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Teaching multiplication and division to learning disabled children

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TEACHING MULTIPLICATION AND DIVISION

TO

LEARNING DISABLED CHILDREN

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TEACHING MULTIPLICATION AND DIVISION
TO
LEARNING DISABLED CHILDREN

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ACTIVITIES FOR TEACHING MULTIPLICATION AND DIVISION
TO
LEARNING HANDICAPPED STUDENTS

INTRODUCTION

Little definitive research has been completed in the area of arithmetic disabilities. The research has been concentrated in the areas of language and reading. This project is designed to provide an over view of the current research available. The research will provide valuable information regarding the design of remediation instruction in the areas of multiplication and division.

A child who is experiencing significant problems in the area of arithmetic is usually labeled as having acalculia. Acalculia is defined as, "The loss of ability to manipulate arithmetic symbols and to perform simple mathematical calculations."[1] Dyscalculia is a term that is more commonly used. "Dyscalculia is a partial loss of the ability to calculate and to manipulate number symbols."[2]

This project is specifically designed to address the needs of the learning disabled student in the areas of multiplication and division. Kirk and Gallagher provided the most acceptable definition of a specific learning disability in 1979.

> "'A specific learning disability is a psychological or neurological impediment to spoken or written language or perceptual, cognitive, or motor behavior. The impediment (1) is manifested by discrepancies among specific behaviors and achievements or between evidenced ability and academic achievement, (2) is of such nature and extent that the child does not learn by the instructional methods and materials appropriate for the majority of children and requires specialized procedures for development and (3) is not primarily due to severs mental retardation, sensory handicaps, or lack of opportunity to learn.'"[3]

The purpose of this project is to fill three general needs. First, to examine the information gleaned from the research on

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teaching arithmetic to learning disabled students. The information will provide important insights into the development of a child's ability to perform arithmetic operations. The information will lead to the formation of conclusions regarding the design of affective teaching techniques and the types of instructional material that will provide the most success. The research will also define the stages of development through which a child's arithmetic ability evolves.

Second, this project will examine some common characteristics of the learning disabled child as they relate to the process of learning arithmetic. This information together with the literature review will help shape the activities offered for use with a learning disabled child.

Third, this project will offer activities to provide instruction and reinforcement in teaching multiplication and division to learning disabled students. The activities will provide alternative strategies to aid students in the mastery of basic multiplication and division concepts. The activities will be based on the information derived from literature.

Learning disabled children require modifications in the teaching strategies and frequent opportunities to succeed. A child with a learning problem in the area of arithmetic will not likely learn the necessary skills in a class with many students and the traditional instructional methods. Learning disabled students require frequent practice on the single skills to internalize the skill and to be able to transfer the learning. The learning disabled student must be taught such basic skills as

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multiplication and division in very small steps. Each step must be designed to motivate and allow the student to feel success. The activities that are suggested in this project provide variations on the same skill practice.

The activities that are suggested will provide guided practice and the opportunities to diagnose ability levels in a small group setting. The learning disabled student will not be faced with the traditional pencil and paper tasks that are presented in the large classroom. Competition is at a minimum and motivation is increased. These activities can also be modified or redesigned to accommodate a particular need a child may have. For example, the instructions for a particular activity might be on a taped cassette for the child who also has difficulty with reading.

The information offered in this project is intended to suggest only basic methods of presentation. The instructor will modify the suggestions to suit individual needs.

LITERATURE REVIEW

Researchers and educators have recognized the problem of arithmetic disabilities in children of normal intelligence since the beginning of this century. Educational researchers have investigated the possible causes and tried to develop successful remediation programs. Though there is a research available on arithmetic disabilities involving learning disabled children, there is considerable agreement within the literature that arithmetic disabilities have not been investigated as exhaustively as reading or language. In fact, many authors agree that there is a lack of conclusive research on the subject. In _Teaching Mildly Handicapped Children: Methods and Material_, the authors addressed this lack of information. "Arithmetic has received far less attention in regular and special education than has reading. The stress on mathematics diminished significantly after the intensity of the 1960's and the influence of the 'new math.'"[4] Gerald Wallace and James Kauffman also cited the lack of information in _Teaching Children with Learning Problems_.

> "The difficulties encountered by children with
> learning problems in arithmetic have not been as
> thoroughly researched or studied as other academic
> handicaps. In comparison to reading, for example,
> teachers will find fewer available tests, remedial
> programs, and instructional materials for helping chil-
> dren with arithmetic problems. It has been suggested
> that the widespread concern for making children liter-
> ate, along with the feelings on the part of some teach-
> ers and parents that arithmetic is not as vital to
> academic success as other areas of the curriculum, may
> be partially responsible for this lack."[5]

Throughout the literature on arithmetic disabilities the authors mention the lack of intense research in the area.

However, there is some information available that has attempted to explain the nature of the ability to comprehend arithmetic, the development of the ability and its implications for teaching the learning disabled child.

This review of the literature will trace the major forces behind the conceptualization of arithmetic abilities and the major arithmetic curriculum trends that have developed. Many of the curriculum trends can be traced directly to some of the most influential educational theorists.

The work of Piaget provides a connection between arithmetic curriculum and cognitive development. Certain cognitive abilities are believed to permit the understanding and uses of specific kinds of arithmetic skills within each stage.[6]

The first stage that Piaget identified is the sensorimotor period, from zero to two years. The period is characterized by the development of perceptual information that has been obtained through the senses of touch, taste, smell, manipulation, sight or hearing. The information obtained through the senses is later incorporated into higher levels of cognition. At the end of the sensorimotor period the child is beginning to understand the concept of time, symbolic thinking, and cause and effect relationships.

The second major developmental period is the preoperational period. In the normally developing child it occurs between the ages of two to seven years. The child begins to realize the use of a symbol system. During this period the child is unable to reverse mental computations, for example, the child may be able to understand that 3+4=7, but may have great difficulty under-

standing that 7-4=3. A child functioning at this stage will also
be unable to conserve quantities.

The concrete operational stage usually includes the ages
from seven to eleven years old. During this stage the child is
able to developed the concept of equivalent sets and is not misled
by the arrangement of the objects. The child at this stage has
developed the ability to conserve. During this period the child
is capable of the higher order skills of subtraction, multiplica-
tion, and division.

The formal operations stage is characterized by the child's
ability to think abstractly. The child is able to use only
mental processes to reason without the use of concrete objects.
The child is able to complete arithmetic operations through
mental calculations.

The following chart summarizes the correlation between
Piaget's developmental stages and the sequence of arithmetic
skills that emerge during those stages.

AGE	CONCEPTS
3 to 5 years	Rudimentary concepts of more, bigger, smaller, etc.
	One-to-one correspondence
6 to 7 years	Cardinality
	Ordinality
	Conceptualization of a set
	Joining sets
	Place value
	Addition
	Concept of equivalency
	Conservation of numbers
	Reversibility
	Rational counting
	Transitivity

```
                          Subtraction
                          Part-to-whole fractions
                          Multiplicative relationships
                          Mastery of geometrical forms

8 to 9 years              Parallelism
                          Three-attribute classification
                          Associative property of addition
                          Distributive property of multiplication
                          Commutative property of addition
                          Fractions

10 to 12 years            Percentages
                          Proportions
                          Probability
                          Conservation of weight and volume
                          Geometry    7
```

The Piagetian stages are important in the instruction and
design of arithmetic curriculum for a learning disabled child.
They provide an understanding and a guide to the limitations and
the uniqueness of a given developmental period. The teacher of a
learning disabled child must continually evaluate the level of
functioning of a student and teach to that level.[8]

In *Science of Education and the Psychology of the Child,*
Piaget directly addresses the teaching of mathematics. He stated
that mathematics is a direct extension of logic. Logical mathe-
matical structures are not naturally or consciously present in
the child's brain. The brain has structures that allow the child
to reason logically that act to direct the logical ability of
the child. Piaget uses the comparison of a person who is able to
sing a tune without being knowledgeable in music theory. The
process is automatic and intuitive not requiring deliberate rea-
soning abilities.[9]

To use mathematics, the child is required to consciously
think about the structure and the processes that require the use

of symbols and abstractions. The use of arithmetic requires the child to make a transition from natural, unconscious abilities to the conscious awareness of the necessary structures.

There are three major implications of Piaget's theories. The first is the necessity to recognize that each child follows an established developmental sequence. Second, each stage of development can be enhanced through the use of manipulatives. Concrete experiences establish concepts and facilitate the transition from concrete to symbolic thought. Third, the arithmetic skill sequence must match the developmental stage of the child. Arithmetic teaching cannot be successful unless the child's developmental thinking allows the information to be assimilated.

Jerome Bruner also presented theories that are directly related to the development of arithmetic teaching theories. Bruner divided knowledge into three components: enactive, iconic, and symbolic. These three forms of knowledge serve to explain how information is stored and retrieved.

Enactive knowledge comes through physical action. It is knowledge entirely on a concrete level. Enactive knowledge may be represented by the ability to walk or to throw a ball. There is no involvement of symbolic interpretation. This level involves the manipulation of objects.

Knowledge at the iconic level has visual or perceptual organization. It may involve communication through pictures or symbols. An example of iconic reasoning is the mental manipulation of concrete objects.

The third stage is the level of symbolic thinking. This

level is the beginning of higher levels of thinking leading to abstractions and problem-solving. Using symbols to calculate an arithmetic problem is a good example of symbolic thinking.

Bruner and Piaget share a common belief that learning is a series of sequential steps. Bruner's stages of development are compatible with Piaget's. Learning takes place in steps and therefore, teaching should follow the same pattern. A learning disabled child must be taught arithmetic skills with these developmental stages clearly understood. All arithmetic knowledge must be founded in a mastery of previous learning. Bruner and Piaget also share the belief that arithmetic education involves physical manipulation, especially in the early stages. Both feel that the manipulation of concrete objects allows the child to internalize relationships as they experience comparisons and discover how to use numbers.

Robert Gagne provided a somewhat different view of learning that has had a major impact on arithmetic curriculum. Gagne calls his approach to learning "instructional" or "learning hierarchies."

Gagne's theory involves the identification of a terminal goal. The goal is stated as the behavior that the student will be able to perform at the conclusion of the instruction. The terminal goal is carefully analyzed and broken down into component skills or enabling skills. This process becomes a task analysis that involves the careful listing of all of the components of a skill or concept that will lead to a mastery of the terminal behavior. Each of these enabling skills has been

the terminal behavior of some previous learning hierarchy.

The third stage of Gagne's learning design is to carefully sequence the enabling behaviors. This final step is the most important step in the design. These sequenced skills are built upon intact skills, those skills already considered mastered and ready for immediate recall.

Gagne's instructional design is commonly reflected in the design of an arithmetic text or workbook. In arithmetic texts the student is taught numerous basic skills before simple multiplication or division is introduced. The developmental level of the child is only of secondary concern as it is reflected in Gagne's theory. However, Gagne's theory is a very successful strategy when it can be used with individual children, especially those children who are learning disabled.

Gagne's task analysis and sequential arrangement of skills provide an outline of the skills that will lead to a final goal. The instructor will be able to continually monitor the progress toward the final goal. Gagne's learning design does not mention the method which is used to teach the skill. His theory is only concerned with the mastery of the required skill.

> "'The activities for each of the major mathematical
> concepts and skills [must be] analyzed into a detailed
> and graduated series of steps [steps more finite and
> elemental than for the normal child]. Each step [must
> aim] at an immediately attainable, limited aspect of
> the concept. More importantly, the criteria by which
> achievement of the objective of each step is determined
> must readily be seen by the child himself. The lessen-
> ing of the learner's reliance on outside authority, and
> resultant increasing self-evaluation, bring benefits to
> self-esteem which cannot be overemphasized.'" 10

The above quote illustrates a teaching technique that has

developed and is commonly used to teach arithmetic skills to learning disabled children. This technique of task analysis seems to come directly from Gagne's learning theories. Each behavior must be broken down into the most basic of skills. The child cannot be taught to multiply if he or she does not possess numerous enabling skills. To perform a simple multiplication problem the child must be able to identify and to write numerals correctly, to have basic directional skills, to understand the process of addition and to have mastery of the operation, the child must be able to recognize the operational sign and to have some understanding of the process goal. These are only a few of the enabling skills that would be necessary to achieve the terminal goal. The step that is equally important is that the skills be precisely sequenced and taught in the correct order. Gagne's theory of analyzing foundation skills and ordering them seems to be the basic theory behind task analysis. Task analysis is used along with diagnostic-perscriptive teaching for learning handicapped children. Both of these strategies take into account the need for developmental teaching and the need for careful sequencing of instruction.

Regardless of the arithmetic operation, the task analysis approach can help teachers pinpoint the specific arithmetic subskill where the child is having difficulty. For example, if the child is having difficulty adding a two digit multiplication problem, the problem may be that the child needs help with visual-spatial memory to help him or her remember in which column to begin adding, the child may need help with remembering basic

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math facts.   Error analysis has become an affective tool to use
in analyzing subskills needed to achieve a final goal.[11]

$$
\begin{array}{r}
26 \\
\underline{\times 12} \\
52 \\
26
\end{array}
$$

A second instructional technique that has developed is diag-
nostic-perscriptive teaching. Johnson noted in 1979 that "'It is
virtually impossible to isolate the diagnostic procedure from the
perscriptive remediation process.'"[12]   Johnson's statement des-
cribes the trend toward diagnostic-perscriptive teaching of the
learning handicapped. Diagnostic-perscriptive teaching involves
a continual process of assessment and reevaluation of the pro-
gram, curriculum, and strategies. Modifications of the teaching
is made thoughout the program. Diagnostic-perscriptive teaching
has been used to teach learning handicapped children for the past
twenty to thirty years. It is a program that assumes that a
child will change and that a single strategy will not be success-
ful every time it is attempted. Diagnostic-perscriptive teaching
also assumes that a series of sequential steps will need to be
followed, the teacher may need to back up to reinforce a founda-
tion skill that has been noted to be particularly weak. The
process is cyclic and never static. It assumes that growth and
change will take place in the child and therefore, depends on
flexibility for its success.

Historical attempts to develop arithmetic remediation
programs are few. In 1943, Grace Fernald offered guidelines for
a remedial arithmetic program in her book Remedial Techniques in

Basic School Subjects. The Fernald program was based on multi-sensory training and intense practice with computational prob-
     13
lems.

In 1947, Strauss and Lehtinen researched arithmetic disabilities. They concluded that the problem involved a disruption in visual spatial organization and emphasized the importance of the child to be able to recognize the parts of a
     14
whole.

In 1965, Jansky wrote a seven page article dealing with arithmetic disabilities after a study of two children. Since that time few authoritative articles or methods have been devel-
                                                    15
oped, and certainly the research has been limited.

In 1966, Elizabeth Friedus stressed the need for recognizing developmental stages and teaching learning disabled children arithmetic skills in a sequential and progressive program. She emphasized the need to teach arithmetic to learning disabled children should be based on concrete experiences. She outlined an arithmetic program that is based upon developmental stages and
                                        16
a sequential ordering of arithmetic skills.

Friedus' first stage is the ability to group objects so that they can be counted. The learning disabled child must first learn to group objects into sets. The idea of "sameness" may be taught at this stage using the characteristics of the objects. The learning disabled child must be able to recognize figure-ground relationships so that a single object may be focused upon.

The second step in Friedus' developmental program includes matching and sorting activities to prepare the learning disabled

child to make comparisons among objects. At this stage the child will learn to make comparisons of bigger to smaller and more or less. With children with learning disabilities such simple relationships usually must be taught carefully and mastered before the students are ready for any more complex tasks.

After the child has learned to compare and to group objects, he or she is ready to begin comparing sets of objects. The sets are first compare informally by estimating, for example, asking the child if the box is big enough for the ball to go into.

The next step is to teach numerals in sequence and match the numerals to sets of objects. The child with a learning disability may find it helpful to move the object as it is counted.

Numerals are then arranged in sequential order. At this step the child should be presented with many sets from which to choose the sets of like value. The child will become comfortable with the concept of "twoness" or "threeness."

The following stage, learning the relationship of the parts to the whole, should include repeated practice and review. The materials should include manipulatives and self-correcting activities. The materials should be designed to allow the learning disabled child to use a discovery process in understanding the relationships.

After mastering the previous sequential steps, the learning disabled child should be ready to learn to manipulate number facts and begin learning arithmetic operations. The addition number facts, one through ten are taught first with concrete activities.

Friedus suggests that the decimal system be taught during the final stages of instruction. She suggests using number lines, ten square counting frames, dimes, pennies, checkers or poker chips.

Friedus felt that if the learning disabled child is taken methodically through these basic steps that arithmetic skills will be learned. She believes that teachers often neglect to assess previous learning experiences and therefore, do not begin instruction at the level or stage that the learning disabled child requires.

The following five points have been summarized by Friedus as her approach to teaching arithmetic skills to the learning disabled:

1. The teacher must be able to break down a skill into its separate, sequential elements.

2. The teacher must have a thorough understanding of how children normally develop the elements of a skill.

3. After analyzing the skill and knowing the sequential stages of development, the teacher must be able to identify the level at which the child's development has been disrupted and what adaptations or compensations the child has made in response to the disruption.

4. The teacher must monitor the results of teaching continuously and must modify procedures when necessary.

5. The teacher must look at the child as an individual, not as a member of a group. The child must be the guide for instruction. 17

The literature agrees that teaching arithmetic to learning disabled children relies on the accurate assessment of the developmental level of the child. Though the child may be developing normally physically, children with perceptual processing deficits have not progressed normally in their

development of fundamental skills. The child must be instructed

at the lowest funtioning level at which some success is achieved.

All of the research on teaching arithmetic agrees that any

curriculum program requires a definite sequence of steps which

build upon previous learning. All of the research completed by

Piaget, Bruner, Gagne, and Friedus is based upon a theory of

logically sequential steps toward a higher level skill. Piaget's

theory is based upon the steps through which cognitive develop-

ment moves. Bruner emphasizes only three basic levels of deve-

lopment which are sequential and dependent on the mastery of

lower level concrete thinking levels. Gagne's learning theory

also depends on an ordering of skills. It is not a developmental

design as with Piaget and Bruner, but an ordering of behaviors

that ultimately lead to a final goal. The sequence of enabling

behaviors must be precise for the final goal to be accomplished.

It can safely be concluded that a major trend in the design

of arithmetic curricula is recognizing the need for a strict

sequencing of learning steps, whether they are developmental or

enabling skill sequences. Another implication is the necessity

for using concrete manipulatives throughout an arithmetic

program. Piaget, Bruner, Gagne, and Friedus base their learning

theories on teaching arithmetic concepts through experience.

Montessori and Fernald also have based their instructional

programs on a manipulation of concrete objects.

Columbia University organized CAMP (Concepts and

Applications of Mathematics Project) in 1968. This project

addressed the problems of teaching arithmetic to learning
handicapped children. The project offered several suggestions to
improve the arithmetic education of the learning handicapped
student. The first suggestion was to design mathematics
laboratories in which multisensory aids, such as overhead
projectors, measuring devices, and calculators could be used.
The second suggestion was to use flow charting as a device to
analyze a problem and to break it down into basic parts. The
flow charting forces the student to ask and to answer questions
about the problem to focus attention and solve the problem in the
correct order.

The third suggestion offered by CAMP was to encourage
students to use calculators. Students would be motivated by the
machines, they would be able to check their work and become
familiar with the equipment.

Fourth, the study suggested the use of tapes with lessons for
drill that are especially affective for students who are non-
readers. Programmed materials were also suggested for use with
learning disabled students because they provide immediate
feedback.
18

In Teaching Mathematics to Children with Special Needs, the
authors offered the results of recent studies addressing the
mathematics education of learning disabled children. The studies
found that the more time that a child was actually involved in
solving an arithmetic task the greater the achievement in
arithmetic. When a student is kept on task and directly involved
in finding the solution of the problem the child's understanding
of the process seemed to increase.

The studies also found that activities should also be designed to lead to success. Success with one learning experience will lead to another. The child's motivation will increase and so will self-esteem.

The diagnostic-perscriptive teaching process was also found to be very successful with learning handicapped students. The teachers with the better diagnostic skills were able to modify teaching strategies and curriculum immediately.

The increased time spent providing students with direct feedback increased both time on task and achievement. Aids, parents, and volunteers also provided increased feedback. Small group instruction may provide the best setting for mathematics instruction. It allows the teacher to provide quicker feedback to a larger number of students and allows the students to receive feedback from their peers.

In 1973, Brainerd introduced research that offered a sequential development of arithmetic skills. The first level is to allow children to manipulate objects. Children should be allowed to touch and to discover number relationships on their own. Talking about arithmetic concepts is not productive at the first level of instruction.

The next level is to allow the child to recognize likenesses and differences among objects. As the child learns to discriminate, he or she will learn to categorize by physical characteristics. When the child is able to accomplish this level, differences can be noted and classified which will lead to abstract thinking.

When the objects can be successfully grouped according to a common characteristic, then the concept of sets can be taught. Attribute blocks or any small objects can be used for this activity. When the child can group objects by characteristics then they are ready to compare sets. One to one correspondence is essential to an understanding of basic arithmetic skills.

The child is then taught ordination. Once the child has mastered the ordering of numbers then cardination will follow. When the child is able to classify and categorize by sets, his experience will then lead to the knowledge of equivalent sets. From this level the child is then able to perform simple addition using the associative and commutative properties of addition. The ability of the child to recognize reversibility and and to use conservation will make this process easier for the child.

Subtraction is the next operation for the child to encounter. Subtraction requires the knowledge of addition and is based in reversibility.

Addition and multiplication are closely related. Multiplication is a higher level skill than addition. If the child has difficulty with the multiplication operation it may mean that the addition operation was never mastered.

It is easy to provide concrete manipulatives to teach addition, subtraction, and division, however, the same is not true for teaching multiplication. Bill Gearheart offered the following pattern as a method for teaching multiplication based
20
on addition.

$$2 + 2 = 4 \quad \text{to} \quad 2 \times 2 = 4$$

$$4 + 4 = 8 \quad \text{to} \quad 2 \times 4 = 8$$

$$8 + 8 = 16 \quad \text{to} \quad 2 \times 8 = 16$$

The above system uses addition as a basic for teaching rather than memorizing the multiplication tables.

The literature also suggests that the perceptual organization of the material presented should also be considered and remain constant throughout instruction. If addition is introduced in a horizontal form then practice activities should remain in the same form until the child has demonstrated the ability to complete the operation vertically without confusion.[21]

The second major implication of learning theories on teaching arithmetic to learning disabled children is the emphasis on the use of manipulatives and concrete learning experiences. Children should be directly involved in the process of learning arithmetic. The use of manipulatives is integrally related to the development of meaning. As children work with objects they begin to see relationships. Piaget, Bruner, Gagne, Friedus and Brainerd all based their theories of development on the importance of concrete experiences.

A research study reported in 1984 found that lessons using manipulatives have a high probability of producing achievement in arithmetic for learning disabled children. Achievement was found to improve at every ability level. The study also showed that children do not necessarily need to manipulate materials themselves for all lessons. Watching the teacher use the materials in a demonstration is sometimes as affective.[22]

The use of manipulatives has been a successful teaching

strategy. A number of activities have been developed to further involve the learning disabled child in learning arithmetic skills. Learning multiplication facts can be frustrating and difficult for a learning disabled child, therefore activities such as, movement activities, storytelling and singing can be used to totally involve the child in the activity without making it a tedious and frustrating experience. The more the child is involved with the instruction, the more the child will learn and retain.

"Chisenbop" is a system of finger counting. It has been used as a method of introducing instruction in addition and subtraction and is only useful after place values have been learned. The fingers on the right hand are the ones and the fingers of the left hand are the tens. The fingers are then depressed to represent numbers. This is a method that was developed as a step between the manipulation of objects and symbolization.[23]

Another method of teaching multiplication and division facts to learning disabled children that has developed is through the use of visual imagery. Using a child's natural ability to fantasize can be a very useful strategy for teaching learning disabled children. A child can be asked to imagine simple objects which represent a specific arithmetic problem and then to think through the solution. This is an affective technique for working story problems as well as operational arithmetic problems.[24]

In an extension of visual imagery and guided fantasy techniques, Bill Gearheart suggested a type of symbolic language to

accompany arithmetic instruction. The technique was first used by Furth in 1966 as a method of teaching arithmetic skills to deaf children. It is very common for arithmetic disorders to be accompanied by language problems, therefore, this technique could be of significant value when used with a child who has multiple learning disabilities.

The following is an example of the symbols that could be used.
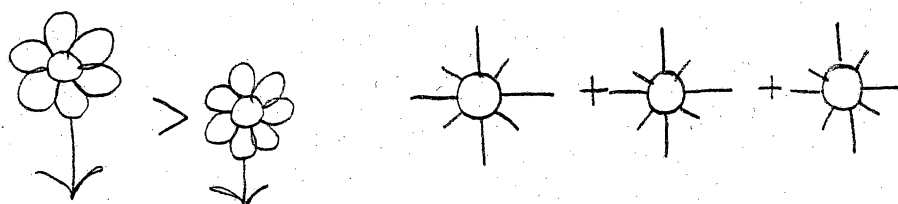
Another method that has been used to reinforce imagery and concrete to symbol association is the overlay of dots on numerals. The dots can be faded as the child learns the numeral.

Teaching the Mildly Handicapped Children offers a logically sequenced program to teach multiplication and division that is based on the use of concrete experiences. Learning disabled children may have great difficulty memorizing basic multiplication facts, therefore, instruction must be based on understanding the basic additive quality of multiplication. The authors suggest the following sequence of steps to teach multiplication to learning disabled children.

1. Concrete experiences should precede graphic representation.

2. Addition should be related to the multiplicative process.

3. Flash cards, the tachistoscope, games, dice and worksheets should only be used to establish competency in immediate recall or mastery of simple combinations after children clearly demonstrate mastery of the underlying process. 26

After children are presented practical application problems, and until the basic simple combinations are mastered, a multiplication table can be used.

Division at the concrete level requires that the child distribute numbers of objects into equivalent sets. Experience with small numbers should be presented numerous times and continually associated with the problem written in the conventional form. Manipulative activities with concrete materials is essential as a prerequisite for for dealing with the computation of a division problem. In Piagetian theory, concrete operational thought is similar to physically manipulating concrete objects. A rapid move from concrete operations to symbolic operations is inconsistent with a learning disabled child's abilities.

Another trend that is being used in the classroom is the practical application of computation to story or word problems. Activities are designed so that the child sees the usefulness of the computation. The activity may also provide motivation and lessen the frustration of tedious computations. Cooking in the classroom is one example of teaching the students the practical applications of multiplication and division operations. There are also materials that require that a child read a restaurant menu to obtain the information for the computations. Resource books, such as Market Math, Menu Math, and Newspaper Math provide

practical applications of multiplication and division. Boys might

be motivated to learn to work a computation to find the average

score of a basketball team's season.

Many children lack sufficient interactive and concrete

experiences necessary for the development of basic arithmetical

concepts. According to investigations by Piaget, actual

manipulation of concrete objects is essential for alteration of
27
mental structures and development of concepts.

There are several arithmetic programs that have been devel-

oped that make use of the basic trends in instruction. The first

is Structural Arithmetic which was developed by Catherine Stern.

The materials include concrete objects, cubes, and pattern

boards. Problem solving is written into the program as a sequen-

tial step.

The Sullivan Arithmetic program is based on programmed

learning. It stresses the sequencing of skills and emphasizes

the mastery of acquisition of computational skills. It provides

a system of immediate feedback and does not require reading

skills.

Cuisenaire rods are frequently used in arithmetic programs

for learning disabled children. They do not constitute a

complete program but, provide supplemental materials. The

materials are rods of different sizes and colors and are used to
28
develop concepts of linear differences.

In conclusion, the literature concerning the teaching of

multiplication and division skills to learning disabled children

seems to present several common trends. The basis of all of the

literature centers on the necessity for a sequential arithmetic
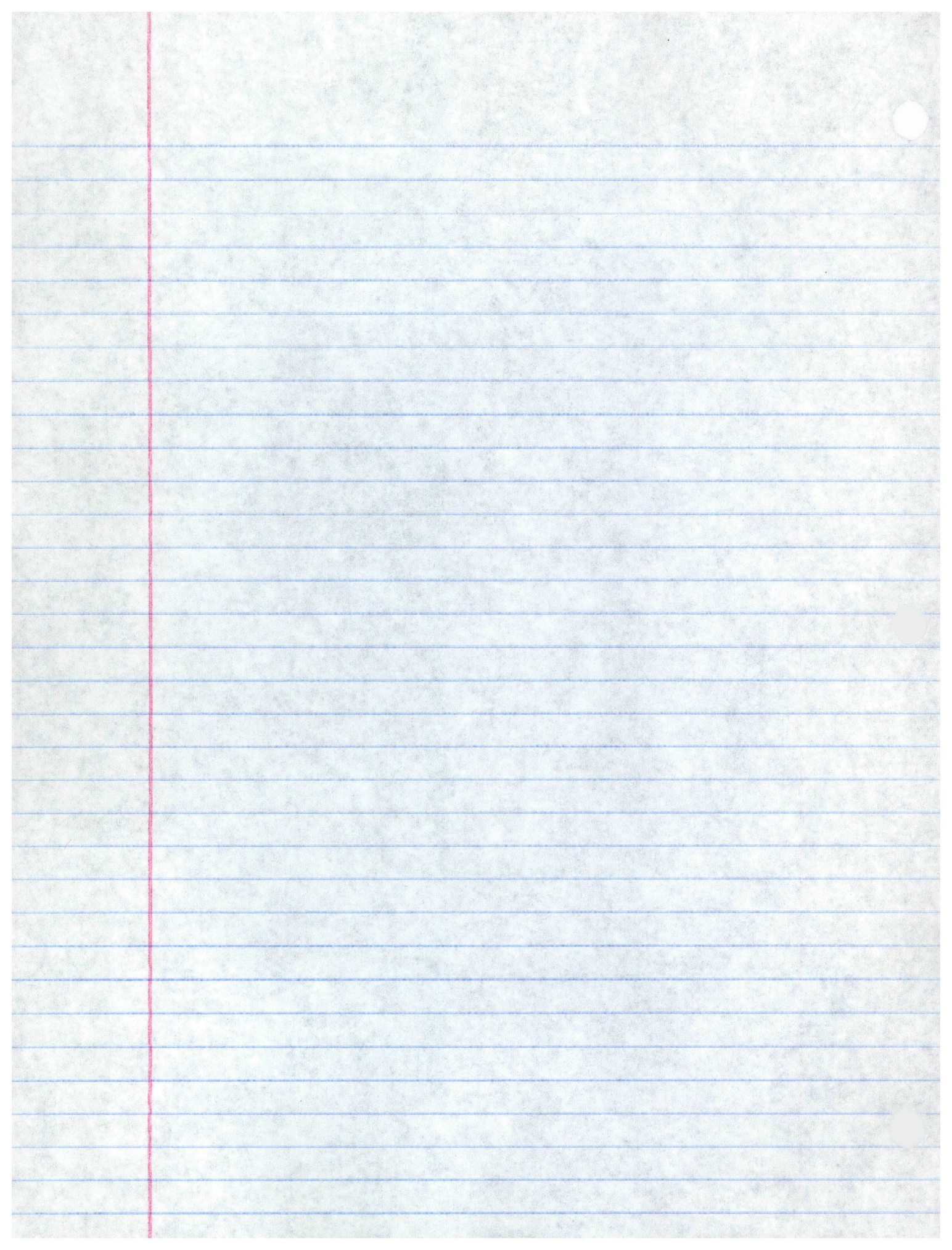
curriculum. The curriculum must accomodate the cognitive devel-

opmental stage of the students and the necessity of mastering

foundation skills before higher level skills, such as, multipli-

cation and division can be learned.

A second thread that connects all of the research is the use

of concrete manipulatives throughout an arithmetic program to

firmly establish basic concepts. The research has shown that

arithmetic achievement is greatly enhanced by the use of manipu-

latives. Other teaching strategies, such as visual imagery,

symbolic arithmentic language and finger counting have been used

successfully as multisensory techniques to increase achievement

in arithmetic. These techniques also have been used to help the

child make the transition from concrete to symbolic thinking.

Diagnostic-perscriptive teaching has developed as an

affective technique for assessing and adjusting the arithmetic

curriculum for learning disabled students. This technique is

based on the assumption that each child has different learning

strategies and a program must have flexibility and be continually

modified to meet the needs of a learning disabled child. During

a lesson, information may suggest that the child does not possess

adequate mastery of a basic skill. If skills are not mastered

the teacher will need to back up and reinforce the weakness

before further learning will be able to take place.

Task analysis has also proven to be a valuable instructional

technique. It breaks down learning into small, sequential steps.

Task analysis helps build success into instruction.

The literature also suggests that multiplication be based in

a mastery of addition. Division should be taught as a disassociative process heavily based in the use of manipulatives. The traditional approach of requiring the child to memorize the basic multiplication and division facts may not be successful with learning disabled students. The instruction should provide alternative reinforcing activities for the child to learn the basic facts without the frustration of rote memorization. A variety of techniques should be used to involve the learning disabled child in the multiplication and division process. Once the child understands the concepts, computation should be related to practical applications. This trend seems to reflect the research of Strauss and Lehtinen, that the learning disabled child must be taught to relate the parts to the whole. By giving the child a purpose for the computation a pattern can be realized and successfully internalized.

The literature has also shown that a modification of a child's routine can increase achievement. A gradual increase in the time on task will increase learning. The activities should be designed to lessen frustration and to increase successful experiences.

> "The quality and the quantity of educational re-
> search into the teaching of arithmetic is low. This a
> serious problem, but it should not deter us from making
> applications of theories and teaching strategies that
> show promise. It should not be surprising to find that
> many problems will yield to intervention of competent
> teaching. Such strategies, would by definition, have
> to be based on the current knowledge of cognitive
> development, arithmetic skills, analysis of error,
> assessment of cognitive development and application of
> appropriate techniques in concert with well designed
> materials. Children cannot wait until research efforts
> are expanded nor can they be expected to make signifi-
> cant gains in arithmetic achievement by concentrating
> on noneducational training." 29

The trends in the literature emphasize the need for a highly structured and well planned arithmetic curriculum. The instructional must be broken down into sequential components to provide an understanding of the basic concepts and applications of the skill. The learning disabled child must be taught each component skill carefully before higher level skills can be attempted.

The basic strategies include the use of manipulatives whenever possible. The instruction should also attempt to involve as many of the child's senses as possible. Verbalizing the steps in a problem or drawing a picture may provide the learning disabled child with the necessary information to complete the problem successfully. The arithmetic curriculum for the learning disabled child should rely on organization, concreteness, and continual review of basic skills.

FACTORS INFLUENCING THE ACQUISTION OF MULTIPLICATION AND DIVISION SKILLS

In designing and modifying arithmetic curriculum to meet the needs of learning disabled or inefficient learners, the types of disabilities that may influence arithmetic learning must also be examined. Since perceptual and processing difficulties are involved with dealing with learning disabled children, an understanding of the possible areas of involvement is essential for meeting the needs of the child and the areas of deficiency which are affecting the child.

Kirk and Gallagher provided the following definition of a specific learning disability:

> "' A specific learning disability is a psychological or neurological impediment to spoken language or percep-tual, cognitive, or motor behavior. The impediment (1) is manifested by discrepancies amoung specific behaviors and achievements or between evidenced ability and aca-demic achievement, (2) is of such a nature and extent that the child does not learn by the instructional methods and materials appropriate for the majority of children and requires specialized procedures for devel-opment and (3) is not primarily due to severe mental retardation, sensory handicaps, enotional problems, or lack of opportunity to learn.'" 30

Figure-Ground Disorders:

Learning disabled students with a figure-ground problem may easily lose their place on a page, not complete all of their work or have difficulty copying arithmetic problems off of the board. In addition, a learning disabled student may become confused by all of the numbers and symbols on a page of arithmetic and copy parts of other problems inappropriately.

Figure-ground disorders severely interfere with the ability

to solve arithmetic problems which require more than one step, such as long division or multiple digit multiplication. Children with figure-ground problems may see individual digits in isolation rather than the total problem.

Some children may also have problems with auditory figure-ground deficits. They cannot filter out extraneous noise. This disorder may also interfere with a student's ability to hear number patterns. Oral counting is a common method of teaching pre-multiplication skills to younger students.

Bold Discrimination:

Students with learning disabilities do not perceive numbers correctly. It is very difficult for these children to copy from the board or write dictated numbers. They tend to write numbers backwards and may not even realize the reversals.

Discrimination disorders may interfere with counting coins and telling time. The child may not be able to tell the differences in the sizes of the coins or the differences in the design. Telling time may be difficult for the child as he or she may not be able to discriminate between the smaller and the larger hand on the clock.

In the following examples the children knew how to complete the operations, yet because of perceptual problems solved them incorrectly. In the first example the student read the 9 as a 6. In the second example the student read the 6 as a 2. The third example illustrates the problem when the sign is misread.

```
                    1                1
      9            46               64
     ±2            x2               ±8
     13           294              492
```

A child with an auditory discrimination problem may be
unable to hear numbers correctly. The Child's ability to count
may be effected. They also may miss the endings of words, they
may develop a counting pattern, such as ... 9, 10, 11, 12, 30,
        31
40, 50.

Reversals:

Children experiencing reversal problems might reverse an
individual numeral or reverse the sequence of numbers. This
problem leads to errors in computation. The most common
reversals occur in two digit numbers, 21 for 12, 32 for 23. The
following example illustrates how reversals can effect
multiplication computation.   In the first problem the student
read 31 for 13. Then the digit 1 was correctly multiplied and a
9 offered as the product. In the second problem the digits were
reversed when they were written down.

```
                          2
        13               58
        X2               X2
        99              477
```

Spatial and Temporal Disabilities:

Learning disabled children experiencing problems with
spatial organization have difficulty learning "right" and "left,"
"up" and "down," "top" and "bottom." The understanding and
location of these spatial designations are very important in
successfully completing arithmetic problems.

Children with problems in temporal organization may be able
to tell time, but may not be able to organize their working time.

Motor Deficits:

Learning disabled children may be unable to write numbers successfully because of motor difficulties. They may expend so much effort trying to form the numeral correctly that they forget what they are supposed to be doing.

Memory Deficits:

Students with memory deficits may not remember what has been taught the day before. The student may have a short term or a long term memory deficit. A child with a short term memory loss may be unable to remember a problem written on the board long enough to copy it on paper. The child will need to check the board continuosly to confirm what is to be copied down.

Visual and auditory short-term memory problems can interfere with a student's ability to solve word problems.

Arithmetic requires a great amount of rote memorization of basic arithmetic facts and basic operational patterns. Children with memory problems will find even basic operations very frustrating. Visualization techniques and overlearning can be used to increase memory skills.

Children with learning disabilities may also have memory sequencig problems. Sequencing is a very important arithmetic skill. The child may be unable to complete a multiplication or division problem successfully because the sequence of steps required to solve the problem cannot be recalled. Subtraction, division and multiplication present the greateds difficulty for children with sequencing problems.

The following example shows a child with a memory disorder.
The child very likely understands the method, but cannot retain
the sequential information long enough to solve the problems.

```
                                          2
        9                    6            5
     4 387                  59            97
        36                 X3Z           X48
        27                 413           776
        24                               388
         3                               4656
```

Intergrative Deficits:

Problems occur when children have problems putting together
what they have learned. A learning disabled child may have
difficulties reading multidigit numbers caused by closure
problems. The child may be able to read a group of numbers in
sequence, but unable to recall a number that is missing.

Children with expressive language problems may not be able
to explain orally how to solve an arithmetic problem, but may be
able to complete a written assignment.

Receptive language disabilities manifest themselves as
difficulty following directions, understanding arithmetic terms,
and difficulties solving problems that deviate from the original
instruction.

Behavior Problems:

There are three main behavior problems that may directly
effect a child's ability to learn multiplication and division
skills, distractability, perseveration and impulsivity.

Distractibility:

Children that are easily distracted may not be able to sustain their attention long enough to complete an entire page of arithmetic problems. These children are distracted by external and internal stimuli that they may be unable to ignore long enough to complete a multiple step multiplication or division problem. A child with an attention span of 120 seconds may experience nearly total failure trying to complete a multi-step multiplication or division problem. Children experiencing such attention deficits should have their work load modified.

Perseveration:

A child with a perseveration disorder may continue to perform the same operation on an entire page without noticing that the signs may be different. The behavior is compulsive and should be corrected immediately.

Impulsivity:

The impulsive child may give irrelevant answers during instruction. Estimating skills are very difficult for the impulsive child to master. An impulsive child may have difficulties completing one problem before moving to another.

The literature regarding learning disabilities and arithmetic skills agrees that the above characteristics are the most common and significant that may affect arithmetic skills in the learning disabled. These characteristics must be considered along with the student's developmental stage as essential indicators of the direction in which the arithmetic curriculum should

be guided and modified. The goal of any curriculum is to meet
the child's needs at whatever level of functioning, and however
the disability manifests itself. The arithmetic curriculum must
be kept flexible and responsive to any changes that might be
necessary to meet the needs of a particular child. The curricu-
lum must be carefully structured to provide a strong an sequen-
tial foundation upon which to base additional learning.

The literature defined two basic categories of arithmetic
disorders, one based on a language disorder and the second based
on a disturbance in qualitative thinking and organization. The
latter category of arithmetic dysfunction, dyscalculia, can
simply be described as an arithmetic disability not due to a lack
                                                        32
of intelligence or a lack of instruction.

Researchers feel that it is rare for dyscalculia to occur in
isolation. It usually is present with other disabilities.
Johnson and Myklebust list the following characteristics as those
most commonly found in children with arithmetic disabilities.

1) Deficiency in visual-spatial organization and nonverbal
integration 2) Extraordinary auditory abilities and often talk
early 3) Many excel in reading 4) Some have faulty or
incomplete Knowledge of body image 5) Some have a disturbance in
visual-motor integration 6) They often have directional
orientation 7) Often show poor distance and time concepts 8)
Often score higher on verbal than nonverbal tests.

Other researchers list spelling deficiencies, left-right
disorientation and spatial-sequential disorientations in addition
to the above list.

In 1974, Kosc identified six specific forms of dyscalculia.

Verbal dyscalculia: People with this form of dyscalculia are unable to name the amounts represented by objects or the value of written numbers although they are able to read or write a dictated number.

Practognostic dyscalculia: This form of dyscalculia is a disturbance in the ability to manipulate real or pictured items. The student may not be able to place the same number of sticks on a table to equal the number in a picture, or arrange objects from smallest to largest.

Lexical dyscalculia: This disorder concerns the inability to read mathematical symbols. The child may be unable to read a single digit or the signs of operation. The child may also interchange similar looking digits.

Graphical dyscalculia: Graphical dyscalculia frequently occurs when dysgraphia or dyslexia is also present. The student may be unable to write a numerical symbol for a dictated number or symbol, but may be able to write the word instead of the number.

Ideognostical dyscalculia: This is the inability to understand mathematical relationships, ideas and mental computation.

Operational dyscalculia: This disorder is the inability to carry out mathematical functions. It may result in the interchange of operations.

A child with a language disorder may have difficulty with arithmetic not because he or she does not understand the principles, but because the child's receptive language may not allow him or her to understand the instruction.

INSTRUCTIONAL STRATEGIES

The arithmetic curriculum for the learning disabled child must be highly structured. The design of the program should be based on the developmental level of the child, previous instruction and the area in which the child exhibits the disability. The curriculum goals should include the introduction of a basic vocabulary and a basic understanding of the foundation or readiness skills necessary to build an arithmetic curriculum.

The educational strategies in this project are offered to facilitate the teaching of multiplication and division to the learning disabled student. The strategies are presented in a sequential manner to build on a child's previous learning and to provide a sequential foundation in which the learning disabled child can learn to understand the multiplication and division operation.

The following is a list of suggestions that should be considered when teaching arithmetic skills to the learning disabled child.

Instructional strategies must have structure built into them. The learning disabled student functions most successfully in an environment that provides strong guidance and direction. The instructional activities must reflect this structure.

The learning disabled child also requires continual repetition and review. New activities should only be introduced after the child has been adequately prepared.

The teaching of arithmetic skills lends itself easily to structure, review and repetition. The activities in this project are offered as reinforcement activities that will support the needed repetition and review of division and multiplication skills.

1. Give a child a small number of problems to do at one time. A child may only be asked to complete one problem.

2. The child should be allowed to use concrete manipulatives whenever possible. The task should be designed to avoid abstractions and be as simple as possible.

3. The child should be given as much time as needed to complete a task. No time limits should be placed on an arithmetic activity.

4. If the child cannot copy a problem from the board give him or her a copy of the problem written on paper.

5. Avoid all distractions, visually and auditorily. The child should be working at a desk or table that is clear of all other materials. Noise distractions should also be eliminated.

6. Give the child a model from which to complete a multiplication or division problem. Each operation may require several steps, a child with a memory deficit may have difficulty remembering the sequential steps.

7. Look for what the child has done correctly, not only for mistakes.

8. If the child has difficulty with expressive language do not require that he or she explain the problems that have been completed.

9. Show the student how to organize his or her paper. Use graph paper, color coding or folding to help the child see rows or columns.

10. For a child with auditory discrimination problems, place the child at the front of the class so he or she can also use visual cues to help understand what is being said.

11. For a child with an auditory memory problem, give directions in small units.

12. Provide the child with specific directions and with easily attainable goals. 33

The learning disabled child presents specific problems that must always be considered when presenting a lesson or designing an arithmetic program. An understanding of the type of disability a child may possess is essential to the success of the instruction. If a child has a difficulty processing auditory information then the initial instruction might be presented visually.

Teaching the learning disabled child requires the use of a variety of teaching strategies. The child's specific disability area must be considered and the instruction designed to compensate for the disability.

PROJECT DESIGN

One of the purposes of this curriculum project is to provide a resource guide offering activities that will help to teach mulitiplication and division skills to learning disabled children. This project does not purport to offer a total arithmetic program, only activities which will enrich and offer a variety of activities directed at the mastery of basic multiplication and division skills.

The project is designed to incorporate the information from the literature review into selected activities. This project will present multiplication first, followed by division. Each section will be divided into three parts. The first part will offer activities to teach the basic concept of the operation. The second part will provide activities to teach and reinforce basic facts that will eventually lead to mastery of the fact. The final part will offer activities that will allow the computational skills to be applied.

A learning disabled student who has been labeled as having dyscalculia or acalculia will have little succes learning multiplication and division in the regular classroom setting. These children require small group instruction that focuses on providing concentrated practice in the areas of need. The learning disabled child may also possess a relatively short attention span, therefore, activities that are motivating and are able to maintain the child's interest are of particular value. This project is designed to suggest small group activities to be used with learning disabled students. The activities were

selected for motivational value, ability to provide concentrated guided practice, lack of direct competitiveness with other students, the use of instructional methods, and the possibilities to modify the activity to suit the individual child.

The project is designed to use information about the learning disabled child and to incorporate the information into useful and productive activities to teach multiplication and division.

The sequence of activities begin with teaching the concept of the operation. These basic activities are heavily dependent upon the use of manipulatives to firmly establish a basis for the operation.

The activities that follow the concept formation are designed to provide practice that will lead to mastery of the basic facts. They will give the learning handicapped child practice that will lead to easy manipulation of the multiplication and division process.

The final sequence offers practical applications of each operation. This section will provide the learning handicapped child with an opportunity to use the knowledge that has been learned and will provide a purpose to each operation.

The activity section is numbered as a separate section of the project so that it may be used apart from the complete project.

FOOTNOTES

1

Faas, Larry A., ^Learning Disabilities: A Competency Based Approach, (Boston: Houghton-Mifflin Co., 1981) pp. 485.

2

Ibid., pp. 487.

3

Faas, Larry A., Children with Learning Problems: A Handbook for Teachers, (Boston: Houghton-Mifflin, 1980) pp. 7.

4

Marsh, George E., Barrie Jo Price, and Tom C. Smith, Teaching the Mildly Handicapped: Methods and Materials (St. C.V. Mosby, 1983) pp. 24.

5

Wallace, Gerald and James M. Kauffman, Teaching Children with Learning Problems, (Columbus, Ohio: Charles E. Merrill, 1987) pp. 10.

6

Gearheart, Bill R., ed. Teaching the Learning Disabled, (St. Louis: C.V. Mosby, 1976) pp. 119.

7

Marsh, Teaching The Mildly Handicapped, pp.242.

8

Ibid., pp. 119

9

Ibid., pp. 121

10

Glennon, Vincent J., The Mathematical Education of Exceptional Children and Youth, (Reston, Virginia: National Council of Teachers, 1981) pp. 55.

11

Bley, Nancy S. and Carol A. Thornton, Teaching Arithmetic to the Learning Disabled, (Rockville, Maryland: Aspen Systems, 1981) pp. 126.

12

Gearheart, Bill, Learning Disabilities: Educational Strategies, (St. Louis: C.V. Mosby, 1981) pp.239.

13

Myers. Patricia I. and Donald Hammill, Methods for Learning Disorders, (New York: John Wiley and Sons, 1969) pp. 140.

14

Gearheart, Teaching the Learning Disabled, pp. 138.

15

    Myers, _Methods for Learning Disorders_, pp. 109.

16

    Myers, _Methods for Learning Disorders_, pp. 111.

17

    Myers, _Methods for Learning Disorders_.
18Shelton, Baker O., _Teaching and Guiding the Slow Learner_, (West Nyack, New York: Parker Publishing Co., 1971) pp.186.

19

    Thornton, Carol A., Benny F. Tucker, and John A. Dossey, _Teaching Math to Children with Special Needs_, (Menlo Park, California: Addison-Wesley, 1983) pp. 95.

20

    Gearheart, _Teaching the Learning Disabled_, pp. 134.

21

    Gearheart, _Teaching the Learning Disabled_.

22

    Suydam, Marilyn N., "Manipulative Materials," _Arithmetic Teacher_, (January 1984) pp. 27.

23

    Marsh, _Teaching Mildly Handicapped Children_, pp. 256.

24

    Grabow, Beverly, "Using Visual Imagery in the Classroom," _Academic Therapy_, (May 1981) pp. 616.

25

    Gearheart, _Teaching the Learning Disabled_, pp. 135.

26

    Marsh, _Teaching Mildly Handicapped Children_, pp. 259.

27

    Marsh, _Teaching Mildly Handicapped Children_, pp. 240.

28

    Gearheart, _Teaching the Learning Disabled_, 134.

29

    Gearheart, _Teaching the Learning Disabled_, 134.

30

    Faas, _Children with Learning Problems: A Handbook for Teachers_, pp. 7.

^{31} Bley, _Teaching Mathematics to the Learning Disabled_, pp. 8.

^{32} Austin, Dan Joe, "Children with Learning Disabilities and Mathematics," _School Science and Mathematics_, (March 1982) pp. 203.

^{33} Glennon, _The Mathematical Education of Exceptional Children and Youth_, 89.

REFERENCES

Armstrong, Thomas, "Chasing Away the Times Tables Blues." Academic Therapy, November 1983.

Austin, Joe Dan, "Children with Learning Disabilities and Mathematics," School Science and Mathematics, March 1982.

Bley, Nancy S. and Thornton, Carol A., Teaching Mathematics to the Learning Disabled. Rockville, Maryland: Aspen Systems, 1981.

Bolduc, Elroy J. "The monsters in Multiplication," Arithmetic Teacher, November 1980.

Danforth, Marion. "Aids for Learning Mathematics," Arithmetic Teacher, January 1984.

Driscoll, Mark. "What Research Says," Arithmetic Teacher, February 1984.

Fass, Larry A., Children with Learning Problems: Handbook for Teachers. Boston: Houghton Mifflin Co., 1980.

Fuys, David J. and Tischler, Rosamond, Welchman, Teaching Math in Elementary School. Boston: Little, Brown and Co., 1979.

Gearheart, Bill, Teaching the Learning Disabled. St. Louis: C.V. Mosby, 1976.

Gearheart, Bill, Learning Disabilities Educational Strategies, St. Louis: C.V. Mosby, 1981.

Glennon, Vincent J. The Mathematics Education of Exceptional Children and Youth. Reston, Va.: National Council of Teachers, 1981.

Grabow, Beverly. "Using Visual Imagery in the Classroom," Academic Therapy, 1981.

Henderson, George. Math Games for Greater Achievement. New York: National Textbook Co., 1972.

Howell, Daisy, Davis, Wilson and Underhill, Leila. Activities for Teaching Math to Low Achievers. Mississippi: University Press, 1974.

Johnson, Stanley, Arithmetic and Learning Disabilities: Guidelines for Identification and Remediation, Boston, Allyn and Bacon, 1979.

Kennedy, Leonard M. and Michon, Ruth S. Games for Individualizing Mathematics Learning. Columbus, Ohio: Charles E. Merrill Co., 1973.

Kirk, Samuel A. and Chalfant, James C., _Academic and Developmental Learning Disabilities_, Denver, Love Publishing Co., 1984.

Lane, George H. "Some Experiences with Slow Learners," _Arithmetic Teacher_, January 1974.

Lessen, Elliott. "Alternatives for Teaching Multiplication Facts," _Arithmetic Teacher_, January 1984.

Marsh, George E., Price, Barrie, and Smith, Tom E.C., _Teaching Mildly Handicapped Children: Methods and Materials_, St. Louis: C.V. Mosby, 1983

Montessori, Maria, _The Montessori Method_, Cambridge, Mass: Robert Bentley, Inc. 1967.

Myers, Patricia and and Hammill, Donald D. _Methods for Learning Disorders_. New York: John Wiley and Sons, 1969.

Richbart, Lynn A. "Remedial Math Program Considerations," _Arithmetic Teacher_, November 1980.

Schminke, C.W. and Dumas, Enoch. _Math Activities for Child Involvement_. Boston: Allyn and Bacon, 1981.

Shelton, Baker O. _Teaching and Guiding the Slow Learner_. West Nyack, New York, 1971.

Stern, Catherine. _Children Discover Arithmetic_. New York: Harper and Row, 1971.

Suydam, Marilyn N., "Manipulative Materials," _Arithmetic Teacher_, January 1984.

Swanson, Lee H. and Reinhart, Henry R. _Teaching Strategies for Children in Conflict_. St. Louis: College Publishers, 1984.

Tanksley, A.E. and Guilliford. _The Education of Slow Learning Children_. London: Routledge and Regan, 1960.

Thornton, Carol A., Tucker Benny F., Dossey, John A., _Teaching Math to Children with Special Needs_, Menlo Park, California: Addison-Wesley, 1983.

Wagner, Guy, Hosier, Max and Gilloley, Laura. _Arithmetic Games and Activities._ Connecticut: Teachers' Publishing Corp., 1964.

Wallace, Gerald and Kauffman, James M. _Teaching Children with Learning Problems._ Ohio: Charles E. Merrill, 1979.

Younie, William J. _Instructional Approaches to Slow Learners._
New York: Teachers' College Press, 1967.

ACTIVITIES FOR TEACHING MULTIPLICATION AND DIVISION
TO LEARNING HANDICAPPED STUDENTS

TABLE OF CONTENTS

ACTIVITIES FOR TEACHING MULTIPLICATION AND DIVISION TO LEARNING HANDICAPPED STUDENTS

MULTIPLICATION

OBJECTIVE: To teach the concept of multiplication to learning handicapped students using arrays.

Activity 1:

Materials: string or cord and counters

Procedure: Give students counters, such as buttons or beans and several pieces of string. Direct the students to arrange the counters in a rectangular display.   Use the string to divide the array into rows or columns.   A variety of arrangements should be made and discussed with students.

When the concept has been established the arrays should be written down in the proper form to form a connection between the concept and the operation.
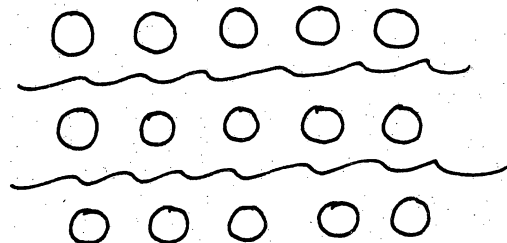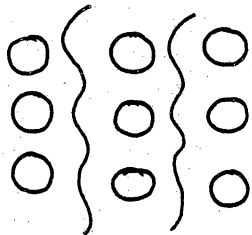
Activity 2:

Materials: graph paper with large squares, pencils, scissors

Procedure: Ask students to fold paper to illustrate various rectangular arrays. Have the folded arrays cut out and labeled as the illustration shows.

This activity can be extended by providing each child with a ten by ten centimeter oaktag board. The students use cover sheets to illustrate various multiplication equations.

2×2

1	2
3	4

2×4

1	2	3	4
5	6	7	8

$4 \times 6 \longrightarrow$

6×4
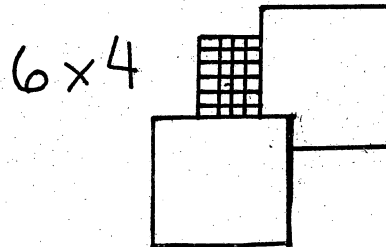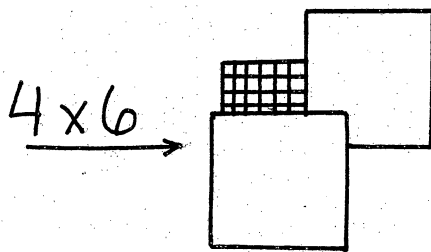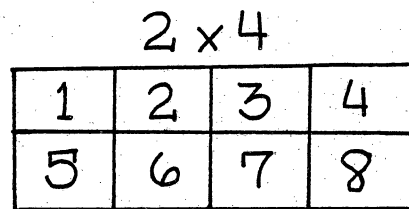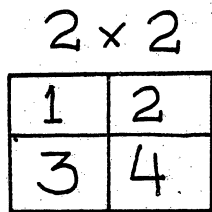
Activity 3:

Materials: pegboard or geoboard

Procedure: Use rubber bands to mark off sections of the pegboard that correspond to a specific multiplication equation. See the appendix for follow-up activities.

OBJECTIVE: To teach multiplication to learning handicapped students using the concept of sets.

Activity 4:

Materials: bottle caps, beans, buttons, blocks

Procedure: Use the counters to illustrate the grouping of equivalent sets. The use of everyday situations is very effective.

2 teams of children in a game.

5 children on each team.
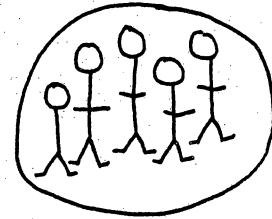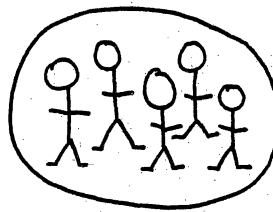
How many children in the game?   2 X 5 = _____

4 sodas, 2 straws in each glass.

How many straws in all?   4 X 2 = _____
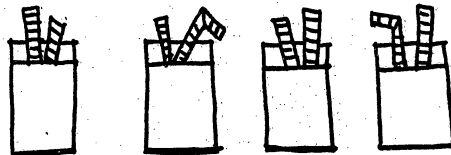
OBJECTIVE: To teach the concept of multiplication to learning handicapped students using the concept of repeated addition.

Activity 5:

Materials: concrete counters

Procedure: Use the counters to demonstrate that multiplication equations represent the combining of groups of objects. It must be emphasized that each group must contain the same number of objects. The student can also be shown that multiplication is a more efficient process when dealing with larger numbers than addition.

OBJECTIVE: To teach multiplication to learning handicapped students using a number line.

Activity 6:

Materials: model of a number line

Procedure: Numbers on the number line are used to demonstrate the solution to a multiplication equation. The following example illustrates the solution to 5 x 3.

5 X 3 would mean 5 jumps of 3 units

```
 |--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|-->
 0  1  2  3  4  5  6  7  8  9  10 11 12 13 14 15
```
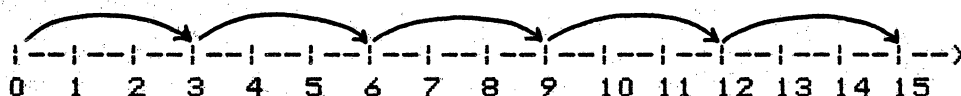
OBJECTIVE: To teach and reinforce basic multiplication facts to learning handicapped students.

There are one hundred basic multiplication facts to memorize. This task may be overwhelming for the learning handicapped student unless the student is taught through a specific system.

First the student is given a multiplication table on which to keep track of the progress in learning the basic facts. The goal is to teach the easiest subsets of facts first. The student can see that the the zero subset and the one subset are very easy to learn. The student can immediately record that two multiplication subsets have been learned.

The next step is to learn the two and five sets of facts. Students can easily learn these facts by "skip counting" or counting on their fingers. When these facts have been learned the student has learned almost half of the one hundred basic facts.

4

The fourth group of facts to learn includes the three, four, six, seven, and eight sets. The nines are learned next.

Activity 7: The Twos

Materials: egg carton, box of crayons (with two rows), buttons, beans

Procedure: Use the model of an egg carton or a box of crayons to illustrate multiplication set of twos. This is a good place to introduce or review the commutative properties of multiplication. Two rows of six equals the same as six rows of two. Instead of learning a single fact the student is actually learning two facts.

Activity 8: Skip Counting

Materials: manipulatives or number line

Procedure: The student can be taught the twos by lining up a row of buttons or beans. Every other button or bean is moved out of the line. Then the student begins counting and notes the number of the button or bean that has been moved. The activity can be done orally. The student can also be directed to write down the number of the object that was moved.

The same procedure can be used with a number line. Noting every other number.

Activity 9: Learning Fives with Money

Materials: pennies, nickels, quarters

Procedure: This activity can be used if the child has an understanding of the value of coins. First review the value of a

penny, nickel, and quarter. Demonstrate using coins that five

pennies equal one nickel and five nickels equal one quarter.

Write out the equations as they are discussed.

     Activity 10: Learning Fives with a Clock

Materials: clock with moveable hands

Procedure: Use this activity to relate the five set to

telling time. Review the number of minutes in an hour, the

position of the half and quarter hours on the clock. Relate the

fives to clock times. Write out equations as they are being

demonstrated on the clock.

    Additional equations can be written for the student to

complete. The student can use the clock to check the product.

5 X 3 = 15   5 X 6 = 30   5 X 4 = 20
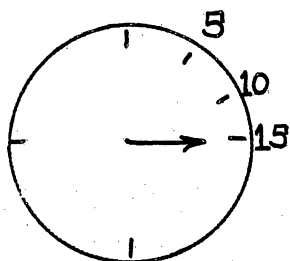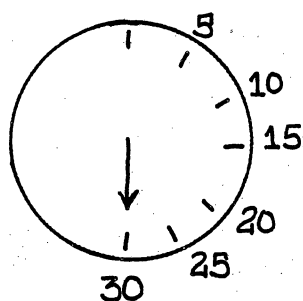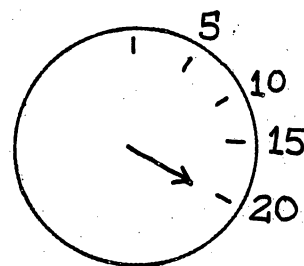
Nines Finger Counting

Procedure: Finger counting is a method used to help figure out the nine facts. Label the fingers on both hands consecutively, 1 through 10. To multiply 3 times 9, bend the third finger. Two fingers represent the tens and seven fingers represent the ones.

To multiply 7 times nine, bend the seventh finger. Six fingers represent the ones. The illustration below shows how finger counting is used.

$3 \times 9 = 27$ $7 \times 9 = 63$
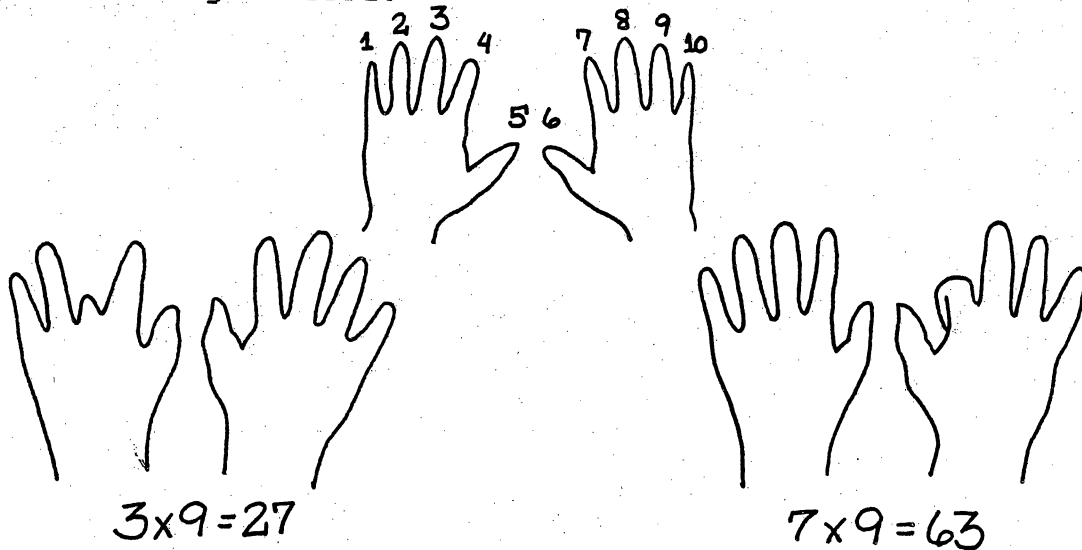
Activity 12: Nine Patterns

Procedure: Children can also be taught the nine facts by remembering patterns. The patterns can be demonstrated by writing the nines facts vertically.

9
18
27
36
45
54
63
72
81
90

The first pattern involves recognizing that if the ones column is followed from the bottom to the top the numbers follow

in sequential order from 0 to 9. The same pattern is present in the tens column, the numbers follow sequentially, top to bottom, 0 to 9.

The second pattern involves matching pairs of numbers, the first with the last, moving toward the center. The pairs of numbers are in reverse order. For example, 09 is the reverse of 90, 18 is the reverse of 81, 27 is the reverse of 72. The pattern follows through all of the products in the nine set.

Activity 13: Threes, Fours, Sixes, and Sevens

Materials: graph paper or blocks

Procedure: Four of the remaining facts can be shown as perfect squares for children with strong visual cueing.

$3 \times 3 = 9$ $4 \times 4 = 16$ $6 \times 6 = 36$

$7 \times 7 = 49$ $8 \times 8 = 64$
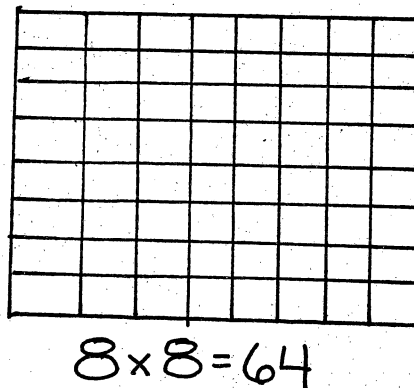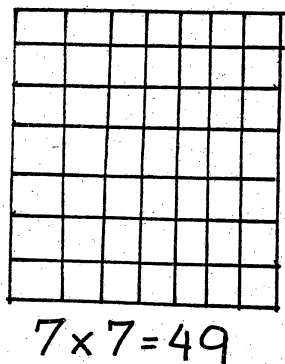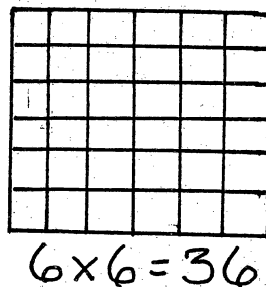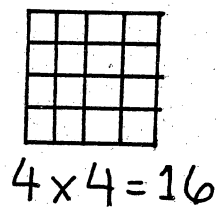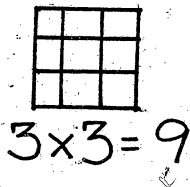
Activity 14: Spin to 200

Materials: spinner with digits 2,3,4,5,6,7,8, and 9 on it

Procedure: Multiply by nine the digit that appears on the spinner. Students keep a running total of their products.

Activity 15: Triangular Flash Cards

Materials: oaktag triangles, the multiplicand written in one corner, the multiplier in a second and the product in the third

Procedure: Children practice basic facts in small groups by showing flash card with one of the parts covered. Another child is to supply the missing element to earn a point.
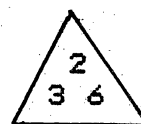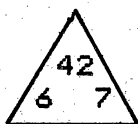
Activity 16: Multi-Fact Find

Materials: laminated gameboard with factors 2-9 arranged in random order, deck of cards with products written on them

Procedure: After the cards have been shuffled, place cards face down. The top card is turned over and placed on the square that corresponds to the factors in the product. Players must play a card adjacent to a card on the board. The winner is the player with the most number of cards on the board.

X	6	8	3	5	9	2	7
4							
3							
9							
6							

Activity 17: Chalkboard Race

Procedure: Divide a group of students into two teams. Write numbers from 0 to 9 in random order on two places on the chalkboard. A player from each team goes to the board. The teacher directs the two players to multiply each number by another number. The first player to write all of the products correctly earns a point for the team. A different multiplier should be selected for each set of players.

Activity 18: Multiplication Grid

Materials: Laminated multiplication grid, grease pencil

Procedure: This activity can be an individual practice activity or a small group activity. Distribute laminated grid to one or more students. Direct the students to fill in the grid as fast as possible. The activity can be timed or used as a self-correcting activity with the answers available.

X	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

Activity 19: Secret Messages

Materials: a list of multiplication equations to complete, a
list of letters that correspond to the products in order, a
secret message.

Procedure: Direct the students to complete the
multiplication equations. Then fill in the secret code with the
letters in the same order as the equations. See appendix.

Activity 20: Troubled Triangles

Materials: a triangle that has been cut apart for the
student to reassemble, factors written along the inside edges of
the triangle which correspond to products written on the opposite
edge, large piece of paper, paste

Procedure: Direct the students to reassemble the triangle
by matching the factors with the product. When they have
constructed the triangle, glue it to the paper.
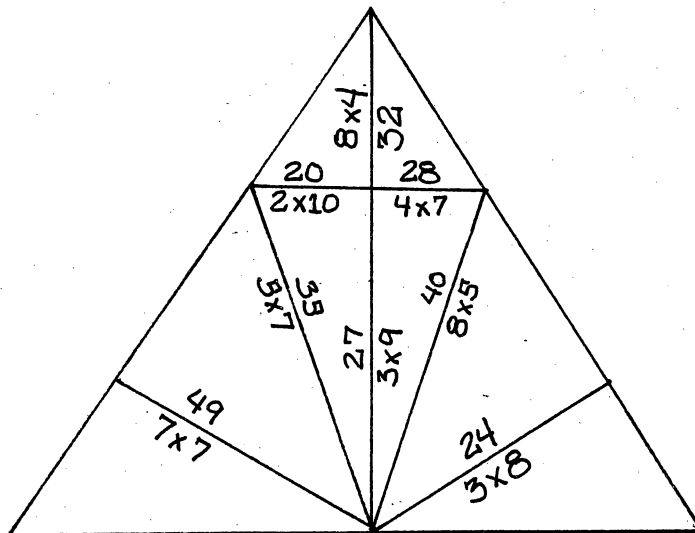
Activity 21: Seven Straight

Materials: gameboard, two sets of markers

Procedure: The first player chooses any square on the board and gives the factors and the product. If the player is correct he places a marker on the square. If the player is incorrect he loses his/her turn. The second player does the same. The winner is the first player to cover seven squares in a row, column, or diagonal.

2x9	5x5	3x3	2x5	8x1	2x5	3x6
6x4	3x5	9x1	2x8	4x8	5x6	6x7
8x5	7x7	6x2	7x4	6x4	6x6	6x3
8x3	3x8	8x6	3x5	9x3	8x3	5x8
9x4	3x3	7x6	6x4	9x8	6x9	5x9
6x2	2x9	6x8	5x8	5x9	9x4	9x1
4x9	8x6	9x4	9x6	9x7	9x3	7x4

Activity 23: Multiplication Match

Materials: 12 multiplication fact cards, 12 answer cards

Procedure: Shuffle the cards and lay them out in 6 x 4 array. The first player turns over two cards. If the multiplication fact matches the product the player keeps the cards. If the cards do not match, the cards are placed face down again. The second player turns over two cards and tries to match a multiplication fact with a product. The game continues until all cards have been matched. The player with the most pairs is the winner.

Activity 24: Cover Up

Materials: gameboard, cards with multiplication facts that correspond to the products on the mat

Procedure: All squares are placed face down. Each player takes ten squares. The first player matches a square with a product and places that square on the appropriate square on the gameboard. The next player must play a square that touches the first covered square, horizontally or vertically. If the player cannot play one of his/her squares another square is drawn from the pile. The winner is the first one to play all of his/her squares.

X	1	2	3	4	5	6	7	8
1								
2								
3								
4								

OBJECTIVE: To teach the practical applications of multiplica-
tion.

   Activity 25: Applications of Multiplication

   Materials: objects to use as examples

   Procedure: Use common objects to illustrate examples of

multiplication. The problems can be presented orally, visually,

or in writing.

   Planning a Party

   5 students
   3 cookies each

   There were five children at the birthday party. Each child
received 3 cookies. How many cookies did the children receive?

   5 children X 3 cookies each = total number of cookies

DIVISION

OBJECTIVE: To teach the concept of division

Activity 26:

Materials: counters

Procedure: Give each student twelve counters. Ask them to divide the counters so that "you and I have the same number of counters." Discuss the results. Ask the students to divide the same number of counters so that there are three equal amounts. Use the same procedure with eighteen counters.

An overhead projector may help if a group of students are participating in the activity.

Activity 27:

Materials: students and objects around the room

Procedure: Ask a student to choose five other students. Ask these students to stand in pairs. Draw chalk circles around each pair. Discuss the results. Ask the same students to arrange themselves into two equal groups. Discuss the results.

Ask a student to gather up all of the coats in the room. Ask the student to place them in two equal piles, then into three equal piles. This activity can be used with any appropriate objects in the classroom.

Activity 28:

Materials: counters, paper and pencil

Procedure: Provide students with a given amount of counters. Ask them to arrange the counters in even rows. An arrangement of fifteen counters might look like the illustration below. Around the counters draw a division bracket with three as the divisor. Discuss the possible answers and what the divisor means in the problem.
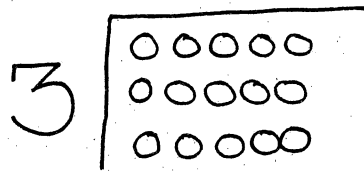
Activity 29:

Materials: rubber bands and geoboard

Procedure: Ask the students how many pegs are on the geoboard. Then ask the students to indicate grouping with rubber bands, how many equal groups are on the geoboard. Discuss how this activity could be written as a division problem. Writing the problem using words first may help the students understand the process they have just completed.
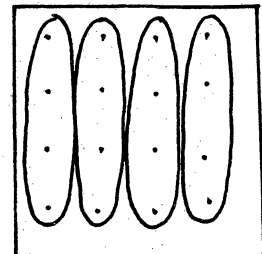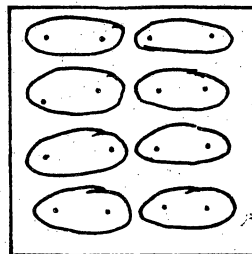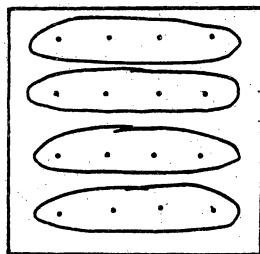
Materials: one square centimeter cubes, pencil and paper

Procedure: Have students arrange twelve cubes in a row. Ask them to remove three cubes at a time and stack them in separate piles. Then record the number of equal piles. Use other groups of cubes and follow the same procedure. Discuss how the division problem could be written.

Activity 31:

Materials: duplicated activity sheets and pencils

Procedure: The illustration below shows how the students can be taught to recognize the equal subgroups in a division problem.
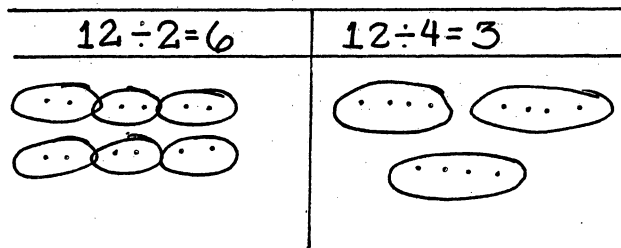
OBJECTIVE: To practice basic division facts.

Activity 32: Division Rummy

Materials: set of twenty-four cards that contain division combinations, set of twenty-four cards that contain their quotients.

Procedure:

1. The dealer shuffles the cards and deals seven to each player. The remaining cards are placed face down to make a draw pile. The top card is turned up.

2. Players check their cards for combinations and matching quotients. All matching pairs are placed face up to form a number sentence.

3. The first player draws either the top card or one from the pile. If the player completes a pair he/she makes a number sentence from them. A card is discarded and placed face up beside the draw pile.

4. Play continues with each player taking his/her turn in order.

5. Play continues until one player has matched all of his/her cards.

6. A point is scored for each number sentence that is made.

Activity 33: Password

Materials: gameboard, markers

Procedure: Each player must answer the division problems correctly before they can pass through the maze.

32÷8
18÷6
12÷4
27÷3
HOME
24÷3
45÷5
15÷3
25÷5

16÷4 20÷4 36÷9 49÷7 9÷3 10÷2 21÷7 35÷5

Start Start Start
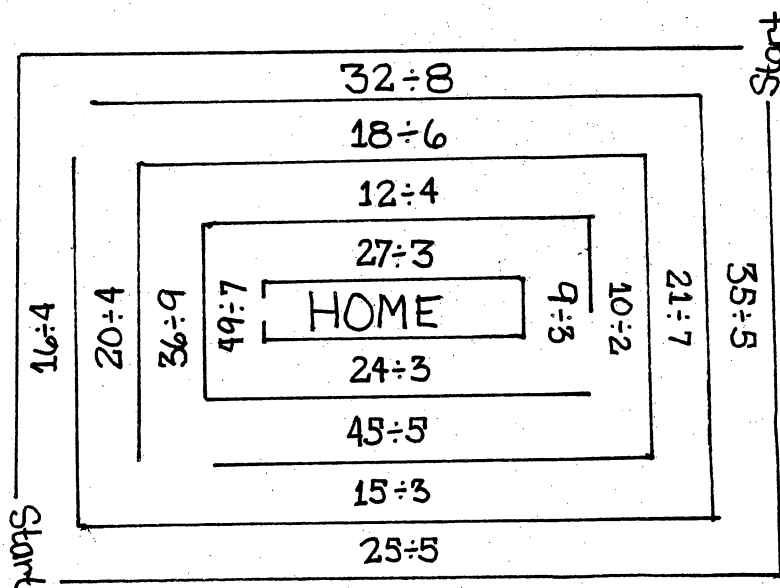
Activity 34: Divide a Square

Materials: game pieces

Procedure: The game pieces have division problems written on the edges whose quotients equal quotients on adjoining edges of the puzzle. The student is asked to assemble the puzzle by matching the equal quotients.
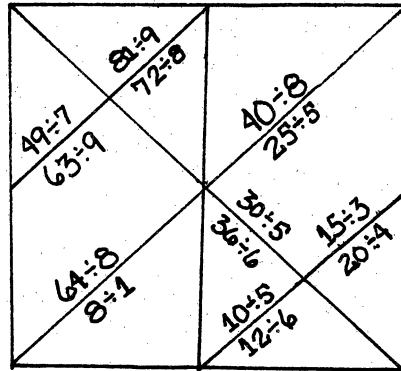
Activity 36: Division Trek

Materials: gameboard, markers, die

Procedure: Each player rolls the die. The player rolling the highest number plays first. The die is rolled by each player to determine the number of spaces to move. The player must correctly answer each space passed in order to advance. If a mistake is made, the player must return to the space where he or she started the turn. The first player to reach the end wins.

42÷6	49÷7	54÷6	48÷6	36÷6	10÷1	16÷8	24÷3
56÷7							27÷3
64÷8		81÷9	63÷9	21÷7	40÷5		30÷5
72÷9		18÷9			45÷5		32÷4
18÷6		18÷3			63÷7		48÷8
6÷2		0÷5			END		42÷7
5÷5		8÷1					56÷8
Start		9÷3	30÷6	30÷5	40÷8	45÷9	54÷9

OBJECTIVE: To teach the practical applications of division.

Activity 36:

Materials: common objects

Procedure: Use common objects and familiar situations to illustrate applications of division operations.

8 candy bars
4 children

How many candy bars will each student get? (Give each student 1 candy bar until there are none left.)

RESOURCES

Bitter, Gary G., Mikesell, Jerald, and Maurdeff, Kathryn. Multiplication and Division Games and Ideas. Pennsylvania: McGraw-Hill, 1976.

Bley, Nancy S. and Thornton, Carol A. Teaching Mathematics to the Learning Disabled. Rockville, Maryland: Aspen Systems, 1981.

Boldoc, Elroy J. "The Monsters in Multiplication." Arithmetic Teacher, November 1980.

Daniel, Charlie and Daniel, Becky. The Multiplication Book. Carthage, Illinois: Good Apple Inc., 1980.

Engelbrecht, Victor M. Math Mysteries. Pennsylvania: Hayes School Publishing Co., 1974.

Fraser, Don. Newspaper Math. Palo Alto, California: Dale Seymour Publications, 1980.

Fuys, David J. and Tischler, Rosamond, Welchman. Teaching Math in Elementary School. Boston: Little, Brown and Co., 1979.

Hyek, Mary. Division Math Mysteries. Rancho Palos Verdes, California: Frank Schaffer Publications, 1977.

Kennedy, Leonard M. and Michon, Ruth S. Games for Individualizing Mathematics Learning. Columbus, Ohio: Charles E. Merrill Co., 1973.

Menu Math. Scottsdale, Arizona: Remedia Publications, 1980.

Mitchell, Kimberly. Bunchy Division. Akron, Ohio: Carson-Dellosa Publishing Co., 1982.

Mitchell, Kimberly. Multiplication Blast Off. Akron, Ohio: Carson-Dellosa Publishing Co., 1982.

Multiplication. San Jose, California: Enrich Publishing Co., 1982.

Schminke, C.W. and Dumas, Enoch. Math Activities for Child
    Involvement. Boston: Allyn and Bacon, 1981.

Stern, Catherine. Children Discover Arithmetic. New York:
    Harper and Row, 1971.

Thornton, Carol A., and Tucker, Benny F. and Dossey, John A.
    Teaching Math to Children with Special Needs. Menlo Park,
    California: Addison-Wesley, 1983.

Wiener, Joy. Multiplication and Division Riddles. Rancho Palos
    Verde, California: Frank Schaffer Publications, 1976.

Van Derford, Jean. Monster Multiplication. Glendale, California:
    Teaching Stuff, 1976.

APPENDIX

Name_____ Date_____

Fill in the multiplication table for easy reference.

X	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

Name_____

Solve each problem in the list below. Use × and = to find the same problems hidden in the puzzle. Circle each hidden problem. Some of the problems share a number.

examples:

3	× 2	= 6	4	8	1	0	3	8	24	10	6	8	5
7	0	5	3	6	7	1	9	3	8	9	6	9	54
21	2	8	6	7	3	5	3	4	5	20	7	1	0
9	6	7	0	0	2	9	8	3	9	0	4	2	6
9	6	2	26	8	34	45	5	1	27	4	9	42	6
1	4	4	18	0	30	8	0	8	4	22	8	16	36
6	5	48	8	5	56	4	5	6	20	64	72	10	4
3	16	8	4	32	10	6	9	48	13	4	24	8	2
3	10	34	12	5	1	5	0	15	29	0	2	2	4
9	3	19	5	31	6	2	35	1	6	23	18	9	8
28	4	1	4	10	0	21	9	27	8	2	16	4	18
16	1	26	7	3	4	12	5	6	45	13	8	6	32
2	0	0	8	47	9	12	3	30	9	18	25	14	6
5	4	37	16	9	36	19	17	11	6	2	17	36	2
10	20	0	7	11	5	22	0	37	54	8	24	0	12

Problem List

2×0=____   3×2=____   9×6=____

3×4=____   3×7=____   3×3=____

5×1=____   3×8=____   4×1=____

8×6=____   5×9=____   0×8=____

9×8=____   7×0=____   4×5=____

2×2=____   6×9=____   6×2=____

1×4=____   2×4=____   8×4=____

2×5=____   8×2=____   6×6=____

4×9=____

1
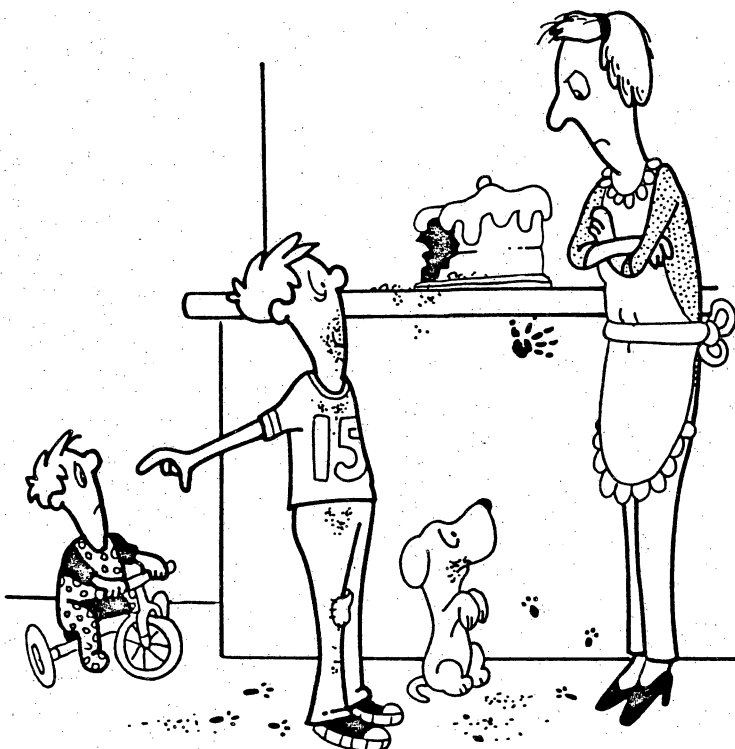
Name_____

Solve the problems in the list. Use ÷ and = to find the same problems hidden in the puzzle. Circle each hidden problem.

examples:

(54 ÷ 9 = 6)	65	1	25	5	5	7	2	24	10	2	36
11	10	18	(36	5	1	61	3	9	7	3	9	16	6
72	9	8	÷ 9	26	1	9	0	0	6	8	10	4	6
0	7	17	= 4)	0	0	8	5	0	63	9	7	4	8
54	7	43	5	10	0	10	24	7	15	6	28	0	9
6	9	64	8	8	0	1	3	24	6	4	41	11	3
9	76	38	9	14	6	12	8	4	19	4	74	3	3
30	9	15	3	5	2	10	27	6	3	25	16	7	10
28	8	9	21	12	5	11	2	34	40	5	8	17	22
7	16	13	10	5	2	11	0	3	13	8	66	65	18
4	14	71	7	15	46	6	0	7	7	7	1	14	2
18	12	24	8	3	4	6	6	1	8	51	7	7	10
6	16	78	5	1	54	1	24	38	20	42	29	2	11
28	4	7	5	49	7	7	81	18	2	9	4	3	23

Problem List

54 ÷ 9 = _____
64 ÷ 8 = _____
28 ÷ 7 = _____
24 ÷ 8 = _____
0 ÷ 0 = _____
49 ÷ 7 = _____
18 ÷ 2 = _____
40 ÷ 5 = _____
7 ÷ 7 = _____
24 ÷ 6 = _____
36 ÷ 6 = _____
63 ÷ 9 = _____
15 ÷ 3 = _____
14 ÷ 7 = _____
25 ÷ 5 = _____
72 ÷ 9 = _____
54 ÷ 6 = _____
9 ÷ 0 = _____
24 ÷ 3 = _____
6 ÷ 6 = _____
28 ÷ 4 = _____
36 ÷ 9 = _____
9 ÷ 3 = _____
2 ÷ 0 = _____
10 ÷ 5 = _____

5

$\begin{array}{r}9\\ \times 9\\\hline\end{array}$
$\begin{array}{r}9\\ \times 7\\\hline\end{array}$
$\begin{array}{r}7\\ \times 7\\\hline\end{array}$
$\begin{array}{r}7\\ \times 4\\\hline\end{array}$
$\begin{array}{r}9\\ \times 5\\\hline\end{array}$

$0 \times 9 =$
$9 \times 9 =$
$5 \times 8 =$

$5 \times 3 =$

$\begin{array}{r}6\\ \times 9\\\hline\end{array}$
$\begin{array}{r}9\\ \times 2\\\hline\end{array}$
$\begin{array}{r}5\\ \times 7\\\hline\end{array}$
$\begin{array}{r}8\\ \times 5\\\hline\end{array}$

$\begin{array}{r}4\\ \times 9\\\hline\end{array}$

$\begin{array}{r}2\\ \times 9\\\hline\end{array}$
$\begin{array}{r}5\\ \times 4\\\hline\end{array}$
$\begin{array}{r}5\\ \times 9\\\hline\end{array}$

$9 \times 4 =$
$3 \times 6 =$

$\begin{array}{r}2\\ \times 8\\\hline\end{array}$

$9 \times 1 =$
$1 \times 9 =$

$\begin{array}{r}8\\ \times 6\\\hline\end{array}$

$\begin{array}{r}2\\ \times 9\\\hline\end{array}$
$\begin{array}{r}5\\ \times 9\\\hline\end{array}$
$\begin{array}{r}7\\ \times 5\\\hline\end{array}$
$\begin{array}{r}7\\ \times 7\\\hline\end{array}$

$\begin{array}{r}6\\ \times 6\\\hline\end{array}$
$\begin{array}{r}8\\ \times 8\\\hline\end{array}$
$\begin{array}{r}6\\ \times 5\\\hline\end{array}$

$5 \times 4 =$
$7 \times 7 =$
$6 \times 8 =$

Basic facts.

If the product has:

0 yellow
5 orange
6 black
8 red
9 green

MM 15

$9 \times 7 =$

6×9

$6 \times 2 =$

$7 \times 9 =$

$21 \div 7 =$

$18 \div 9 =$

$8 \times 8 =$

$7 \times 8 =$

$9 \times 4 =$

$24 \div 6 =$

$9 \times 2 =$

$8 \times 7 =$

$4 \times 9 =$

$6 \times 4 =$

$2 \times 7 =$

$36 \div 6 =$

$21 \div 7 =$

$64 \div 8 =$

$9 \times 8 =$

$27 \div 9 =$

$3 \times 1 =$

$48 \div 6 =$

$72 \div 9 =$

$9 \times 6 =$

$32 \div 8 =$

$54 \div 9 =$

$24 \div 8 =$

$6 \times 7 =$

$7 \times 9 =$

$8 \times 8 =$

$56 \div 7 =$

$4 \times 8 =$

$8 \times 6 =$

$2 \times 6 =$

Mixed practice.

If the number in the one's place is:

2 green   3 yellow   4 red   6 purple   8 orange

MM 18

Name _____

1	2	3	4	5
blue	green	yellow	brown	orange

$2 \div 2 =$

$2\sqrt{2}$

$2\sqrt{2}$

$2\sqrt{4}$

$4 \div 2 =$

$2\sqrt{8}$

$8 \div 2 =$

$8 \div 2 =$

$10 \div 2 =$

$6 \div 2 =$

$2\sqrt{8}$

$8 \div 2 =$

$10 \div 2 =$

$2\sqrt{10}$

2

4 $2\sqrt{6}$

$2\sqrt{10}$

$2\sqrt{10}$ $10 \div 2 =$

$2\sqrt{8}$

$2\sqrt{6}$

$2\sqrt{4}$

$8 \div 2 =$

$4 \div 2 =$

2

1

Math Mysteries - Division
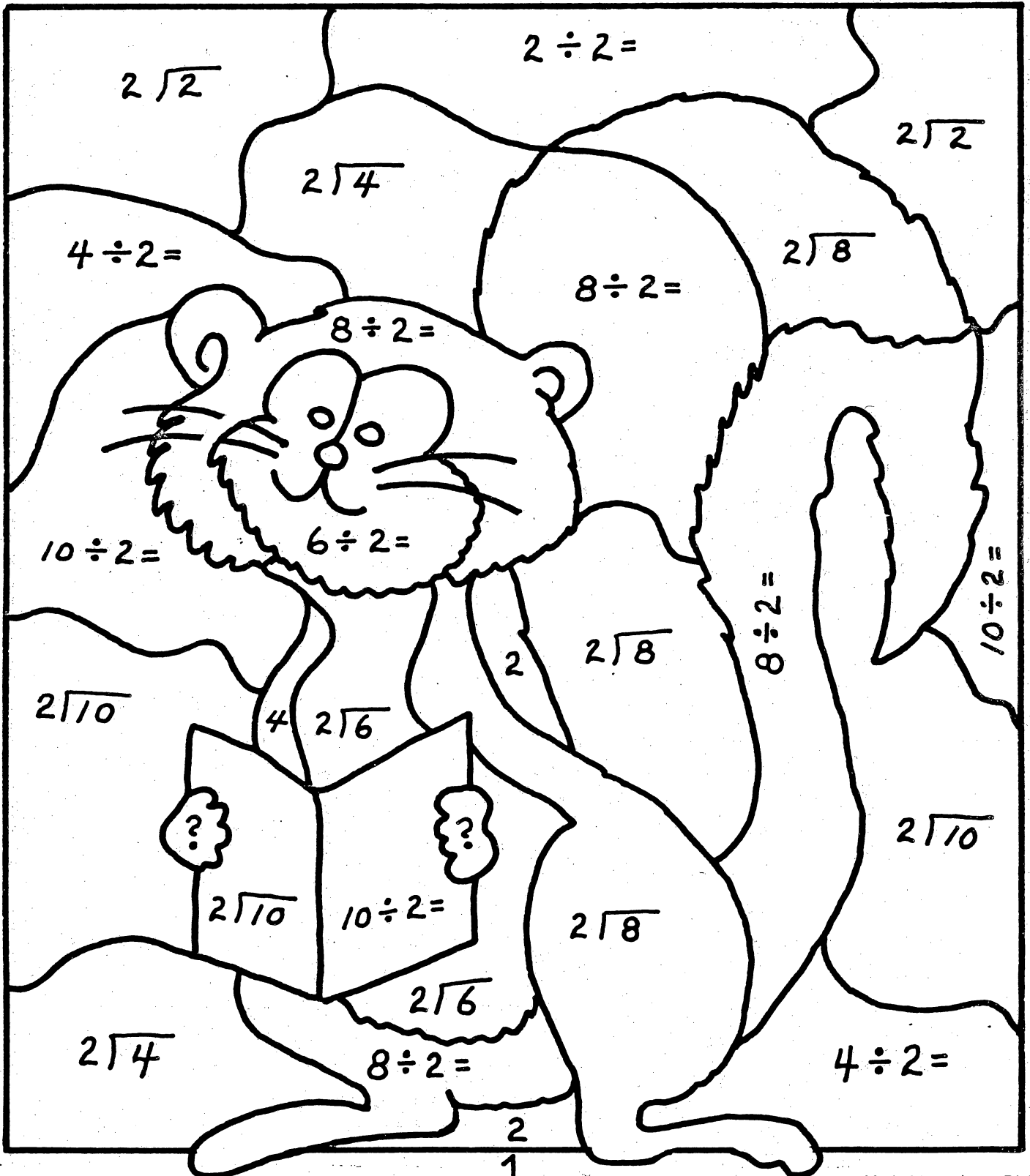
I thought that you liked me. Sniff.
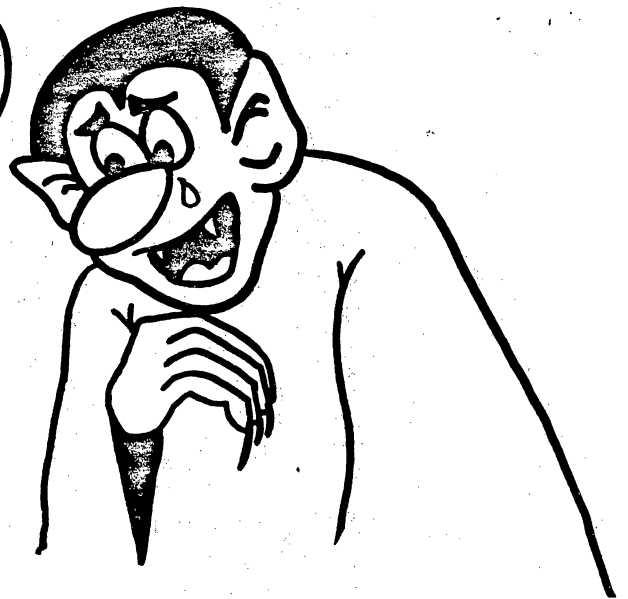
Daffynition: Vampire—

$\overline{87}$ $\overline{56}$ $\overline{70}$ $\overline{125}$ $\overline{56}$ $\overline{150}$ $\overline{125}$ $\overline{0}$ $\overline{180}$ $\overline{56}$

$\overline{75}$ $\overline{87}$ $\overline{140}$ $\overline{288}$ $\overline{75}$ $\overline{81}$ $\overline{105}$ $\overline{140}$ $\overline{75}$ $\overline{150}$

$\overline{75}$ $\overline{150}$ $\overline{114}$ $\overline{180}$ $\overline{125}$ $\overline{150}$ $\overline{125}$ $\overline{115}$ $\overline{176}$

A	B	C	E	G
35×4	32×9	23×5	25×5	27×3
H 45×4	**I** 25×3	**K** 22×8	**M** 14×5	**N** 25×6
O 28×2	**P** 35×3	**S** 29×3	**T** 38×3	**W** 28×0

Riddle:

16 26 80   39 96 16 39 60 46

16 26 70 46 15 96 48 46   39 15

16 80 44 36 ?

39   66 80 80 15 28 39 96 96

55 80 39 55 26 .

A	B	C	E	F
13 × 3	14 × 2	11 × 5	12 × 4	11 × 6
H 13 × 2	**I** 10 × 7	**K** 12 × 3	**L** 32 × 3	**O** 20 × 4
R 22 × 2	**S** 23 × 2	**T** 15 × 1	**W** 16 × 1	**Y** 10 × 6

4

Concept: Division, 1 and 2 digit divisors, no remainders.

Riddle: What is red and bright has wire wheels?

"There it goes!"
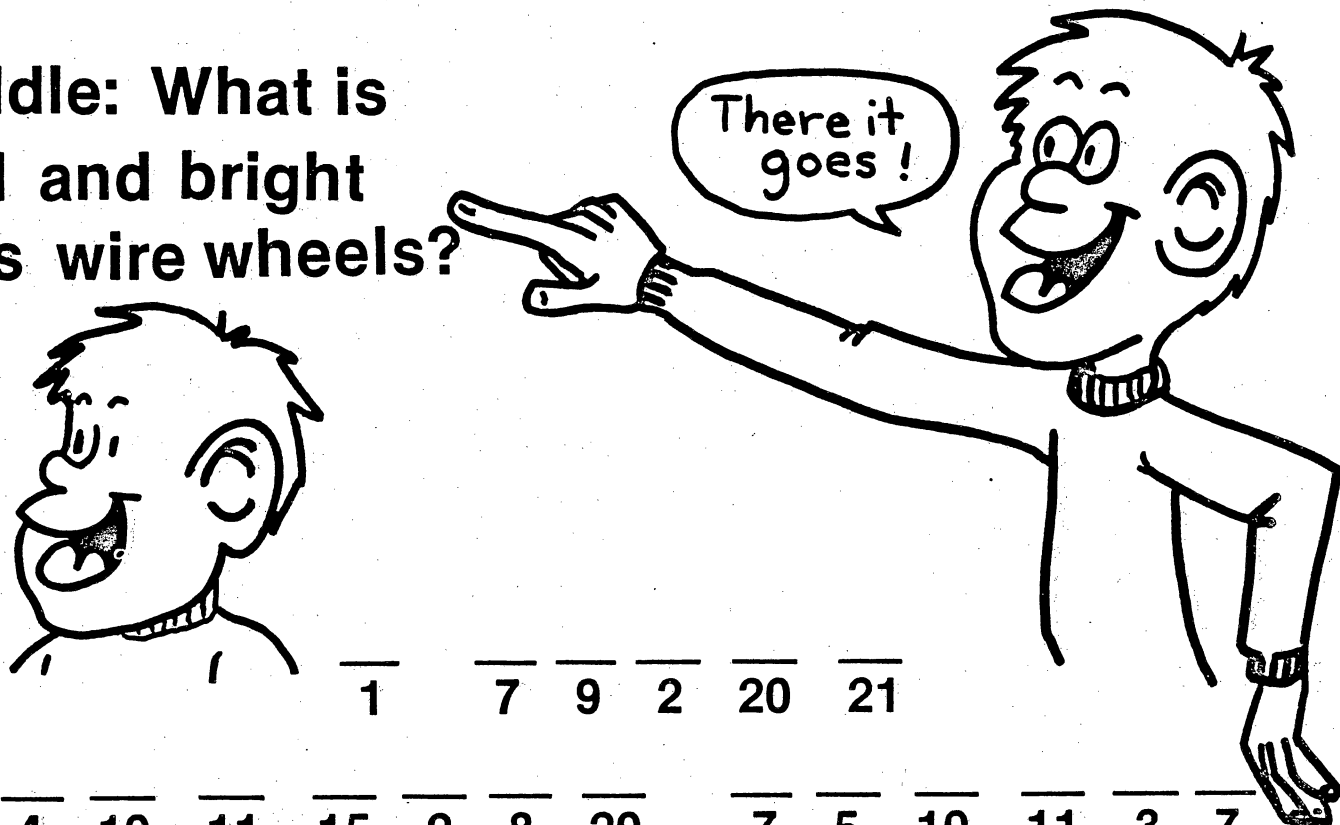
$\overline{}\ \overline{}\ \overline{}\ \overline{}\ \overline{}\ \overline{}$
1   7   9   2   20   21

$\overline{}\ \overline{}\ \overline{}\ \overline{}\ \overline{}\ \overline{}\ \overline{}\ \overline{}\ \overline{}\ \overline{}\ \overline{}\ \overline{}\ \overline{}$
4   10   11   15   2   8   20   7   5   10   11   3   7

7   3   11   1   12   6   15   11   11   21 .

A $25\overline{)25}$	B $9\overline{)54}$	E $1\overline{)15}$	F $12\overline{)48}$	G $6\overline{)48}$
H $9\overline{)81}$	I $14\overline{)28}$	N $4\overline{)80}$	O $5\overline{)50}$	P $12\overline{)60}$
R $3\overline{)33}$	S $6\overline{)42}$	T $30\overline{)90}$	W $3\overline{)36}$	Y $2\overline{)42}$

25

SECRET MESSAGES

1. 4x4=   8. 8x4=   15. 3x3=
2. 7x2=   9. 3x4=   16. 5x8=
3. 5x7=   10. 0x8=   17. 3x3=
4. 4x5=   11. 3x5=   18. 2x2x2=
5. 3x8=   12. 4x7=   19. 3x3x5=
6. 7x3=   13. 5x6=   20. 2x5=
7. 5x1=   14. 6x7=   21. 9x2=

A=14   F=18   M=16   S=21
B=9   G=27   N=12   T=35
C=28   H=20   O=15   U=32
D=8   I=24   P=42   V=10
E=0   J=30   R=45   W=40
                              Y=5

_____   _____   _____   _____
16 4 2 3   18 11   7   11 8 19   6 4 11 10 6

_____   _____   _____
15 10 12 11 1 10   16 4 10 9   7 11 8

_____   _____   _____   _____   _____
6 3 10 14   5 9 3 11   2   15 5 17   14 8 18 18 13 10

Answer: _____
          16 10 3

Twos

Circle every other number.

1 ② 3 ④ 5 ⑥ 7 8 9 10 11 12

13 14 15 16 17 18 19 20 21 22 23 24

$2 \times 2 = 2 + 2 =$ $7 \times 2 =$

$3 \times 2 = 2 + 2 + 2 = 6$ $8 \times 2 =$

$4 \times 2 =$ $9 \times 2 =$

$5 \times 2 =$ $10 \times 2 =$

$6 \times 2 =$ $11 \times 2 =$

          $12 \times 2 =$

Write the addition problems and answers.
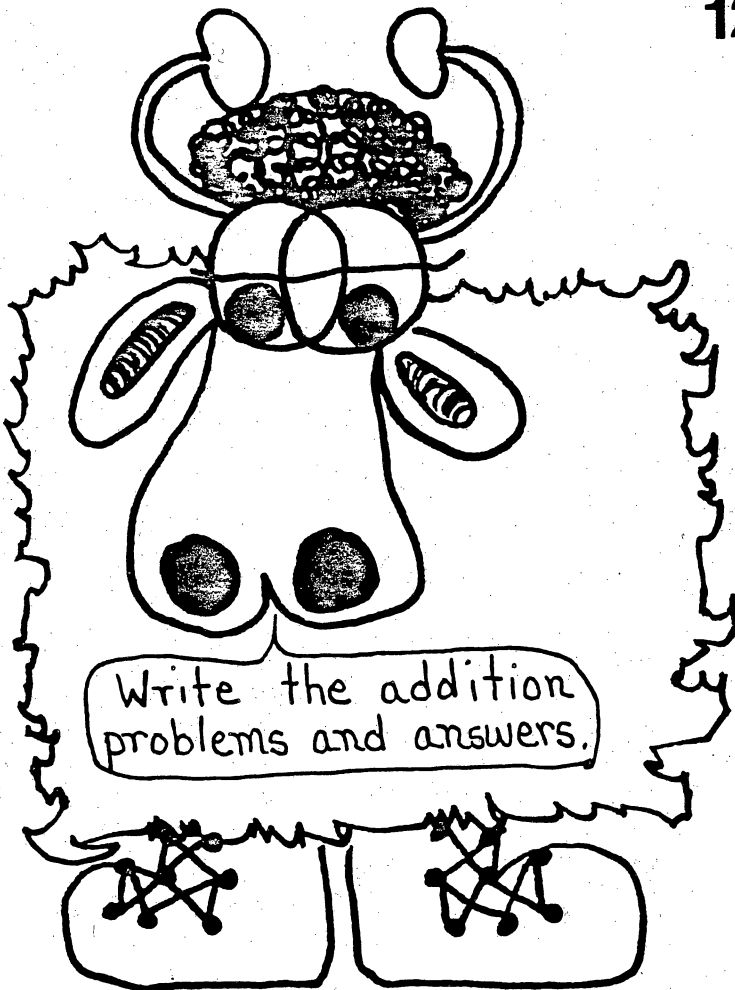
Draw 2 sets of two flowers.

Draw 2 sets of 2 balls.

3

Threes

Circle every third number.

1   2   ③   4   5   ⑥   7   8   9   10   11   12

13   14   15   16   17   18   19   20   21   22   23   24

25   26   27   28   29   30   31   32   33   34   35   36

$1 \times 3 = 3$

$2 \times 3 = 3 + 3 = 6$

$3 \times 3 =$

$4 \times 3 =$

$5 \times 3 =$

$6 \times 3 =$

$7 \times 3 =$

$8 \times 3 =$

$9 \times 3 =$

$10 \times 3 =$

$11 \times 3 =$

$12 \times 3 =$

Write the addition problem and answer for each multiplication problem.

Draw dots to prove that 2 X 3 = 6.

Draw dots to prove that 3 X 2 = 6.

5

MULTIPLY BY 8

**SPACE MONSTERS
MULTIPLY BY 8**

•

©TEACHING STUFF

Directions to Student:

1. Color and cut out the Space Monster on **Page 13** that goes with this disc.
2. Cut along dotted lines on Space Monster.
3. Cut out this disc.
4. Connect the Space Monster and this disc with a brass paper fastener through the center at the • and + .
5. HAVE FUN LEARNING YOUR FACTS. Lift up flap to check your answer.

2⟌4   2⟌24   2⟌8   2⟌6   2⟌12   2⟌20   2⟌14   2⟌16   2⟌18   2⟌10   2⟌22   2⟌2

3⟌27   3⟌6   3⟌12   3⟌33   3⟌15   3⟌3   3⟌18   3⟌21   3⟌9   3⟌30   3⟌36   3⟌24

4⟌4   4⟌16   4⟌12   4⟌24   4⟌20   4⟌32   4⟌28   4⟌40   4⟌36   4⟌48   4⟌44   4⟌8

5⟌30   5⟌10   5⟌20   5⟌50   5⟌35   5⟌40   5⟌55   5⟌15   5⟌25   5⟌5   5⟌45   5⟌60

6⟌72   6⟌12   6⟌30   6⟌54   6⟌36   6⟌24   6⟌42   6⟌60   6⟌18   6⟌66   6⟌6   6⟌48

7⟌35   7⟌21   7⟌14   7⟌28   7⟌42   7⟌56   7⟌49   7⟌70   7⟌63   7⟌77   7⟌7   7⟌84

8⟌16   8⟌8   8⟌24   8⟌40   8⟌32   8⟌48   8⟌64   8⟌56   8⟌72   8⟌88   8⟌80   8⟌96

9⟌9   9⟌27   9⟌63   9⟌36   9⟌54   9⟌72   9⟌99   9⟌81   9⟌45   9⟌108   9⟌18   9⟌90

10⟌70   10⟌50   10⟌80   10⟌40   10⟌90   10⟌30   10⟌100   10⟌20   10⟌110   10⟌10   10⟌120   10⟌60

11⟌77   11⟌11   11⟌132   11⟌22   11⟌121   11⟌33   11⟌110   11⟌44   11⟌99   11⟌55   11⟌88   11⟌66

12⟌24   12⟌132   12⟌36   12⟌12   12⟌48   12⟌72   12⟌120   12⟌84   12⟌60   12⟌108   12⟌96   12⟌144

Name_____ Date_____

	(a)	(b)	(c)	(d)	(e)
1)	1 5 × 6	2 6 × 3	1 6 × 4	2 7 × 3	2 5 × 3
2)	1 5 × 2	2 6 × 2	3 7 × 2	1 4 × 5	4 6 × 2
3)	1 2 × 7	1 3 × 6	1 1 × 7	1 4 × 3	4 7 × 2
4)	1 6 × 5	1 9 × 4	3 6 × 2	3 9 × 2	1 7 × 3
5)	2 3 × 4	2 8 × 2	2 9 × 3	1 8 × 5	4 8 × 2

Skill: Multiplying two-place numbers by one-place numbers.

1

ARRAYS

Name_____

Activity 1

Outline rectangles to show the arrays and answer the questions that follow.

1.   Outline a 4 x 5 array.   Outline a 6 x 4 array.
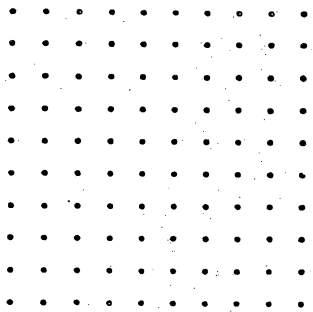
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
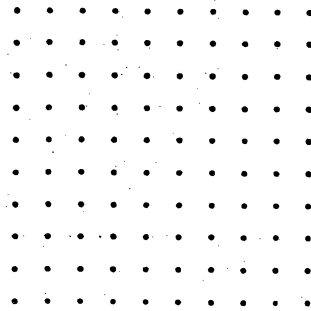
2.   How many dots are in each array outlined above?

     _____   _____

3.   How many different arrays can you make with 12 objects? _____

4.   How many dots are shown in each of these arrays?

. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
                                              . . . .

     _____   _____   _____
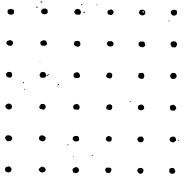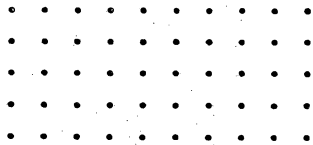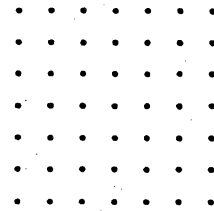
5.   Two other arrays with 36 dots are 12 x 3 and 3 x 12. Name two more arrays with 36 dots.
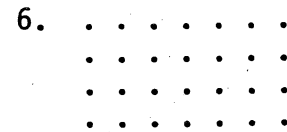
     _____

Name _____

Activity 2

```
. . . . .
. . . . .
. . . . .
. . . . .
. . . . .
```
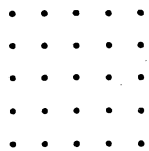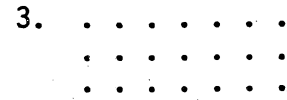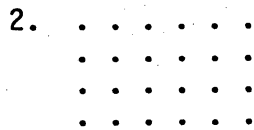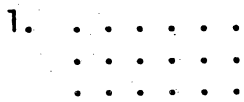
5 x 5 = 25

The number of dots in this array can be described with the number sentence given underneath it. Write a sentence for each array given below.

1.
```
. . . . . . .
. . . . . . .
. . . . . . .
```
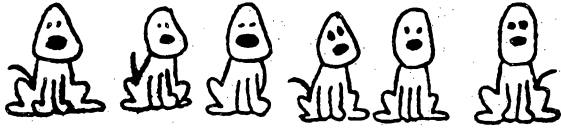_____

2.
```
. . . . . . .
. . . . . . .
. . . . . . .
. . . . . . .
```
_____

3.
```
. . . . . . .
. . . . . . .
. . . . . . .
```
_____

4.
```
.
.
.
.
.
```
_____

5.
```
. . . . . . .
. . . . . . .
. . . . . . .
. . . . . . .
. . . . . . .
```
_____

6.
```
. . . . . . . .
. . . . . . . .
. . . . . . . .
. . . . . . . .
```
_____

7.
```
. . . . . . . .
```
_____

8.
```
. . . . . . . . .
. . . . . . . . .
```
_____

9.
```
. . . . . . . .
. . . . . . . .
```
_____

10.
```
. . . . . . .
. . . . . . .
. . . . . . .
. . . . . . .
. . . . . . .
. . . . . . .
```
_____

11.
```
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
```
_____

12.
```
. . . . . . .
. . . . . . .
. . . . . . .
. . . . . . .
. . . . . . .
```
_____
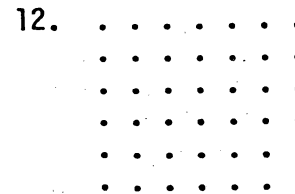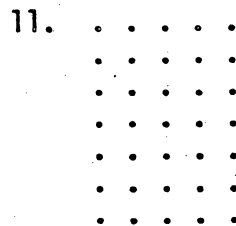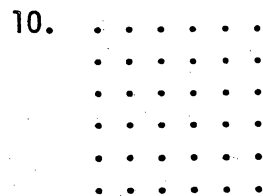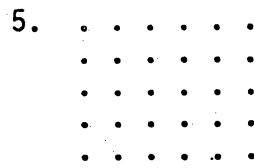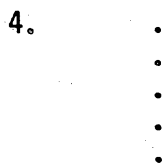
ARRAYS   Name_____

Activity 3

Complete the tables.
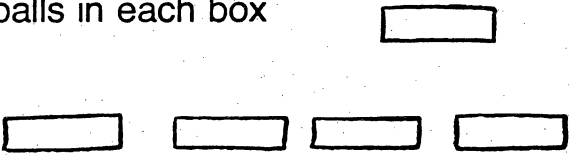
Number of Rows	4	5		8	7		4
Number of Columns	3	4	6		6	8	
Number of Objects	12		24	24		56	28
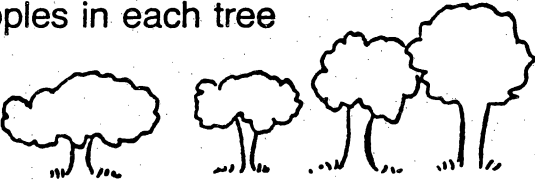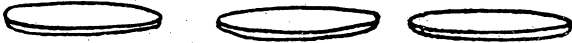
| Picture of the Array | ```
* *
* *
* *
* *
* *
* *
``` | ```
@ @ @ @
@ @ @ @
@ @ @ @
@ @ @ @
@ @ @ @
@ @ @ @
@ @ @ @
``` | | | |
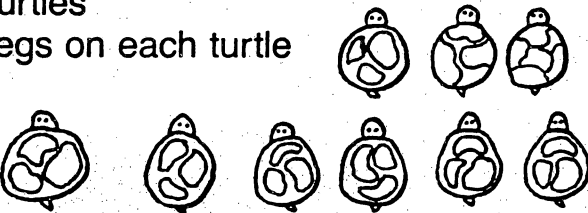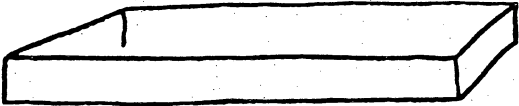|---|---|---|---|---|---|
| Name of the Array | 6 x 2 | | 3 x 4 | 4 x 8 | 6 x ___ |
| Number of Objects in the Array | 12 | | | | 42 |

25

Name _____

Finish the pictures. Find the answers. Label your answers. The first one has been done for you.

EXAMPLE: 3 boys
Each boy has 2 eyes.

How many eyes? ____6 eyes____

We can also say it like this: __3 × 2 = 6__ .

---

6 puppies   4.
2 ears on each puppy

How many ears? _____

We can also say it like this: _____ .

---

5 boxes   1.
3 balls in each box

How many balls? _____

We can also say it like this: _____ .

---

7 cones   5.
2 scoops of ice cream on each cone

How many scoops? _____

We can also say it like this: _____ .

---

4 trees   2.
6 apples in each tree

How many apples? _____

We can also say it like this: _____ .

---

3 dishes   6.
4 oranges in each dish

How many oranges? _____

We can also say it like this: _____ .

---

9 turtles   3.
4 legs on each turtle

How many legs? _____

We can also say it like this: _____ .

---

1 box   7.
9 pencils in the box

How many pencils? _____

We can also say it like this: _____ .

Name_____

Solve each story problem.

a. Joey uses 8 towels each time he washes his dad's car. He has washed his dad's car 3 times this week. How many towels did he use in all?

_____ = _____

b. Bob washed 9 cars today. Tony washed 4 times as many cars as Bob washed today. How many cars did Tony wash?
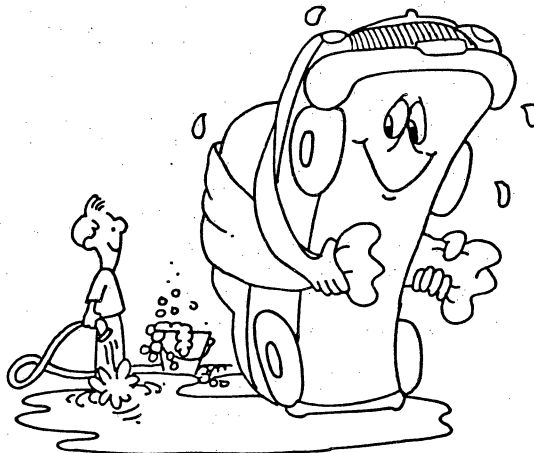
_____ = _____

c. Jim uses 6 buckets of water to wash his dad's car. He washed his dad's car 4 times this week. How many buckets of water did Jim use altogether?

_____ = _____

d. Pam, Sue and Debbie each washed 7 cars today. How many cars did they wash in all?

_____ = _____

e. Patty used 6 jars of wax last week to wax cars. This week she used 7 times as many jars of wax as she did last week. How many jars of wax did she use this week?

_____ = _____

f. Jay's baseball team washed cars all day yesterday to raise money for new uniforms. They washed 8 cars during each of the 8 hours they worked. How many cars did they wash in all?
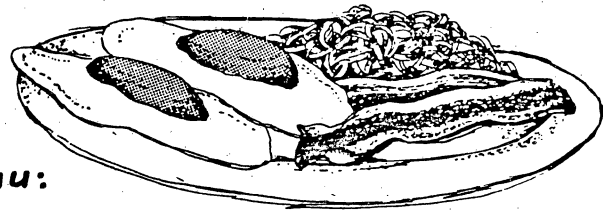
_____ = _____

g. Mike spent 7 minutes cleaning the front window on his dad's car. Julie cleaned the back window. She spent 8 times as many minutes as Mike cleaning the window. How many minutes did it take Julie to clean the back window?

_____ = _____

Answer Key: a. 24   b. 36   c. 24   d. 21   e. 42   f. 64   g. 56

4

OBJECTIVE: To solve division
   Story problems.
Directions: Solve the following
   story problems without the menu:

1. The bill for hamburgers and Colas came to $41.60. Thirty-two people shared the cost equally. How much did each person pay?

2. Pie and Coffee for members of the Ladies' Club was $48.60. The 27 members shared the cost equally. What amount did each lady owe?

3. By working at the Hamburger Hut, Gary saved $768.00. He did this by putting the same amount of money in the bank each time he was paid. There were 24 paydays. How much did Gary save out of each paycheck?

4. In May, the Hamburger Hut used 465 gallons of milk. What was the average amount used each day?

5. Mark earned $6,672.00 working at The Hut during 1979. How much was that per month?

6. Last year 10,800 hamburgers were sold. What was the average number of burgers sold every month?

7. Elizabeth worked 27 days in July. Her paycheck came to $702.00. What was the average amount she earned each day?

OBJECTIVE: Multiplying and dividing to solve problems (answers exclude sales tax).
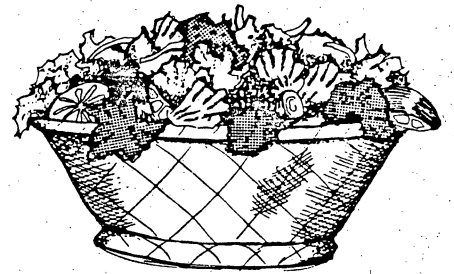
Directions: Solve the following problems using the menu. Place an answer in each box.

ORDER	COST	Divided	COST EACH
1. 4 Chef Salads		Five Ways	
2. 8 Avocado Burgers		Three Ways	
3. 10 Colas		Five Ways	
4. 6 Cheese Sandwiches		Six Ways	
5. 8 Beef Stew Dinners		Four Ways	
6. 6 Waldorf Salads		Eight Ways	
7. 4 Steak Dinners		Seven Ways	
8. 16 Chicken Sandwiches		Ten Ways	
9. 6 Shrimp Salads		Three Ways	
10. 7 Bacon and Eggs		Five Ways	
11. 8 Reuben Sandwiches		Four Ways	
12. 8 Chicken Dinners		Six Ways	
13. 7 Chili Burgers		Five Ways	
14. 9 Tomato Salads		Seven Ways	

OBJECTIVE: To solve multiplication
   story problems.
Directions: Use the menu to solve the
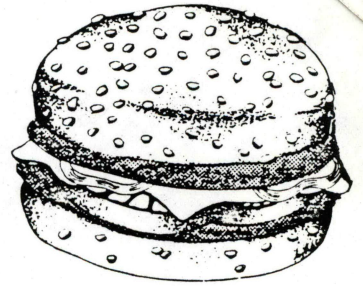   following story problems (all answers
   exclude sales tax):

1. What would be the cost of 6 Steak Dinners?

2. How much must you pay for 5 glasses of Orange Juice?

3. Randy bought 4 Bacon Burgers. How much did he pay?

4. Find the cost of 8 Chicken Sandwiches.

5. A man orders 7 Hut Burgers with cheese. How much will
   his check total?

6. Find the cost of 4 Fish Dinners.

7. What amount of money would you need if you wanted to
   buy 6 pieces of Cherry Pie?

8. What would your check total if you ordered 5 Cheese
   Sandwiches?

9. How much would 9 Steak Dinners cost?

10. What would you pay for 7 Spanish Omelettes?

OBJECTIVE: Multiplying
  to find cost when buying
  more than one of the same item.
Directions:
  1. Use the menu to find the item price.
  2. Multiply to find total price.

$2.25
x   5
$11.25

	item price	total price
1. Five Hut Burgers	$2.25	$11.25
2. Six Colas		
3. Eight Jumbo Burgers		
4. Ten Tuna Sandwiches		
5. Four Beef Stew Dinners		
6. Thirteen Ham and Eggs		
7. Sixteen Tomato Salads		
8. Twelve Steak Dinners		
9. Nine Root Beers		
10. Seventeen Bacon Burgers		
11. Twenty-seven Puddings		
12. Thirty-four Ham Sandwiches		
13. Forty-two Plain Omelettes		
14. Twenty-nine Patty Melts		

28