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**ASSISTING THE LEARNING DISABLED THROUGH THE USE OF
MATH MANIPULATIVES**

A Project
Presented to the
Faculty of
California State University,
San Bernardino

In Partial Fulfillment
of the Requirements for the Degree
Master of Arts
in
Education: Special Education Option

by
Joan Horton McBryde

June 1993

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June 1993

Approved by:


Dr. Patricia Tefft Cousin, First Reader

6-8-93
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Dr. Barbara Larrivee, Second Reader

ABSTRACT

The purpose of this project was to evaluate the effectiveness of the use of math manipulative materials with the learning handicapped student. Frequently the student with a learning handicap dislikes math or has difficulty with math. Progress is slow and somewhat limited. However, studies of math programs revealed that the use of math manipulatives help the learning handicapped student adopt a tangible, active approach to learning mathematics. The reluctance of some educators to use manipulative materials in math is cause for concern. As a result there was a need for further investigation and documentation of the value of using manipulative materials for the learning handicapped student in math programs.

The major goal of this project was to emphasize the importance of the use of math manipulatives for the learning handicapped student. In the literature review, several significant questions concerning math manipulative materials were examined. This review indicated that students with a learning handicap do benefit from the use of math manipulative material.

The second goal of this project was to provide strategies for elementary teachers to initiate and use math manipulatives in the classroom. While many teachers believe very strongly in the benefits of using math manipulatives, there are a number of constraints and obstacles that inhibit their use. However, teachers should strive to overcome the obstacles due to strong evidence that the use of math manipulative materials greatly support and assist children in building a firm understanding of mathematical concepts. Relative to the classroom use of math manipulatives, it is the intent of this project to document their benefits, create incentives for their use, and provide basic strategies for their implementation.

This project was written in an effort to emphasize the importance of the use of math manipulatives in the classroom with the learning handicapped or disabled student. Several questions were addressed in order to understand the importance of the use of math manipulatives in the classroom. They were:

What is a math manipulative?

What are the popular kinds of manipulatives currently in use?

What are the important characteristics of math manipulatives?

What is the history of math manipulatives?

Why should math manipulatives be used?

How effective are manipulatives?

When should manipulatives be used?

How can manipulatives be used with the learning handicapped student?

Through answering these questions, it was evident that learning handicapped students should be allowed and encouraged to conceptualize abstract ideas through the use of math manipulatives. The results of the studies on using math manipulatives by theorists such as Piaget, Montessori, Brownell, Bruner, Skemp, Dienes and Gagne are persuasive. These learning theorists suggest that children whose mathematical learning is firmly grounded in manipulative experiences will be more likely to bridge the gap between the world in which they live and the abstract world of mathematics. The student with a learning handicap especially needs to have the concrete models in order to better understand mathematical concepts.

Dedicated with love to my husband, Ron, and my children, Beau and Britni,
whose constant love, support and understanding
have made this project possible.

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INTRODUCTION

Most learning disabled students have difficulty organizing information that they wish to learn. The learning disabled student may have developmental lags, poor retention, brief attention spans, perceptual problems, logical-thinking deficiencies or linguistic difficulties. Any or all of these negatively impact the abstract and logical thinking skills required to learn and perform mathematics. Even though each student is different, most have one trait in common: they usually benefit from a carefully structured, active approach to learning mathematics (Thornton & Wilmot, 1986).

As learners with handicaps are guided in their work with materials and are given a chance to verbalize what they do, they begin to internalize and make sense of concepts in their own minds. Structured situations involving manipulatives help these students organize their thinking so that they begin to see relationships or follow the flow of a computational procedure (Thornton & Wilmot, 1986).

The results from a number of studies indicate that manipulative materials improve children's understanding of the mathematics they learn. Further, many teachers believe very strongly in the benefits of using math manipulatives. A few teachers view them with skepticism and are less inclined to use them. Yet strong evidence supports the use of manipulative materials to help children build a firm understanding of mathematical concepts (Kennedy, 1986).

This project was written in an effort to emphasize the importance of the use of math manipulatives in the classroom with the learning disabled student. Of course, math manipulatives are beneficial to all students. Consequently, this information might be applied to the regular classroom student as well.

This project includes a definition and brief history of the development of manipulatives. A summary of the principal kinds of manipulatives is incorporated. The first section of the project will address why manipulatives should be used for the learning disabled student by detailing the need and effectiveness of these learning tools. The first section will conclude with a discussion on when to use manipulatives by examining such issues as appropriateness, limitations and problems generated.

The second section of this project will be devoted strictly to strategies for using math manipulatives. This section will be augmented by examples of manipulatives currently being utilized.

LITERATURE REVIEW

There are several questions which should be addressed in order to understand the importance of the use of math manipulatives in the classroom. They are: (a) what is a math manipulative? (b) what are the popular kinds of manipulatives currently in use? (c) what are the important characteristics of math manipulatives? (d) what is the history of math manipulatives? (e) why should math manipulatives be used? (f) how effective are manipulatives? (g) when should manipulatives be used? (h) what are the limitations of manipulatives? and (i) how should manipulatives be used with the learning handicapped? These components will be described in detail.

What is a math manipulative?

Manipulative materials are objects that appeal to several senses that can be touched, moved about, arranged, and otherwise handled by children. These materials are objects which represent mathematical ideas that can be grasped through physical involvement from the environment, such as money or measurement instruments, or materials specifically designed to teach mathematical concepts, such as base-ten blocks and balances (Kennedy, 1986).

Manipulative materials are concrete models that incorporate mathematical concepts. This definition implies that students will have such objects or materials in their possession to utilize or manipulate. It is not sufficient for students to observe a demonstration of the use of an aid. The act of manipulating the aid allows students to experience the patterns and relations that are the focus of mathematics (Haynes, 1986).

What are the popular kinds of manipulatives currently in use?

The following list is a collection of suggested manipulative material for developing math concepts (Marzola 1987). These are available from distributors whose addresses are provided in the Appendix..

Materials

Multipurpose Manipulatives

Cuisinaire Rods (Cuisinaire Company; Creative Publications)

Multilinks (Creative Publications)

Recipe for Math (Book-Lab)

Stern Structural Arithmetic (Educators Publishing Service)

Unifix materials and stamps (Creative Publications, Didax)

Place Value and Operations

Arithmetic Tube (Ideal)

Base Ten Blocks (Creative Publications; Didax)

Base Ten Group Set (Ideal)

Base Ten Rubber Stamps (Creative Publications)

Chip Trading (Scott Resources, Creative Publications)

Fundamath (Ideal)

Math Balance (Creative Publications; Ideal)

Modern Computing Abacus (Ideal)

Number Blox (Creative Publications)

Place Value Activity Cards (DLM)

Place Value Board (Ideal)

Place Value Dominoes (Didax)

Place Value Stamps (Janian; DLM)

Money

Cash Box (Ideal)

Coin Matching Cards (Didax)

Coin Stamps (Creative Publications; Didax)

Coins and Bills (Didax)

Dollars and Cents Kit (Ideal)

Money Dominoes (Didax)

Money Flash Cards (Ideal)

U.S. Bills Stamps (Didax)

Time

Clock Dominoes (Didax)

Clock Puzzles (DLM)

Clock Stamps (DLM)

Geared Clock Face (Didax)

Student's Individual Clocks (Didax)

Weights and Measurement

Centicube (Didax)

Gram-Lok Cubes (Ideal)

Labscale (Ideal)

Measuring Up (DLM)

Metric Bottle Set (Ideal)

Plastic Stacking Weights (Didax)

Trundle Wheel (Ideal)

Platform Scale: Metric (Ideal)

Super Beamer Balance (Didax)

Fractions

Comparative Fraction Blocks (Educational Teaching Aids)

Comparative Fractions Strips (Didax)

Equivalent Fractions Matching Cards (Didax)

Fraction Circles and Squares (Ideal)

Fraction Dominoes (Creative Publications; Didax)

Fraction Factory (Creative Publications)

Fraction Flip Books and Activity Cards (Didax)

Fraction Pieces (Creative Publications)

Fraction Stamps (Didax)

Fraction Tiles (Activity Resources Co.)

Moving Up in Fractions (DLM)

Pizza Party (Creative Publications; Didax)

Decimals

Decimal Squares (Creative Publications)

Moving Up in Decimals (DLM)

Percents

Moving Up in Percents (DLM)

Geometry

Angle Study Dominoes (Didax)

Clear Plastic Geometric Volume Set (Didax)

Geoboards (Creative Publications; Didax; Ideal)

Geofix Solid Geometry Construction Set (Didax)

Geometry Models (Didax)

Geometry Dominoes (Didax)

Geometry Resource Kit (Didax)

Volume Relationship Sets (Ideal)

Wooden Geometric Solids (Didax)

What are the important characteristics of math manipulatives?

There are a number of criteria to consider before selecting the proper manipulative for a particular purpose. The following is a list of characteristics of effective manipulative materials (Haynes, 1986).

1. *Use clear representation of mathematical ideas.*

Manipulatives can help children understand mathematical concepts only if the material selected clearly represents the idea in question.

The representation should be natural and easily understood. If the

goal is to teach subtraction with regrouping, materials that can actually be regrouped are necessary.

2. *Select materials for students' developmental level.*

It is important that manipulatives be selected on the basis of the child's ability to operate effectively with the apparent attributes of the manipulative material. Materials selected for use must also be consistent with motor development of the child.

3. *Select manipulative materials which arouse interest.*

The physical characteristic of the material often promotes interest in itself. In addition, interest is highest when each student can manipulate the material.

4. *Select manipulative materials which have versatility.*

Some materials stimulate a number of senses at the same time and can be used to teach many mathematical concepts at various grade levels.

5. *Consider the physical characteristic of the manipulative material.*

Usually, information about the physical appearance of materials can be obtained in commercial catalogs and brochures before purchasing manipulative material. One should consider the durability, simplicity, attractiveness, ease of manageability, and

storage requirements. The cost should also be considered. When pricing the material, the buyer should consider their use, versatility, and life expectancy (Haynes, 1986).

What is the history of math manipulatives?

The following people had an influence in the history of math manipulatives. This chart provides one with the understanding of manipulative materials in relationship to the timeline in history and also indicates that the use of manipulative material has been taking place for a long time.

- | | | |
|---------------------|-----------|-------------|
| • Friedrich Froebel | 1782-1852 | Germany |
| • John Dewey | 1859-1952 | New York |
| • Maria Montessori | 1870-1952 | Italy |
| • Jean Piaget | 1896-1980 | Switzerland |

In addition, the contributions of other individuals, who are continuing to have an influence on the use of manipulative materials today, will be discussed. These include Brownell, Bruner, Skemp, Dienes and Gagne.

It was about one hundred and fifty years ago that Friedrich Froebel invented the term "kindergarten" and provided his students with wooden geometric shapes with which to play. These shapes were used as models for drawing from different perspectives and for exploration (Morrow, 1990). This might possibly be the beginning of the use of math manipulatives.

The use of manipulatives was advocated by Montessori and Dewey in the early 1900s (Cohen, 1981). Montessori sought to teach children by supplying concrete materials and organizing situations conducive to learning with these materials. Montessori discovered that certain simple materials aroused an interest in young children not previously thought possible. These materials included beads arranged in graduated-number units for premathematics instruction, small slabs of wood designed to train the eye in left-to-right reading movements, and graduated series of cylinders for small muscle training. The materials used were designed specifically to encourage individual rather than cooperative effort. Montessori scorned conventional classrooms, where "children, like butterflies mounted on pins, are fastened each to his place" (Gwinn & Norton, 1991, 28).

Dewey believed that the educational process must begin with and build on the interests of the child. The process must provide opportunity for the interplay of thinking and doing in the child's classroom experience. Dewey believed that the teacher should be a guide and co-worker with the pupils rather than a taskmaker assigning a fixed set of lessons and recitations. The goal of education is the growth of the child in all aspects of its being. Compared to Montessori, Dewey's approach involved direct guidance by the teacher and, in addition, the coaching that could be given to the student by serving as a colleague in the learning and discovery process.

The mental-discipline and stimulus-response theory of the nineteenth and early twentieth century gave way to meaning theory, espoused by William Brownell in the 1930s. This theory was based on the belief that children must understand the basic concepts that underlie what they are learning if learning is to be permanent (Kennedy, 1986).

More recent studies of human learning by Jean Piaget (1952) and Richard Skemp (1982) led them to conclude that all individuals pass through stages as they mature. Piaget believed that cognitive development occurs in four stages. Mental images and abstract ideas are based on experiences. Therefore, students who see and manipulate a variety of objects have clearer mental images and can represent abstract ideas more completely than those whose experiences are limited (Kennedy, 1986). Piaget proposed a comprehensive theory of cognitive development that suggests, among other things, that children of seven through eleven years of age need to experience, through hands-on, concrete actions, the ideas which symbols represent (Morrow, 1990). Piaget contended that experience is one of the factors mediating cognitive development, the other three being maturation, social transmission, and equilibration. According to Piaget, knowledge is linked with action. To know an object a subject must act upon it, and thus transform it; that is displace, connect, combine, take apart and reassemble it (Cohen, 1981). Skemp believed that the two level director system governs learning. At the first level, the manipulation of objects in and out of the school environment provides the learner with physical activities that form the basis for further learning and the internalization of ideas. At the second level, activities build on those of the first and are mental rather than physical (Kennedy, 1986).

Jerome Bruner, a psychologist and educator, whose work on perception, learning, memory, and other aspects of cognition in young children, along with the related work of Piaget, has influenced the American education system. Bruner's studies helped to introduce Piaget's concept of developmental stages of cognition into the classroom. He argued that any subject can be taught to any child at any stage of development, if it is presented in the proper manner. According to

Bruner, all children have natural curiosity and a desire to become competent at various learning tasks. When a task as presented to them is too difficult, however, they become bored. Therefore, a teacher must present schoolwork at a level so as to challenge the child's current developmental stage (Gwinn & Norton, 1991).

The use of manipulatives has also been supported by theorists such as Dienes and Gagne in the 1960s. Dienes advocated the use of manipulative material by children. He described the use of multibase blocks and other materials to build children's understanding of numbers. Each of the devices supplies a proper concrete representation of a concept. He advocated the use of "multiple embodiments" rather than a single representation of a concept. The multiple representation of place value, for example, occurs when children use loose and bundled sticks, ten frames, abacuses, and other place value materials to learn the meaning of the numeration system (Kennedy, 1986).

The strength of the arguments of these and other theorists for using manipulative materials is persuasive. Learning theories suggest that children whose mathematical learning is firmly grounded in manipulative experiences will be more likely to bridge the gap between the world in which they live and the abstract world of mathematics. The manipulatives help children understand both the meanings of mathematic ideas and the applications of these ideas to real world situations (Kennedy, 1986).

Why should math manipulatives be used?

Manipulatives bridge the gap between the concrete and the abstract. The stage between the concrete and abstract can be described as the semiconcrete stage and the semiabstract stage (Heddens, 1986). The semiconcrete level is a representation of a real situation, pictures of the real items themselves. The semiabstract level involves a symbolic representation of concrete items, but the symbols or pictures do not look like the objects for which they stand. Helping children to bridge this gap is crucial because many children cannot move from concrete to abstract without the teacher's assistance (Heddens, 1986). Manipulatives are one way to bridge this gap.

The gap is bridged by modeling ideas. A concrete-operational child cannot handle abstract concepts. However, with manipulatives it is possible for such a student to take the first steps toward exploring the concepts; manipulatives are concrete introductions to abstract ideas. Lessons in which teachers use math manipulatives have a higher probability of achievement than traditional lessons (Chester & Davis & Reglin, 1991).

The experience with manipulative materials helps provide a strong basis for conceptual understanding, whether it be of later procedural skills or an appreciation of properties and relationship. A proper use of manipulatives may remove the need for later remediation.

To be sure, using manipulative materials is potentially valuable for the student in need of remediation and the learning disabled student. It may well be that misconceptions, misunderstandings, and an inability to use certain procedures

are based on an originally weak conceptual understanding of the subject matter (Moser, 1986).

How effective are manipulatives?

A confining, narrow textbook approach to teaching mathematics is inadequate and inappropriate in today's schools. Math manipulatives are effective in the sense that manipulatives motivate students. Manipulatives stimulate students to think mathematically, and manipulatives informally introduce "big" ideas in mathematics (Herbert, 1985). Manipulatives allow teachers to create situations that involve the students actively in the creation of mathematics, resulting in improvements in motivation, involvement, understanding, and achievement (Herbert, 1985).

In a study by Chester, Davis, and Reglin (1991), it was found that the use of math manipulatives increased student achievement in the area of mathematics. Further, understanding was retained longer through discovery. Exploration and discovery techniques required teachers to spend time examining the cognitive structures of the concepts and then to create appropriate experiences in which students could discover the concepts. Using math manipulatives required students to actively do math instead of passively completing paper and pencil activities. Concrete materials helped students understand concepts by helping them understand the reason for the rules, not just how to apply them (Chester, Davis & Reglin, 1991).

In another study by Karp (1990), the use of math manipulatives has been identified as an important factor to investigate. The purpose of the study was to

document the student achievement differences between three mathematical programs ranging from a strict manipulative focus to an abstract, non-manipulative approach. The data obtained indicated that a non-traditional, abstract mathematics program may be successful in increasing student achievement at all performance levels in first grade (Karp, 1990).

When should math manipulatives be used?

Certainly, manipulatives should not be used for teaching every concept. Careful use of manipulatives provides opportunities for generalizing across multiple instances of concepts. The main emphasis that needs to be made in teaching with manipulatives is that symbols and manipulatives always reflect the same concept. Students do not always grasp this connection (Bright, 1996).

The use of manipulatives has been beneficial in a wide variety of math topics and across diverse grade levels. Manipulatives can be used to teach concepts, relationships, operations, and problem solving (Marzola, 1987).

Teachers sometimes resist the use of manipulatives because they fear their students will become too dependent on them. While it is important to move students from concrete to symbolic representations, it is more important to solidly establish the connections between actions on objects, the concepts they illustrate, accurate verbal versions of these, and the related symbolic notation. As children become more adept at handling concepts symbolically, use of concrete materials may decrease (Marzola, 1987). It is important not to abandon manipulatives too quickly. Manipulatives can and should be used on a regular basis in the

mathematics classroom along with a variety of other tools and methods (Herbert, 1985).

What are the limitations of math manipulatives?

The primary limitations on the use of math manipulatives include:

- Financial Constraints
- Time Constraints
- Classroom Control and Management
- Lack of Teacher Commitment

Even with long-standing and widespread support, manipulative materials do not appear to be extensively used (Worth, 1986). Many complex factors inhibit the extensive use of manipulative materials. One reason may be financial constraints on education today. Tight budgets may prevent the purchase of required materials and objects. Teachers also frequently claim that not enough time is available to use manipulatives (Herbert, 1985). The utilization of manipulatives does require time, but the results justify the time requirement. Planning time is time well spent. Some feel that using manipulatives is the same as "playing". It is a discovering time for the teacher and student, but that is what makes teaching so rewarding. Another fear expressed by teachers is the loss of control of the classroom and the difficulty of maintaining classroom management. However, manipulatives do not have to be difficult to manage. For example, a class can be divided into groups with the teacher supervising each group by

circulating around the room. This allows the students to struggle and experiment on their own (Herbert, 1985).

Simply using manipulative materials does not guarantee meaningful learning. Like any tool, manipulative material must be used judiciously and carefully for positive results. If used inappropriately or without skill, they may not yield the desired impact. Thoughtful use of manipulative materials entails consideration of the following questions: Can pupils use this manipulative in such a way that it connects with their existing knowledge and, hence, is meaningful to them? Is the manipulative used in such a way that it requires reflection or thought on the part of the students? (Baroody, 1989).

Because we are still learning about manipulative materials, that, in itself, is a limitation. We must be aware of the importance of keeping an open mind about using manipulatives.

How should manipulatives be used with the learning handicapped?

As stated previously, structured situations involving manipulatives help the students with learning handicaps to organize their thinking so that they begin to see relationships or follow the flow of a computational procedure (Thornton, 1986). There are four things that might provide this structure.

They are:

Vary the way manipulatives are presented in a lesson to capitalize on students' learning strength. Stagger the presentation of visual, verbal, and tactile information when introducing or summarizing a manipulative activity.

Help students build important self-monitoring skills. Children with learning handicaps must also be taught "how to learn" in addition to important concepts and skills. Central to this perspective is the development of self-monitoring skills, which are critical when students are under pressure (as in tests), are tired, or are under stress. Appropriate questions repeated during active instruction with materials, bring responses during initial instruction. These same questions often are internalized and used later by the students themselves as the basis for selecting correct principles or procedures for completing a mathematical task.

Provide for consistency. It is critical that the special educator and the classroom teacher cooperate in planning and implementing the mathematics curriculum for the student with a learning handicap.

Carefully plan the use and the phasing out of manipulative materials both in concept and skill learning. The use of the following six step teaching sequence may be helpful with most students with learning handicaps:

- **Manipulate-Talk**

Use materials to model an example. Talk through each step.

- **Manipulate-Write**

Use manipulatives to model the example. Write down each step as it is completed.

- **Talk-Write-Manipulate**

Talk through each step, write what you say, and use manipulatives after each step is recorded to check. This helps the handicapped student to internalize appropriate language and thinking.

- **Talk-Write-Manipulate: entire solution**

Talk through the entire solution, recording as you go. Then model the entire example as a check. Pay close attention to the student's use of materials, and then check the written answer. When students can correctly demonstrate and explain the thinking process they used to complete an example, then manipulatives are no longer needed.

- **Discriminate**

Help students discriminate new from previous learning. For example, have the student circle and do just the problems that require regrouping when they are mixed with those that require no regrouping.

- **Compute-Model**

From time to time, use manipulatives to check written work. The goal is maintenance. If this step is neglected, learning handicapped children will narrowly focus on getting answers, make conceptual errors, and fail in the transfer to related learning (Thornton & Wilmot, 1986).

CONCLUSION

This project was written in an effort to emphasize the importance of the use of math manipulatives in the classroom with the learning handicapped student. Several questions were addressed in order to understand the importance of the use of math manipulatives in the classroom. They were:

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How can manipulatives be used with the learning handicapped student?

Through answering these questions, it was evident that learning handicapped students should be allowed and encouraged to conceptualize abstract ideas through the use of math manipulatives. The results of the studies by theorists such as Piaget, Montessori, Brownell, Bruner, Skemp, Dienes and Gagne for using manipulative material is persuasive. Learning theorists suggest that children whose mathematical learning is firmly grounded in manipulative experiences will be more likely to bridge the gap between the world in which they live and the abstract world of mathematics. The student with a learning handicap especially needs to have the concrete models in order to better understand mathematical concepts.

CURRICULUM PROJECT

The purpose of this section is to provide strategies in implementing the use of math manipulatives in the classroom. These suggested strategies are only a few of a broad number of strategies that can be used. It is the intention of the writer that the following strategies provide an introduction into the use of math manipulatives. Many of these strategies utilize manipulatives that can be made without purchasing expensive kits or materials.

There are a few guidelines which might be helpful in using these strategies, or any strategy incorporating manipulative material:

1. *Free Exploration* is necessary whenever new manipulative material is introduced. The learning disabled student will be naturally curious about the new material. The desire to investigate in their own manner is stronger than the desire to follow the teacher's directions. Lack of exploration time will jeopardize the best-planned activity.

2. *Package and organize the materials* according to the purpose of the lesson. Learning disabled students may not be patient while parts of manipulative material are counted out. Have the students help organize the materials. This will be a learning opportunity in itself. Store materials so that students can distribute and put away the manipulative materials without extensive directions.

3. *Clear expectations and guidelines* should be established. The teacher must be able to articulate the purpose of the lesson and how the student may use the materials. The student needs a clear understanding of why the materials are important and what is acceptable when using the manipulatives.

4. "*Thinking aloud*" should be modeled by the teacher and, in turn, the student should be encouraged to verbalize their actions with the manipulative materials. The student must be encouraged to summarize their activities, talk about the idea, and discover through the use of the materials.

The ultimate goal, as a teacher, is to help students solve problems without the use of manipulatives. It is important that there be no rush in removing manipulatives. Give the student time to become comfortable in forming a mental picture of the math concept. Then use that picture to talk about the concept. Once the student understands this process, the concept is understood. Therefore, the following strategies can be used to introduce the concept and support the student's development and understanding of the concept.

Addition and Subtraction Strategies

Children need to know that addition is putting groups of objects together to determine a sum and subtraction is taking objects away from a group to determine how many are left (or comparing groups to determine the difference between them). Children need to learn that certain words and language patterns such as *total*, *sum*, *difference*, and *equals* are used in describing these processes that are familiar occurrences in their everyday lives (Richardson, 1984).

The only way to get a sense of the addition or subtraction problem is to experience the quantity of real objects and then learn to associate said quantities with oral and written symbols.

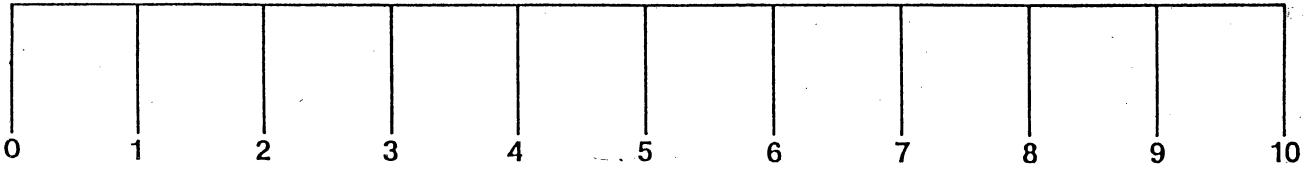
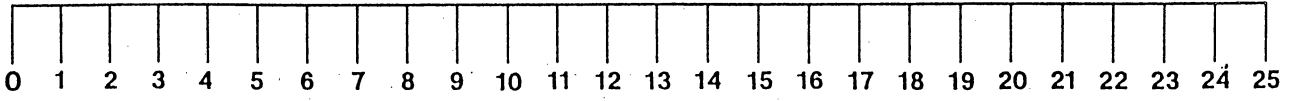


Figure 1. Number Lines.

Strategy: Addition and Subtraction Using Number Line (grades 1-3)

Use the number line when adding and subtracting numbers up to 25 (see figure number 1). Problems such as $6+6=12$, $15-7=8$. Even a problem, such as $6+?=10$, can be demonstrated and solved on the number line.

Strategy: Addition and Subtraction Using Counters (grades k-2)

Another good way to add or subtract numbers is by using beads on a straightened coat hanger wire. Use wire cutters to cut the bottom length of a coat hanger. Put 10 or 20 beads on the wire and bend the ends with pliers. This can be easily made and students can push or move the beads to solve or make an equation.

Strategy: Addition Using Blocks (grades k-2)

Make a stack of any number of blocks between 3 and 10. Have students break down the stack into as many combinations as possible to make mathematical equations. Have students record and discuss the results. Differences and similarities between the pairs such as $6+2=8$ and $2+6=8$ should be discussed.

Place Value Strategies

The concept of "place value" is not an easy idea for the learning disabled student. The basic concept children must grasp is that our number system is based on the formation of groups of ten. When those ten groups are combined, there is one hundred, and when those one hundred are combined, there is one thousand, and so on.

Counting "groups" requires a different kind of thinking than counting single objects. When dealing with numbers above ten, the student is required to count groups as though they were individual objects. The use of manipulatives can help the student in dealing with place value concepts.

Strategy: Unifix Cubes (grades 1-3)

The unique qualities of the Unifix Cubes can make them an effective tool. Children can join ten cubes into a single quantity so that ten single objects become one object. Figure 2 shows how Unifix Cubes can be joined together into a train from single units. Joining these ten cubes into one train does not cause the units to disappear. The student can still count and check, if necessary, to see how many units there are (Richardson, 1984).

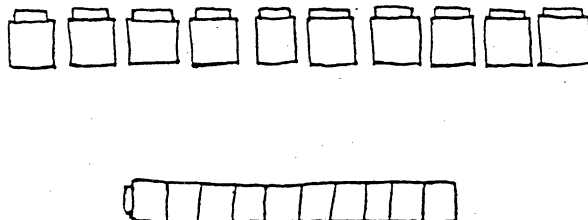


Figure 2. Unifix Cubes.

Ten Unifix Cubes can be joined together to form a unit (a train).

Strategy: Cheerios (grades k-6)

Cheerio counters can be made and used instead of purchasing base ten blocks. Each child can have their own set and can make them on their own. A single Cheerio represents the one unit, 10 Cheerios glued on a Popsicle stick makes a *long*, 10 longs glued on two perpendicular sticks, raft style, make a *flat*, and 10

flats stacked on top of each other make a *box*. Let the students dye the Cheerios before making the sets by putting several drops of food coloring in a large bowl with water. Quickly dip the Cheerios in the dye and immediately spread the Cheerios on a cookie sheet and bake at 250 degrees for 30 minutes. The Cheerios will become hardened and become brighter after removed from the oven. Two boxes of Cheerios are needed for a class of 25 (Scheer, 1984).

Strategy: Trading Post (grades k-3)

This two to six player game helps students learn the relationships among the Cheerio counters and the ones, tens, and hundreds places. Base ten blocks can be used. You will need a collection of Cheerio counters and two dice, one white and one another color. Players take turns rolling the dice and taking the number of units shown on the white die and the number of longs shown on the colored die from the pile of counters. When each player accumulates enough smaller Cheerio counters, he or she trades for a larger counter(10 units for a long, 10 longs for a flat). The first one to exchange for a flat wins (Scheer, 1984).

Strategy: I Know My Place (value) (grades 2-6)

You will need a ditto game board for each child, with a series of spaces in any pattern and spaces marked *start* and *end*. Make cards with one-, two-, three-, and four-digit numerals; a spinner with 1, 10, 100, and 1,000 printed in four sections; and Cheerios counters(or base 10 blocks). Each student puts a marker on *start*. The first player picks the top card from the pile and spins. The player uses Cheerio counters to build the number indicated on the card. The player then points to the Cheerios that represent the place indicated on the spinner and moves

that number of spaces on the game board. For example, if the spinner points to 10, the player would point to the *longs* in the number just built and move 10 spaces. The winner is the first to reach the end.

Multiplication Strategies

Many learning disabled students consider learning how to do "multiplication" a dreaded task. Although a child may be able to rattle off "ten times ten is one hundred", he or she may not be able to express exactly what that means. The student may not know that what he or she is saying means something in the real world. Knowing the answer and being able to "know the times table" is only useful if that knowledge can be easily interpreted and used for problem solving.

The first experience of the student should be one that will develop the language of multiplication. These words should be words with natural language from the real world such as "rows of," "groups of," and "stacks of." The following strategies should be repeated many times so that the language of multiplication becomes comfortable to use.

Strategy: Pinto Beans (grades k-6)

A sack of uncooked pinto beans can be readily utilized to convey the concept of multiplication. In the problem of 2×3 , the student would make two stacks of three beans, and then count up to get an answer of six.

Strategy: Circles and Stars (grades 3-6)

Divide the class into pairs. You will need one die per pair of students. The first partner rolls the die, and draws that many circles on a piece of blank paper. For example, a student rolling a 5 would draw 5 circles. The other student rolls the die, and draws that number of stars inside each circle. For example, if the student rolled a 4, 4 stars would be drawn inside each circle. At the bottom of the paper, the students are to make a multiplication problem. In this example, it would be $5 \times 4 = 20$. Display all the drawings and share. Write all the problems that were created.

Division Strategies

There are two types of situations in the process of division: determining how many groups (the grouping process), and determining how many in each group (the sharing process). The "grouping process" is the dividing of a quantity of objects into smaller groups. The "sharing process" is the dividing of a quantity of objects into a particular number of groups to determine the number of objects in each group. Do not attempt to distinguish between these processes; just provide experiences with both. It is also important to deal with "remainders" from the beginning. This is a natural part of the division process (Richardson, 1984).

Strategy: Acting Out Division Stories (grades 4-6)

Tell the students stories, and ask them to "act out" the story using real things in the classroom. One example would be: Pat had twenty pieces of paper. Each child

needs two pieces of paper to make a book cover. How many children will get paper from Pat? (see Figure 3).

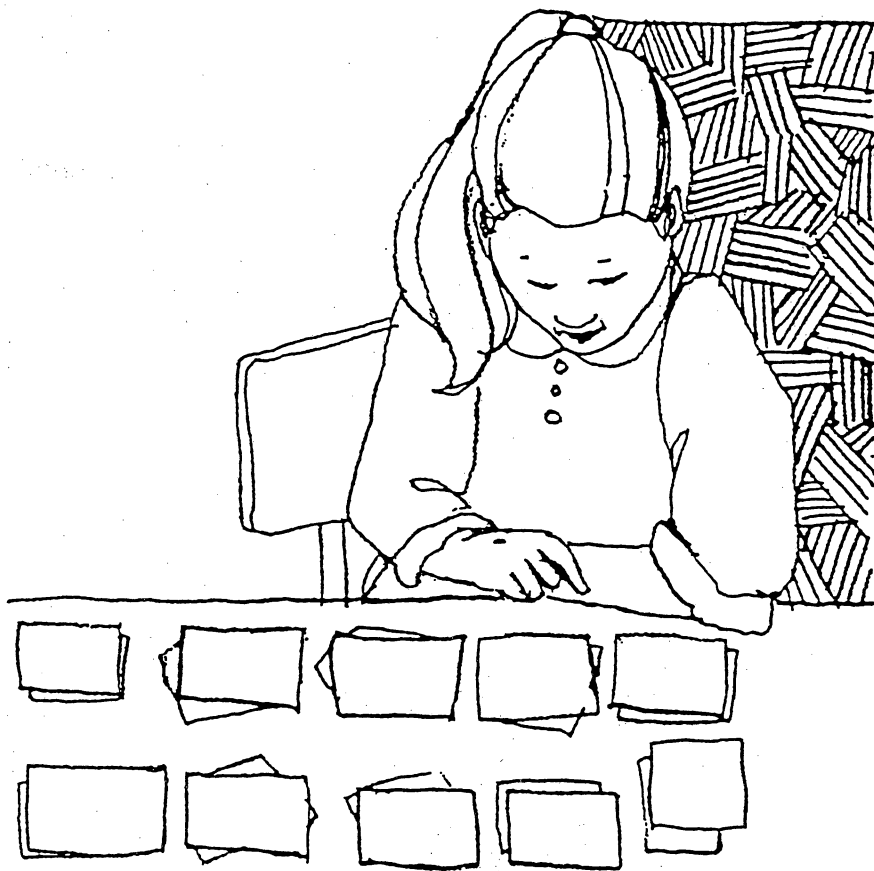


Figure 3. Acting Out Division

Pat can give paper to ten children.

Strategy: Beans (grades k-6)

Pinto beans may also be used for the concept of division. For the problem such as 15 divided by 3, give the student 15 beans. Tell the student to sort the beans into three equal groups. When the beans are all sorted, the student should see that there are five beans in each group. Have the student explain what happened to the beans and then write out the equation.

Fraction Strategies

Fractions are conceptualized as being part of a whole or part of a set. Fraction circles lend themselves to discussing fractions as a part of a whole. The geoboard, or any whole object that can be divided into equal parts is good to use. Unifix Cubes or Multilink Cubes lend themselves most appropriately to discussing fractions as a part of a set.

Strategy: Food Fractions (grades k-6)

Take Kraft individually wrapped cheese squares to discuss the concept of 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$ and so on. Ask students to divide the square into equal thirds. Try to make equivalent fractions such as $\frac{1}{2}$ and $\frac{2}{4}$. After the student has demonstrated the fraction, it can be eaten. Apples can be used in the same way.

Strategy: Simple Fraction Concepts (grades 2-4)

For this activity, you will need fraction pieces. Make the fraction pieces from sheets of stiff, colored paper. Cut six inch circles into halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and leave one whole. Divide the students into small groups. Have each group use the fraction pieces to do these activities.

Show $1/2$, $3/4$, $5/12$, $1\ 2/3$, and $3\ 3/5$.

If you have three "wholes", how many halves will you need to cover them?

If you have two "wholes", how many fifths will you need to cover them?

How many ways can you cover a $1/2$ piece with other pieces?

Use the pieces to complete these equations:

$$1/2 = \text{how many } 1/4\text{s?}$$

$$1/2 = \text{how many } 1/6\text{s?}$$

$$1/2 = \text{how many } 1/8\text{s?}$$

$$1/2 = \text{how many } 1/10\text{s?}$$

$$1/2 = \text{how many } 1/12\text{s?}$$

Do you see a pattern? Have the students explain.

Geometry Strategies

Geometric shapes and patterns are all around us. Learning disabled students should be given an opportunity to explore this area of math to help them become aware of sizes, shapes and patterns. Geometry in the classroom should take an informal approach. Geometry should be taught intuitively, allowing the student time to explore and experiment with manipulatives.

Strategy: Home On the Range (determining area) (grades 3-6)

Take two large green sheets of paper and place a toy horse and four equal-sized cube blocks on each sheet, at random. Ask students if there are the same number of blocks on each paper. How do they know? After they count the blocks, tell the students the papers represent green fields of grass for each horse and the blocks represent barns. On one field, place all four of the blocks in a flat square together. On the other, make two groups of two blocks each. Tell the students the farmer has built his barn in two fields. In one field, all the barns are together. In the other, there are two groups of barns. Which horse has the most grass to eat? Ask how they can find out. Let the students move blocks or draw pictures to figure out the solution (Scheer, 1984).

Strategy: Geoboards (determining area and perimeter) (grades k-3)

Use rubber bands and pegs to make square shapes on the board. Each square is one unit. Find the area and perimeter of a one-unit square, then a 4-unit square. Find how many different squares and rectangular shapes can be made, and record the area and perimeter. For the older student, when rectangles and squares are mastered, stretch a rubber band diagonally, to make a triangle, challenge the students to find the area of the triangle. The student should soon see that the area of a triangle is half the area of the rectangle with the same length and width (see Figure 4).

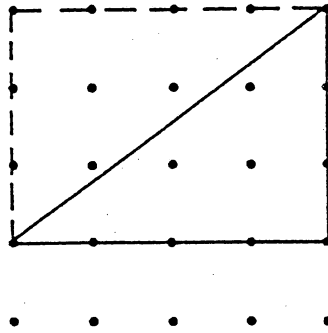


Figure 4. Area of a Triangle.

The area of the square is 12 square units. The area of the triangle, therefore, is $\frac{1}{2}$ of the 12, or 6 square units.

Time and Money Strategies

The learning disabled student needs the hands-on experience of moving the clock hands to learn how to tell time. The same is true with money. There is no substitution for using clocks and play money to help the student with time and money. Parents should be encouraged to allow the student to make change and count money when going to the store. Time should be practiced at home with the analog clock. The more practice with time and money in the real world, the more comfortable the student will become.

Strategy: What Time Is It? (grades 2-3)

Have each student make a clock as shown in Figure 5. Divide the class into teams of 4. Child one from each team is the recorder at the chalkboard. Child two uses a clock to display the time. Call out a time. child one must record the time, and

child two must display it. The other children on the team may advise the teammates. The first team to get both correct win one point. Five points win the game. Use clocks with minute marks for determining time to the minute (see Figure 6).

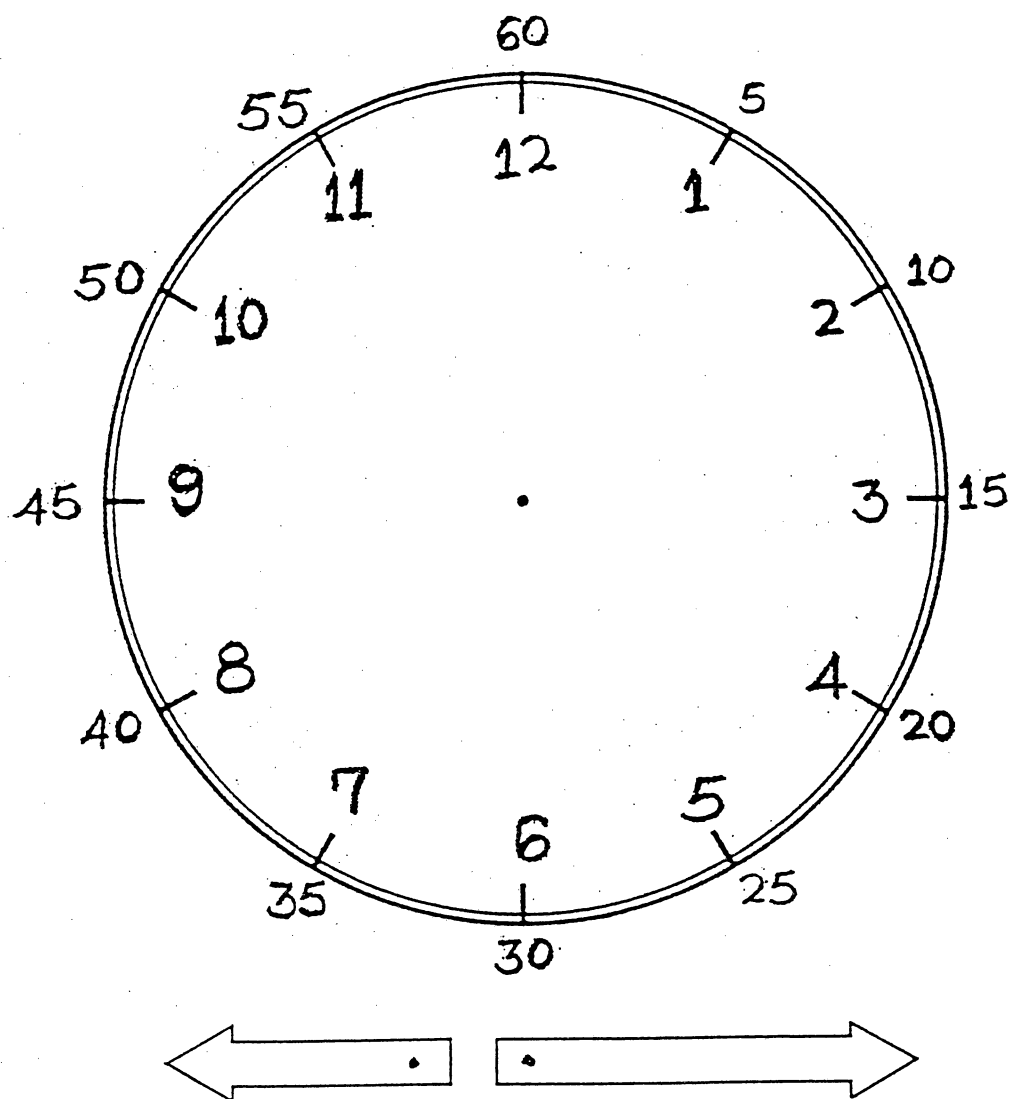


Figure 5. Clock (Five Minute Intervals).

Use this clock for determining time to five minute intervals.

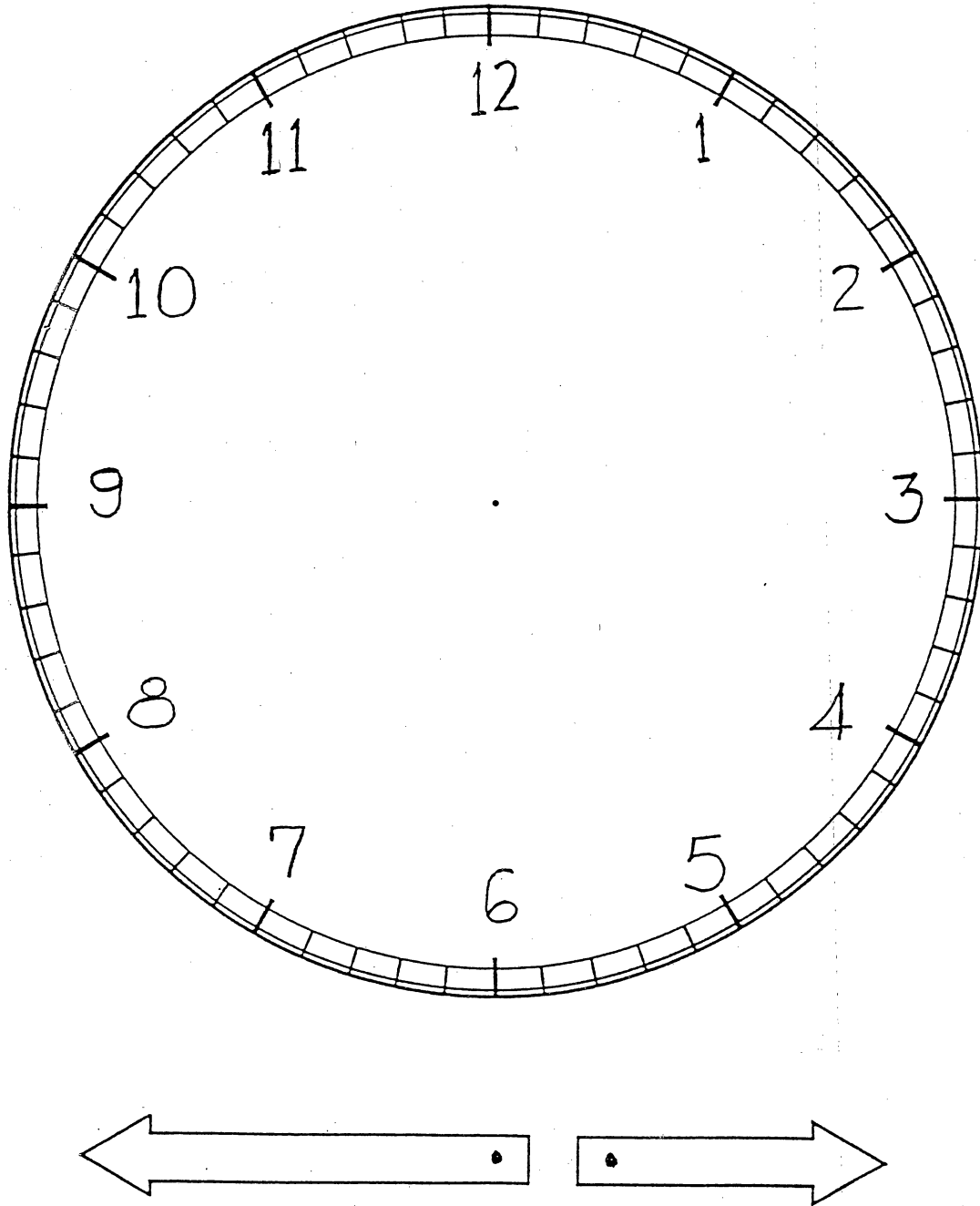


Figure 6. Clock (One Minute Intervals).

Use this clock for determining time to the nearest minute.

Strategy: Money: Heads or Tails (grades 1-3)

Ask the student to make rubbings of each coin, "heads" and "tails". On the paper, below the rubbing of each coin, ask the student to write the name of the coin and the value of each coin. When the student is familiar with each coin, pair up the students. Give each pair of students a set of all the coins in a paper sack. Each student should have a pencil and paper. One student reaches in the bag and pulls out a coin. The student will flip the coin in the air, but first, a prediction is made by the student whether the coin will be "heads" or "tails" when it lands. If the student is correct after tossing the coin, he or she receives the value of the coin in points. Each student takes turns until 100 points, is reached by one student(the winner).

Measurement Strategies

This is another area that can be an informal type of learning experience. Real world objects should be used when exploring and experimenting as much as possible. Use the ruler or yardstick when learning about inches and feet. Use cup, pint, quart and gallon containers when learning about volume. Use thermometers when learning about temperature.

Strategy: Measuring/Graphing Using Cuisenaire Rods (grades 1-3)

Ask each child to trace his/her hand on a blank piece of paper. Using white, purple, and orange rods to measure the length of the outline. Leave the rods on the paper while they record their numbers. Arrange each outline in order, from shortest to longest. Make a graph by cutting strips of black construction paper to correspond to the lengths of the children's hands. This could be displayed on a

bulletin board. This same activity can be done with the foot. Extend the lesson by asking the students to find out how many different combinations of rods can be used to measure the length . Use colored paper to correspond to the color of the rods. The resulting graph will illustrate the equivalent measurements.

Strategy: Straws and String (determining length) (grades 3-6)

Take two straws and two pieces of string the same length. Cut one of the straws into several pieces and form curves in one of the pieces of string. Show the students the two straws and the two strings and ask if the cut and curved samples are the same length as their whole counterpart. Ask students how they can find out if the straws and strings are the same length. Give each pair of students a sample, and allow them to play with the material. Hopefully, someone will suggest stretching the string and putting the straw back in line (Scheer, 1986).

Using the Calculator and Manipulatives

Calculators can be used in conjunction with manipulative materials to develop number concept, counting skills, and place-value concepts. As pupils listen to coins, counters or other objects in a jar, one by one, the students can enter the numeral to display the total on the calculator. The student may also use the constant-function feature of a calculator to count the sounds they hear. The student enters "+1". When "=" is pressed, the calculator displays the total number of sounds or counters. The constant-function feature can also be used to learn the patterns for counting base-ten materials.

By using the calculator to explore the various arrangements of the same place-value pieces, students build strong and flexible images of numbers that can then be used to construct meaningful procedures for mental computation (see Figure 7).

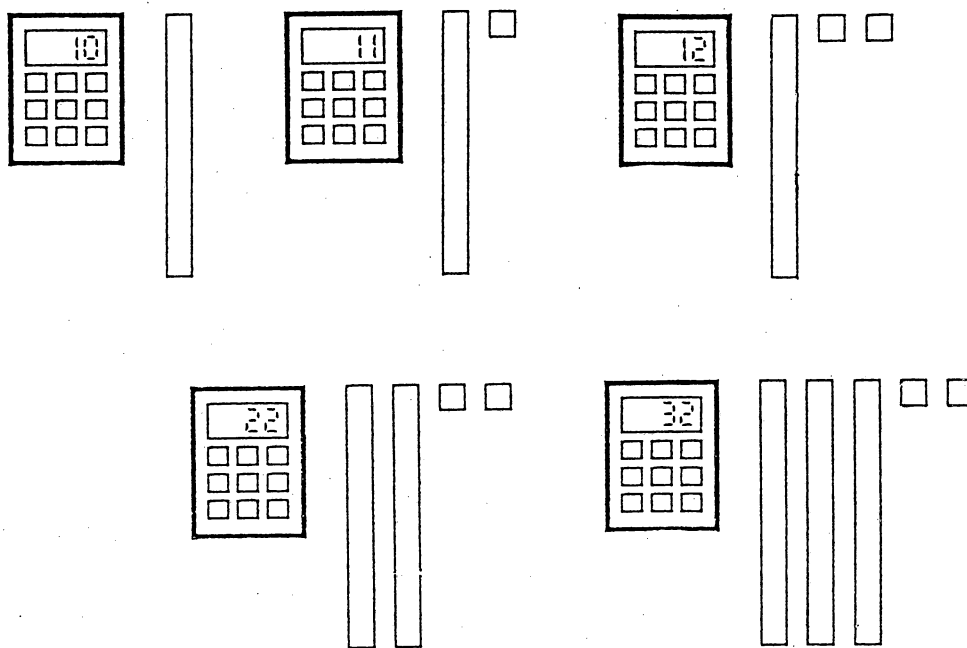


Figure 7. Using the Calculator.

Using the calculator reinforces place-value concepts.

The calculator can be used to generate data to investigate patterns in and relationships between numbers. How does subtracting one over and over from ten compare to subtracting two over and over? What happens if you keep subtracting after you reach zero? What happens if you repeatedly subtract two starting at eleven, then sixteen? How are these patterns alike and different? Students can keep a visual record of their investigation by coloring the appropriate squares on a hundred chart (see Figure 8).

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

Figure 8. Hundred Charts.

Patterns can be generated on a hundred chart by repeatedly subtracting 2 from 10, 11 and 16.

The calculator can be used to investigate the operations of addition and subtraction. Again, the calculator should be used as a recording device, along with the manipulative material. Involvement in calculator activities to develop number and operation sense enhances rather than prevents learning of basic computation facts. It also promotes development of better reasoning skills in computation (Schielack, 1991).

Using the Computer and Manipulatives

Appropriate computer software can also offer the support needed for the learning disabled student. Many software packages on number recognition, counting, basic facts, and other traditional topics have been developed that are visually appealing to the student. Many software packages have been developed to involve students in problem solving and higher order thinking. The computer is a useful tool to enhance instruction and brings an experience which can happen only with technology.

The following is a list of programs appropriate for students with learning disabilities. All of these programs are MECC programs (see appendix for MECC address).

Addition Logican: Designed for third grade students, this program focuses on whole number addition involving regrouping.

Arithmetic Critters: Features four programs that expand on counting skills for grades k-1.

Circus Math: This program is designed for second and third grade students and focuses on whole number addition.

Conquering Decimals: There are two separate discs, one for addition and subtraction, and one for multiplication and division. Drill and practice sessions and educational games give practice in this area for grades 4 and up.

Conquering Fractions: There are two separate discs, as in Conquering Decimals. One provides practice with problems in addition and subtraction of fractions, and includes two content-related games that may be used for reinforcement. The other provides students with practice in multiplying and dividing combinations of fractions, mixed numbers, and whole numbers. Two games reinforce the instructional objectives introduced in the programs for grades 4 and up.

Conquering Whole Numbers: Includes practice in the four basic math operations with multiple digits, regrouping, problem-solving, and factoring skills with new improved versions of "Bagels" and "Tax Collector" for grades 3-6.

Counting Critters: Five colorful programs drill students on beginning counting skills for grades k-1.

Decimal Concepts: This program introduces the student to the meaning of decimal numbers and provides practice with naming, comparing, and rounding decimal numbers, and place value for grades 3-6.

Early Addition: Drill and practice in whole number addition for grades k-3.

Fraction Concepts, Inc.: Beginning fraction concepts are introduced for grades 4-6.

Fraction Munchers: Provides practice in proper and improper fractions, equivalent fractions, and fraction types for grades 4-6.

Measure Works: This program introduces concepts of liquid and weight measurement (English or metric). Basic concepts of perimeter and area are also presented for grades 1-3.

Money Works: This is an animated program that provides practice in identifying and counting coins, determining value of coins and giving change for grades 1-4.

Multiplication Puzzles: This program focuses on whole number multiplication for grades 3-4.

Number Munchers: A program that features an educational game to reinforce basic math skills and concepts for grades 3 and up.

Path Tactics: Contains basic math operations for whole numbers and is ideal for all grades.

Problem-Solving Strategies: Problem-solving strategies are introduced for grades 4-6.

Problem-Solving With Nim: A program, based on the ancient game of Nim, that helps with problem-solving strategies and logical thinking skills.

Quotient Quest: Students in grades 4-6 enjoy this program that focuses on whole number division.

Space Subtraction: This program focuses on whole number subtraction.

Speedway Math: Provides students with practice in the quick recall of math facts.

Teacher opinions allow content to be modified for grades 1-6.

Subtraction Puzzles: Whole number subtraction is the focus for this third grade program.

CONCLUSION

The purpose of the curriculum project was to provide strategies for implementing the use of math manipulatives in the classroom. These suggested strategies are only a few of the infinite number of strategies that can be used. The areas of strategies discussed were addition, subtraction, multiplication, division, fractions, geometry, time and money, measurement, calculators, and computers.

SUMMARY

This project was written in an effort to emphasize the importance of the use of math manipulatives in the classroom with the learning handicapped student. Several questions were addressed in order to understand the importance of the use of math manipulatives in the classroom. They were:

What is a math manipulative?

What are the popular kinds of manipulatives currently in use?

What are the important characteristics of math manipulatives?

What is the history of math manipulatives?

Why should math manipulatives be used?

How effective are math manipulatives?

When should manipulatives be used?

How can manipulatives be used with the learning handicapped student?

Through answering these questions, it was evident that learning handicapped students should be allowed and encouraged to conceptualize abstract ideas through the use of math manipulatives. The results of the studies by theorists such as Piaget, Montessori, Brownell, Bruner, Skemp, Dienes and Gagne for using manipulative material is persuasive. Learning theorists suggest that children whose mathematical learning is firmly grounded in manipulative experiences will be more likely to bridge the gap between the world in which they live and the abstract world of mathematics. The student with a learning handicap especially needs to have the concrete models in order to better understand mathematical concepts.

APPENDIX

Distributors of Materials

Activity Resources Co., Inc., Box 4875, Hayward CA 94540

Book-Lab, 500 74th Street, North Bergen, NJ 07407

Burt Harrison & Co., P.O. Box 732, Weston, MA 02193-0732

Creative Publications, 5005 West 110th Street, Oak Lawn, IL 60453

DLM Teaching Resources, P.O. Box 4000, One DLM Park, Allen, TX 75002

Didax Educational Resources, 5 Fourth Street, Peabody, MA 01960

Educational Teaching Aids, 159 West Kinzie St., Chicago, IL 60610

Educators Publishing Service, 75 Moulton Street, Cambridge, MA 02138

Ideal School Supply Co., 11000 South Lavergne Ave., Oak Lawn, IL 60453

Janian, Box C16, Port Roberts, WA 98281

MECC, 6160 Summit Drive North, Minneapolis, MN 55430-9851

Scott Resources, Inc., Box 2121, Fort Collins, CO 80522

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