Pace Law Review

Volume 37
Issue 1 *Fall 2016*

Article 7

March 2017

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Recommended Citation

Jonathan Barry Forman and Michael J. Sabin, *Survivor Funds*, 37 Pace L. Rev. 204 (2017)
Available at: https://digitalcommons.pace.edu/plr/vol37/iss1/7

Survivor Funds

Jonathan Barry Forman[1] & Michael J. Sabin[2]

ABSTRACT

This Article explains how to create "survivor funds"—short-term investment funds that would pay more to those investors who live until the end of the fund's term than to those who die before then. For example, instead of just investing in a ten-year bond and dividing the proceeds among the investors at the end of the bond term, a survivor fund would invest in that ten-year bond but divide the proceeds only among those who <u>*survived*</u> *the full ten years. These survivor funds would be attractive investments because the survivors would get a greater return on their investments, while the decedents, for obvious reasons, would not care.*

Survivor funds would work like short-term tontines. Basically, a tontine is a financial product that combines features of an annuity and a lottery. In a simple tontine, a group of investors pools their money together to buy a portfolio of investments, and, as investors die, their shares are forfeited, often with the entire fund going to the last survivor. For example, imagine that ten 65-year-old men each contribute

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$1000 to a fund that buys a large diamond for $10,000 and that the men agree that the last "survivor will get the diamond. Accordingly, after the ninth man dies, the tenth man gets the diamond, and he can keep it or sell it.

Of course, the survivor principle—that the share of each, at death, is enjoyed by the survivors—can be used to design financial products that would benefit multiple survivors, not just the last survivor. For example, elsewhere, we showed how tontines could be used to create so-called "tontine annuities" and "tontine pensions" that would benefit lots of retirees. In this Article, we show how the survivor principle can be used to create survivor funds that would only make payments to those who survive for a specified number of years.

INTRODUCTION

"Steve and Mark are camping when a bear suddenly comes out and growls. Steve starts putting on his tennis shoes. Mark says, 'What are you doing? You can't outrun a bear!' Steve says, 'I don't have to outrun the bear—I just have to outrun you!'"[3]

This Article explains how to create *survivor funds*—short-term investment funds that would pay more to those investors who live until the end of the fund's term than to those who die before then. For example, instead of just investing in a ten-year zero coupon bond[4] and dividing the proceeds among the

3. *You Can't Outrun a Bear*, BOYS' LIFE, http://boyslife.org/jokes/6953/you-cant-outrun-a-bear/ (last visited Oct. 30, 2016).

4. *Zero Coupon Bonds*, U.S. SEC. & EXCH. COMM'N., (Mar. 29, 2010), http://www.sec.gov/answers/zero.htm ("Zero coupon bonds are bonds that do not pay interest during the life of the bonds. Instead, investors buy zero coupon bonds at a deep discount from their face value, which is the amount a bond will be worth when it "matures" or comes due. When a zero coupon bond matures, the investor will receive one lump sum equal to the initial investment plus the imputed interest . . ."). Historically, bonds were issued in the form of bearer certificates, with interest coupons printed on the certificate. *See, e.g.*, *Coupon Bond*, INVESTINGANSWERS, http://www.investinganswers.com/financial-dictionary/bonds/coupon-bond-1039 (last visited Oct. 30, 2016). As each interest payment came due, the bearer would detach ("clip") the coupon and exchange it for the interest payment. *Id.*

investors at the end of the bond term, a survivor fund would invest in that ten-year zero coupon bond but divide the proceeds only among those who *survived* the full ten years. These survivor funds would be attractive investments because the survivors would get a greater return on their investments, while the decedents, for obvious reasons, would not care.

Survivor funds would work like short-term *tontines*.[5] Basically, "[a] tontine is a financial product that combines the features of an annuity and a lottery."[6] "In a simple tontine, a group of investors pool their money together to buy a portfolio of investments and, as investors die, their shares are forfeited, with the entire fund going to the last surviving investor."[7] "For example, in an episode of the popular television series *M*A*S*H*, Colonel Sherman T. Potter, as the last survivor of his World War I unit, got to open the bottle of cognac that he and his buddies brought (and share it with his Korean War compatriots)."[8] Similarly, in the reality television show *Survivor*, contestants are stranded in a remote location, and the last "survivor" gets a million-dollar prize.[9]

Of course, the *survivor principle*—"that the share of each, at his death, is enjoyed by the survivors"[10]—can be used to design a variety of financial products that would benefit multiple survivors, not just the last survivor. In the seventeenth and eighteenth centuries, for example, many European governments used multi-beneficiary tontines to raise

5. Tontines are named after Lorenzo de Tonti, the seventeenth century Italian banker who came up with the idea. *See*, *e.g.*, MOSHE A. MILEVSKY, KING WILLIAM'S TONTINE: WHY THE RETIREMENT ANNUITY OF THE FUTURE SHOULD RESEMBLE ITS PAST 42 (2015).

6. Jonathan Barry Forman & Michael J. Sabin, *Tontine Pensions*, 163 U. PA. L. REV. 755, 757 (2015) [hereinafter Forman & Sabin, *Tontine Pensions*]. "An annuity is a financial instrument (e.g., an insurance contract) that converts a lump sum of money into a stream of income payable over a period of years, typically for life. The person holding an annuity is called an annuitant." *Id.* at 757 n.1.

7. *Id.* at 757.

8. *Id.* (referring to *M*A*S*H: Old Soldiers* (CBS television broadcast Jan. 21, 1980)).

9. Survivor *(TV Series 2000-)*, http://www.imdb.com/title/tt0239195/ (last visited Oct. 30, 2016).

10. *Tontine*, WORDNIK, https://www.wordnik.com/words/tontine (last visited Oct. 30, 2016).

money.[11] Similarly, elsewhere, we have described how tontines could be used to create so-called *tontine annuities* and *tontine pensions* that would benefit lots of retirees.[12]

In this Article, we show how the survivor principle can be used to create survivor funds that would only make payments to those who survive for a specified number of years. We believe that many investors would be attracted to these survivor funds. These are people who want the higher returns that survivors could get with survivor funds and who would be willing to accept the losses that they would incur if they died before the survivor-fund term ended. In short, we believe that there would be a demand for short-term survivor funds, and given that demand, there would be companies that want to offer them.

Indeed, we believe that many investors would be attracted to survivor funds, and this Article explains how those survivor funds would work. At the outset, Part I of this Article discusses the *individual-cohort method* for reallocating the balances in the accounts of those who die before the survivor-fund term ends. Part II explains the *date-aligned method* for reallocating those account balances. Part III discusses the difficulty of selecting the correct mortality table for a survivor fund to use (when one is needed), and Part IV shows how the *age-aligned method* for reallocating account balances losses can avoid that difficulty. Finally, Part V shows that survivor funds could work with all types of investments, and Part VI discusses how to resolve some of the technical problems that would arise with survivor funds.

11. *See*, *e.g.*, MILEVSKY, *supra* note 5; ROBERT W. COOPER, AN HISTORICAL ANALYSIS OF THE TONTINE PRINCIPLE (1972); Kent McKeever, *A Short History of Tontines*, 15 FORDHAM J. CORP. & FIN. L. 491 (2009).

12. *See* Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 790-801 (tontine annuities), 802-07 (tontine pensions); Michael J. Sabin, *Fair Tontine Annuity* (Mar. 26, 2010) (unpublished manuscript), http://ssrn.com/abstract=1579932 [hereinafter Sabin, *Fair Tontine Annuity*].

I. A SIMPLE SURVIVOR FUND: THE INDIVIDUAL-COHORT METHOD

A. Longevity Risk, Annuities, and Bonds

"Longevity risk—the risk of outliving one's retirement savings—is probably the greatest risk facing current and future retirees."[13] Individuals use a variety of approaches to "hedge" against living too long. Traditional lifetime annuities offer a particularly good way for individuals to generate income throughout their retirement years. For example, for a 65-year-old man who purchased a $100,000 immediate, level-payment annuity without inflation protection as of December 1, 2015, the annual payment would be around $6540 (6.54% of the annuity's purchase price.[14]

Of course, people rarely choose to buy annuities voluntarily.[15] Even if they have no particular desire to leave assets to their heirs,[16] many investors just hate the idea that

13. Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 761. *See, e.g.*, Youngkyun Park, *Retirement Income Adequacy With Immediate and Longevity Annuities*, EBRI ISSUE BRIEF, No. 357, May 2011, http://www.ebri.org/pdf/briefspdf/EBRI_IB_05-2011_No357_Annuities.pdf; *Common Retirement Risks,* AMERIPRISE FIN., https://www.ameriprise.com/retirement/retirement-planning/common-retirement-risks/ (last visited Oct. 30, 2016).

14. *See Immediate Annuities Update*, ANNUITY SHOPPER BUYER'S GUIDE, Jan. 2016, at 8, 17 tbl.5, https://www.immediateannuities.com/pdfs/as/annuity-shopper-2016-01.pdf (showing average payments to a 65-year-old man of $545 per month, $6540 per year). Because women tend to live longer than men, the annual payments for a 65-year-old woman who elected an immediate, level-payment annuity as of December 1, 2015 would be only $6132, or 6.13% of the annuity's purchase price (showing an average payment to 65-year-old woman of $511 per month). *Id.* Inflation-adjusted annuities offer an even better way to hedge against living too long. With inflation-adjusted annuities, annual payments would start out lower than level-payment annuities, but could end up higher. For example, if the hypothetical 65-year-old man in the last paragraph instead chose an annuity stream with a 3% annual escalator, the initial annual payment would be just $4728, but, eventually, the annual payments would exceed the $6540 per year fixed under the level-payment life annuity. *Id.* (showing average monthly payments to 65-year-old men with a 3%-cost-of-living adjustment of $394 per month in the first year of his retirement [$4728 in the first year]).

15. Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 800.

16. Economists call this a bequest motive. *See, e.g.*, *Bequest Motive*,

an early death seems to benefit the insurance company.

Instead, many investors prefer to invest in stocks and bonds. Pertinent here, many investors buy bonds. While bonds can often be sold without penalty before they mature, many investors—especially retired investors—hold their bonds until maturity, and then they either reinvest or spend the proceeds as needed. In order to minimize interest-rate risk and increase liquidity, it can make sense to create a "laddered bond" portfolio where the bonds' maturity dates are evenly spaced over several years so that bonds are maturing and being reinvested at regular intervals.[17] For example, rather than buying just one bond worth $100,000 that matures in ten years, a retiree might buy ten bonds worth $10,000 each that were laddered so that one bond matured in each of the next ten years.

All in all, there is a large demand for bonds,[18] and there is relatively little demand for lifetime annuities.[19] That is where

ANNUITY DIGEST, http://www.annuitydigest.com/bequest-motive/definition (last visited Oct. 30, 2016). People with a "bequest motive" value the prospect of leaving wealth to family, friends, or good causes. *See, e.g.*, Lee M. Lockwood, *Bequest Motives and the Annuity Puzzle*, AMER. RISK & INS. ASSOC. (Dec. 7, 2010), http://www.aria.org/meetings/2011%20papers/Bequest%20Motives%20and%20the%20Annuity%20Puzzle.pdf.

17. *See, e.g.*, *How and Why to Build a Bond Ladder,* FIDELITY (Nov. 24, 2015), https://www.fidelity.com/viewpoints/bond-ladder-strategy.

18. For example, in 2014, there was $39,034 billion in outstanding U.S. bond market debt. *US Bond Market Outstanding* tbl. 2.1 (Outstanding U.S. Bond Market Debt, last updated Dec. 23, 2015) SIFMA (last visited Jan. 15, 2016), http://www.sifma.org/uploadedFiles/Research/Statistics/StatisticsFiles/CM-US-Bond-Market-SIFMA.xls?n=27525. Also, in 2015, $6,436.9 billion in new U.S. bonds were issued. *US Bond Market Issuance*, Monthly tbl. 1.1 (Issuance in the U.S. Bond Markets, last updated Jan. 14, 2015) SIFMA (last visited Jan.15, 2016), http://www.sifma.org/uploadedFiles/Research/Statistics/StatisticsFiles/CM-US-Bond-Market-SIFMA.xls?n=27525.

19. In 2014, just $229.4 billion of annuities were sold in the U.S. *IRI Issues Fourth-Quarter and Year-End 2014 Annuity Sales Report*, INSURED RET. INST. (Mar. 25, 2015), http://www.irionline.org/research/research-detail-view/iri-issues-fourth-quarter-and-year-end-2014-annuity-sales-report. On the relatively low demand for annuities, *see, e.g.*, Shlomo Benartzi, Alessandro Previtero & Richard H. Thaler, *Annuitization Puzzles*, 25 J. ECON. PERSP. 143 (2011); Franco Modigliani, *Life Cycle, Individual Thrift, and the Wealth of Nations*, 76 AM. ECON. REV. 297 (1986); Menahem E. Yaari, *Uncertain Lifetime, Life Insurance, and the Theory of the Consumer*, 32 REV.

survivor funds come in. As more fully described below, with a survivor fund, the investor would get a higher rate of return than she would get with a regular bond—as long she is willing to lose her investment if she does not survive until the survivor fund matures. As with regular bonds, an investor could hold a single survivor fund or a laddered portfolio of survivor funds. At bottom, with a survivor fund, an investor could take on some modest mortality risk over the short term of the survivor fund, as opposed to the lifetime mortality risk that comes from investing in a lifetime annuity.[20]

B. The Survivor Principle and a Simple Survivor Fund

In a simple tontine, investors "contribute equally to buy a portfolio of investments that is awarded entirely to the last surviving member[;] [a]lternatively, each time a member of a tontine pool dies, her account balance could be divided among the surviving [investors]."[21] "The key point is that variations on the [survivor] principle—that the share of each, at death, is enjoyed by the survivors—can be used to create a variety of attractive" financial products, including survivor funds.[22]

1. A Simple Zero Coupon Bond

At the outset, imagine that 1000 otherwise identical 65-year-old men each contribute $1000 to an investment fund that invests $1,000,000 in 10-year zero coupon bonds, each with a 5% yield to maturity. At maturity in ten years, those bonds would have a maturity value of $1,628,894.63,[23] and, at the end

ECON. STUD. 137 (1965).

20. Here, we use the term "mortality risk" to refer to the risk that an investor will lose all or a portion of her investment because of death. There is relatively little risk that an investor will lose her investment over the short-term of a survivor fund. On the other hand, when an investor buys a lifetime annuity, she has a fairly high probability of dying before she has fully recovered her initial investment. *See infra* note 27 and accompanying text (explaining mortality gains and losses).

21. Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 774.

22. *Id.*

23. *See, e.g.*, *Zero Coupon Bond Yield Calculator*, FINANCE FORMULAS,

of the 10-year period, each investor (or his heirs) would get around $1629 on his $1000 investment.[24]

2. A Simple Survivor Fund Using the Individual-Cohort Method

Now imagine that we instead divide that $1,628,894.63, but only among the men who *survived* those ten years. For example, suppose that only 800 of our original group of 1000 men survived ten years to age 75.[25] Then, each *survivor* would get around $2036,[26] while those who died during the term of this *survivor fund* would lose their investments. That is, the "winners" who survived all ten years would have *mortality gains* of around $407,[27] while the "losers" who died during the 10-year period would have *mortality losses* of around $1629.[28] In short, some "outran" the bear, and some got eaten.[29]

For the survivors, it is as if they earned a yield to maturity of around 7.37% on this survivor-fund investment rather than

http://www.financeformulas.net/Zero-Coupon-Bond-Effective-Yield.html#Calc-Header (last visited Oct. 31, 2016). $1,628,894.63 = $1,000,000 × (1.00 + 0.05).

24. $1628.89 = $1,628,894.63/1000. In this Article, computations involving money are usually rounded to the nearest penny, and many other computations are rounded to two digits after the decimal point.

25. At this point in the Article, suffice it to say that 20% is a very reasonable mortality rate, *but see* Part III, *infra*, for a discussion about what mortality tables are appropriate for survivor funds.

26. $2036.12 = $1,628,894.63/800.

27. $407.23 = $2036.12– $1628.89. "Individuals who invest in annuity-like products have mortality gains and losses depending on when they die. Individuals who live longer than their peers get mortality gains from those who precede them, while individuals who die earlier than their peers suffer mortality losses." Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 776 n.105 (citing David Blake, *Annuity Markets: Problems and Solutions*, 24 GENEVA PAPERS ON RISK & INS. 358, 371 (1999) (explaining that a mortality cross-subsidy "arises because some annuitants will die shortly after taking out an annuity thereby releasing a 'mortality profit' which insurance companies share with longer-surviving annuitants . . . "). In the specific case of a survivor fund, individuals who survive to the end of the period get mortality gains from those who died during the period.

28. Regardless of when a loser dies, the amount he loses is $1628.89, the final value of his investment.

29. *See supra* note 3.

just the 5% earned on the underlying zero coupon bond.[30] That is, the survivors would get a 2.37% higher yield to maturity.[31] Put differently, the yield to maturity for this survivor fund[32] would be 47% higher than the yield to maturity on the underlying zero coupon bonds.[33]

The *survivors* cannot possibly do worse investing in this survivor fund than they would by holding the underlying zero coupon bonds directly;[34] and the more investors that die, the better the survivors would do. In short, assuming that they survive, investors in a survivor fund have nothing to lose and everything to gain. For an investor with no "bequest motive"[35] (e.g., no heirs) who currently buys and holds short-term bonds, buying a short-term survivor fund, instead, would make a lot of sense.[36]

We call the simple method that we used to create the hypothetical 65-year-old-man survivor fund above the "individual-cohort method." A *cohort* is defined as a group of people who are the same age[37] (and, perhaps, the same gender, i.e., 65-year-old men). Theoretically, an investment company wishing to offer survivor funds could simply create a separate survivor fund for each and every cohort, just as we did for the 65-year-old-male cohort above.

30. 7.36935% = ($2036.12/$1000.00)$^{0.1}$– 1.

31. 2.37% = 7.37% – 5.00%.

32. As this survivor fund is invested entirely in zero coupon bonds, we could call it a "survivor bond" instead.

33. 47.4% = (7.37% – 5.00%)/5.00%. Put differently, the survivors get 25% more from the survivor fund than from the underlying 5% zero coupon bonds. 1.25 = $2036.12/$1628.89. *See also* Jonathan Barry Forman & Michael J. Sabin, *Using Survivor Funds to Boost 401(k) returns*, PENSIONS & INVESTMENTS (June 27, 2016, 12:01 AM), http://www.pionline.com/article/20160627/ONLINE/160629917/using-survivor-funds-to-boost-401k-returns.

34. If, miraculously, none of the investors died during the 10-year bond period, all of the investors would get the 5.00% yield to maturity on the underlying zero coupon bonds.

35. *See supra* note 16 and accompanying text.

36. Almost all of the survivor fund examples in this Article refer to 10-year investment periods, and we refer to these investments as "short-term." Of course, the same principles could be used to create survivor funds with longer or shorter durations.

37. *Cohort*, VOCUBULARY.COM, https://www.vocabulary.com/dictionary/cohort (last visited Nov. 7, 2016).

These individual-cohort survivor funds would certainly be fair to all investors, as each investor in a given survivor fund would be the same age (and, perhaps, the same gender), each investor would make the same initial contribution, and each survivor would get an equal share of from the accounts of the investors who died during the survivor-fund term. Unfortunately, the individual-cohort method has some practical problems.

C. Projecting Investment Outcomes for the Survivors in a Survivor Fund Using the Individual-Cohort Method

At the outset, an individual-cohort survivor fund would have trouble accurately projecting the investment outcomes that the survivors could anticipate receiving. Under the individual-cohort method, the actual investment outcomes that the survivors would receive would depend on two principal factors: 1) the underlying performance of the investment assets; and 2) the *actual* mortality experience of the survivor-fund investors.[38] These are discussed in turn.

1. The Underlying Performance of the Investment Assets

First, for now, there is not much to say about the underlying performance of the investment assets. While, in principle, a survivor fund could invest in anything from simple zero coupon bonds to complicated hedge funds,[39] at this point in

38. Administrative costs could also play a role in investment outcomes, but we assume that these would be trivial. *See infra* Part V.B.2.

39. *See infra* Part V. *See Hedge Fund*, INVESTOPEDIA, http://www.investopedia.com/terms/h/hedgefund.asp (last visited Nov. 7, 2016). The definition reads:

> Hedge funds are alternative investments using pooled funds that may use a number of different strategies in order to earn active return, or alpha, for their investors. Hedge funds may be aggressively managed or make use of derivatives and leverage in both domestic and international markets with the goal of generating high returns (either in

the Article, it is simplest to assume that all of our hypothetical survivor funds would invest in 10-year zero coupon bonds with a 5% yield to maturity.[40]

2. The Actual (and Projected) Mortality Experience of Survivor-fund Investors

Second, however, there is a great deal to say about how the actual mortality experience of the survivor-fund investors will affect their actual investment outcomes. In that regard, in making a *projection* of the anticipated investment outcomes for survivors of a specific survivor fund, an investment company would have to make some assumptions about the mortality experience of the survivor-fund investors (based on some kind of mortality table), but—and this is critical—the *actual* investment outcomes for the survivors will depend on the *actual* mortality experience of the survivor-fund investors and not on the investment company's mortality assumptions.

For example, in Part 2 above, we assumed that *1000* 65-year-old men each invested $1000 in a 10-year survivor fund that invested in 10-year zero coupon bonds with a 5% yield to maturity, and we further assumed that 800 of them would live to age 75—that is, 200 would die before then. As a result, each of the 800 survivors got $2036 on their $1000 investments,[41] which is a 47% larger yield to maturity than if they had instead invested $1000 directly in the underlying 5% zero coupon bonds.[42]

Pertinent here, if fewer than 200 investors had died during the 10-year term of that survivor fund, the survivor fund's yield

> an absolute sense or over a specified market benchmark). Because hedge funds may have low correlations with a traditional portfolio of stocks and bonds, allocating an exposure to hedge funds can be a good diversifier.

Id.

40. We note, in passing, that if the underlying investments in a survivor fund generated a higher-than-anticipated yield to maturity, the survivors would have better investment outcomes, and if the underlying investments generated a lower-than-anticipated yield to maturity, the survivors would have worse investment outcomes.

41. *See supra* note 26.

42. *See supra* note 33 and accompanying text.

to maturity for the survivors would have been lower.[43] On the other hand, if more than 200 investors had died, the survivor fund's yield to maturity would have been higher. To be sure, it is unlikely that *exactly* 200 investors would die, but it is incumbent upon the investment company to base its projected yield to maturity on the correct mortality table. After all, U.S. securities laws require that investment companies provide accurate information (and projections) to investors,[44] and using an incorrect mortality table for a given population of investors would result in inaccurate projections of the projected (and advertised) yields to maturity.

We have much more to say about how to select the correct mortality table when one is needed.[45] For now, it is enough to reiterate that the individual-cohort method's reliance on mortality tables could lead to significant discrepancies between the projected and the actual investment outcomes for survivors.

D. The Individual-Cohort Method Would Require Way Too Many Survivor Funds

To be sure, there is a much larger practical problem with the individual-cohort method: *an investment company using the individual-cohort method would have to offer way too many*

43. The effect of mortality experience is more fully discussed in Part III, *infra*. *See* Table 12 and accompanying text.

44. *See, e.g.*, *What We Do,* U.S. SEC. & EXCH. COMM'N (June 10, 2013), http://www.sec.gov/about/whatwedo.shtml#.VQm1w010zPQ. The Commission states:

> The laws and rules that govern the securities industry in the United States derive from a simple and straightforward concept: all investors, whether large institutions or private individuals, should have access to certain basic facts about an investment prior to buying it, and so long as they hold it. To achieve this, the SEC requires public companies to disclose meaningful financial and other information to the public. This provides a common pool of knowledge for all investors to use to judge for themselves whether to buy, sell, or hold a particular security. Only through the steady flow of timely, comprehensive, and accurate information can people make sound investment decisions.

Id.

45. *See infra* Part III.

different survivor funds. After all, in addition to having to offer different survivor funds for each and every age cohort (and possibly for each and every age-gender cohort), an investment company would also want to offer each investor a choice of investments. Some people would want to invest in government bonds, but others would want to invest in corporate bonds, large cap stocks,[46] small cap stocks,[47] global equities, real estate, commodities, hedge funds, or other alternative investments.[48]

Moreover, not every investor would want to invest exactly the same amount. Some would want to invest $1000, but others would want to invest $10,000 or even $100,000. Furthermore, as the easiest way to design a survivor fund is to have all the investors come in at roughly the same time, an investment company would need to create new and separate survivor funds every month or, at least, every quarter. All in all, if an investment company tried to rely on the individual-cohort method, it could easily end up having to offer hundreds of different survivor funds. That would certainly drive up the company's administrative costs, and it is likely that many of the survivor funds would be undersubscribed and have a very tough time performing as projected.

We believe that investment companies would want to offer a much smaller number of survivor funds, and in Parts II and

46. *See Large Cap - Big Cap*, INVESTOPEDIA, http://www.investopedia.com/terms/l/large-cap.asp (last visited Nov. 6, 2016). "Large Cap - Big Cap" is a term used by the investment community to refer to companies with a "market capitalization value of more than $5 billion." *Id.* Large cap is an abbreviation of the term "large market capitalization." *Id.* "Market capitalization is calculated by multiplying the number of a company's shares outstanding by its stock price per share." *Id.*

47. *See Small Cap*, INVESTOPEDIA, http://www.investopedia.com/terms/s/small-cap.asp (last visited Nov. 6, 2016). "Small Cap" refers to stocks with a "relatively small market capitalization." *Id.* "The definition of small cap can vary among brokerages, but it is generally a company with a market capitalization of between $300 million and $2 billion." *Id.*

48. *See Alternative Asset*, INVESTOPEDIA, http://www.investopedia.com/terms/a/alternativeassets.asp (last visited Nov. 6, 2016). "Alternative Asset" refers to any "non-traditional asset with potential economic value that would not be found in a standard investment portfolio." *Id.* "Due to the unconventional nature of alternative assets, valuation of some of these assets can be difficult." *Id.*

III below we offer some viable alternatives to the individual-cohort method. Before moving on to our explanations of those alternatives, however, we will use the much-simpler individual-cohort method to explain the troublesome *variability* in investment outcomes that would occur whenever a survivor fund has relatively few investors.[49]

E. The Statistics of Getting Survivor Funds to Behave as Investors Expect

Investment companies offering survivor funds would very much want to assure investors that their survivor funds would have predictable outcomes. For example, an investment company might want to tell a 65-year-old man that, if he survives to age 75, his investment in a 10-year survivor fund would have a projected yield to maturity of, say, 7.6%, and that there would be a 95% chance that the actual yield to maturity would be in the range from 7.3% to 7.9% (i.e., 7.6% ± 0.3%). Unfortunately, because of the variability in mortality experience that would inevitably occur with any survivor fund that has relatively few investors, our hypothetical investment company would have a hard time relying on the individual-cohort method to achieve investment outcomes in that small of a range.[50]

49. *See* *Variability*, INVESTOPEDIA, http://www.investopedia.com/terms/v/variability.asp (last visited Nov. 6, 2016). "Variability" refers to "the extent to which data points in a statistical distribution or data set diverge from the average, or mean, value as well as the extent to which these data points differ from each other. There are four commonly used measures of variability: range, mean, variance and standard deviation." *Id.*

50. Depending on the underlying investments, variability in investment performance would also be an issue for survivor funds. To be sure, at this point in the Article, we are assuming that all survivor funds would invest in 5% zero coupon bonds. On the other hand, in Part V, *infra*, we explain how survivor funds could be designed to invest in all types of asset classes, many of which can be quite volatile. For now, it is enough to say that if the rates of return on the underlying investments are volatile, then investment outcomes would vary both because of variability in mortality experience and also because of that investment volatility. For example, if an investment company were to market a survivor fund that invested in small cap stocks instead of zero coupon bonds, the actual investment performance would be more variable, and it would be far less likely that 95% of the investment

This is a practical problem that is best understood in terms of some simple mathematics (statistics, to be precise). It comes down to this: with a small number of investors in a given survivor fund, the actual yields to maturity for the survivors could vary dramatically from the projected yield to maturity. That is where the *law of large numbers* comes into play.[51] If the number of investors in a survivor fund is large, then the actual mortality experience of the investors in that fund would tend to be very close to what was expected, and, as a result, the actual yield to maturity would tend to be very close to what was projected. On the other hand, if the number of investors is small, then the actual mortality experience of the investors would be likely to vary rather significantly from what was expected, and that would cause the actual yield to maturity to vary significantly from what was projected. Simply put, the more investors there are in a survivor fund, the smaller the variance from the projected yield to maturity.

At the outset, consider a hypothetical survivor fund with *100* 65-year-old men, and assume that the mortality rates of these 100 men follow the 2010 Social Security Administration area population mortality table shown in Appendix Table 1.[52]

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outcomes would fall within ± 0.3% of the expected rate of return on those small cap stocks.

51. *See, e.g.*, *Law Of Large Numbers*, INVESTOPEDIA, http://www.investopedia.com/terms/l/lawoflargenumbers.asp (last visited Jan. 11, 2016). A principle of probability and statistics which states that "as a sample size grows, its mean gets closer and closer to the average of the whole population." *Id.* The law of large numbers in the financial context has a different connotation, which is that a "large entity which is growing rapidly cannot maintain that growth pace forever. The biggest of the blue chips, with market values in the hundreds of billions, are frequently cited as examples of this phenomenon." *Id.*

52. Appendix Table 1 is drawn from the Social Security Administration's 2010 period life table for the Social Security area population included in an e-mail from K. Mark Bye, Soc. Sec. Admin., to Jonathan Barry Forman (Dec. 3, 2014, 10:03 a.m. EST) (on file with authors). *Cf. Actuarial Life Table,* SOC. SEC. ADMIN. (2013), https://www.ssa.gov/oact/STATS/table4c6.html (last visited Nov. 6, 2016). The 2013 table states:

> A period life table is based on the mortality experience of a population during a relatively short period of time. Here we present the 2013 period life table for the Social Security area population. For this table, the period life expectancy at

Loosely speaking, that table suggests that 21.59% of 65-year-old men are expected to die before reaching age 75.[53] Obviously, in a group of 100 men, it is not possible for exactly 21.59% of them to die, since the number of deaths must be a whole number; for example, twenty-one of the men could die, or twenty-two, but not 21.59. Indeed, it is possible that the number of deaths could be any whole number in the range from zero to 100. Of course, the number of deaths is likely to be some whole number close to 21.59. In that regard, Figure 1 shows that the number of deaths over the 10-year term of this survivor fund would take the approximate form of a *normal distribution*[54] around the *mean* value of 21.59,[55] and we will

> a given age is the average remaining number of years expected prior to death for a person at that exact age, born on January 1, using the mortality rates for 2013 over the course of his or her remaining life.

Id.

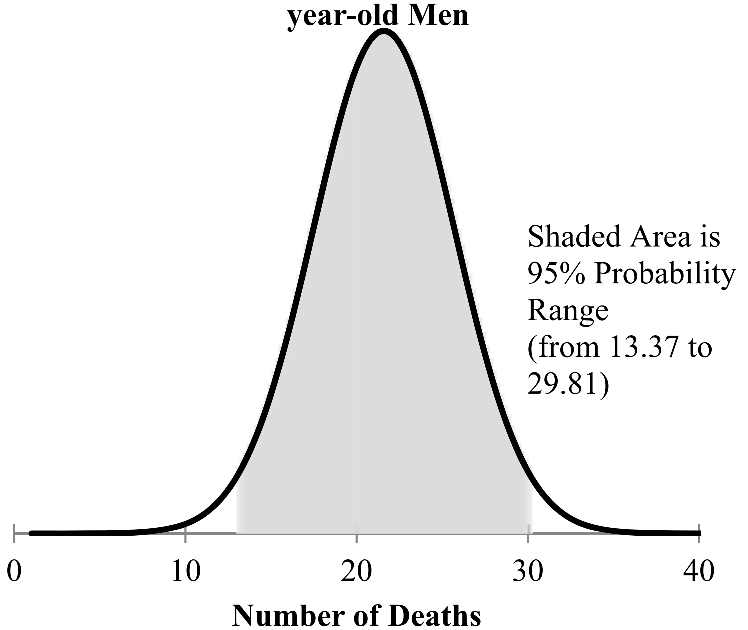
53. Appendix Table 1 implies that of the 100,000 live male births in the table, 80,729 made it to age 65 and 63,300 of them made it to age 75. That is, 17,429 (21.58952%) of the 80,729 men alive at age 65 would die before reaching age 75 (17,429 = 80,729 – 63,300; 21.58952% = 17,429/80,729). We say "implies" because the table only offers one-year death probabilities, and while we think that this footnote offers a reasonable way to approximate the 10-year death probability for 65-year-old men, we assume that, if called upon to do so, the actuaries would use a much better approach and get a slightly different number. An individual's (one-year) death probability is his probability of dying within one year. For example, Appendix Table 1 shows that a 65-year-old male has a 1.6% chance of dying before his next birthday (death probability, q_i, of 0.015927).

54. *Normal Distribution*, INVESTOPEDIA, http://www.investopedia.com/terms/n/normaldistribution.asp (last visited Nov. 6, 2016). "Normal Distribution" is "the probability distribution that plots all of its values in a symmetrical fashion, and most of the results are situated around the probability's mean." *Id.* "Values are equally likely to plot either above or below the mean. Grouping takes place at values that are close to the mean and then tails off symmetrically away from the mean." *Id.* A "Normal Distribution" is also known as the Gaussian distribution or "bell curve." *Id. See also* Deborah J. Rumsey, *How to Find the Normal Approximation to the Binomial with a Large Sample* N*,* DUMMIES, http://www.dummies.com/how-to/content/how-to-find-the-normal-approximation-to-the-binomi.html (last visited Nov. 6, 2016).

55. *See, e.g.*, *Arithmetic Mean*, INVESTOPEDIA, http://www.investopedia.com/terms/a/arithmeticmean.asp (last visited Nov. 6, 2016). "The arithmetic mean is a mathematical representation of the typical value of a series of numbers, computed as the sum of all the numbers in the series divided by the count of all numbers in the series. The arithmetic mean is sometimes referred to as the average or simply as the mean." *Id.*

work with such fractional numbers throughout this Article.

Figure 1. Probability Density of Number of Deaths Over 10 Years for a Group of *100* 65-year-old Men



With *100* 65-year-old men, the *standard deviation*[56]—which is a measure of the dispersion of the number of deaths about the mean—is 4.11.[57] In a normal distribution, 95.45% of

56. The standard deviation is a mathematical measure of variance. Literally, the standard deviation is the square root of the variance, and the variance is the average of the squared differences between population values and their mean. *See, e.g.*, *Standard Deviation and Variance*, MATH IS FUN, http://www.mathsisfun.com/data/standard-deviation.html (last visited Nov. 6, 2016); *see also Standard Deviation*, INVESTOPEDIA, http://www.investopedia.com/terms/s/standarddeviation.asp (last visited Nov. 6, 2016). "Standard deviation is a measure of the dispersion of a set of data from its mean." *Id.* The more spread apart the data, the higher the deviation. *Id.* "Standard deviation is calculated as the square root of variance. . . ." *Id.*

57. Calculated using the formula for the standard deviation of a binomial distribution: $4.11445 = \sigma = \sqrt{}(np(1-p) = (np(1-p))^{0.5} = (100 \times 0.2159 \times (1 - 0.2159))^{0.5}$,where n is the sample size and p is the individual probability.

the outcomes fall within two standard deviations of the mean; that is, there is a 95.45% probability that the actual number of deaths in a 100-man sample would be in the range from 13.37 to 29.81.[58] For ease of wording, however, we will usually abbreviate "95.45%" as "95%" and the "95.45% range" as the "95% range." In summary, for our hypothetical *100*-65-year-old-man survivor fund, we project that the mean number of deaths will be 21.59 and that there is a 95% probability that the number of deaths occurring will be in the range from 13.37 to 29.81.

Continuing with our hypothetical example, we further assume that each of the *100* 65-year-old men invested $1000 in a survivor fund that would hold 10-year zero coupon bonds with a 5% yield to maturity, so that the value of each man's original investment would grow to $1628.89 at the end of the 10-year survivor-fund term. As we projected that 21.59 of those men would die during the 10-year term of the zero coupon bonds, the 78.41 men who survived the ten years would have a projected mortality-gain distribution of $448.51,[59] and, therefore, we project that each of the survivors would collect $2077.40 at that time ($2077.40 = $1628.89 + $448.51). Figure 2 shows the near-normal distribution of the projected mortality-gain distributions for the survivors of this survivor fund.[60]

58. Two standard deviations below the mean is 13.37 = 21.59 – (2 × 4.11), and two standard deviations above the mean is 29.81 = 21.59 + (2 × 4.11).

59. $448.51 = (21.59 × $1628.89)/78.41. Recall that in *supra* note 53, we showed that 21.59% of 65-year-old men are expected to die before reaching age 75, which means that 78.41% are expected to survive. 21.59 = 100 × 21.59%; 78.41 = 100 × 78.41%; 78.41% = 100% – 21.59%.

60. The formula for mortality-gain distributions is $1628.89 × $d/(100 – d)$, where d is the number of deaths. As we discussed in note 52 and accompanying text, the number of deaths (d) is normally distributed. *See supra* Figure 1. On the other hand, the function ($d/(100 – d)$) is *not* normally distributed, but it does appear to be near normal.

Figure 2. Probability Density of Mortality-gain Distribution for the Survivors of an Individual Cohort of *100* 65-year-old Men

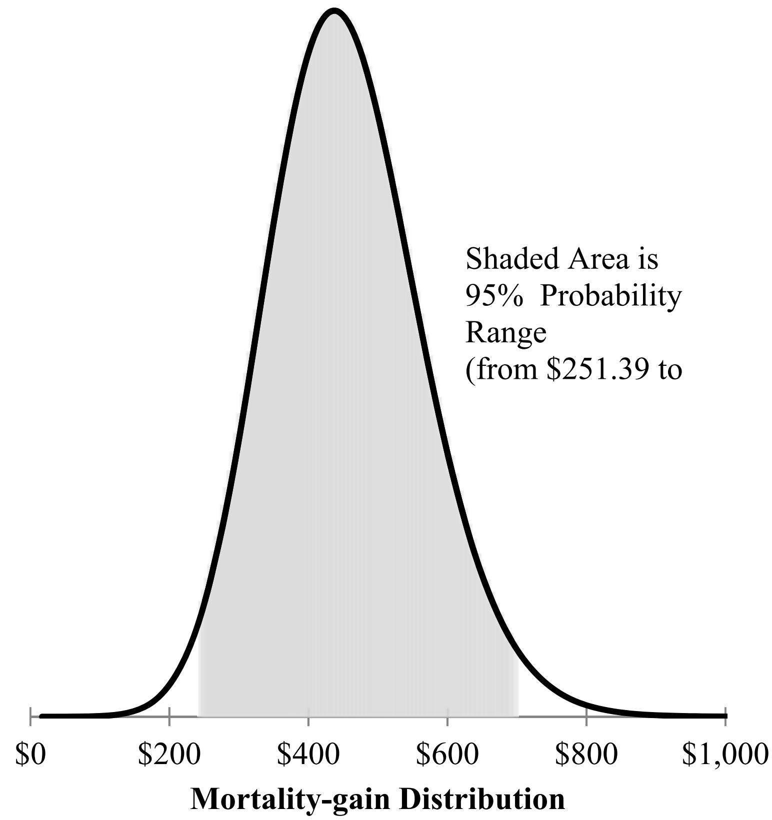


Figure 2 also shows the 95% range of mortality-gain distributions, based on the 95% range of deaths from Figure 1 (from 13.37 to 29.81 deaths). At one end of the 95% range, if there were 13.37 deaths, then each of the 86.63 survivors would receive a mortality-gain distribution of $251.39.[61] At the other end of the 95% range, if there were 29.81 deaths, then each of the 70.19 survivors would receive a mortality-gain distribution of $691.80.[62] In summary, the investment company could fairly advertise that there would be a 95%

61. $251.39 = (13.37 × $1628.89)/86.63; 86.63 = 100 – 13.37.
62. $691.80 = (29.81 × 1628.89)/70.19; 70.19 = 100 – 29.81.

probability that survivors would receive a mortality-gain distribution of from \$251 to \$692.

Finally, the investment company could also express these survivor-fund investment outcomes in terms of yields to maturity on the survivor's original \$1000 investments. That is, the investment company could fairly advertise that investors could anticipate a projected yield to maturity of around 7.59%[63] and that there would be a 95% probability that those survivors would receive a yield to maturity of from 6.5% to 8.8%.[64] See Figure 3.

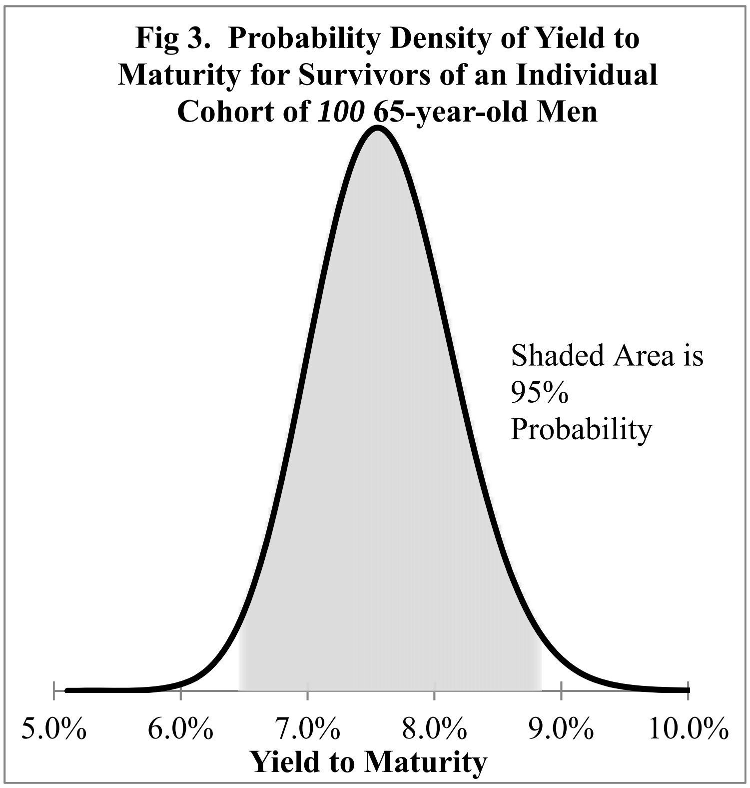


Fig 3. Probability Density of Yield to Maturity for Survivors of an Individual Cohort of *100* 65-year-old Men

63. 7.58507% = ((\$1628.89 + \$448.51)/\$1000.00)$^{0.1}$ – 1. Recall that the projected yield to maturity happens when the actual mortality-gain distribution equals the projected mortality-gain distribution. *See supra* note 59 and accompanying text (for \$1628.89 and \$448.51).

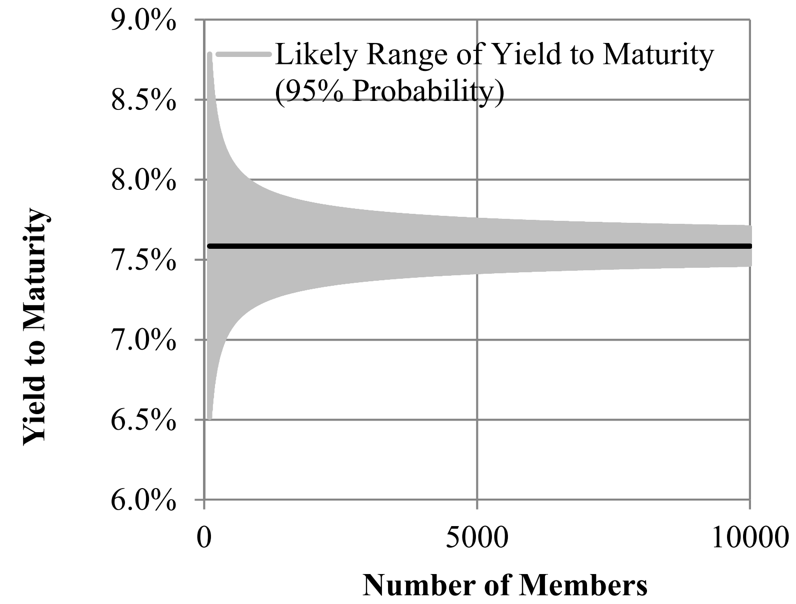
64. 6.51782% = ((\$1628.89 + \$251.39) /1000.00)$^{0.}$ – 1; 8.78317% = ((\$1628.89 + \$691.80) /1000.00)$^{0.1}$ – 1.

Unfortunately, 6.5% to 8.8% is a rather large 95%-yield-to-maturity range for an investment company that is trying to promote an investment product. This 95%-yield-to-maturity range is large because, with only 100 investors, the standard deviation of the number of deaths (4.11) is sizable relative to the mean number of deaths (21.59). If the survivor fund could attract more than 100 investors, however, the 95%-yield-to-maturity range would shrink. Basically, the larger the sample size, the smaller the standard deviation of the sample.[65] For example, with *1000* 65-year-old men, the 95%-yield-to-maturity range would shrink to just 7.2% to 7.9%.[66] More generally, Figure 4 shows how the 95%-yield-to-maturity range would shrink as the number of 65-year-old male investors in a survivor fund using the individual-cohort method increases.

65. The formula is $\sigma_S = \sigma/\sqrt{n}$, where σ_S is the standard deviation of the sample, σ is the standard deviation of the population, and n is the sample size. *See, e.g.*, Douglas G. Altman & J Martin Bland, *Standard Deviations and Standard Errors*, 331 BRITISH MED. J. 7521 (2005), http://www.ncbi.nlm.nih.gov/pmc/articles/PMC1255808/; Deborah J. Rumsey, *How Sample Size Affects Standard Error*, DUMMIES, http://www.dummies.com/how-to/content/how-sample-size-affects-standard-error.html (last visited Nov. 6, 2016).

66. For *1000* 65-year-old male investors, the mean number of deaths would be 215.9, and the standard deviation of the number of deaths would be $13.01 = (1000 \times 0.2159 \times (1 - 0.2159))^{0.5}$. Two standard deviations below the mean would be $189.88 = 215.9 - (2 \times 13.01)$, for a lower mortality-gain distribution of $\$381.79 = 189.88/(1000 - 189.88) \times \$1628.89$, and a lower yield to maturity of $7.23444\% = ((\$1628.89 + \$381.79)/\$1000.00)^{0.1} - 1$. Two standard deviations above the mean would be $241.92 = 215.9 + (2 \times 13.01)$, for an upper mortality-gain distribution of $\$519.81 = 241.92/(1000 - 241.92) \times \$1628.89$, and an upper yield to maturity of $7.94874\% = ((\$1628.89 + \$519.81)/\$1000.00)^{0.1} - 1$.

Figure 4. Yield-to-maturity Range for Survivors of an Individual Cohort of 65-year-old Men



It seems reasonable to conclude from Figure 4 that a survivor fund using the individual-cohort method could operate with as few as 500 identical investors; that is to say that the 95%-yield-to-maturity range for such a fund would be acceptably narrow. Still, we doubt that investment companies would want to offer a different survivor fund for each age-gender cohort, nor would they want to require that each investor put in exactly the same amount in exactly the same type of investment. For example, we would be surprised if an investment company could even find *500* 73-year-old women who were willing to each invest exactly $1000 in the 10-year survivor fund that started on May 1, 2017 and that invested in small cap stocks. We doubt that any investment company would be willing to take on that kind of *marketing* risk.

Instead, we believe that investment companies offering survivor funds would want to offer just a few different survivor funds for investors each month (or quarter). To reduce the number of survivor funds that an investment company would need to offer, we need to find a fair way to combine different cohorts of investors together into the same survivor fund, and we also need to find a fair way to combine investors who make different levels of investment.

Ideally, then, separate survivor funds should *not* be created based on individual investor characteristics like age, gender, or investment amount. Instead, separate survivor funds should combine various cohorts of investors, but be designed only to offer different kinds of underlying investments. For example, we can imagine that each month an investment company would offer a handful of 10-year survivor funds where each fund invested in different assets, such as government bonds, corporate bonds, large cap stocks, small cap stocks, and so on. The individual-cohort method could not be used to fairly reallocate mortality gains in such multi-cohort survivor funds, but both the date-aligned method that we discuss in Part II below and the age-aligned method that we discuss in Part IV below could be used to achieve such fair reallocations.

II. DESIGNING SURVIVOR FUNDS THAT ARE FAIR TO ALL TYPES OF INVESTORS: THE DATE-ALIGNED METHOD

In a simple survivor fund, when an investor dies, the balance in her account (i.e., her contribution plus investment earnings) would be divided equally among the surviving investors. Unfortunately, if the survivor fund included a variety of investors, that approach would result in an unfair situation, for example, because it would favor younger investors who are likely to live longer and receive more mortality-gain distributions. On the contrary, if a survivor fund has investors with different ages, genders, and investment levels, "the surviving investors should *not* get equal portions of a dying member's balance. Instead, the [mortality-gain] distributions should be made in *unequal* portions, carefully chosen to provide fair bets for all investors. In short, a [survivor] fund should be governed by a *fair transfer-plan* that accounts for each [investor's mortality risk] (i.e., death probability)[67] and investment level."[68] In this Part, we describe how such survivor funds could be designed.

A. *The Basic Design and Operation of a Survivor Fund: The Date-Aligned Method*

This subpart explains in greater detail the basic design and operation of survivor funds. More specifically, this subpart develops the so-called "date-aligned method" that can be used to design survivor funds that *are* fair to a broad range of investors, including those with different ages, genders, and investment levels.[69]

At the outset, Table 1 shows the membership of a simple survivor fund. To keep this initial example as simple as possible (and also to keep it consistent with the simple, single-cohort survivor fund that we described in Part 2 above), we assume: 1) that there are just ten investors in this survivor fund; 2) that each member (i) is a 65-year-old male who has contributed \$1000 (c_i) to an account that invests in 10-year zero coupon bonds; and 3) that each zero coupon bond has a 5% yield to maturity leading to a final account value (s_i) for each member of \$1628.89.[70] That \$1628.89 is the amount that each member would receive after ten years if none of the ten investors died. We do not have to worry about the value of the account at any intermediate time; it suffices (and it is more convenient) to just work with the account value at maturity (i.e., $t = 10$).

67. The term "fair transfer-plan" is derived from Sabin, *supra* note 12, at 5. *See also* Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 776. Recall that an individual's death probability is his probability of dying within one year. *See supra* note 54 and accompanying text.

68. Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 776 (emphasis added).

69. The approach parallels what we did in Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 776-84.

70. \$1628.89 = \$1000.00 × (1.00 + 0.05).

Table 1. A Simple Survivor Fund with Ten 65-year-old Male Members

Member *(i)*	*Age* *(x_i)*	*Gender* *(g_i)*	*Contribution* *(c_i)*	*Account Value at Maturity* *(s_i)*
1	65	male	$1000	$1628.89
2	65	male	$1000	$1628.89
3	65	male	$1000	$1628.89
4	65	male	$1000	$1628.89
5	65	male	$1000	$1628.89
6	65	male	$1000	$1628.89
7	65	male	$1000	$1628.89
8	65	male	$1000	$1628.89
9	65	male	$1000	$1628.89
10	65	male	$1000	$1628.89

The survivor fund starts with the members all making their contributions at time 0, and nothing happens until the end of the 10-year investment term (i.e., until the maturity date of the underlying zero coupon bonds). At that time (i.e., time 10), the survivor fund will create a fair transfer-plan to reallocate the balances in the accounts of any members who died before time 10 among the survivors in a way that provides fair bets for all of the members.

For example, suppose that two of the ten members (20%) died during the 10-year-survivor-fund term.[71] At time 10, we would reallocate the $1628.89 in the accounts of those two decedents, one at a time, and in the order that their deaths occurred. We call this the "date-aligned method," and here is how it would work:

Suppose that member 3 was the first member to die and that his death occurred 3.33 years after the start of the survivor fund (when all ten members of the survivor fund were 68.33 years-old, 68.33 = 65 + 3.33). Since this would be the first death to occur, all ten members would compete equally in the first fair transfer-plan, and we would use that fair transfer-

71. Again, 20% is a very reasonable mortality rate for a population of 65-year-old men. *See supra* note 25 and accompanying text, and recall that we showed in *supra* note 53 that approximately 21.59% of 65-year-old men in the 2010 Social Security area population mortality table could be expected to die before reaching the age of 75.

plan to distribute dying member 3's $1628.89 to the nine survivors. Table 2 shows the results after that first death.

Table 2. Accounting for the First Death, Date-aligned Method

Member (i)	*Account Value at Maturity (s_i)*	*Fair-transfer-plan Weight (wi)*	*Account Balance After the First Distribution (b1i)*
1	$1628.89	0.1	$1809.88
2	$1628.89	0.1	$1809.88
3	$1628.89	0.1	$ 0.00
4	$1628.89	0.1	$1809.88
5	$1628.89	0.1	$1809.88
6	$1628.89	0.1	$1809.88
7	$1628.89	0.1	$1809.88
8	$1628.89	0.1	$1809.88
9	$1628.89	0.1	$1809.88
10	$1628.89	0.1	$1809.88

Column 1 of Table 2 shows the member (i) and column 2 shows the account value in that member's account at maturity (s_i).[72] Column 3 of Table 2 shows a parameter that we call the *fair-transfer-plan weight* (w_i). When a member of a survivor fund dies, the maturity value of his account is forfeited and divided among the survivors, with each survivor receiving some fraction of the decedent's account which is based on that survivor's fair-transfer-plan weight (w_i).

More specifically, if member j dies, each surviving member i would receive some fraction of j's $1628.89 account value at maturity. Mathematically, the fraction that each member i would receive of member j's account (s_j) is equal to $w_i/(1 - w_j)$, for $i \neq j$. The fair-transfer-plan weights (w_i) are positive values that sum to one, so the denominator $(1 - w_j)$ is the sum of all fair transfer-plan weights (w_i) except that of member j. Meanwhile, member j would forfeit all $1628.89 in his account.

Pertinent here, since all ten members in our simple survivor fund were the same age when member 3 died (68.33 years-old), the same gender (male), and made the same contribution ($1000), all ten members would have the same fair-transfer-plan weight, $w_i = 0.1 = 1/10$ (see column 3 of Table

72. These values come from column 5 of Table 1.

2). Accordingly, at time 10, when it is time to divide the \$1628.89 in member 3's account among the *nine* members who survived him, each of those nine survivors would receive \$180.99 = $s_3 \times w_i/(1 - w_3)$ = \$1628.89 × 0.1/(1 – 0.1),[73] and, of course, member 3 would forfeit the \$1628.89 in his account. In short, each of the nine survivors would get a mortality gain of \$180.99 and would enter the next distribution with an account balance after the first distribution (*b1i*) of \$1809.88 = \$1628.89 + \$180.99 (see column 4 of Table 2).[74] We refer to the \$180.99 distributions to members 1, 2, and 4-10 as *mortality-gain distributions*; meanwhile, member 3 had a \$1628.89 mortality loss.

Now suppose that member 7 was the second member to die and that his death occurred 6.67 years after the start of the survivor fund (when the nine surviving members of the survivor fund were 71.67 years-old (71.67 = 65 + 6.67). As before, we would create a fair transfer-plan, but this time just for the nine members who survived to age 71.67. Those nine survivors would compete equally in the second fair transfer-plan, and we would use that fair transfer-plan to reallocate dying member 7's \$1809.88 to the *eight* members who survived him. Table 3 shows the results after that second mortality-gain distribution.

73. Put differently (and, perhaps, more simply), \$180.99 is \$1628.89 divided equally among the nine identical survivors (\$180.99 = \$1628.89/9).

74. \$1809.88 = $s_i + (s_3 \times w_i/(1 - w_3))$ = \$1628.89 + \$180.99.

Table 3. Accounting for the Second Death, Date-aligned Method

Member (i)	*Account Balance After the First Distribution (b1i)*	*Fair-transfer-plan Weight (wi)*	*Account Balance After the Second Distribution (b2i)*
1	$1809.88	0.111111	$2036.11
2	$1809.88	0.111111	$2036.11
3	$ 0.00	dead@68.33	$ 0.00
4	$1809.88	0.111111	$2036.11
5	$1809.88	0.111111	$2036.11
6	$1809.88	0.111111	$2036.11
7	$1809.88	0.111111	$ 0.00
8	$1809.88	0.111111	$2036.11
9	$1809.88	0.111111	$2036.11
10	$1809.88	0.111111	$2036.11

Column 1 of Table 3 shows the member (*i*), and column 2 of Table 3 shows the value of each member's account balance after the first distribution (*b1i*).[75] Column 3 of Table 3 shows the applicable fair-transfer-plan weights (w_i) for each of the nine members who survived until time 6.67. As all nine surviving members were the same age when the second member died (71.67 years-old), the same gender (male), and have the same balances in their account after the first mortality-gain distribution (*b1i* = $1809.88); all nine have the same fair transfer-plan weight, w_i = 0.111111 = 1/9. Accordingly, at time 10, when it is time to divide the $1809.88 in member 7's account among the remaining *eight* members who survive him, each of those eight survivors will receive a mortality-gain distribution of $226.23 = $b17 \times w_i/(1 - w_7)$ = $1809.88 × 0.111111/(1 – 0.111111),[76] and each ends up with an account balance after the second distribution (*b2i*) of $2036.11 = $1809.88 + $226.23 (see column 4 of Table 3).[77] Of course, member 7 would forfeit his $1809.88 and end up with a zero account balance (see column 4 of Table 3).

75. These values come from column 4 of Table 2.

76. Put differently (and, perhaps, more simply), $226.243 is $1809.88 divided equally among the eight remaining survivors ($226.235 = $1809.88/8).

77. $2036.11 = $b1i + (b17 \times w_i/(1 - w_7))$ = $1809.88 + $226.23.

As we assumed that there were no further deaths during the 10-year survivor-fund term, that \$2036.11 account balance after the second distribution (*b2i*) would also be the ending account balance (*bei*) for the eight surviving members (see column 5 of Table 4), and each of these eight survivors would have that \$2036.11 distributed to them at that time (again, at $t = 10$). Column 6 of Table 4 also shows that the yield to maturity (*yi*) for these eight survivors would be 7.37%, as opposed to the 5% yield to maturity on the underlying zero coupon bonds.[78]

Table 4. The End of a 10-man Survivor Fund with Eight Survivors, Date-aligned Method

Member (i)	*Ending Age (x_i)*	*Gender (g_i)*	*Initial Contribution (ci)*	*Ending Account Balance (bei)*	*Yield to Maturity (yi)*
1	75	Male	\$1000	\$2036.11	7.37%
2	75	Male	\$1000	\$2036.11	7.37%
3	dead@68.33	Male	\$1000	\$ 0.00	n/a
4	75	Male	\$1000	\$2036.11	7.37%
5	75	Male	\$1000	\$2036.11	7.37%
6	75	Male	\$1000	\$2036.11	7.37%
7	dead@71.67	Male	\$1000	\$ 0.00	n/a
8	75	Male	\$1000	\$2036.11	7.37%
9	75	Male	\$1000	\$2036.11	7.37%
10	75	Male	\$1000	\$2036.11	7.37%

Not surprisingly, with this group of investors who are identical in age, gender, and investment level, the date-aligned method reaches the exact same results as the individual-cohort method would. More specifically, the date-aligned method here got the same \$2036 ending account balance and the same 7.37% yield to maturity that we found in Part 2 above (where we divided the \$1,628,894.63 proceeds from \$1,000,000 worth of 10-year, zero coupon bonds among the 800 survivors of that *1000*-65-year-old-man, individual-cohort survivor fund).[79]

Indeed, both the individual-cohort method and the date-aligned method would result in fair reallocations any time all of

78. $7.36930\% = (\$2036.11/\$1000.00)^{0.1} - 1$.

79. There is a one-cent difference in the ending account balances here (\$2036.11) and there (\$2036.12) due to rounding of intermediate calculations.

the survivor-fund members are identical (i.e., have the same age and gender and have made exactly the same level of contributions). As we will see in the next few subparts of this Article, however, only something like the date-aligned method that we developed in this subpart could be used to achieve fair reallocations when members of a survivor fund have different ages, genders, or investment levels.

B. Survivor Funds Using the Date-Aligned Method Would Be Fair to Investors of Different Ages

This subpart shows that survivor funds using the date-aligned method would be fair to investors of different ages. For example, Table 5 shows another 10-man survivor fund. We again assume that each member (i) has invested \$1000 (c_i) in the survivor fund and that the survivor fund invests in zero coupon bonds that mature in ten years with a 5% yield to maturity for a final value at maturity for each member of \$1628.89 (s_i), but, this time, *our ten male investors are different ages*. This survivor fund again starts at time 0, and nothing happens until the end of the 10-year, survivor-fund term. At that time (time 10), the survivor fund would again use a fair transfer-plan to reallocate the balances in the accounts of the members who died before time 10 among the survivors.

Table 5. A Survivor Fund with 10 Male Members of Different Ages, Date-aligned Method

Member (i)	*Age (x_i)*	*Gender (g_i)*	*Account Value at Maturity (s_i)*	*Life Expectancy (years) (e_i)*	*Death Probability (q_i)*
1	65	male	$1628.89	17.57	0.015927
2	66	male	$1628.89	16.84	0.017370
3	67	male	$1628.89	16.13	0.018895
4	68	male	$1628.89	15.43	0.020484
5	69	male	$1628.89	14.75	0.022191
6	70	male	$1628.89	14.07	0.024139
7	71	male	$1628.89	13.40	0.026364
8	72	male	$1628.89	12.75	0.028808
9	73	male	$1628.89	12.12	0.031480
10	74	male	$1628.89	11.49	0.034442

Because our members are different ages, however, we would use death probabilities from a mortality table to create the fair transfer-plan.[80] As a starting point, column 2 of Table 5 shows the age (x_i) of each man, column 5 shows his initial life expectancy at that age (e_i),[81] and column 6 shows his death probability (q_i) at that age. These values are drawn from the 2010 Social Security Administration area population mortality table in Appendix Table 1. For example, member 3 in Table 5 is a 67-year-old man (columns 2 & 3) who has a life expectancy (e_i) of 16.13 years (column 5) and a 1.9% chance of dying before reaching age 68 (column 6; i.e., a death probability, q_i, of 0.018895).

To make comparisons easy, we again assume that two members of this survivor fund would die over the 10-year survivor-fund term:[82] member 3 would die at time 3.33, and member 7 would die at time 6.67. We again use the date-

80. Recall that an individual's death probability is his probability of dying within the next year. *See supra* note 53 and accompanying text.

81. Recall that life expectancy at a given age is the average remaining number of years expected prior to death for a person at that exact age. *See supra* note 52 and accompanying text.

82. We note, in passing, that this example is not quite as reasonable as the one in Part I.A (Table 1): given the relatively older population in the survivor fund in Table 5, it is likely that *more than* two (out of 10) members would die over the 10-year, survivor-fund term.

aligned method that we developed in Part I.A. As all ten members would be alive when the first death occurs at time 3.33, all ten would compete in the first fair transfer-plan, and we would again create a fair transfer-plan to reallocate dying member 3's $1628.89 to the nine survivors. Table 6 shows the results after that first mortality-gain distribution.

Table 6. Accounting for the First Death in a Survivor Fund with Members of Different Ages, Date-aligned Method

Member (i)	*Age at the First Death (xi)*	*Gender (gi)*	*Account Value at Maturity (si)*	*Force-of-Mortality Probability (fi)*	*Fair-transfer-plan Weight (wi)*	*Account Balance After the First Distribution (b1i)*
1	68.33	male	$1628.89	0.020697	0.061656	$1737.32
2	69.33	male	$1628.89	0.022441	0.067253	$1747.16
3	dead@70.33	male	$1628.89	0.024435	0.073742	$ 0.00
4	71.33	male	$1628.89	0.026718	0.081294	$1771.86
5	72.33	male	$1628.89	0.029231	0.089769	$1786.76
6	73.33	male	$1628.89	0.031986	0.099266	$1803.46
7	74.33	male	$1628.89	0.035049	0.110095	$1822.50
8	75.33	male	$1628.89	0.038590	0.123002	$1845.20
9	76.33	male	$1628.89	0.042620	0.138252	$1872.02
10	77.33	male	$1628.89	0.047020	0.155671	$1902.65

Column 1 of Table 6 shows the member (*i*), and column 2 shows the ages of the members at the time of the first death (t = 3.33), ranging from 68.33-years-old for member 1 (68.33 = 65 + 3.33) to 77.33-years-old for member 10 (77.33 = 74 + 3.33). Column 4 of Table 6 shows the $1628.89 account value at maturity (s_i) for each of the founding investors in this survivor fund.

Pertinent here, as the ten members are different ages, they should have *different* fair-transfer-plan weights. In general, a fair transfer-plan should reallocate more of each decedent's mortality losses to older survivors than to younger survivors precisely because younger members are more likely to survive to collect more mortality-gain distributions than older members; that is, older survivors should receive more than younger survivors to ensure that all members get a fair bet. Here is how it works:

At the outset, Column 5 of Table 6 also shows a parameter known as the *force-of-mortality probability* (f_i).[83] These force-of-mortality probabilities indicate the *relative* probability of death for each member of the survivor fund at time *t*. In that regard, if, at the instant that a member died, one member has a force-of-mortality probability with a value *f*, and another has a value 2*f*, then the second member is twice as likely as the first to be the one who died. For example, member 10, who would be 77.33 years-old at time 3.33, had a relatively large force-of-mortality probability (0.047) at that time, while member 1, who was just 68.33 years-old then, had a relatively small force-of-mortality probability (0.021). In short, member 10 would clearly be the more likely of the two to have died at time 3.33. Indeed, of the ten members in Table 5, member 10 would be the *most* likely to have died first. These force-of-mortality probabilities (f_i) are relatively easy to compute from the death probabilities (q_i) in a mortality table (i.e., column 6 of Table 5).[84]

We then use these force-of-mortality probabilities (f_i) to determine the fair-transfer-plan weights (w_i) in column 6 of Table 6.[85] Then, as in Part I.A above, we use those fair-transfer-plan weights (w_i) to divide the account value at maturity of the member who died at time 3.33. Recall that we again assumed that it would be member 3 who died at time 3.33. As column 7 of Table 6 shows, member 3 would lose the \$1628.89 in his account, and that \$1628.89 would be divided among the *nine* survivors and added to their accounts. For example, member 1 would receive mortality-gain distribution of \$108.43 = $s_3 \times w_1/(1 - w_3)$ = \$1628.89 × 0.061656/(1 – 0.073742);

83. This explanation follows Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 777-78.

84. For a more detailed explanation of the computation of force-of-mortality probabilities (f_i), *see*, *id.* at 778 n.111.

85. For a more detailed explanation of the computation of fair-transfer-plan weights (w_i), *see id.* at 778 n.112.

and that \$108.43 would be added to his starting account value at maturity (s_1) of \$1628.89 to get an account balance after the first distribution ($b1_1$) of \$1737.32 (see column 7 of Table 6).[86] On the other hand, member 10 would get a mortality-gain distribution of \$273.76 and end up with an account balance after the first distribution ($b1_{10}$) of \$1902.65.[87]

Next, to account for the death of member 7 at time 6.67, we would create a second fair transfer-plan to reallocate dying member 7's \$1822.50 (*b17*, from column 7 of Table 6) among the *eight* members who survived him. Table 7 shows the results after that second mortality-gain distribution.

Table 7. Accounting for the Second Death in a Survivor Fund with Members of Different Ages, Date-aligned Method

Member (i)	*Age at the Second Death (x_i)*	*Gender (g_i)*	*Account Balance After the First Distribution (b1i)*	*Fair-transfer-plan Weight (wi)*	*Account Balance After the Second Distribution (b2i)*
1	71.67	male	\$1737.32	0.060657	\$1862.87
2	72.67	male	\$1747.16	0.067208	\$1886.27
3	dead@70.33	male	\$ 0.00	dead@70.33	\$ 0.00
4	74.67	male	\$1771.86	0.083144	\$1943.95
5	75.67	male	\$1786.76	0.093354	\$1979.98
6	76.67	male	\$1803.46	0.105478	\$2021.78
7	dead@77.67	male	\$1822.50	0.119464	\$ 0.00
8	78.67	male	\$1845.20	0.135718	\$2126.11
9	79.67	male	\$1872.02	0.155220	\$2193.29
10	80.67	male	\$1902.65	0.179756	\$2274.71

Column 1 of Table 7 shows the member (*i*), and column 2 shows the age of the member at the time of the second death (t = 6.67). Column 4 of Table 7 shows the account balance for each member after the first distribution ($b1_i$),[88] and column 5 of Table 7 shows each member's fair-transfer-plan weight (w_i) at

86. \$1737.32 = $s_1 + (s_3 \times w_1/(1 - w_3))$ = \$1628.89 + \$108.43.

87. \$1902.65 = $s_{10} + (s_3 \times w_{10}/(1 - w_3))$ = \$1628.89 + (\$1628.89 × 0.155671/(1 – 0.073742)) = \$1628.89 + \$273.76.

88. These values come from column 7 of Table 6.

that time.[89] As columns 4 and 6 of Table 7 show, member 7 would lose the $1822.50 in his account, and it would be divided among the eight survivors and added to their accounts. For example, member 1 would have $125.55 added to his $1737.32 account balance after the first distribution ($b1_1$) to get an account balance after the second distribution ($b2_1$) of $1862.87.[90]

As we assumed there would be no further deaths during the 10-year survivor-fund term, the account balances after the second distribution ($b2_i$) are also the ending account balances (be_i) for the eight final members (see column 5 of Table 8), and each of the eight survivors would have his balance distributed to him at that time. Column 6 of Table 8 also shows that the yields to maturity (y_i) for all eight survivors would be significantly higher than the 5% yield to maturity on the underlying zero coupon bonds.

Table 8. The End of a Ten-man Survivor Fund with Eight Survivors of Different Ages, Date-aligned Method

Member (i)	*Ending Age (x_i)*	*Gender (g_i)*	*Initial Contribution (c_i)*	*Ending Account Balance (bei)*	*Yield to Maturity (yi)*
1	75	male	$1000	$1862.87	6.42%
2	76	male	$1000	$1886.27	6.55%
3	dead@70.33	male	$1000	$ 0.00	n/a
4	78	male	$1000	$1943.95	6.87%
5	79	male	$1000	$1979.98	7.07%
6	80	male	$1000	$2021.78	7.29%
7	dead@71.67	male	$1000	$ 0.00	n/a
8	82	male	$1000	$2126.11	7.83%
9	83	male	$1000	$2193.29	8.17%
10	84	male	$1000	$2274.71	8.57%

In short, the date-aligned method can accommodate investors of different ages. To be sure, columns 5 and 6 of Table 8 make it appear that older survivors would do better

89. Determining the values of these fair transfer plan weights (w_i) is more complicated, as the surviving members now differ in both age and account balance.

90. $1862.87 = $b1i + (b17 \times w_1/(1 - w_7))$ = $1737.32 + ($1822.50 × 0.060657/(1 – 0.119464)) = $1737.32 + $125.55.

than younger survivors, but remember that older members would actually be less likely to survive all ten years than younger survivors. Accordingly, a fair transfer-plan must give more of each decedent's mortality losses to older survivors than to younger survivors—precisely because the younger members would be more likely to survive all ten years, and all of the members should get a fair bet. The key to a date-aligned, fair transfer-plan is to use death probabilities to generate fair-transfer-plan weights (w_i) that offer fair bets to members of all ages. Moreover, while the survivor fund that we used as our example only involved investors aged 65 to 74, theoretically, a survivor fund could accommodate members of all ages.[91]

C. Survivor Funds Using the Date-Aligned Method Could Be Fair to Both Men and Women

Similarly, survivor funds could be designed to take gender into account.[92] In that regard, "[w]omen tend to live longer than men and have lower death probabilities than same-aged men."[93] For example, Appendix Table 1 shows that while a 65-year-old man in the 2010 Social Security area population had a 17.57-year life expectancy and a 1.6% death probability, a 65-

91. Pertinent here, we doubt that short-term survivor funds would be very attractive to young investors. Younger *survivors* often have such low death probabilities that their yields to maturity from survivor funds would be only slightly higher than the yields to maturity on the underlying investments. For example, we doubt that many parents would use survivor funds to save for their children's college education. Imagine that the parents of *1000* 8-year-old male children each contributed $1000 to a survivor fund that held $1,000,000 worth of 10-years zero coupon bonds earning a 5% yield to maturity. Appendix Table 1 suggests that of the 100,000 live male births in the table, 99,176 could be expected to make it to age 8, and 98,906 of them could be expected to make it to age 18. In effect, the table suggests that just 270 (0.27%) of those 8-year-old boys would die before reaching age 18 (270 = 99,176-98,906; 0.27222% = 270/99,176). It follows that out of the *1000* 8-year-old boys whose parents invested in that survivor fund, approximately 997 of them would still be alive at age 18 (997.28 = 1000 × 98,906/99,176). On average, the winners who survived those ten years could anticipate receiving negligible mortality-gain distributions of just $5 ($1633.80 = $1,628,894.63/997; $4.91 = $1633.80 – $1628.89).

92. *See generally* Forman & Sabin, *Tontine Pensions, supra* note 6, at 780-81.

93. *Id.*

year-old woman had a 20.20-year life expectancy and just a 1.0% death probability. While a survivor fund that included both men and women could use a unisex mortality table to determine the fair-transfer-plan weights (w_i) that would result in their final account balances, a survivor fund could instead use a gender-based mortality table to take gender into account. To be sure, we are not advocating that survivor funds should take gender into account, but only noting that they could.[94]

D. Survivor Funds Using the Date-Aligned Method Would Fairly Accommodate Investors with Differing Levels of Investment

Survivor funds could also be designed to allow members to make differing levels of contributions.[95] For example, Table 9 shows another 10-man survivor fund, but this time we assume that these otherwise-identical men would have different contribution levels (c_i) ranging from $1000 to $10,000 (column 4). As before, we assumed that two members of the survivor fund would die over the 10-year survivor-fund term: member 3 would die at time 3.33, and member 7 would die at time 6.67. We again used the date-aligned method that we developed in Part I.A , and Table 9 shows the final results. Again, the yields to maturity (y_i) for the eight survivors would be significantly higher than the 5% yield to maturity on the underlying zero coupon bonds; meanwhile, members 3 and 7 would forfeit their investments (see columns 5 & 6 of Table 9). In short, the date-aligned method would accommodate investors with different investment levels.

94. *Cf.*, Mary L. Heen, *Nondiscrimination in Insurance: The Next Chapter*, 49 GA. L. REV. 1 (2014) (arguing that gender nondiscrimination laws should be expanded to *prevent* insurance companies from selling gender-based annuities). *See also* Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 823-26 (discussing a number of similar gender issues).

95. *See generally* Forman & Sabin, *Tontine Pensions, supra* note 6, at 782-83.

Table 9. A Survivor Fund with Members Making Different Contributions, Date-aligned Method

Member (i)	*Ending Age (x_i)*	*Gender (g_i)*	*Initial Contribution (c_i)*	*Ending Account Balance (bei)*	*Yield to Maturity (yi)*
1	75	male	$1000	$ 1942.07	6.86%
2	75	male	$2000	$ 3895.58	6.89%
3	dead@68.33	male	$3000	$ 0.00	n/a
4	75	male	$4000	$ 7842.63	6.96%
5	75	male	$5000	$ 9839.82	7.00%
6	75	male	$6000	$11,856.02	7.05%
7	dead@71.67	male	$7000	$ 0.00	n/a
8	75	male	$8000	$15,959.60	7.15%
9	75	male	$9000	$18,057.30	7.21%
10	75	male	$10,000	$20,196.18	7.28%

E. The Date-Aligned Method is Practical

In short, a survivor fund could use the date-aligned method to include investors with different ages, genders, and investment levels. That means that an investment company could succeed in offering just a few survivor funds each month (or quarter). For example, an investment company could offer a handful of 10-year survivor funds each month where each fund invests in a specific type of assets, such as government bonds, corporate bonds, large cap stocks, small cap stocks, and so on. Each of these survivor funds would be open to investors of any age, gender, or investment level, and an investor could choose to invest in one or more of the funds. With only a handful of funds created each month (or quarter), the number of investors in each fund should be reasonably high, and the administrative costs incurred by the investment company should be relatively low.

It turns out that there is another huge advantage to having a survivor fund use the date-aligned method: more predictable investment outcomes. Again, we will have to delve into the world of statistics to see why.

Recall that in Part III above, we showed how to calculate the 95%-mortality-gain-distribution range for a survivor fund that used the individual-cohort method. In that example, *100* 65-year-old men each invested $1000 in a survivor fund that held 10-year zero coupon bonds with a 5% yield to maturity, and the mortality rates of those 100 men were taken from the 2010 Social Security area population mortality table in Appendix Table 1. For that hypothetical 100-man, individual-cohort survivor fund, we calculated that: 1) the projected mortality-gain distribution would be $448.51;[96] and 2) there would be a 95% probability that the actual mortality-gain distribution that a survivor received would be between $251 and $692.[97] We also explained that the 95% range for that would be narrower if that individual-cohort survivor fund had a larger number of investors, but because of the requirement that all investors in that individual-cohort survivor fund had to be 65-year-old males willing to invest exactly $1000, we thought that it would be difficult to attract a greater number of identical investors.[98]

On the other hand, if a survivor fund instead used the date-aligned method, the survivor fund could be open to investors of any age, gender, or investment level, and that would make it easier to attract a large number of investors. With more investors, the 95%-mortality-gain-distribution range would be narrower. To be sure, it is not feasible to *calculate* the 95%-mortality-gain-distribution range for a survivor fund that uses the date-aligned method and has investors of different ages, genders, or investment levels; the analytic formulas are hopelessly complex. It is, however, possible to *simulate* such a date-aligned survivor fund and to measure the 95%-mortality-gain-distribution range that results from repeated simulations, which is what we did instead.

96. *See supra* note 59 and accompanying text.
97. *See supra* notes 61-62 and accompanying text.
98. *See supra* the discussion accompanying Figure 4.

Specifically, we simulated a simple, multi-cohort survivor fund with 1000 male investors, 100 at each age from 65 through 74. We again assumed that each man invested $1000 in the survivor fund and that the survivor fund invested all those contributions in 10-year zero coupon bonds with a 5% yield to maturity. We again used the mortality rates implied by the 2010 Social Security area population mortality table in Appendix Table 1. We ran 1000 simulations using the date-aligned method to reallocate the mortality losses of those who died during the 10-year survivor-fund terms of those 1000 simulations, and the results for all ten cohorts are listed in Appendix Table 2A. For the 65-year-old men, the average mortality-gain distribution over the simulation runs was $449.26 (see column 6 of Appendix Table 2A), which is very close to the projected mortality-gain distribution of $448.49 (see column 5 of Appendix Table 2A).[99] The 95%-mortality-gain-distribution range for these men was from $395 to $505, meaning that 95% of the time, the actual mortality-gain distribution that a survivor received was between $395 and $505 (see columns 7 and 8 of Appendix Table 2A).[100] This is a much narrower 95% mortality-gain-distribution range than the $251 to $692 range that we calculated would be experienced by the *100* 65-year-old men in hypothetical, individual-cohort survivor fund that we developed in Part I above.[101]

Thus, both the date-aligned and the individual-cohort method produce average mortality-gain distributions that are close to the projected mortality-gain distribution of $448.49.[102] This is not surprising, since both methods are designed to be fair. Where they differ, however, is in their variability, i.e., in the 95% mortality-gain-distribution range that investors would experience.

99. *See supra* note 59 and accompanying text. There is a two-cent difference in the projected value listed here ($448.49) and there ($448.51) due to rounding of the intermediate calculations.

100. In each simulation run, *100* 65-year-old men were alive at the start of the 10-year period, for a total of 100 × 1000 = 100,000 initial investors over the 1000 simulation runs. A total of 78,454 of these survived the 10-year period. Of those who survived, 74,531 = 0.95 × 78,454 (95%) received a mortality-gain distribution in the range from $395 to $505.

101. *See supra* notes 61-62 and accompanying text.

102. *See supra* note 99.

An insightful way to compare that variability is to ask: what size of an individual cohort would produce a 95%-mortality-gain-distribution range that matched what a member would experience with the date-aligned method? It turns out that it is fairly easy to calculate this *equivalent-individual-cohort size* for each cohort in our date-aligned-method simulations.[103] For example, for the *100* 65-year-old men in our multi-cohort, date-aligned-method simulation, the equivalent-individual-cohort-method size would be 1,572 (see column 9 of Appendix Table 2A).[104] Similarly, for the *100* 74-year-old men, the equivalent-individual-cohort size would be 762 (see column 9 of Appendix Table 2A).

The equivalent-individual-cohort size shows how much less variance the date-aligned method produces compared to the individual-cohort method. The advantage is greatest for the *100* 65-year-old men—if they were removed from the multi-cohort date-aligned survivor fund and placed in an individual-cohort survivor fund, they would need to increase the number of investors by a factor of about 15.7 in order to enjoy a 95%-mortality-gain-distribution range as narrow as they would experience with the date-aligned fund.[105] That would be a huge

103. Determining the equivalent-individual-cohort size in this manner uses trial-and-error. We try a specific cohort size, calculate the 95% range that would result from using the individual-cohort method, and then measure the percentage of simulation results that lie within this calculated range. If the percentage is greater than 95.45%, we know the cohort size we tried is too small, and we try a larger size. Conversely, if the percentage is less than 95.45%, we know the cohort size we tried is too large, and we try a smaller size. With this approach we can quickly determine the cohort size which produces the 95% range in which exactly 95.45% of the simulation results lie.

104. In a group of *1572* 65-year-old men, the mean number of deaths is $339.39 = 1572 \times 0.2159$, and the standard deviation of the number of deaths is $16.31 = 1572 \times 0.2159 \times (1 - 0.2159))^{0.5}$. Two standard deviations below the mean is $306.77 = 339.39 - (2 \times 16.31)$, for a lower mortality gain of $\$394.94 = 306.77/(1572 - 306.77) \times \$1628.89$. Two standard deviations above the mean is $372.01 = 339.39 + (2 \times 16.31)$, for an upper mortality-gain distribution of $\$504.97 = 372.01/(1{,}572 - 372.01) \times \$1628.89$. Thus the 95% range (95.45% probability) for the mortality-gain distributions using the individual-cohort method is from $394.94 to $504.97. When we examine the date-aligned simulation results, we find that 95.45% of the age-65 survivors in the simulations received a mortality credit between $394.94 and $504.97. Thus, the simulation results have an equivalent-individual-cohort size of 1572.

105. 15.72 = 1572/100.

increase in the required number of identical investors. The advantage of the date-aligned method decreases with age, being the least for the 74-year-old men—they would need to increase the number of investors by a factor of just 7.6 in order to enjoy a 95%-mortality-gain-distribution range as narrow as they would experience with the date-aligned fund.[106] Still, even that would be a very large increase in the required number of identical investors.

Summing the equivalent-individual-cohort sizes for all of the individual-cohorts works out to a total of 11,109 equivalent investors in ten individual-cohort funds—about 11.1 times more than the 1000 investors we would have in the single, multi-cohort date-aligned fund (see column 9 of Appendix Table 2A).[107] This equivalent-individual-cohort size comparison illustrates the motivation for using the date-aligned method rather than the individual-cohort method: the date-aligned method would result in a narrow 95%-mortality-gain-distribution range using far fewer investors (here, a similar 95% range using a number of investors that is an order of magnitude smaller).[108]

Accordingly, the date-aligned method would also result in a narrow 95%-yield-to-maturity range using far fewer investors (see columns 11 and 12 of Appendix Table 2A). For example, while both date-aligned and individual-cohort survivor funds would produce projected yields to maturity for 65-year-old men of 7.58% (see column 10 of Appendix Table 2A),[109] the multi-cohort date-aligned survivor fund would do so with much less variance per investor. For example, while the *100* 65-year-old men who invested in our hypothetical 1000-man, multi-cohort

106. 7.62 = 762/100.

107. 11.11 = 11,109/1000.

108. Of course, everything said here is specific to our example population. The results would be different for different population groups. Still, if we have a diverse population with many different cohorts, it is clear that a single survivor fund using the date-aligned method would provide investors with a 95%-mortality-gain-distribution range that is much narrower than they could get with multiple separate survivor funds using the individual-cohort method.

109. *See also supra* note 63 and accompanying text. There is a small difference in the projected yield to maturity here ($7.58%) and there (7.59%) due to rounding of intermediate calculations.

date-aligned fund experienced a 95%-yield-to-maturity range from 7.3% to 7.9% in our simulations (see columns 11 and 12 of Appendix Table 2B), in Part I above, we calculated that *100* 65-year-old men who invested in a free-standing, 100-man individual-cohort fund would experience a much larger 95%-yield-to-maturity range of from 6.5% to 8.8%.[110] As investment companies that market survivor funds would want to advertise predictable outcomes and small 95%-yield-to-maturity ranges for their products,[111] we expect that those companies would certainly prefer using the date-aligned method to the individual-cohort method.

F. Process All the Deaths at the Same Time: The Rough-Justice Method

Before moving on, it is worth considering another possible method that a survivor fund could use to reallocate the account forfeitures of dying members. Instead of processing deaths one at a time and in the order that they occurred, we might process all of the deaths at one time with a single transfer. We call this the "rough-justice method" as it is not as nuanced as the date-aligned method.

Recall that in Table 5 in Part I.B above, we hypothesized that ten men of different ages each invested $1000 ($c_i$) in a survivor fund that invested in zero coupon bonds that matured in ten years, and that the value of each man's account at maturity (s_i) was $1628.89. We then used the date-aligned method and the death probabilities (q_i) from the 2010 Social Security area population mortality table in Appendix Table 1 to calculate fair-transfer-plan weights (w_i) that were used to reallocate the accounts of member 3 who died at time 3.33 and then of member 7 who died at time 6.67, and Table 8 showed the final results.

110. Again, in our examples, it would take 15.7 times as many 65-year-old men in the individual-cohort survivor fund to have a 95%-yield-to-mortality range as narrow as what we found in the date-aligned survivor fund. *See supra* note 105 and accompanying text.

111. *See supra* Part I.

With that reminder about how the date-aligned method works, we can now use the same hypothetical facts to show how to design a survivor fund that instead makes a single, simultaneous reallocation of the amounts in the accounts of both dying members 3 and 7. For reasons that will become apparent later in this subpart, we call this the *rough-justice method*, and it is essentially the method used by the College Retirement Equity Fund (CREF)[112] and by the Group Self Annuity (GSA)[113] to determine annuity payouts.

For example, Table 10 shows another 10-man survivor fund like the one in Table 5 above. We again assume that each member (i) has invested \$1000 (c_i) in the survivor fund, that the ten men are different ages (see column 2 of Table 10), and that the survivor fund invests in zero coupon bonds that mature in ten years with a 5% yield to maturity for a final value at maturity for each member of \$1628.89 (s_i) (see column 4 of Table 10). The survivor fund again starts at time 0, and nothing happens until the end of the 10-year survivor-fund term. At that time, this survivor fund will use the rough-justice method to reallocate the balances in the accounts of the members who died before time 10 among the survivors, and here is how it works:

112. *See*, *e.g.*, Forman & Sabin, *Tontine Pensions, supra* note 6, at 798. "Basically, within each investment account, CREF periodically adjusts annuity payments so that the present value of the aggregate amount expected to be paid out over the participants' remaining lifetimes matches the current value of the assets in the [CREF] account." *Id.* At the time of adjustment, each participant's annuity payment is scaled by a factor common to all participants, that factor chosen to make the *aggregate* expected payout match the value of assets. "If participants in the fund 'live longer . . . than expected, the amount payable to each will be less than if they as a group die sooner than expected.'" *Id.* (citations omitted).

113. *See generally* John Piggott, Emiliano A. Valdez & Bettina Detzel, *The Simple Analytics of a Pooled Annuity Fund*, 72 J. RISK & INS. 497 (2005).

Table 10. The Start of a Survivor Fund with Ten Members of Different Ages, Rough-justice Method

Member (i)	*Age (x_i)*	*Gender (g_i)*	*Account Value at Maturity (s_i)*	*Probability of Surviving 10 Years ($su10_i$)*	*Expected Ending Account Balance for the Survivors (ebei)*
1	65	male	$1628.89	0.784105	$2077.39
2	66	male	$1628.89	0.766628	$2124.75
3	67	male	$1628.89	0.747630	$2178.74
4	68	male	$1628.89	0.727023	$2240.49
5	69	male	$1628.89	0.704772	$2311.23
6	70	male	$1628.89	0.680785	$2392.66
7	71	male	$1628.89	0.654951	$2487.04
8	72	male	$1628.89	0.627143	$2597.32
9	73	male	$1628.89	0.597364	$2726.80
10	74	male	$1628.89	0.565667	$2879.59

For each member, we calculated the probability of surviving ten years until the survivor fund ends ($su10_i$) using the 2010 Social Security area population mortality table in Appendix Table 1. For example, column 5 of Table 10 suggests that the 65-year-old male has a 78.41% chance of living ten years and reaching aged 75 ($su10_1$ = 0.784105); put differently, he has a 21.59% probability of dying before the survivor fund ends (10-year death probability, $q10_1$ = 0.215895 = 1 – 0.784105).[114] On the other hand, the 74-year-old has only a 56.57% chance of living ten years to age 84 (and a 43.43% 10-year death probability) (see column 5 of Table 10).

Finally, column 6 of Table 10 shows the projected ending account balance (*ebei*) that each member could anticipate if he survived to the end of the 10-year survivor fund and if the actual mortality experience of the members of the survivor fund comported with the 2010 Social Security area population mortality table in Appendix Table 1. For example, if our

114. *See supra* notes 53, 59 and accompanying text.

hypothetical 65-year-old man lives ten years to age 75, he could expect to collect \$2077.39 (see column 6 of Table 10).[115]

As we did in explaining the date-aligned method, we again assume that two members of the survivor fund would die over the 10-year survivor-fund term: member 3 would die at time 3.33 and member 7 would die at time 6.67, and each would lose the \$1628.89 in his account at time 10.[116] Under the rough-justice method, at time 10, the total amount \$16,288.90 in the survivor fund (\$16,288.90 = \$1628.89 × 10) would be simultaneously split among the eight surviving members in proportion to each survivor's expected ending account balance (*ebei*) (see column 6 of Table 10); Table 11 shows the results.

115. More precisely, the expected ending account balance is the amount a survivor would expect to collect if he participated in an individual-cohort survivor fund whose actual mortality experience comported with the 2010 Social Security area population mortality table in Appendix Table 1. For the 65-year-old, 78.4105% of the members in an individual-cohort fund would survive, so each survivor would receive \$2077.39 = \$1628.89/0.784105. For the 74-year-old, 56.5667% of the members in an individual-cohort fund would survive, so each survivor would receive \$2879.59 = \$1628.89/0.565667.

116. Since member 3 was 67 at time 0, he was 70.33 at time 3.33 (70.33 = 67 + 3.33). Similarly, since member 7 was 71 at time 0, he was 77.67 at time 6.67 (77.67 = 71 + 6.67).

Table 11. The End of a Ten-man Survivor Fund with Eight Survivors of Different Ages, Rough-justice Method

Member (i)	*Age (x_i)*	*Gender (g_i)*	*Account Value at Maturity (s_i)*	*Mortality Gain or (Loss) (mg_i)*	*Ending Account Balance (bei)*	*Yield to Maturity (yi)*
1	75	male	$1628.89	$119.84	$1748.73	5.75%
2	76	male	$1628.89	$159.71	$1788.60	5.99%
3	dead@70.33	male	$1628.89	($1628.89)	$0.00	n/a
4	78	male	$1628.89	$257.14	$1886.03	6.55%
5	79	male	$1628.89	$316.69	$1945.58	6.88%
6	80	male	$1628.89	$385.24	$2014.13	7.25%
7	dead@77.67	male	$1628.89	($1628.89)	$0.00	n/a
8	82	male	$1628.89	$557.52	$2186.41	8.14%
9	83	male	$1628.89	$666.51	$2295.40	8.66%
10	84	male	$1628.89	$795.13	$2424.02	9.26%

At the outset, Column 6 of Table 11 shows the ending account balance (be_i) for each of the ten members. For example, member 1's ending account balance (be_1; i.e., his share of the $16,288.90 total fund amount) is $1748.73, computed as follows:

be_1= \$16288.90 × ebe_1 /(ebe_1 + ebe_2 + ebe_4 + ebe_5 + ebe_6 + ebe_8 + ebe_9 + ebe_{10})
= \$16288.90 × \$2077.39/(\$2077.39 + \$2124.75+ \$2240.49 + \$2311.23 + \$2392.66 + \$2597.32 + \$2726.80 + \$2879.59)
= \$1748.73.

Next, column 5 of Table 11 shows the mortality gains and losses (mg_i) for the ten members.[117] For example, member 1's mortality-gain distribution (mg_1) is \$119.84, computed as follows:

$$mg_1 = be_1 - s_1$$
$$= \$1748.73 - \$1628.89$$
$$= \$119.84.$$

Finally, column 7 of Table 11 shows that all eight survivors have higher yields to maturity (y_i) than the 5% yield to maturity on the underlying zero coupon bonds.

The problem with the rough-justice method is that it does not treat all members fairly, at least not exactly. Recall that in Part I.B above, we showed that in a survivor fund with members of different ages, a dying member's account should be distributed to the surviving members in portions that are carefully chosen to provide fair bets for all investors; that is, the distribution should be made according to a fair transfer-plan. We then showed how the date-aligned method—which processes deaths one at a time and in the order that they occur, using a fair transfer plan at each death—provided fair bets for members of different ages.

On the other hand, the rough-justice method—which processes all of the deaths at once without using a fair transfer-plan—injects a small bias that favors certain ages over others.[118] Indeed, that is why we call it the rough-justice method. The bias inherent in the rough-justice method has not been studied extensively and is not well understood.[119] In simulations that we ran (but which are not presented here), we

117. These mortality gains and losses are provided largely for comparison sake. Under the rough-justice method, these mortality gains and losses do not have to be computed in order to get to a member's ending account balance (be_i). Instead, the rough-justice method computes the ending account balances (be_i) directly and finds the mortality-gain distributions indirectly (by subtracting the account value at maturity (s_i) from that member's ending account balance (be_i)).

118. The rough-justice method would also inject a small bias when investors differ in other ways, for example, if they had invested different amounts.

119. As far as we know, there are only two mentions of this bias in the literature: Sabin, *Fair Tontine Annuity*, *supra* note 12, at Appendix I; Catherine Donnelly, *Actuarial Fairness and Solidarity in Pooled Annuity Funds*, 45 ASTIN BULL. 49 (2015).

found that the age bias inherent in the rough-justice method dropped quickly as the number of members in the survivor fund increased, and the bias was negligible once there were at least a few hundred members. Catherine Donnelly performed comparable simulations and reported similar findings.[120] All in all, these findings suggest that in funds with many hundreds or thousands of members (like those run by TIAA-CREF), the bias inherent in the rough-justice method largely disappears.

III. THE TROUBLE WITH MORTALITY TABLES

Mortality *is* what makes a survivor fund a survivor fund. There are several ways that a survivor fund typically takes mortality into account. First, and most obviously, investors who die during the term of the survivor-fund lose their investments. Second, reallocations occur only if at least some investors in the survivor fund actually *do* die during the survivor-fund term. Third, investment companies will inevitably use mortality tables to project the yields to maturity that investors could anticipate.[121] Finally, we note that when an investor in a survivor fund does die, under the date-aligned method we described in Part II above, the balance in her account would be reallocated to the survivors based on fair-transfer-plan weights (w_i) that would be based on the mortality-table death probabilities of all of the members who competed in the survivor fund at the time of that member's death.[122] This Part discusses which mortality tables survivor funds should use when mortality projections are needed.

120. *See* Donnelly, *supra* note 119.

121. *See*, *supra* Part I.

122. *See*, *supra* Part I.B. On the other hand, *infra* Part IV, shows an alternative method for making fair transfer-plan distributions that is based entirely on the *actual* mortality experience of the investors rather than on projections from a mortality table.

A. The Role of Mortality Tables in an Individual-Cohort Survivor Fund

At the outset, consider an individual-cohort survivor fund that consists of *1000* 65-year-old men that each invest $1000 in a 10-year survivor fund that invests in 10-year zero coupon bonds with a 5% yield to maturity. Recall that in Part III, above, we described how to calculate both the projected mortality-gain distribution and the 95%-mortality-gain-distribution range for such an individual-cohort survivor fund. We used the 2010 Social Security area population mortality table in Appendix Table 1 to calculate that each hypothetical 65-year-old man would have a 21.59% probability of dying during the 10-year term of the survivor fund (see row 1, column 2 of Table 12).[123] Accordingly, we would expect that 215.9 of our *1000* 65-year-old men would die before reaching age 75 (see row 2 of Table 12),[124] and that the 784.1 survivors could each expect to receive a projected mortality-gain distribution of $448.51 (see row 5 of Table 12).[125] The actual number of deaths would follow a normal distribution with a mean of 215.9 (see row 2 of Table 12) and a standard deviation of 13.01 (see row 3 of Table 12), and it follows that the 95% range of the actual number of deaths would be from 189.88 to 241.92 (see row 4 of Table 12), for a 95%-mortality-gain-distribution range from $381.79 to $519.81 (see row 6 of Table 12).[126] Finally, the projected yield to maturity for each survivor would be 7.59% (see row 7 of Table 12), and the 95%-yield-to-maturity range would be from 7.23% to 7.95% (see row 8 of Table 12).[127]

123. *See supra* note 53.
124. 215.9 = 1000 × 21.59. *See supra* note 59.
125. $448.51 = ($1,628.89 × 215.9)/784.1. *See supra* note 59.
126. *See supra* note 66.
127. *Id.*

Table 12. Forecast of Outcomes for an Individual-Cohort Survivor Fund, Based on Mortality-Table Death Probability and True Death Probability

	Mortality Table	*True Mortality*
Probability of death	21.59%	15.00%
Mean number of deaths	215.9	150.0
Standard deviation of deaths	13.01	11.29
95% range of deaths	189.88 – 241.92	127.42 – 172.58
Projected mortality gain	$448.51	$287.45
95%-mortality-gain-distribution range	$381.79 – $519.81	$237.86 – $339.75
Projected yield to maturity	7.59%	6.72%
95% y=Yield-to-maturity range	7.23% – 7.95%	6.44% – 7.01%

But what if our *1000* 65-year-old male investors are actually healthier than those in the 2010 Social Security area population mortality table in Appendix Table 1? For example, imagine that the investors' *true* 10-year death probability is only 15.00%, not 21.59% (see row 1, column 3 of Table 12). Then, the projected number of deaths in the group would be only 150.0, not 215.9 (see row 2 of Table 12),[128] and the standard deviation of the number of deaths would be 11.29, not 13.0 (see row 3 of Table 12).[129] The projected 850 survivors could each expect to receive a projected mortality-gain distribution of just $287.45, not $448.51 (see row 5 of Table 12),[130] and their 95%-mortality-gain-distribution range would also be smaller—just $237.86 to $339.75 (see row 6 of Table 12).[131] All in all, the 850 survivors' projected yield to maturity would be just 6.72% (see row 7 of Table 12),[132] and their 95%-

128. 150.0 = 1000 × 15.00%.

129. $11.29159 = (1000 \times 0.1500 \times (1 - 0.1500))^{0.5}$.

130. $287.45 = ($1,628.89 × 150.0)/850.0.

131. Two standard deviations below the mean number of deaths is 127.42 = 150.0 – (2 × 11.29), for a lower mortality-gain distribution of $237.86 = $1628.89 × 127.42/(1000 – 127.42). Two standard deviations above the mean number of deaths is 172.58 = 150.0 + (2 × 11.29), for an upper mortality-gain distribution of $339.75 = $1628.89 × 172.58/(1000 – 172.58).

132. $6.72035\% = ((\$1628.89 + \$287.45)/\$1000.00)^{0.1} - 1$.

yield-to-maturity range would be just 6.44% to 7.01% (see row 8 of Table 12).[133]

This example illustrates the importance of choosing the correct mortality table for an individual-cohort survivor fund, as the accuracy of the forecast of projected investment greatly depends on how accurately the mortality table represents the true probability of death for that particular group of investors. In the hypothetical example in Table 12, the death probabilities from the 2010 Social Security area population mortality table in column 2 of Table 12 overstated the true death probabilities of the actual investors in the survivor fund (21.59% versus 15.00%). As a result, the projected yield to maturity looked more favorable than it truly was (7.59% versus 6.72%), and the 95%-yield-to-maturity range also looked more favorable than it truly was (7.23% to 7.95%, versus 6.44% to 7.01%). A surviving investor in this fund would likely be disappointed with the investment company's projections, as he would have anticipated a yield to maturity of around 7.59%, but he would likely have ended up with a yield to maturity of around 6.72%.

This example shows how important it is for an investment company to choose the correct mortality table—that is, one that can reasonably be expected to match the actual mortality experience of the actual investors in the particular survivor fund offered.

B. The Even Greater Importance of Selecting the Correct Mortality Table when a Survivor Fund Includes Investors of Different Ages

Choosing the correct mortality table is even more important when a survivor fund includes investors of different ages and uses the date-aligned method to reallocate forfeitures. As we saw in Part I.B above, when a survivor fund uses the date-aligned method, the transfer-plan weights (w_i) used to reallocate the accounts of the investors of different ages who

133. 6.44092% = (($1628.89 + $237.86)/$1000.00)$^{0.1}$ – 1; 7.00809% = (($1628.89 + $339.75)/$1000.00)$^{0.1}$ – 1.

die during the term of the survivor-fund would be based on the differing death probabilities (q_i) of those investors. Of course, those death probabilities would be based on the mortality table selected by the investment company that offered the survivor fund. If the survivor fund used a different mortality table, the transfer-plan weights (w_i) used to reallocate the accounts of the investors who died during the term of the survivor-fund would change, and that would change the actual mortality-gain-distribution amounts that the surviving investors would receive. This result is in sharp contrast to what happens with a survivor fund that uses the individual-cohort method: for a survivor fund that uses the individual-cohort method, the mortality table only affects the *projected* mortality-gain distribution to be received by the survivors, not the *actual* mortality-gain distributions that they receive, as the mortality table plays *no* role in how the accounts of investors who die are actually reallocated. In short, when a survivor fund with members of different ages uses the date-aligned method, the mortality table selected affects both the projected mortality-gain distributions and the *actual* mortality-gain distributions that members would receive would receive; and here is a more detailed explanation:

Recall that in Part II above, we described a hypothetical survivor fund in which there were 1000 male investors, 100 at each age from 65 to 74. Each invested $1000 in 10-year zero coupon bonds with a 5% yield to maturity. We ran 1000 simulations of that survivor fund, using the date-aligned method to reallocate the forfeitures of those who died during the 10-year period. In those simulation results, we found that the average mortality-gain distribution received by a survivor was nearly identical to the projected mortality gain that he would have experienced in an individual-cohort fund. We also found that the 95%-mortality-gain-distribution range that a survivor experienced in the simulations was much narrower than the 95%-mortality-gain-distribution range that he would have experienced had his cohort been placed in its own individual-cohort survivor fund. In those simulations, the simulated death times for each investor were randomly picked using probabilities taken from the 2010 Social Security area population mortality table in Appendix Table 1. In other words, we simulated the case where the 2010 Social Security area population mortality table accurately reflected the death probabilities for the investor population. Had we instead simulated a case with a mortality table that was inaccurate for the investor population, for example, by simulating death times for each investor using probabilities taken from a mortality table that assumed that the investor population was healthier (or sicker) than the death probabilities in the 2010 Social Security area population mortality table, the results for each cohort's mortality-gain distributions would have been different. Some cohorts would have received larger mortality-gain distributions than they would have if the fair-transfer-plan weights (w_i) had been based on the true mortality table, and some cohorts would have received smaller mortality-gain distributions.

This illustrates the greater role of mortality tables when the date-aligned method is used to accommodate investors of different ages in a single survivor fund. The date-aligned method allows the fund to claim that each cohort of investors enjoys a better experience than it would if it were placed in its own individual-cohort fund. That is because a cohort's projected mortality-gain distribution would be the same as if it had been placed in its own individual-cohort fund, but the 95%-mortality-gain-distribution range and the 95%-yield-to-maturity range would be much narrower. However, that "better-experience-with-a-date-aligned-survivor-fund" claim is only valid if the mortality table used to calculate the fair transfer-plan weights accurately represents the true death probabilities of the investors in the survivor fund. If the mortality table that is relied upon by a date-aligned survivor fund does not accurately represent the actual death probabilities of its investor population, then some cohorts would do better if placed in an individual-cohort fund, and some would do worse. Pertinent here, the next subpart of this Article discusses what the correct mortality table to use is.

C. What is the Correct Mortality Table to Use?

So far, when we have needed to use a mortality table, we have used the 2010 Social Security area population mortality table in Appendix Table 1. That table is certainly a good starting point as it offers pretty good estimates of the life expectancies and death probabilities of average Americans today. Unfortunately, it probably does not represent the population of people who would actually invest in survivor funds.

We expect that the individuals who would actually buy into survivor funds would likely be healthier and live longer than the average Americans in the 2010 Social Security area population mortality table. Accordingly, survivor funds should use a mortality table that takes that "healthiness" *moral hazard* into account.[134] Pertinent here, we reiterate that U.S. securities laws require that investment companies provide accurate information to investors and that using an incorrect mortality table for a given population of investors would result in inaccurate projected (and advertised) yields to maturity and 95%-mortality-gain-distribution ranges.[135]

In that regard, we know that people who voluntarily purchase annuities tend to live longer than those who do not.[136] As there is every reason to believe that survivor funds would also attract more healthy investors than unhealthy ones,[137] it would be more appropriate to estimate the projected mortality-gain distributions from a survivor fund—and hence the projected yields to maturity—using a mortality table for the

134. *See Moral Hazard*, INVESTOPEDIA, http://www.investopedia.com/terms/m/moralhazard.asp (last visited Nov. 8, 2016) (defining "moral hazard" as "the risk that a party to a transaction has not entered into the contract in good faith, has provided misleading information about its assets, liabilities or credit capacity, or has an incentive to take unusual risks in a desperate attempt to earn a profit before the contract settles").

135. *See supra* note 41 and accompanying text.

136. Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 800-01.

137. People who think they will live a long time would have every reason to invest in a survivor fund, while people who think they are on their death bed would not.

healthy population, rather than a mortality table for the entire population.

While this Article does not offer any suggestions about which specific mortality table would be the correct one for a given survivor fund to use, Appendix Table 3 does compare the death probabilities for males in the 2010 Social Security area population mortality table in Appendix Table 1 with two plausible alternatives. The first set of alternative death probability estimates in Appendix Table 3 is from the Society of Actuaries (i.e., the so-called RP-2014 Mortality Tables for retirement plans),[138] and the second set of estimates is from the National Association of Insurance Commissioners [NAIC]) (i.e., the so-called 2012 Individual Annuity Mortality Period Life [2012 IAM Period] Tables).[139] For example, column 2 of Appendix Table 3 shows that a 65-year-old male in the 2010 Social Security area population had a 1.6% death probability (q_i = 0.015927). On the other hand, column 4 of Appendix Table 3 shows that a *healthy* 65-year-old male annuitant in the Society of Actuaries' RP-2014 mortality table had a death probability of just 1.1% (q_i = 0.011013). Similarly, column 6 of Appendix Table 3 shows that a 65-year-old male in the National Association of Insurance Commissioners' 2012 IAM Period Table had a death probability of just 0.8% (q_i = 0.008106).

All in all, the lower death probabilities associated with healthier investors suggest fewer deaths over the term of any survivor fund and, therefore, lower projected yields to maturity for survivors. Accordingly, in order to provide accurate

138. *RP-2014 Rates; Total Dataset*, SOC'Y OF ACTUARIES (2014), https://www.soa.org/Files/Research/Exp-Study/research-2014-rp-mort-tab-rates.xlsx. *See also RP-2014 Mortality Tables Report* 5 n.2, SOC'Y OF ACTUARIES (Nov. 2014), https://www.soa.org/Files/Research/Exp-Study/research-2014-rp-report.pdf; American Academy of Actuaries Pension Committee, *Selecting and Documenting Mortality Assumptions for Pensions,* AM. ACAD. OF ACTUARIES (June 2015), http://actuary.org/files/Mortality_PN_060515_0.pdf.

139. *NAIC Model Rule for Recognizing a New Annuity Mortality Table for Use in Determining Reserve Liabilities for Annuities,* NAT'L ASS'N OF INS. COMMISSIONERS (Jan. 2013), http://www.naic.org/store/free/MDL-821.pdf [hereinafter *NAIC Model Rule*]. The 2012 Individual Annuity Reserving (IAR) Mortality Tables are designed for use in determining the minimum standard of valuation for individual annuity or pure endowment contracts issued after the effective date of the rule. *Id.* at § 4.D.

information to prospective investors, survivor funds in the real world would need to use mortality tables that would correctly project the actual mortality experience that would be expected from their investor populations.[140]

IV. ANOTHER FAIR DESIGN: THE AGE-ALIGNED METHOD

In Part I.B above we showed how a survivor fund that was open to investors of different ages (and genders) could use the date-aligned method to take age (and gender) into account by using the death probabilities (q_i) from a mortality table to determine the fair-transfer-plan weights (w_i) needed to make fair reallocations. As Part III showed, however, there are real challenges involved in selecting the correct mortality table for the population of investors covered by a survivor fund. In this Part, we show that a survivor fund would not have to rely on a mortality table to reallocate mortality losses. Instead, it could use the so-called "age-aligned method" to base reallocations only on *actual mortality experience*.[141]

140. While life expectancies and death probabilities change over time, this should not be much of an issue for the short-term survivor funds that we discuss in this Article. For long-term funds, however, it might be necessary for survivor-fund designers to build in a rule about whether to stay with the original mortality table or to change mortality tables as newer versions become available. *See, e.g.*, Felicitie C. Bell & Michael L. Miller, Life Tables for the United States Social Security Area 1900-2100, SOC. SEC. ADMIN., 14 fig.4a (2005), http://www.ssa.gov/oact/NOTES/pdf_studies/study120.pdf; Jonathan Barry Forman & Yung-Ping Chen, *Optimal Retirement Age*, N.Y.U. REV. OF BENEFITS & EXEC. COMP. § 14.02 (2008), http://retirement2020.soa.org/Files/washington-forman-opt.pdf (outlining the effect of increased life expectancy on current pension plans). *See also Mortality Improvement Scale MP-2015*, SOC'Y OF ACTUARIES (2015), https://www.soa.org/Files/Research/Exp-Study/research-2015-mp-report.pdf (presenting the Retirement Plans Experience Committees' update to the Mortality Improvement Scale MP-2014 that was released in Oct. 2014).

141. While the age-aligned method would result in different reallocations than those made by the date-aligned method, as this Part will show, the age-aligned method is also based on a fair transfer-plan.

A. How the Age-Aligned Method Works

As an initial matter, again recall that in Table 5 in Part I.B above, we hypothesized that ten men of different ages each invested $1000 ($c_i$) in a survivor fund that invested in zero coupon bonds that matured in ten years and that the value of each man's account at maturity (s_i) was $1628.89. We then used the date-aligned method and the death probabilities (q_i) from the 2010 Social Security area population mortality table in Appendix Table 1 to calculate fair-transfer-plan weights (w_i) that were used to reallocate the accounts of member 3 who died at time 3.33, and then of member 7 who died at time 6.67, and Table 8 showed the final results.

With that reminder about how the date-aligned method worked, we can now use the same hypothetical example to show how to design a survivor fund that would also be fair to members of different ages but that would *not* use information from any mortality table to reallocate forfeitures. We call this the *age-aligned method*, and here is how it works:

Again, nothing happens until the end of the 10-year survivor-fund term. At that time, the survivor fund would again use a fair transfer-plan to reallocate the balances in the accounts of the members who died before time 10 among the survivors. This time, however, we focus on the *age* of each death rather than the *time* of that death. The key feature of the age-aligned method is that it processes deaths one at a time and *from the youngest age of death to the oldest*. The method is easy to apply as, once again, all of the "processing of deaths" occurs at the end of the survivor-fund term (i.e., t = 10), and, at that time, we would know who had died and how old they were when they died.

The key assumption of the age-aligned method is that all members who were alive at any given *age* are assumed to have the same force-of-mortality probability (f_i) *at that age*. We do not make *any* assumption about what the value of that force-of-mortality probability would be; we only assume that it would be the same for all members at that age. As a result, we will see that no mortality table is needed or used.

We again assume that member 3 died at age 70.33 and that member 7 died at age 77.67. It happens that member 3 died at time 3.33 (70.33 = 67 + 3.33) and that member 7 died at time 6.67 (77.67 = 71 + 6.67), but in the age-aligned method, it simply does not matter *when* members 3 and 7 died because the age-aligned method processes deaths based on the *age of death* (not the time of death), and from the youngest age of death to the oldest. Here, for example, we would process the death that occurred at age 70.33 before the death that occurred at age 77.67, *regardless* of which death occurred first and which occurred second.

In short, under the age-aligned method, we first determine the *ages* at which deaths occurred. We then place those deaths in order from the youngest age of death to the oldest, and, finally, we create fair transfer-plans to reallocate the forfeitures that result from those deaths.

In our hypothetical example, deaths occurred at ages 70.33 and 77.67, and the age-aligned method would process those two deaths in that order. First, we would create a fair transfer-plan *but only among the members of the survivor fund who could have died at age 70.33* during the 10-year term of the survivor fund—that is, only for members who actually reached age 70.33 at some time during the 10-year term of the survivor fund, and Table 13 shows the results after the first mortality-gain distribution.

Table 13. Accounting for the First Death, Age-aligned Method

Member (i)	*10-year Age Range*	*Gender (g_i)*	*Account Value at Maturity (s_i)*	*Fair Transfer-plan Weight (wi)*	*Account Balance After the First Distribution ($b1i$)*
1	65-75	Male	$1628.89	0.166667	$1954.67
2	66-76	Male	$1628.89	0.166667	$1954.67
3	67-77	Male	$1628.89	0.166667	$ 0.00
4	68-78	Male	$1628.89	0.166667	$1954.67
5	69-79	Male	$1628.89	0.166667	$1954.67
6	70-80	Male	$1628.89	0.166667	$1954.67
7	71-81	Male	$1628.89	0	$1628.89
8	72-82	Male	$1628.89	0	$1628.89
9	73-83	Male	$1628.89	0	$1628.89
10	74-84	Male	$1628.89	0	$1628.89

Only members 1 through 6 would compete in the first fair transfer-plan; members 7 through 10 would be excluded. Members 1 through 6 were younger than 70.33 at the start of the 10-year term of the survivor fund, and each would be older than 70.33 at the end of the survivor-fund term if he survives. Thus, it is possible that any one of those members 1 through 6 could be the one who died at age 70.33, so each would be included in the first fair transfer-plan. On the other hand, members 7 through 10 were all *older* than 70.33 when the survivor-fund began and so could *not* have died at age 70.33 during the 10-year term of the survivor fund; accordingly, members 7 through 10 would be excluded from the first fair transfer-plan.[142]

142. It happens that member 3 was the one who died at age 70.33, at time 3.33. Readers might be puzzled by the fact that members 1 and 2 had not *yet* reached age 70.33 at time 3.33, yet they were *included* in the first fair transfer-plan. Since the youngest age of death was, in fact, 70.33, it follows that members 1 and 2 *must have* lived past age 70.33 during the survivor-fund term—otherwise the youngest age of death would have been less than 70.33. For example, if member 1 had died at time 4 when he was 69 (69 = 65 + 4), then *his* death would have been the youngest death to occur, and it would have triggered the very first fair transfer-plan (this time, just for members 1 through 5, since these were the only members who were alive at the age of 69 at some time during the 10-year term of the survivor-fund and could have been the ones who died at that age; members 6 through 10 could not have been the ones who died at age 69 because they were all older than 69 when the survivor fund started). The death of member 3 at age 70.33

We assume that all six of the competitors who could have died at age 70.33 (members 1 through 6) were equally likely to be the ones who, in fact, died at that age; that is, we assume that they each had the same force-of-mortality probability at that age.[143] Therefore, we created the first fair transfer-plan by simply using the account values at maturity (s_i) (see column 4 of Table 13) held by these six competitors without any consideration of the particular value of the force-of-mortality probability. From these account values we computed the fair-transfer-plan weights (w_i) which, here, all turn out to be 0.166667 = 1/6 (see column 5 of Table 13). Accordingly, at time 10, when it would be time to divide the $1628.89 in member 3's account among the *five* surviving competitors, each of those five survivors would receive $325.78 = $s_3 \times w_i/(1 - w_3)$ = $1628.89 × 0.166667/(1 − 0.166667),[144] and, of course, member 3 would forfeit his $1628.89. In short, each of the five surviving competitors would get a mortality-gain distribution of $325.78 and would enter the next distribution with an account balance after the first mortality-gain distribution ($b1i$) of $1954.67 (see column 6 of Table 13).[145]

Next, we would create a second fair transfer-plan but, this time, only among the members of the survivor fund who could have died at age 77.67 during the 10-year term of the survivor fund—that is, only for members who actually reached age 77.67 at some time during the 10-year term of the survivor fund, and Table 14 shows the results after the second mortality-gain distribution.

would then trigger the second fair transfer-plan, and the death of member 7 at age 77.67 would trigger a third-and-final fair transfer-plan.

143. To be sure, we understand that the actual force-of-mortality probabilities at any given age can change slightly over time. Over the short, 10-year terms of the survivor funds that we are talking about in this Article, however, we believe that the differences in force-of-mortality probabilities would be relatively insignificant, and, in any event, we have chosen to ignore those differences in this Article.

144. In this case, where all six competing members had the same account balances immediately before the first-death distribution, the result is equivalent to dividing up the decedent's amount equally among the survivors ($325.78 = $1628.89/5), but this will not generally happen (e.g., if the competing members started with different account balances).

145. $1954.67 = $1628.89 + $325.78.

Table 14. Accounting for the Second Death, Age-aligned Method

Member (i)	Age Range	*Gender (g_i)*	*Account Balance After the First Distribution (b1i)*	*Fair Transfer-plan Weight (wi)*	*Account Balance After the Second Distribution (b2i)*
1	65-75	Male	$1954.67	0	$1954.67
2	66-76	Male	$1954.67	0	$1954.67
3	67-dead@70.33	Male	$ 0.00	0	$ 0.00
4	68-78	Male	$1954.67	0.160981	$2255.82
5	69-79	Male	$1954.67	0.160981	$2255.82
6	70-80	Male	$1954.67	0.160981	$2255.82
7	71-81	Male	$1628.89	0.129264	$ 0.00
8	72-82	Male	$1628.89	0.129264	$1870.71
9	73-83	Male	$1628.89	0.129264	$1870.71
10	74-84	Male	$1628.89	0.129264	$1870.71

This time, the competition would be among members 4 through 10, as these would be the only ones who could have died at age 77.67. Member 3 would be excluded because he died before reaching age 77.67. Members 1 and 2 would also be excluded as they would still be *younger* than age 77.67 at the end of the 10-year term of the survivor fund. As we did at the first death, we then computed fair-transfer-plan weights (w_i) (see column 5 of Table 14), again, assuming that each of the seven competitors was *equally likely* to be the one who died at age 77.67. This time, however, the competitors do not have equal balances in their accounts (*b1i*) because some had benefitted from mortality-gain distributions after the first death and some had not (see column 4 of Table 14).[146] Accordingly, when member 7's $1628.89 account value is divided among the *six* surviving competitors in this second fair transfer-plan, some would get more than others, and Column 6 of Table 14 shows the account balances after the second distribution (*b2i*) for these seven competitors.

146. These values come from column 6 of Table 13.

Finally, Table 15 shows the ending account balance (*bei*) for the eight surviving members of this survivor fund (see column 5).[147] Again, the yields to maturity (y_i) for the eight survivors (see column 6 of Table 15) would be significantly higher than the 5% yield to maturity on the underlying zero coupon bonds.

Table 15. The End of a 10-man Survivor Fund with Eight Survivors, Age-aligned Method

Member (i)	*Age (x_i)*	*Gender (g_i)*	*Initial Contribution (c_i)*	*Ending Account Balance (bei)*	*Yield to Maturity (yi)*
1	75	Male	$1000	$1954.67	6.93%
2	76	Male	$1000	$1954.67	6.93%
3	dead@70.33	Male	$1000	$ 0.00	n/a
4	78	Male	$1000	$2255.82	8.48%
5	79	Male	$1000	$2255.82	8.48%
6	80	Male	$1000	$2255.82	8.48%
7	dead@77.67	Male	$1000	$ 0.00	n/a
8	82	Male	$1000	$1870.71	6.46%
9	83	Male	$1000	$1870.71	6.46%
10	84	Male	$1000	$1870.71	6.46%

In short, we used the 10-member fund from Table 5 to show how the age-aligned method works. Like the date-aligned method, the age-aligned method is fair to all ten investors.

With such a small number of investors, however, the results here are somewhat noisy and anomalous. As we saw in column 6 of Table 8, in a survivor fund with members of different ages, older survivors generally get higher yields to maturity than younger survivors; but here, it was "middle-aged" members 4, 5, and 6 who got the highest yields to

147. These values come from column 6 of Table 14.

maturity (see column 6 of Table 15). The reason that the older survivors here did not get higher yields than the younger survivors has to do with our somewhat anomalous assumptions: we only had ten investors and we arbitrarily picked the two investors who died and when they died. In the real world, however, we would expect that a survivor fund using the age-aligned method would have hundreds or thousands of investors. As a result, the actual mortality experience of most survivor funds would be much closer to the mortality experience of the general population. Accordingly, older survivors in such large survivor funds would, in fact, get higher yields to maturity than younger survivors.[148]

B. The Age-Aligned Method is Also Quite Practical

Like the date-aligned method, the age-aligned method is quite practical. A survivor fund could use the age-aligned method to include investors with different ages, genders, and investment levels. That again means that an investment company could succeed in offering just a few survivor funds each month (or quarter), as we saw with the date-aligned method.[149] Again, it turns out that, for any given number of investors, the age-aligned method would generate more predictable outcomes than the individual-cohort method, although the outcomes would not be quite as predictable as they were with the date-aligned method. We again have to delve into the world of statistics to explain this phenomenon.

Recall that in Part III above, we described how to calculate the 95%-mortality-gain-distribution range for a survivor fund that used the individual-cohort method. We posed a

148. In that regard, both Table 10 and Appendix Table 1 show that older individuals have higher death probabilities than younger individuals. For example, Table 10 suggests that while 78.4% of 65-year-old men would live ten years to age 75, just 56.6% of 74-year-old men would live ten years to age 84. As survivor funds generally make larger mortality-gain distributions to survivors who have higher death probabilities, older survivors should do better than younger survivors.

149. *See supra* Part II.

hypothetical example in which *100* 65-year-old men each invested $1000 in a survivor fund that held 10-year zero coupon bonds with a 5% yield to maturity and, in which, the mortality rates of those 100 men were taken from the 2010 Social Security area population mortality table in Appendix Table 1. We then calculated that: 1) the projected mortality-gain distribution would be $448.49[150] and 2) there would be a 95% probability that the actual mortality-gain distribution that a survivor receives will be between $251 and $692.[151] We also explained that the 95% mortality-gain-distribution range would be narrower if the survivor fund had a larger number of investors, but because of the requirement that all investors in that individual-cohort survivor fund had to be 65-year-old males willing to invest exactly $1000, we thought that it would be difficult to attract a greater number of identical investors.[152]

On the other hand, if a survivor fund instead used the age-aligned method, it could be open to investors of any age, gender, or investment level, and that would make it easier to attract a large number of investors. With more investors, the 95%-mortality-gain-distribution range would be narrower. Unfortunately, it is once again not feasible to calculate the 95% range for a survivor fund that uses the age-aligned method when we have investors of differing ages, genders, or investment levels; the analytic formulas are hopelessly complex, just as they were with the date-aligned method.[153] However, it is again possible to simulate our hypothetical survivor fund and measure the 95%-mortality-gain-distribution range that would result from repeated simulations, so that is again what we did instead.[154]

As before,[155] we simulated a simple survivor fund with 1000 male investors, 100 at each age from 65 through 74. Each man invested $1000 in a survivor fund that invested in 10-year zero coupon bonds with a 5% yield to maturity, and we again used the mortality rates implied by the 2010 Social Security

150. *See supra* note 99 and accompanying text.
151. *See supra* notes 61-62 and accompanying text.
152. *See supra* discussion accompanying Figure 4.
153. *See supra* Part II.
154. *Id.*
155. *Id.*

area population mortality table in Appendix Table 1. We again ran 1000 simulations. This time, however, we used the age-aligned method to reallocate the mortality losses of those who died during the 10-year, survivor-fund terms of those 1000 simulations, and the results for all ten age cohorts are shown in Appendix Table 2B. For the 65-year-old men, the average mortality-gain distribution over the simulation runs was $449.53 (see column 6 of Appendix Table 2B), which is again very close to the projected mortality gain of $448.49.[156] The 95%-mortality-gain-distribution range for these men over the simulation runs was from $348 to $560 (see columns 7 and 8 of Appendix Table 2B). This is a much narrower range than the 95% range from $243 to $698 that we calculated would be experienced by *100* 65-year-old-men if the individual-cohort method were used. However, it is not as narrow as the 95% mortality gain distribution range that they experienced in the simulation under the date-aligned method (from $395 to $505).

Similarly, the age-aligned method would also result in a narrow 95%-yield-to-maturity range (see columns 11 and 12 of Appendix Table 2B). For example, while both the age-aligned and individual-cohort survivor funds would produce projected yields to maturity for 65-year-old men of 7.58% (see column 10 of Appendix Table 2B),[157] the multi-cohort age-aligned survivor fund would do so with much less variance per investor. For example, while the *100* 65-year-old men who invested in our hypothetical 1000-man, multi-cohort age-aligned survivor fund experienced a 95%-yield-to-maturity range from 7.1% to 8.2% in our simulations (see columns 11 and 12 of Appendix Table 2B), we calculated that the *100* 65-year-old men who invested in the 100-man individual-cohort survivor fund in Part I above would have experienced a significantly larger 95%-yield-to-maturity range of from 6.5% to 8.8%.

The equivalent-individual-cohort size again shows how much less variance the age-aligned method produces than the individual-cohort method.[158] And again, it is fairly easy to calculate the equivalent-individual-cohort size for each cohort

156. *See supra* note 99 and accompanying text.
157. *See supra* note 109 and accompanying text.
158. *See supra* Part IV.

in our age-aligned-method simulation results. These results are listed in column 9 of Appendix Table 3B. This time, for the *100* 65-year-old men in our age-aligned-method simulation, the equivalent-individual-cohort size is 426. Similarly, for the 100 age-69 men, the equivalent-individual-cohort size is 761, and for the *100* 74-year-old men it is 241. Thus, this time the advantage is greatest for the *100* 69-year-old men—if they were removed from the multi-cohort age-aligned survivor fund and placed in an individual-cohort survivor fund, the number of investors would have to increase by a factor of about 7.6 for those 100 65-year-old men to enjoy a 95%-mortality-gain-distribution range as narrow as they would experience with the age-aligned fund.[159] That would be a very large increase in the required number of investors. The advantage of the age-aligned method is smaller for ages other than 69, because men of those ages would be likely to have fewer competitors in their fair transfer-plans than the age-69 men.[160]

Summing the equivalent-individual-cohort sizes for all the cohorts works out to a total of 5666—about 5.7 times more than the 1000 investors we would have in the single, multi-cohort age-aligned fund.[161] This illustrates the case for using the age-aligned method rather than the individual-cohort method.[162]

For now, it is enough to say that the age-aligned method is a very interesting and fair way to design a survivor fund. Accordingly, investment companies that want to offer survivor funds should seriously consider using the age-aligned method.

159. 7.61 = 761/100.

160. The advantage is least for the 74-year-old men—they would need to increase the number of investors by a factor of just 2.4 if they moved to the individual-cohort method. 2.41 = 241/100.

161. 5.67 = 5666/100.

162. To be sure, we could do even better using the date-aligned method (i.e., have even less variance), but then we would have to rely on a mortality table that may, or may not, accurately reflect our investor population.

V. SURVIVOR FUNDS COULD WORK WITH ALL TYPES OF INVESTMENTS

The simple 10-year survivor funds that we have talked about so far have all been invested in zero coupon bonds that had a 5% yield to maturity. In this Part, we show how survivor funds could work with alternative investments.

A. How Variations in Market Interest Rates Affect Survivor Fund Yields to Maturity

At the outset, however, we consider how variations in the rate of return on the underlying assets in a survivor fund would affect the yield to maturity for the surviving members. In that regard, Table 16 builds on the survivor fund that we described in Part 2 above. That is, Table 16 assumes that: 1) *1000* 65-year-old men each invested $1000 in a survivor fund that bought $1,000,000 worth of zero coupon bonds, and 2) that just 800 of the original 1000 investors survived ten years until the bonds matured.

Table 16. How Variations in Market Interest Rates Affect Survivor Fund Yields

Zero Coupon Bond Yield to Maturity	*Survivor Fund Yield to Maturity*	*Increase in Yield to Maturity*	*Percentage Increase in Yield to Maturity*
3%	5.32%	2.32%	77%
5%	7.37%	2.37%	47%
7%	9.41%	2.41%	34%
10%	12.48%	2.48%	25%

Basically, Table 16 shows how the yield to maturity of a survivor fund would vary depending on the yield to maturity for the underlying investments. As we showed in Part 2 above, for zero coupon bonds with a yield to maturity of 5% (see row 2, column 1 of Table 16), each of the 800 survivors would get around $2036, $407 more than they would have gotten if the $1,628,894.63 maturity value had been divided among all 1000 original investors (or their heirs), and that translated into a yield to maturity of 7.37% (see row 2, column 2 of Table 16).[163] That is a 2.37% increase in the yield to maturity for the survivors (see row 2, column 3 of Table 16).[164] Basically, column 3 shows that all of the yields to maturity in Column 2 are *around* 2.4% larger than the yields to maturity on the underlying zero-coupon-bond investments in column 1.

In that regard, however, Column 4 of Table 16 reveals an especially interesting relationship between the yield to maturity on zero coupon bonds and the yield to maturity on the survivor funds that hold them. We have already seen that the yield to maturity for the survivor fund that invested in 5% zero coupon bonds was 7.37% for the survivors and that that 7.37% yield to maturity was 47% higher than the 5% yield to maturity yield to maturity on the underlying zero coupon bonds.[165] Column 4 shows that 47% result, and it also shows the percentage increases in yields to maturity for survivor funds that hold bonds with higher and lower yields to maturity. Looking at the entries in columns 1 and 4 of Table 16 reveals a very interesting pattern: the lower the yield to maturity on the underlying zero coupon bonds, the greater the *percentage increase* in yield to maturity for the survivor funds that holds them. The implication is that the survivor funds are likely to be especially attractive during periods of low interest rates, due to the relatively large percentage increase in yield to maturity from investing in a survivor fund that holds low-interest-rate bonds as opposed to just investing directly in those low-interest-rate bonds.[166]

163. *See supra* notes 26-30 and accompanying text.

164. *See supra* note 31.

165. *See supra* note 33.

166. We note, in passing, that now is, in fact, a period of historically low interest rates. *See, e.g.*, Neil Irwin, *Why Very Low Interest Rates May Stick Around*, N.Y. TIMES (Dec. 14, 2015), http://www.nytimes.com/2015/12/15/upshot/why-very-low-interest-rates-may-stick-around.html.

B. Survivor Funds Could Work With Almost Any Kind of Investment

1. Survivor Funds Could Work with Almost Any Kind of Bond

While the simple survivor funds that we have talked about so far were designed to invest in zero coupon bonds and to make a single distribution of the proceeds at maturity, a survivor fund could instead invest in other types of bonds.

For example, instead of investing in a zero coupon bond, a survivor fund could invest in a regular coupon bond that made periodic interest payments which the survivor fund would promptly distribute.[167] For example, imagine a survivor fund that buys a single $1,000,000 regular coupon bond with ten annual interest coupons attached, each paying $50,000 (i.e., 5%).[168] As in the case of the "single-payment" survivor funds that we have talked about so far, with a "periodic-payment" survivor fund, at maturity the balance in the survivor fund would be divided among the survivors. The difference here is that with a periodic-payment survivor fund, there would also be periodic distributions along the way; that is, a periodic-payment survivor fund would promptly divide and distribute the periodic interest payments as soon as they are received.

Those $50,000-per-year periodic interest payments could be divided among all of the members of the fund, living or not. Alternatively, those periodic interest payments could be divided but only among the members who were alive on the date that the survivor fund received each particular interest payment.

167. We use the term "regular coupon bond" to mean a bond that pays interest periodically over the course of its life, as opposed to a zero coupon bond that only makes a single payment at the end of the bond term.

168. Of course, a bond could have interest coupons that were paid out semi-annually, quarterly, or even monthly. *See How does the money from the interest on my bond get to me?,* INVESTOPEDIA, http://www.investopedia.com/ask/answers/174.asp (last visited Nov. 8, 2016).

We believe that most investors would prefer the first approach—a periodic-payment survivor fund that would: 1) divide the first nine years of interest coupon payments among all members, living or not, and 2) divide the final interest coupon payment and the redemption principal among those that survived all ten years.[169] For example, our hypothetical 10-year, $1,000,000 regular coupon bond would divide the first nine $50,000 interest payments equally among the 1000 original investors (or their heirs), with each receiving $50 at the end of years 1 through 9. At the end of year 10, however, the final $50,000 interest payment and the original $1,000,000 investment would be divided equally among those members who survived all ten years. For example, if we again assume that only 800 of the original 1000 investors survive ten years (as we did in Part 2 above), each investor (or her heirs) would receive $50 at the end of each of the first nine years, and each of the *800* 10-year survivors would receive $1312.50 at the end of year 10 (with the heirs of the 200 decedents receiving nothing at time 10).[170] All in all, the survivors in our periodic-payment-survivor-fund example would earn a yield to maturity of 6.91%,[171] which turns out to be somewhat less than the 7.37% yield to maturity earned by investors in the zero-coupon-bond-fund example in Part 2 above.[172] This slightly-lower yield to maturity reflects the fact that the survivors here would not receive any mortality gains on the first nine interest coupon payments, as the decedents (or their heirs) already each received the full $450 from those nine $50 payments.[173]

169. In other words, only the redemption principal and the final coupon payment would be at risk of forfeiture, and survivors would earn a mortality-gain distribution only on that amount. There would be no risk of forfeiture of nine coupon payments made prior to redemption time, nor would any mortality-gain distributions be earned on them.

170. $1312.50 = ($50,000 + $1,000,000)/800.

171. Authors' calculations use the TIME VALUE OF MONEY CALCULATOR, http://www.zenwealth.com/businessfinanceonline/TVM/TVMCalculator.html (–$1000 present value, $50 annual payment, ten periods, $1262.50 future value; results in a nominal interest rate of 6.91%; $1262.50 = $1312.50 – $50.00).

172. *See supra* note 31 and accompanying text.

173. $450 = 9 × $50. In passing, we note that had we calculated the yield to maturity for the 200 decedents, it would have been a negative percentage as the $450 that each received is less than her original $1000

Alternatively, a periodic-payment survivor fund could divide each year's interest coupon payment, but only among those members who were alive at the time that the fund received that payment. This would increase the mortality-gain distributions received by the surviving members. However, this alternative approach would increase the cost of operating the periodic-payment survivor fund, as it would require the fund to process death records at the end of *each* year, rather than just once at the end of the tenth year.[174] We suspect that the additional expense of taking actual mortality experience into account each year would outweigh any perceived benefits from doing so. Moreover, we believe that most investors would prefer the periodic-payment survivor fund that we first described—that is, one that would effectively guarantee a nine-year income stream even if they do not live that long.[175]

investments.

174. For example, assume that our *1000* 65-year-old investors had an actual mortality experience that mimicked that of the 65-year-old males in the 2010 Social Security area population mortality table in Appendix Table 1. That table suggests that, of the 100,000 live male births, 80,729 would have made it to age 65, 79,444 of them would make it to age 66, 78,064 would make it to age 67 and so on until just 63,300 would make it to age 75. A fair-transfer plan could be used to divide the $50,000 annual interest payments—and the $1,000,000 principal sum—among those who survived to the relevant ages.

175. Having a definite, future income stream can often make financial planning easier. For example, when an investor has a home with a mortgage, she may want her heirs to have a definite source of income to cover the mortgage payments for a certain number of years, even after her death. Consider what happens if our investor is also a member of a periodic-payment survivor fund that holds a coupon bond that makes annual distributions to all members or their heirs. If she lived the full ten years, she would get $50 a year, and she would get $1312.50 at the end of year ten (her original $1000.00 investment plus a $312.50 mortality gain). *See supra* note 170. If she died, for example, in year 7, her heirs would at least get the $50 distributions at the end of years 7, 8, and 9, and those distributions would help them make the often difficult financial transition that accompanies the death of a loved one. *See, e.g.*, Mark Colgan, *Surviving a Loss: Financial Planning for Widows and Widowers*, AM. ASS'N. OF INDIVIDUAL INV., http://www.aaii.com/journal/article/surviving-a-loss-financial-planning-for-widows-and-widowers (last visited Nov. 8, 2016).

2. Survivor Funds Could Invest in Stocks or Other Assets, and Fees Could Be Very Low

So far, to keep it simple, we have discussed only survivor funds that invest in bonds, but survivor funds could easily be designed to invest in large cap stocks, small cap stocks, global equities, real estate, commodities, hedge funds, or other alternative investments.[176] Although these survivor funds would provide investors with mortality-gain distributions, they would not, strictly speaking, be insurance products.[177] Accordingly, these survivor funds could be managed by mutual-fund houses and discount brokers, and no money would have to be set aside for insurance agent commissions or insurance company reserves or risk-taking.[178] In particular, we think that a survivor fund could be designed around families of mutual fund, like those currently offered by Fidelity,[179] Schwab,[180] and Vanguard.[181]

To be sure, a mutual-fund house or discount broker would have expenses associated with managing a survivor fund. In particular, there would be the one-time, administrative expenses of processing death records at the end of the survivor-fund term, but we believe that these administrative expenses would be very small compared to the much higher fees that

176. Although it is beyond the scope of this Article, we believe that it is possible to design a survivor fund that would allow members to direct their own investments, just as people often do today with their self-directed 401(k) plans and IRAs and with their mutual fund and brokerage accounts. *See, e.g.*, Chris Gay, *Some 401(k) Plans Let You Take the Wheel—If You Dare*, U.S. NEWS (June 18, 2012, 9:30 AM), http://money.usnews.com/money/personal-finance/mutual-funds/articles/2012/06/18/some-401k-plans-let-you-take-the-wheelif-you-dare.

177. *See, e.g.*, Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 818-22. *See also infra* Part VI.A.

178. This discussion follows Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 796-97.

179. *See Mutual Funds*, FIDELITY, https://www.fidelity.com/mutual-funds/overview (last visited Nov. 8, 2016).

180. *See Mutual Funds*, CHARLES SCHWAB, http://www.schwab.com/public/schwab/investing/accounts_products/investment/mutual_funds (last visited Nov. 8, 2016).

181. *See Investment products*, VANGUARD, https://investor.vanguard.com/investing/investment-products (last visited Nov. 8, 2016).

insurance companies charge for their products.[182] For example, imagine a survivor fund that invested entirely in an S&P 500 stock index fund. We know that many discount brokers offer an S&P 500 index fund with expense ratios of 0.10% or less,[183] and we believe that the management and record-keeping functions associated with a survivor fund could be performed for no more than 0.20% of assets under management.[184] That means that the total costs of running a survivor fund invested in an S&P 500 stock index fund could be less than 0.30% of assets under management.

The point here is simply that it would be possible for a survivor fund to hold stocks or mutual funds, as opposed to bonds, and for it to be offered by a mutual-fund house or discount broker at a cost that is advantageous to investors. Similar logic suggests that it would also be possible to design low-cost survivor funds that invested in real estate, private equity, commodities, or other alternatives.

VI. SOLVING THE TECHNICAL PROBLEMS OF CREATING SURVIVOR FUNDS

A. The Regulation of Survivor Funds

Just how survivor funds would be regulated is a matter of some importance. To the extent that survivor funds are viewed as securities, they would, of course, be subject to state and federal securities laws such as those administered by the

182. Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 796 (footnote omitted) ("For example, a recent Morningstar survey of 2037 variable annuities showed an average administrative fee in 2014 of 1.33% of assets under management, and that fee is on top of the cost of managing the underlying investments, which itself can easily run another 1.0%."). A "variable annuity" is an annuity that offers a range of investment options. Accordingly, the value of the annuity and the monthly payments will vary depending on the performance of the underlying investments. *See*, *e.g.*, *Variable Annuities: What You Should Know*, U.S. SEC. & EXCH. COMM'N (Apr. 18, 2011), http://www.sec.gov/investor/pubs/varannty.htm.

183. *See, e.g.*, *Fidelity 500 Index Fund—Investor Class*, FIDELITY, https://fundresearch.fidelity.com/mutual-funds/summary/315911206 (last visited Nov. 8, 2016) (offering a 0.10% gross expense ratio; i.e., 0.10% of assets under management).

184. *See, e.g.*, Forman & Sabin, *Tontine Pensions*, *supra* note 6, at 797.

Securities and Exchange Commission (SEC).[185] The more interesting question is whether survivor funds might also be subject to regulation by state insurance departments.[186] To be sure, survivor funds look a lot like annuities, and, in general, companies offering annuities are subject to a comprehensive regulation by state insurance departments.[187]

Like annuities, survivor funds would make payments that depend on mortality experience: for example, investors who die forfeit the balance in their account and do not receive any further payments or distributions.[188] Unlike typical annuities, however, survivor funds do not make any *guarantees*. In a survivor fund, all of the risks fall on the investors, whereas with a typical annuity, an insurance company bears the risk of making certain guaranteed payments. Because insurance companies typically bear such risks, they are heavily regulated, and they must maintain adequate reserves.[189] On the other hand, the investment companies (e.g., mutual-fund houses and discount brokers) that would sell survivor funds would bear *no* risk, and they would not need to maintain any reserves. In our view, then, survivor funds are *not* annuities and should not be subjected to regulation by state insurance departments, but we

185. *See supra* note 44 and accompanying text.

186. Both the SEC and state insurance departments regulate so-called variable annuities; however, the SEC does not view fixed annuities as securities and so does not regulate them. *Annuities*, U.S. SEC. & EXCH. COMM'N (Apr. 6, 2011), http://www.sec.gov/answers/annuity.htm.

187. *See, e.g.*, *State Regulation of Annuities*, INSURED RETIREMENT INST., http://www.irionline.org/government-affairs/annuities-regulation-industry-information/state-regulation-of-annuities (last visited Nov. 8, 2016).

188. *See, e.g.*, the discussion in Part III.

189. The National Association of Insurance Commissioners Model Standard Valuation Law generally requires insurance companies to maintain annuity reserves according to the Commissioners' Annuity Reserve Method (CARVM). *See, e.g.*, *Special Issues for Variable Annuities*, AM. ACAD. ACTUARIES (1999), https://www.actuary.org/files/publications/Practice_Note_Special_Issues_for_Variable_Annuities_july1999.pdf; Standard Valuation Law 820, §§ 5a, 6 (2010), http://www.naic.org/store/free/MDL-820.pdf. *See also* Kush Kotecha, Ben Yahr & James Collingwood, *Statutory Reserving for Fixed Indexed Annuities with Guaranteed Lifetime Withdrawal Benefits*, FIN. REP., at 3 (Sept. 2012) https://www.soa.org/library/newsletters/financial-reporter/2012/september/frn-2012-iss90-kotecha.aspx; Keith P. Sharp, *Commissioners Annuity Reserve Valuation Method (CARVM)*, 7 J. ACTUARIAL PRAC. 107 (1999), http://www.jofap.org/documents/vol7/v7_verall.pdf.

can well imagine that some states might dispute that conclusion. The issue could be especially important to the states that impose insurance premium taxes on annuities.[190]

Along the same lines, we also imagine that some states might view survivor funds as violating their gambling laws. Really, however, a survivor fund is not all that different from several other forms of investment that are never considered to be gambling. For example, property law has long recognized that property can be held in a joint tenancy with a right of survivorship.[191] For example, if two people hold a joint bank account with a right-of-survivorship feature, while both are alive, they share the account, but when one dies, the survivor inherits the other member's share.[192] The law of intestacy has also long recognized the use of wills and trusts to divide assets among survivors.[193] Here again, these legal forms often make the division of property contingent on survival, but no one thinks of them as gambling.[194] The survivor funds described in this Article are not all that different, and they also should not be viewed as violating state gambling laws.

All in all, while we readily admit that survivor funds combine features of an annuity and a lottery,[195] we do not believe that survivor funds should be regulated by state insurance departments or barred by state gambling laws.

190. *See, e.g.*, Darla Mercado, *Beware State Premium Taxes on Annuities*, INVESTMENTNEWS (June 18, 2014, 12:01 AM), http://www.investmentnews.com/article/20140618/FREE/140619928/beware-state-premium-taxes-on-annuities; Hersh Stern, *State Premium Tax*, IMMEDIATE ANNUITIES (Oct. 15, 2016), https://www.immediateannuities.com/state-premium-tax/.

191. *See, e.g.*, *Joint Tenancy*, THE FREE DICTIONARY, http://legal-dictionary.thefreedictionary.com/Joint+Tenancy (last visited Nov. 9, 2016).

192. *Id.*

193. *See, e.g.*, *What Happens If You Die Without a Will?*, FINDLAW, http://files.findlaw.com/pdf/estate/estate.findlaw.com_wills_what-happens-if-i-die-without-a-will-.pdf (last visited Nov. 9, 2016).

194. Still other mechanisms for designing survivor funds should be considered. *Cf.,* Druce Vertes, *Tontines: Strange Name, Great Idea for Retirement (So Good They're Illegal)*, HUFFINGTON POST (Oct. 1, 2015, 3:25 PM), http://www.huffingtonpost.com/druce-vertes-cfa/tontines-strange-name-gre_b_8227082.html (suggesting that renegade entrepreneurs should use a Kickstarter-like website for tontines and "dare the authorities to shut it down").

195. *See supra* note 6 and accompanying text.

Instead, survivor funds should be regulated just like other securities and mutual funds.

B. The Tax Treatment of Survivor Funds

Survivor funds also raise a variety of important tax questions. While virtually all income is taxed under the federal income tax, as more fully explained below, different tax rules apply to interest income, gains, and annuities. The question here is just which set of rules should apply to survivor funds.

1. Taxing Survivor Funds Like Other Market Investments

In general, income is taxed when it is received.[196] When it comes to interest income, however, it is usually taxed when it is earned.[197] For example, interest in a bank savings account is taxed in the year that it is earned and credited to an investor's savings account rather than when it is withdrawn.[198] Also, when the interest rate on a bond is below the market interest rate, so-called "original issue discount" rules apply.[199] These rules generally require the owner of the bond to report the

196. 26 U.S.C. §§ 61, 451 (2012).

197. INTERNAL REVENUE SERV., DEP'T OF TREASURY. PUBLICATION NO. 17, TAX GUIDE 2015 62-63 (2015), https://www.irs.gov/pub/irs-pdf/p17.pdf [hereinafter INTERNAL REVENUE SERV.].

198. The Tax Guide 2015 states:

> Constructive receipt. You constructively receive income when it is credited to your account or made available to you. You do not need to have physical possession of it. For example, you are considered to receive interest, dividends, or other earnings on any deposit or account in a bank, savings and loan, or similar financial institution, or interest on life insurance policy dividends left to accumulate, when they are credited to your account and subject to your withdrawal. This is true even if they are not yet entered in your passbook.

Id. at 63.

199. *See, e.g.*, *id.* at 62; INTERNAL REVENUE SERV., DEP'T OF TREASURY, PUBLICATION NO. 1212, GUIDE TO ORIGINAL ISSUE DISCOUNT (OID) INSTRUMENTS (2014), https://www.irs.gov/pub/irs-pdf/p1212.pdf.

interest as it "accrues" over the term of the bond.[200] For example, when an investor buys a zero coupon bond, she cannot usually wait until the end of the bond term to report the interest income: in most cases, a portion of the interest must be reported as income each year.[201] Pertinent here, if an investor bought a 10-year zero coupon bond with a 5% yield to maturity for $1000 and later received $1629 at maturity,[202] she would have to report a portion of that $629 of interest each year rather than reporting all $629 of interest income when it is actually received at the end of ten-year bond terms.

A different set of rules applies to gains and losses that result from investments. Gains on investments are typically taxed only when they are realized at a sale or exchange.[203] For example, if an investor bought some stock for $1000 at time 0 and then sold it for $1629 ten years later at time 10, the entire

200. INTERNAL REVENUE SERV., *supra* note 197, at 62.

201. *See, e.g.*, *STRIPS*, TREASURY DIRECT (Mar. 6, 2015), https://www.treasurydirect.gov/instit/marketables/strips/strips.htm ("Income must be reported even though it is not received until maturity or the STRIPS are sold."); *U.S. Treasury Securities*, RAYMOND JAMES, http://www.raymondjames.com/fixed_income_us_treasury_bonds.htm (last visited Nov. 9, 2016); Tim Plaehn, *Tax on US Treasury STRIPS*, ZACKS, http://finance.zacks.com/tax-treasury-strips-7266.html (last visited Nov. 9, 2016).

202. *See, e.g.*, *supra* note 25 and accompanying text.

203. 26 U.S.C. §§ 61, 1001 (2012). To be sure, it could make sense to tax many gains on an annual basis, rather than waiting until the property is sold (and the gain is "realized"). For example, with respect to publicly-traded stocks and bonds, there is no reason why they could not be valued at the end of each year. Taxpayers could then report their gains and losses on an annual basis, and the Treasury would collect billions more in taxes. This is the so-called "mark-to-market" approach to taxing accrued gains. *See, e.g.*, David S. Miller, *A Progressive System of Mark-to-Market Taxation*, 109 TAX NOTES 1047 (2005); Nohel B. Cunningham & Deborah H. Schenk, *Colloquium on Capital Gains: The Case for a Capital Gains Preference*, 48 TAX L. REV. 319 (1993); Jeff Strnad, *Periodicity and Accretion Taxation: Norms and Implementation*, 99 YALE L.J. 1817 (1990); David J. Shakow, *Taxation Without Realization: A Proposal for Accrual Taxation*, 134 U. PA. L. REV. 1111 (1986). As for nonpublicly-traded assets, a deemed mark-to-market regime could be used to "impose a tax that would leave the taxpayer with the after-tax amount that would have resulted had the asset appreciated constantly at the pre-tax yield, the taxpayer had been taxed annually on a mark-to-market basis, and the taxpayer had sold enough of the asset to pay the tax." David S. Miller, *A Comprehensive Mark-to-Market Tax for the 0.1% Wealthiest and Highest-Earning Taxpayers*, 4 (Jan. 4, 2016), http://papers.ssrn.com/abstract=2710738.

$629 of gain would be reported in year 10. Moreover, as stock is usually a "capital asset," the gain would be taxed at a reduced "capital gains" rate.[204] On the other hand, losses are usually deductible.[205] For example, if an investor bought some stock for $1000 at time 0 and sold if for $400 ten years later, she could typically deduct a $600 capital loss in year 10.[206] Similarly, if her investment became worthless at some point, say, because the underlying company went out of business, she could probably deduct all $1000 in the year her stock became worthless.[207]

All in all, the simplest approach might be to tax survivor funds like most other investments in which there are gains and losses. For example, if an individual invested $1000 in a survivor fund and ten years later collected $2036, she would have no income until year 10, and then she would have $1036 of income in Year 10. On the other hand, if she died before year 10 and so forfeited her $1000 survivor-fund investment, she could be allowed to deduct her $1000 loss in the year of her death.[208] Moreover, survivor funds could be treated as capital assets, in which case, the favorable capital gains rates would apply to the gains, while the usual limitations on capital losses would apply to any losses.[209]

On the other hand, a powerful argument can be made that survivor funds should instead be taxed like annuities. The income tax system generally provides fairly favorable tax treatment for investments in annuities.[210] Although the value of an annuity investment grows over time, no tax is imposed until annuity distributions commence. In short, there is no tax on the so-called "inside buildup" until the "annuity starting

204. 26 U.S.C. §§ 1(h), 1221 (2012).

205. 26 U.S.C. §§ 165(a)-(c), (2)(f); 1001(a); 1211(b) (2012).

206. For individuals, capital losses are typically deductible to the extent of capital gains, plus as much as another $3000 of capital losses each year. *See* 26 U.S.C. § 1211(b).

207. *See* 26 U.S.C. § 165(g) (2012).

208. As her investment was $1000, her "basis" for computing her loss would be $1000. *See* 26 U.S.C. § 1012.

209. *See supra* note 206 and accompanying text.

210. *See* 26 U.S.C. § 72 (2012). *See also* INTERNAL REVENUE SERV., DEP'T OF TREASURY, PUBLICATION NO. 575, *Pension and Annuity Income* (2016), https://www.irs.gov/pub/irs-pdf/p575.pdf.

date."[211] Even then, the annuitant can exclude a fraction of each benefit payment from income. That fraction (the "exclusion ratio") is based on the amount of premiums or other after-tax contributions made by the individual.[212] The exclusion ratio enables the individual to recover her own after-tax contributions tax free and to pay tax only on the remaining portion of benefits which represents income. For example, if an individual invested $1000 in an annuity and ten years later received an annuity distribution of $2036, she would have no income until year 10, and then she would have $1036 of income in Year 10. That $1036 would be ordinary income, not eligible for preferential capital gains rates.[213]

Like investors in annuities, surviving investors in survivor funds would receive both mortality gains and interest.[214] For example, in Part 2 above, we assumed that *1000* 65-year-old men each invested $1000 in a 10-year survivor fund that invested in a 10-year zero coupon bond with a 5% yield to maturity, and we further assumed that 800 of them made it to age 75—that is, 200 died. As a result, each of the 800 survivors got $2036 on their $1000 investments, rather than just $1629. In effect, each survivor got $629 in interest and got a mortality gain of $407. Theoretically, it might make sense to tax the interest income as it is earned (applying the original issue discount rules), while the mortality gain could instead be taxed in year 10, perhaps even at a reduced rate as a capital gain. On the other hand, survivor funds are so much like annuities that we believe that they should probably be taxed like annuities. That is, income taxation should be deferred until distributions are made at the end of the survivor-fund term.

To be sure, elsewhere, one of us (Forman) has argued for repeal of the favorable tax treatment of annuities;[215] however,

211. *See, e.g.*, 26 U.S.C. § 72(c)(4) (2012); *The Tax Expenditure for Life Insurance Inside Buildup*, U.S. DEP'T OF THE TREASURY, OFFICE OF TAX ANALYSIS (Sept. 28, 2016), https://www.treasury.gov/resource-center/tax-policy/tax-analysis/Documents/Life-Insurance-Inside-Buildup.pdf.

212. *See* 26 U.S.C. § 72(b) (2012).

213. On the other hand, if you lose money on an annuity, the capital loss limitation rules in 26 U.S.C. § 1211(b) do not apply.

214. On the other hand, investors in survivor funds who die before the end receive nothing.

215. Jonathan B. Forman, *Reconsidering the Tax Treatment of Pensions*

as long as the federal tax system provides favorable tax treatment for annuities, we see no reason why these same rules should not also be applied to survivor funds.[216]

2. The Tax Treatment of Companies Administering Survivor Funds

The tax treatment of the companies selling survivor funds is also relevant. For income tax purposes, insurance companies typically report all the premiums and other income that they receive and deduct their payouts.[217] Trusts also include the income that they receive, but they can typically deduct the amounts that they distribute to beneficiaries.[218] Similar rules apply to mutual fund companies.[219] All of this suggests, that survivor funds may present some important tax issues for the companies that sell them. If those companies postpone making distributions, say, for the full 10-year term of a survivor fund, then presumably, the government will want to tax those companies on the income from the underlying investments as that income is earned and delay the deduction for distributions until those companies actually make those distributions.

and Annuities, 18 CHAP. L. REV. 221, 232-33 (2014).

216. To be sure, an anti-abuse rule might be needed to prevent survivor funds from becoming a tool to avoid the original issue discount rules. Today, if an individual buys a zero coupon bond, the original issue discount rules force her to report the interest on that bond as it is earned. On the other hand, if she instead invests in a survivor fund that invests in zero coupon bonds and the annuity tax rules were to apply, she would not report any income until the end of the survivor-fund term. In effect, she would have avoided the imputed interest rules. Accordingly, an anti-abuse rule might tax her on that imputed interest unless there is a meaningful risk that she would not live until the end of the survivor-fund term. For example, 65-year-olds have a meaningful risk of dying before age 75 (21.59%); on the other hand, however, 8-year-olds do not have a meaningful risk of dying before age 18 (0.27%). *See supra* notes 54 & 92.

217. *See* 26 U.S.C. § 831 et seq.

218. *See, e.g.*, *Abusive Trust Tax Evasion Schemes - Facts (Section II): Basic Trust Taxation Rules*, INTERNAL REVENUE SERV. (Aug. 15, 2016), https://www.irs.gov/Businesses/Small-Businesses-&-Self-Employed/Abusive-Trust-Tax-Evasion-Schemes-Facts-Section-II.

219. *See, e.g.*, Andriy Blokhin, *Do mutual fund companies pay taxes?*, INVESTOPEDIA, (Sept. 11, 2015, 1:15 PM), http://www.investopedia.com/ask/answers/091115/do-mutual-fund-companies-pay-taxes.asp.

All in all, the tax rules affecting the companies selling survivor funds and the tax rules affecting the investors who buy them will have an impact on their after-tax rate of return. Whatever those tax rules are, however, we are confident that survivor funds would be very attractive investment vehicles, and we look forward to helping the market develop.

VII. CONCLUSION

In this Article, we showed how the survivor principle could be used to create survivor funds that only make payments to those who survive for a specified number of years. For example, instead of just investing in 10-year zero coupon bonds and dividing the proceeds among the investors at the end of 10 years, a survivor fund would invest in those 10-year zero coupon bonds but divide the proceeds only among those who survived the full ten years. We also showed how survivor funds could be designed to invest in stocks or mutual funds instead of bonds.

This Article also explained several methods for designing survivor funds, including the individual-cohort method, the date-aligned method, and the age-aligned method. All three of these methods would provide fair bets for all investors, but only the age-aligned and date-aligned methods are practical, as these methods could easily accommodate real-world investors with different ages, genders, and investment levels (while the individual-cohort method could not).

All in all, survivor funds would be very attractive investments because the survivors would get significantly greater returns on their investments, while the decedents, for obvious reasons, would not care. The origins of investing on Wall Street date back to around 1793 when the *Tontine Coffee House* was established there as a meeting place for stockbrokers.[220] We think survivor funds will also find a new home on Wall Street—with today's investment companies and mutual fund houses.

220. *See, e.g.*, *Tontine Coffee House*, AARON BURR ASS'N, http://www.aaronburrassociation.org/tontine.htm (last visited Nov. 9, 2016).

APPENDIX

Appendix Table 1. Period Life Table, 2010.[221]

	Male			*Female*		
Age (i)	*Death Probability (q_i)*	*No. of Lives (n_i)*	*Life Expectancy (l_i)*	*Death Probability (q_i)*	*No. of Lives (n_i)*	*Life Expectancy (l_i)*
0	0.006680	100,000	76.10	0.005562	100,000	80.94
1	0.000436	99,332	75.62	0.000396	99,444	80.39
2	0.000304	99,289	74.65	0.000214	99,404	79.43
3	0.000232	99,259	73.67	0.000162	99,383	78.44
4	0.000172	99,235	72.69	0.000132	99,367	77.46
5	0.000155	99,218	71.70	0.000117	99,354	76.47
6	0.000143	99,203	70.71	0.000106	99,342	75.47
7	0.000131	99,189	69.72	0.000099	99,332	74.48
8	0.000115	99,176	68.73	0.000093	99,322	73.49
9	0.000096	99,164	67.74	0.000090	99,313	72.50
10	0.000082	99,155	66.74	0.000090	99,304	71.50
11	0.000086	99,147	65.75	0.000096	99,295	70.51
12	0.000125	99,138	64.76	0.000111	99,285	69.52
13	0.000205	99,126	63.76	0.000137	99,274	68.52
14	0.000319	99,106	62.78	0.000170	99,261	67.53
15	0.000441	99,074	61.80	0.000207	99,244	66.54
16	0.000562	99,030	60.82	0.000245	99,223	65.56
17	0.000690	98,975	59.86	0.000282	99,199	64.57
18	0.000820	98,906	58.90	0.000318	99,171	63.59
19	0.000949	98,825	57.95	0.000352	99,139	62.61
20	0.001085	98,731	57.00	0.000388	99,105	61.63
21	0.001213	98,624	56.06	0.000423	99,066	60.66
22	0.001304	98,505	55.13	0.000454	99,024	59.68
23	0.001345	98,376	54.20	0.000476	98,979	58.71
24	0.001350	98,244	53.27	0.000494	98,932	57.74
25	0.001342	98,111	52.34	0.000511	98,883	56.77
26	0.001340	97,980	51.41	0.000531	98,833	55.79
27	0.001342	97,848	50.48	0.000553	98,780	54.82
28	0.001356	97,717	49.55	0.000579	98,726	53.85
29	0.001380	97,584	48.62	0.000608	98,668	52.88
30	0.001408	97,450	47.68	0.000641	98,608	51.92
31	0.001435	97,313	46.75	0.000677	98,545	50.95
32	0.001466	97,173	45.82	0.000719	98,479	49.98
33	0.001499	97,031	44.88	0.000765	98,408	49.02
34	0.001539	96,885	43.95	0.000818	98,332	48.06

221. Bye, *supra* note 52.

	Male			*Female*		
Age *(i)*	*Death Probability* *(q_i)*	*No. of Lives* *(n_i)*	*Life Expectancy* *(l_i)*	*Death Probability* *(q_i)*	*No. of Lives* *(n_i)*	*Life Expectancy* *(l_i)*
35	0.001592	96,736	43.02	0.000879	98,252	47.10
36	0.001660	96,582	42.08	0.000948	98,166	46.14
37	0.001741	96,422	41.15	0.001022	98,073	45.18
38	0.001837	96,254	40.22	0.001100	97,972	44.23
39	0.001953	96,077	39.30	0.001185	97,865	43.27
40	0.002084	95,889	38.37	0.001279	97,749	42.32
41	0.002241	95,689	37.45	0.001387	97,624	41.38
42	0.002439	95,475	36.53	0.001518	97,488	40.43
43	0.002686	95,242	35.62	0.001676	97,340	39.50
44	0.002975	94,986	34.72	0.001858	97,177	38.56
45	0.003297	94,704	33.82	0.002055	96,997	37.63
46	0.003639	94,392	32.93	0.002262	96,797	36.71
47	0.003997	94,048	32.05	0.002480	96,578	35.79
48	0.004366	93,672	31.17	0.002709	96,339	34.88
49	0.004750	93,263	30.31	0.002947	96,078	33.97
50	0.005156	92,820	29.45	0.003209	95,795	33.07
51	0.005596	92,342	28.60	0.003484	95,487	32.18
52	0.006078	91,825	27.76	0.003751	95,155	31.29
53	0.006605	91,267	26.93	0.004000	94,798	30.40
54	0.007174	90,664	26.10	0.004246	94,418	29.52
55	0.007805	90,013	25.29	0.004520	94,017	28.65
56	0.008464	89,311	24.48	0.004836	93,593	27.77
57	0.009095	88,555	23.69	0.005185	93,140	26.91
58	0.009676	87,750	22.90	0.005570	92,657	26.04
59	0.010245	86,901	22.12	0.006001	92,141	25.19
60	0.010865	86,010	21.34	0.006489	91,588	24.34
61	0.011592	85,076	20.57	0.007046	90,994	23.49
62	0.012444	84,090	19.81	0.007686	90,352	22.65
63	0.013451	83,043	19.05	0.008419	89,658	21.83
64	0.014608	81,926	18.30	0.009249	88,903	21.01
65	0.015927	80,729	17.57	0.010201	88,081	20.20
66	0.017370	79,444	16.84	0.011255	87,182	19.40
67	0.018895	78,064	16.13	0.012372	86,201	18.62
68	0.020484	76,589	15.43	0.013538	85,135	17.84
69	0.022191	75,020	14.75	0.014793	83,982	17.08
70	0.024139	73,355	14.07	0.016233	82,740	16.33
71	0.026364	71,584	13.40	0.017882	81,397	15.59
72	0.028808	69,697	12.75	0.019693	79,941	14.87
73	0.031480	67,689	12.12	0.021671	78,367	14.16
74	0.034442	65,558	11.49	0.023866	76,669	13.46
75	0.037855	63,300	10.89	0.026437	74,839	12.77
76	0.041725	60,904	10.30	0.029368	72,860	12.11
77	0.045932	58,363	9.72	0.032519	70,721	11.46

	Male			*Female*		
Age (i)	*Death Probability (q_i)*	*No. of Lives (n_i)*	*Life Expectancy (l_i)*	*Death Probability (q_i)*	*No. of Lives (n_i)*	*Life Expectancy (l_i)*
78	0.050469	55,682	9.17	0.035870	68,421	10.83
79	0.055465	52,872	8.63	0.039555	65,967	10.21
80	0.061179	49,939	8.10	0.043828	63,357	9.61
81	0.067698	46,884	7.60	0.048808	60,580	9.03
82	0.074923	43,710	7.11	0.054434	57,624	8.47
83	0.082891	40,435	6.65	0.060762	54,487	7.93
84	0.091725	37,084	6.21	0.067889	51,176	7.41
85	0.101575	33,682	5.78	0.075926	47,702	6.91
86	0.112568	30,261	5.38	0.084968	44,080	6.44
87	0.124795	26,854	5.00	0.095093	40,335	5.99
88	0.138305	23,503	4.64	0.106352	36,499	5.56
89	0.153107	20,253	4.30	0.118777	32,617	5.17
90	0.169195	17,152	3.99	0.132384	28,743	4.80
91	0.186543	14,250	3.70	0.147181	24,938	4.45
92	0.205115	11,592	3.44	0.163161	21,268	4.13
93	0.224867	9,214	3.20	0.180314	17,798	3.84
94	0.245744	7,142	2.98	0.198615	14,588	3.58
95	0.266454	5,387	2.79	0.217125	11,691	3.34
96	0.286625	3,952	2.62	0.235558	9,153	3.13
97	0.305869	2,819	2.47	0.253602	6,997	2.94
98	0.323783	1,957	2.34	0.270923	5,222	2.76
99	0.339972	1,323	2.22	0.287178	3,807	2.60
100	0.356971	873	2.10	0.304409	2,714	2.45
101	0.374819	562	1.99	0.322673	1,888	2.31
102	0.393560	351	1.88	0.342033	1,279	2.17
103	0.413238	213	1.78	0.362555	841	2.03
104	0.433900	125	1.68	0.384309	536	1.91
105	0.455595	71	1.59	0.407367	330	1.79
106	0.478375	39	1.50	0.431809	196	1.67
107	0.502293	20	1.41	0.457718	111	1.56
108	0.527408	10	1.32	0.485181	60	1.45
109	0.553778	5	1.24	0.514292	31	1.35

	Male			*Female*		
Age (i)	*Death Probability (q_i)*	*No. of Lives (n_i)*	*Life Expectancy (l_i)*	*Death Probability (q_i)*	*No. of Lives (n_i)*	*Life Expectancy (l_i)*
110	0.581467	2	1.17	0.545149	15	1.26
111	0.610541	1	1.09	0.577858	7	1.17
112	0.641068	0	1.02	0.612530	3	1.08
113	0.673121	0	0.95	0.649282	1	1.00
114	0.706777	0	0.89	0.688238	0	0.92
115	0.742116	0	0.83	0.729533	0	0.84
116	0.779222	0	0.77	0.773305	0	0.77
117	0.818183	0	0.71	0.818183	0	0.71
118	0.859092	0	0.66	0.859092	0	0.66
119	0.902047	0	0.60	0.902047	0	0.60

Appendix Table 2. Simulation Results

Start age	*Gender*	*Starting number of members*	Account value	*Projected mortality gain*	*Average mortality gain*	*95% range low mortality gain*	*95% range high mortality gain*	*Equivalent - individual - cohort size*	*Projected YTM*	*95% range low YTM*	*95% range high YTM*
						A. Date-aligned Method					
65	m	100	$1,628.89	$448.49	$449.26	$394.93	$504.97	1,572	7.58%	7.30%	7.87%
66	m	100	$1,628.89	$495.84	$496.70	$436.04	$559.20	1,452	7.83%	7.52%	8.15%
67	m	100	$1,628.89	$549.84	$550.87	$482.59	$621.52	1,331	8.10%	7.76%	8.45%
68	m	100	$1,628.89	$611.58	$612.70	$535.58	$693.12	1,218	8.40%	8.03%	8.79%
69	m	100	$1,628.89	$682.33	$683.56	$596.01	$775.63	1,113	8.74%	8.33%	9.17%
70	m	100	$1,628.89	$763.75	$765.04	$665.65	$870.62	1,026	9.12%	8.66%	9.59%
71	m	100	$1,628.89	$858.15	$859.88	$746.06	$981.35	946	9.54%	9.03%	10.07%
72	m	100	$1,628.89	$968.42	$970.54	$839.65	$1,111.35	874	10.02%	9.46%	10.61%
73	m	100	$1,628.89	$1,097.89	$1,100.32	$949.59	$1,264.29	815	10.55%	9.94%	11.21%
74	m	100	$1,628.89	$1,250.75	$1,253.28	$1,078.84	$1,445.96	762	11.16%	10.47%	11.89%
						B. Age-aligned Method					
65	m	100	$1,628.89	$448.49	$449.53	$347.98	$559.78	426	7.58%	7.05%	8.15%
66	m	100	$1,628.89	$495.84	$496.68	$402.29	$598.42	574	7.83%	7.34%	8.34%
67	m	100	$1,628.89	$549.84	$551.20	$455.63	$652.97	661	8.10%	7.62%	8.60%
68	m	100	$1,628.89	$611.58	$612.66	$513.27	$719.36	713	8.40%	7.92%	8.91%
69	m	100	$1,628.89	$682.33	$682.60	$578.74	$796.12	761	8.74%	8.24%	9.26%
70	m	100	$1,628.89	$763.75	$764.63	$643.61	$897.31	671	9.12%	8.56%	9.71%
71	m	100	$1,628.89	$858.15	$858.16	$726.91	$1,004.88	679	9.54%	8.95%	10.17%
72	m	100	$1,628.89	$968.42	$970.55	$809.51	$1,149.47	560	10.02%	9.32%	10.76%
73	m	100	$1,628.89	$1,097.89	$1,101.73	$886.05	$1,348.70	380	10.55%	9.66%	11.53%
74	m	100	$1,628.89	$1,250.75	$1,254.72	$958.64	$1,617.20	241	11.16%	9.97%	12.50%

Appendix Table 3. Mortality Rates for Males, Aged 50-80.[222]

Age (i)	*Social Security Universe, 2010 (q_i)*	*Society of Actuaries, RP-2014*			*NAIC 2012 (q_i)*
		Employee (q_i)	*Healthy Annuitant (q_i)*	*Disabled Retiree (q_i)*	
50	0.005156	0.001686	0.004064	0.020395	0.002057
51	0.005596	0.001871	0.004384	0.021016	0.002302
52	0.006078	0.002072	0.004709	0.021621	0.002545
53	0.006605	0.002289	0.005042	0.022210	0.002779
54	0.007174	0.002527	0.005384	0.022791	0.003011
55	0.007805	0.002788	0.005735	0.023369	0.003254
56	0.008464	0.003079	0.006099	0.023953	0.003529
57	0.009095	0.003407	0.006478	0.024557	0.003845
58	0.009676	0.003779	0.006877	0.025190	0.004213
59	0.010245	0.004204	0.007305	0.025868	0.004631
60	0.010865	0.004688	0.007771	0.026604	0.005096
61	0.011592	0.005240	0.008284	0.027414	0.005614
62	0.012444	0.005867	0.008854	0.028312	0.006169
63	0.013451	0.006577	0.009492	0.029314	0.006759
64	0.014608	0.007377	0.010209	0.030433	0.007398
65	0.015927	0.008277	0.011013	0.031685	0.008106
66	0.017370	0.009175	0.011916	0.033081	0.008548
67	0.018895	0.010171	0.012930	0.034633	0.009076
68	0.020484	0.011275	0.014067	0.036353	0.009708
69	0.022191	0.012498	0.015342	0.038253	0.010463
70	0.024139	0.013854	0.016769	0.040346	0.011357
71	0.026364	0.015357	0.018363	0.042647	0.012418
72	0.028808	0.017023	0.020141	0.045170	0.013675
73	0.031480	0.018870	0.022127	0.047935	0.015150
74	0.034442	0.020918	0.024345	0.050965	0.016860
75	0.037855	0.023188	0.026826	0.054287	0.018815
76	0.041725	0.025704	0.029608	0.057934	0.021031
77	0.045932	0.028493	0.032735	0.061945	0.023540
78	0.050469	0.031585	0.036258	0.066363	0.026375
79	0.055465	0.035012	0.040232	0.071235	0.029572
80	0.061179	0.038811	0.044722	0.076616	0.033234

222 Bye, *supra* note 52; *RP-2014 Rates; Total Dataset*, *supra* note 138; *NAIC Model Rule*, *supra* note 139, at Appendix II.