print(ssfa)
summary(ssfa)
```

`u.ssfa` *SSFA inefficiency*

Description

This function returns the specific inefficiency of each producer (without local spatial effects) calculated by the Jondrow et al. (JLMS) (1982) formulation modified by using an autoregressive specification in the inefficiency term.

Usage

```
u.ssfa(object, ...)
```

Arguments

`object`	an object of class `ssfa`.
`...`	further arguments for methods.

Value

Inefficiency of each producer (without local spatial effects).

References

Fusco, E. and Vidoli, F. (2013) *Spatial stochastic frontier models: controlling spatial global and local heterogeneity* , International Review of Applied Economics, 27(5) 679-694. Kumbhakar, S. C., and C. A. K. Lovell. (2000) *Stochastic Frontier Analysis*, Cambridge University Press.

Jondrow, J., C. A. Knox Lovell, I. S. Materov, and P. Schmidt. (1982). *On the Estimation of Technical Inefficiency in the Stochastic Frontier Production Function Model.* Journal of Econometrics 19 (2-3): 233-238.

See Also

`eff.ssfa`

Examples

```
library(ssfa)
data(SSFA_example_data)
data(Italian_W)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data,
            data_w=Italian_W, form = "production", par_rho=TRUE)
ineff <- u.ssfa(ssfa)
```

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6.3 Vignette

Spatial Stochastic frontier models: Instructions for use

Elisa Fusco & Francesco Vidoli

June 9, 2015

In the last decade stochastic frontiers traditional models (see Kumbhakar and Lovell, 2000 for a detailed introduction to frontier analysis) have been extended with the aim to take into account firm specific heterogeneity (see *e.g.* Greene, 2004, Greene, 2005b, Greene, 2005a). If firm specific heterogeneity is not accounted, in fact, a considerable bias in the inefficiency estimates can be endogenously created.

`ssfa` package allows to include heterogeneity in a different way with respect to traditional techniques: *"instead of identifying ex-ante a multitude of determinants, often statistically and economically difficult to detect [...] this approach allow the evaluation of the conjoint effect of a multitude of determinants"* (Fusco and Vidoli, 2013) considering spatial proximities; more particularly `ssfa` package implements the Spatial Stochastic Frontier Analysis (SSFA), an original method introduced by Fusco and Vidoli (2013) with the aim to test and depurate the spatial heterogeneity in Stochastic Frontier Analysis (SFA) models by splitting the inefficiency term into three terms: the first one related to spatial peculiarities of the territory in which each single unit operates, the second one related to the specific production features and the third one representing the error term.

The main idea is that spatial dependence refers to how much the level of technical inefficiency of farm i depends on the levels set by other farms $j = 1, ..., n$, under the assumption that part of the farm i inefficiency (u_i) is linked to the neighbour DMU j's performances ($j \neq i$).
Denoting y_i as the single output of producer i, x_i the inputs vector and f a generic parametric function, the Normal / Half-Normal cross-sectional production frontier model can be respectively written[1]:

$$\begin{aligned} log(y_i) &= log(f(x_i;\beta_i)) + v_i - u_i \\ &= log(f(x_i;\beta_i)) + v_i - (1-\rho\sum_i w_{i.})^{-1}\widetilde{u_i} \end{aligned}$$

where

$$\begin{aligned} & v_i \sim \mathcal{N}(0,\sigma_v^2) \\ & u_i \sim \mathcal{N}^+(0,(1-\rho\sum_i w_{i.})^{-2}\sigma_{\widetilde{u}}^2) \qquad (1) \\ & u_i \text{ and } v_i \text{ are independently distributed of each other,} \\ & \quad \text{and of the regressors} \\ & \widetilde{u_i} \sim \mathcal{N}(0,\sigma_{\widetilde{u}}^2) \\ & w_{i.} \text{ is a standardized row of the spatial weights matrix} \\ & \rho \text{ is the spatial lag parameter } (\rho \in [0,1]) \end{aligned}$$

[1]For simplicity's sake and to make the notation more consistent with the SFA literature, we did not write the model in matrix form, but for each company i.

ssfa package allows to estimate both the "*production*" form (as shown in equation (1) and the "*cost*" form of the frontier *i.e.*:

$$log(C_i) = log(f(y_i, w_i; \beta_i)) + v_i + u_i$$

where (2)

C_i is the cost
w_i are the input prices.

Introducing a variable *sc* that defines the form of the frontier:

$$\begin{cases} 1 \text{ for production function} \\ -1 \text{ for cost function} \end{cases} \quad (3)$$

ssfa model can be written as:

$$log(y_i) = log(f(x_i; \beta_i)) + v_i - sc \cdot u_i \quad (4)$$

In order to estimate the ssfa model we have to install and load the package:

```
> #install.packages("ssfa")
> library(ssfa)
```

In this package, the SSFA_example_data and Italian_W datasets have been included in order to better illustrate and comment the model.

- The first dataset contains the simulated data used by Fusco and Vidoli (2013) to test the model. Data Generating Process (DGP) follows the construction criteria proposed by Banker and Natarajan (2008), also used by Johnson and Kuosmanen (2011), with the addition of a strong spatial correlation ($\rho = 0.80$) in the inefficiency term through a spatial lag parameter and the contiguity matrix Italian_W.
- The second dataset is the Italian provinces contiguity matrix for the year 2008 containing 107 x 107 row-standardized distances.

```
> data(SSFA_example_data)
> data(Italian_W)
> names(SSFA_example_data)

[1] "DMU"   "log_y" "log_x"
```

The variable log_y is the log-transformed output, log_x is the log-transformed input and DMU is the Decision Making Unit name.

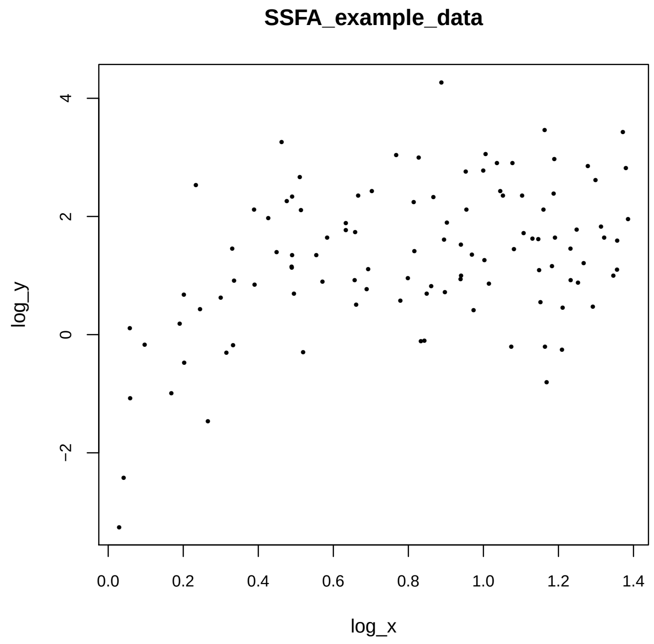


Figure 1: Example simulated data

`ssfa` package allows to easily compare the Spatial Stochastic Frontier (SSFA) with the classical Stochastic Frontier (SFA) by setting the parameter `par_rho` as `TRUE` to estimate the SSFA or `FALSE` to estimate the classical SFA.

In order to compare the SSFA estimation versus the SFA one, a standard SFA production frontier has been first estimate by setting, into the `ssfa` function, command `form="production"` and `par_rho="FALSE"`:

```
> sfa <- ssfa(log_y ~ log_x , data = SSFA_example_data, data_w=Italian_W,
+              form = "production", par_rho=FALSE)
> summary(sfa)


Stochastic frontier analysis model

          Estimate Std. Error z value Pr(>|z|)
Intercept 1.185847   0.450144 2.63437 0.008429 **
log_x     1.273394   0.302062 4.21567  2.5e-05 ***
sigmau2   1.319260   0.915815 1.44053 0.149717
sigmav2   0.779320   0.311867 2.49889 0.012458 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

sigma2 = 2.098581

Inefficiency parameter Lambda (sigmau/sigmav): 1.30109

Moran I statistic: 0.457094
```

```
Mean efficiency: 0.485295

LR-test: sigmau2 = 0 (inefficiency has no influence to the model)
H0: sigmau2 = 0 (beta_ssfa = beta_ols)

     Value Log-Lik
ssfa     -163.6215
ols      -164.1653

Value LR-Test: 1.088 p-value 0.148

AIC: 335.2431, (AIC for lm: 332.3306)
```

In the standard SFA framework (`par_rho="FALSE"`), `ssfa` function returns, in addition to the *intercept* and the `log_x` coefficient, the estimation of the variance of the two error components `sigmau2` and `sigmav2`. Other useful information about efficiency estimation are reported:

- `sigma2`: the estimate of the *total variance* where $\sigma^2 = \sigma_u^2 + \sigma_v^2$;
- `lambda`: the ratio of the standard deviation of the inefficiency term to the standard deviation of the stochastic term *i.e.* $\frac{\sigma_u}{\sigma_v}$;
- the mean of efficiency estimated;
- the results of the test on the influence of the inefficiency on the model. This is a test of the null hypothesis $H_0 : \sigma_u^2 = 0$ against the alternative hypotheses $H_1 : \sigma_u^2 > 0$. If the null hypothesis is true, the stochastic frontier model is reduced to an OLS model with normal errors. For this example, the output shows $LR = 1.088$ with a p-value of 0.148. There are several possible reasons for the failure to this test, including for example the uncontrolled spatial dependence of the inefficiency term.

In addition to previous statistics, `summary` function displays information about the spatial autocorrelation of the SFA residuals, the Moran's I statistic. For example, in this application $I = 0.457$ showing a positive and significant ($p - value < 2.2e - 16$) global autocorrelation among residuals.

```
> moran.test(residuals(sfa), listw=sfa$list_w)


        Moran's I test under randomisation

data:  residuals(sfa)
weights: sfa$list_w

Moran I statistic standard deviate = 8.3329, p-value < 2.2e-16
alternative hypothesis: greater
sample estimates:
Moran I statistic       Expectation          Variance
      0.457093893      -0.009433962       0.003134475
```

Autocorrelation among residuals can be tested also locally thanks to `plot_moran` function that enables you to assess how similar an observed value is to its neighbouring observations; its horizontal axis is based on the values of the observations and is also known as the response axis, while the vertical Y axis is based on the weighted average or spatial lag of the corresponding observation on the horizontal X axis. This function need a neighbours list: it can be easily calculate thanks to the `nb2listw` function of `spdep` package from the contiguity matrix `Italian_W`.

```
>  plot_moran(sfa, listw=sfa$list_w)
```

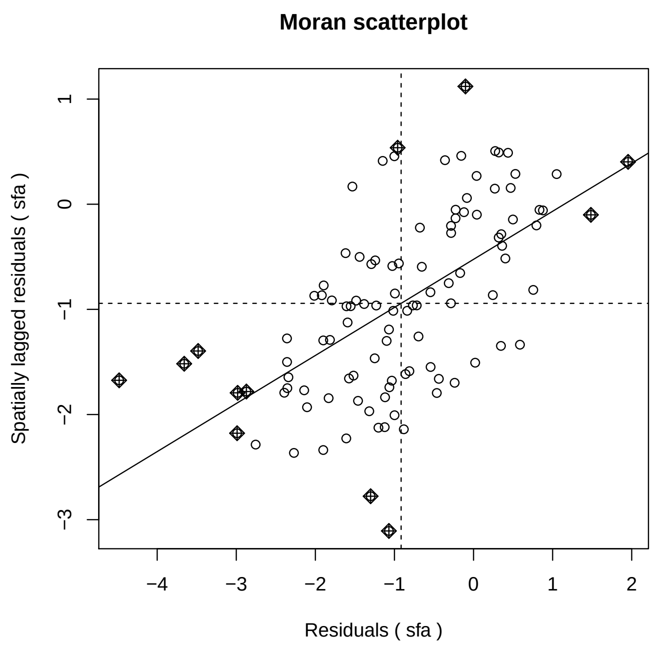


Figure 2: SFA Moran scatterplot

Finally, `summary` function reports the AIC value for the `ssfa` model and the `lm` model.

Having estimated the SFA model as baseline, the spatial production frontier SSFA can be carried on by setting command `form="production"` and `par_rho="TRUE"`:

```
> ssfa <- ssfa(log_y ~ log_x , data = SSFA_example_data, data_w=Italian_W,
+              form = "production", par_rho=TRUE)
> summary(ssfa)


Spatial Stochastic frontier analysis model

            Estimate Std. Error z value Pr(>|z|)
Intercept   3.445042   2.153384 1.59983 0.109637
log_x       1.633247   0.226971 7.19585  < 2e-16 ***
sigmau2_dmu 0.596074   0.650270 0.91666 0.359322
sigmav2     0.474248   0.214914 2.20668 0.027336 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Pay attention:
1 - classical SFA sigmau2 = sigmau2_dmu + sigmau2_sar: 0.882803 where sigmau2_sar: 0.286729
2 - sigma2 = sigmau2_dmu + sigmau2_sar + sigmav2: 1.357051

Inefficiency parameter Lambda = sigmau_dmu/sigmav: 1.256883
```

```
Spatial parameter Rho: 0.778393

Moran I statistic: -0.189043

Mean efficiency: 0.571884

LR-test: sigmau2_dmu = 0 (inefficiency has no influence to the model)
H0: sigmau2_dmu = 0 (beta_ssfa = beta_ols)

     Value Log-Lik
ssfa     -138.9479
ols      -164.1653

Value LR-Test: 50.435 p-value 0

AIC: 297.8958, (AIC for lm: 332.3306)
```

The output of `ssfa` (with `par_rho="FALSE"`) returns the *intercept*, the `log_x` coefficient and the estimation of the variance of the two error components not spatially correlated *i.e.* `sigmau2_dmu` and `sigmav2`.

In this case, the model decomposes the inefficiency variance `sigmau2` into `sigmau2_dmu` and `sigmau2_sar`, respectively the part of inefficiency variance due to DMU's specificities and to the spatial dependence, *i.e.* $\sigma_u^2 = \sigma_{u_{dmu}}^2 + \sigma_{u_{sar}}^2$. Consequently, the *total variance* is given by $\sigma^2 = \sigma_{u_{dmu}}^2 + \sigma_{u_{sar}}^2 + \sigma_v^2$.

In this application, (`lambda` $= 1.257$) is smaller than the SFA one (`lambda` $= 1.301$) because the production unit inefficiency is sterilized from the influence of the neighbourhood performances.

In addition, the `summary` function reports the estimated spatial parameter ρ that in this case is 0.778 very close to the true simulation parameter (0.80); Moran's $I = -0.189$ is no more significant ($p-value = 0.9993$).

```
> moran.test(residuals(ssfa), listw=ssfa$list_w)


        Moran's I test under randomisation

data:  residuals(ssfa)
weights: ssfa$list_w

Moran I statistic standard deviate = -3.2046, p-value = 0.9993
alternative hypothesis: greater
sample estimates:
Moran I statistic       Expectation          Variance
     -0.189042939      -0.009433962       0.003141349


> plot_moran(ssfa, listw=sfa$list_w)
```

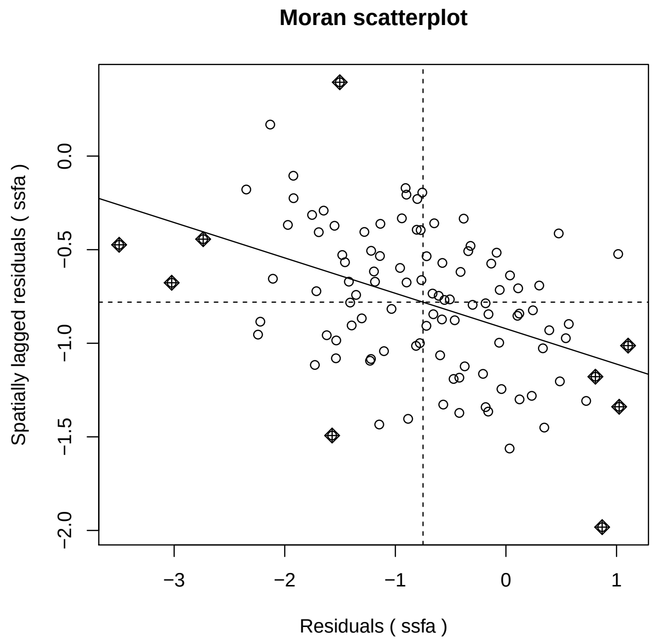


Figure 3: SSFA Moran scatterplot

In this application it can be easily note that the likelihood-ratio test is highly significant ($LR = 50.435$ with a p-value $= 0.000$); these findings, support the conclusion that the SSFA model is able to correctly estimate the inefficiency component of the error term.

Other functions are available into `ssfa` package:

- `fitted.ssfa`: this function calculates the fitted values of the original data used to estimate the SSFA model.

```
> ssfa_fitted <- fitted.ssfa(ssfa)
> sfa_fitted <- fitted.ssfa(sfa)
```

- `plot_fitted`: plots the original data, the SSFA fitted frontier and optionally the SFA fitted frontier with the aim to compare models colouring points according to the efficiency values.

```
> plot_fitted(SSFA_example_data$log_x, SSFA_example_data$log_y, ssfa, pch=16, cex=0.5,
+               xlab="X", ylab="Y", cex.axis=0.8 )
> points(SSFA_example_data$log_x, SSFA_example_data$log_y, pch=16, cex=0.5,
+   col= ifelse(eff.ssfa(ssfa)<=quantile(eff.ssfa(ssfa), 0.20) , "#D7191C",
+         ifelse(eff.ssfa(ssfa)>quantile(eff.ssfa(ssfa), 0.20)
+           &eff.ssfa(ssfa)<=quantile(eff.ssfa(ssfa), 0.4) ,"#FF8C00",
+         ifelse(eff.ssfa(ssfa)>quantile(eff.ssfa(ssfa), 0.4)
+           &eff.ssfa(ssfa)<=quantile(eff.ssfa(ssfa), 0.6) ,"#FFFF00",
+         ifelse(eff.ssfa(ssfa)>quantile(eff.ssfa(ssfa), 0.6)
+           &eff.ssfa(ssfa)<quantile(eff.ssfa(ssfa), 0.8) ,"#ADFF2F",
+         ifelse(eff.ssfa(ssfa)>quantile(eff.ssfa(ssfa), 0.8)
+           &eff.ssfa(ssfa)<=quantile(eff.ssfa(ssfa), 1),"#008B00", "#2F4F4F"))))))
> lines(sort(SSFA_example_data$log_x),sfa_fitted[order(SSFA_example_data$log_x)],
+             col="red")
```

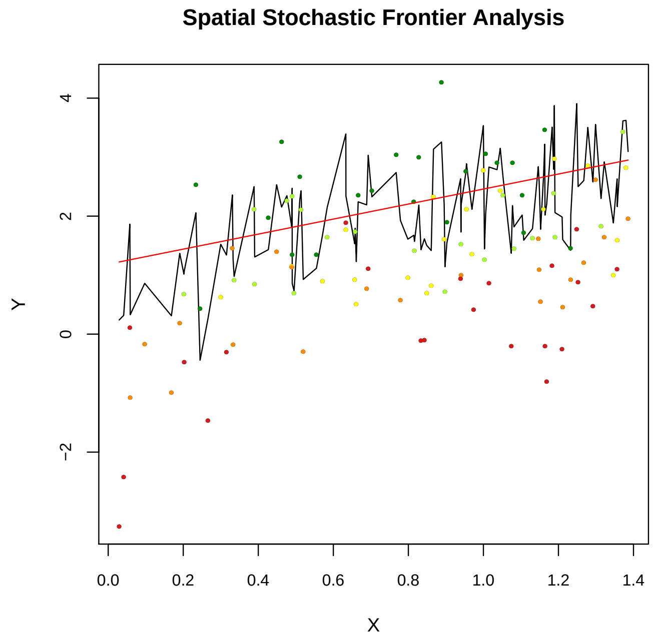
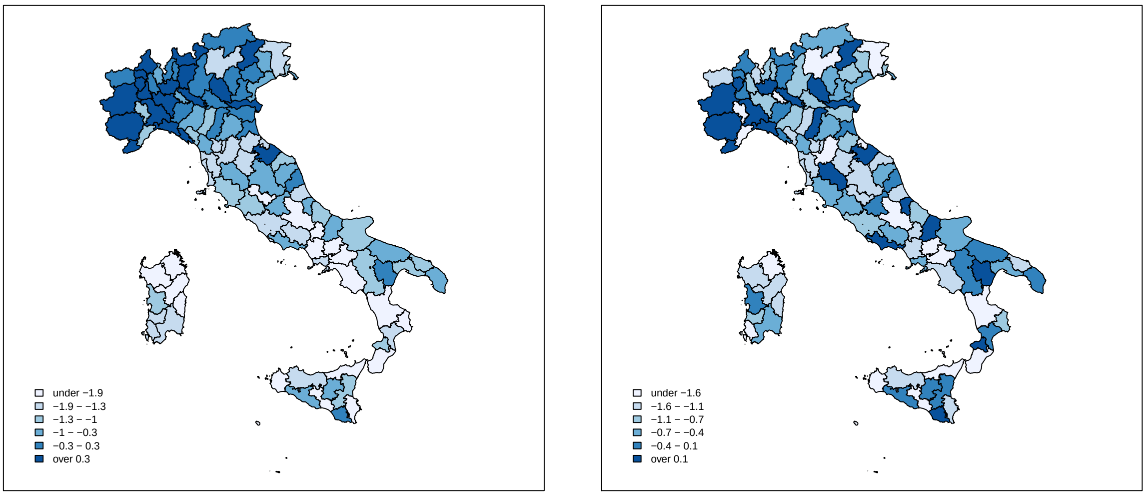


Figure 4: Plot data, SSFA and SSFA frontiers

- `residuals.ssfa`: calculates the SSFA model residuals.

  > ssfa_residuals <- residuals.ssfa(ssfa)
  > sfa_residuals <- residuals.ssfa(sfa)

  With residuals estimation we can compare SFA and SSFA results, for example, with maps like the following:



(a) SFA (b) SSFA

Figure 5: Spatial residuals distribution by method

Figure 5 shows that the spatial dependence present in SFA residuals (a) is fully neutralized by the SSFA model (b).

- `eff.ssfa`: calculates the efficiency (Battese and Coelli (1988) formulation) and inefficiency (Jondrow et al. (1982) formulation) estimated.

```
> ssfa_eff <-  eff.ssfa(ssfa)
> #sfa_eff <-  eff.ssfa(sfa)
>
> #summary(sfa_eff)
> #summary(ssfa_eff)
>
> ssfa_u <- u.ssfa(ssfa)
> #sfa_u <- u.ssfa(sfa)
>
> #summary(ssfa_u)
> #summary(sfa_u)
```

References

Banker, R. and Natarajan, R. (2008). Evaluating contextual variables affecting productivity using data envelopment analysis. *Operations research*, 56(1):48–58.

Battese, G. E. and Coelli, T. J. (1988). Prediction of firm-level technical efficiencies with a generalized frontier production function and panel data. *Journal of Econometrics*, 38(3):387–399.

Fusco, E. and Vidoli, F. (2013). Spatial stochastic frontier models: controlling spatial global and local heterogeneity. *International Review of Applied Economics*, 27(5):679–694.

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Part III

Frontier models for aggregating problems

Chapter 7

COMPOSITE INDICATORS

7.1 Introduction

In this part, the analysis will be focused on another way to use frontier models, that is a particular approach for constructing Composite Indicators (CI). Firstly, a definition of CI is given and an overview of the main issues of the aggregation process and on the previous proposed methods is discussed, with the aim to allow the comprehensibility of the innovative subsequents chapters.

"Composite indicator is formed when individual indicators are compiled into a single index, on the basis of an underlying model of the multi-dimensional concept that is being measured"

(OECD, 2005)

Therefore, constructing a composite indicator consists in the aggregation of an appropriate number of simple indicators representing different multidimensional aspects of the same issue.

Interest in CIs, as a useful tool to support decision-makers in policy analysis and policy communication (Nardo et al., 2005), is rapidly growing thanks to their capability to summarise multi-dimensional issues, to rank countries in benchmarking analysis and to their ease of interpretation.

On the other hand, the construction of a CI is a very complex and delicate process because if the CI is poorly constructed or misinterpreted it may send misleading and non-robust policy messages leading to simplistic policy conclusions. For these reasons Freudenberg (2003) has proposed a series of subsequent steps for obtaining a good CI, among which:

1. the systematisation of a theoretical framework for the identification of relevant analysis dimensions;

2. the standardisation of the simple indicators with the aim of transforming them into pure, dimensionless numbers and to invert possible opposite polarities/signs (*e.g.* air pollution in OECD Better Life Index) in order to allow comparisons;

3. the imputation of missing data;

4. the weighting of simple indicators;

5. the succeeding sensitivity analysis on the robustness of the aggregation.

Moreover, the construction of CIs involves some stages where judgments have to be made: the selection of sub-indicators, the choice of the model, the choice of the weights for aggregating indicators, the treatment of missing values and so on.

In particular, a critical step in this sense and focus of the following sections and chapters, is the weighting of simple indicators (Step 4), in fact, Joint Research Centre of European Commission asserts that *"no uniformly agreed methodology exists to weight individual indicators before aggregating them into a composite indicator"*[9]. In this framework, the two main issues to be considered are: *(i)* how to find weights, *i.e.* if in a subjective or objective manner (see Section 7.2); *(ii)* if a trade-off relation exists among simple indicators *i.e.* the possibility to compensate a disadvantage on some simple indicators, with a sufficiently large advantage on the others (see Chapter 8).

[9] http://composite-indicators.jrc.ec.europa.eu/S6_weighting.htm

7.2 Weighting issue

In order to provide an answer to the first problem, a large number of researchers identify weights subjectively in cooperation with experts who know the theoretical context well (please see *e.g.* ONS, 2002, WMRC, 2001), others on the contrary, use objective methods in order to avoid arbitrariness problems.

The first group of proposed methods, includes the *Budget Allocation Processes* (BAP - Jesinghaus in Moldan et al., 1997) based on a subjective allocation of a "budget" of one hundred points to a set of indicators; the *Analytic Hierarchy Processes* (AHP - Forman, 1983, Saaty, 1987) in which weights are the trade-offs across indicators; the *Conjoint Analysis* (CA - Green and Srinivasan, 1978, Hair, 1995, McDaniel and Gates, 1998) that studies the evaluations (preferences) given by the respondents on a set of alternative scenarios representing different values for the individual indicators.

The second group includes: the *Principal Component Analysis* (PCA - Manly, 1994) and the *Factor Analysis* (FA - Thurstone, 1931) that groups collinear simple indicators with the aim to capture the common information among them; however, weights cannot be estimated with these methods if weak correlation exists among indicators; the *Unobserved Components Model* (UCM - Kaufmann et al., 1999, Kaufmann et al., 2003) that assumes the dependence of the simple indicator on an unobserved variable plus an error term in order to identify the relationship between the composite and its components; the DEA models (see Subsection 3.2.2 for a mention on the methodology) and in particular the *Benefit of Doubt Approach* (BoD - Melyn et al., 1991) based on the identification of an efficiency frontier.

In general, in the CI framework, there are no functional relationships among single indicators, covering different aspects of a specific economic or social phenomenon, and so, nomic casuality cannot be assumed (Born, 1949), meaning that certain or probabilistic general functions covering relations among instances cannot be assumed. Moreover, in a nonparametric perspective, it is not even useful to introduce constraints, parametric functional forms or penalties which may be linked to a specific theoretical model because results would be clearly linked to the theoretical model that has generated them.

From a logical perspective, therefore, the claim of "*back to the details*" suggested by Nardo et al. (2005) appears as misleading, because the resulting composite indicator is clearly linked to the model that generated it.

This happens because, especially in this context, "*in the mathematical knowledge the consideration [the assumptions] is an operation that, for the objects, comes from* *outside; so it follows that the real object is altered*"[10] (Hegel, 1995). Knowledge, is considered as a bias with respect to the expected measure, and the measure itself influences the analysis, that is, there are not neutral actions without consequences: or the thought explains the evolution of the object (*adaequatio intellectus ad rem*), or the object itself is deformed by the thought (*adaequatio rei to intellectus*).

Given this premise, the choice among different weighting functions, models or evaluation frameworks can be done or in an axiomatic way or according to the required statistical properties. In the following section (7.3) the BoD approach is analyzed in detail since it determines the weights endogenously and consequently avoids the main critical remark on the subjectivity involved in the choice of the weights set.

7.3 Frontier approach: the Benefit of the Doubt

DEA techniques have been applied in the construction of many socio-economic CIs, like the evaluation of public policies: see *e.g.* Prieto and Zofio (2001) and Zafra-Gomez et al. (2010) for local government performance evaluation; Storrie and Bjurek (2000), for the European labor market analysis; Cherchye et al. (2004) for social inclusion policies at EU level and Takamura and Tone (2003) for government agencies evaluation. They have been used also in analysis of country performance through macroeconomic indicators: see *e.g.* Lovell (1995), Lovell et al. (1995), Cherchye (2001) and Cherchye et al. (2005) for internal market policies; for the construction of a environmental and ecological performance indicator: see *e.g.* Zhou et al. (2007), Bellenger and Herlihy (2009), Lo (2010), Sahoo et al. (2011), Rogge (2012) and Zanella et al. (2013) and finally for the calculation of the Human Development Index: see *e.g.* Mahlberg and Obersteiner (2001), Despotis (2005*a*), Despotis (2005*b*) and Cherchye et al. (2008).

In Section 1.1 the *production set* Ψ was introduced such as:

$$\Psi = \{(\mathbf{x}, \mathbf{y}) : \mathbf{x} \text{ } can \text{ } produce \text{ } \mathbf{y}\} \tag{7.1}$$

satisfying the usual assumptions as in Shephard (1970) and in Färe et al. (1985) described in detail in the same section.

As highlighted by Witte and Rogge (2009), the classical BoD approach is a particular case of the CCR-DEA model (Charnes et al., 1978) presented in Subsection

[10] In Italian "*Nel conoscere matematico la considerazione è un operare che, per la cosa, vien da fuori; ne segue quindi che la cosa vera viene alterata*".

3.2.2 where x is univariate and constant equal to $\mathbb{1}$ and $\mathbf{y}$ is a vector of k simple indicators in $[0,1]$ (from this point denoted by $\mathbf{I}$).

The BoD estimator of the output efficiency score ϕ (see Subsection 1.1.2 for a definition) for a given unit o is obtained by solving the following linear program:

$$\begin{aligned} \max_{\phi,\gamma_1,\dots,\gamma_n} \quad & \phi \\ s.t. \quad & \phi \mathbf{I_o} \leq \sum_{i=1}^{n} \gamma_i \mathbf{I_i} \\ & \gamma_i \geq 0 \end{aligned} \tag{7.2}$$

where γ_i are the weights searched.

CIs calculated through equation (7.2) satisfy the following desirable properties:

1. Weights are **endogenously** determined by the observed performances and the benchmark is not based upon theoretical bounds, but on a linear combination of observed best performances;

2. BoD CI is **weak monotone**: *Let CI an unbalanced-adjusted aggregation function of Q simple indicators I_q, CI is weakly positive monotone if for each $c > 0$, $CI(I_1,...,I_q,...,I_Q) \leq CI(I_1,...,I_q + c,...,I_Q)$* (see *e.g.* Chakravarty, 2003, Casadio Tarabusi and Guarini, 2013);

3. Weighting scheme is the **highest possible**; this property is particularly "*useful in policy arena, since policy-makers could not complain about unfair weighting: any other weighting scheme would have generated lower composite scores*" (Nardo et al., 2005).

Figure 7.2 shows the properties described above graphically, in particular, the solid lines indicate the level curves of the CI obtained by combining two indicators I_1 and I_2 that are positive monotone, in fact, CI increases whenever any of the simple indicators increase and the others are left unchanged. Moreover, the frontier is determined by points A,B and C that are the benchmarks, *i.e.* the units in the sample that yields the maximal CI value equal to 1 (given the constraints on weights). Finally, the weights set of other units, and in particular of point D, is the one that maximizes units CI value with respect the benchmark identified as the unit that minimize the distance from the single unit and the frontier (*e.g.* the point B for D).

Therefore, the optimal set of weights (if it exists) guarantees the corresponding unit the best position with respect to all the other units in the sample.

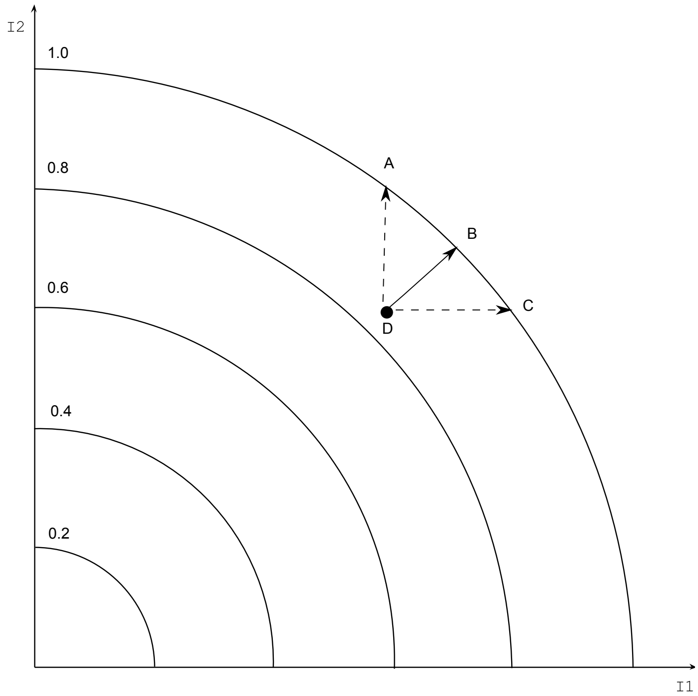


Figure 7.1: BoD

However, this method inherits all the hypothesis and drawbacks of the DEA model some of which are described below.

One of the main hypothesis is the *preferential independence* among simple indicators as it is a linear aggregation method.
In fact, in the construction of CIs, the first assumption to make is on the functional form for the underlying aggregation rule (see *e.g.* Diewert, 1976) that is generally *linear* (Freudenberg, 2003) *i.e.* $I = \sum_{i=1}^{N} w_i x_i$, where x_i is a scale adjusted variable normalized in $[0,1]$ and w_i is the related weight (usually $\sum_{i=1}^{N} w_i = 1$, $0 \le w_i \le 1$).
This hypothesis is acceptable only under the condition of this theorem: "*given the variables $x_1, x_2, ..., x_n$, an additive aggregation function exists if and only if these variables are mutually preferentially independent*" (Debreu, 1960, Keeney and Raiffa, 1993, Krantz et al., 1971).
Note that a subset of indicators I is *preferentially independent* of $I^c = Q$ (the complement of I) only if any conditional preference among elements of I, holding all elements of Q fixed, remain the same, regardless of the levels at which Q are held.
The variables $x_1, x_2, ..., x_n$ are *mutually preferentially independent* if every subset I of these variables is preferentially independent of its complementary set of evaluators.
Preferential independence is a very strong condition implying the independence between the trade-off ratio of two variables $S_{x,y}$ and the values of the $n-2$ other variables, *i.e.* $\frac{\partial S_{x,y}}{\partial q} = 0$, $\forall x, y \in I, q \in Q$ (Ting, 1971). An additive aggregation function permits the evaluation of the marginal contribution of each variable separately

and so the possibility to sum together the single contributions to obtain a total value. However, in empirical applications there often exists collinearity among variables, in this case a linear aggregation could generate biased CIs and so it is better to use nonlinear aggregation rules.

Other main hypothesis are the positive monotonicity and convexity (see Subsection 3.2.2) of the aggregation function.

The principal drawbacks regard, instead, the obtained weights being country specific, thus cross-country comparisons are not possible and so without imposing constraints on weights multiplicity of equilibria problem arises *i.e.* weights are not uniquely determined (multiple solutions have been proposed: please see Allen et al., 1997, Thanassoulis et al., 2004, Estellita-Lins et al., 2007, Cooper et al., 2009 for some methods that incorporate the "value judgment" of the specialists (bounds on the weights) in the classical DEA specification and Takamura and Tone, 2003, Lauer et al., 2004, Mazziotta and Vidoli, 2009, Rogge, 2012 in the case of BoD; or see *e.g.* Kao et al., 2008 for a method that introduces a *priori* weights); the frontier is sensitive to extreme values and outliers (please see Cazals et al., 2002, Daraio and Simar, 2005 for a robust version of nonparametric frontier estimations).

Finally, the BoD model assumes the *compensability* among simple indicators, namely allowing lower values in some indicators to be compensated by higher values in the others, but this property is not even verified in the practical application, especially if they have to be interpreted as *importance coefficients* (Bouyssou and Vansnick, 1986; Bouyssou, 1986; Vansnick, 1986; Keeney and Raiffa, 1993; Podinovskii, 1994; Munda and Nardo, 2005).

Compensability and sensitivity to extreme values and outliers issues will be the focus of the following innovative Chapters 8 and 9.

CHAPTER 8

NON-COMPENSATORY ISSUE IN CIS FRAMEWORK: DIRECTIONAL BOD

8.1 Introduction

In the previous Chapter (7) the BoD drawback of the *Compensability* has been highlighted as a strong assumption of the model unverified when in practical application a preference structure on indicators exists.

In this section the main proposals existing in literature will be presented and a new method in the field of CI, which enables taking into account the preference structure among simple indicators in a BoD framework will be proposed.

"A preference relation is compensatory if weights are considered as intensities or non-compensatory if weights are considered as importance coefficients" (please see Munda and Nardo, 2005, Munda and Nardo, 2009, Munda and Saisana, 2011, Munda, 2012*a*, Munda, 2012*b* for a recent discussion).

Recently, multiple solutions have been proposed to avoid the compensability assumption introducing weight constraints, weighting each tensor that links the single point to the frontier (see *e.g.* Tsutsui et al., 2009) or including a penalty according to the different mix of simple indicators.

In particular, considering the third approach, Vidoli and Mazziotta (2013) suggested to incorporate the *Method of Penalties for Coefficient of Variation* (MPVC - De Muro et al., 2010) idea in the basic BoD method, *i.e.*:

$$\text{BoD-PVC}_i = BoD_i \cdot (1 - cv_i^2), \forall i = 1, ..., n \tag{8.1}$$

where cv_i^2 represents the coefficient of variation for the unit i among all indicators. This approach, therefore, allows to take into account the benchmark units on the frontier (as in BoD) and to penalize, in the case of non-compensatory issues, the presence of unbalanced data (as in MPVC).
In this method, however, given the chosen penalty criteria, the aggregate function does not always satisfy the *weakly positive monotonicity property* (see Section 7.2). Figure 8.1 shows how the BoD-PVC (solid line) modifies the BoD (dashed line) level curves; in some cases the BoD-PVC does not satisfy the monotonicity property (*e.g.* if the simple indicator I_2 increases from point B to point A, the value of CI decreases).

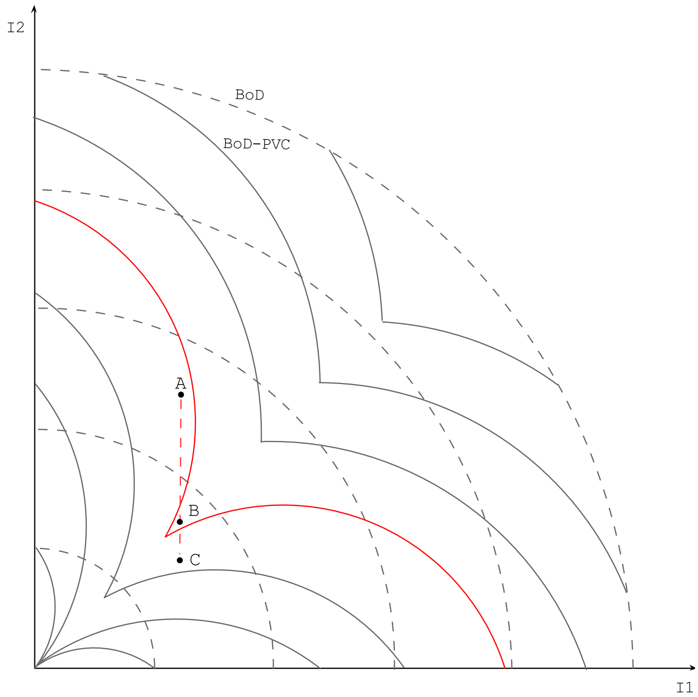


Figure 8.1: Comparison between BoD and BoD-PVC

Therefore, with the aim of finding an increasing non-compensatory CI it is important that the resulting aggregation function is monotone positive.

8.2 Directional BoD (D-BoD)

Fusco, E. (2015) *Enhancing non-compensatory composite indicators: A directional proposal.* European Journal of Operational Research, 242, 620 - 630, DOI:10.1016/j.ejor.2014.10.017.

Cited in:

Sahoo, B. K., Singh, R., Mishra, B., and Sankaran, K. (2016). *Research productivity in management schools of india during 1968-2015: A directional benefit-of-doubt model analysis.* Omega:
[...]"The solutions proposed in the literature for dealing with the foregoing problems of multiple and/or zero weights (cf. Fusco [74] on the detailed references on these) include value judgments by either imposing bounds on the weights or setting a priori weights. Since such value judgments vary across analysts/experts, the weights suffer from obvious arbitrariness. Therefore, we adjudged the ratings based on the arbitrary weight restrictions principle as unacceptable. Moreover, as Podinovski [104] also pointed out, the BOD model imposes the compensatory preference relation among individual indicators without actually verifying whether this relation actually exists in the data. We saw merit in following the advice of Fusco [74] who recommended including directional penalties in the BOD model. More specifically, the directional distance function (DDF) of Chamber et al. [105] accommodates the non-compensatory preference relations among indicators rather well.[...].

Amado, Carla A.F., Sao Jose, Jose M.S. and Santos, Sergio, (2016). *Measuring active ageing: A Data Envelopment Analysis approach*, European Journal of Operational Research, 255, issue 1, p. 207-223.

Van Puyenbroeck, Tom and Rogge, Nicky. (2017). *Geometric mean quantity index numbers with Benefit-of-the-Doubt weights*, European Journal of Operational Research, 256, issue 3, p. 1004-1014.

In order to take into account the preference structure among simple indicators and to apply a monotone positive aggregation function, Fusco (2015) suggested including in the BoD model (equation 7.2) a "directional" penalty using the *directional distance function* introduced by Chambers et al. (1998) (see Section 2.3 for a methodological discussion):

$$\overrightarrow{D}_T(\mathbf{x},\mathbf{y};\mathbf{g}) = sup\{e : (x - eg_x, y + eg_y) \in \Psi\} \tag{8.2}$$

where $\mathbf{g} = (g_x, g_y)$ is the directional vector.

Given that, as again assumed, x is fixed and equal to $\mathbb{1}$ and $\mathbf{y}$ is a vector of k simple indicators in $[0,1]$ (from this point denoted by $\mathbf{I}$), and it is considered a directional *output* distance function, where the directional vector is $\mathbf{g} = (0, g_1, ..., g_k)$. As a consequence, following Bogetoft and Otto (2011), the output distance of a specific unit o to the frontier in *g-units* is evaluated as:

$$\overrightarrow{D}(\mathbb{1},\mathbf{I_o};\Psi,\mathbf{g}) = \max\{e \in \mathbb{R}_+ | (\mathbb{1},\mathbf{I_o} + e\mathbf{g}) \in \Psi\}. \tag{8.3}$$

The Directional BoD (D-BoD - Fusco, 2015) estimator of the output distance e is obtained by solving the following linear program:

$$\begin{aligned} \max_{e,\gamma_1,...,\gamma_n} \quad & e \\ s.t. \quad & \mathbf{I_o} + e\mathbf{g} \leq \sum_{i=1}^{n} \gamma_i \mathbf{I_i} \\ & \gamma_i \geq 0 \end{aligned} \tag{8.4}$$

Finally, the Shepard output distance for the specific unit o function can be derived from the directional distance function as:

$$D(\mathbb{1},\mathbf{I_o};\Psi) = \frac{1}{\overrightarrow{D}(\mathbb{1},\mathbf{I_o};\Psi,\mathbf{g}) + 1}, \forall o \tag{8.5}$$

For the sake of simplicity, and in order to better visually illustrate the method, from this point, a bivariate case of two simple indicators *i.e.* $\mathbf{I} = (I_1, I_2)$ will be considered.
Figure 8.2 compares the CI scores obtained with BoD (dashed line) and D-BoD (solid line) formulation in a hypothetical case in which the direction favors greater values of indicator I_1 with respect I_2. The two straight lines represent the directions underlying the models: $g_{BoD} = (I_1, I_2)$ and $g_{D-BoD} = (I_1, I_2 \cdot 0.5)$.

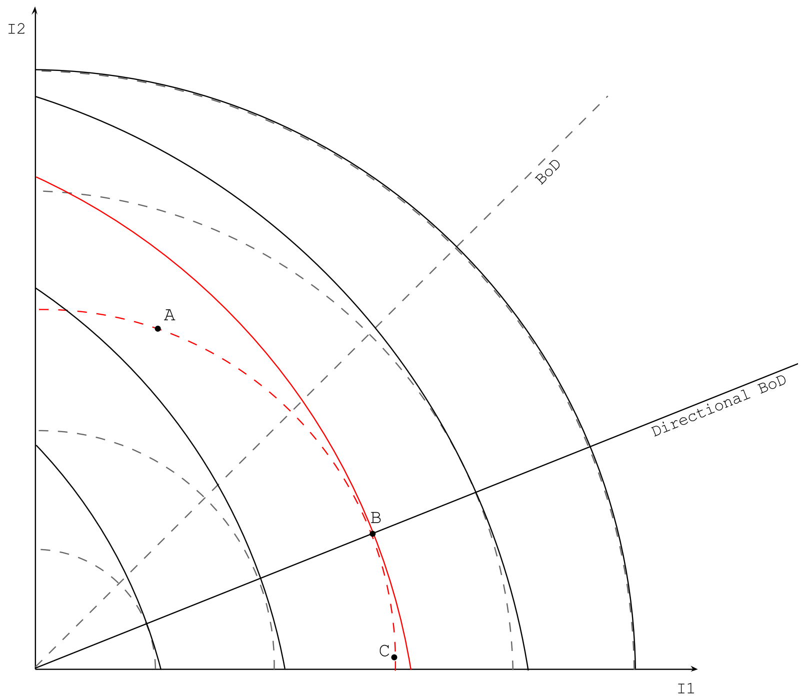


Figure 8.2: Comparison between BoD and Directional BoD

Given this representation, point A, B and C lie on the same level curve (the red dashed line) in a BoD model, while in a D-BoD model points A and C have a lower level of the CI score than B.
As a matter of fact, the D-BoD model rewards the combinations of I_1 and I_2 on the main direction (point B) and penalizes, in a different way, the combinations of low values of I_1 and high values of I_2 (point A) with respect to combinations of high values of I_1 and low values of I_2 (point C).

It can be observed that on the main direction the BoD level curve coincides with the D-BoD one and that the two curves are overlaid on the frontier. The proposed model is, therefore, a more general formulation of the basic BoD model where I_1 and I_2 have the same importance, *i.e.* $g_{BoD} = (I_1, I_2)$.

Against this background, in literature, a crucial question in a directional framework is the correct choice of the direction **g** in which inputs have to be contracted and/or outputs have to be expanded to reach the boundary.
Some authors (see *e.g.* Briec and Lesourd, 1999, Färe et al., 2005) suggested choosing $\mathbf{g} = (1, ..., 1)$ which is mathematically equivalent to seeking the Chebyshev distance l_∞ to the frontier of the technology; Bogetoft and Otto (2011) conversely proposed four approaches: *(i)* to use the direction of the actual value of input consumption (output production), *i.e.* $g_x = x_o$ (or $g_y = y_o$), *(ii)* to fix a part of the input-vector (output-vector) *i.e.* $g_x = (1, ..., 1, 0, ..., 0)$ (or $g_y = (1, ..., 1, 0, ..., 0)$), *(iii)* to use the *potential*

improvements or *multi-directional efficiency* analysis based on the bargaining theory and *(iv)* consider the subjective user point of view.
In the fourth approach methods able to identify a preference structure can also be included, as the *multi-criteria* approach (please see *e.g.* Munda, 2004, Roy, 1996, Figueira et al., 2005, Munda, 2014) that evaluates economic, social or environmental issues by establishing objectives that could be translated into the direction vector.

In the D-BoD method all of above proposals could be used to determine the direction, but with the drawback, however, of assuming exogenous choices of the researcher. Therefore, Fusco (2015) proposed to find the direction vector directly from the data, estimating the endogenous preference structure among indicators, getting through to PCA.
The preference structure estimated is hypothesized to be based on the variability of each indicator by following the Mazziotta and Vidoli (2009) idea. Following this criteria, a simple indicator with a high variability is more *"important"* than an indicator with a low variability in discriminating units.

In order to strengthen the estimate, the variability is evaluated by calculating a robust kernel variance of all indicators projected onto all principal components. As matter of fact, PCA allows to create a ranking in which the first principal component has the largest variance and each succeeding component has the highest variance possible under the constraint that it is orthogonal to the preceding components.

In our framework, therefore, the slope of the first principal component gives the direction $\mathbf{g}$ and the ratio between the kernel variances of the indicators projected onto the principal components $\widehat{\mathbf{I}}$ gives the intensity of the average rates of substitution among indicators, *i.e.*:

$$\mathbf{g} = \left(I_{PC1}, I_{PC2} \cdot \frac{var(\widehat{I_{PC2}})}{var(\widehat{I_{PC1}})}, ..., I_{PCk} \cdot \frac{var(\widehat{I_{PCk}})}{var(\widehat{I_{PC1}})} \right) \tag{8.6}$$

where *e.g.* I_{PC1} is the original simple indicator most correlated with the PC1.

This approach, consequently, allows to derive both the preference structure and the direction from the data avoiding subjective judgments of the researcher.

Therefore, the D-BoD adds the following properties to the classical BoD model:

4. **non-compensability property**: the directional vector g_y rewards units along a generic direction (not only along the bisector direction) by penalizing units far from the chosen direction; therefore full compensability can be seen as a special case when $g_y = \mathbf{I}$.

5. **Translation property**: D-BoD is invariant to the chosen mean normalization method, *i.e.* $D(\mathbb{1}, \mathbf{y} + \alpha g_y; g_y) = D(\mathbb{1}, \mathbf{y}; g_y) - \alpha$ for $\alpha \in \mathbb{R}_+$.

As highlighted in Section 7.3 a drawback of DEA and consequently of BoD and D-BoD is the sensitivity to extreme values and outliers that will be discussed in the following Chapter (9).

8.3 Simulation

In order to test and to better explain the model proposed in Section 8.2 a simulation, has been conducted, on a standardized unbalanced data set (Figure 8.3) composed of two groups:

- A major homogeneous group ($G1$) containing units that follow a specific direction, where I_1 is a vector of 3,000 multivariate normal random numbers, *i.e.* $N_{1...1000}(0.5, 0.3)$, $N_{1001...2000}(0.4, 0.1)$, $N_{2001...3000}(0.7, 0.1)$ and I_2 is a multiple of I_1 plus a noise term, *i.e.* $I_2 = 0.1 \cdot I_1 + 0.1 \cdot N(0, 0.2)$;

- An isolated minor group ($G2$) containing units with a different preference structure, where I_1 and I_2 are two vectors of 100 multivariate normal random numbers, *i.e.* $I_1 = N_{3001...3100}(0, 0.05)$ and $I_2 = N_{3001...3100}(0.3, 0.02)$.

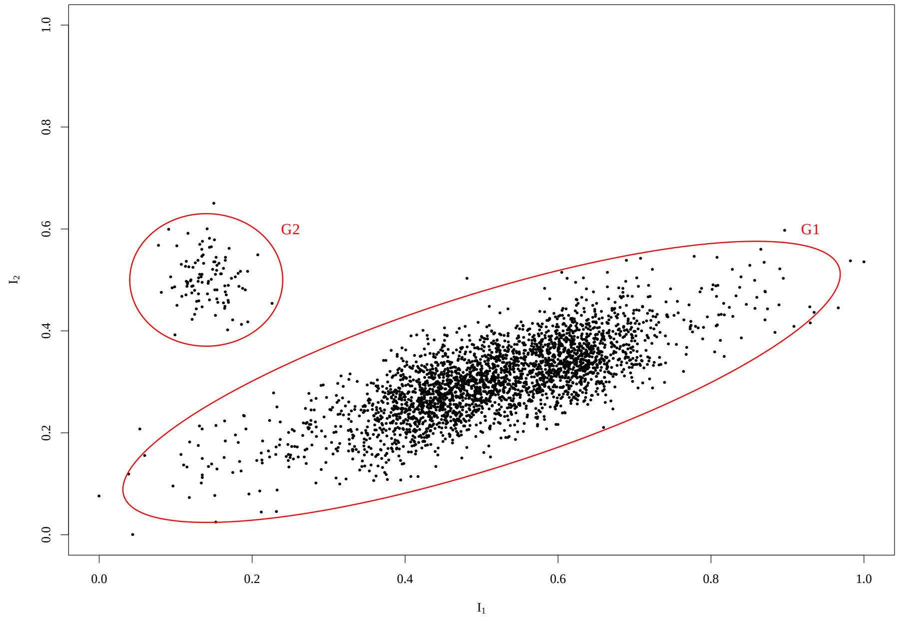


Figure 8.3: Simulated data

The first step of the analysis is intended to find the main preference structures between I_1 and I_2 through PCA. In this simulation it has been obtained that 98.39% of the total variance is explained by the first principal component given that most of the information is contained in the first eigenvalue. Figure 8.4 shows the two principal components $PC1$ and $PC2$.

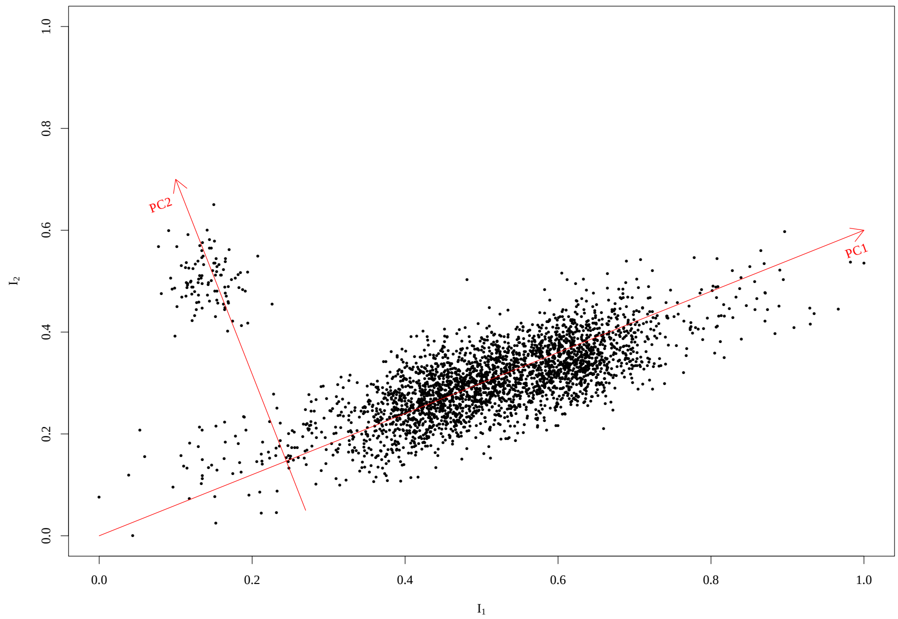


Figure 8.4: Principal components $PC1$ and $PC2$

Afterward, the robust bivariate kernel density of the rotated data points and the kernel variance of the projected values of I_1 and I_2 onto the principal components denoted by $\widehat{I_1}$ and $\widehat{I_2}$ are calculated. In this way, the direction and the intensity of the rates of substitution between I_1 and I_2 can be obtained from equation (8.6), *i.e.* $\mathbf{g} = \left(I_1, I_2 \cdot \frac{var_k(\widehat{I_2})}{var_k(\widehat{I_1})} \right) = (I_1, I_2 \cdot 0.325)$.

Figure 8.5 shows the kernel densities and the directions in the case of a simple BoD model where I_1 and I_2 have the same importance (blue line) and in the case of D-BoD model where I_1 is the most important in the construction of the CI (red dashed line).

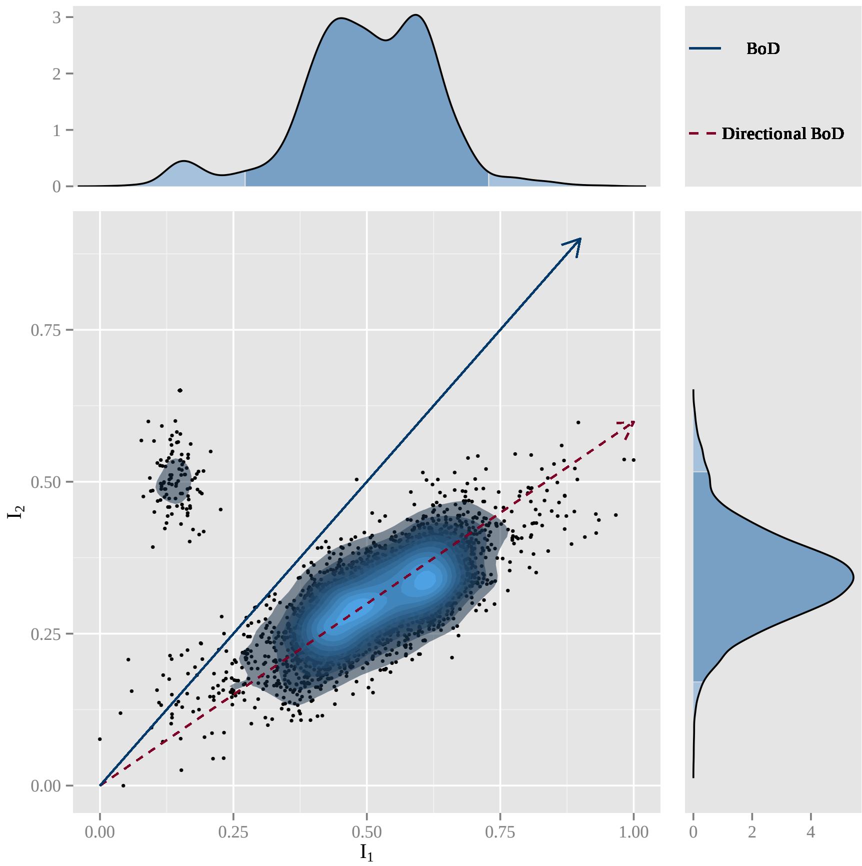
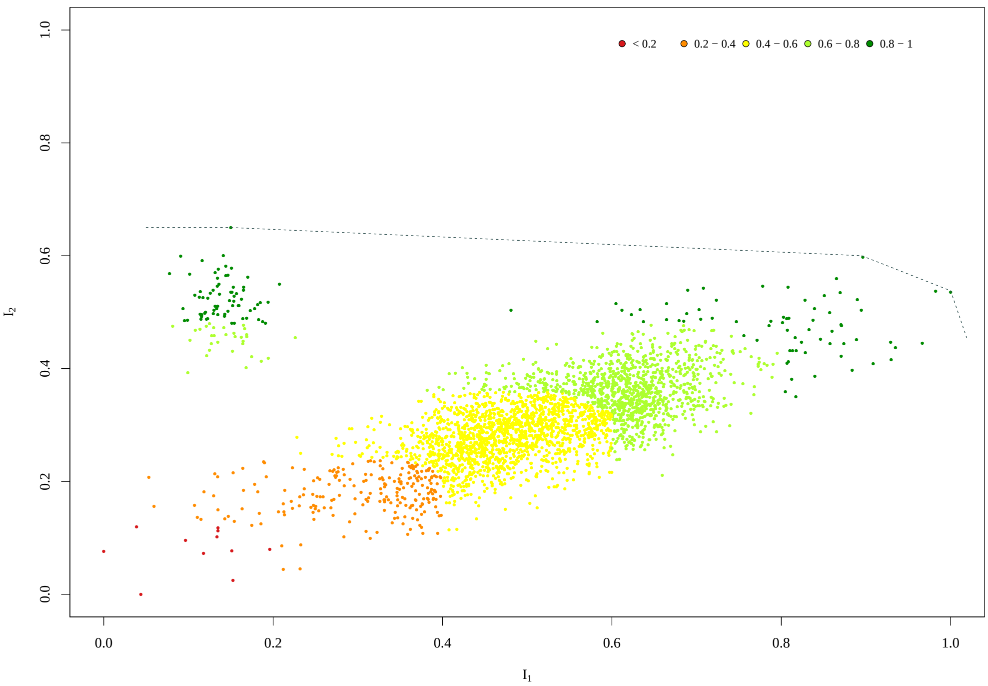


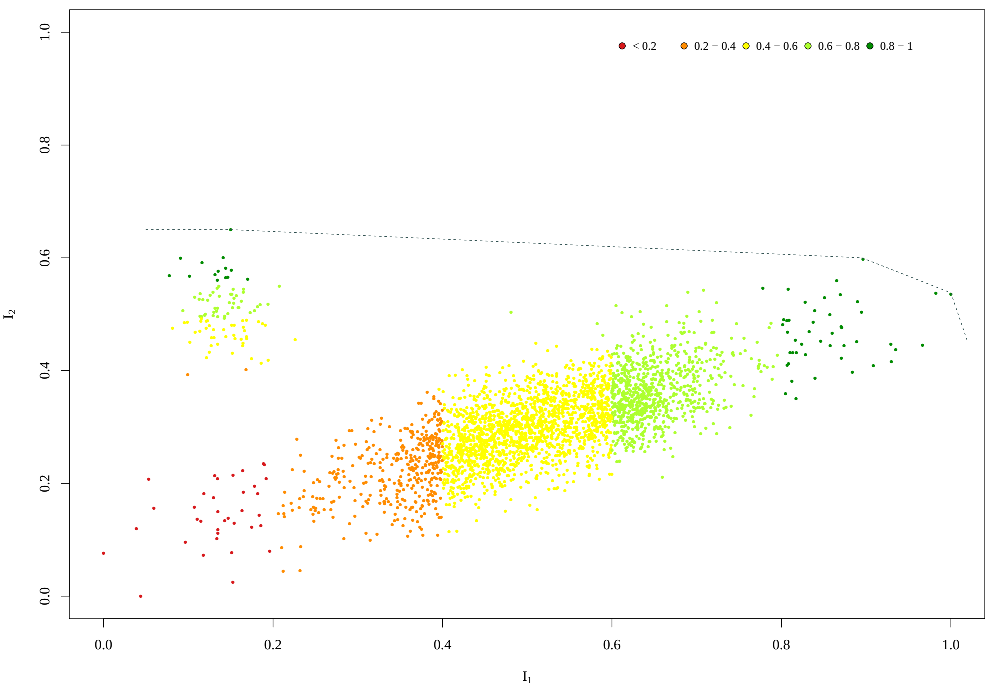
Figure 8.5: Kernel density

Finally, having estimated **g** the CI score can be calculated in a Shephard formulation.
Figure 8.6 compares BoD and D-BoD models results confirming that the directional approach rewards units nearby the main direction. For $G2$ group, in fact, a reduction of the average efficiency from 83.46% to 63.76% has been obtained . In addition, it can be observed that the biggest differences are detected, as expected, for the units with low values of I_1 and especially in the units of the isolated group ($G2$) with low

values in both indicators where the CI score falls from 65.40% to 38.17%.



(a) *BoD*



(b) *D-BoD*

Figure 8.6: Comparison between BoD and D-BoD on simulated data

Chapter 9

ROBUSTNESS ISSUE IN D-BOD APPROACH: ROBUST DIRECTIONAL BOD

9.1 Introduction

The D-BoD (Fusco, 2015) as highlighted in Section 8.2 still suffers a serious drawback: the lack of robustness with respect to outliers, that shifts the frontier biasing the CI of all units in the sample, as shown in Figure 9.1.

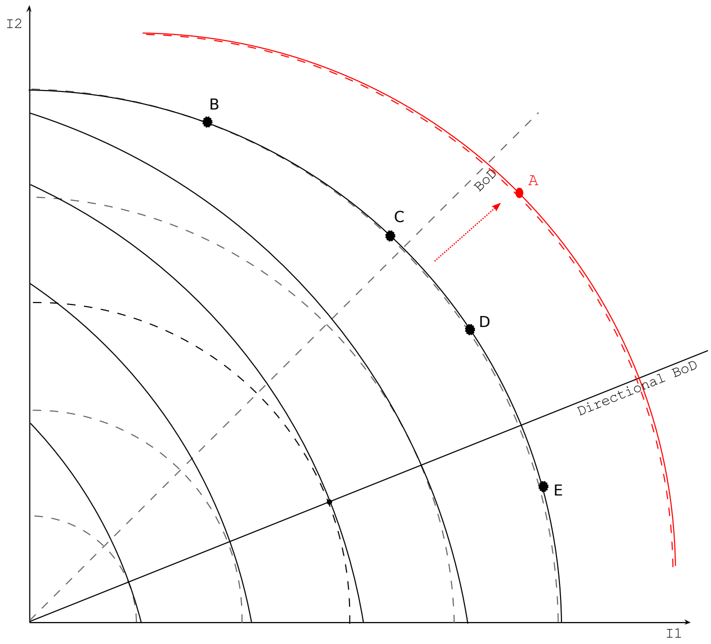


Figure 9.1: BoD and D-BoD sensitivity to outliers

To overcome this significant shortcoming, as discussed in Section 3.2.3 (in an input-oriented framework), the production set Ψ can be translated in a probabilistic framework following Daraio and Simar (2005)'s proposal.

9.2 Robust directional BoD (RD-BoD)

Vidoli, F., Fusco, E., and Mazziotta, C. (2015). *Non-compensability in composite indicators: A robust directional frontier method.* Social Indicators Research, 122(3):635–652., DOI:10.1007/s11205-014-0710-y.

Cited in:

González, E.; Cárcaba, A. & Ventura, J. (2016). *Weight Constrained DEA Measurement of the Quality of Life in Spanish Municipalities in 2011.* Social Indicators Research.

In an output-oriented framework, where x is univariate and constant equal to $\mathbb{1}$ and $\mathbf{y}$ is a vector of k simple indicators in $[0,1]$ (denoted by $\mathbf{I}$), considering a sample of m random variables with replacement $S_m = \{\mathbf{I_i}\}_{i=1}^{m}$ drawn from the density of $\mathbf{I}$, the random set $\widetilde{\Psi}_m$ is defined as:

$$\widetilde{\Psi}_m = \{(\mathbb{1}, \mathbf{I}) \in \mathbb{R}_{+}^{1+Q} | X \equiv \mathbb{1}, \mathbf{I_i} \geq \mathbf{I}\}. \tag{9.1}$$

Therefore, the effect of an abnormal or outlier unit is dampened, in fact, the single unit is not compared to all the others but to a sample subset of size m.

This generalization allows to calculate iteratively the sample subset of size m (for $b = 1, ..., B$ times) and for each b iteration the *output* directional distance function, where the directional vector is $\mathbf{g} = (0, g_1, ..., g_k)$.

The output distance at iteration b for the single unit from the maximum values can be defined as:

$$\widetilde{D}_m^b(\mathbb{1}, \mathbf{I}; \widetilde{\Psi}_m, \mathbf{g}) = \max\{e \in \mathbb{R}_+ | (\mathbb{1}, \mathbf{I} + e\mathbf{g}) \in \widetilde{\Psi}_m\}, \forall b = 1, \dots, B \tag{9.2}$$

More specifically equation (9.2) can be practically computed, in a multivariate setting, as suggested by Daouia et al. (2010), by using the dimensionless transformation by minimum for the $\mathbf{I}^* = \mathbf{g} \cdot \mathbf{I}$:

$$\widetilde{D}_m^b(\mathbb{1}, \mathbf{I}; \widetilde{\Psi}_m, \mathbf{g}) = \max_{i=1,\dots,m} \left\{ \min_{k=1,\dots,K} \left(\frac{I_{ik}^{*b}}{I_{.k}^{*}} \right) \right\} \tag{9.3}$$

and the order-m directional distance estimator $\widetilde{D}_m(\mathbb{1}, \mathbf{I}; \widetilde{\Psi}_m, \mathbf{g})$ *i.e.* the *Robust Directional BoD* (RD-BoD - Vidoli, Fusco and Mazziotta, 2015) as:

$$\widetilde{D}_m(\mathbb{1}, \mathbf{I}; \widetilde{\Psi}_m, \mathbf{g}) = E\left[\widetilde{D}_m^b(\mathbb{1}, \mathbf{I}; \widetilde{\Psi}_m, \mathbf{g})\right] \tag{9.4}$$

Following Cazals et al. (2002), finally, the order-m directional distance estimator can be approximated - even in a Shephard formulation - by computing the empirical mean over B:

$$\widehat{D}_m(\mathbb{1}, \mathbf{I}; \widetilde{\Psi}_m, \mathbf{g}) = 1/\widehat{E}\left[\widetilde{D}_m^b(\mathbb{1}, \mathbf{I}; \widetilde{\Psi}_m, \mathbf{g})\right] \tag{9.5}$$

The RD-BoD adds the following property to classical BoD and D-BoD models:

6. **External robustness property**: RD-BoD allows to remove outliers influence on the estimated frontier and, as a consequence, on the resulting CI ranking.

For the sake of clarification, Section 9.3 illustrates a graphical representation of the method using simulated data, in order to highlight that units' scores are not affected by outliers.

9.2.1 Robust PCA directional BoD

In the previous section in the RD-BoD a generic direction **g** has been included, but as highlighted at page 118 for practical computation the direction **g**, can be seen, for example, as the marginal rate of substitution among indicators; following this criteria, in case of additional information, it may be imposed by the researcher or derived directly from the data, for example through a PCA as in Fusco (2015).

PCA, in fact, allows for identifying the first principal component with the largest variance showing the internal structure and the main pattern of the data with the advantages of ease of calculation.
Despite this attractive feature, however, PCA models have several shortcomings; among others, all the classical PCA algorithms - based on least squares techniques - are set up on the assumption that outliers are not included in the dataset.
In order to bypass PCA outliers drawback, several robust versions of PCA have been developed by a modification of the *covariance matrix* (see *e.g.* Campbell, 1980), by *Projection Pursuit* (see *e.g.* Li and Chen, 1985) or by weighting *Singular Value Decomposition* (SVD) (see *e.g.* Gabriel and Zamir, 1979).
In particular, the Robust PCA by Projection Pursuit is preferable with respect to use a more robust covariance matrix that can be computationally intensive, especially if the involved covariance matrices must be estimated in a robust way (see in particular Croux et al., 2007 and Filzmoser et al., 2006).

This particular choice of **g**, based on PCA by Projection Pursuit, adds the following property to the RD-BoD:

7. **Internal robustness property**: Robust PCA directional BoD Robustness allows to remove outliers influence on the main direction estimated.

Simulation in Section 9.3 highlights the robustness of the main direction estimated in presence of outliers.

9.3 Simulation

In the present section multiple simulations are introduced with the aim of testing the validity of RD-BoD approach and illustrating the required properties of the suggested methods, rather than proving the stability towards another one.

More specifically, a two-dimensional dataset of 300 units i with a marginal rate of substitution between first and second indicator equal to 0.5 has been generated *i.e.*:

$$\begin{cases} I_{i1} \in N(0.5, 0.1) \\ I_{i2} = 0.5 \cdot I_{i1} + N(0, 0.05) \end{cases}$$

In order to test the robustness of the measure, an outlier set with a marginal rate of substitution between first and second indicator higher than 1.5 has been added to the dataset.

More specifically, in the following Figures 9.2 and 9.3, the simulated simple indicators I_1 and I_2 with an increasing size and color for higher values of CI are plotted and the composite indicator isoquants are also displayed (light green for lower values of the composite indicator, dark green for higher values).

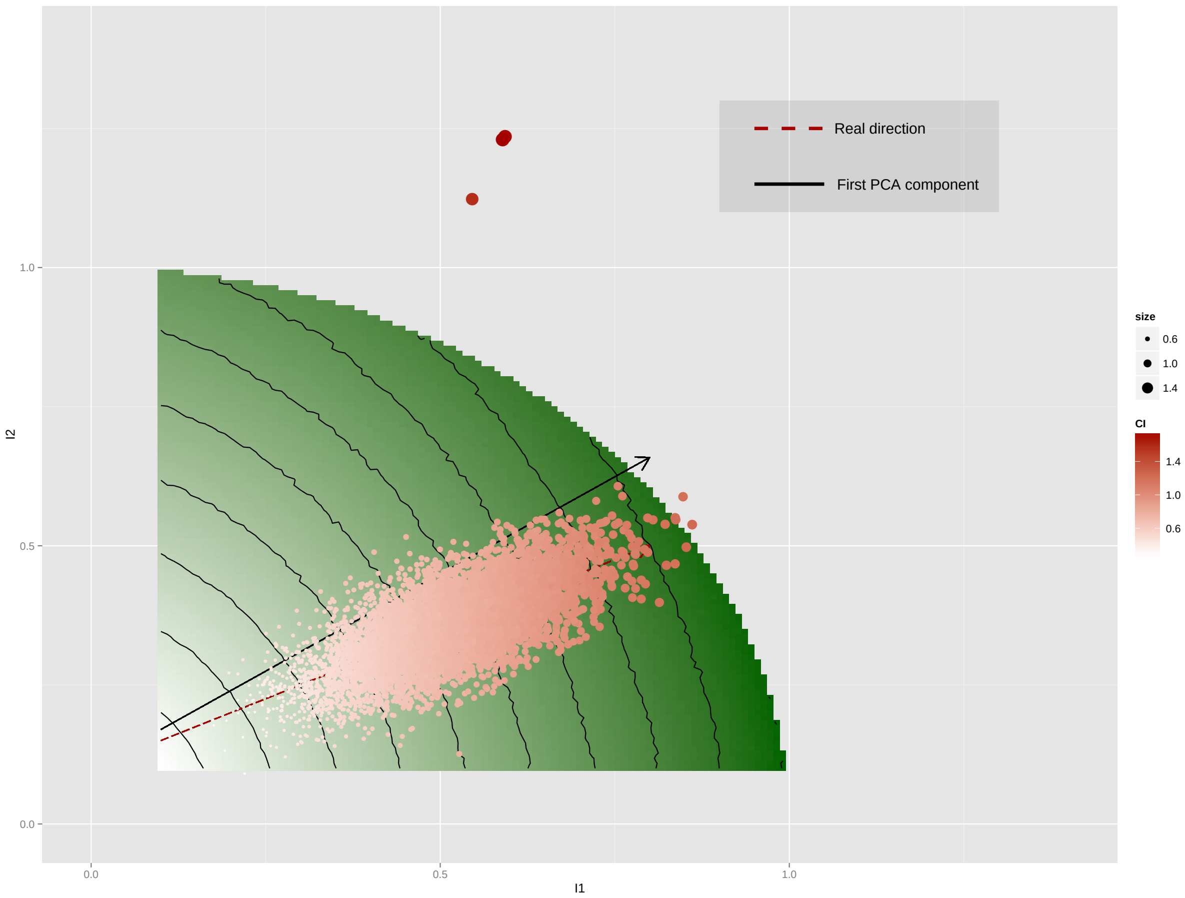


Figure 9.2: Robust directional BoD on simulated data

Figure 9.2 confirms that RD-BoD is able to satisfy the requested properties outlined in Chapter 8; in particular, it can be highlighted that - without imposing a priori compensability or non-compensability among indicators - directional measure lets to obtain a monotonic increasing of CI when I_1 or I_2 increase.

At the same time, it can be observed how outliers clearly influence (estimated rate of substitution equal to 0.71) the estimated direction **g** (internal robustness property) when classical PCA is used, while they do not have any impact on the frontier estimation (external robustness property) and on the relative ranking.

Finally, it can be noted that the CI is greater than 1 for the outlier units; this finding (in agreement with the order-m model) does not affect the scores of the other units and it is indicative of its characteristic.

With the aim of correctly estimating the real direction the Robust PCA by Projection Pursuit to the same dataset has been applied obtaining an estimated rate of substitution equal to 0.51 (see Figure 9.3) very similar to the real one (0.5).

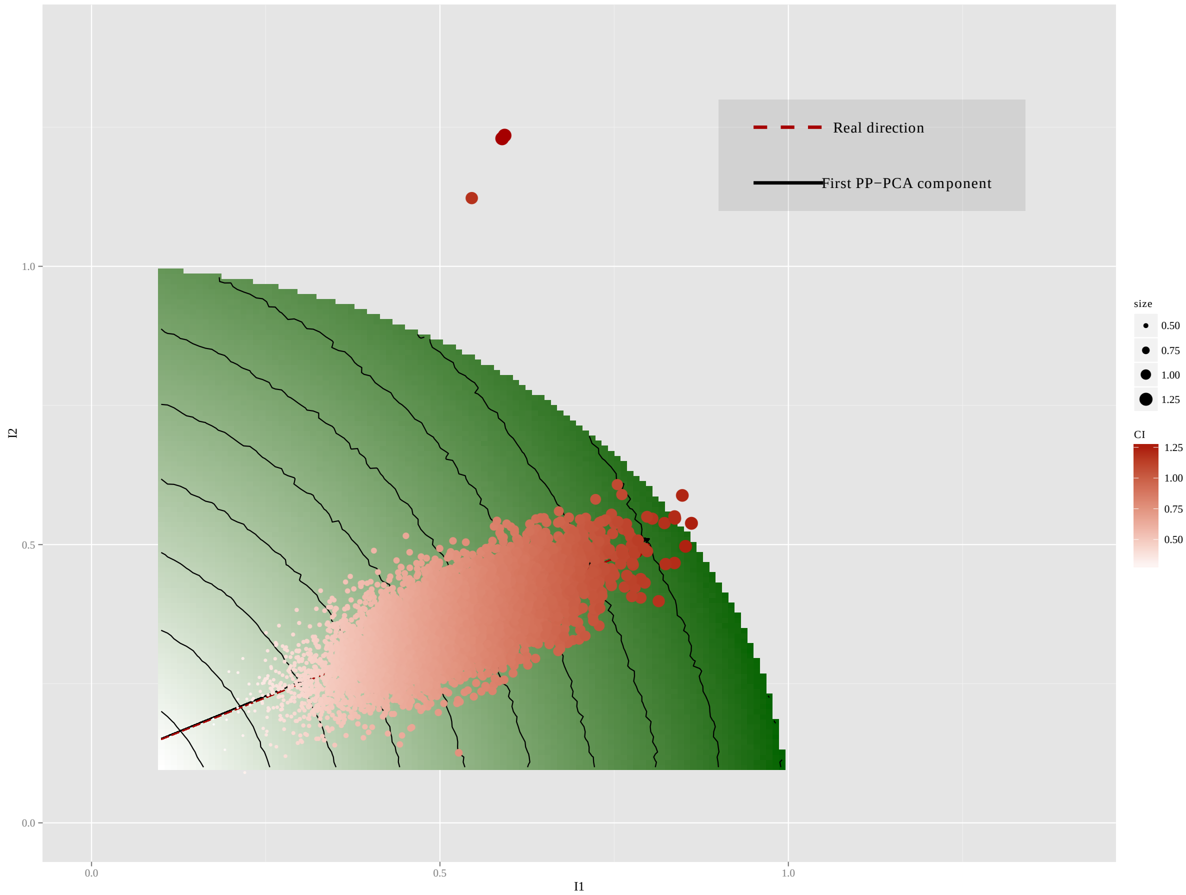


Figure 9.3: Robust PCA (by Projection Pursuit) directional BoD on simulated data

9.4 Strengths, drawbacks and possible enhancements

In conclusion, D-BoD is an innovative method, in a CI framework, for constructing a CI with fewer subjective choices as possible.

In fact, it allows to obtain a CI where *(i)* weights are *endogenously* determined by the observed performances and the benchmark is not based upon theoretical bounds, but on a linear combination of observed best performances; *(ii)* CI is *positive monotone*; *(iii)* the weighting scheme is the *highest possible*; *(iv)* CI allows considering a preference structure *i.e.* the *non-compensability* among indicators; *(v)* CI is invariant to the chosen mean normalization method.
Moreover, by adding the ulterior property *(vi)* the external robustness of the RD-BoD is obtained allowing to remove outliers influence on the estimated frontier and, as a consequence, on the resulting CI ranking.

Another element of objectivity is introduced in the choice of the direction, that represents the relationship among simple indicators, by hypothesizing a preference structure based on the variability of each indicator. To identify the importance ranking of simple indicators in terms of variability a PCA in the D-BoD and a robust PCA in the RD-BoD have been used by adding in the latter also the *(vii)* property (internal robustness property) that allows to remove outliers influence on the main estimated direction.

A possible enhancement of the RD-BoD, but also of BoD and D-BoD, is to consider instead of the constant return to scale case the other specifications discussed in Subsection 3.2.2 as proposed in Sahoo et al. (2016) that has cited the paper Fusco (2015), in Omega journal, in a positive way and changed the D-BoD constraint to have a variable return to scale.

Chapter 10

COMPIND R PACKAGE

10.1 Introduction

To implement the D-BoD and RD-BoD approaches proposed in Chapters 8 and 9, a new package named *Compind* has been implemented in R. *Compind* allows also to construct CI with a plurality of other methods proposed in literature, focusing on the normalisation and weighting-aggregation steps and to supports researchers into robustness analysis through repeated simulations on subsamples of units or variables.

Below are reported the "*Reference manual*" and the vignette entitled " *Compind: Composite indicators functions based on frontiers in R*" published on CRAN.

10.2 Reference manual

Package 'Compind'

June 27, 2016

Type Package

Title Composite Indicators Functions

Version 1.1.2

Date 2016-06-21

Author Francesco Vidoli, Elisa Fusco

Maintainer Francesco Vidoli <fvidoli@gmail.com>

Description Contains several functions to enhance approaches to the Composite Indicators methods, focusing, in particular, on the normalisation and weighting-aggregation steps.

Depends Benchmarking, psych, boot

Imports Hmisc, MASS, GPArotation, lpSolve, nonparaeff

License GPL-3

Suggests R.rsp

VignetteBuilder R.rsp

NeedsCompilation no

Repository CRAN

Date/Publication 2016-06-27 13:07:48

R topics documented:

`Compind-package` *Composite Indicators - Compind*

Description

Compind package contains functions to enhance several approaches to the Composite Indicators (CIs) methods, focusing, in particular, on the normalisation and weighting-aggregation steps.

Details

Package:	Compind
Type:	Package
Version:	0.1
Date:	2014-01-01
Depends:	Benchmarking, Hmisc, MASS, ggplot2, psych, GPArotation, lpSolve, nonparaeff, boot
License:	GPL-3
Built:	R 3.0.2; ; 2014-02-06 13:14:40 UTC; unix

Index:

```
Compind-package         Composite Indicators - Compind
EU_2020                 Europe 2020 indicators
EU_NUTS1                EU NUTS1 Transportation data
ci_bod                  Benefit of the Doubt approach (BoD)
ci_bod_dir              Directional Benefit of the Doubt (D-BoD) model
ci_bod_var_w            Variance weighted Benefit of the Doubt approach
                        (BoD variance weighted)
ci_factor               Weighting method based on Factor Analysis
ci_mean_geom            Weighting method based on geometric aggregation
ci_mpi                  Mazziotta-Pareto Index (MPI) method
ci_rbod                 Robust Benefit of the Doubt approach (RBoD)
ci_rbod_dir             Directional Robust Benefit of the Doubt
                        approach (D-RBoD)
ci_wroclaw              Wroclaw Taxonomic Method
normalise_ci            Normalisation and polarity functions
plot_influent           Plot influents units in terms of horizontal
                        mean and variability
```

Author(s)

Francesco Vidoli, Elisa Fusco Maintainer: Francesco Vidoli <fvidoli@gmail.com>

References

Daraio, C., Simar, L. "*Introducing environmental variables in nonparametric frontier models: a probabilistic approach*", Journal of productivity analysis, 2005, 24(1), 93?121.

Fusco E., "*Enhancing non compensatory composite indicators: A directional proposal*", 2013, unpublished.

OECD, "*Handbook on constructing composite indicators: methodology and user guide*", 2008.

Mazziotta C., Mazziotta M., Pareto A., Vidoli F., "*La sintesi di indicatori territoriali di dotazione infrastrutturale: metodi di costruzione e procedure di ponderazione a confronto*", Rivista di Economia e Statistica del territorio, n.1, 2010.

Melyn W. and Moesen W.W., "*Towards a synthetic indicator of macroeconomic performance: unequal weighting when limited information is available*", Public Economic research Paper 17, CES, KU Leuven, 1991.

Simar L., Vanhems A., "*Probabilistic characterization of directional distances and their robust versions*", Journal of Econometrics, 2012, 166(2), 342?354.

UNESCO, "*Social indicators: problems of definition and of selection*", Paris 1974.

Vidoli F., Fusco E., Mazziotta C., "*Non-compensability in composite indicators: a robust directional frontier method*", Social Indicators Research, Springer Netherlands.

Vidoli F., Mazziotta C., "*Robust weighted composite indicators by means of frontier methods with an application to European infrastructure endowment*", Statistica Applicata, Italian Journal of Applied Statistics, 2013.

`ci_bod` *Benefit of the Doubt approach (BoD)*

Description

Benefit of the Doubt approach (BoD) is the application of Data Envelopment Analysis (DEA) to the field of composite indicators. It was originally proposed by Melyn and Moesen (1991) to evaluate macroeconomic performance.

Usage

```
ci_bod(x,indic_col)
```

Arguments

`x`	A data.frame containing simple indicators.
`indic_col`	A numeric list indicating the positions of the simple indicators.

Value

An object of class "CI". This is a list containing the following elements:

`ci_bod_est`	Composite indicator estimated values.
`ci_method`	Method used; for this function ci_method="bod".
`ci_bod_weights`	Raw weights assigned to the simple indicators (Dual values - prices - in the DUAL Dea formulation).

Author(s)

Vidoli F.

References

OECD, *Handbook on constructing composite indicators: methodology and user guide*, 2008.

Melyn W. and Moesen W.W., "*Towards a synthetic indicator of macroeconomic performance: unequal weighting when limited information is available*", Public Economic research Paper 17, CES, KU Leuven, 1991.

Witte, K. D., Rogge, N. "*Accounting for exogenous influences in a benevolent performance evaluation of teachers*". Tech. rept. Working Paper Series ces0913, Katholieke Universiteit Leuven, Centrum voor Economische Studien, 2009.

See Also

`ci_bod_dir`,`ci_rbod`

Examples

```
i1 <- seq(0.3, 0.5, len = 100) - rnorm (100, 0.2, 0.03)
i2 <- seq(0.3, 1, len = 100)   - rnorm (100, 0.2, 0.03)
Indic = data.frame(i1, i2)
CI = ci_bod(Indic)
 # validating BoD score
w = CI$ci_bod_weights
Indic[,1]*w[,1] + Indic[,2]*w[,2]


data(EU_NUTS1)
data_norm = normalise_ci(EU_NUTS1,c(2:3),polarity = c("POS","POS"), method=2)
CI = ci_bod(data_norm$ci_norm,c(1:2))
```

`ci_bod_dir` *Directional Benefit of the Doubt (D-BoD) model*

Description

Directional Benefit of the Doubt (D-BoD) model enhance non-compensatory property by introducing directional penalties in a standard BoD model in order to consider the preference structure among simple indicators.

Usage

```
ci_bod_dir(x, indic_col, dir)
```

Arguments

`x`	A data.frame containing score of the simple indicators.
`indic_col`	Simple indicators column number.
`dir`	Main direction. For example you can set the average rates of substitution.

Value

An object of class "CI". This is a list containing the following elements:

`ci_bod_dir_est`	Composite indicator estimated values.
`ci_method`	Method used; for this function ci_method="bod_dir".

Author(s)

Vidoli F., Fusco E.

References

Fusco E., *Enhancing non compensatory composite indicators: A directional proposal*, 2013, unpublished

See Also

`ci_bod`, `ci_rbod`

Examples

```
i1 <- seq(0.3, 0.5, len = 100) - rnorm (100, 0.2, 0.03)
i2 <- seq(0.3, 1, len = 100)   - rnorm (100, 0.2, 0.03)
Indic = data.frame(i1, i2)
CI = ci_bod_dir(Indic,dir=c(1,1))

data(EU_NUTS1)
data_norm = normalise_ci(EU_NUTS1,c(2:3),polarity = c("POS","POS"), method=2)
CI = ci_bod_dir(data_norm$ci_norm,c(1:2),dir=c(1,0.5))
```

`ci_bod_var_w` *Variance weighted Benefit of the Doubt approach (BoD variance weighted)*

Description

Variance weighted Benefit of the Doubt approach (BoD variance weighted) is a particular form of BoD method with additional information in the optimization problem. In particular it has been added weight constraints (in form of an Assurance region type I (AR I)) endogenously determined in order to take into account the ratio of the vertical variability of each simple indicator relative to one another.

Usage

```
ci_bod_var_w(x,indic_col,boot_rep = 5000)
```

Arguments

`x`	A data.frame containing score of the simple indicators.
`indic_col`	Simple indicators column number.
`boot_rep`	The number of bootstrap replicates (default=5000) for the estimates of the non-parametric bootstrap (first order normal approximation) confidence intervals for the variances of the simple indicators.

Details

For more informations about the estimation of the confidence interval for the variances, please see function *boot.ci*, package *boot*.

Value

An object of class "CI". This is a list containing the following elements:

`ci_bod_var_w_est`	Composite indicator estimated values.
`ci_method`	Method used; for this function ci_method="bod_var_w".

Author(s)

Vidoli F.

References

Vidoli F., Mazziotta C., "*Robust weighted composite indicators by means of frontier methods with an application to European infrastructure endowment*", Statistica Applicata, Italian Journal of Applied Statistics, 2013.

See Also

ci_bod, ci_rbod

Examples

```
i1 <- seq(0.3, 0.5, len = 100) - rnorm (100, 0.2, 0.03)
i2 <- seq(0.3, 1, len = 100)   - rnorm (100, 0.2, 0.03)
Indic = data.frame(i1, i2)
CI = ci_bod_var_w(Indic)
```

---

ci_bod_vrs   *Benefit of the Doubt approach (BoD) VRS*

Description

Benefit of the Doubt approach (BoD) is the application of Data Envelopment Analysis (DEA) to the field of composite indicators. It was originally proposed by Melyn and Moesen (1991) to evaluate macroeconomic performance.

Usage

```
ci_bod_vrs(x,indic_col)
```

Arguments

x	A data.frame containing simple indicators.
indic_col	A numeric list indicating the positions of the simple indicators.

Value

An object of class "CI". This is a list containing the following elements:

ci_bod_vrs_est	Composite indicator estimated values.
ci_method	Method used; for this function ci_method="bod_vrs".
ci_bod_vrs_weights	Raw weights assigned to the simple indicators (Dual values - prices - in the DUAL Dea formulation).

Author(s)

Vidoli F.

References

OECD, *Handbook on constructing composite indicators: methodology and user guide*, 2008.

Melyn W. and Moesen W.W., "*Towards a synthetic indicator of macroeconomic performance: unequal weighting when limited information is available*", Public Economic research Paper 17, CES, KU Leuven, 1991.

Witte, K. D., Rogge, N. "*Accounting for exogenous influences in a benevolent performance evaluation of teachers*". Tech. rept. Working Paper Series ces0913, Katholieke Universiteit Leuven, Centrum voor Economische Studien, 2009.

See Also

ci_bod,ci_rbod

Examples

```
i1 <- seq(0.3, 0.5, len = 100) - rnorm (100, 0.2, 0.03)
i2 <- seq(0.3, 1, len = 100)   - rnorm (100, 0.2, 0.03)
Indic = data.frame(i1, i2)
CI = ci_bod_vrs(Indic)
 # validating BoD score
w = CI$ci_bod__vrs_weights
Indic[,1]*w[,1] + Indic[,2]*w[,2]


data(EU_NUTS1)
data_norm = normalise_ci(EU_NUTS1,c(2:3),polarity = c("POS","POS"), method=2)
CI = ci_bod_vrs(data_norm$ci_norm,c(1:2))
```

---

ci_factor *Weighting method based on Factor Analysis*

Description

Factor analysis groups together collinear simple indicators to estimate a composite indicator that captures as much as possible of the information common to individual indicators.

Usage

```
ci_factor(x,indic_col,method="ONE",dim)
```

Arguments

x	A data.frame containing score of the simple indicators.
indic_col	Simple indicators column number.

`method`	If method = "ONE" (default) the composite indicator estimated values are equal to first component scores; if method = "ALL" the composite indicator estimated values are equal to component score multiplied by its proportion variance; if method = "CH" it can be choose the number of the component to take into account.
`dim`	Number of chosen component (if method = "CH", default is 3).

Value

An object of class "CI". This is a list containing the following elements:

`ci_factor_est`	Composite indicator estimated values.
`loadings_fact`	Variance explained by principal factors (in percentage terms).
`ci_method`	Method used; for this function ci_method="factor".

Author(s)

Vidoli F.

References

OECD, *Handbook on constructing composite indicators: methodology and user guide*, 2008

See Also

`ci_bod`, `ci_mpi`

Examples

```
i1 <- seq(0.3, 0.5, len = 100) - rnorm (100, 0.2, 0.03)
i2 <- seq(0.3, 1, len = 100)   - rnorm (100, 0.2, 0.03)
Indic = data.frame(i1, i2)
CI = ci_factor(Indic)

data(EU_NUTS1)
CI = ci_factor(EU_NUTS1,c(2:3), method="ALL")

data(EU_2020)
data_norm = normalise_ci(EU_2020,c(47:51),polarity = c("POS","POS","POS","POS","POS"), method=2)
CI3 = ci_factor(data_norm$ci_norm,c(1:5),method="CH", dim=3)
```

`ci_mean_geom` *Weighting method based on geometric aggregation*

Description

Geometric aggregation lets to bypass the full compensability hypothesis using geometric mean.

Usage

```
ci_mean_geom(x, indic_col, na.rm=TRUE)
```

Arguments

`x`	A data.frame containing simple indicators.
`indic_col`	Simple indicators column number.
`na.rm`	Remove NA values before processing; default is TRUE.

Value

An object of class "CI". This is a list containing the following elements:

`ci_mean_geom_est`	Composite indicator estimated values.
`ci_method`	Method used; for this function ci_method="mean_geom".

Author(s)

Vidoli F.

References

OECD, *Handbook on constructing composite indicators: methodology and user guide*, 2008.

See Also

`ci_bod`, `ci_factor`

Examples

```
i1 <- seq(0.3, 0.5, len = 100) - rnorm (100, 0.2, 0.03)
i2 <- seq(0.3, 1, len = 100)   - rnorm (100, 0.2, 0.03)
Indic = data.frame(i1, i2)
CI = ci_mean_geom(Indic)

data(EU_NUTS1)
CI = ci_mean_geom(EU_NUTS1,c(2:3))
```

`ci_mean_min` *Mean-Min Function*

Description

The Mean-Min Function (MMF) is an intermediate case between arithmetic `mean`, according to which no unbalance is penalized, and `min` function, according to which the penalization is maximum. It depends on two parameters that are respectively related to the intensity of penalization of unbalance (α) and intensity of complementarity (β) among indicators.

Usage

```
ci_mean_min(x, indic_col, alpha, beta)
```

Arguments

`x`	A data.frame containing simple indicators.
`indic_col`	Simple indicators column number.
`alpha`	The intensity of penalisation of unbalance among indicators, $0 \leq \alpha \leq 1$
`beta`	The intensity of complementarity among indicators, $\beta \geq 0$

Value

An object of class "CI". This is a list containing the following elements:

`ci_mean_min_est`	Composite indicator estimated values.
`ci_method`	Method used; for this function ci_method="mean_min".

Author(s)

Vidoli F.

References

Casadio Tarabusi, E., & Guarini, G. (2013) "*An unbalance adjustment method for development indicators*", Social indicators research, 112(1), 19-45.

See Also

`ci_mpi`, `normalise_ci`

Examples

```
data(EU_NUTS1)
data_norm = normalise_ci(EU_NUTS1,c(2:3),c("NEG","POS"),method=2)
CI = ci_mean_min(data_norm$ci_norm, alpha=0.5, beta=1)
```

ci_mpi *Mazziotta-Pareto Index (MPI) method*

Description

Mazziotta-Pareto Index (MPI) is a non-linear composite index method which transforms a set of individual indicators in standardized variables and summarizes them using an arithmetic mean adjusted by a "penalty" coefficient related to the variability of each unit (method of the coefficient of variation penalty).

Usage

```
ci_mpi(x, indic_col, penalty="POS")
```

Arguments

x	A data.frame containing simple indicators.
indic_col	Simple indicators column number.
penalty	Penalty direction; Use "POS" (default) in case of 'increasing' or 'positive' composite index (e.g., well-being index)), "NEG" in case of 'decreasing' or 'negative' composite index (e.g., poverty index).

Value

An object of class "CI". This is a list containing the following elements:

ci_mpi_est	Composite indicator estimated values.
ci_method	Method used; for this function ci_method="mpi".

Author(s)

Vidoli F.

References

De Muro P., Mazziotta M., Pareto A. (2011), "*Composite Indices of Development and Poverty: An Application to MDGs*", Social Indicators Research, Volume 104, Number 1, pp. 1-18.

See Also

ci_bod, normalise_ci

Examples

```
data(EU_NUTS1)

# Please, pay attention. MPI can be calculated only with two standardizations methods:
# Classic MPI - method=1, z.mean=100 and z.std=10
# Correct MPI - method=2
# For more info, please see references.

data_norm = normalise_ci(EU_NUTS1,c(2:3),c("NEG","POS"),method=1,z.mean=100, z.std=10)
CI = ci_mpi(data_norm$ci_norm, penalty="NEG")

data(EU_NUTS1)
CI = ci_mpi(EU_NUTS1,c(2:3),penalty="NEG")
```

`ci_rbod` *Robust Benefit of the Doubt approach (RBoD)*

Description

Robust Benefit of the Doubt approach (RBoD) is the robust version of the BoD method. It is based on the concept of the expected minimum input function of order-*m* so "*in place of looking for the lower boundary of the support of F, as was typically the case for the full-frontier (DEA or FDH), the order-m efficiency score can be viewed as the expectation of the maximal score, when compared to m units randomly drawn from the population of units presenting a greater level of simple indicators*", Daraio and Simar (2005).

Usage

```
ci_rbod(x,indic_col,M,B)
```

Arguments

`x`	A data.frame containing score of the simple indicators.
`indic_col`	Simple indicators column number.
`M`	The number of elements in each of the bootstrapped samples.
`B`	The number of bootstrap replicates.

Value

An object of class "CI". This is a list containing the following elements:

`ci_rbod_est`	Composite indicator estimated values.
`ci_method`	Method used; for this function ci_method="rbod".

Author(s)

Vidoli F.

References

Daraio, C., Simar, L. "*Introducing environmental variables in nonparametric frontier models: a probabilistic approach*", Journal of productivity analysis, 2005, 24(1), 93 - 121.

Vidoli F., Mazziotta C., "*Robust weighted composite indicators by means of frontier methods with an application to European infrastructure endowment*", Statistica Applicata, Italian Journal of Applied Statistics, 2013.

See Also

ci_bod, ci_bod_var_w

Examples

```
i1 <- seq(0.3, 0.5, len = 100) - rnorm (100, 0.2, 0.03)
i2 <- seq(0.3, 1, len = 100)   - rnorm (100, 0.2, 0.03)
Indic = data.frame(i1, i2)
CI = ci_rbod(Indic,B=10)

data(EU_NUTS1)
data_norm = normalise_ci(EU_NUTS1,c(2:3),polarity = c("POS","POS"), method=2)
CI = ci_rbod(data_norm$ci_norm,c(1:2),M=10,B=20)
```

---

ci_rbod_dir — *Directional Robust Benefit of the Doubt approach (D-RBoD)*

Description

Directional Robust Benefit of the Doubt approach (D-RBoD) is the directional robust version of the BoD method.

Usage

```
ci_rbod_dir(x,indic_col,M,B,dir)
```

Arguments

x	A data.frame containing score of the simple indicators.
indic_col	Simple indicators column number.
M	The number of elements in each of the bootstrapped samples.
B	The number of bootstap replicates.
dir	Main direction. For example you can set the average rates of substitution.

Value

An object of class "CI". This is a list containing the following elements:

`ci_rbod_dir_est`
: Composite indicator estimated values.

`ci_method`
: Method used; for this function ci_method="rbod_dir".

Author(s)

Fusco E., Vidoli F.

References

Daraio C., Simar L., "*Introducing environmental variables in nonparametric frontier models: a probabilistic approach*", Journal of productivity analysis, 2005, 24(1), 93 121.

Simar L., Vanhems A., "*Probabilistic characterization of directional distances and their robust versions*", Journal of Econometrics, 2012, 166(2), 342 354.

Vidoli F., Fusco E., Mazziotta C., "*Non-compensability in composite indicators: a robust directional frontier method*", Social Indicators Research, Springer Netherlands.

See Also

`ci_bod`, `ci_rbod`

Examples

```
data(EU_NUTS1)
data_norm = normalise_ci(EU_NUTS1,c(2:3),polarity = c("POS","POS"), method=2)
CI = ci_rbod_dir(data_norm$ci_norm, c(1:2), M = 25, B = 50, c(1,0.1))
```

`ci_wroclaw` *Wroclaw Taxonomic Method*

Description

Wroclaw taxonomy method (also known as the dendric method), originally developed at the University of Wroclaw, is based on the distance from a theoretical unit characterized by the best performance for all indicators considered; the composite indicator is therefore based on the sum of euclidean distances from the ideal unit and normalized by a measure of variability of these distance (mean + 2*std).

Usage

```
ci_wroclaw(x,indic_col)
```

Arguments

`x`	A data.frame containing simple indicators.
`indic_col`	Simple indicators column number.

Details

Please pay attention that *ci_wroclaw_est* is the distance from the "ideal" unit; so, units with higher values for the simple indicators get lower values of composite indicator.

Value

An object of class "CI". This is a list containing the following elements:

`ci_wroclaw_est`	Composite indicator estimated values.
`ci_method`	Method used; for this function ci_method="wroclaw".

Author(s)

Vidoli F.

References

UNESCO, "*Social indicators: problems of definition and of selection*", Paris 1974.

Mazziotta C., Mazziotta M., Pareto A., Vidoli F., "*La sintesi di indicatori territoriali di dotazione infrastrutturale: metodi di costruzione e procedure di ponderazione a confronto*", Rivista di Economia e Statistica del territorio, n.1, 2010.

See Also

`ci_bod`, `ci_mpi`

Examples

```
i1 <- seq(0.3, 0.5, len = 100) - rnorm (100, 0.2, 0.03)
i2 <- seq(0.3, 1, len = 100)   - rnorm (100, 0.2, 0.03)
Indic = data.frame(i1, i2)
CI = ci_wroclaw(Indic)

data(EU_NUTS1)
CI = ci_wroclaw(EU_NUTS1,c(2:3))

data(EU_2020)
data_selez = EU_2020[,c(1,22,191)]
data_norm = normalise_ci(data_selez,c(2:3),c("POS","NEG"),method=3)
ci_wroclaw(data_norm$ci_norm,c(1:2))
```

`EU_2020` *Europe 2020 indicators*

Description

Europe 2020, a strategy for jobs and smart, sustainable and inclusive growth, is based on five EU headline targets which are currently measured by eight headline indicators, Headline indicators, Eurostat, year 1990-2012 (Last update: 21/11/2013).

For more info, please see http://ec.europa.eu/europe2020/index_en.htm.

Usage

```
data(EU_2020)
```

Format

EU_2020 is a dataset with 30 observations and 12 indicators (190 indicator per year).

geo EU-Member States including EU (28 countries) and EU (27 countries) row.

employXXXX Employment rate - age group 20-64, year XXXX (1992-2012).

perc_GDPXXXX Gross domestic expenditure on R&D (GERD), year XXXX (1990-2012).

gas_emissXXXX Greenhouse gas emissions - base year 1990, year XXXX (1990-2011).

share_renXXXX Share of renewable energy in gross final energy consumption, year XXXX (2004-2011).

prim_enerXXXX Primary energy consumption, year XXXX (1990-2011).

final_energyXXXX Final energy consumption, year XXXX (1990-2011).

final_energyXXXX Early leavers from education and training - Perc. of the population aged 18-24 with at most lower secondary education and not in further education or training, year XXXX (1992-2012).

tertiaryXXXX Tertiary educational attainment - age group 30-34, year XXXX (2000-2012).

risk_povertyXXXX People at risk of poverty or social exclusion - 1000 persons Perc. of total population, year XXXX (2004-2012).

low_workXXXX People living in households with very low work intensity - 1000 persons Perc. of total population, year XXXX (2004-2012).

risk_povertyXXXX People at risk of poverty after social transfers - 1000 persons Perc. of total population, year XXXX (2003-2012).

deprivedXXXX Severely materially deprived people - 1000 persons Perc. of total population, year XXXX (2003-2012).

Author(s)

Vidoli F.

References

http://ec.europa.eu/europe2020/index_en.htm

Examples

```
data(EU_2020)
```

EU_NUTS1 *EU NUTS1 Transportation data*

Description

Eurostat regional transport statistics (reg_tran) data, year 2012.

For more info, please see http://ec.europa.eu/eurostat/data/browse-statistics-by-theme.

Usage

```
data(EU_NUTS1)
```

Format

EU_NUTS1 is a dataset with 34 observations and two indicators describing transportation infrastructure endowment of the main (in terms of population and GDP) European NUTS1 regions: France, Germany, Italy, Spain (United Kingdom has been omitted, due to lack of data concerning railways).

roads Calculated as (2 * Motorways - Kilometres per 1000 km2 + Other roads - Kilometres per 1000 km2)/3

trains Calculated as (2 *Railway lines double+Electrified railway lines)/3

Author(s)

Vidoli F.

References

Vidoli F., Mazziotta C., "*Robust weighted composite indicators by means of frontier methods with an application to European infrastructure endowment*", Statistica Applicata, Italian Journal of Applied Statistics, 2013.

Examples

```
data(EU_NUTS1)
```

`normalise_ci` *Normalisation and polarity functions*

Description

This function lets to normalise simple indicators according to the polarity of each one.

Usage

```
normalise_ci(x, indic_col, polarity, method=1, z.mean=0, z.std=1, ties.method ="average")
```

Arguments

`x` A data frame containing simple indicators.

`indic_col` Simple indicators column number.

`method` Normalisation methods:

- **1** (default) = standardization or z-scores using the following formulation:

$$z_{ij} = z.mean \pm \frac{x_{ij} - M_{x_j}}{S_{x_j}} \cdot z.std$$

  where $\pm$ depends on *polarity* parameter and *z.mean* and *z.std* represent the shifting parameters.
- **2** = Min-max method using the following formulation:
  if *polarity*="POS":
  $$\frac{x - min(x)}{max(x) - min(x)}$$
  if *polarity*="NEG":
  $$\frac{max(x) - x}{max(x) - min(x)}$$
- **3** = Ranking method. If *polarity*="POS" ranking is increasing, while if *polarity*="NEG" ranking is decreasing.

`polarity` Polarity vector: "POS" = positive, "NEG" = negative. The polarity of a individual indicator is the sign of the relationship between the indicator and the phenomenon to be measured (e.g., in a well-being index, "GDP per capita" has 'positive' polarity and "Unemployment rate" has 'negative' polarity).

`z.mean` If method=1, Average shifting parameter. Default is 0.

`z.std` If method=1, Standard deviation expansion parameter. Default is 1.

`ties.method` If method=3, A character string specifying how ties are treated, see `rank` for details. Default is "average".

Value

`ci_norm` A data.frame containing normalised score of the choosen simple indicators.

`norm_method` Normalisation method used.

Author(s)

Vidoli F.

References

OECD, "*Handbook on constructing composite indicators: methodology and user guide*", 2008, pag.30.

See Also

ci_bod, ci_mpi

Examples

```
data(EU_NUTS1)

# Standard z-scores normalisation #
data_norm = normalise_ci(EU_NUTS1,c(2:3),c("NEG","POS"),method=1,z.mean=0, z.std=1)
summary(data_norm$ci_norm)

# Normalisation for MPI index #
data_norm = normalise_ci(EU_NUTS1,c(2:3),c("NEG","POS"),method=1,z.mean=100, z.std=10)
summary(data_norm$ci_norm)

data_norm = normalise_ci(EU_NUTS1,c(2:3),c("NEG","POS"),method=2)
summary(data_norm$ci_norm)
```

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10.3 Vignette

Compind: Composite indicators functions based on frontiers in R

(Compind package version 1.1)

F. Vidoli and E. Fusco

September 4, 2015

Introduction

CI's methods are increasingly recognized as a useful tool in policy analysis and public communication (Nardo *et al.*, 2005) for a variety of policy matters such as public units benchmark, industrial competitiveness, sustainable development, quality of life assessment, globalization and innovation. They provide simple comparisons of units that can be used to illustrate complex and sometimes elusive issues in wide ranging fields, *e.g.* the environmental, economical, social or technological development. These indicators often seem easier to interpret by the general public finding a common trend in many separate indicators and have proven useful in benchmarking country performance.
Along such lines the Joint Research Centre of European Commission asserts that *"no uniformly agreed methodology exists to weight individual indicators before aggregating them into a composite indicator"*[1].

Several steps are involved in creating composite indicators: *investigating the structure* of simple indicators by means of multivariate statistics, handling the problem of *missing data* that can be missing either in a random or in a non-random fashion, bringing the indicators to the same unit by *normalization* and finally selecting an appropriate *weighting and aggregation* model. (for a complete explanation of every step, please see Nardo *et al.*, 2005).
A much wider ranging literature is found for the *aggregation* methods than the one regarding *weight* systems; however, the two aspects are related and interwoven and often lead to the same solutions.
Several weighting techniques exist in literature[2], derived both from statistical methodologies, such as factor analysis, *DEA* and unobserved components models (UCM), or from more specific methods like budget allocation processes (BAP), analytic hierarchy processes (AHP) or conjoint analysis (CA).

[1] `http://composite-indicators.jrc.ec.europa.eu/S6\_weighting.htm`

[2] For a complete review, please see Nardo *et al.* (2005) and Freudenberg (2003) for major applications and papers.

1

The applicative difficulties in applying composite indicators (CI) methods derived from the production frontier analysis (*i.e.* Benefit of the Doubt - BoD) have often discouraged the practical adoption of the more complex methods, while having desirable properties.
`Compind` package make comparable and easily calculable composite indicators developed with a plurality of methods and supports researcher into robustness analysis through repeated simulations on subsamples of units or variables.
Given that, the first question is: why a frontier CI package in R? Answer is easy: R is the most comprehensive statistical analysis package available (over 4800 packages), R is free, cross-platform and open source software, but especially R is a programming language (no specific pull-down menu software) allowing to rethinking CI not only as an evaluation tool, but as a part of the main research flow making easy carry on sensitivity analysis through bootstrap replications.

So the subsequent question become: how design a CI package in R? In our opinion, the package would have these basis properties:

- It has to be as simple as possible to use;
- The syntax has to be easy and independent (as possible) from the chosen method;
- Package must cover several steps of the CI calculation (not only the weighting and aggregation step).

Given these premises, `Compind` R package contains a plurality of methods can be divided into:

- Frontier methods;
- Non frontier methods;
- Utilities.

1 Frontier methods

1.1 Benefit of the Doubt approach

"The Benefit of the Doubt approach is formally tantamount to the original input-oriented CRS-DEA[3] *model of Charnes* et al. *(1978), with all questionnaire items considered as outputs and a dummy input equal to one for all observations"*, Witte & Rogge (2009).

In particular BoD approach offers several advantages:

1. Weights are *endogenously determined* by the observed performances and benchmark is not based on theoretical bounds, but it's a linear combination of the observed best performances.

[3]Constant Returns to Scale Data Envelopment Analysis.

2. Principle is *easy to communicate*: since we are not sure about the right weights, we look for "benefit of the doubt" weights (such that your overall relative performance index is as high as possible).

3. BoD CI is *weak monotone*.

So, let's draw a sample of 100 units for two simple indicators $i1$ and $i2 \in [0, 1]$ and two "particular" rows: the first one is an outlier, while the second one have a `NA` on the second indicator.

```
i1 <- seq(0.3, 0.5, len = 100) - rnorm (100, 0.2, 0.05)
i2 <- seq(0.3, 1, len = 100)   - rnorm (100, 0.2, 0.05)
dati = data.frame(i1, i2)
random1 = data.frame(i1=0.6, i2=1)
random2 = data.frame(i1=0.5, i2=NA)
Indic = rbind(dati,random1,random2)
```

As pointed out by the OECD Handbook on Constructing Composite Indicators, dataset must not contain missing data; to overcome this issue researcher can make imputation or delete the observations. For this reason, all the `Compind` functions alert users to the presence of missing values within the data (depending on the function the calculation can stop or not).

```
CI1 = ci_bod(Indic)

## Pay attention:  NA values at column:  102 , row 2 .  Composite indicator
has been computed, but results may be misleading, Please refer to OECD
handbook, pg.  26.
```

Given that, in this example, missing row has been deleted and the BoD composite indicator by `ci_bod` function recalculated; Figure 1 show the sample data highlighting the contribution of the outlier on the composite scores of the other units.

```
Indic = Indic[complete.cases(Indic),]
CI1 = ci_bod(Indic)
Indic_CI = data.frame(Indic, CI_est= CI1$ci_bod_est)
ggplot(data = Indic_CI, aes(x = i1, y = i2)) +
       geom_point(aes(colour = CI_est),size=3)
```

It may be readily noted that the BoD composite score depends exclusively on the frontier's distance; in this framework one drawback is directly linked with the *DEA* problem solution: since the weights are unit specific, cross-unit comparisons are not possible and the values of the scoreboard depend on the benchmark performance.
There are also three other drawbacks we will discuss in the following paragraphs: the multiplicity of equilibria, the lack of robustness and perfect non compensability among indicators.

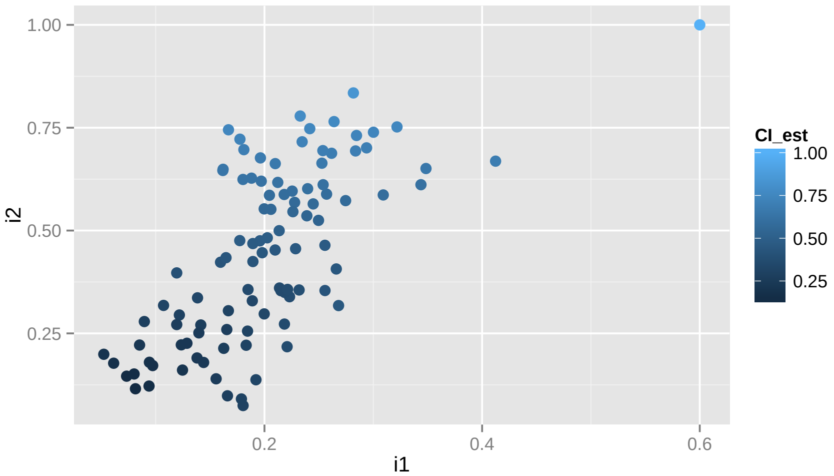

Figure 1: Simple indicators and BoD CI

1.2 Multiplicity of equilibria: Variance weighted BoD

As pointed out before, BoD formulation can hide the problem of the multiplicity of equilibria thus weights are not uniquely determined (even though the CI is unique). The weight values for the units are to be chosen from many (infinite) possibilities. It is also worth noting that multiple solutions are likely to depend upon the set of constraints imposed on the weights of the maximization problem: the wider the range of the variation of weights, the lower the possibility of obtaining a unique solution.

The optimization process could lead to many zero weights (see table 1) if no restrictions on the weights are imposed.

	Weights	Freq
1	0 - 1	75
2	1.667 - 0	26

Table 1: BoD weights

There is a wide choice for incorporating "*value judgements*" in a DEA classical model and in general in efficiency analysis (please see Allen *et al.*, 1997, Estellita-Lins *et al.*, 2007 and Thanassoulis *et al.*, 2004); three basic approaches are the most used:

- Direct restrictions on the weights;
- Adjustment of the observed input-output levels;
- Restrictions on the virtual inputs and outputs.

In recent years many additional weighting schema have been proposed (*i.e.* Rogge, 2012); Mazziotta & Vidoli (2009), for example, proposed the inclusion of additional "Assurance regions", type I (AR I) constraints in order to highlight indicators with a higher sample variance than the others.

The basic thesis involves weighting simple indicators by their own sample variance; thus, indicators with a high variability will strongly affect the composite indicator. There are however consequences to this approach: our measurement has to be read as a "gap indicator" among the unit characteristics. The preliminary hypothesis is that every single indicator $I_q, q = 1, \ldots, Q$ is a probabilistic variable, following a Normal Gaussian distribution[4]:

$$I_q \sim N(\mu_{I_q}, \sigma_{I_q}), \forall q = 1, \ldots, Q \tag{1}$$

In this way, the variance of each indicator can be computed in a standard probabilistic setting and the unbiased variance confidence interval is:

$$P(\frac{n-1}{\chi^2_{n-1,1-\alpha/2}}\overline{S}^2 < \sigma^2 < \frac{n-1}{\chi^2_{n-1,\alpha/2}}) = 1 - \alpha \tag{2}$$

which, for the sake of compactness, can be written:

$$P(low_{I_q} < \sigma^2 < high_{I_q}) = 1 - \alpha \tag{3}$$

Even when the underlying distribution is not Normal, the procedure can be still used to obtain the approximate confidence bounds for the variance estimated. If the distribution is not too far from the Normal one, we have tested the robustness of our procedure. We can use low_{I_q} and $high_{I_q}$ for each indicator to reconstruct the marginal rates of substitution among indicators:

$$\frac{low_{I_i}}{high_{I_j}} \leq \frac{w_{I_i}}{w_{I_j}} \leq \frac{low_{I_j}}{high_{I_i}}, \forall i, j = 1, \ldots, Q \tag{4}$$

When the confidence interval inferior limit of the variance is contrasted with the maximum of another, one assumes a *"benefit of doubt"* attitude in that an exact relationship among weights is not imposed, thereby establishing a range in which every unit obtains the maximum relative weight.

In `Compind` package the implementation of this model thought the `ci_bod_var_w` function is easy and quite similar to the BoD model; Figure 2 shows how the variance weighted CI is, for construction, lower than the BoD one.

```
CI_w1 = ci_bod_var_w(Indic)
Indic_CI2 = data.frame(Indic_CI, CI_w_est= CI_w1$ci_bod_var_w_est)
ggplot(data = Indic_CI2, aes(x = CI_est, y = CI_w_est)) +
       geom_point(size=3)+
       geom_abline(intercept = 0, slope = 1, linetype="dashed")+
       xlab("BoD estimated CI") +
       ylab("Variance weighted BoD estimated CI")
```

[4]To bypass this assumption, future developments of this methodology may involve the analysis of the kernel density estimate of the simple indicators and their own sample variance.

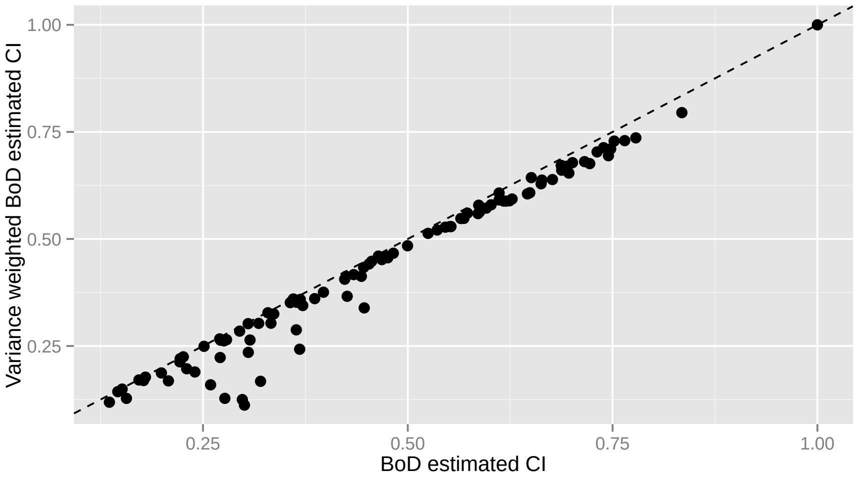

Figure 2: BoD and Variance weighted BoD estimated CI

1.3 Robust BoD

As mentioned in paragraph 1.1, one of the main drawbacks of *DEA*/*FDH* non-parametric estimators is their sensitivity to extreme values and outliers.

To introduce Robust BoD we first expose the simplified idea (based on the *Order-m* idea, Daraio & Simar, 2005).

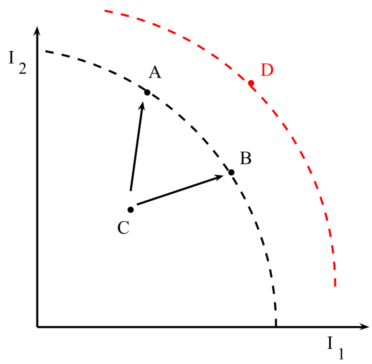

Figure 3: Outlier effects in a frontier framework

We extend the Daraio & Simar (2005) idea into CI's framework by repeatedly and with replacement drawing m observations from the original sample of n observations, choosing only from those observations which are obtaining higher basic indicators (I_1, I_2) - *red lines* - than the evaluated observation C.

In other words and practically speaking:

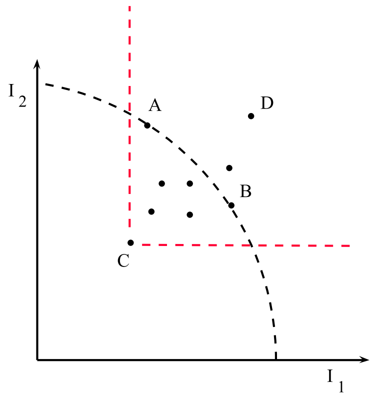

Figure 4: Support of the generic unit C

- we draw m observation only from those observations which are obtaining higher basic indicators than the evaluated observation C;
- we label this set as SET_{bm};
- we estimate BoD scores relative to this sub-sample SET_{bm} for B times;
- having obtained the B scores, we compute the arithmetic average.

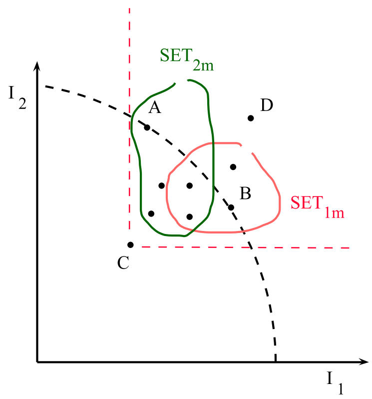

Figure 5: Order-m calculation criteria

This is certainly a less extreme benchmark for the unit C than the *"absolute"* maximum achievable level of output.
Unit C is compared to a set of m peers (potential competitors) having higher basic indicators than its level and we take as a benchmark, the expectation of the maximum achievable CI in place of the absolute maximum CI.

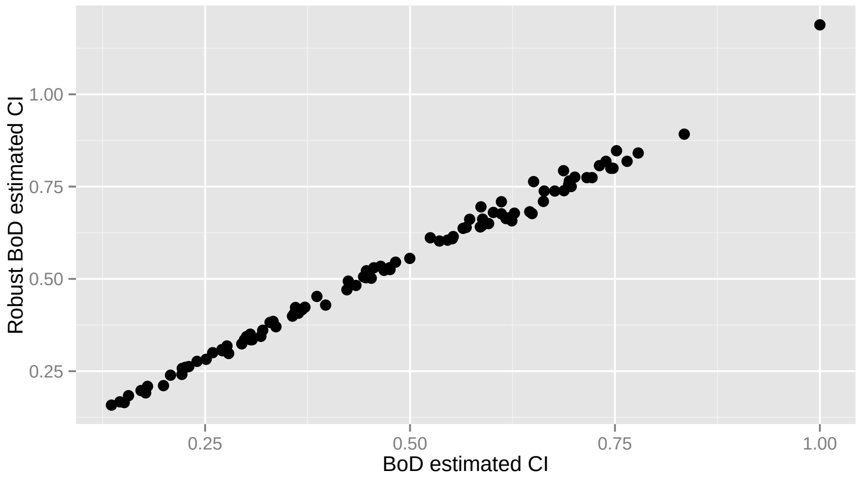

Figure 6: BoD and Robust BoD estimated CI

`Compind` package lets to calculate Robust BoD via `ci_rbod` function; two other options, respect to the `ci_bod` function, are available: `M` to fix the number of peers for the generic unit i in each sample and `B` to indicate the number of bootstrap replicates.

```
CI_r1 = ci_rbod(Indic, B=100)
Indic_CI3 = data.frame(Indic_CI2, CI_r_est= CI_r1$ci_rbod_est)
ggplot(data = Indic_CI3, aes(x = CI_est, y = CI_r_est)) +
       geom_point(size=3)+
       xlab("BoD estimated CI") +
       ylab("Robust BoD estimated CI")
```

Figure 6 allows to detect the outlier (with robust score greater than 1) and, above all, to obtain a score distribution (see Figure 7) not affected by outliers.

```
per_plot = melt(data.frame(Indic_CI3$CI_est,Indic_CI3$CI_r_est))
ggplot(per_plot, aes(x=value, fill=as.factor(variable))) +
       geom_density(alpha=.5)+
       labs(x = "Composite indicator", y="Kernel density")+
       theme(legend.position="bottom")+
       scale_fill_manual(values=c("#999999", "#E69F00"),
                    name="CI estimated value",
                    labels=c("BoD", "Robust BoD"))
```

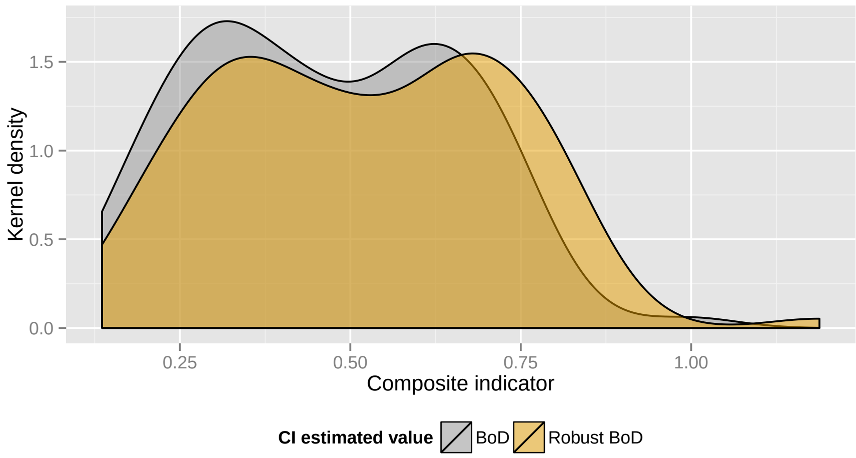

Figure 7: BoD and Robust BoD CI kernel density

1.4 Directional BoD

Most of aggregation methods assume, in weighting phase, the compensability among simple indicators (Bouyssou & Vansnick, 1986) namely allowing lower values in some indicators to be compensated by higher values in others. This property, even not verified in the practical application, is not appropriate especially if CI has to be interpreted as "importance coefficients" (Munda & Nardo, 2005).

In last years multiple solutions have been proposed to avoid this strong assumption introducing weight constraints, weighting each tensor that links the single point to the frontier (see *e.g.* Tsutsui *et al.*, 2009) or including a penalty according to the different mix of simple indicators (De Muro *et al.*, 2010).

Given that in practical application most often exist a preference structure and with the aim to respect the weakly positive monotonicity property (Casadio Tarabusi & Guarini, 2013), Fusco (2015) suggest to include in the BoD model a "directional" penalty using the directional distance function introduced by Chambers *et al.* (1998).

Even if in literature a crucial question in a directional approach is the correct choice of the direction, this issue is irrelevant with the illustration of this package and for this reason it's left to the research decisions.

To better illustrate the characteristics of the Directional BoD method the European regional transport data, year 2012, for 34 NUTS1 regions has been used[5]; Figure 8 relates the kilometres of roads and railways highlighting as, for most of the regions, the "desired" ratio can be set equal to 2 to 10.

[5]In the ode below function *normalise_ci* has been used; see paragraph 3 for more info.

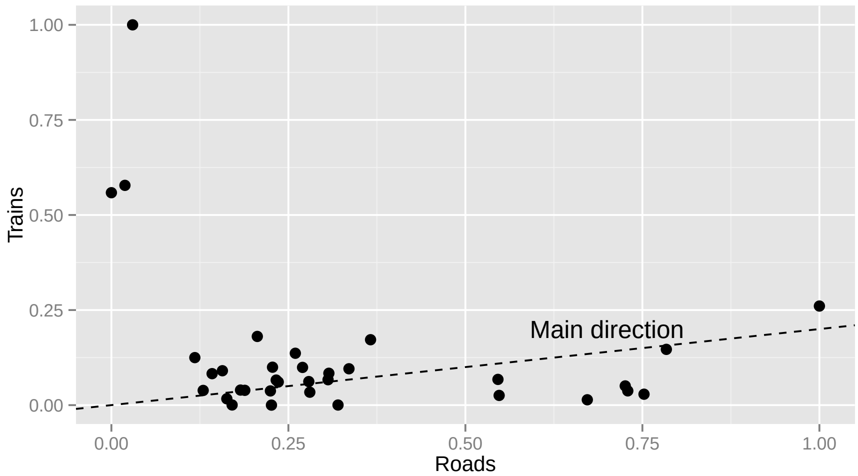

Figure 8: Eu regional transport data, year 2012

```
data(EU_NUTS1)
data_norm = normalise_ci(EU_NUTS1,c(2:3),
                         polarity = c("POS","POS"), method=2)
ggplot(data = data_norm$ci_norm, aes(x = roads, y = trains)) +
       geom_point(size=3)+
       geom_abline(intercept=0, slope=0.2, linetype="dashed")+
       annotate("text", x=0.7, y=0.2, label="Main direction")+
       xlab("Roads") +
       ylab("Trains")
```

Function `ci_bod_dir` allows to calculate Directional BoD given a direction `dir`, expressed as the ratio between the first and the second indicator; Figure 9 highlight as the main differences between BoD CI and Directional BoD CI occur for the units with the lowest values along the chosen direction.

```
CI_bod_est     = ci_bod(data_norm$ci_norm,c(1:2))
CI_bod_dir_est = ci_bod_dir(data_norm$ci_norm,c(1:2),
                            dir = c(1,0.2))
Diff = CI_bod_dir_est$ci_bod_dir_est - CI_bod_est$ci_bod_est
Indic_tot = data.frame(data_norm, Diff)

ggplot(data = Indic_tot,
       aes(x = ci_norm.roads, y = ci_norm.trains)) +
       geom_point(aes(colour = Diff),size=3)+
       theme(legend.position="bottom")+
```

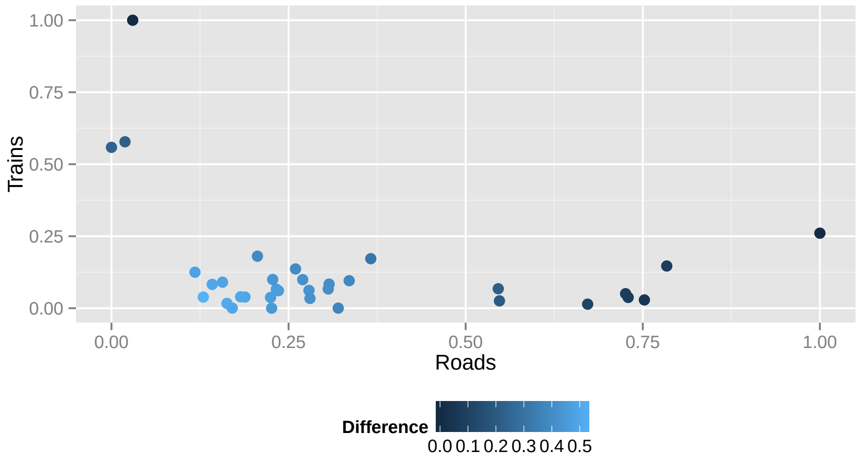

Figure 9: Eu regional transport data - difference between BoD and Directional BoD

```
        scale_colour_continuous(name="Difference")+
        xlab("Roads") +
        ylab("Trains")
```

1.5 Directional Robust BoD

Directional Robust BoD method, proposed in Vidoli *et al.* (2015), is the logical union between the Robust BoD and the directional BoD methods; Figure 10 compares the directional measure with the directional robust one, highlighting how, even in this case, the main differences occur for the units with the lowest values.

```
CI_rbod_dir_est = ci_rbod_dir(data_norm$ci_norm,c(1:2),
                              dir = c(1,0.2))
Indic_tot = data.frame(data_norm,
            CI_dir  = CI_bod_dir_est$ci_bod_dir_est,
            CI_rdir = CI_rbod_dir_est$ci_rbod_dir_est)
ggplot(data = Indic_tot, aes(x = CI_dir, y = CI_rdir)) +
        geom_point(size=3)+
        geom_abline(intercept = 0, slope = 1, linetype="dashed")+
        xlab("Directional BoD estimated CI") +
        ylab("Directional Robust BoD estimated CI")
```

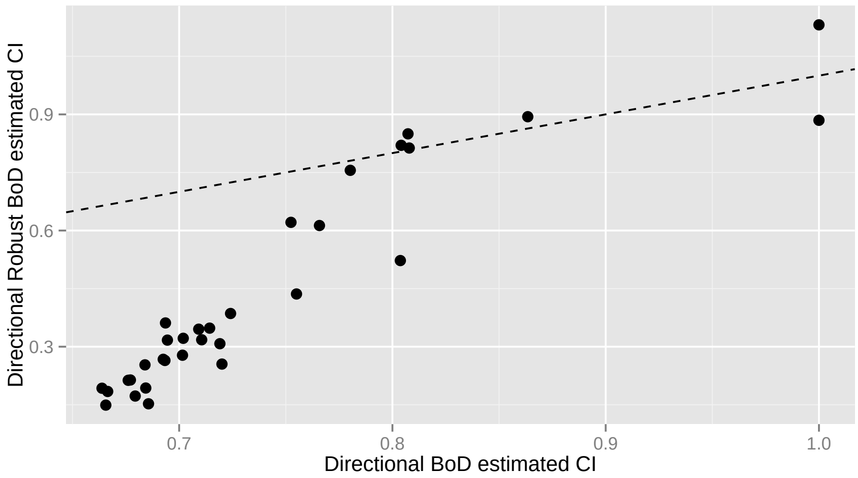

Figure 10: Directional BoD vs Directional Robust BoD estimated CI

2 Non frontier methods

This section provides some functions commonly used in the calculation of composite indicators; `Compind` implements the main methodologies proposed in the OECD manual more closely linked with mathematical procedure avoiding all methods which in some way would provide for a subjective choice of the weights.

2.1 Weighting method based on Factor Analysis

Factor Analysis (FA) aims to describe a set of Q indicators $i1, i2, \ldots, iQ$ in terms of a smaller number of m factors and to highlight the relationship between these variables. Contrary to the Principal Component Analysis, the FA model assumes that the data is based on the underlying factors of the model, and that the data variance can be decomposed into that accounted for by common and unique factors.
On the issue of how factors should be retained in the analysis without losing too much information, methodologists are divided; `Compind` package with the `ci_factor` function offers three possibilities: 1) `method="ONE"` (default) the composite indicator estimated values are equal to first component scores; 2) `method="ALL"` the composite indicator estimated values are equal to component scores multiplied by its proportion variance and 3) `method="CH"` it can be choose the number of the component to take into account.

After choosing five indicators it was applied factorial analysis choosing to weigh the scores on the three components with the associated loadings.

```
data(EU_2020)
data_norm=normalise_ci(EU_2020,c(47:51),
                      polarity = c("POS","POS","POS","POS","POS"),
                      method=2)
CI1 = ci_factor(data_norm$ci_norm,c(1:5),method="CH", dim=3)
summ = summary(as.data.frame(CI1$ci_factor_est))
print(xtable(summ,caption = "Factor Analysis scores based
             on first 3 components",label="tab_factor1"),
      include.rownames=FALSE)
```

V1
Min. :-1.6549
1st Qu.:-0.4842
Median : 0.1630
Mean : 0.0000
3rd Qu.: 0.4416
Max. : 1.1475

Table 2: Factor Analysis scores based on first 3 components

The associated loadings ..

```
round(CI1$loadings_fact,3)
```

[1] 0.698 0.285 0.010

The robustness of the results can be tested even varying the number of components; in this case it was decided to retain only the first factor (`method="ONE"`).

```
CI2 = ci_factor(data_norm$ci_norm,c(1:5),method="ONE")
summ2 = summary(as.data.frame(CI2$ci_factor_est))
print(xtable(summ2,caption = "Factor Analysis scores based
             on first component",label="tab_factor2"),
      include.rownames=FALSE)
```

CI2$ci_factor_est
Min. :-3.1144
1st Qu.:-0.2607
Median : 0.1247
Mean : 0.0000
3rd Qu.: 0.6264
Max. : 1.3446

Table 3: Factor Analysis scores based on first component

It can be noted however very good correlation between the two scores (0.926).

2.2 Weighting method based on geometric aggregation

Geometric aggregation (GA) is a simple method less compensatory approach than the additive ones; in other terms, units with low scores in some indicators would prefer a linear rather than a geometric aggregation, that is an increase in an indicator value would have higher marginal utility on the composite indicator if the indicator value is low.
Since in GA compensability degree is not constant, because is higher for composite indexes with high values and vice versa, units with low scores tend to prefer use of linear aggregation, trying to improve their position in ranking.
The implementation in `Compind` package is trivial.

```
data(EU_NUTS1)
CI_geom_estimated = ci_mean_geom(EU_NUTS1,c(2:3))
```

2.3 Mazziotta-Pareto Index (MPI) method

The MPI is a non-linear composite index which, starting from a linear aggregation, introduces a penalty for the units with unbalanced values of the indicators (De Muro *et al.*, 2010). It is composed of two parts (a measure of the mean level and a measure of the amount of unbalance) and, differently from other methods, may be used for building both "positive" and "negative" composite indices (`penalty` direction).
MPI method need to normalize simple indicator following two standardizations methods:

- For classic MPI it must use `normalize_ci` function with `method=1`, `z.mean=100` and `z.std=10`;
- For Correct MPI it must use `normalize_ci` function with min-max standardization (`method=2`).

```
data(EU_NUTS1)
data_norm = normalise_ci(EU_NUTS1,c(2:3),
                         c("NEG","POS"),
                         method=1,z.mean=100, z.std=10)
CI_pi_estimated = ci_mpi(data_norm$ci_norm, penalty="NEG")
```

2.4 Mean-min Function

The Mean-Min Function (MMF), proposed by Casadio Tarabusi & Guarini (2013), can be seen as an intermediate method between arithmetic `mean`, according to which no unbalance is penalized, and `min` function, according to which the penalization is maximum. It depends on two parameters that are respectively related to the intensity of penalization of unbalance ($\alpha, 0 \leq \alpha \leq 1$)

and the intensity of complementarity ($\beta, \beta \geq 0$) among indicators.
MMF index can be expressed as:

$$MMF_i = M_{Z_i} - \alpha(\sqrt[2]{(M_{Z_i} - min_j(z_{ij})) + \beta^2} - \beta) \qquad (5)$$

where Z is the normalized matrix of the data.
The function reduces to the arithmetic `mean` for $\alpha = 0$ (in this case β is irrelevant) and to the minimum function for $\alpha = 1$ and $\beta = 0$. Moreover, with $\alpha = 1$ the function has incomplete compensability; with $\beta = 0$ and $0 \leq \alpha \leq 1$ it has proportional compensability.
Therefore, authors write that: "*by choosing the values of parameters appropriately one can obtain the form of this aggregation function that best suits the specific theoretical approach*".
Once fixed α and β, the implementation in `Compind` package is trivial.

```
data(EU_NUTS1)
CI_mean_min_estimated = ci_mean_min(EU_NUTS1,c(2:3),
                                    alpha=0.5, beta=1)
```

2.5 Wroclaw Taxonomic Method

Wroclaw Taxonomic Method is a technique originally developed at the University of Wroclaw, which has experienced a fairly widespread in Italy, especially for the development of economic and social indicators (see *e.g.* Schifini D'Andrea, 1982; Quirino, 1990; Mazziotta, 1998) and recently by Cwiakala-Malys, 2009.
It's based on a very simple principle: the benchmark is the one that has the least distance from an "ideal" unit, characterized by the best performance for all the indicators considered; following the calculation of (Euclidean) distances of all units by the "ideal" one, it can build a list in which the different units are ordered in proportion with the distance from the optimum situation.
The implementation in `Compind` package is trivial.

```
data(EU_NUTS1)
CI_wroclaw_estimated = ci_wroclaw(EU_NUTS1,c(2:3))
```

3 Utilities: Normalisation and polarity functions

Although presented at the end, the `normalize_ci` is a crucial function that lets to normalise simple indicators according to the polarity of each one.
`Compind` provides three different methods: the standardization or z-scores (`method=1`), the min-max method (`method=2`) and the ranking method (`method=3`); each method provides for the indication of the polarity of the single indicator in order to obtain standardized indicators with the same polarity.

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Part IV

Empirical applications

Chapter 11

SSFA: TWO APPLICATIONS ON AGRICULTURAL SECTOR IN ITALY

11.1 Spatial stochastic frontier models:controlling spatial global and local heterogeneity

Fusco, E. and Vidoli, F. (2013). *Spatial stochastic frontier models: controlling spatial global and local heterogeneity*. International Review of Applied Economics, 27(5):679–694, http://dx.doi.org/10.1080/02692171.2013.804493.

11.1.1 Introduction

The SSFA model outlined in Chapter 5 has also been applied to the agricultural sector in Italy for 2009.

The main reason for this choice is that agricultural production, as well as the individual firm's productivity, can be strongly influenced by random factors such as climatic and local factors related to the specific characteristics of the region or technical specifications, such as "protected designation of origin" (PDO).[11]

In particular, the Organisation for Economic Co-operation and Development (OECD) (see *e.g.* OECD (2011)) suggest that the main determinants of agricultural competitiveness are either the farm size and the farm specialization linked to consumer demand, either the natural environment and the presence of a specialized agri-food industry (for the complete literature, please see Latruffe, 2010). These key determinants can be ideally divided into two groups: *external* factors that involve the firm's production, on which the company cannot act and therefore suffers, and *internal* factors on which the company may develop policies for improvements.

Agricultural production efficiency, therefore, as also noted by Coelli (1995) in his pioneering work, must be estimated *"using the stochastic frontier method because measurement error, missing variables and weather play a significant role in this field"*.

In addition to random factors outlined by Coelli (1995), there are many other factors which affect, both qualitatively and quantitatively, the production and its commercialization like the soil conformation, the rigid production rules in the PDO or in protected geographical indication (PGI) products areas or the presence of a specific local district.

[11] Council Regulation (EEC) No 2081/92 of 14 July 1992.

11.1.2 Variables and data

These are the specific spatial factors that we want to highlight, analyzing Italian FADN[12] data, year 2009, specifically focusing our attention on 975 wine companies[13] located throughout the country, but with well-defined spatial cluster, as shown in Figure 11.1 for the Trentino-Alto Adige (north), Tuscany (centre) and Apulia (south) regions.

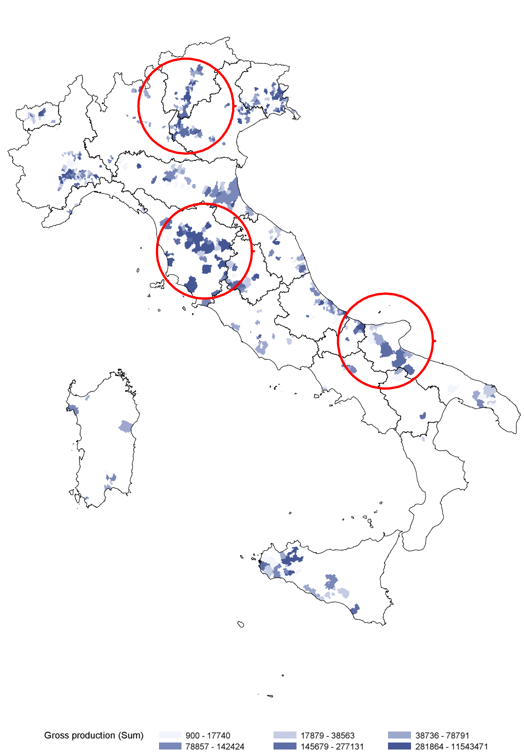


Figure 11.1: Wine gross production at the municipal level, (source: INEA 2009)

In Italy, FADN survey is carried out every year by the National Institute of Agricultural Economics (INEA), as the liaison agency between the EU and each Member State. The European Commission provides guidelines to define the instructions and recommendations for the design of the FADN selection plan. It must

[12]The Farm Accountancy Data Network (FADN) is an annual survey carried out by the Member States of the European Union and it represents an instrument to evaluate the income of agricultural holdings and the impacts of the Common Agricultural Policy. Derived from national surveys, FADN is the only source of micro-economic data that is harmonised (in other words the bookkeeping principles are the same in all countries). For further information on the FADN methodology for the selection and extrapolation to the population, please see the EU FADN at http://ec.europa.eu/agriculture/rica/methodology1_en.cfm.

[13]The Community typological scheme provides 58 different combinations of production which are grouped into three successive levels of detail: General, Main and Particular farm type. "Specialist wine" correspond to Main farm type = 31, (Particular farm type: 3110, 3120, 3130).

ensure the representativeness of the returning holdings as a whole and defines the number of farms to be selected by region, farm type and classes of economic size and also specifies the rules applied for selecting the holdings. The Italian FADN sample is selected using the stratified random sampling technique and comprises only commercial farms, or those with an economic size of more than 4 Economic Size Units (ESU).[14]

Following recommendations outlined in earlier studies (see *e.g.* Hani et al., 2003 and Reig-Martinez et al., 2011) concerning production in agriculture, we have built a basic analysis framework schematically illustrated in Table 11.1.

Table 11.1: Variables involved in the estimation.

Variable	Role in the analysis	Abbreviation
Gross production	Output	PL
Land farm capital	Input	CAP_L
Operating farm capital	Input	CAP_O
Agricultural employment	Input	LAB
Energy and Water consumption	Input	E_W

As shown in Table 11.1, the response variable (PL) represents gross production. It would have been preferable to choose a measure of physical output in order to sterilize the influence of agricultural prices, which differ greatly in various Italian regions, but it was not possible due to a lack of data.[15] Labor (LAB, measured in terms of total number of hours worked per year) and capital (CAP_L, land farm capital and CAP_O, operating farm capital) are the aggregate inputs usually included in the production function estimation. We have also included one other input variable that, from an economic point of view, characterizes environmental sustainability also: E_W, Energy and water consumption.

Since the farms' geolocation[16], were not included in the FADN database, we have calculated a distance matrix between municipalities using the shape file provided by Italian National Institute of Statistics at the municipal level and selecting a threshold distance of 50 km radius. Since different farms could belong to the same municipalities (we provide an extract in Table 11.2 as an example), we have calculated

[14]EEC Regulation No. 1859/82 establishes the minimum threshold of economic size for inclusion in the FADN field of observation. The economic size of a farm is defined by the total standard gross margin expressed in ESU, where 1 ESU corresponds to 1,200 €.

[15]This critical issue, however, is partially mitigated in our analysis in that we had considered an homogeneous production sector.

[16]This is a shortcoming that in the future will be resolved.

a 975×975 distance matrix (called W_2), which contains the distance value between each municipal centroid, if the companies are based in different municipalities, and 0 otherwise.[17] This matrix was finally standardized by row, dividing each value for its row total Tot_i.

Table 11.2: Matrix W_2: an extract. Distance in km

Firm		Municipality				
		Alba	Barbaresco	Barbaresco	Barolo	...
ID_1	Alba	0	$5.084/Tot_1$	$5.084/Tot_1$	$10.730/Tot_1$	...
ID_2	Barbaresco	$5.084/Tot_2$	0	$1/Tot_2$	$15.798/Tot_2$	...
ID_3	Barbaresco	$5.084/Tot_3$	$1/Tot_3$	0	$15.798/Tot_3$	...
ID_4	Barolo	$10.730/Tot_4$	$15.798/Tot_4$	$15.798/Tot_4$	0	...
ID_5	...	...	...	...	...	...

11.1.3 Estimation

In order to test the absence of spatial heterogeneity, we used the Moran's I index (see Anselin, 1995); in our application we found a positive global spatial autocorrelation (value: 0.12).

However global spatial auto-correlation statistics, like Moran's I, are based on the assumption of stationarity or structural stability over space, which is unrealistic in many contexts. Spatial association can also be detected using local spatial auto-correlation indices which allow for *local instabilities* in overall spatial association, instabilities that are the most interesting "object" in our application.

Anselin (1995) has shown that Moran's I can be decomposed into local values, introducing the LISA index.

[17]If two companies belonged to the same municipality, we chose to assign a dummy distance of 1 km.

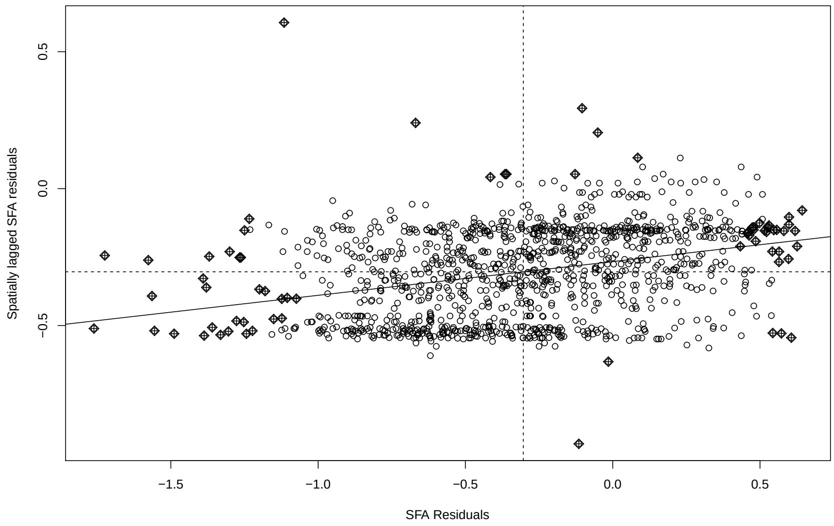


Figure 11.2: Moran LISA plot - SFA residuals VS Spatially lagged SFA residuals

Figure 11.2 shows a positive relation indicating positive spatial autocorrelation.[18] In particular, the first and the third quadrants (high-high, low-low) show spatial cluster, while the second and the fourth (high-low, low-high) show spatial outliers.

Instead of simulated data example, where we induced a strong global spatial auto-correlation, the global Moran's I in the agricultural data appears very low; however, we are interested in removing the effects of global heterogeneity, and above all, in sterilizing the local effects on some specific spatial clusters of units.

Following steps given in the previous example, we applied both SFA and SSFA methods, obtaining the results shown in Table 11.3.

Once the relative robustness of SFA estimates are verified on the *"average level"*, we are interested in the presence of a local spatial heterogeneity. To test local perturbations we analyzed the differences in terms of efficiency estimated between the SFA model with and without spatial interactions, calculating the following measure of distance d_i:

$$d_i = \frac{\text{Eff}_{SFA} - \text{Eff}_{SSFA}}{\text{Eff}_{SFA}} * 100, \forall i = 1, \ldots, n \tag{11.1}$$

The measure d_i allows the spatial evaluation of two effects: the absolute magnitude of the effect of territory on the efficiency of each company, while the sign shows whether the interdependencies between firms are positive or not (a negative d_i shows

[18] The slope of linear scatterplot smoother is equal to the global Moran's I.

Table 11.3: Estimation results by method. Wine firms.

	SFA	SSFA
Intercept	1.9422	2.0704
β_1 - Agricultural employment	0.8388	0.8535
β_2 - Land farm capital	0.0423	0.0411
β_3 - Operating farm capital	0.1857	0.1823
β_4 - Energy and Water consumption	0.0549	0.0552
σ_u	0.5165	0.5002
σ_v	0.2514	0.2537
Moran's I	0.12	0.02
ρ	-	0.2734

a positive local effect of territory); this is because even in the face of a global index of high spatial dependence, dependencies do not occur uniformly over the whole territory examined.

To test the results obtained, especially with regard to the existence of spin-offs between contiguous firms, we have chosen three traditionally important wine regions: Trentino-Alto Adige, Tuscany and Apulia.

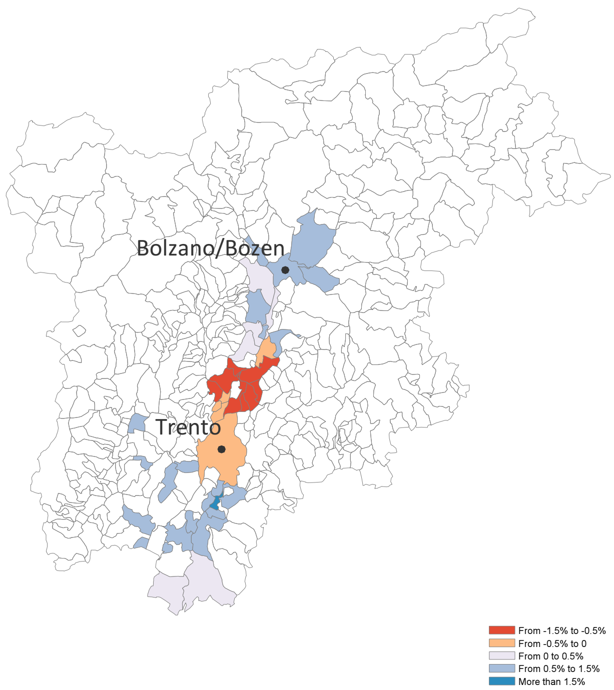


Figure 11.3: Percentage of difference between non-spatial vs spatial SFA - Region: Trentino-Alto Adige, (source: INEA, 2009)

Regarding the region of Trentino-Alto Adige, our analysis shows evident spin-offs between contiguous firms in the so-called *"South Tyrolean Wine Road"*[19], a production zone in the province of Trento. These spatial spinoffs, however, seem to gradually decline as we move away from a cluster of highly specialized municipalities.

Another well-known production area is where some of the most famous Italian red wines come from, *"Chiantishire"* in the province of Siena.

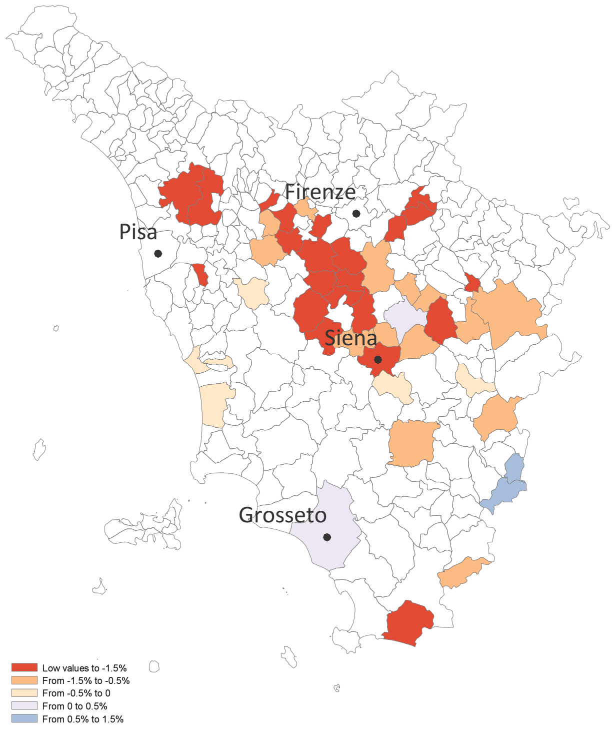


Figure 11.4: Percentage difference between non-spatial vs spatial SFA - Region: Tuscany, (source: INEA, 2009)

In Tuscany, the presence of production consortia and very strict rules of production, create a common way of producing with strong local links over a wide area.

[19]This area is particularly famous for its white and sparkling wine production.

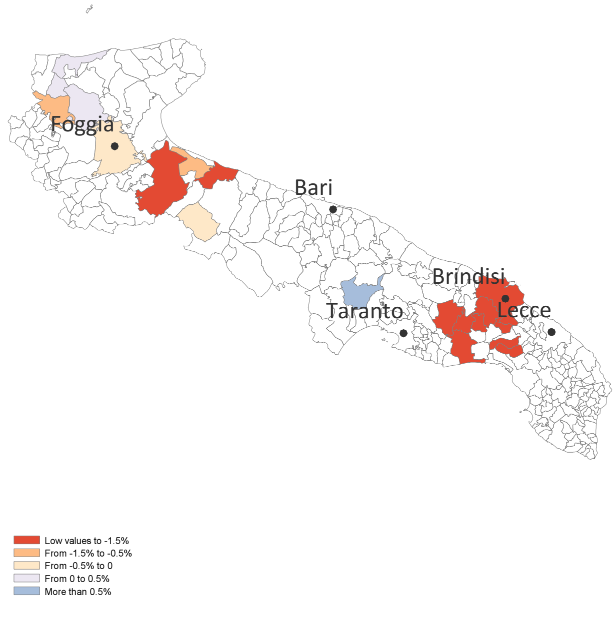


Figure 11.5: Percentage difference between non-spatial vs spatial SFA - Region: Apulia, (source: INEA, 2009)

In the last region, the Apulia,(see Figure 11.5) the production of "protected designation of origin" wines is geographically very localized, in particular the areas of *Negramaro* and *Primitivo di Manduria* wine production, near Brindisi.

11.2 Spatial nonstationarity in the stochastic frontier model: an application to the Italian wine industry

Vidoli, F., Cardillo, C., Fusco, E., and Canello, J. (2016). ***Spatial nonstationarity in the stochastic frontier model: an application to the italian wine industry.*** **Regional Science and Urban Economics - accepted for publication.**
http://dx.doi.org/10.1016/j.regsciurbeco.2016.10.003

11.2.1 Introduction

The globalization of productive processes and liberalization of trade activities have generated a strong competition between regional economic systems: paradoxically, rather than drastically reducing the role of spatial proximity, this new open scenario has shed new light on the key relevance of local and agglomeration externalities in the generation of competitive advantage (Porter, 2000). The relevance of these aspects is particularly evident in certain sectors, such as wine production, where the rapid transformations which have taken place in the last few decades have fostered a rapid process of technological change in which firms are constantly required to be at the forefront of the productive process in order to survive in the competitive arena (Cusmano et al., 2010). In this context, the role of intangible factors associated with the 'business climate' is crucial in stimulating the process of knowledge accumulation and learning through continuous interaction with peers located in close proximity: in several circumstances, the presence of these mechanisms ensures the diffusion of new productive practices that prevents local firms from increasing their gap with the technological frontier.

The classical stream of literature linking productive efficiency to territorial determinants assumes that the dynamic process leading firms to concentrate in specific subregions is only associated to specific tangible aspects: this assumption leads to neglecting the role of spatial non-stationarity, intended as "*a condition in which a simple global model cannot explain the relationships between some sets of variables*" (Brunsdon et al., 1996). This problem is particularly evident in the parametric frontier framework, where it is essential to specify a priori an explicit functional form of the boundary of the production set: however, in the early contributions the spatial dependence among productive units has often been ignored and associated to the stochastic error. A number of recent works have attempted to address this issue by specifically including a set of contextual factors in the model (see *e.g.* Hughes et al., 2011,Brehm, 2013): however, such a strategy is not always effective as it ignores the fact that the relationship between the dependent variable and the covariates (a) tends to vary in a *continuous* rather than a discrete manner among spatial units and (b) may not be necessarily related to measurable local factors. This problem is particularly evident in specific spatial contexts, such as industrial districts, characterized by the presence of global intangible factors that cannot be measured empirically (Vidoli and Canello, 2016).

Ignoring spatial autocorrelation among residuals limits the validity of the empirical investigation for several reasons. First, it causes serious consequences to statistical inference, reducing both the efficiency and consistency of the estimations and generating a negative impact on the validity of testing procedures and on the predicting capability of the model. This drawback generates significant distortions in the interpretation of the stochastic frontier model, as higher values of the inefficiency term may be associated with a territorial effect rather than the ability of a productive unit to generate more output with the same amount of inputs. In this respect, the inclusion of spatial autocorrelation into the stochastic frontier production framework has been the subject of a lively debate in the econometrics literature in the recent past, generating a multitude of approaches aimed to address this issue: in this context, the SSFA specification proposed by Fusco and Vidoli (2013) appears to be particularly suitable in that the spatial autoregressive specification is modelled in the error term, generating results that can be directly compared with those of the classic stochastic frontier approach.

In this paper, the above mentioned spatial stochastic frontier approach is implemented in a sample of Italian firms specialized in wine production, using data extracted from the 2013 FADN Survey. This archive is particularly suitable for the scope of the analysis, as it allows to account for a wide variety of structural and economic factors that are believed to influence the territorial effects: moreover, the presence of specific reference to a wide variety of inputs allows to build a solid production function with several benefits for the estimation process. The aim of the empirical exercise proposed in this paper is to evaluate the contribution of both tangible and intangible factors in influencing the performance of these firms, discussing how the space can play a different role for the different members of a local network. In this respect, the specification proposed is of particular use as it allows to isolate the local intangible factors, often statistically and economically difficult to capture through specific proxies, that nonetheless are determinant in influencing the firms productivity. The role of tangible factors is nonetheless evaluated through a second stage estimation.

11.2.2 The recent trends in the Italian wine industry and the role of agglomeration effects

Italy has played, together with France, a key role in the wine industry for several decades, dominating the international scenario in terms of both exported volumes and values. This established pattern has radically changed since the 1990s, when the entrance of the New World producers (United States, Australia, Chile, Argentina, South Africa) in the global market has fostered a radical transformation of the existing competitive arena (Cusmano et al., 2010). The increased complexity of the new global environment has further been influenced by several exogenous risk factors, such as the increased climate variability and the radical changes in wine consumption habits, with a shift in preferences towards high quality wines (Bardaji and Iraizoz, 2015). In this context, the sector has experienced a process of rapid modernization and technological change, identified by Crowley, 2000 as a "*wine revolution*". This radical transformation process is pushing wine producers to adopt improvement strategies in the quality and production process and acquire new knowledge in order to effectively respond to the volatile needs of the global markets. The potential gains from selecting an effective strategy are especially important in the wine sector where, despite the existence of a moderate correlation between price and quality, several price setting possibilities are available for wine producers given the incomplete quality information held by consumers (Oczkowski and Doucouliagos, 2015).

The current scenario generates several challenges for the Italian wine sector, which has recently faced a significant downturn in domestic demand and is characterized by a higher degree of fragmentation relative to other countries, such as Australia or Chile (Cusmano et al., 2010). Italian wine producers are often small and medium businesses that lack the financial and managerial resources to handle the increased complexity of the surrounding environment. This limit is especially problematic in the new global context, characterized by the constant need to update productive knowledge and acquire new skills and competences. In fact, small businesses are not generally equipped to gather relevant information outside the locality in which they are embedded; moreover, they cannot rely on the same formal channels used by leader firms, such as formal collaborations with research institutions (Giuliani et al., 2010) and interaction with foreign competitors, often through the presence of foreign subsidiaries (Felzensztein and Deans, 2013) or simply through the creation of relational networks with producers that are at the forefront of the industry (Turner, 2010). Given these opportunities are not generally accessible to small producers, the main source of learning and developing new competencies is the community in which these entrepreneurs are embedded. The role of this factor is especially relevant in Italy, where contrary to other countries, such as Australia, the wine sector cannot count on institutional assets and top-down measures to stimulate the above mentioned upgrading process.

Although several contributions have stressed the increasing importance of codified knowledge in the wine industry (*e.g.* Giuliani, 2007), the sector is still dependent on context-specific and localized informal practices of learning, that are crucial to take advantage of the specificities of each *terroir* (Turner, 2010). The effect generated by the local business climate and the informal interaction among local actors can be explained through the concept of “industrial atmosphere”, that has generally been used in the industrial districts literature (Marshall, 1920): other than generating tangible benefits such as reputation, greater international demand and access to skilled labour pool, spatial proximity among wine producers stimulates everyday interaction, facilitating the opportunities for face-to-face contacts that are crucial to generate tacit knowledge flows and incremental learning. In this respect, wine clusters can be seen as communitarian networks, characterized by resource sharing and continuous informal interaction (Turner, 2010). The presence of interpersonal networks can be beneficial in many respects: producers can be rapidly informed of the presence of new business opportunities, but also of new sellers or providers that can form new partnerships and generate further spillovers. Inter-firms market cooperation can also foster marketing collaboration strategies, facilitating development of joint sales in foreign markets and allowing to overcome the limited exporting capabilities of several small and medium firms (Felzensztein and Deans, 2013). More importantly, the presence of a collaborative environment can allow small producers to fill the technological gap with competitors, as collaboration can foster the shared use of new technology, exchange of technical advices and information on the effective use of machinery and inputs (Morrison and Rabellotti, 2009).

A certain number of agglomeration externalities generates spontaneously as a consequence of spatial proximity between wine producers. For example, the successful performance of neighbouring wineries stimulates the development of positive marketing-related externalities for the whole area (Giuliani and Bell, 2005): these positive spillovers in terms of reputation for the neighbouring producers have been classified by Beebe et al. (2013) as “*halo*” effect. However, spatial proximity itself is not sufficient to guarantee the diffusion of agglomeration externalities among all the members of a local community. Indeed, two elements are required to enhance this process, *i.e.* the willingness of givers to share their knowledge and the absorptive capacity of the receiver: these conditions are generally met when the cognitive proximity among the members of a network is present (Boschma, 2005). The presence of diversified abilities/attitudes to access to local informal knowledge has been documented in different regional contexts in the wine industry (Giuliani and Bell, 2005; Morrison and Rabellotti, 2009): according to Giuliani and Bell (2005), the presence of barriers to knowledge exchange is testified by the presence of different production methods within the same wine cluster. What is the profile of those firms which are more often engaged in networking activities? The core of these local networks is generally represented by small firms, which are generally more inclined to cooperate and share information in order to overcome their structural limits: the lack of competencies among small firms act as stimulus to share different experiences and spread knowledge among the community. On the other hand, large firms tend to be located at the periphery of the local network and provide a limited contribution to the local learning system: these actors generally have stronger connections with external sources of knowledge and often prefer to share the acquired competencies with a restricted number of partners that are directly involved in their production process (Morrison and Rabellotti, 2009). This trend is confirmed by the empirical investigation of Turner (2010), who has shown that small wine producers are more interested with marketing practices associated with the territory while large firms are more interested in developing their own brand.

The brief review presented in this section has shown that the tangible and intangible local effects play a key role in determining the performance of wine producers. Against this background, the aim of the following section is to propose an empirical framework that can be effectively used to account for both effects in the estimation of productive efficiency, allowing to evaluate the role of the different spatial factors in a consistent manner.

Therefore, in this Section, SSFA is used also to evaluate the role of local effects in the Italian wine industry: the recent technological advances in the sector have increased the importance of both the tangible and intangible factors associated with the specific territorial effects that cannot always be captured by the inclusion of contextual variables. In this respect, the spatial technique proposed in the previous section appears to be especially effective in accounting for these factors and evaluating their role in influencing firm-level efficiency. The empirical investigation is focused on year 2013 and is implemented on a detailed database that includes a wide variety of economic and structural variables: the main features of this database are presented in the following subsection.

11.2.3 Variables and data

The Farm Accountancy Data Network (FADN) is a yearly survey carried out by the Member States of the European Union and established in 1965 by the Council

Regulation No 79/65/EEC: this measure was aimed to establish a network for the systematic collection of accountancy data on incomes and business operations of agricultural holdings in the European Economic Community. According to Regulation n. 1859/82, this database includes all the agricultural holdings having an economic size equal to or greater than a minimum threshold, *i.e.* that identified to be considered commercial: in Italy, this threshold is set in 4,000 €of standard output. The selection of the holdings that take part in the survey is carried out according to sampling plans defined at the national level, following the guidelines and recommendations provided by the European Commission. The sampling procedure must ensure the representativeness of the identified subset and defines the number of farms to be selected, specifying the approach followed to select the productive units. According to FADN methodology, stratification variables are territorial location, economic size and type of farming.

The Italian section of the survey is based on the Agricultural Census, updated on a two-year basis by the Farm Structure Survey (FSS) carried out by ISTAT: this main data source is complemented with further sources of agricultural statistics. The Italian FADN sample is selected using the stratified random sampling technique described above: in particular, the territorial location corresponds to the 21 administrative regions; the economic size is expressed in terms of Standard Output and defined through several classes, the lowest of which starts from 4,000 €and the highest refers to those with more than 3 million €; the type of farming corresponds to the particular level grouped according to the importance of the specific agricultural activity in the region. According to the procedure, some types of farming and some classes of Standard Output could be aggregated in order to have a sufficient number of observations in each strata. Following the above mentioned approach, the 2013 version of the Italian FADN survey, which is the one used in this paper, includes a total number of about 700,000 farms.

The productive units of the survey are allocated in each stratum according to strategic variables such as Standard Output, Utilized Agricultural Area, Livestock Units and Working days. To get the desired level of precision for each strategic variable sampling errors are fixed, in terms of percentage of coefficients of variation[20], they represent the errors that may possibly occur, with a fixed probability, estimating a variable compared to its real value, hence they determine the reliability of estimates. Sample size and its distribution among the strata are established by setting the precision required in terms of percentage of coefficients of variation for strategic variables, both at national and at regional level. The methodology used to allocate

[20]The coefficient of variation of a variable is the ratio between the standard deviation of the variable layer and the estimate of the total layer of variable.

the sample among the strata is a combination of Neyman and Bethel methods (Bethel, 1989).

The main benefits associated with the use of this database can be summarized by the following two aspects:

- harmonization: FADN is the only source of micro-economic data that is harmonised at European level, *i.e.* the book-keeping principles are the same in all countries, and it represents an important tool for the evaluation of the income of agricultural holdings and the impacts of the Common Agricultural Policy. In Italy the FADN survey is carried out by the Center for Policy and Bio-economy of the Council for Research in Agriculture and the Agricultural Economics Analysis - CREA[21], as liaison agency between EU and Member State.

- information assets: The FADN survey collects more than 1,000 variables that refer to physical and structural data, such as location, crop areas, livestock units, labour force. It also contains economic and financial data, such as the value of production of the different crops, stocks, sales and purchases, production costs, assets, liabilities, production quotas and subsidies, including those connected with the application of CAP measures and recently information linked to environmental aspects was also added. These variables are extremely convenient for the purpose of this paper, as they allow to create both a solid production function and include a wide variety of local effects that are associable with the performance of these firms.

In this paper a sub sample of 853 wineries has been extracted from the Italian FADN database, which includes a total number of 11.319 farms in year 2013. Using this data, the application presented in the following sections compares the results of the traditional specifications of the production function with those of the SSFA model, showing the benefits associated with the use of the latter approach. An important caveat relates to the variable used to evaluate output, *i.e.* the litres of wine produced by each unit: given the information available it is not possible to evaluate the qualitative aspects of production (which are nonetheless relevant in the sector), the concept of efficiency should be interpreted from a technical point of view, avoiding any considerations on the quality of output produced.

[21] Previously National Institute of Agricultural Economics - INEA.

11.2.4 Estimation

The production function of the Italian wine firms has been initially estimated using a simple OLS approach, choosing a Cobb-Douglas[22] log-log functional form and relating the produced quantity of output with labour, machinery, water-energy-fuel and land capital inputs. The basic statistics of the variables used in the analysis and the relative units of measurement are given in Table 11.4.

Statistic	N	Mean	St. Dev.	Min	Max
Production output (Physical units) (log)	853	6.599	1.236	2.890	10.953
Labour input (Hours) (log)	853	8.071	0.746	6.125	11.613
Machinery capital input (Kw) (log)	853	4.672	0.786	1.609	7.553
Water, energy and fuel input (€) (log)	853	5.837	2.331	0.000	12.468
Land capital input (Ha) (log)	853	6.806	1.097	3.434	10.787
OTE = 3510 (yes)	853	0.715	0.452	0	1
Physical disadvantage (slope) (yes=1)	853	0.216	0.412	0	1
Physical disadvantage (climate) (yes=1)	853	0.284	0.451	0	1
Biophysical disadvantage (yes=1)	853	0.699	0.459	0	1
Economic infrastructure indicator (index)	853	96.038	57.679	26.668	397.647
Network infrastructure indicator (index)	853	86.356	30.926	18.400	187.976
Scholastic drop-out indicator (index)	853	13.499	4.482	8.394	24.026
DOCG production area (yes=1)	853	0.356	0.479	0	1
EU subsidies (1,000 €)	853	3.339	8.093	0.000	87.108
Financial charges (1,000 €)	853	−0.313	2.771	−51.183	0.000
Family owned (Direct family members=1)	853	0.431	0.496	0	1
Gender (M=1)	853	0.779	0.415	0	1
Young owner (yes=1)	853	0.117	0.322	0	1
Diversified production (yes=1)	853	0.109	0.312	0	1
Organic production (yes=1)	853	0.036	0.187	0	1

Table 11.4: Production function variables - main statistics

The results of the estimation (Table 11.5) appear to confirm the validity of the specification, given the significance of all covariates and the high $R^2 = 0.593$; it is also worth noting that the intercept is negative and statistically significant.

Using OLS as baseline for the analysis, the stochastic frontier model has been estimated (Table 11.6) and the results of the two specifications compared: the analysis confirms the stability of the latter model, since the values of the coefficients are similar in the two cases, except for the intercept that decreases in absolute value. This trend is expected and can be explained by the fact that the production function has been shifted from the average values to efficient ones without affecting the relationship between output and inputs. The specific parameters of SFA (σ^2, γ and the average efficiency equal to 0.59) confirm the validity of the proposed model.

[22] This model has also been estimated using a Translog specification for the production function. However, given the lack of significance of composite terms, a simpler model has been chosen for this part of the analysis.

	Dependent variable: production output (log)
Intercept	−2.068*** (0.309)
Labour input	0.641*** (0.055)
Machinery capital input	0.307*** (0.045)
Water, energy and fuel input	0.067*** (0.014)
Land capital input	0.245*** (0.037)
Observations	853
R^2	0.593
Adjusted R^2	0.591
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 11.5: Wine production function - OLS estimators

In particular, $\gamma = \frac{\sigma_u^2}{\sigma^2}$ depends on two relevant parameters, σ_u^2 and σ_v^2, that are the variances of the noise and inefficiency effects. Note that γ varies from 0 to 1: when the value is close to zero deviations from the frontier are attributed to noise, while in the opposite case the deviations are entirely explained by the technical inefficiency of the firm.

	Dependent variable: production output (log)			
	Estimate	Std. Error	z value	$Pr(>\|z\|)$
Intercept	-1.411159	0.322292	-4.3785	1.195e-05 ***
Labour input	0.637796	0.055230	11.5480	< 2.2e-16 ***
Machinery capital input	0.314146	0.044574	7.0477	1.819e-12 ***
Water, energy and fuel input	0.071348	0.013375	5.3346	9.573e-08 ***
Land capital input	0.233026	0.037678	6.1847	6.224e-10 ***
σ^2	1.003306	0.116481	8.6135	< 2.2e-16 ***
γ	0.581710	0.097161	5.9871	2.136e-09 ***
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 11.6: Wine production function - SFA estimator

As discussed in Chapter 5, the SFA model is based on the hypothesis of mutual independence among the productive units: therefore, this specification ignores the role of any spatial effects that may be present in the data. However, the evolutionary trend emerging from the brief overview of the Italian wine industry presented in this paper suggests that efficiency in this sector could be influenced by a multiplicity of tangible and intangible local factors. To evaluate the role of these effects, a formal statistical test has been implemented to verify the presence of spatial correlation among residuals: specifically, the global and local indicators proposed by Geary (1954) have been used, previously specifying a distance matrix to map the neighbourhood of each production unit. The correct definition of the matrix is crucial to ensure the consistency of the spatial analysis: in this respect, the identification of a correct unit of distance must be driven by economic considerations associated with the peculiarities of the sector under investigation. In this specific case, characterized by the presence of productive units often concentrated in narrow geographical areas, a particularly close neighbourhood (nearest neighbour, $n = 10$) has been chosen in order to account for the specificities of the wine industry: the contiguity matrix resulting from the application of this criterion is graphically represented in Figure 11.6.

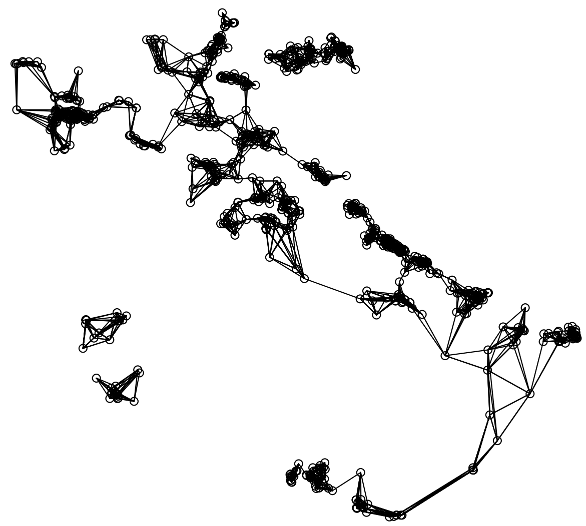


Figure 11.6: Contiguity matrix, nearest neighbour ($n = 10$)

Using this distance matrix, the presence of spatial autocorrelation among residuals for the SFA model has been formally tested using the Geary C statistic[23]: the estimated value of this variable (0.733) leads to reject the null hypothesis of mutual independence among firms, confirming the presence of a positive neighbourhood effect among the Italian wineries that cannot be isolated and estimated through the traditional stochastic frontier model. This scenario motivates the need to use a spatial stochastic frontier approach with the data at disposal: in this respect, the SSFA model proposed in equation (5.3) seems a particularly effective tool to isolate and evaluate the territorial component separately from the individual performance of the productive units. The results of the estimation are reported in Table 11.7: in all

[23]The value of Geary C lies between 0 and 2. Values lower than 1 demonstrate increasing positive spatial autocorrelation, whereas values higher than 1 indicate increasing negative spatial autocorrelation. C=1 is consistent with no spatial autocorrelation in the data.

cases, the value of the coefficients for the inputs are consistent with those obtained from the SFA specification, with the expectation of the intercept that becomes not significant; however, this result is expected as the spatial specification generates a further shift in the production curve with respect to the SFA as a consequence of the isolation of the spatial effect, transforming the average value of β_0 into a multiplicity of individual effects. Interestingly, the value of the γ parameter (0.433) is lower than the one estimated with the SFA model: this evidence supports the hypothesis that part of the technical inefficiency was mistakenly attributed to the production process rather than to neighbourhood effects.

	Dependent variable: production output (log)			
	Estimate	Std. Error	z value	Pr($>\|z\|$)
Intercept	-0.0707772	0.7387170	-0.09581	0.923671
Labour input	0.6081372	0.0525574	11.57091	$<$ 2e-16 ***
Machinery capital input	0.3328368	0.0436632	7.62282	$<$ 2e-16 ***
Water, energy and fuel input	0.0500784	0.0132213	3.78770	0.000152 ***
Land capital input	0.3250007	0.0401612	8.09240	$<$ 2e-16 ***
σ^2_{dmu}	0.3253159	0.1250316	2.60187	0.009272 **
σ^2_v	0.3591916	0.0456068	7.87584	$<$ 2e-16 ***
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 11.7: Wine production function - SSFA estimator

The benefits of the proposed framework are also evident in terms of global and local spatial autocorrelation: indeed, the global Geary C statistic equal to 1.048 suggests that global autocorrelation has been effectively removed from the residuals, while local c_i (for each unit i) associated with the SSFA estimates is significantly lower with respect to the unconditional SFA scenario (Figure 11.7).

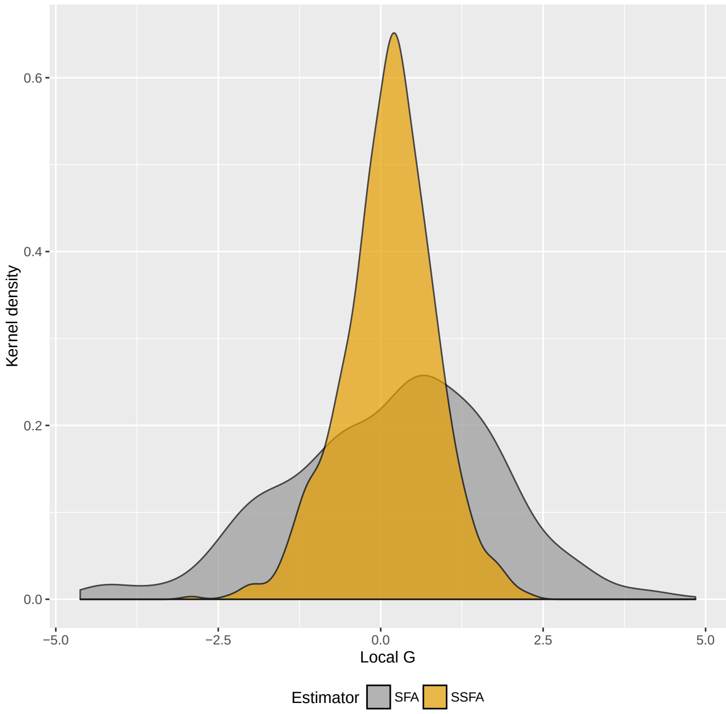


Figure 11.7: Local c_i kernel density of the SFA and SSFA efficiency

11.2.5 Explaining the spatial effect through the analysis of territorial imbalances

The analysis presented in the previous section has confirmed the presence of a spatial effect in the data that has been successfully isolated using the SSFA specification proposed in Chapter 5. In this part of the empirical investigation, the focus is moved to the spatial effect itself, in an attempt to explain its structural characteristics and interpret its nature in light of the considerations emerged in the brief review of the wine sector presented in section 11.2.2. In order to do so, the analysis is focused on the territorial imbalances, defined as the difference between the efficiency term estimated in the SFA specification and that identified with the SSFA approach[24]: in general, higher values of territorial imbalance suggest the presence of a stronger territorial component.

A geographical representation of the territorial imbalances is presented in (Figure 11.8): the map shows the presence of a heterogeneous distribution of the spatial effect, with areas characterized by a strong territorial factors while in other cases the role of the locality appears to be negligible in determining the performance of the productive units.

[24]this term is generally positive, given in the SFA the spatial effect is mistakenly incorporated into the error term, generating higher values relative to those estimated with the SSFA.

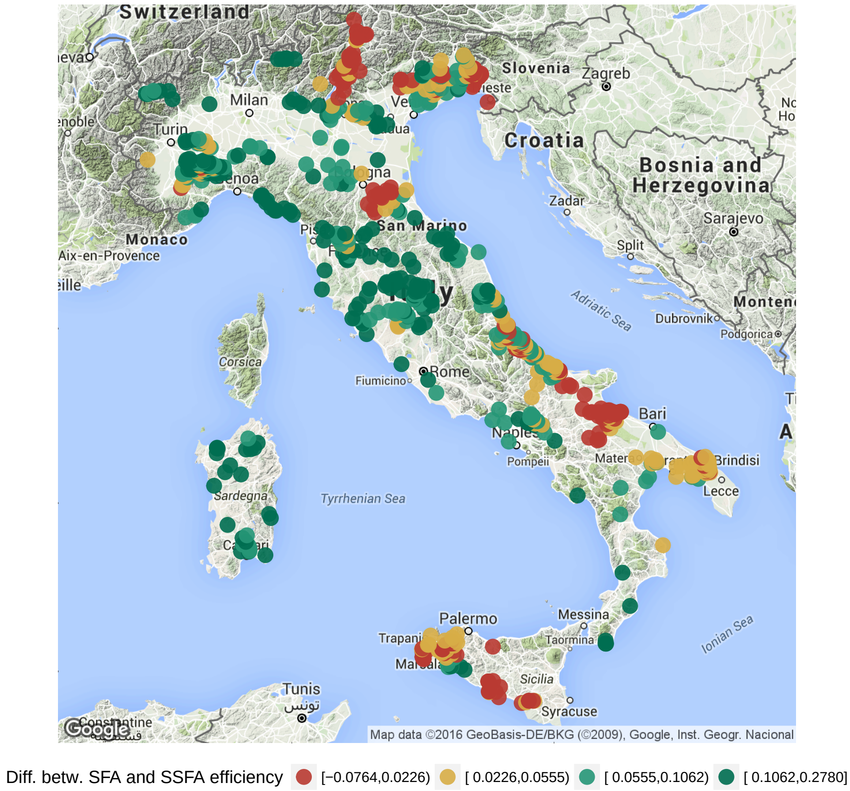


Figure 11.8: Differences between SFA and SSFA efficiencies per quantile, $q = 4$

Having established the presence of a relevant and heterogeneous spatial effect in the Italian wine industry, the immediate question is whether this effect can be satisfactorily explained by a plurality of tangible local factors. To address this issue, a second stage analysis has been implemented regressing the territorial imbalances on a plurality of contextual variables, following previous findings in earlier studies (see *e.g.* Hani et al., 2003, Reig-Martinez et al., 2011 and Bardaji and Iraizoz, 2015) and incorporating other determinants that are believed to have an impact on firm-level productivity in agriculture. In this respect, the wide variety of variables available in the FADN database can be effectively used to incorporate a number of relevant factors that are believed to explain the spatial effect in the wine industry[25]: specifically, the set of covariates include *(i)* endogenous factors linked to the productive process or to the corporate characteristics, *(ii)* exogenous physical factors and *(iii)* exogenous economical indicators related to the local supply factors.

The results of the estimation are reported in Table 11.8: in most cases, the coefficients are significant and the sign is that expected. As far as the endogenous factors are concerned, the first interesting result is determined by the key role played

[25]Note that the physical/contextual data used in the estimation do not exhaust the multiplicity of issues that characterize the production within a territory.

by quality and reputation: in fact, firms producing a better wine quality (OTE = 3510, wineries specialized in the production of quality wine) appear to benefit from a larger territorial effect. Moreover, being located in an area characterized by a higher reputation of the production process (i.e. DOCG production area) is also associable with an increase in territorial imbalances, irrespective of the quality of wine produced by the firm. The data also reveals the importance of the territorial effect for family-owned firms (Family owned) and those productive units characterized by an higher degree of product differentiation.

Regarding the exogenous physical factors[26] it is worth noting that the coefficients of both Physical disadvantage (climate) and the Biophysical disadvantage are negative and statistically significant: this result is not surprising considering the beneficial effects associated with a temperate climate, an advantageous slope inclination and slope exposure on wine production.

Finally, a set of exogenous economical factors[27] have been included in the model, in order to account for the role of elements external to firms and internal to the region that generate a competitive advantage among economic agents. The results are consistent with the expectations, showing that a higher level of the surrounding economic and network infrastructure has indirect beneficial effects for the productive units specialized in wine production; on the other hand, the presence of a lower level of human capital in the region (higher Scholastic drop-out indicator) has a negative impact on the territorial imbalances.

Despite the results of the estimation confirming the important role played by the above mentioned variables, it is worth noting that the presence of these tangible factors is not sufficient itself to explain the variance of the territorial imbalances ($R^2 = 0.214$). This evidence indirectly confirms the presence of intangible factors associated with context specific and informal practices of learning that cannot be evaluated through the mere inclusion of specific contextual variables in the model: in this respect, the implementation of a SSFA approach can effectively address this issue, allowing to isolate the intangible effects associated with tacit knowledge flows and incremental learning that are peculiar to the wine industry and cannot be merely proxied through the inclusion of specific contextual variables.

[26]This data is available at an extremely detailed territorial level, *i.e.* the municipality (CREA, 2013).

[27]These composite indicators are present in the Istituto Tagliacarne (2013) database and defined at a narrow territorial level, *i.e* the municipality.

	Dependent variable:
	Difference between SSFA and SFA efficiency
Constant	0.076*** (0.012)
Endogenous factors	
OTE = 3510 (yes=1)	0.021*** (0.005)
DOCG production area (yes=1)	0.016*** (0.004)
Gender (M=1)	−0.018*** (0.005)
Family owned	0.015*** (0.004)
Diversified production (yes=1)	0.014** (0.006)
EU subsidies (1,000 €)	0.001** (0.0002)
Financial charges (1,000 €)	−0.0004 (0.001)
Young owner (yes=1)	−0.005 (0.006)
Organic production (yes=1)	0.001 (0.010)
Exogenous physical factors	
Physical disadvantage (slope)	0.022*** (0.005)
Physical disadvantage (climate)	−0.030*** (0.006)
Biophysical disadvantage	−0.011** (0.005)
Exogenous economical factors	
Economic infrastructure indicator	0.0001** (0.00004)
Network infrastructure indicator	0.0002** (0.0001)
Scholastic drop-out indicator	−0.002*** (0.001)
Observations	853
R^2	0.214
Adjusted R^2	0.200
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 11.8: Determinant of the SFA - SSFA differences, OLS estimator

11.2.6 Do spatial effects vary with size? The different role played by the local network in small and large firms' efficiency

The analysis implemented in the previous section has highlighted that a combination of tangible and intangible factors explain the presence of a spatial effect in the Italian wine industry. In this scenario, the role of intangible effects appears to be particularly significant and possibly associable with the local business climate and informal interaction among local actors: in this respect, the results support the empirical evidence which has emerged in previous contributions focusing on case studies in specific wine regions, suggesting the presence of communitarian networks, characterized by resource sharing and informal interaction.

However, as several recent contributions have shown, access to these networks appears not to be uniform among local actors: not infrequently, small firms tend to be more inclined to engage in informal interaction practices with local peers, given the higher expected benefits, while large firms tend to acquire new knowledge from external sources, using the wide variety of formal channels at their disposal.

The possible presence of this differential has been evaluated by plotting the difference between the SFA and the SSFA efficiencies against the size of the firm. Figure 11.9 confirms that an inverse relationship exists between the two variables, with a higher territorial effect for small firms that tends to decrease when firm size becomes larger: however, the presence of a negligible spatial effect among large wine producers seems not to be associated with lower levels of technical efficiency: in fact, Figure 11.9 shows that these firms are more efficient than small producers.

This apparently contrasting trend can be easily interpreted in light of the patterns already identified in Section 11.2.2: the presence of a negligible spatial effect among large firms should not be motivated by their difficulty to access local networks, but rather by a voluntary choice aimed at focusing on alternative sources of knowledge, such as internal learning, interaction with producers located outside the neighbourhoods and formal collaborations with institutional actors. The choice of these alternative forms of learning enables these firms to stay at the forefront of technical development, focusing on the most efficient technologies and maintaining high levels of technical efficiency. On the other hand, small producers who cannot access external knowledge are required to rely on the informal learning practices associated with continuous interaction with the local community, generating the spatial effect identified for this subset of the firm population in the model. Although these intangible factors allow to reduce the gap with the leaders, small firms still display a lower level of efficiency relative to the larger ones.

The main finding introduced in this section confirms that spatial proximity does not necessarily generate knowledge spillovers. The main reasons explaining this pattern are probably two: on the one hand, large producers generally do not need to link to local networks to access informal sources of learning; on the other hand, leader firms may not be willing to share the knowledge acquired through access to external or formal sources, generating positive externalities for small firms located in the close neighbourhood. In this respect, the behaviour of large firms could be interpreted as a rational strategy aimed at retaining a competitive advantage in the production process, ensuring higher levels of technical efficiency.

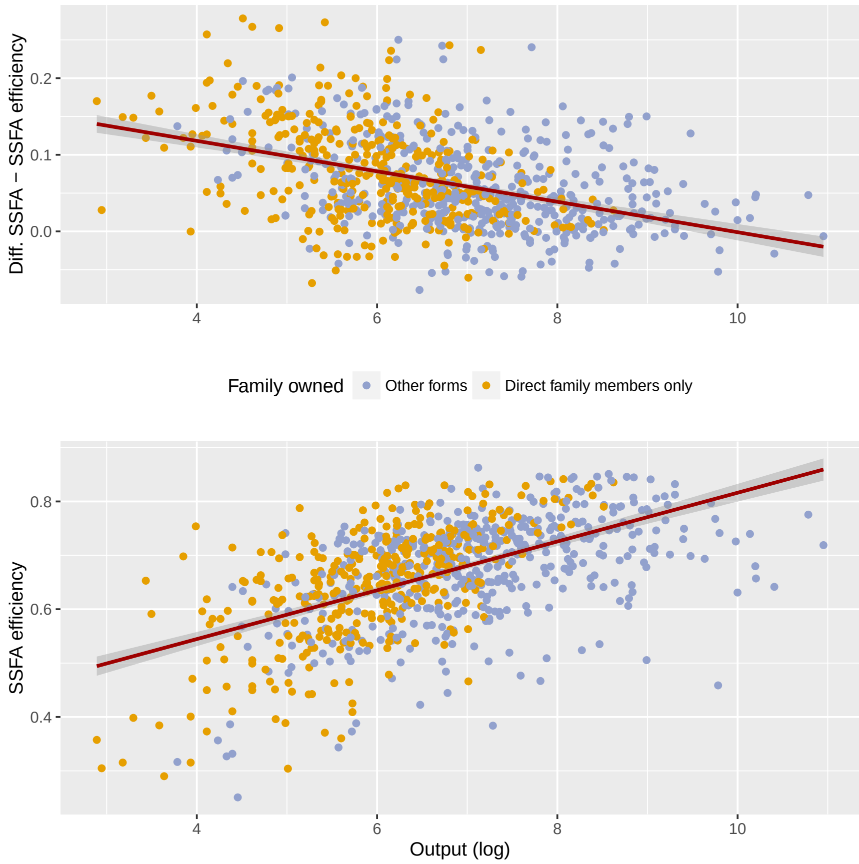


Figure 11.9: SSFA efficiency and difference with SFA per produced output (log) and ownership

11.2.7 Concluding remarks

The empirical exercise, implemented on a sample of wine producers extracted from the Italian FADN survey, shows that the spatial specification proposed by Fusco and

Vidoli (2013) is extremely effective in disentangling the spatial effect that is present in the data, isolating a specific component that is erroneously attributed to the error term in the standard SFA approach.

The main features of the spatial effect are evaluated through a second stage analysis, in which the territorial imbalances (*i.e.* the difference between the inefficiency term calculated with the SFA specification and that identified with the SSFA approach) are regressed against a set of contextual variables that are generally associated with the presence of a stronger effect: although the role of these factors is confirmed by the results of the estimation, a relevant share of the variance in the model remains unexplained, suggesting that a key role is played by intangible factors that cannot be formally included in the model. Following the recent findings in the literature, it is reasonable to assume that this intangible component is the consequence of a network effect associable with the local business climate: in most localities, the presence of an embedded community stimulates a process of local learning that generates the diffusion of tacit knowledge through continuous interaction among the local actors. Although the investigation does not allow to evaluate whether this flow relates more to business information of technical knowledge, the identification of such aspect is by itself a key finding of the contribution.

The analysis of the degree to which the spatial effect varies with firm size provides evidence of a clear tendency of this effect to be significantly lower among large firms. This finding is in line with previous research, confirming that firms interacting in economic networks are not an homogeneous entity, but play different roles in the local scenario: although it is not a direct consequence of the results of the paper, it can be speculated that the different size of the territorial effect found in small and large wine producers is associated with different abilities and willingness to interact and share knowledge with neighbours located in close vicinity: such a scenario would confirm the trend already identified in case studies on wine sector (Giuliani, 2007; Morrison and Rabellotti, 2009), showing that large firms have a strong tendency to access to external and formal sources of knowledge, sharing the information acquired from outside with a small number of firms who collaborate on a regular basis. The regular interaction with external sources of knowledge enable these firms to stay at the forefront of the technological frontier, enabling them to face the challenges required by the rapid technological change: such a trend would be consistent with the higher levels of technical efficiency found among large firms in the empirical analysis.

The results of this investigation open some interesting avenues for further research. The SSFA specification can be extended to accommodate the use of panel data: the implementation of such an approach would be particularly convenient to control for seasonal or other unobserved factors that can influence harvesting in a particular year, such as the presence of parasites or other transient factors.

CHAPTER 12

INPUT DISTANCE FUNCTIONS: LEVEL OF SERVICES, SPATIAL DEPENDENCE AND ALLOCATIVE EFFICIENCY IN THE LOCAL GOVERNMENTS EXPENDITURE NEEDS FRAMEWORK

Under review on: Empirical Economics

12.1 Introduction

Recent economic crisis, different territorial dynamics in innovation, as well as, the international shifting of the production centres, are having a profound impact on the macro economics dynamics and therefore on the long-term sustainability of public national accounts.
The crisis of the social state induced by the economic slowdown, by demographic changes together with the construction of the European Monetary Union lead to tighter fiscal disciplines in Member States, put under pressure the government spending schemes and, in particular, the equalisation transfers to the local level reinforcing the trends towards decentralisation.
The focus of public national accounts is, therefore, shifting to the need, both in the short term, but especially in the medium-long term, to build stable, flexible and proactive control spending systems, both at central level and, mainly, at local level.
Given these constraints, the real challenge for governments is to build spending perequation and account control systems that allow to maintain public service levels, as much as possible, unchanged by minimizing the overspending due to inefficiency, incorrect allocations of production factors or chronic misalignments in optimal Local Authorities (LAs) sizing.

Over the recent years, the general trend in Europe has been towards increased decentralisation.
In several EU15 Member States, decentralisation dates back to the late 1970s - early 1980s when the first legislation improvements were implemented and proceeded even further over the following decade (*e.g.* 1978 in Spain, 1982 and 1983 in France, 1988 in Belgium, 1990 in Italy). The process of decentralisation was pursued further during the following decades.
Although in OECD countries fiscal equalisation absorbs 5% of total public expenditure, it represents the primary aspect and most delicate of all fiscal and administrative decentralization processes, since equalisation transfers limit the territorial imbalances amplified by devolution. This issue is particularly important in countries, such as Italy, characterized by wide territorial differences.
A greater institutional decentralization implies a greater financial autonomy for LAs and, on the other hand, it assigns a portion of richer communities resources to the disadvantaged ones; in other terms, institutional decentralization could enhance

the horizontal equalisation systems compensating territorial imbalances avoiding excessive public spending or allocations of public goods characterized by uncertainty and misallocation across districts, as pointed out by Besley and Coate (2003).
Horizontal equalisation policies, as a matter of fact, have the aim to reduce resources and/or charges disparities between LAs; these policies, principally refer to the seminal concept of "*territorial equity*" (Buchanan 1950), consist in equalising the purchasing power of LAs in local public services (*i.e.* the ratio benefit/tax effort, Thurow 1970). Generally, financial equalisation policies contribute to a certain equal opportunity between LAs: moderating the vertical imbalances, diminishing the tax competition (even between regional and local governments, Zhuravskaya, 2000), limiting the risks of uncertainties and also maintaining social cohesion.

Due to these needs, an innovative autonomy path for LAs has started in Italy in 2009 linked to a fundamental change in the funding criteria. The statutory law on fiscal federalism (Law No. 42, May 5^{th} 2009) and the determination of standard requirements for LAs (Municipalities, Provinces and Metropolitan cities) issued through Legislative Decree No. 216, November 26^{th} 2010, are allowing to modernize intergovernmental financial relations and to minimize LAs cost (overall 77 billion Euro in 2010, equal to 5% of GDP and 10% of consolidated public spending).
This constitutional reform intersects, also, with a more contingent need caused by the structural Italian public finances imbalances, removing disparity conditions at territorial and national level. In 2012, the Legislative Decree No. 95 (also called "*spending review*") emphasized the necessity to adopt in the short term "*urgent measures for the reduction of public spending keeping constant level of services*".[28] However, efficiency analysis and local grants perequation criteria still remain separated both in the theoretical systematization and in the practical implementation.

Against this background, in the existing literature (see *e.g.* OECD 1981, Reschovsky 2006, Reschovsky 2007, Blochliger et al. 2007, Blochliger and Charbit 2008, Dafflon and Mischler 2007) local governments expenditure needs, cost efficiency and the levels of services provided by LAs have been analysed separately.
An integrated model of cost and allocative efficiency equalization would lead to a greater territorial equity and, at the same time, it would empower LAs to provide better services to users.

The main purpose of this paper is, therefore, to combine in a single framework, the need for a more accurate allocation of intergovernmental transfers for providing a higher level of service thanks to the increment of production factors efficiency. Equity and efficiency are two aspects that must be *pursued and evaluated together*

[28]Whereas, in practice, the levels of service were not taken into account in the spending actual cuts criteria.

with the aim of increasing the collective well-being.

From a methodological point of view, departing from previous approaches based on regression techniques or on traditional frontier models, a comprehensive method for the estimation of standard needs which simultaneously takes into account the different production efficiency of LAs is proposed. In the classical standard needs techniques, in fact, the allocative inefficiency is not included in the analysis.
Another issue, often neglected, is the spatial autocorrelation of the residuals in the classical estimation linear models; this issue, besides representing a serious problem from a statistical point of view, may involve the systematic allocation of more transfer to some territories compared to other ones.

The procedure is based on a *three steps* approach (for further details, see section 12.3). In the **first** step standard level of services, according to demand and supply local factors, is evaluated; in the **second** step, the relationship among the level of service (output) and the inputs is identified by estimating a parametric frontier, and then the allocative efficiency is estimated taking advantage of the duality theory for the inputs shadow prices computation.
Finally, in the **third** step, by using the the allocative efficiency previously determined and given the standard level of services, a spatial autoregressive cost function is estimated.

12.2 Cost and efficiency of Local Governments

In a decentralized public sector the achievement of a reform is strictly related to the measurement of the local government performance.[29]

As a matter of fact, a comparative performance evaluation ensures transparency and accountability of services providers to citizens, it is useful in the decision making process, encourages the audit of managerial performance and highlights the contextual dimensions influencing the operating environment.

However, there are several critical issues in measuring accurately the performance (in terms of costs and services) in the public local sector. The complexity and/or the multiplicity of outputs, the difficulty in establishing cause-effect relations between services and final outcomes, the interplay of many institutional levels that require a wide range of governmental performance information and, finally, the stakeholders restrictions that influence the theoretical ability to improve performance, represent

[29] Please see Worthington and Dollery (2000) for a complete survey of frontier efficiency measurement techniques in local public sector.

some of the main issues.

The performance evaluation can be assessed through two different dimensions: the *effectiveness* that provides information about the achievement of the policy objectives in terms of appropriateness, accessibility and quality of the services and the *efficiency* that describes how a government manages its resources in producing services and that is generally estimated by parametric and/or non-parametric frontier techniques.

In fact, as underlined by Worthington and Dollery (2000), *"the complex politicised milieu of local government implies that the effectiveness of services is at least as important as economic efficiency in gauging the success of specific municipalities facing different demands. Frontier efficiency measurement is concerned only with the dimensions of economic efficiency and takes no account of the effectiveness of service provision. It is thus, at best, only a partial view of the operations of councils"*.

The construction of effectiveness indicators in the public sector face with several difficulties, especially related to the definition of objectives and the verification of their achievement since: *(i)* public services often pursue simultaneously multiple goals requiring a weighting criterion, *(ii)* public outputs are not always related with a "measurable" outcome and *(iii)* outcomes may occur over a long time period.

The efficiency assessment in the public sector, although with its specific peculiarities, is less complex and starting from the '90s has been widely developed from both a theoretical and applied point of view.

In early studies, the Stochastic Frontiers Analysis (SFA) have been applied to examine municipal service efficiency in terms of cost (see *e.g.* Hayes and Chang 1990, Deller 1992) or production (see *e.g.* Jayasuriya and Wodon 2003), while from '90s, the most employed approach to investigate the local public sector efficiency has been the nonparametric techniques such as Data Envelopment Analysis (DEA) (see *e.g.* Cook 1991, Deller 1992, Prieto and Zofio 2001, Afonso and Fernandes 2006) and Free Disposal Hull (FDH) approach (see *e.g.* Vanden Eeckaut et al. 1993, De Borger and Kerstens 1996*b*, Herrera and Pang 2005).

Empirical applications to local public services highlight three important critical issues: *(i)* the choice about the inclusion of non-discretionary inputs/outputs in the analysis: Banker and Morey (1986) and Golany and Roll (1993) suggested a "single-stage" procedure based on the maximisation of the sub-vector of discretionary inputs/outputs; De Borger (1994), instead, proposed a "two-stage" approach in order to obtain a residual as a "pure" allocative efficiency (see *e.g.* also Ruggiero 2004, Balaguer-Coll et al. 2007, Afonso and Fernandes 2008); *(ii)* the choice of the input or output orientation in efficiency measures: many contributions have focused the analysis on input efficiency measures (see *e.g.* Hayes and Chang 1990 and Vanden Eeckaut et al. 1993), while De Borger (1994) affirmed that the exact formulation depends on the particular empirical context; *(iii)* the robustness of the efficiency measure results (rankings): De Borger and Kerstens (1996*a*) suggested using a broad variety of methods as sensitivity analysis.

Finally, from a territorial point of view, previous studies analysed only a sample of Local Governments (*e.g.* 235 Belgian Municipalities, Vanden Eeckaut et al. 1993; 589 Belgian municipalities, De Borger and Kerstens (1996*a*); 172 Greek municipalities, Athanassopoulos and Triantis 1998; 1,103 Brasilian municipalities, Sousa and Ramos 1999; 166 Australian municipalities, Worthington 2000; 209 Spanish municipalities from Castilla and Leon, Spain Prieto and Zofio 2001; 353 Finnish municipalities, Loikkanen and Susiluoto 2005; 278 Portuguese municipalities, Afonso and Fernandes 2008; 262 Italian municipalities, Boetti et al. 2012; 414 Spanish municipalities, Balaguer-Coll et al. 2007) or carried on comparative analysis at international level.

However, these levels of analysis are deeply influenced by specific issues linked to the local supply and demand being affected by sample size bias drawbacks in local studies, or, in the matter of international analysis, not avoiding the comparative analytical problems related to different structures of the administrative decentralization.

In conclusion, national level, as also suggested by Wolman (2008), is a good trade-off between local and international level, especially, as in the present case, where cost, input and output data are available for all Italian LAs.

12.3 Methodologies for the estimation of Local governments' expenditure needs

The evaluation of local governments' expenditure needs is the core of any intergovernmental fiscal equalisation system which aims to remove differences in the actual costs of providing local public services and it is the foundation of intergovernmental fiscal relations (Blochliger et al., 2007).

There is a wide variety of methods for the estimation of local governments expenditure requirements. This heterogeneity is not only due to the multiplicity of available techniques, but also to the specific arrangements in the concrete experiences of different countries, which adopt variants and particular features based on specific needs and on their own historic and cultural traditions.

However, international literature (Dafflon and Mischler, 2007) discerns various primary methodologies in which the practical experiences of different countries can relate.

According to the complexity and the amount of data required, the allocation techniques can be summarized into two sub-groups: *(i)* basic techniques: historical expenditure, uniform per capita expenditure or requirements weighted indexes expenditure and *(ii)* advanced standard needs techniques: for example Representative Expenditure System (RES) or Regression Cost Base Approach (RCA).
The two major approaches differ considerably in terms of information and calculation requirements. The basic techniques do not require the support of “hard” statistical and/or econometric methodology, thus minimizing the need for data; in this case, standard needs are established in relation to what was previously accounted for in the last available budget or in a certain number of past financial accounts.

The advanced standard needs criteria, instead, are based on the idea that the financial needs of a LA are linked to the territorial and socio-demographic characteristics of its resident population. These aspects, on the one hand, have an impact on the needs of citizens and consequently on the demand of services; on the other hand, they influence directly the production costs.

OECD suggests the adoption of standard needs as a “best practice" to be used in the planning of financial systems for local government. In particular, it is argued that provisions of transfers based on mathematical formulas are preferable to systems based on historical expenditures or on discretionary criteria, because they guarantee greater transparency, more equity in the redistribution of resources and greater efficiency thanks to more rigid budget constraints for local government.

According to these principles, standard needs may be obtained by following a RES (or bottom-up) approach, *i.e.* identifying a minimum basket of goods/services in order to ensure a basic level of benefits[30], or by using an RCA (or top-down) approach in which the standard needs for each authority are obtained according to the relevant factors of demand variability between regions and to the local services production costs.

The RCA approach, adopted by the majority of developed countries, implies the use of a structural model of demand and supply of local public good, in which the method of instrumental variables is used to solve the endogeneity issues related to the historical level of output.

[30]Requirements for each authority are determined by the linear combination of goods/services with a plurality of weights (prices); usually the choice of these factors is entrusted to experts or it is submitted to a political decision.

More specifically, following an *RCA* approach, the selection of variables is based on a theoretical model in which the demand for public services expressed by citizens interacts with the supply of public services expressed by the local government. In this framework, the efficient cost for each service depends on three basic dimensions: the optimal quantity of service offered, the prices for the inputs used in the production process (primarily labour costs) and the contextual demand and supply covariates.[31] Formally, the *RCA* basic model can be expressed[32] in terms of unitary cost (y) as:

$$y = f(\mathbf{o}, \mathbf{w}, \mathbf{x}) \tag{12.1}$$

where $\mathbf{o}$ are the exogenous outputs or workloads, $\mathbf{w}$ the labour and capital input prices vector (w_l, w_k) and $\mathbf{x}$ are the demand factor or the supply contextual variables that represent morphological and socio-economic constraints, which affect the unitary service costs.

12.4 The estimation strategy

The estimation strategy of the equation (12.1) has to face essentially with three critical issues:

(i) *the endogeneity of the output variables*; the presence of the output variables in the estimation model provides three disadvantages: *(i)* outputs are not always measurable and there may be a very serious lack of information, *(ii)* they might be external to the initial theoretical model (*e.g.* if individual LA provides specific service in accordance with the Central Authorities), *(iii)* they might be endogenous, since their optimal quantity is determined simultaneously in terms of cost.
Given these premises, several studies (see *e.g.* Andrews et al., 2014) suggest using the Instrumental Variable (IV) approach in order to obtain unbiased estimates of the outputs ($\mathbf{o}_{iv}$); this two-stage estimation technique helps to estimate $\mathbf{o}_{iv}$ given the endogenous values of the outputs $\mathbf{o}$ and the correct instrumental variables $\mathbf{iv}$:

$$\begin{cases} \mathbf{o}_{iv} = g(\mathbf{iv}) \\ y = f(\mathbf{o}_{iv}, w_l, w_k, \mathbf{x}) \end{cases} \tag{12.2}$$

[31] For example, the external factors that, all things being equal, can favour or hinder the supply of local public goods, such as the morphological characteristics of the territory or the surface area.

[32] Please note that the notation is different from the standard one in order to to be consistent with the following paragraphs.

Moreover, $\mathbf{o}_{iv}$ may be referred to an average or a frontier level, as a function of supply and demand covariates, as a single composite indicator of the outputs (see *e.g.* Porcelli et al., 2016) or by setting, over the years, higher level of services estimated taking into account different quantiles of the output distribution.[33]

After estimating each o_{iv} through the instrumental variables[34], unbiased standard needs can be estimated by usual regression techniques. In this framework, all cost differentials are determined within a model that can correctly represent the real determinants of the expenditure requirements (especially when LAs are extremely heterogeneous).

(ii) *the non-inclusion of allocative efficiency in the model*; equation (12.2) hides the effect of different relationships between labour (I_l) and capital input (I_k) and the output produced, assuming a ratio (or more generally an efficiency) equal for every unit; this issue appears a serious lack, because the different efficiency among units deeply impacts on output and consequently on cost. In other terms, LAs best practices are not taken into account given that the efficiency of each LA is neglected both in the definition of $\mathbf{o_{iv}}$ and in the setting of the input prices $\mathbf{w}$.

Kumbhakar and Lovell (2000) suggested the use of the shadow price approach since *"[...] estimating cost inefficiency was easy, but decomposing estimated cost inefficiency into its technical and allocative component was not. Decomposition was impossible in a single equation framework, and while decomposition is theoretically possible in a system of equations framework, it proved to be difficult econometrically. Perhaps the shadow price approach will be more productive"*. Given that, the allocative efficiency (E_{all}) has been calculated as the result of a shadow-pricing model formulated by a Shephard input distance function (see Chapter 2), in terms of the market capital price.[35]

$$\begin{cases} \mathbf{o}_{iv} = g(\mathbf{iv}) \\ E_{all} = D_I(\mathbf{o}, I_l, I_k, w_k) \\ y = f(\mathbf{o}_{iv}, w_l, w_k, \mathbf{x}, E_{all}) \end{cases} \tag{12.3}$$

"Comparing shadow price ratio with actual price ratio" (Fried and Lovell, 2008) allows to obtain a measure of the inefficient use of resources that in a perequation framework should not be granted to the inefficient LAs.

[33]This approach is suitable for policies which aim to improve the overall performance of the LAs as a whole by progressively increasing the standard level of services given the available macro budget.

[34]In the section 12.5 we'll refer, without loss of generality, to an univariate output o_{iv}.

[35]The model is specular in case of optimal capital price given the labour one.

(iii) *the criteria of selection of the local supply and demand covariates* $\mathbf{x}$ in order to catch all the territorial characteristics; it can be noted that a uniform agreement about a consistent criteria is not yet reached in literature even if two main practical approaches are generally followed: the first one introduces into analysis a multitude of variables in order to capture all the cost variability, while the second one introduces regional dummies in order to catch all the territorial regularities not included directly in the analysis.

In this paper, a Spatial AutoCorrelation (SAC, Kelejian and Prucha, 1998) estimation model has been proposed (see Subsection 4.3.3) with the aim of combining model simplicity and unbiased estimates. This estimator, named f_{SAC}, lets to include the omitted variables and more generally all the intangible factors as a spatial lag and, more generally, to reduce the covariate terms; on the other side, it allows to control the spatial autocorrelations of the error term obtaining unbiased estimates.

$$\begin{cases} \mathbf{o}_{iv} = g(\mathbf{iv}) \\ E_{all} = D_I(\mathbf{o}, I_l, I_k, w_k) \\ y = f_{SAC}(\mathbf{o}_{iv}, w_l^{eff}, w_k, \mathbf{x}, E_{all}) \end{cases} \tag{12.4}$$

Equation (12.4), finally, allows to estimate expenditure standard needs y in an unitary approach that simultaneously takes into account the standard (or desired, target, minimum) level of services $\mathbf{o_{iv}}$ and the allocative efficiency E_{all} where spatial dependence between territorial authorities is controlled.

In the following subsections the fundamentals behind the methodologies used in steps *ii)* and *iii)* are briefly outlined.

12.5 Empirical analysis

12.5.1 Variables and data

In 2009, Italian government started a challenging reform process in order to equalise transfers to LAs according to standard needs criteria; the statutory law on fiscal federalism (Law No. 42, May 5^{th} 2009) and the subsequent Legislative Decree No. 216, November 26^{th} 2010 assigned these tasks to SOSE S.p.A., a subsidiary of Italian Ministry of Economy and Finance and Bank of Italy.
The standard needs assessment for each LA (Municipalities, Provinces and Metropoli-

tan cities) has started with the construction of a very extended database[36] covering a plurality of information for each service provided.

Structural data collected through the questionnaires has allowed to collect extremely detailed information about specific factors of supply and demand, expenditures and revenues, inputs (personnel employed, instrumental allotments, local units used, etc.), output produced (services implemented) and procedures for the implementation of services (*e.g.* forms of association between Municipalities) into a unitary and coherent framework for each "essential" function.[37]
Structural data, after having passed stringent consistency and quality checks in order to correct anomalies and serious inconsistencies, has been integrated with structural and accounts information drawn from official sources.
In the following empirical analysis, the focus has shifted on a sub-function of the General Administration, more in particular on services delivered by the Municipal Registry office; this is essentially because the quality of services provided can be assumed as uniform in all 6,702 municipalities analysed while providing a variety of services.[38]
Finally, data (year 2009) are freely downloadable at:
`http://www.opencivitas.it/open-data`.

12.5.2 Estimation of the standard level of services o_{iv}

The first step of our analysis involves the estimation of the standard level of services o_{iv} according to the equation (12.2).
For sake of simplicity and in the absence of a desired or regulatory level, in this application, vector $\mathbf{o}_{iv}$ is set equal to a single output estimated as the mean level of the historical composite outputs; this approach can be made more general by imposing a multi-output – multi-input system of linear equations where the expected level of each output is set as dependent by some covariates, but also by the interrelationships with the other ones (for more details please see Porcelli et al., 2016).

Table 12.1 shows the OLS estimation results (and in particular the most complete model OLS3) highlighting the positive contribution of the local demand covariates related to the effects of the population (both domestic - `Daytime population` - and foreign - `Tourist presence`) and the complexity of the Municipality (`Surface`

[36] More information at: `http://tinyurl.com/cz9stat`.
[37] The "essential" functions for the Italian Municipalities are: General Administration, Local Police, Education (complementary services), Public Roads and Transport, Planning and Environment and Social care.
[38] In public services, in fact, we are often in the presence of multi-input and multi-output processes; in this case the different quality of the output can heavily affect the estimation of efficiency.

area) on the level of services.

	Dependent variable:		
	Historical output		
	(OLS1)	(OLS2)	(OLS3)
Daytime population	0.587***	0.545***	0.541***
Surface area		0.372***	0.362***
Tourist presence			0.015***
Constant	0.318***	-0.014	-0.012
Observations	4,336	4,336	4,336
R^2	0.659	0.659	0.660
Adjusted R^2	0.659	0.659	0.660
Residual Std. Error	1.040	1.040	1.038
F Statistic	44,185.790***	4,185.790***	2,807.336***
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 12.1: Estimation results - Standard level of services o_{iv}

Studentized Breusch-Pagan test[39] equal to 652.69 (p-value $< 2.2e-16$) and Jarque Bera test[40] (X-squared = 1335200, p-value $< 2.2e-16$) show a good adaptation of the estimated model to the primary OLS assumptions.

12.5.3 Estimation of the allocative efficiency E_{all}

The second step involves the calculation of the allocative efficiency E_{all} through the estimation of the labour input shadow price w_l^{sh}; as pointed out in equation (2.9), a frontier production model must be initially estimated in which the inverse of a single input (in this case, the capital k since the relative price[41] was not affected by errors) is correlated with the ratio between labour (w, number of the employees[42] divided by its average) and capital input k (office, square meters divided by its average) and the ratio between the output level (o, historical output) and the capital input k.

$$log(1/k) = f(log(w/k), log(o/k)) \tag{12.5}$$

[39] It test the heteroskedasticity of the residuals, namely if the estimated variance of the residuals are dependent on the values of the independent variables.

[40] It is a goodness-of-fit test, testing the normality of the residuals through the skewness and kurtosis.

[41] Estimation based on the cost of renting properties at the provincial level.

[42] Data source: IX General census of industry and services - Survey public institutions, ISTAT. The elementary data, available only for the total of the municipality, has been re-proportioned by the percentage of the costs of the Municipal Registry function on the total expenses for employees.

The SFA [43] results are reported in table 12.2; in particular a satisfying covariate fitting can be noted and a very significant percentage of total error variance due to inefficiency ($\sigma_U^2/\sigma^2 = \lambda^2/(\lambda^2+1) = 75\%$, mean efficiency = 0.61) showing that the 75% of the total variation is due to inefficiency and that the remaining 25% is random variation.

	Estimate	Std. Error	z value	Pr(>|z|)
Intercept	0.955355	0.023745	40.2339	$< 2.2e-16$ ***
$log(w/k)$	0.525679	0.020999	25.0333	$< 2.2e-16$ ***
$log(o/k)$	-0.122719	0.017275	-7.1038	$1.214e-12$ ***
σ_U	0.719213	0.026516	27.1234	$< 2.2e-16$ ***
σ_V	0.406405	0.014242	28.5355	$< 2.2e-16$ ***
λ	1.769697	0.119584	14.7988	$< 2.2e-16$ ***

Table 12.2: Stochastic frontier analysis results

Finally, the estimation of the stochastic frontier has been functional to the calculation of the derivatives $\partial D_I(\mathbf{I},\mathbf{o})/\partial I_k$ and $\partial D_I(\mathbf{I},\mathbf{o})/\partial I_l$ and subsequently to the estimation of E_{all} by equation (2.10).

12.5.4 Estimation of the spatial cost function given o_{iv} and E_{all}

The last step of our analysis involves the estimation of the standard cost given standard level of service (o_{iv}) and the allocative efficiency (E_{all}) depurated by the autocorrelation both in the residuals (SAC) both in the explicative variables (SAC-mixed).
In a second step the fitted values (the standard cost) of the estimated cost function has been calculated penalizing the units presenting the lowest efficiency values.

Results[44] are reported in table 12.3; please note that the cost function follows the specification of equation (12.4) in which the demand factors $\mathbf{x}$ are represented by the advanced computer equipment and by the decentralized office (these factors indeed affect the cost positively). Note that because of the dependence structure of the SAC model, coefficient estimates do not have the same interpretation as in OLS. The β parameter reflects only[45] the short-run direct impact of x_i on y_i.

Some issues can be noticed: (i) the estimated coefficients look stable and significant

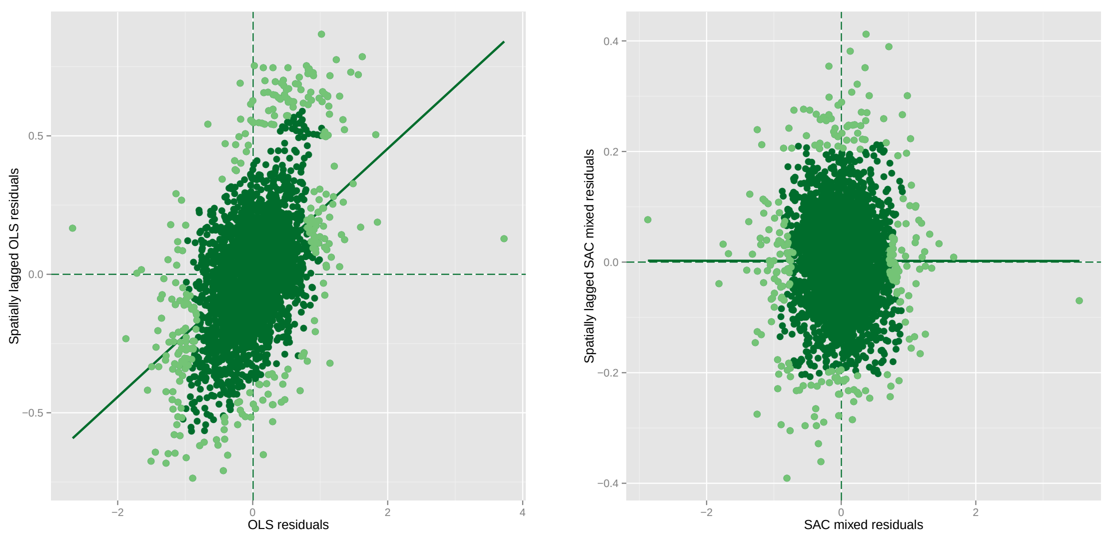
[43] In the error components frontier formulation, see Battese and Coelli (1992).
[44] In the estimation dependent variables and covariates are logged and divided by the mean.
[45] The indirect impacts of x_i on y_i (that y_i exerts on its neighbours y_j, which in turn feeds back into y_i) are not yet available for SAC models in R *spdep* package.

	Dependent variable:					
	Per capita cost					
	(OLS1)	(OLS2)	(OLS3)	(OLS4)	(SAC)	(SAC mixed)
Constant	0.075***	−6.030***	−6.208***	−2.606***	−4.333***	3.733*
Output IV	0.586***	0.527***	0.494***	0.462***	0.487***	0.486***
w_k		0.446***	0.437***	0.557***	0.533***	0.528***
w_l		0.512***	0.524***	0.197***	0.359***	0.351***
Decentralized office			0.179***	0.195***	0.168***	0.168***
Advanced equipment			0.217***	0.206***	0.180***	0.180***
E_{all}				−0.349***	−0.271***	−0.279***
Lagged Output IV						−0.401***
Lagged w_k						−0.214***
Lagged w_l						−0.715***
Lagged Decentralized office						0.027
Lagged Advanced equipment						-0.072
Lagged E_{all}						−0.168*
AIC	4582.448	4224.015	4091.897	3831.333	3228.728	3202.994
R^2	0.545	0.597	0.616	0.648		
Adjusted R^2	0.545	0.597	0.615	0.648		
F Statistic	3,523.516***	1,455.743***	941.934***	903.485***		
λ (deterministic part)					0.043	0.694***
ρ (error part)					0.741***	0.035
Note:						*p<0.1; **p<0.05; ***p<0.01

Table 12.3: Results

in all models; (*ii*) the minimum AIC is reached by increasing the complexity of the model; (*iii*) the coefficient of the allocative efficiency (E_{all}) is negative confirming that an improved efficiency decreases the cost of the service;(*iv*) the coefficient of the capital price (w_k) is increasingly statistically significant with the increasing complexity of the model, but in particular it becomes economically significant in the SAC-mixed model; (*v*) finally ρ is positive and significant in the SAC formulation[46] while it becomes not significant in the SAC-mixed formulation for the benefit of λ; this result shows that the strong residuals autocorrelation, initially in the SAC attributed to the positive demand effects of the omitted variables, was linked, as was expected, to the spatial dependence of the output and especially the prices. In addition to this, note that the spatial lagged covariates more related to the single implementation of each Municipality (`Decentralized office`, `Advanced equipment` and `Allocative efficiency`) are not significant.

The incorporation of the local spatial factors into the estimation is necessary as it is confirmed in figure 12.1; the SAC-mixed residuals - right figure - unlike the OLS ones - left figure - in fact are not correlated with the own spatially lagged residuals, that is, there is no more global spatial autocorrelation in estimation SAC residuals.



(a) *Moran plot OLS4 residual*

(b) *Moran plot SAC residual*

Figure 12.1: Comparing the estimation residuals - Moran plot

The benefits of the proposed estimation method are evident even in terms of local spatial autocorrelation: as shown in figure 12.2, the proposed method allows to reduce substantially the local spatial autocorrelation with respect to the unconditional OLS estimations.

[46] Please note that we used the Kelejian and Prucha notation.

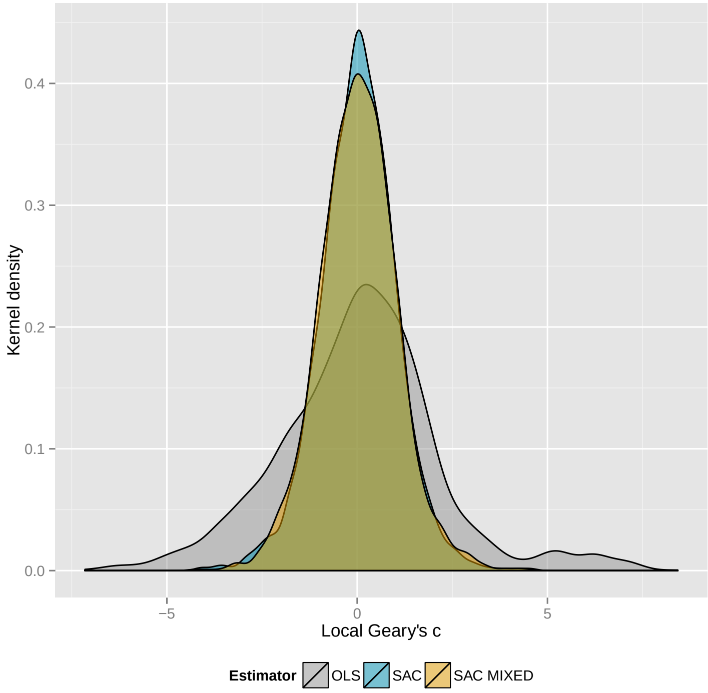


Figure 12.2: Kernel density of local Geary c - OLS, SAC and SAC mixed

Limitations related to the OLS static model (from a territorial point of view) are also evident analysing the single local covariates importance on cost varying location; but how demonstrate this? GWR (see Subsection 4.3.4) is a valid answer given that performs a series of weighted least squares regressions on subsets of the data, where the influence of an observation i decreases with the Euclidean distance to a regression point j. These distance-dependent weights are determined by a kernel function and the range of the input data is set according to a specific bandwidth in order to carry on a LWR (Fotheringham et al., 2002) for each spatial subset.
The aim of this part, therefore, is not so much to estimate the cost locally, but to highlight how and where the covariates coefficients remain stable; in other words, stable coefficients in space justify the adoption of a static model as OLS, while variable coefficients suggest the need for spatial models.

Figure 12.3 and table 12.4 show how, especially for the prices, the impact on cost is not homogeneous from a territorial point of view pushing to sharply reject a prediction model that does not take into account the different organizational models that insist on heterogeneous territories.

After identifying the correct estimation model, the last task of our empirical analysis is the application of the estimated SAC-mixed model[47] to a dataset in which

[47]The authors thanks Roger Bivand and Martin Gubri for their advice regarding the predictions

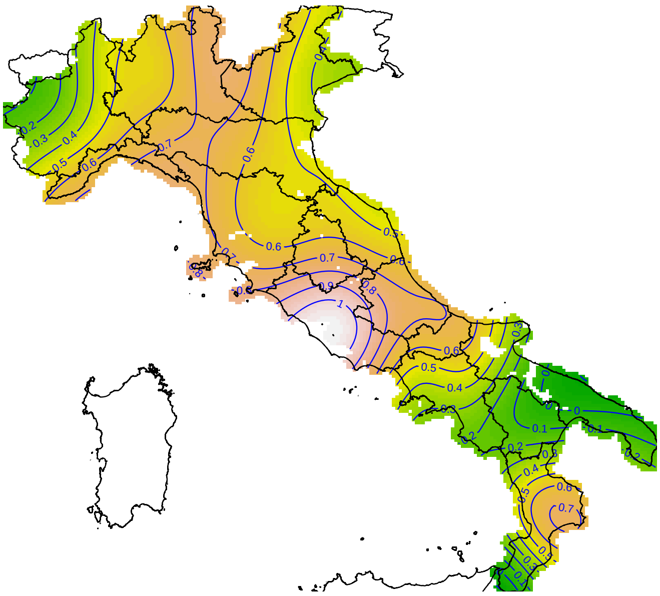


Figure 12.3: Semi-parametric smoothing function of the GWR w_l coefficient

Statistic	N	Mean	St. Dev.	Min	Max
Output coefficient	3,413	0.545	0.045	0.393	0.683
w_k coefficient	3,413	0.185	0.142	−0.319	0.733
w_l coefficient	3,413	0.522	0.210	−0.383	1.109

Table 12.4: Summary of the GWR coefficients

all variables had the same historical values except allocative efficiency that was equal to maximum for all units; this means that, in the application phase more inefficient units receive a transfer minor than the value assigned to the efficient ones (see figure 12.4) given that the efficiency coefficient is negative.

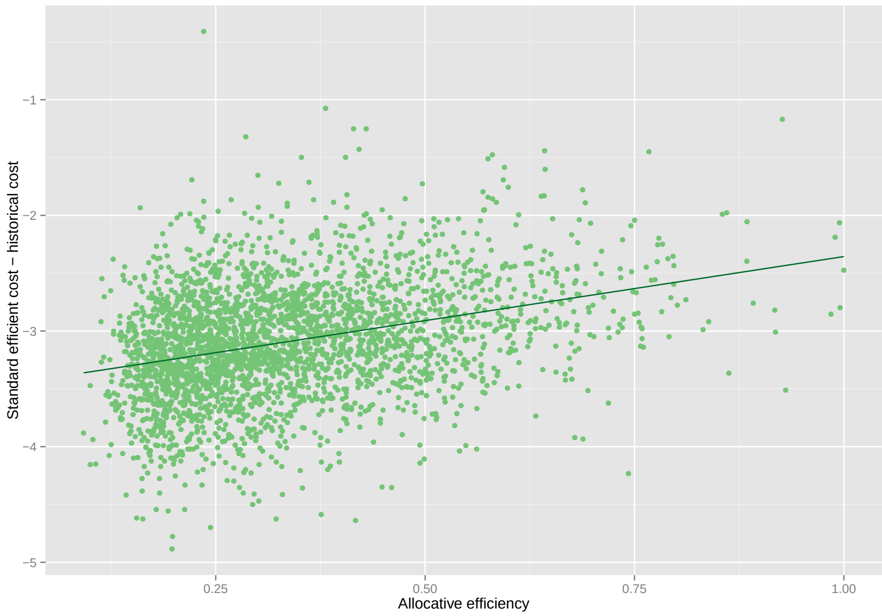


Figure 12.4: Estimated cost saving (€ per capita) and allocative efficiency

Finally, figure 12.5 show the cost saving territorial distribution highlighting that major savings could be obtained in metropolitan areas near Milan and Naples and in the Apulia region.

12.6 Concluding remarks

This paper presents a novel framework to incorporate spatial dependence, allocative efficiency and standard level of service into a public cost setting: a more accurate allocation of intergovernmental transfers let reach a higher level of equity and efficiency, two aspects that must be pursued and evaluated together.
Without taking into account spatial dependence, in fact, full territorial equity cannot be attained; without fixing a given standard level of service nothing guarantees that the differences in terms of social equity can be mitigated; without estimating allocative efficiency between output and input in terms of prices a lower cost of

in spatial autoregressive models; more in particular we used the *sppred* function - still not public in R *spdep* package - with the KP3 (Kelejian and Prucha, 2007) predictor in order to obtain the best linear unbiased prediction (BLUP).

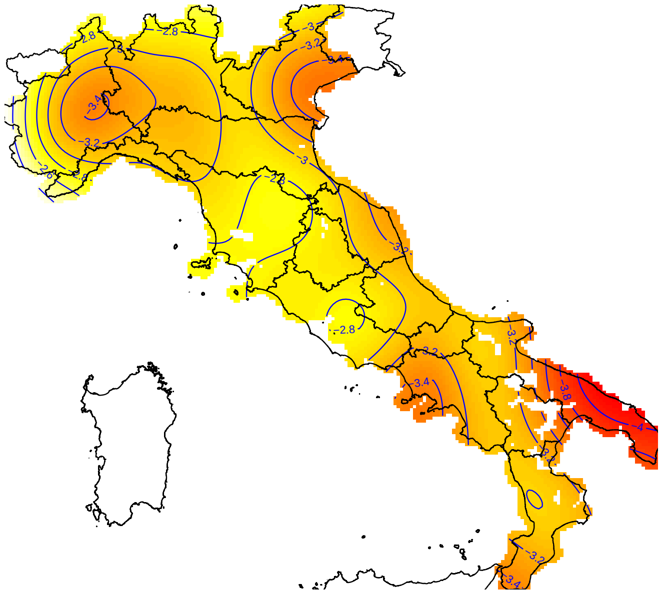


Figure 12.5: Semi-parametric smoothing function of per capita estimated cost saving

service can not be reached; in other words, without taking into account these issues nothing guarantees reaching the main aim of the public action.

The proposed approach has been tested on a novel very extended dataset covering more than 6.000 Italian Municipalities highlighting the heterogeneous relationship among cost, prices, output and demand factors and the need to explain this complexity through flexible empirical models.

It is possible and desirable - especially in Italy - to finally address the issue of financing LAs with a new approach: *(i)* more transparent straightforward by basing equalization standard on objective and correct criteria, *(ii)* more equitable by taking into account the standard level of service without penalizing LAs that produce a good level of service and *(iii)* more consistent by maintaining a stable evaluation system over time.

CHAPTER 13

OUTPUT DISTANCE FUNCTIONS: "BANK FINANCIAL WORLD CRISIS: INEFFICIENCIES AND RESPONSIBILITIES"

Working paper:
Fusco E. and Maggi B. (2016). *Bank Financial world crisis: Inefficiencies and Responsibilities.* DSS Empirical Economics and Econometrics Working Papers Series, DSS-E3 WP 2016/2, University of Rome La Sapienza.
http://www.dss.uniroma1.it/RePec/sas/wpaper/20162_fm.pdf

13.1 Introduction

Untill recently, most of the literature regarding banking systems efficiency neglected the question of problem loans. Under the influence of the 2008-9 crisis, this question started receiving growing consideration. Berger and DeYoung (1997) pioneered this field trying to face the study of the relations between problem loans and efficiency by means of the Granger-causality method, Hughes and Mester (1993) considered problem loans inside the frontier function. However, both these attempts are not satisfactory because, the former is a mere statistical tool based on the VAR methodology and so deprived of an economic interpretation of causality, the latter is incoherent since an increase of efficiency may be due simply to an increasing number of regressors. Only at the beginning of the first decade of the 20s with the works of Pastor (2002) and Pastor and Serrano (2005) the question under consideration has been addressed properly with a non parametric approach, which is of less powerful insight from a modelization point of view. Pastor and Serrano (2006) adopted a parametric approach but did not find a functional relationship between stochastic frontier and non performing loans (NPLs) and focused their investigation on the connection between NPLs and X-efficiency. Maggi and Guida (2011) addressed this point by considering an indirect function linking NPLs with stochastic frontier.

With the present work we go further by directly inserting the NPLs variable in the stochastic frontier as a negative output, taking advantage of the fact that our definition of efficiency relies on the concept of the distance function. In this way we are capable of assessing on the quality of the problem loans adopted by banks and on their responsibility in the risk management. The former question is addressed by calculating the price of non performing loans per year, bank and country considered in our dataset, the latter by comparing the management - in terms of variance analysis - of NPLs and their price across geographic areas and bank dimension over time. In doing so we provide a methodology which allows on one hand to alert in advance on an incumbent state of crisis and, on the other hand to evaluate the responsibility to be imputed to the main actors in the credit sectors, *i.e.* banks and local governments. Furthermore, an economic policy in terms of regulatory activities focused on the NPLs price comes out naturally as an implication of the analysis implemented. In fact, notably, the NPLs price is unknown and therefore not normally observable. Instead, our methodology allows its calculation from the first order conditions underlying the estimation performed. Indeed, in Maggi and Guida (2011) there is also the possibility of evaluating a similar indicator. However, from the cost function there considered, the marginal cost calculated cannot be assumed as a price-quality indicator in that this would have been possible only with perfect competition which is not the case for a credit market. Moreover, that methodology passes through the definition of a density function which inevitably involves a degree of arbitrariness in its form of definition. From an econometric point of view, we are -to our knowledge- the first to adopt the semi-nonparametric Fourier specification which, among the functional-flexible-form alternatives, is capable of guaranteeing the convergence of the estimated parameters and the related X-efficiency to the true ones (Gallant, 1981, Berger et al., 1997).

Our goals consist in: 1) finding a map of the responsibilities of the last financial crisis, 2) finding a road to regulate the risk in the credit market, 3) alerting the crisis period, 4) providing a rigorous method to calculate efficiency in case of a production function with undesired outputs. In order to cope with the need of monitoring the credit sector, for the reasons mentioned above, our prime necessity is to calculate the shadow-price applied to NPLs.

We intend to derive the X-inefficiency and a closed form solution for price of problem loans conceived as a negative output. Such a closed form will be used for the estimation in the next section. The representative commercial bank uses a positive vector of N inputs, denoted by $\mathbf{x} = (x_1, ..., x_N)$, $\mathbf{x} \in \mathbb{R}^N_+$ to produce a positive vector of M outputs, denoted by $\mathbf{u} = (u_1, ..., u_M)$, $\mathbf{u} \in \mathbb{R}^M_+$. The production technology of the bank can be defined by the output set, $P(\mathbf{x})$ that can be produced by means of the input vector $\mathbf{x}$, *i.e.*, $P(\mathbf{x}) = \left\{\mathbf{u} \in \mathbb{R}^M_+ : \mathbf{x} \textit{ can produce } \mathbf{u}\right\}$. It is also assumed that technology satisfies the usual axioms initially proposed by Shephard (1970), which allow to define the distance function -in terms of output- as the reciprocal of the maximum radial expansion of a given output vector proportional to the maximum output attainable. In such a way the resulting output vector remains within $P(\mathbf{x})$, being attainable using the available resources and technology. The output distance can be formally defined as [48]:

$$D_o(\mathbf{x},\mathbf{u}) = inf\left\{\theta : \left(\frac{\mathbf{u}}{\theta}\right) \in P(\mathbf{x})\right\} \tag{13.1}$$

where $D_o(\mathbf{x},\mathbf{u})$ is the distance from the bank's output set to the frontier, and $\theta \in [0,1]$ is the corresponding level of efficiency. The output distance function seeks the largest proportional increase in the observed output vector $\mathbf{u}$ provided that the expanded vector $(\frac{\mathbf{u}}{\theta}))$ is still an element of the original output set (Färe and Primont, 1995). Such an expression defines the *weak* disposability of outputs and therefore the inefficiency, which could explain, in our context, the presence of NPLs (undesirable outputs) that banks generate in their production processes, and that cannot be freely eliminated either because it would require a greater use of inputs, and/or because resources would have to be diverted from marketable production.

In fact, by considering the NPLs as an output of a production process, other than giving the advantage of deriving the correspondent price, it eliminates the empirical complications that would have occurred using a cost function approach. In fact, in this case a simultaneity problem would have arisen between inefficiency and therefore costs- and NPLs considered as an explicative variable. Our approach exploits the duality of maximum revenue problem, expressed in terms of distance function (13.1), where the correspondence between the primal and the dual problems relies on efficiency and output prices. Furthermore, such an approach allows the definition of inefficiency as a function of outputs and prices, included that one of NPLs, on which the empirical analysis is focused. More specifically, undesirable outputs, such as NPLs, have non-positive shadow prices that may be obtained empirically by exploiting the above mentioned duality. Now we set primal and revenue function problems in order to find the two corresponding shadow prices vectors in natural numbers and normalized for the revenue function, respectively. Then, we find the NPLs price in natural numbers from the revenue function (see Section 2.2 for a methodological discussion).

13.2 Variables and data

Data are from Bankscope and are referred to 517 Commercial Banks in Europe and 2404 in the U.S., the sample period is 2000-2008. Europe includes the Euro system plus UK, Sweden, Norway and Turkey. The list of countries considered is reported in the following Tables 13.1 and 13.2. The large database used allows for a very specific

[48]This expression is equivalent to the reciprocal of the output oriented efficiency measure of Farrell (Farrell, 1957 and Fare and Knox Lovell, 1978).

assessment both on the responsibilities of the single country policy and legislation and on the bank discipline during the last financial crisis.

The specification we adopt for the distance function is the *production approach* with three outputs and two inputs. Among *desirable* outputs we consider deposits (u_1), loans (u_2) and services (u_3), NPL (u_4) is the *undesirable* output and inputs are capital (x_1) and labor (x_2). Deposits are regarded as an output, rather than an input, for the diminishing importance of the corresponding interest rate still in the commercial banking system. All variables are expressed in nominal (dollar) values at constant prices (year 2000). The labor price is calculated as total personnel cost divided by the number of employees. Fixed assets have been transformed from the historical cost evaluation of balance sheet (International Accounting Standards 16) to current cost. As for capital price we estimate the following indirect function where the total capital is proxied[49]:

$$log(CapitalCost_{kt}) = \sum_{k=1}^{K} d_{pck} \cdot log(pc_{k0}) + \beta_1 log\left(\frac{A_t + L_t}{2}\right) + \varepsilon_{kt} \tag{13.2}$$

for $k = 1, ..., K, t = 2000, ..., 2008$
where A_t stands for total assets, L_t for total liabilities, pc_k are estimated coefficients of the dummy variables d_{pck} representing the capital price for each branch and $\beta_1 log\left(\frac{A_t+L_t}{2}\right)$ is the proxied total capital.
The services variable is constructed as the total value of "net" services.

Importantly, NPLs have different definitions across European countries and in the U.S.. In particular, the U.S. definition includes only the protested credits whilst a more prudential definition is adopted in Europe where uncertain loans are also considered. We may now calculate a first indicator of the banking system risk consisting in the empirical NPLs failure probability for loans given by NPLs out of loans and reported in Figure 13.1.

The descriptive statistics are shown below. We consider the mean and the standard deviation of the variables used in the estimation.

[49]We tried also direct functions both linear and logarithmic and other indirect functions with less qualitative results available upon request.

Table 13.1: Descriptive statistics: *Europe**

Country	**Statistics**	**Deposits** (u_1)	**Loans** (u_2)	**Services** (u_3)	**Capital** (x_1)	**Labour** (x_2)	**Non Performing loans** (u_4)
Austria	Mean	1747,04	1213,82	18,32	5985,93	267	12,09
	St. Dev.	5977,25	4553,54	32,11	18095,67	797	27,11
Belgium	Mean	2031,03	1306,25	20,79	4488,13	404	17,84
	St. Dev.	2170,53	1677,11	26,58	4974,75	407	15,84
Denmark	Mean	4661,29	4252,55	47,77	15587,65	885	27,34
	St. Dev.	17543,28	14334,46	151,75	55413,08	2459	60,31
Finland	Mean	21048,81	14555,98	238,64	67574,18	3183	91,98
	St. Dev.	23227,00	15318,52	294,19	87932,30	3372	68,33
France	Mean	13656,28	6885,30	164,02	37467,77	1777	37,99
	St. Dev.	68586,04	30746,82	799,17	205861,30	6327	88,48
Germany	Mean	8495,53	5686,36	64,33	21259,51	815	30,50
	St. Dev.	43770,04	25874,97	331,86	107016,10	3456	81,58
Great Britain	Mean	1425,02	661,53	18,11	3747,84	134	9,62
	St. Dev.	3006,67	1646,89	41,26	9392,71	209	14,44
Greece	Mean	20988,58	14953,53	225,88	47050,90	5307	95,79
	St. Dev.	19250,46	13110,13	247,78	43570,73	4192	61,60
Ireland	Mean	1628,32	876,27	4,16	11277,91	28	14,63
	St. Dev.	1351,37	752,11	13,56	18714,80	24	9,34
Italy	Mean	9368,70	8047,37	151,08	29230,47	2156	51,09
	St. Dev.	25245,42	21729,86	405,87	83840,79	5555	76,63
Luxembourg	Mean	6482,70	1791,80	51,02	14633,63	253	21,08
	St. Dev.	9543,99	2645,64	86,90	20937,75	429	21,58
Norway	Mean	8917,08	8516,49	91,95	20680,95	1067	57,94
	St. Dev.	12749,99	11497,77	127,96	28475,77	1309	62,61
Holland	Mean	1739,90	961,61	19,54	3634,53	181	16,90
	St. Dev.	573,13	327,12	17,51	1263,83	181	3,88
Portugal	Mean	12354,38	9385,98	183,55	32140,49	1924	58,88
	St. Dev.	17147,59	13751,15	214,46	45620,39	2535	71,89
Spain	Mean	28746,29	21868,65	305,62	73064,57	6056	107,96
	St. Dev.	50308,91	35403,95	654,43	140552,40	12693	120,13
Sweden	Mean	12186,94	6861,50	123,59	31601,42	1401	39,05
	St. Dev.	28378,89	16713,39	290,24	74676,69	3093	75,51
Switzerland	Mean	8456,13	3261,29	150,64	19532,63	677	17,27
	St. Dev.	65926,16	22076,50	1158,58	151058,40	4533	66,77
Turkey	Mean	6663,87	3679,48	177,47	15626,92	6039	33,50
	St. Dev.	7067,57	4082,52	197,98	16380,99	7292	34,09
Europe	Mean	8864,66	5420,21	105,41	23524,42	1246	32,79
	St. Dev.	43645,11	21728,41	588,05	116975,50	4685	72,43

**Note*: Time average data are expressed in millions of Dollars

Table 13.2: Descriptive statistics: *U.S.**

Country	**Statistics**	**Deposits** (u_1)	**Loans** (u_2)	**Services** (u_3)	**Capital** (x_1)	**Labour** (x_2)	**Non Performing loans** (u_4)
Alabama	Mean	138,33	105,40	1,28	461,65	60	1,46
	St. Dev.	153,33	126,36	1,71	509,40	69	2,05
Alaska	Mean	1069,90	744,94	17,38	3502,90	484	9,98
	St. Dev.	457,51	219,85	12,38	1661,77	225	2,51
Arizona	Mean	104,74	93,13	0,67	332,77	35	1,26
	St. Dev.	54,12	45,70	0,55	175,67	21	1,07
Arkansas	Mean	216,39	165,79	2,69	698,86	99	2,40
	St. Dev.	277,49	219,59	4,65	893,72	121	3,19
California	Mean	317,58	274,19	3,38	1050,45	107	3,73
	St. Dev.	429,59	424,74	5,87	1487,87	134	5,45
Colorado	Mean	291,00	227,30	3,38	910,52	99	2,96
	St. Dev.	368,11	313,67	4,69	1131,38	107	4,50
Connecticut	Mean	359,14	337,63	1,90	1224,77	105	3,25
	St. Dev.	216,80	236,87	2,30	770,70	54	2,16
Delaware	Mean	572,83	440,43	17,46	1859,01	138	7,05
	St. Dev.	795,49	601,74	30,20	2570,17	131	10,10
Florida	Mean	219,11	179,88	2,28	704,21	80	2,35
	St. Dev.	274,42	239,94	5,50	887,63	112	3,25
Georgia	Mean	162,21	134,68	1,65	519,78	64	2,01
	St. Dev.	159,96	142,80	2,46	513,98	64	2,41
Idaho	Mean	259,08	216,61	3,64	827,18	143	3,47
	St. Dev.	199,31	179,38	3,34	631,46	102	3,07
Illinois	Mean	250,90	191,98	2,69	806,28	87	2,46
	St. Dev.	535,00	396,56	7,18	1728,90	169	6,08
Indiana	Mean	302,54	252,21	4,19	1009,05	127	2,90
	St. Dev.	327,11	276,36	8,03	1093,48	142	3,21
Iowa	Mean	122,67	103,12	1,15	410,98	45	2,46
	St. Dev.	146,09	134,27	1,76	509,43	49	10,93
Kansas	Mean	111,72	88,82	1,75	364,69	52	1,23
	St. Dev.	128,16	105,72	4,76	423,43	64	1,62
Kentuchy	Mean	156,75	130,23	1,84	514,18	68	1,68
	St. Dev.	175,14	156,02	3,35	559,22	68	2,06
Louisiana	Mean	166,92	122,56	2,14	530,11	92	1,80
	St. Dev.	137,55	109,59	2,13	432,44	77	2,43
Maine	Mean	357,92	332,59	3,99	1239,64	133	3,99
	St. Dev.	218,25	184,48	4,15	752,53	47	2,63
Maryland	Mean	256,04	225,10	2,50	843,20	121	2,63
	St. Dev.	253,44	220,72	3,81	851,96	148	2,58
Massachusetts	Mean	170,91	140,21	1,38	572,73	59	1,60
	St. Dev.	228,99	184,21	2,72	791,37	73	2,49
Michigan	Mean	232,37	209,43	2,26	771,86	91	3,01
	St. Dev.	323,53	294,21	3,85	1081,65	135	4,49
Minnesota	Mean	168,90	142,17	1,46	544,25	55	1,92
	St. Dev.	226,04	184,37	2,14	732,36	52	2,69
Mississipi	Mean	207,18	158,74	2,65	671,60	99	2,24
	St. Dev.	227,22	195,03	4,30	742,07	118	2,54
Missouri	Mean	167,02	136,90	1,94	536,07	70	1,86
	St. Dev.	212,98	172,58	5,02	689,62	85	2,80
Montana	Mean	152,25	126,38	1,44	501,07	64	1,88
	St. Dev.	141,65	124,18	1,69	469,14	51	2,00
Nebraska	Mean	157,92	134,32	1,70	507,59	65	2,09

Continued on Next Page.

Table 13.2: Descriptive statistics: *U.S.**

Country	**Statistics**	**Deposits** (u_1)	**Loans** (u_2)	**Services** (u_3)	**Capital** (x_1)	**Labour** (x_2)	**Non Performing loans** (u_4)
	St. Dev.	234,49	207,96	3,77	748,72	90	3,83
Nevada	Mean	296,77	232,23	0,09	930,84	81	3,57
	St. Dev.	450,03	367,39	8,84	1412,74	97	6,81
New Hampshire	Mean	278,76	236,47	2,15	923,97	114	2,89
	St. Dev.	180,83	148,04	1,83	593,58	75	2,46
New Jersey	Mean	403,32	291,87	2,06	1311,36	103	3,86
	St. Dev.	722,25	431,84	5,49	2398,59	195	7,58
New Mexico	Mean	177,40	135,37	1,77	568,59	75	1,79
	St. Dev.	211,24	190,84	2,11	684,07	65	2,12
New York	Mean	503,54	302,44	4,61	1653,79	135	4,19
	St. Dev.	938,82	420,23	8,81	3146,15	153	7,61
North Carolina	Mean	388,06	340,69	3,74	1277,65	149	4,70
	St. Dev.	316,63	286,39	3,73	1022,51	127	4,33
North Dakota	Mean	170,11	145,40	2,10	545,12	76	2,04
	St. Dev.	248,47	234,69	4,28	794,56	107	3,21
Ohio	Mean	155,69	124,61	1,47	518,19	68	1,45
	St. Dev.	202,46	161,72	3,26	706,08	86	2,24
Oklahoma	Mean	144,26	113,47	1,91	461,54	73	1,31
	St. Dev.	167,05	140,41	2,70	537,30	78	1,76
Oregon	Mean	298,07	272,26	3,35	951,79	147	3,82
	St. Dev.	298,40	302,82	3,34	956,51	127	5,42
Pennsylvania	Mean	306,48	238,52	2,95	1040,64	121	2,66
	St. Dev.	244,16	190,12	3,81	823,73	112	2,24
Rhode Island	Mean	844,72	740,07	7,07	3027,55	245	10,01
	St. Dev.	121,37	130,17	1,71	615,99	29	1,39
South Carolina	Mean	216,28	185,88	2,37	716,97	88	2,32
	St. Dev.	220,79	211,66	3,45	712,27	95	2,80
South Dakota	Mean	215,88	179,46	3,33	689,67	88	2,11
	St. Dev.	354,76	300,89	11,95	1102,97	122	3,30
Tennessee	Mean	173,61	140,24	1,77	554,16	74	5,09
	St. Dev.	202,39	176,61	2,14	646,64	63	26,92
Texas	Mean	255,25	177,31	4,20	809,56	115	2,18
	St. Dev.	559,52	427,80	11,81	1813,10	217	4,98
Utah	Mean	223,56	187,50	3,01	713,14	99	2,95
	St. Dev.	178,44	153,63	3,03	557,56	87	3,58
Vermont	Mean	200,52	163,03	2,18	644,51	102	1,94
	St. Dev.	73,33	67,46	1,53	259,36	34	0,95
Virginia	Mean	243,88	199,92	2,94	790,76	111	2,48
	St. Dev.	197,69	169,57	4,81	664,15	90	2,43
Washington	Mean	275,15	233,72	3,37	898,31	111	3,24
	St. Dev.	401,96	368,43	6,59	1317,89	151	5,52
West Virginia	Mean	230,27	187,14	2,14	761,62	101	2,25
	St. Dev.	507,27	415,63	6,14	1764,97	219	5,21
Wisconsin	Mean	243,59	204,84	2,54	794,74	77	2,73
	St. Dev.	1095,49	900,07	16,11	3614,55	309	12,84
Wyoming	Mean	117,28	79,66	0,94	363,87	45	0,82
	St. Dev.	85,91	60,73	1,10	265,02	37	0,49
U.S.	Mean	219,53	174,83	2,44	714,60	85	2,43
	St. Dev.	426,82	332,39	6,68	1405,78	137	6,91

**Note*: Time average data are expressed in millions of Dollars

Figure 13.1, coherently with the wider definition of non performing loans in

Europe, shows that the ratio $NPLs/LOANS$ is always higher in Europe with an average of 2.7% compared with 1.4% of the U.S.. We note however that in Europe the ratio decreases during the years considered in contrast to the U.S. where it increases especially in 2008, raised from 1.36% to 1.57%.

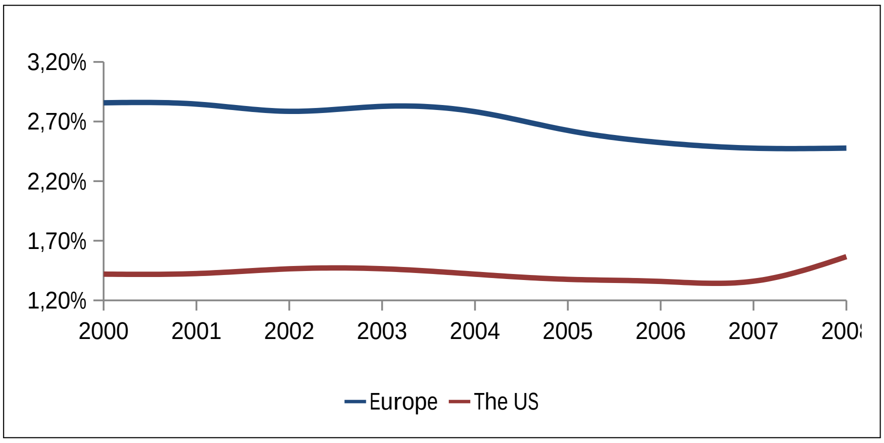


Figure 13.1: NPLs/Loans series of Europe and the U.S. (years 2000-2008)

13.3 Empirical methodology

In order to be able to calculate the shadow prices of the NPLs as described in section 2.2, in this section we estimate with a *Feasible Generalized Least Squares regression (FGLS)* the distance function. Following Aigner and Chu (1968) the problem to be solved is:

$$max \sum_{k=1}^{K} \Big[logD_o(x^k, u^k) - log(1)\Big] \tag{13.3}$$

where $k = 1, ..., K$ indexes individual banks.

This function is subject to the following constraints:

(i) $logD_o(x^k, u^k) \leq 0, k = 1, ..., K$

(ii) $\frac{\partial logD_o(x^k, u^k)}{\partial logu_m^k} \geq 0, m = 1, ..., h; k = 1, ..., K$

(iii) $\frac{\partial logD_o(x^k, u^k)}{\partial logu_m^k} \leq 0, m = h+1, ..., M; k = 1, ..., K$

(iv) $\sum_{m=1}^{M} \alpha_m = 1$, $\sum_{m=1}^{M} \alpha_{mm'} = \sum_{m=1}^{M} \gamma_{nm} = 0,$

(v) $\beta_{nn'} = \beta_{n'n}, \alpha_{mm'} = \alpha_{m'm}, \gamma_{nm} = \gamma_{mn}, \delta_{ij} = \delta_{ji}, \lambda_{ij} = \lambda_{ji},$
$m = 1, ..., M, n = 1, ..., N, i, j = 1, ..., N + M.$

where the first h outputs are desirables and the next *(M-h)* outputs are undesirables.

The objective function "minimizes" the sum of deviations of individual observations from the frontier of technology. The set of restrictions in (i) implies that each observation is located either on or below the technological frontier; the restrictions contained in (ii) ensure that desirable outputs will have nonnegative shadow prices for all firms, while (iii) undesirable outputs will have nonpositive shadow prices, also for all firms. The assumption of weak disposal of outputs is introduced by restriction (iv) that imposes homogeneity of degree 1 in outputs; finally (v) imposes symmetry.

The specification we adopt is the Fourier Flexible Functional form (FFF) which can globally approximate the unknown true function[50]. In order to test the robustness of our results, we also estimate the Translog (TL) being the most broadly used flexible functional form[51].

The FFF can be expressed as follows:

$$\begin{aligned} lnD_o = \alpha_0 &+ \sum_{n=1}^{N} \beta_n \cdot lnx_n + \sum_{m=1}^{M} \alpha_m \cdot lnu_m + \frac{1}{2} \sum_{n=1}^{N} \sum_{n'=1}^{N} \beta_{nn'} \cdot (lnx_n) \cdot (lnx_{n'}) \\ &+ \frac{1}{2} \sum_{m=1}^{M} \sum_{m'=1}^{M} \alpha_{mm'} \cdot (lnu_m) \cdot (lnu_{m'}) + \sum_{n=1}^{N} \sum_{m=1}^{M} \gamma_{nm} \cdot (lnx_n) \cdot (lnu_m) \\ &+ \sum_{i=1}^{M+N} \delta_i \cdot sin(z_i) + \sum_{i=1}^{M+N} \lambda_i \cdot cos(z_i) + \sum_{i=1}^{M+N} \sum_{j=1}^{M+N} \delta_{ij} \cdot sin(z_i + z_j) \\ &+ \sum_{i=1}^{M+N} \sum_{j=1}^{M+N} \lambda_{ij} \cdot cos(z_i + z_j) + \sum_{i=1}^{M+N} \sum_{j=1}^{M+N} \sum_{l=1}^{M+N} \delta_{ijl} \cdot sin(z_i + z_j + z_l) \\ &+ \sum_{i=1}^{M+N} \sum_{j=1}^{M+N} \sum_{l=1}^{M+N} \lambda_{ijl} \cdot cos(z_i + z_j + z_l) + \epsilon \end{aligned} \tag{13.4}$$

As for the determination of the frontier, D_o needs to be equal to unity and, in that case, the logarithm of the term on the left side of the equation (13.4) will equate zero.

[50]The FFF, developed by Gallant (1981), combines the standard TL with the non-parametric Fourier form. The number of trigonometric terms in the FFF has been chosen, following the rule of thumb expounded in Eastwood and Gallant (1991) to get a total number of parameters equal to the number of the observations raised to the power of two-thirds. Such a rule serves to obtain consistent and asymptotically normal estimates. However, as suggested in Gallant (1981), the effective number of coefficients may be corrected, by reducing the number of trigonometric terms, to avoid possible multicollinearity consequences.

[51]Results available upon request.

Consequently, it is necessary that outputs meet the homogeneity condition of degree 1 in order to satisfy the restriction (iv). Following Lovell, Travers, Richardson and Wood (1994), this condition has been imposed by normalising the distance function with one of the outputs. This starts from the assumption that homogeneity implies that:

$$D_o\left(\mathbf{x}, \frac{\mathbf{u}}{u_M}\right) = \frac{D_o(\mathbf{x}, \mathbf{u})}{u_M} \tag{13.5}$$

Substituting $u^*_m = \frac{u_m}{u_M}, m = 1, ..., M-1$ in (13.4) we obtain a regression of the general form:

$$ln(D_o/u_M) = FFF(\mathbf{x}, \mathbf{u}^*, \alpha, \beta, \gamma, \lambda, \delta) \tag{13.6}$$

where $\mathbf{u}^* = (\frac{u_1}{u_M}, \frac{u_2}{u_M}, ..., \frac{u_{M-1}}{u_M})$.
Equation (13.6) can be written as:

$$-ln(u_M) = FFF(\mathbf{x}, \mathbf{u}^*, \alpha, \beta, \gamma, \lambda, \delta) - ln(D_o) \tag{13.7}$$

In equation (13.7) the $-ln(D_o)$ can be interpreted as an error term which captures the technical inefficiency.

Finally, in order to improve the quality of the FFF approximation, and to have a reference with the Taylor expansion, outputs ($\mathbf{u}$) and inputs ($\mathbf{x}$) are all expressed as differences from the sample mean.

Therefore, the estimated FFF is:

$$\begin{aligned}
-lnu_M = {} & \alpha_0 + \sum_{n=1}^{N} \beta_n \cdot lnx_n + \sum_{m=1}^{M-1} \alpha_m \cdot lnu^*_m + \frac{1}{2} \sum_{n=1}^{N} \sum_{n'=1}^{N} \beta_{nn'} \cdot (lnx_n) \cdot (lnx_{n'}) \\
& + \frac{1}{2} \sum_{m=1}^{M-1} \sum_{m'=1}^{M-1} \alpha_{mm'} \cdot (lnu^*_m) \cdot (lnu^*_{m'}) + \sum_{n=1}^{N} \sum_{m=1}^{M-1} \gamma_{nm} \cdot (lnx_n) \cdot (lnu^*_m) \\
& + \sum_{i=1}^{M-1+N} \delta_i \cdot sin(z_i) + \sum_{i=1}^{M-1+N} \lambda_i \cdot cos(z_i) + \sum_{i=1}^{M-1+N} \sum_{j=1}^{M-1+N} \delta_{ij} \cdot sin(z_i + z_j) \\
& + \sum_{i=1}^{M-1+N} \sum_{j=1}^{M-1+N} \lambda_{ij} \cdot cos(z_i + z_j) + \sum_{i=1}^{M-1+N} \sum_{j=1}^{M-1+N} \sum_{l=1}^{M-1+N} \delta_{ijl} \cdot sin(z_i + z_j + z_l) \\
& + \sum_{i=1}^{M-1+N} \sum_{j=1}^{M-1+N} \sum_{l=1}^{M-1+N} \lambda_{ijl} \cdot cos(z_i + z_j + z_l) + \epsilon
\end{aligned} \tag{13.8}$$

where $u^*_m = \frac{u_m}{u_M}, m = 1, ..., M-1$ and $\epsilon = -ln(D_o) + ln(v)$.

For coherency purposes we have transformed the original independent variables in radiants to be used in the trigonometric part of the function as in Berger et al. (1997): $z_i = 0.2 \cdot \pi - \mu \cdot a + \mu \cdot ln(y_i)$ where $\mu \equiv \frac{0.9 \cdot 2\pi - 0.1 \cdot 2\pi}{(b-a)}$ and $[a, b]$ is the range of $ln(y_i)$. In this case $ln(y_i)$ with $i = 1, ..., 6$ refers to the sequence of deposits, loans, services, NPLs, capital and labor.
Once the distance function is estimated, we calculate the efficiency by adopting the *"Free efficiency"* method (see Berger, 1993):

$$TE_k = exp\left\{ - \left[\max_k(\widehat{\epsilon_{.k}}) - \widehat{\epsilon_{.k}} \right] \right\} \tag{13.9}$$

where $\widehat{\epsilon_{.k}} = \sum_t \epsilon_{tk} / \text{T}$.

Then the shadow price of NPLs may be found according to the procedure expounded above.
Hence, we estimate the price of loans by assuming that its shadow price is equal to its market price. So, we compute normalized shadow prices $r^*(\mathbf{x}, \mathbf{u})$ of desirable and undesirable outputs for each bank, using (2.16), and we calculate the shadow revenue R using the (2.18). Given the shadow revenue, we derive absolute shadow prices for NPLs using the (2.19).

13.4 Estimation

In this section we report the results of our estimation. All variables have been divided by its sample mean so that the first-order coefficients can be interpreted as distance elasticities evaluated at the sample means. The linear homogeneity in outputs is imposed using the output "Deposits" as a numeraire[52]. Due to multicollinearity we consider the Fourier approximation till the third term and drop some of the regressors[53] [54].

13.4.1 Europe

[52] The choice of the output is arbitrary and the resulting estimates are invariant to the normalization (see Cuesta and Orea, 2002).
[53] All estimations and calculations have been done with Stata 11 software.
[54] Please see section 13.2 at page 224 for the meaning of the variables notations.

Table 13.3: Distance function estimation: *Europe*

Dependent variable: $ln(1/u_1)$	**Coef.**	**Std. Err.**	**z**	$P > z$	**95% Conf.**	**Interval**
$ln(u_2/u_1)$	0.349	0.113	3.080	0.002	0.127	0.572
$ln(u_3/u_1)$	0.153	0.049	3.120	0.002	0.057	0.249
$ln(u_4/u_1)$	-0.352	0.204	-1.730	0.084	-0.751	0.047
$ln(x_1)$	-0.466	0.064	-7.240	0.000	-0.593	-0.340
$ln(x_2)$	-0.771	0.033	-23.030	0.000	-0.836	-0.705
$ln(u_2/u_1)^2$	0.084	0.011	7.810	0.000	0.063	0.105
$ln(u_2/u_1)ln(u_3/u_1)$	-0.050	0.006	-8.360	0.000	-0.062	-0.039
$ln(u_2/u_1)ln(u_4/u_1)$	0.017	0.003	4.970	0.000	0.010	0.024
$ln(u_3/u_1)^2$	-0.007	0.016	-0.470	0.640	-0.039	0.024
$ln(u_3/u_1)ln(u_4/u_1)$	0.008	0.002	3.760	0.000	0.004	0.012
$ln(u_4/u_1)^2$	-0.052	0.020	-2.630	0.009	-0.092	-0.013
$ln(x_1)^2$	-0.029	0.017	-1.690	0.092	-0.062	0.005
$ln(x_1)ln(x_2)$	-0.043	0.006	-7.730	0.000	-0.054	-0.032
$ln(x_2)^2$	-0.106	0.020	-5.180	0.000	-0.146	-0.066
$ln(u_2/u_1)ln(x_1)$	-0.017	0.008	-1.970	0.048	-0.033	0.000
$ln(u_2/u_1)ln(x_2)$	0.011	0.009	1.180	0.236	-0.007	0.029
$ln(u_3/u_1)ln(x_1)$	-0.049	0.015	-3.370	0.001	-0.078	-0.021
$ln(u_3/u_1)ln(x_2)$	-0.048	0.012	-3.940	0.000	-0.072	-0.024
$ln(u_4/u_1)ln(x_1)$	0.005	0.003	1.980	0.047	0.000	0.010
$ln(u_4/u_1)ln(x_2)$	-0.029	0.003	-10.350	0.000	-0.034	-0.023
$sin(z_2)$	-0.029	0.549	-0.050	0.957	-1.106	1.047
$sin(z_4)$	-1.879	1.011	-1.860	0.063	-3.860	0.102
$sin(z_5)$	-0.054	0.272	-0.200	0.844	-0.586	0.479
$cos(z_{22})$	-0.124	0.086	-1.440	0.149	-0.292	0.044
$sin(z_{22})$	-0.526	0.209	-2.520	0.012	-0.935	-0.116
$cos(z_{33})$	0.326	0.059	5.490	0.000	0.210	0.443
$sin(z_{33})$	-0.465	0.042	-10.960	0.000	-0.548	-0.382
$cos(z_{44})$	-0.244	0.152	-1.610	0.108	-0.542	0.053
$sin(z_{44})$	-0.515	0.264	-1.950	0.051	-1.033	0.003
$cos(z_{55})$	0.127	0.039	3.230	0.001	0.050	0.204
$sin(z_{55})$	-0.224	0.090	-2.490	0.013	-0.401	-0.048
$cos(z_{66})$	0.092	0.077	1.200	0.231	-0.058	0.242
$sin(z_{66})$	-0.212	0.068	-3.130	0.002	-0.344	-0.079
$cos(z_{23})$	-0.176	0.043	-4.110	0.000	-0.259	-0.092
$sin(z_{23})$	0.251	0.023	10.840	0.000	0.206	0.297
$sin(z_{24})$	-0.121	0.030	-4.040	0.000	-0.180	-0.062
$cos(z_{25})$	-0.095	0.043	-2.220	0.026	-0.180	-0.011
$sin(z_{25})$	0.036	0.024	1.500	0.135	-0.011	0.082
$cos(z_{26})$	0.005	0.042	0.130	0.898	-0.076	0.087
$sin(z_{26})$	-0.054	0.027	-2.010	0.045	-0.108	-0.001
$cos(z_{35})$	-0.271	0.074	-3.650	0.000	-0.417	-0.125
$sin(z_{35})$	-0.270	0.042	-6.420	0.000	-0.353	-0.188
$cos(z_{36})$	-0.050	0.059	-0.850	0.396	-0.165	0.065
$sin(z_{36})$	-0.256	0.035	-7.380	0.000	-0.324	-0.188
$cos(z_{56})$	-0.132	0.022	-5.880	0.000	-0.176	-0.088
$sin(z_{56})$	0.032	0.018	1.760	0.079	-0.004	0.069
$cos(z_{222})$	-0.099	0.039	-2.540	0.011	-0.176	-0.023
$cos(z_{333})$	0.260	0.030	8.610	0.000	0.201	0.319
$cos(z_{444})$	-0.115	0.051	-2.250	0.024	-0.215	-0.015
$cos(z_{555})$	-0.010	0.016	-0.640	0.524	-0.043	0.022
$cos(z_{666})$	0.078	0.023	3.410	0.001	0.033	0.123
$sin(z_{222})$	-0.175	0.060	-2.940	0.003	-0.292	-0.058
$sin(z_{333})$	-0.079	0.021	-3.770	0.000	-0.120	-0.038

Continued on Next Page.

Table 13.3: Distance function estimation: *Europe*

Dependent variable: $ln(1/u_1)$	**Coef.**	**Std. Err.**	**z**	$P > z$	**95% Conf.**	**Interval**
$sin(z_{444})$	-0.088	0.054	-1.620	0.106	-0.194	0.018
$sin(z_{555})$	-0.017	0.026	-0.650	0.517	-0.069	0.035
$sin(z_{666})$	-0.132	0.030	-4.450	0.000	-0.190	-0.074
t	-0.005	0.003	-1.670	0.096	-0.011	0.001
$t(u_2/u_1)$	-0.007	0.002	-4.700	0.000	-0.010	-0.004
$t(u_3/u_1)$	-0.001	0.001	-0.450	0.652	-0.003	0.002
$t(u_4/u_1)$	0.005	0.001	4.940	0.000	0.003	0.006
$t(x_1)$	0.006	0.001	4.110	0.000	0.003	0.008
$t(x_2)$	-0.003	0.001	-1.870	0.061	-0.006	0.000
Austria	-0.928	0.150	-6.180	0.000	-1.223	-0.634
Belgium	-1.019	0.151	-6.730	0.000	-1.316	-0.722
Denmark	-0.756	0.150	-5.050	0.000	-1.050	-0.463
Finland	-1.031	0.158	-6.540	0.000	-1.340	-0.722
France	-0.988	0.150	-6.600	0.000	-1.282	-0.695
Germany	-1.108	0.150	-7.390	0.000	-1.402	-0.814
Greece	-0.718	0.153	-4.690	0.000	-1.018	-0.418
Great Britain	-0.954	0.151	-6.320	0.000	-1.251	-0.658
Ireland	-1.496	0.173	-8.650	0.000	-1.835	-1.157
Italy	-1.042	0.150	-6.950	0.000	-1.336	-0.748
Luxembourg	-1.443	0.151	-9.560	0.000	-1.739	-1.147
Netherlands	-0.641	0.163	-3.920	0.000	-0.961	-0.321
Norway	-1.285	0.160	-8.030	0.000	-1.598	-0.971
Portugal	-0.844	0.159	-5.320	0.000	-1.155	-0.533
Spain	-1.038	0.152	-6.850	0.000	-1.336	-0.741
Sweden	-1.136	0.151	-7.540	0.000	-1.431	-0.841
Switzerland	-1.151	0.150	-7.660	0.000	-1.446	-0.857
Turkey	(omitted)					
Constant	2.570	0.303	8.490	0.000	1.976	3.163

We get estimates significant and coherent with the literature (see among others Cuesta and Orea (2002)). In particular, looking at the elasticities of the first order terms, we find positive coefficients for desirable outputs (Loans, deposits and services)[55] and negative for the undesirable output (NPLs). The negative sign of the latter represents the opportunity cost measurable in terms of the loss in desirable outputs production that banks would incur in case of compliance with a regulation directed to compensate the NPLs.

As regards the management of inputs, the labor factor has a greater impact (0.771) on the production possibilities frontier than capital (0.466).

Looking at the country dummies, we can identify the spatial effect on efficiency (with respect to Turkey, the reference state). These differences can be caused by factors not considered in the analysis such as technology, environmental factors, externalities, etc.

The dummies are all negative, which means that they increase -other things being equal- the efficiency with respect to the base country (Turkey).

[55]Remember that the function is homogeneous of degree 1 in outputs, so that the coefficient of deposits is given by $1 - \sum_{m=1}^{M-1} \alpha_m$.

From this analysis it would seem that Ireland is the most efficient country compared to Turkey, and that the most inefficient are Netherlands, Greece and Denmark. These results seem implausible in light of the past financial crisis. Hence, we think that the distance function lacks considering, among inputs, the risk arising from the amount of NPLs.
Finally, the time variable (trend) has a negative coefficient (-0.01), which suggests that over time, there was on average an efficiency decrease of 1%.

13.4.2 The U.S.

Table 13.4: Distance function estimation: *U.S.*

Dependent variable: $ln(1/u_1)$	**Coef.**	**Std. Err.**	**z**	$P > z$	**95% Conf.**	**Interval**
$ln(u_2/u_1)$	0.302	0.043	7.040	0.000	0.218	0.386
$ln(u_3/u_1)$	0.165	0.024	6.800	0.000	0.117	0.212
$ln(u_4/u_1)$	-0.275	0.124	-2.210	0.027	-0.518	-0.032
$ln(x_1)$	-0.215	0.023	-9.380	0.000	-0.260	-0.170
$ln(x_2)$	-0.862	0.023	-38.100	0.000	-0.907	-0.818
$ln(u_2/u_1)ln(u_3/u_1)$	0.012	0.009	1.330	0.185	-0.006	0.029
$ln(u_2/u_1)ln(u_4/u_1)$	-0.060	0.009	-6.360	0.000	-0.078	-0.041
$ln(u_3/u_1)^2$	-0.087	0.017	-5.210	0.000	-0.120	-0.055
$ln(u_3/u_1)ln(u_4/u_1)$	-0.010	0.005	-1.800	0.072	-0.020	0.001
$ln(x_1)^2$	0.028	0.016	1.820	0.069	-0.002	0.059
$ln(x_1)ln(x_2)$	-0.137	0.005	-29.180	0.000	-0.147	-0.128
$ln(x_2)^2$	-0.118	0.024	-4.920	0.000	-0.166	-0.071
$ln(u_2/u_1)ln(x_1)$	0.217	0.020	10.970	0.000	0.179	0.256
$ln(u_2/u_1)ln(x_2)$	-0.203	0.034	-5.910	0.000	-0.270	-0.136
$ln(u_3/u_1)ln(x_1)$	-0.078	0.016	-4.970	0.000	-0.109	-0.047
$ln(u_3/u_1)ln(x_2)$	-0.123	0.004	-28.490	0.000	-0.132	-0.115
$ln(u_4/u_1)ln(x_1)$	-0.032	0.003	-11.820	0.000	-0.038	-0.027
$ln(u_4/u_1)ln(x_2)$	-0.040	0.003	-11.320	0.000	-0.046	-0.033
$cos(z_2)$	0.436	0.256	1.710	0.088	-0.065	0.937
$sin(z_4)$	-0.811	0.385	-2.100	0.035	-1.566	-0.055
$cos(z_{22})$	0.094	0.143	0.660	0.510	-0.186	0.374
$sin(z_{22})$	-0.033	0.021	-1.580	0.115	-0.074	0.008
$sin(z_{33})$	-0.214	0.022	-9.650	0.000	-0.257	-0.171
$cos(z_{44})$	0.169	0.016	10.600	0.000	0.138	0.200
$sin(z_{44})$	-0.199	0.112	-1.780	0.075	-0.417	0.020
$cos(z_{55})$	0.104	0.023	4.410	0.000	0.058	0.150
$sin(z_{55})$	0.201	0.015	13.210	0.000	0.171	0.231
$cos(z_{66})$	-0.476	0.035	-13.450	0.000	-0.546	-0.407
$sin(z_{66})$	0.061	0.043	1.440	0.150	-0.022	0.145
$cos(z_{25})$	0.279	0.025	10.970	0.000	0.229	0.329
$sin(z_{25})$	-0.126	0.015	-8.210	0.000	-0.156	-0.096
$cos(z_{26})$	-0.245	0.033	-7.530	0.000	-0.309	-0.181
$sin(z_{26})$	0.116	0.027	4.340	0.000	0.063	0.168
$cos(z_{34})$	-0.025	0.013	-1.910	0.057	-0.051	0.001
$sin(z_{34})$	0.008	0.009	0.940	0.345	-0.009	0.025
$cos(z_{35})$	-0.431	0.042	-10.180	0.000	-0.514	-0.348
$sin(z_{35})$	-0.136	0.013	-10.270	0.000	-0.162	-0.110
$cos(z_{222})$	-0.064	0.042	-1.520	0.128	-0.145	0.018
$cos(z_{333})$	-0.041	0.005	-7.910	0.000	-0.051	-0.031
$cos(z_{444})$	0.052	0.007	7.320	0.000	0.038	0.066
$cos(z_{555})$	0.018	0.009	2.050	0.040	0.001	0.036
$cos(z_{666})$	-0.125	0.010	-12.550	0.000	-0.144	-0.105
$sin(z_{333})$	-0.121	0.011	-11.020	0.000	-0.142	-0.099
$sin(z_{444})$	-0.013	0.024	-0.550	0.582	-0.060	0.034
$sin(z_{555})$	0.038	0.007	5.680	0.000	0.025	0.051
$sin(z_{666})$	0.115	0.017	6.870	0.000	0.082	0.148
t	0.013	0.001	9.860	0.000	0.010	0.015
$t(u_2/u_1)$	0.008	0.002	3.650	0.000	0.004	0.013
$t(u_3/u_1)$	0.008	0.001	8.700	0.000	0.006	0.010

Continued on Next Page.

Table 13.4: Distance function estimation: *U.S.*

Dependent variable: $ln(1/u_1)$	**Coef.**	**Std. Err.**	**z**	$P > z$	**95% Conf.**	**Interval**
$ty(u_4/u_1)$	-0.002	0.001	-3.380	0.001	-0.004	-0.001
$t(x_1)$	0.014	0.001	16.570	0.000	0.012	0.015
$t(x_2)$	-0.011	0.001	-11.030	0.000	-0.013	-0.009
Alabama	0.106	0.010	10.620	0.000	0.087	0.126
Alaska	-0.004	0.038	-0.110	0.916	-0.079	0.071
Arizona	-0.053	0.023	-2.300	0.022	-0.098	-0.008
Arkansas	0.095	0.010	9.640	0.000	0.075	0.114
California	-0.036	0.010	-3.740	0.000	-0.055	-0.017
Colorado	0.031	0.012	2.450	0.014	0.006	0.055
Connecticut	-0.148	0.024	-6.240	0.000	-0.194	-0.101
Delaware	-0.167	0.034	-4.910	0.000	-0.233	-0.100
Florida	0.000	0.010	0.000	1.000	-0.019	0.019
Georgia	0.009	0.008	1.140	0.255	-0.007	0.025
Idaho	0.182	0.016	11.670	0.000	0.151	0.212
Illinois	0.018	0.008	2.430	0.015	0.004	0.033
Indiana	0.097	0.010	9.960	0.000	0.078	0.117
Iowa	0.031	0.008	3.700	0.000	0.015	0.047
Kansas	0.100	0.009	11.420	0.000	0.083	0.117
Kentuchy	0.085	0.009	9.300	0.000	0.067	0.103
Louisiana	0.151	0.009	15.900	0.000	0.132	0.169
Maine	-0.014	0.019	-0.740	0.459	-0.051	0.023
Maryland	0.125	0.016	7.950	0.000	0.094	0.155
Massachusetts	0.030	0.008	3.900	0.000	0.015	0.045
Michigan	0.044	0.011	4.120	0.000	0.023	0.066
Minnesota	-0.027	0.011	-2.540	0.011	-0.048	-0.006
Mississipi	0.057	0.011	5.140	0.000	0.035	0.079
Missouri	0.113	0.008	14.430	0.000	0.097	0.128
Montana	0.082	0.014	5.970	0.000	0.055	0.108
Nebraska	0.057	0.010	5.640	0.000	0.037	0.077
Nevada	-0.024	0.024	-1.040	0.300	-0.070	0.022
New Hampshire	0.053	0.024	2.230	0.026	0.006	0.100
New Jersey	-0.038	0.012	-3.090	0.002	-0.062	-0.014
New Mexico	0.097	0.019	5.140	0.000	0.060	0.134
New York	-0.004	0.011	-0.340	0.731	-0.025	0.018
North Carolina	0.006	0.012	0.510	0.613	-0.017	0.029
North Dakota	0.089	0.012	7.230	0.000	0.065	0.114
Ohio	0.126	0.011	11.490	0.000	0.104	0.147
Oklahoma	0.144	0.009	16.480	0.000	0.127	0.161
Oregon	0.179	0.016	11.090	0.000	0.147	0.211
Pennsylvania	0.048	0.010	4.640	0.000	0.028	0.068
Rhode Island	-0.052	0.058	-0.910	0.364	-0.166	0.061
South Carolina	0.007	0.012	0.560	0.574	-0.017	0.031
South Dakota	0.058	0.017	3.430	0.001	0.025	0.092
Tennessee	0.068	0.009	7.240	0.000	0.050	0.087
Texas	0.080	0.008	10.230	0.000	0.065	0.096
Utah	-0.064	0.030	-2.140	0.032	-0.123	-0.005
Vermont	0.157	0.020	7.780	0.000	0.117	0.197
Virginia	0.114	0.011	10.110	0.000	0.092	0.136
Washington	0.056	0.012	4.640	0.000	0.033	0.080
West Virginia	0.133	0.014	9.600	0.000	0.106	0.160
Wyoming	(omitted)					
Constant	1.279	0.205	6.250	0.000	0.877	1.680

Also in this case the coefficients of inputs and outputs are significant and with correct sign.
Making a comparison with Europe we can say that in the U.S. there is a greater impact of labor (-0.86 vs -0.77) and the opposite for capital (-0.22 vs -0.47). This means that the labor factor (capital factor) in the U.S. performs more (less) than in Europe, in fact, increasing the latter the negative effect on efficiency is more limited. This is probably due to the lower dimension of capital employed in Europe compared to labor.
If we analyze the spatial dummies, there are two groups of countries placed above and below the baseline country (Wyoming) in terms of efficiency level. This leads us to question the inefficiencies and responsibilities of countries and banks.

13.5 Inefficiencies and responsibilities

In this section we report and comment the results of efficiency evaluations obtained for Europe and the U.S..
First, the distance function satisfies all constraints listed in section 13.3.
Table 13.5 shows the ranking of efficiency of banks in Europe[56] and Table 13.6 in the U.S.. The efficiency score for Europe and the U.S. has been calculated as average over banks.

Table 13.5: Efficiency: *Europe* (Time average data)

Country	**Efficiency**	**Country**	**Efficiency**
Great Britain	0.8136	*Denmark*	0.8116
Germany	0.8133	*Sweden*	0.8116
Austria	0.8132	*Portugal*	0.8113
France	0.8132	*Ireland*	0.8112
Belgium	0.8129	*Luxembourg*	0.8108
Italy	0.8128	*Switzerland*	0.8101
Greece	0.8120	*Spain*	0.8078
Netherlands	0.8117	***Europe***	**0.8120**

[56] We omit in table 13.5 Finland, Norway and Turkey given the limited available number of banks for these countries.

Table 13.6: Efficiency: *U.S.* (Time average data)

Country	Efficiency	Country	Efficiency	Country	Efficiency
Wyoming	0.8507	*Arkansas*	0.8456	*Virginia*	0.8445
Nevada	0.8496	*Georgia*	0.8453	*New Jersey*	0.8445
Connecticut	0.8480	*Alaska*	0.8453	*North Carolina*	0.8443
New Mexico	0.8476	*Missouri*	0.8452	*Wisconsin*	0.8443
Louisiana	0.8470	*Iowa*	0.8452	*Michigan*	0.8443
Montana	0.8466	*Oregon*	0.8451	*Alabama*	0.8439
North Dakota	0.8465	*Texas*	0.8451	*Florida*	0.8438
California	0.8465	*Maryland*	0.8450	*Minnesota*	0.8437
Mississipi	0.8464	*Illinois*	0.8450	*Vermont*	0.8436
Tennessee	0.8461	*Oklahoma*	0.8450	*Colorado*	0.8436
Ohio	0.8461	*Pennsylvania*	0.8450	*Maine*	0.8435
Idaho	0.8460	*Nebraska*	0.8450	*New York*	0.8433
South Carolina	0.8459	*Kansas*	0.8449	*Washington*	0.8430
Massachusetts	0.8459	*Kentuchy*	0.8449	*Delaware*	0.8409
South Dakota	0.8458	*West Virginia*	0.8448	*Utah*	0.8374
New Hampshire	0.8457	*Indiana*	0.8447		
Rhode Island	0.8456	*Arizona*	0.8446	***U.S.***	**0.8451**

We note that the average value of efficiency is high both in Europe (0.812) and even more in the U.S. (0.845) with a slight difference of about 3%. However, such a result may be due once again - as seen for the efficiency performance of Ireland in Europe - to the fact that the distance function does not consider the default risk. Actually, the NPLs price may well be different for the banks of these two countries even if the efficiency is similar. To verify this it suffices to think that the efficiency is defined by the distance function while the NPLs price (normalized) by its first derivative. Pastor and Serrano (2006) arrived to the same conclusion although with a non parametric approach, which shows that our result is not peculiar to the methodology here adopted. With the aim to consider a measure of the default risk we calculate the shadow prices of the NPLs. In Tables 13.7[57] and 13.8 shadow prices of the NPLs are shown in absolute value.

Table 13.7: Shadow price of NPL: *Europe* (Time average data)

Country	P^o_{NPL}	Country	P^o_{NPL}
Sweden	0.3281	*Denmark*	0.1394
Netherlands	0.2684	*Austria*	0.1167
Belgium	0.2392	*Germany*	0.1167
Switzerland	0.2138	*Italy*	0.1084
Luxembourg	0.2042	*Spain*	0.1057
France	0.1822	*Greece*	0.0821
Great Britain	0.1770	*Portugal*	0.0371
Ireland	0.1551	***Europe***	**0.1580**

[57] We omit in table 13.7 Finland, Norway and Turkey given the limited available number of banks for these countries.

Table 13.8: Shadow price of NPL: *U.S.* (Time average data)

Country	P^o_{NPL}	**Country**	P^o_{NPL}	**Country**	P^o_{NPL}
New Hampshire	0.3987	*Mississipi*	0.2395	*Michigan*	0.2060
Vermont	0.3852	*Arizona*	0.2367	*Delaware*	0.2055
Rhode Island	0.3795	*California*	0.2327	*Tennessee*	0.2051
Maine	0.3722	*Louisiana*	0.2316	*North Carolina*	0.2047
New Jersey	0.3511	*South Carolina*	0.2302	*Idaho*	0.2041
Pennsylvania	0.3300	*Washington*	0.2290	*Arkansas*	0.1979
Alaska	0.2988	*Iowa*	0.2273	*Georgia*	0.1969
Connecticut	0.2986	*Ohio*	0.2261	*Nebraska*	0.1963
Wyoming	0.2691	*Illinois*	0.2248	*Nevada*	0.1913
Virginia	0.2688	*Kentuchy*	0.2232	*Alabama*	0.1877
Indiana	0.2679	*West Virginia*	0.2228	*Oklahoma*	0.1872
Maryland	0.2639	*Montana*	0.2221	*Kansas*	0.1868
Oregon	0.2586	*Minnesota*	0.2205	*New Mexico*	0.1865
New York	0.2583	*Missouri*	0.2199	*North Dakota*	0.1740
Massachusetts	0.2575	*Florida*	0.2153	*Utah*	0.1672
Wisconsin	0.2513	*Colorado*	0.2143		
South Dakota	0.2406	*Texas*	0.2138	***U.S.***	**0.2272**

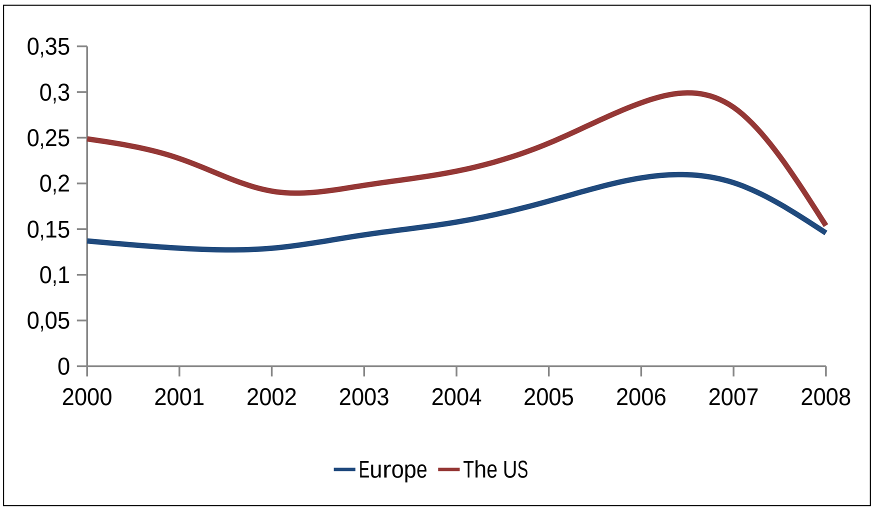


Figure 13.2: Shadow price of NPLs series of Europe and the U.S. (years 2000-2008)

As can be seen from Tables 13.7 and 13.8 even if the banks in our sample are very efficient and even more so than those in the U.S., this result may be reasonably due to a risky management of the credit activity. In fact, the average shadow price of NPL is quantitatively relevant in both areas but more costly in the U.S.. More specifically, on average, the cost of the debt collection amounts to 22% in the U.S. and 16% in Europe.

The graph in Figure 13.2 shows how in both cases the price of NPLs has greatly risen since 2002 with a peak between years 2006 and 2007 and has fallen during 2008 for the regulatory actions of the governments as a consequence of the crisis. In 2008 the two NPLs prices become almost equal.

But who is the responsible between countries and banks?

To answer this question we move from the consideration that if the bank risk is controlled across countries by each single bank, then the banking system would be reliable. On the other hand, if the risk doesn't vary across banks per each single country, then the regulation imposed by countries is effective. If the autonomy of the banks to manage the risk across countries is high, then the responsibility is more referable to banks. This problem can be analyzed in terms of between-variances, applied to the variable representing the risk, evaluated across countries ($\sigma^2_{B_{countries}}$) and across banks ($\sigma^2_{B_{banks}}$): the former represents the capacity of the banks to control the risk and the latter the capacity of counties to set appropriate regulations capable to control the risk. We normalize the former to the latter to make a comparison. We consider two variables of interest, $NPLs/L$ and NPLs price, and conclude that the higher is the $\sigma^2_{B_{countries}}/\sigma^2_{B_{banks}}$ ratio the less the attention devoted by countries to the control the risk of the banking system.

We set up an analysis of variance of both the ratio $NPLs/L$ and NPLs shadow prices by decomposing the total variance for *banks* and *countries*, and found that in both cases the between variance is the largest one.

Table 13.9: Analysis of variance of $NPLs/L^*$

Year	**Europe**	**U.S.**
2000	2.358	0.036
2001	2.062	0.033
2002	2.232	0.024
2003	2.384	0.010
2004	2.367	0.002
2005	2.299	0.001
2006	2.302	0.001
2007	1.972	0.002
2008	1.950	0.014

$^*\left(\sigma^2_{B_{countries(NPLs/L)}}/\sigma^2_{B_{banks(NPLs/L)}}\right)$

Table 13.10: Analysis of variance of the shadow price of the $NPLs^*$

Year	**Europe**	**U.S.**
2000	0.431	5.516
2001	0.096	3.180
2002	0.098	1.264
2003	0.202	1.704
2004	0.199	2.049
2005	0.221	3.020
2006	0.286	2.897
2007	0.221	2.088
2008	0.362	0.383

$^*\left(\sigma^2_{B_{countries}(P_{NPLs})}/\sigma^2_{B_{banks}(P_{NPLs})}\right)$

Tables 13.9 and 13.10 show that European countries are careful with the NPLs management ($\sigma^2_{B_{countries}}(NPLs/L) > \sigma^2_{B_{banks}}(NPLs/L)$), while the quality of NPLs - *i.e.* their price - is defined by the banks ($\sigma^2_{B_{countries}}(P_{NPLs}) < \sigma^2_{B_{banks}}(P_{NPLs})$).

Surprisingly we obtain for the two variables two unequivocally opposite evidences attesting the U.S. system, compared to Europe, as more vigilant on banks default when considering $NPLs/L$ and the opposite for the NPLs price. The explanation of such apparently different results is that the definition of NPLs in the U.S. is not so prudent as in Europe in that in the former case the NPLs refer only to the loans declared officially non reimbursable while in the latter one is much more cautious including also the loans declared protested against. Actually the main difference between the two regulatory systems is that, under the US the *generally accepted accounting principles* (GAAP), the *statement financial accounting standard* (SFAS) n.5 defines a very broad and vague criterion to detect the NPLs, based on the “probable” and “reasonably estimated” loss. As a consequence the loss provision becomes a strategic variable for banks which may increase it in case of bad evaluation from the markets to show a greater credibility or, on the contrary, may enhance it in the opposite case in order to improve profits by reducing the tax base. This of course artificially lowers or raises the variance across countries of $NPLs/L$ of the banks in the U.S. and Europe respectively [58]. Instead, according to the Basel agreements II and III, in Europe there is a lower bound of 1.25% of the “risk weighted asset” for the loss provision and an upper bound of 50% of the “regulatory capital requirements”[59].

[58] Note that such a result does emerge notwithstanding we considered the different definitions of the NPLs, in the two countries under exam, after having normalized between the variances.

[59] Moreover, still in the definition of the “risk weighted asset” the weights are more compelling in Europe than in the US.

13.6 Final Remarks and Policy implications

In the analysis developed we discuss the credit market, country's policy actions and efficiency of the banking system.

With regard to the credit market our analysis identifies an increasing NPLs price in the considered period as shown in Figure 13.2 and we underline that in a usual risk analysis it is difficult to take properly into account this trend as the NPLs price is not normally observable.

Moreover, comparing the NPLs price with the interest rate of loans, we reckon that the banks measure incorrectly the real risk and the cost of recovery of the NPLs by fixing an interest rate that does not contain adequately the effective NPLs price. Given such an excessive cost of NPL's recovery, it would be appropriate to monitor the lending banks policy with apposite regulations which take into account the NPLs price as a margin to be stored in case of loss.

A second point is that there is the necessity to homogenize the definition of NPLs through countries in order to avoid the ratio $NPLs/L$ being systemically and artificially lower in the US than in Europe. Contrarily our analysis shows that the NPLs price is always higher in the U.S. with respect to the European one.

Another significant result of our study is the importance of countries in explaining the recent financial crisis. In fact, from Table 13.10 in the U.S. the $\sigma^2_{B_{countries}}(P_{NPLs})/\sigma^2_{B_{banks}}(P_{NPLs})$ ratio is very high from 2000 to 2007 showing a great responsibility of the countries in not having preserved a homogeneous risk management of banks (low $\sigma^2_{B_{banks}}(P_{NPLs})$). This is confirmed by the low $NPLs/L$ ratio obtained in 2008 when the U.S. government intervened by introducing market-wide support measures and assisting failing financial institutions. In light of these facts, legislative measures for monitoring the banks would be important to avoid future crisis. In Europe, instead, this supervision was already effective as shown by the low $\sigma^2_{B_{countries}}(P_{NPLs})/\sigma^2_{B_{banks}}(P_{NPLs})$ ratio. Therefore, in order to improve the quality of loans in Europe the other direction is to look for some improvements in terms of efficiency. In effect, in such a respect we note that in Europe the risk strategies, concerning the ratio between non performing loans and loans, are very different among banks (high $\sigma^2_{B_{countries}}(NPLs/L)/\sigma^2_{B_{banks}}(NPLs/L)$), which is likely to be referred to different levels of efficiency in the loans management. Actually, Tables 13.5 and 13.6 show a pronounced lower efficiency in Europe than in the U.S.. A possible explanation of this fact is that European banks try to bypass the stricter rules on the NPLs registration by improving profits with a reduction in the regulatory capital[60]. A way to solve this problem would be to penalize risky banks by asking them to pay the NPLs price as a penalty. This would be a counterincentive to the expansion of NPLs as a strategy to gather more funds irrespective of the risk.

13.7 Conclusion

The analysis conducted in this paper showed that the recent bank crisis could have been anticipated if appropriate indicators had been used. We propose here the NPLs price which is not observable. Our econometric methodology based on the Fourier expansion validates significantly the theoretical set up adopted. Actually we found that the market interest rates do not adequately account for the risk of loans loss. Further, Europe and the U.S. have different peculiarities concerning the inefficiencies of the bank system and the responsibilities of the two countries. We found more countries' responsibility in terms of low regulations for the U.S. and a slightly more inefficiency for Europe. A proposal to monitor both aspects is to penalize risky banks by asking to pay the NPLs price as a penalty.

[60] Slovik (2012) finds the same result on the base of an analysis of the ratio between the risk-weighted assets to total asset.

Chapter 14

ENHANCING NON-COMPENSATORY COMPOSITE INDICATORS: A DIRECTIONAL PROPOSAL

Fusco, E. (2015) ***Enhancing non-compensatory composite indicators: A directional proposal.*** **European Journal of Operational Research, 242, 620 - 630, DOI:**10.1016/j.ejor.2014.10.017.

14.1 Introduction

The D-BoD model outlined in section 8.2 has also been applied to the terrestrial transport infrastructure endowment in European Regions following Vidoli and Mazziotta (2013) proposal.

14.2 Variables and data

The data set[61] includes information on two simple indicators[62] concerning roads (I_{Roads}) and railways (I_{Trains}) endowment for France, Germany, Italy and Spain.

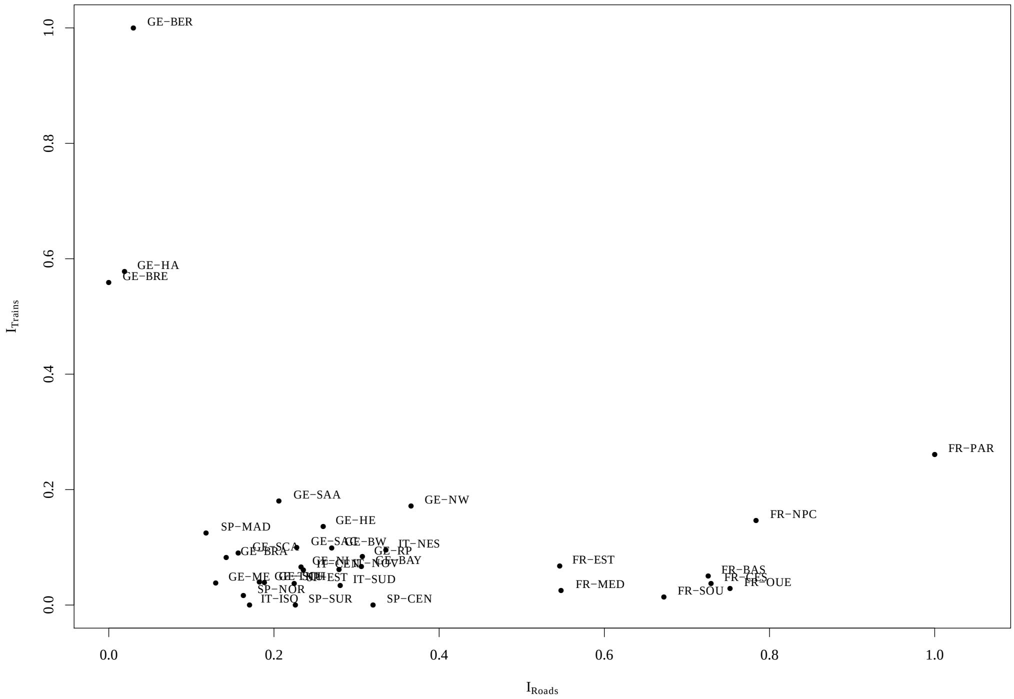


Figure 14.1: European infrastructure endowment - Data

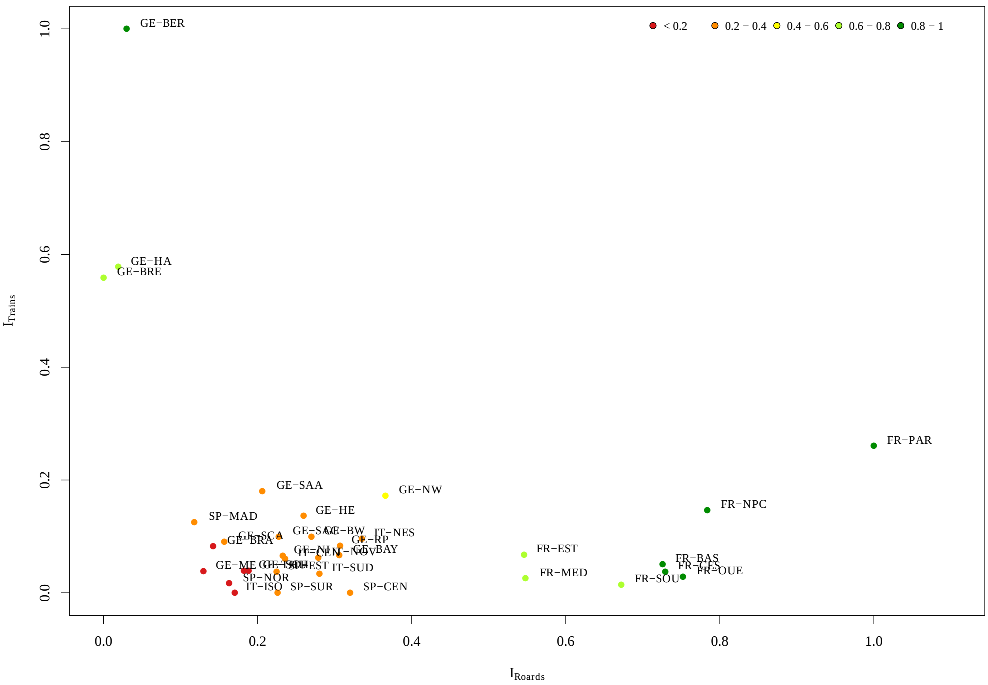
14.3 Results

Figure 14.1 shows a similar path to the simulation in section 8.3 with three departments (GE-BER- Berlin, GE-BRE - Bremen and GE-HA - Hamburg) with low values of I_{Roads} and high values of I_{Trains}. Table 14.1 and Figure 14.2 show results obtained with BoD and D-BoD methods. In particular, while the Spearman Index between the two approaches is very high (equal to 0.938), the average CI score of the isolated departments falls (please see Table 14.1) from 55.86% to 14.19% in GE-BRE - Bremen and from 57.92% to 17.31% in GE-HA - Hamburg.
Finally, we highlight that Berlin remains at the top of the ranking because on the frontier the BoD level curve coincides with the D-BoD one (please see Figure 8.2).

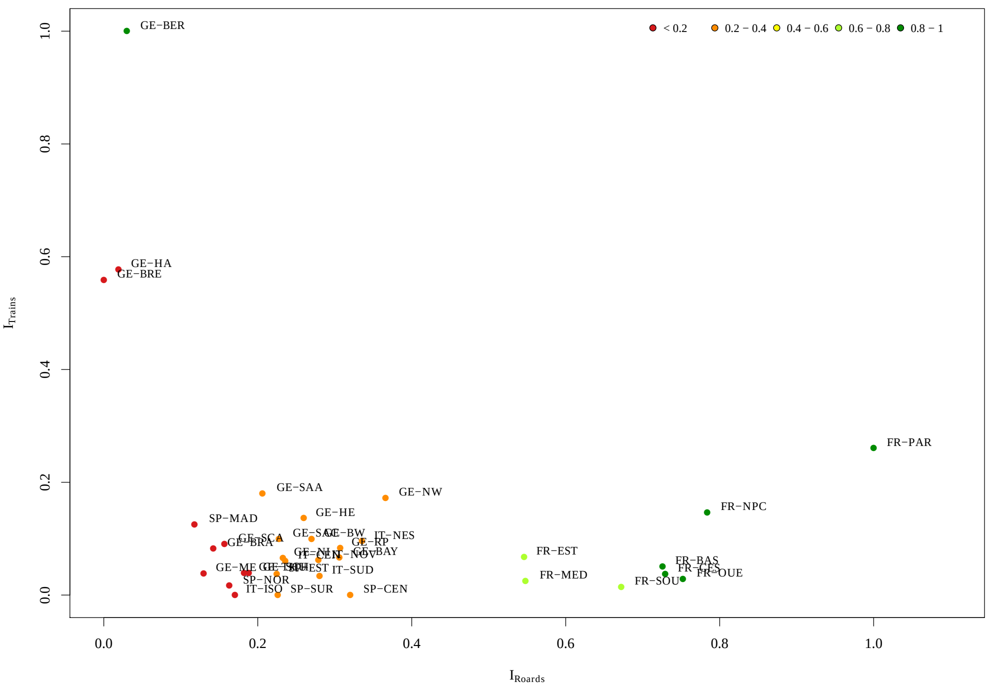
[61] Source: Eurostat, Statistics by theme, 2012
[62] For variables and the method of construction of simple indicators, please see Vidoli and Mazziotta (2013).

Table 14.1: European infrastructure endowment: Comparison between BoD and D-BoD

NUTS2	**Department**	**Country**	**Bod**	**D-Bod**
FR-BAS	Bassin Parisien	France	0.72579	0.72579
FR-CES	Centre-Est (FR)	France	0.72935	0.72935
FR-EST	Est (FR)	France	0.54597	0.54597
FR-MED	Méditerranée	France	0.54767	0.54767
FR-NPC	Nord - Pas-de-Calais	France	0.78384	0.78384
FR-OUE	Ouest (FR)	France	0.75227	0.75227
FR-PAR	Île de France	France	1.00000	1.00000
FR-SOU	Sud-Ouest (FR)	France	0.67219	0.67219
GE-BAY	Bayern	Germany	0.30602	0.30602
GE-BER	Berlin	Germany	1.00000	1.00000
GE-BRA	Brandenburg	Germany	0.18692	0.14226
GE-BRE	Bremen	Germany	0.55858	0.14190
GE-BW	Baden-Württemberg	Germany	0.29817	0.26998
GE-HA	Hamburg	Germany	0.57922	0.17308
GE-HE	Hessen	Germany	0.32695	0.25981
GE-ME	Mecklenburg-Vorpommern	Germany	0.13439	0.12965
GE-NI	Niedersachsen	Germany	0.23789	0.23278
GE-NW	Nordrhein-Westfalen	Germany	0.44095	0.36605
GE-RP	Rheinland-Pfalz	Germany	0.31083	0.30717
GE-SAA	Saarland	Germany	0.33017	0.20871
GE-SAC	Sachsen	Germany	0.26706	0.22757
GE-SCA	Sachsen-Anhalt	Germany	0.20525	0.15678
GE-SCH	Schleswig-Holstein	Germany	0.18842	0.18842
GE-THU	Thüringen	Germany	0.18239	0.18239
IT-CEN	Centro (IT)	Italy	0.23567	0.23567
IT-ISO	Isole	Italy	0.17049	0.17049
IT-NES	Nord-Est	Italy	0.34354	0.33554
IT-NOV	Nord-Ovest	Italy	0.27881	0.27881
IT-SUD	Sud	Italy	0.28023	0.28023
SP-CEN	Centro (ES)	Spain	0.32009	0.32009
SP-EST	Este (ES)	Spain	0.22460	0.22460
SP-MAD	Comunidad de Madrid	Spain	0.20997	0.11784
SP-NOR	Noreste (ES)	Spain	0.16303	0.16303
SP-SUR	Sur (ES)	Spain	0.22607	0.22607



(a) *BoD*



(b) *D-BoD*

Figure 14.2: European infrastructure endowment: Comparison between BoD and D-BoD

Chapter 15

RD-BOD: SUPPLY LEVELS IN ITALIAN HEALTH SYSTEM

Vidoli, F., Fusco, E., and Mazziotta, C. (2015). *Non-compensability in composite indicators: A robust directional frontier method.* Social Indicators Research, 122(3):635–652., DOI:10.1007/s11205-014-0710-y.

15.1 Introduction

The evaluation and the improvement of the national health system performance has become a key policy subject in most developed nations; in recent years many national authorities, as the United Kingdom National Health Service (NHS) and the Canadian Institute for Health Information, or research analysis on national health system (see *e.g* Jencks et al. (2000), Kwon (2003) or Nuti et al. (2011) for the Italian

health care system), have put in place.
Many subjects and objectives may be considered behind a health system evaluation framework: the scope of performance indicators, for this reason, can be ranged considerably for examining the state of a nation's health system to reflect the experiences of the individual patients. Performance, efficiency or analysis on the supply side could be carried out at international, national, regional, local or institutional level Ibrahim (2001).

So a key question, especially in the complex healthcare system is: what should be measured? In general, health care systems can be evaluated with respect to the quality/quantity of care, to the access to care or to the cost (which, however, are not the only analytical dimensions); Paakkonen and Seppala (2014), for example, takes into account in its analysis the accessibility, the efficiency and equality.

But the real challenge in the evaluation models is the multidimensionality of health systems closely linked with the development of composite measures; in order to assess, compare and improve performance, quality or supply between or inside countries it is crucial to dispose of: i) a set of measurable and reliable indicators built up from a good information system, ii) a robust and stable method to integrate indicators in a composite one and to set benchmarks.
Smith (2002), more specifically, discusses three methodological issues in the health sector related to composite indices: i) the development of a set of weights, ii) the treatment of exogenous influences on system performance and iii) the modelling of efficiency; he notes that there isn't a wide consensus regarding methodology issues, such as the weights to be used to form the composite index. In addition, composite measures of health system performance "*lack precision and combine uncertain weighting systems, imprecision arising from the potential non-comparability of component measures, and misleading reliability in the form of whole-population averages that mask distribution issues*" Bankauskaite and Dargent (2007).
Moreover, in our opinion, there are other issues related to the correct choice of the weights not yet fully highlighted in the literature; these matters are related to the public nature of the service: i) optimal weights may not coincide with the marginal utility of the citizen and ii) weights can not be assumed as invariant to the increase of the Local Authorities dimension (see the results of Tiebout (1956)).

Given these premises, it seems even more appropriate to propose a method with these particular characteristics: i) weight endogeneity (property #1), variance of optimal weights among units (linked to the property #3), iii) robustness to outliers (property #6) and easily extendible to a model that also controls the effect of contextual variables (see Vidoli and Mazziotta (2013)).

From an economic point of view, in this paper research questions are linked to administrative changes occurred in the last 15 years in Italy; we focus, therefore, on structural changes and on the different trends - only on the supply side of the Italian regions - due to the administrative devolution from the central government to the single local authority. More specifically, the aim of our application is the measurement of the variations on the supply side and the evaluation of territorial differences between richest regions (northern ones) and the less developed regions (southern ones).

15.2 Variables and data

For this purpose, we used the database of indicators regarding the health system in Italy[63], provided by ISTAT for the years 1998-2010, containing more than 4,000 indicators on the socio-demographic aspects, lifestyles, disabilities and dependencies, monetary and input resources and health care supply.
Specifying that the accurate analysis of the whole health system is beyond the scope of this paper, we focus our attention, as previously mentioned, exclusively on the per capita outputs evaluating if the regional spending and legislative autonomy of each region has brought the healthcare system supply towards a territorial balance or not. In order to avoid collinearity among the elementary indicators, linked to the multidimensionality nature of the informative setting, in a first step we get two main independent and informative factors through principal component analysis[64] (see table 15.1).

15.3 Results

Principal component analysis highlights two factors: the first one can be interpreted as the dimensional factor, while the second one appears to be more dependent on the proxies of the quality of the health system.

Figures 15.1 and 15.2 show a different temporal evolution among different regions: in fact, while supply, in terms of dimension, remains fairly stable or slightly decreasing (due to the national spending review laws), quality seems to be very differentiated.

In figure 15.3 we plot the main $[g_y = (1, 1.24)]$ and the compensative $[g_y = (1, 1)]$

[63] This database is named "Health for all", available at http://www.istat.it/it/archivio/14562.

[64] Total variance explained by the two factors: 77%; printed values are multiplied by 100 and rounded to the nearest integer; values greater than 0.6 are marked with "*"; values less than 0.3 are not printed.

Table 15.1: Principal component analysis, output provided, source: ISTAT

Elementary indicators	Factor1		Factor2	
Total hospitalization (number)	96	*	.	
Inpatient acute care (number)	96	*	.	
Inpatient private hospitals (number)	95	*	.	
Acute care hospitalization (days)	94	*	.	
Total hospitalization (days)	94	*	.	
Inpatient private hospitals (days)	93	*	.	
Inpatient rehabilitation private hospitals (number)	82	*	.	
Inpatient rehabilitation hospitals (number)	80	*	.	
Inpatient rehabilitation hospitals (days)	80	*	.	
Inter-hospital mobility (active-passive) - for 10,000 inhabitants	.		73	*
Inter-hospital mobility (active-passive) - for admissions residents	.		73	*
Average number of resident assisted by a physician	.		48	*
Utilization rate of hospital beds	.		48	*
Average number of 0-14 y. resident assisted by a paediatrician	.		48	*

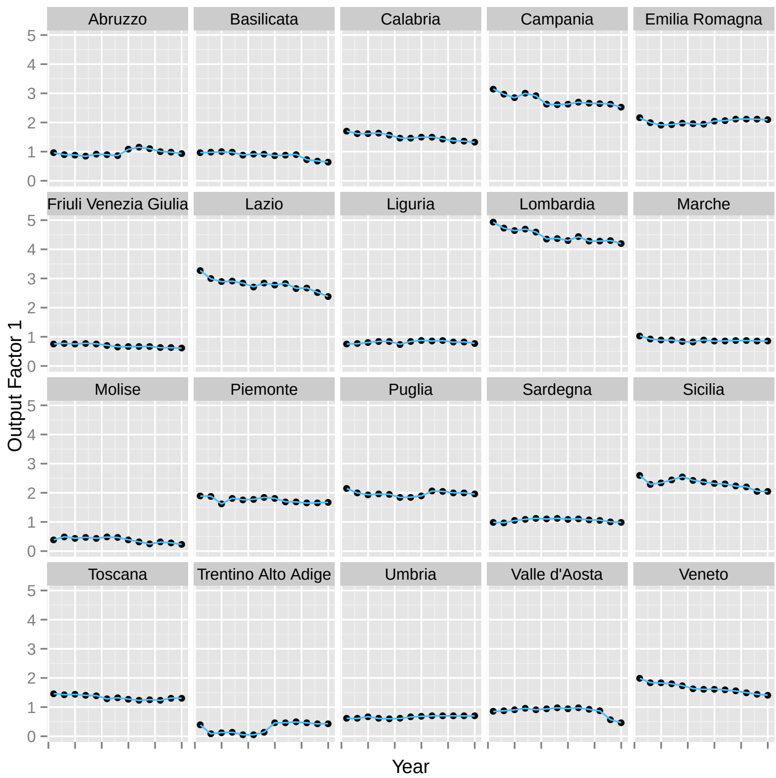


Figure 15.1: Factor 1 annual evolution (dimension) per region

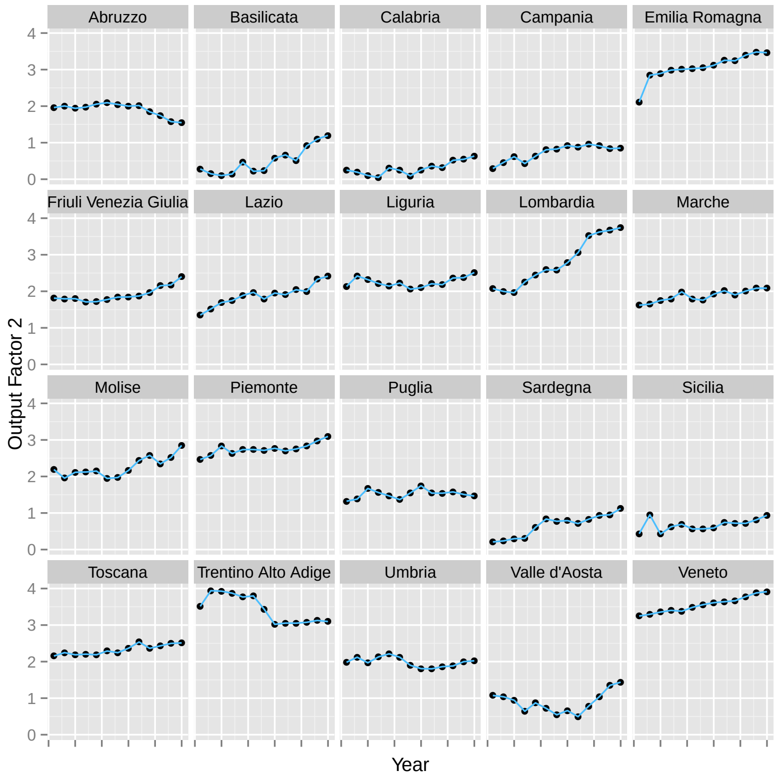


Figure 15.2: Factor 2 annual evolution (quality) per region

direction on Factor 1 (dimension) and Factor 2 (quality) by regional code (lower values = northern regions, higher values = southern regions); given this plot, we expect that, introducing the main direction respectively in the BoD and in R-BoD models, the overall composite indicators remain stable.
Table 15.2 confirms this conjecture showing how, at least in ranks, the composite indicator is very robust to changes in the estimation model.

Table 15.2: Spearman's rank correlation coefficient by methods

	BoD	RBoD	Dir. BoD	Dir. RBoD
BoD	.	0.95	0.92	0.89
RBoD	0.95	.	0.92	0.99
Dir. BoD	0.92	0.92	.	0.92
Dir. RBoD	0.89	0.99	0.92	.

Even if in most regions there was an increase of the provided output - in terms of composite indicator - from a relative point of view, we observe a highest level of performance in the northern regions (Lombardia, Veneto, Emilia Romagna and Piemonte, with a maximum in Lombardia, see figure 15.4). The chosen dataset

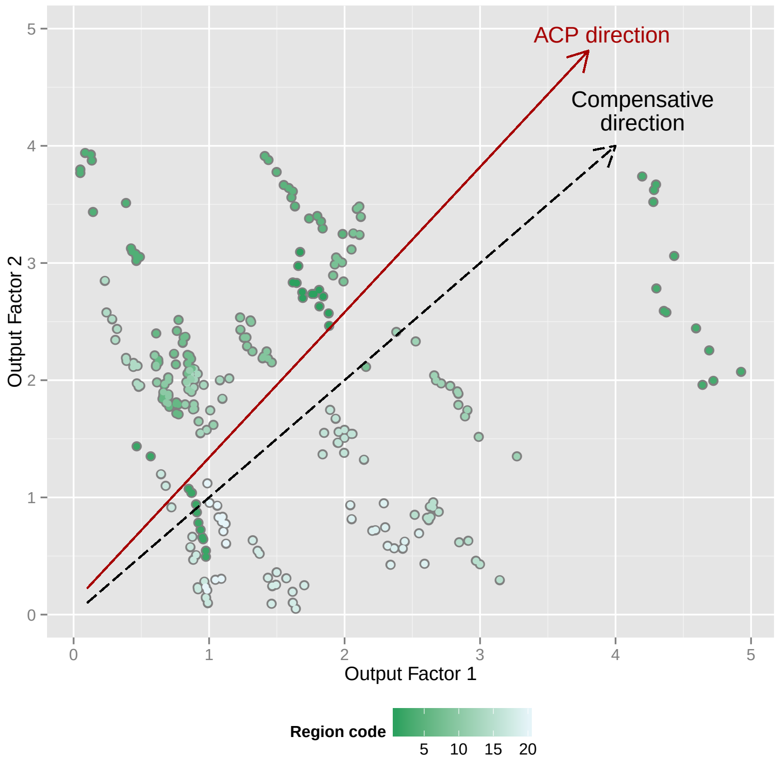


Figure 15.3: Directions on Factor 1 (dimension) and Factor 2 (quality) by regional code

highlights one of the limitations of our approach: not being able to disentangle the increase due to technical progress (frontier shifting) and due to the improvement of a single region; in this setting, in other terms, all regions are analysed in a cross section framework, *i.e.* all units are compared to a single frontier.

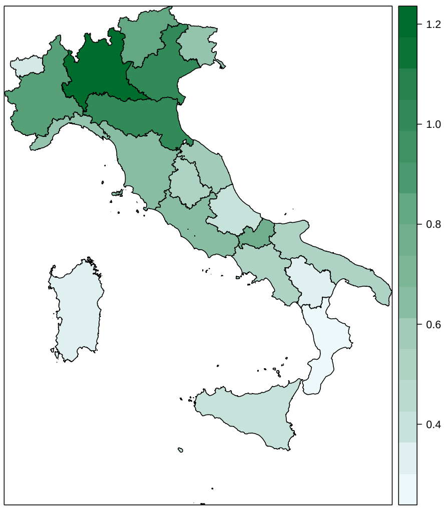


Figure 15.4: Output composite indicator (Directional RBoD) by regions, year 2010

From an economic point of view, it is interesting to verify if the higher level of service provided by the northern regions in the last year (2010) is due to better initial conditions or whether it is the result of better management of resources over time; for this purpose we have computed, for each region i, the main trend of the composite indicator β_i regressing the directional robust composite indicator on years t:

$$Dir\_RBoD_{it} = \alpha + \beta_i \cdot t, \forall i \tag{15.1}$$

In figure 15.5 it is straightforward to note that northern regions[65] have increased their supply level while the southern ones have, even more, decreased their services both in qualitative and quantitative terms.

The last research question concerns the robustness and reliability of the results; in fact, even if there is a good correlation between methods in global terms (see table 15.2), figure 15.6 shows that this result is not always verified for all regions; more specifically, in terms of rank differences (between the RD-BoD and BoD), the variation of the weighting scheme seems to have a lower impact in northern regions (closer to the frontier), while it is more consistent for the southern ones.

[65]Except for Trentino-Alto Adige - regional code 4 - since it's a autonomous status region.

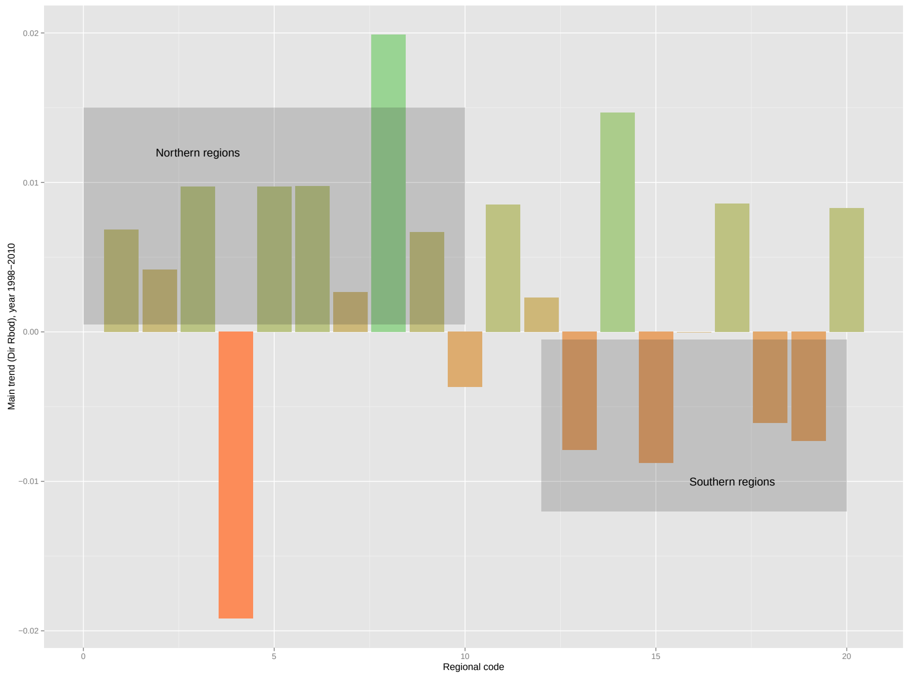


Figure 15.5: Composite indicator (Dir RBoD) main trend 1998-2010 by regional code

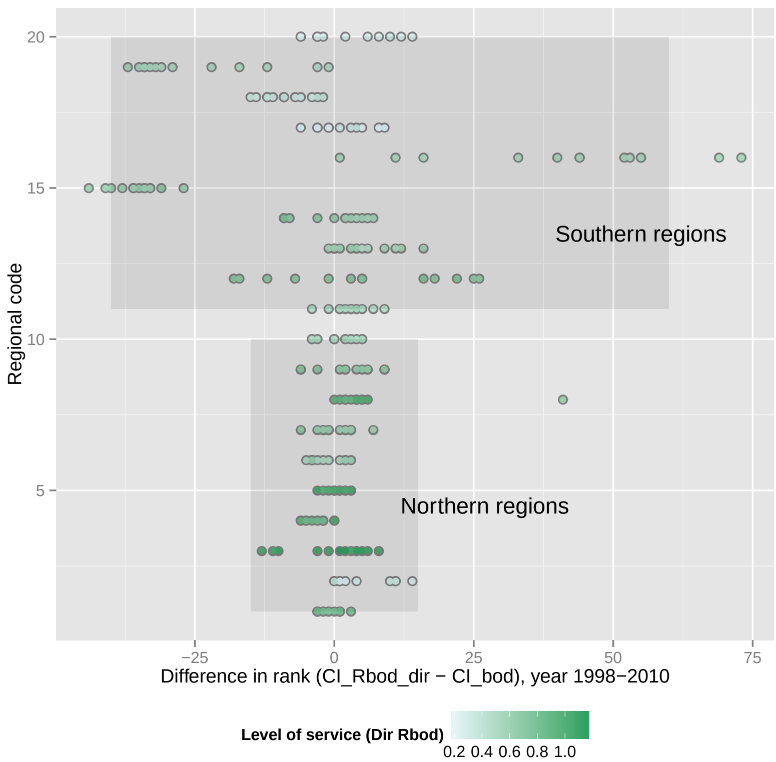


Figure 15.6: Difference in rank by regional code, year 1998-2010

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