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Helping children understand fractions

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California State University

San Bernardino

HELPING CHILDREN UNDERSTAND FRACTIONS

A Project Submitted to

The Faculty of the School of Education

In Partial Fulfillment of the Requirements of the Degree of

Master of Arts

in

Education: Elementary Option

by

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San Bernardino, California

1986

APPROVED BY:

Advisor

Committee Member

INTRODUCTION

Research on children and their understanding of fractions is limited. Mathematics educators and curriculum writers have placed emphasis on the learning of rules for operations on fractions. This project is designed to provide an overview of the current research available. The literature review will provide valuable information regarding the design of instruction in the area of fractions.

The need to find answers to problems not involving whole numbers led man to invent fractional numbers. The word fraction is derived from a Latin word meaning "broken," suggesting that fractions deal with parts, or pieces of wholes. Fractions have two meanings: equal parts (partitioning) and ratio (comparison by division). Also there are two kinds of fractions: common fractions and decimal-fractions. Decimal fractions and common fractions are based on the same ideas and differ only in notation. Although children in their everyday concerns think more in terms of common fractions than of decimals, they cannot avoid meeting decimal fractions in their daily living, since decimals are such an integral part of the science and industry of our civilization. However, for purposes of this project, common fractions and their understanding for children will be the thrust.

Methods for finding common denominators, reducing fractions to lowest terms, cross multiplying, and so on, come in endless strings of rules and procedures. Typically students search for rules in

order to find one they think will work. For children with weak grasps of these concepts, the rules become mysterious, confusing, and frustrating. Some students have a group of isolated, inflexible, specific rules that are not synthesized and which allow for very little transfer.

Currently, it is believed that teaching children to understand arithmetic is more effective than merely encouraging them to memorize specific arithmetic combinations and relationships. Learning is achieved if it is retained over a period of time beyond the concluding day of the training period.

This project is specifically designed to address the need for certain stated subconcepts as being prerequisite to others. Unfortunately, fractions do not represent a simple extension of familiar skills. Since the application of fraction skills depends on an understanding of fractional numbers, initial instruction in the concepts and conventions characterizing fractions is critical.

The purpose of this project is to fill two general needs. First, it will examine the information gleaned from the research on helping children understand fractions. The information will provide important insights into the development of a child's ability to develop a fractional concept and learn to compute with fractional numbers. The research will provide a basis for teaching fractions that will provide the most success.

Second, this project will formulate a skill hierarchy based on

the information derived from the literature, designed to see the interrelationship among various fraction skills. Methods, procedures, and activities to help children form a better foundation in the meaning of fractions rather than more work with rules and procedures for computation will be provided. The emphasis needs to shift from the learning of rules for operations on fractions to the unveiling of a conceptual basis for fractions (14, 21, 23).

Although the language of fractions is spoken daily by most children, their concepts of fractional values may be cloudy or distorted. Presenting fractions to children in a meaningful manner has always been a difficult and challenging task for the teacher. Many fractional ideas and operations are complex and frequently have to be demonstrated and explained to some children in many ways. The highly creative teacher is constantly searching for activities and frames of reference into which mathematics can be projected.

As with other areas of mathematics, educational games and activities will help maintain students' interest in fractions and offer practice and reinforcement opportunities. The activities suggested can be modified or redesigned to accommodate a particular need a child may have. A creative teacher will devise some of his/her own games and activities as well as incorporating the suggested activities or commercially made games. There are many commercially produced games dealing with fractions available.

The information offered in this project is intended to suggest

only basic methods of presentation. The suggestions can be modified

to suit individual needs.

LITERATURE REVIEW

Researchers and educators have recognized the problem of children's understanding of fractions for many years. Educational researchers have investigated the possible causes and have tried to develop successful programs for the teaching of fraction concepts. While research has been conducted to show that operations are more difficult when applied to fractions than when applied to whole numbers (28), research on teaching fraction operations is difficult to interpret. More carefully controlled research is needed to sort out the relative importance of particular strategies (29). The increased attention and need for research on children's acquisition of rational number concepts is shown in results from national assessments. Behr et al. (1) indicated that in a recent national assessment, 30% of the nation's 13 year-olds added the numerators and denominators to find the sum of 1/2 and 1/3. They suggest that these national assessments demonstrate that more insight into children's thinking about rational number concepts is needed.

Peck and Jencks (21) state that the difficulties children have with fractions are conceptual. Children appear to be going through the motions of operations of fractions but they have not been exposed to the kinds of experiences that could provide them with necessary understandings. They suggest that mathematics educators and curriculum writers need to shift emphasis from the learning of

rules for operations on fractions to the unveiling of a conceptual basis for fractions. In their study with sixth grade children, it was found that when they were given physical materials, and asked to make their own decisions, nearly all of them were able to conceptualize and make logical predictions. However, when it came to assessing the children's understanding of fractions, the children were confused. About 55% of the students interviewed were unable to show that they possessed a meaningful concept of fractions. About 35% of the students appeared to have a correct concept of fractions.

At the time of entering school, many children have already acquired some ideas of fractional parts. They may have used expressions such as "half a glass of milk," "half an apple," and "half an hour." Children have many uses for fractions in daily life. At an early age they learn to break a candy bar into halves. They see pies or cakes cut into fourths, fifths, or sixths. Gradually, through experience and guidance, they understand that fractions are needed for measurement and to make possible the division of any two whole numbers.

Presumably, older children are better able to understand fractions than younger children. However, there is no reason for delaying the teaching of fractions. The reason for difficulty is that common fractions are too frequently taught in terms of operations on the symbols. Rules for operations are stressed before children have adequate conceptual understanding anchored in

perceptual experience (20). Presenting fractions to children in a meaningful manner has always been a difficult and challenging task for the teacher. Many fractional ideas and operations are complex and frequently have to be demonstrated and explained in many ways.

Often times pupils become quite expert at the manipulation of fractions and yet have little understanding of their meaning or of the operations used in combining them. Several researchers (14, 17, 23) agree that it is relatively easy to teach children to manipulate fractions as numerals so that they arrive at a correct answer but have little or no understanding of the ideas behind the manipulation. There needs to be extensive use of concrete teaching materials that permit the child to actually see and feel fractional divisions and ratios. Young children should have some experience dividing objects into fractional parts.

The foundation for meaningful and efficient computation of fractions should be laid slowly and thoroughly. Children should not be rushed into the use of the symbolism of fractions. They should be permitted to have a great many first-hand contacts with a variety of everyday materials in concrete situations involving fractions and fractional parts. Informal experience with fraction expressions builds toward understanding, but misconceptions may also be acquired through these unplanned and incidental experiences.

Children should be encouraged to utilize concrete models representing complicated rational operations until adequate

conceptual understanding has been developed. Symbolic manipulation should take on deeper meaning as a result (1). The concrete media is important in building clear conceptual constructs. Behr et al. (1) found that strategies used by children suggest that they base their thinking on a mental image of their experience with manipulative aids.

The literature (1, 14, 17) indicates that the printed textbook and teacher's manual are still the primary bases for most arithmetic instruction. There are two main areas of concern regarding the way fractions are taught in basal programs. They are (a) lack of adequate practice for students to develop mastery and (b) lack of structured teaching (23). Typically in grade one through three books, only one to three weeks are devoted to fraction skills. Minimal review is presented on fraction skills after the fraction unit is presented. Because of this minimal review, it is highly likely that many students will not retain fraction-related skills taught in these earlier grades. In intermediate grades, the critical fraction analysis skills taught in early grades are briefly reviewed prior to teaching new skills. The amount of practice provided in the programs must be significantly supplemented if the students are to develop mastery (29).

Researchers (1, 18) feel that more instructional time is required to develop an understanding in the area of fractions. There is a need for careful spiraling of concepts through several

grades. The concepts and experiences with different kinds of fractional parts need to be developed continually during each school year. Many of the textbooks wait until almost the end of the book to deal with fractions and then only one chapter, or at the most two, is devoted to working with fractions.

Research has been conducted to show that operations are more difficult when applied to fractions than when applied to whole numbers (28). Research on teaching fraction operations is difficult to interpret. For example, Miller (17) compared a traditional approach to teaching the multiplication of fractions with an experimental program in which students worked from automated devices that provided immediate feedback to the students. Although students in the experimental groups answered significantly more problems on a post-test, the F ratio of 7.60 exceeded the critical ratio of 3.93, the reason for the difference is difficult to determine. Possibly immediate feedback accounted for the difference, but other aspects of the treatment could have been responsible. Miller felt that there were positive findings regarding the superiority of the experimental approach versus the traditional approach.

A wide variety of methods for teaching arithmetic are in use today. Teachers tend to use whatever method seems most efficient to them. Teachers are usually satisfied with the method used when measurable learning takes place. The type of learning that takes place may be of little concern. Tests generally measure learning

requiring computational ability but do not measure the extent of the child's understanding of the process.

Invariably, division of a fraction by a fraction has been taught through presentation of the rule invert the divisor and multiply. Then drill is used for reinforcement. Krich (14) feels that there needs to be sharp delineation between methods stressing meaningful teaching and those stressing mechanical methods of teaching (drill).

Krich's (14) study of two methods for teaching the process of division of a fraction by a fraction concluded that when arithmetic is taught meaningfully, children can and do retain the material. The two methods he used were: (1) the meaningful method, which allowed children the opporutnity to understand the arithmetic processes used in dividing a fraction by a fraction and (2) the mechanical method or rote learning, where the child was given a rule (invert the divisor and multiply) to use when dividing and then was presented with drill material.

The most commonly used semi-concrete models for fractions in elementary school textbooks are geometric regions, sets, and the number line. According to a study by Larson (15) it is more difficult for students to associate a proper fraction with a point on a number line than associating a proper fraction with a part-whole or part-group model. When responding to test items where the number line was of length 2, 15 to 25% of the sample chose fractions

that indicated that they considered the whole number line the unit and not just the segment from 0 to 1. For example, 25% of the students selected 2/12 as the correct response when the number line was of length two and each segment was separated into six equivalent line segments. Results of the study indicated that some students do not have a very flexible concept of equivalent fractions. The data collected suggested that many students do not associate the name 1/3 with a point indicated by 2/6 on a number line. Perhaps some students have a group of isolated, inflexible, specific rules that are not synthesized and which allow for very little transfer.

Behr et al. (1) found in their study with children's understanding of equivalent fractions that most children did not show evidence of being aware that both the numerator and the denominator must be considered when judging the equivalence of two fractions. These researchers also found that the performance of the children was dominated by their knowledge of the ordering of whole numbers.

In agreement with Piaget, Pothier and Sawada (23) found that young children master halves and fourths prior to thirds. It was found that in attempts to partition rectangular and circular regions into thirds and fifths, the dominance of the halving algorithm was evidenced. It seemed that many children were incapable of deviating from employing a halving line as the initial cut.

The young child, even though frequently hearing expressions

such as "Break it in half" and "Here's half," does not know the meaning of half in a number sense, and the necessary characteristics of evenness and one of two parts are not understood. The concept of rational number is rote learning at this stage (18).

There are certain stated subconcepts which are prerequisite to others. Novillis (8) came to this conclusion using a hierarchy of selected subconcepts which she developed based on Gagne's (7) procedure for developing a hierarchy. She constructed The Fraction Concept Test with all items at the knowledge or comprehension level. It was necessary for a student to attain a score at the 75% level of accuracy in order to attain criterion on each subtest. If a student attained criterion on a subtest, then it was assumed that he/she had acquired the related subconcept.

The hierarchy was constructed on the premise that students' familiarity with part-group and part-whole models in various situations developed in a similar sequence. The results supported this premise. The author developed two ratios to analyze the data (based on the work of Gagne and Walbessen). Ratio 1 was at the 0.75 level. Ratio 2 was computed only when Ratio 1 was at the 0.75 level (it reached the 0.90 level). Ratio 2 was defined as an adequate test of the hypothesis (18).

The results of students' scores on some of the subtests lead to the inference that the instances of the fraction concept that students are exposed to in elementary school are not of sufficient

variety to encourage generalization of the fraction concept. Another inference Novillis made was that students do not come into contact with an adequate number of negative instances of the fraction concept. An example of this would be a problem similar to the following: "If two rectangular regions have been separated into five parts such that in one case the parts are congruent and in the other case the parts are neither congruent nor equal in area, and in each case one of the parts is shaded, then many students associate the fraction 1/5 with each of these regions and indicate that 1/5 of each region is shaded (18).

When teaching any mathematical process, it is sound psychology to bring into the explanation previous learnings a child has had, provided they are germane to the process. Many students have no idea how the symbols of mathematics relate to their previous experience.

A knowledge of common fractions is important. A wide variety of occupations depends on a knowledge of common fractions. Anyone who omits or fails to teach fundamental ideas of common fractions is guilty of blocking students' development, curtailing career opportunitites, and limiting career choices (20).
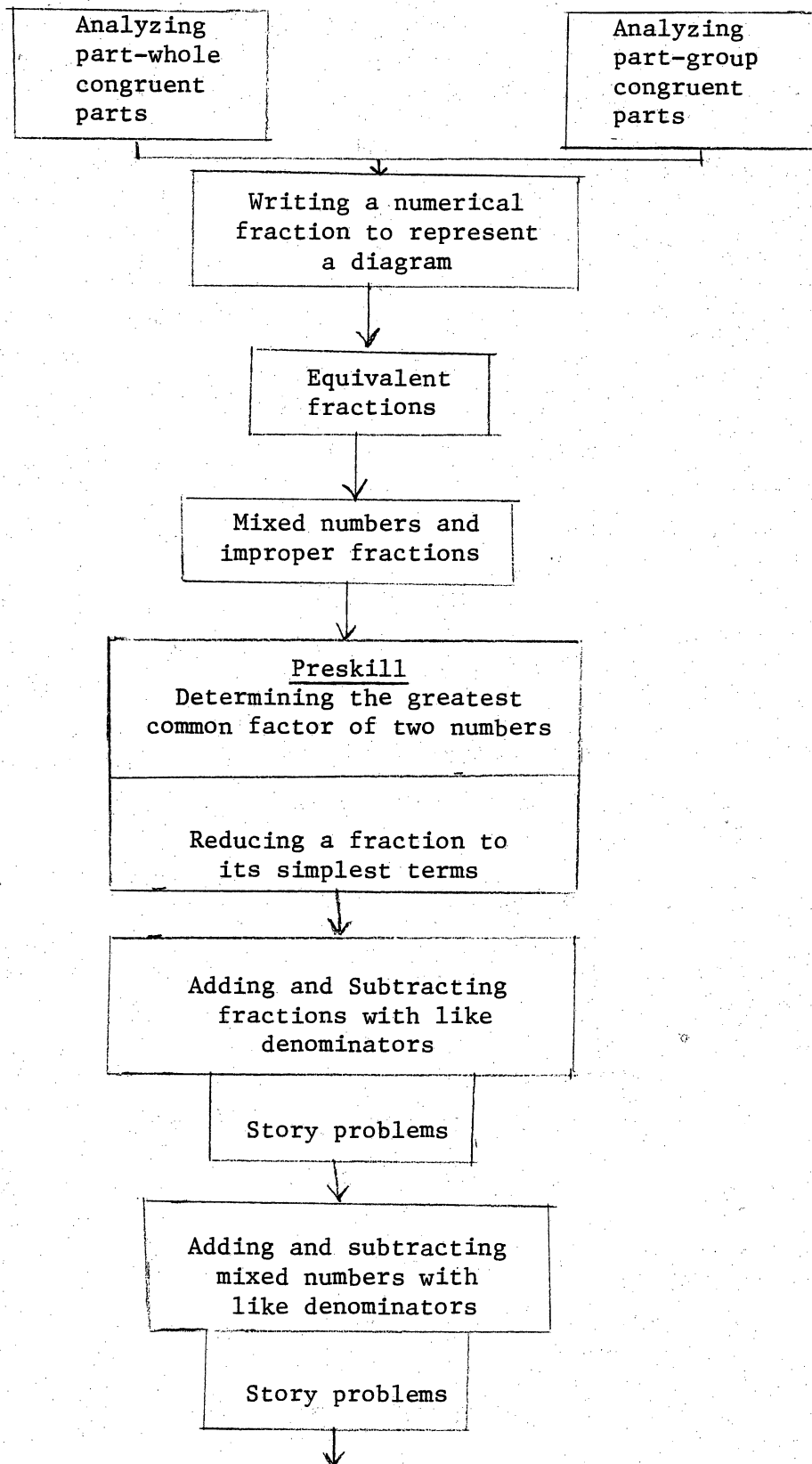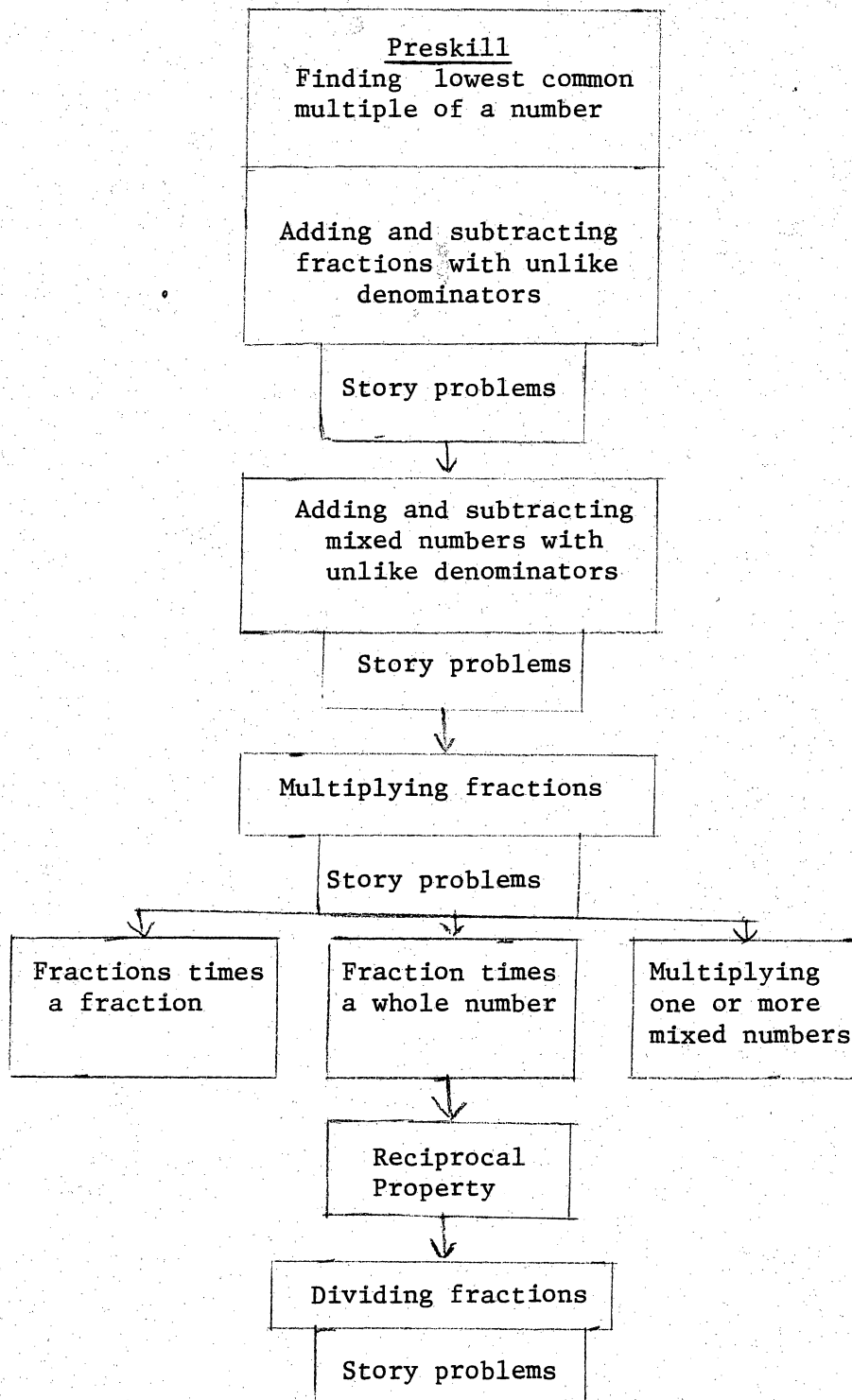
FRACTIONS: A SKILL HIERARCHY

Following the ideas presented by researchers (1, 14, 18), it can be said that in the area of fractions a skill hierarachy to show the interrelationship among various fractions skills and that certain subconcepts are prerequisite to others is basic and essential. A sequence for teaching fractions must be arranged so that all component skills for an advanced problem type have been presented before the advanced problem type is presented. The writer of this project has developed a skill hierarchy designed to help the reader see the interrelationship among various fraction skills. The skills appearing at the beginning of the sequence chart (see Fig. 1) lay the foundation for a conceptual understanding of fractions.

Motivation comes from the real world. Children do not have out-of-classroom experience for what they are doing in school mathematics. It is important that we make fractions relevant to everyday life.

Textbooks do not place enough emphasis on the learning of fractions. Concepts need to be slowly introduced throughout the textbook and reviewed continually. This is better than throwing the concept at the children for a matter of ten pages or less. A careful spiraling of the fraction concept through the grades is essential (1). We, as teachers, can help students with this by making fractions an ongoing process throughout the year in our classrooms.

FIGURE 1. SKILL HIERARCHY

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┌─────────────────┐                    ┌─────────────────┐
│ Analyzing       │                    │ Analyzing       │
│ part-whole      │                    │ part-group      │
│ congruent       │                    │ congruent       │
│ parts           │                    │ parts           │
└─────────────────┘                    └─────────────────┘
          │                                    │
          └──────────────┬─────────────────────┘
                         ▼
          ┌─────────────────────────────┐
          │  Writing a numerical         │
          │  fraction to represent       │
          │        a diagram             │
          └─────────────────────────────┘
                         │
                         ▼
              ┌────────────────────┐
              │    Equivalent      │
              │    fractions       │
              └────────────────────┘
                         │
                         ▼
          ┌─────────────────────────────┐
          │  Mixed numbers and           │
          │  improper fractions          │
          └─────────────────────────────┘
                         │
                         ▼
     ┌──────────────────────────────────────┐
     │            Preskill                    │
     │   Determining the greatest             │
     │  common factor of two numbers          │
     │                                        │
     ├──────────────────────────────────────┤
     │                                        │
     │      Reducing a fraction to            │
     │        its simplest terms              │
     └──────────────────────────────────────┘
                         │
                         ▼
     ┌──────────────────────────────────────┐
     │      Adding and Subtracting            │
     │        fractions with like             │
     │           denominators                 │
     └──────────────────────────────────────┘
                         │
              ┌──────────────────────┐
              │   Story problems     │
              └──────────────────────┘
                         │
                         ▼
          ┌─────────────────────────────┐
          │   Adding and subtracting     │
          │     mixed numbers with       │
          │      like denominators       │
          └─────────────────────────────┘
                         │
              ┌──────────────────────┐
              │   Story problems     │
              └──────────────────────┘
                         │
                         ▼
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                    ┌─────────────────────────────┐
                    │          Preskill           │
                    │ Finding  lowest common      │
                    │ multiple of a number        │
                    ├─────────────────────────────┤
                    │                             │
                    │ Adding and subtracting      │
                    │ fractions with unlike       │
                    │      denominators           │
                    └─────────────────────────────┘
                        │  Story problems         │
                        └────────────┬────────────┘
                                     ▼
                    ┌─────────────────────────────┐
                    │  Adding and subtracting     │
                    │     mixed numbers with      │
                    │    unlike denominators      │
                    └─────────────────────────────┘
                          │ Story problems  │
                                ▼
                    ┌─────────────────────────────┐
                    │    Multiplying fractions    │
                    └─────────────────────────────┘
                          │ Story problems  │
        ┌──────────────────────┼──────────────────────────┐
        ▼                      ▼                           ▼
┌──────────────┐    ┌──────────────────┐        ┌──────────────────┐
│ Fractions times│  │  Fraction times  │        │   Multiplying    │
│  a fraction   │    │  a whole number  │        │  one or more     │
│              │    │                  │        │  mixed numbers   │
└──────────────┘    └──────────────────┘        └──────────────────┘
                            ▼
                    ┌──────────────────┐
                    │    Reciprocal    │
                    │    Property      │
                    └──────────────────┘
                            ▼
                    ┌──────────────────┐
                    │ Dividing fractions│
                    └──────────────────┘
                      │ Story problems  │
```

Analyzing Part-Whole and Part-Group Parts

Students need to begin their study of fractions by discriminating between the number of parts in each whole unit and the number of whole units. Lola May (16) feels that beginning around first grade, students should work with whole units divided into many different numbers of congruent parts. She says it is important to plan the exercises so that pupils learn first to notice the total number of congruent parts.

A concrete model that can be used to show fractional parts of a whole is food cut into pieces. A group of students is given an apple, a pizza, or some other easily partitioned food and asked to divide it so that all students involved are satisfied with their share.

A concrete model for showing students the part-group concept is to give groups of students counters and have the students divide the counters into piles for the number of students in the group. Another example suggested by Clark and Eads (3) is to use sixteen boys in the classroom. Questions to be asked are: If half of the boys go to the library, how many boys will go? Or replace half with one-fourth or one-eighth. Another example is: Jerry guessed five of the ten words correctly. What part of the words did he guess correctly? The richer the first hand experiences children have with people and things, the more children will feel the need to understand and use fractions.

At the representational stage for analyzing part-whole and part-group fractions, the teacher places circles (or other geometric shapes) on the board and divides each into an equal number of parts. The teacher tells the students that each circle or group of circles (depending on whether part-whole or part-group fractions are being emphasized) is called a whole and then leads the students through determining how many parts in each whole. Reys et al. (24) feels that a variety of shapes should be used so that children do not think that a fraction is always a "part of a pie." The rectangle is probably the easiest for children to draw and partition.

Underlying the idea of part-whole is the meaning of part and of whole. The whole is whatever is specified as the unit. The part must be an "equal" part and students must learn to partition the whole into equal parts. It is imperative that students first learn to notice the total number of congruent parts.

Activities

1-1. To help children learn about fractional parts, cut some strips of paper, each a fractional part of a foot. Fold into uniform size but with its designation on the outside, and place in a pocket chart. Children may be instructed to arrange fractions in order of size. If they cannot decide between 2/3 and 3/4 for example, they can unfold strips and compare lengths.

1-2. Half a Rod (5).

Purpose: to provide practice in using Cuisenaire rods to show one-half of even whole numbers less than 12.

Number of Players: 1 or more.

Materials: Cuisenaire rods.

Procedure: Tell players that, at a signal, they are to show all rods or combinations of rods that measure one-half the length of some other rod, e.g., red is one-half the length of purple; green and white are one-half the length of brown. Some rods such as black, will have no half-length combinations.

Variation: Do the same with one-third or one-fourth the length of rods up to orange and yellow.

Give rods number values (one for white, two for red, three for green, etc.) and ask players to record on paper all natural numbers or combinations of natural numbers which equal one-half or one-third of the number (1/2 of 10 = 4 + 1 = 3 + 2 = 1 + 4, etc.).

1-3. Geoboard Fractions (27).

Purpose: to provide practice in using geoboards to show one-half.

Materials: Geoboards, rubber bands of different colors.

Procedure: The teacher puts a rubber band of one color around the outside edge of each geoboard. Each student is asked to divide his geoboard in half with a different-colored rubber band. The board may be divided in half in a variety of ways. Students should be asked to find as many different ways as they can. Encourage them to define some criterion for "halfness" that all of these representations have in common.

This activity can be repeated with one-fourth or one-eighth.

Writing Numerical Fractions to Represent Diagrams

After the meaning of the symbol is taught then the symbol is introduced. May (16) suggests that when beginning to introduce the symbol for fractional numbers, one ask the group of students how they would write 1 out of 2, or what we usually call one-half. She feels that many creative answers will be received and that it is important to give pupils credit for all these efforts. Then tell them that man invented a way of writing fractional numbers many years ago. We all have to use the method because people all over the world understand this method of writing fractional numbers.

Students learn that the bottom number of a fraction tells how many parts in each whole while the top number of a fraction represents how many parts are special.

$$\frac{1}{2} \quad \begin{array}{l} \text{Number of special parts} \\ \text{Number of congruent parts altogether} \end{array}$$

At this point students can learn the terms numerator and denominator. Students can be told that the line in the symnbol can be read as "of the," indicating the division of a unit into congruent parts.

A couple of guidelines are important for an appropriate example selection for this skill. First, the number of parts in each whole unit, the number of whole units, and the number of special parts should vary from example to example. Second, the examples should include a mixture of proper and improper fractions. Examples should

include some fractions that equal less than a whole unit:
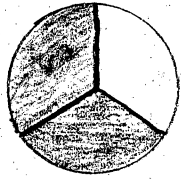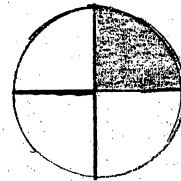
 $= \frac{2}{3}$ $= \frac{1}{4}$

some examples that equal more than one unit:
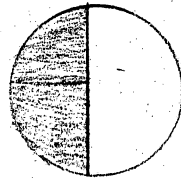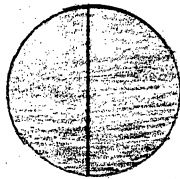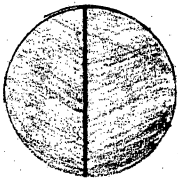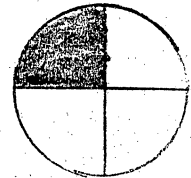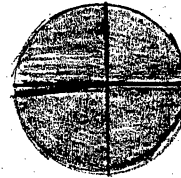
 $= \frac{5}{2}$ $= \frac{5}{4}$

and a few examples that equal one unit:
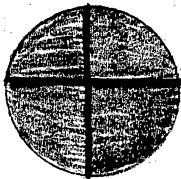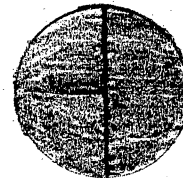
 $= \frac{4}{4}$ $= \frac{2}{2}$

Special attention should be given to examples containing a series of units that are not divided:
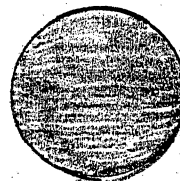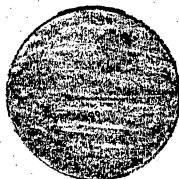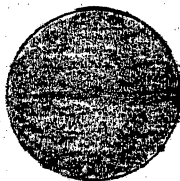
 $= \frac{3}{1}$

These diagrams will need special explanation. The teacher should point out that if a whole is not divided into parts, students should write a 1 on the bottom. The 1 tells that there is only one part in

the whole unit. Examples which yield 1 as a denominator should not be introduced when fractions are initially presented but can be introduced about a week later. Thereafter, about 1 in every 10 diagrams should be an example with 1 as a denominator. These examples are important since they present a conceptual base of exercises in which students convert a whole number to a fraction (e.g. 8 = 8/1).

When students have learned to accurately fill in the numerals to represent a fraction, they can learn the simple procedure of translating numerical fractions into diagrams. For 3/4, the teacher would say "Touch the bottom number. What does it tell you?...Draw four parts in each whole...Touch the top number...What does it tell you?...Shade in three parts."

Activities

2-1. Fract Match (12).

Purpose: to provide practice matching numeral fractions to
diagrams.

Number of Players: 2-5.

Materials: Prepare a set of three-by-five-inch cards that contains
numerals for common fractions - 1/2, 1/4, 2/3, 5/6, 7/8
- and any others with which children are working. Make
another set of cards that contains pictures of sets,
regions, and line segments separated into parts to
represent the common fractions. There should be more
than one picture for each fraction card. Make a key so
players can check answers during the game.

Procedure:

1. The dealer shuffles the fraction cards and puts them face
down. He shuffles the picture cards and deals all of them.
Each player puts his picture cards face down in a pile.

2. The dealer turns over the top fraction card.

3. At the dealer's signal each player turns over his top card.
If any one or more of the picture cards match the fraction
card, the players say "Fract Match." The first player to
say it gets all the picture cards that have been turned up.

4. Play continues with the dealer turning fraction cards and
players turning picture cards until all picture cards have

been played. (When players turn over their last picture
cards without making a match with the top fraction card,
each one shuffles his remaining picture cards and puts them
face down again so they can continue to play.)

5. The winner is the player who collects the most cards.

Equivalent Fractions

To find an equivalent fraction, the teacher wants the generalization that both the numerator and denominator may be multiplied (or divided) by the same number. A paper-folding model, symbolically describing what is happening should be used as a first step in this process.

Picture A   Picture B
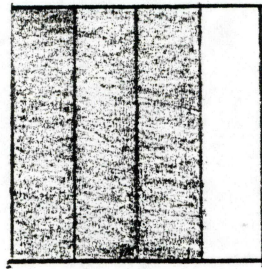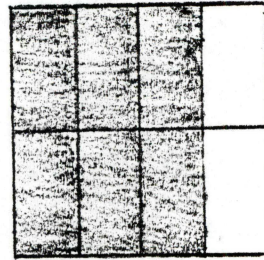
$\dfrac{3}{4}$ $\dfrac{6}{8}$

Make a model of 3/4 by folding a piece of paper (picture A); then fold it in half the other way (picture B). Establish that 3/4 = 6/8. Now, look at what happened when the paper was folded in half. Twice as many equal parts (or 2 x 4) were created and twice as many shaded parts (or 2 x 3). This can be written as:

$$\frac{2 \times 3}{2 \times 4} \quad \text{or} \quad \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

After more examples, the students should make the generalization that both the numerator and denominator may be multiplied by the same number and the resulting fraction is equivalent.

Conversely, one could begin with picture B and describe how to get to picture A. In picture B, we began with 8 equal parts and grouped them by 2, or $8 \div 2$. This also groups the number of parts under consideration by 2, or $6 \div 2$. Thus,

$$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}"$$

Again, this type of example should lead to the generalization that the numerator and denominator may be divided by the same number.

A basic rule to be reviewed at this point is the property of one. Ask students what the fraction 2/2 means. The students should say it means that some whole unit has been divided into two congruent parts and that both parts are being considered. Thus their answer should be that another number for 2/2 would be one whole unit or the number 1. After several questions like this, pupils should begin to see that when any number is divided by itself, it stands for the whole number 1.

Once the students have made the generalization that both the numerator and denominator may be multiplied or divided by the same number, then the students are ready to move to problems such as:

$$\frac{2}{3} = \frac{}{12} \qquad \frac{4}{6} = \frac{}{3}$$

In the first example, the students need to think, "What is 3 multiplied by to get 12?" Once they have established it is 4, they should write:

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

In the second example, they should realize that 6 was divided by 2 to obtain 3 so that 4 would also have to be divided by 2, or

$$\frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

The first example is the type of thinking one needs in finding a common denominator; the second, the type needed in simplifying many problems.

May (16) believes that teachers should try an intermediate stage of adding like numbers to show equivalent fractions before they get to the stage where students are asked to multiply both the numerator and the denominator by the same number. May says that many pupils for the first time will understand why they can multiply by the same number without changing the value of the number.

The addition stage that May is suggesting has students trying to discover a pattern with equivalent fractions. After a pattern has been found, one-half can be presented in another way. Is one-half equal to seven divided by seven plus seven? (Yes, it is.) $\frac{1}{2} = \frac{7}{7+7}$. Now pupils can fill in missing numbers in similar problems: $\frac{1}{2} = \frac{15}{15+}$, $\frac{1}{2} = \frac{}{17+17}$. By filling in the numbers the pupils are seeing that one-half means two of some number in the denominator and one of the same number in the numerator. This leads to the next stage. If $\frac{1}{2} = \frac{7}{7+7}$ then does $\frac{1}{2} = \frac{1 \times 7}{2 \times 7}$? The answer is yes because 7 + 7 means 2 x 7 and 7 means 7 x 1. Using this method

to name equivalent fractions of 2/3 would look like this:

$$2/3 = \frac{5+5}{5+5+5} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

To find an equivalent fraction, we want the generalization that both
the numerator and denominator may be multiplied (or divided) by the
same number.

Lola May (16) suggests the test of cross multiplying to prove
whether fractions are equal. Cross multiplying involves using the
numerator of the fraction on the left and multiplying it by the
denominator of the fraction on the right; then multiply the
denominator of the fraction on the left by the numerator of the
fraction on the right. If the cross multiplications are equal, then
the fractions are equal.

$$\frac{3}{4} \stackrel{?}{=} \frac{6}{8} \qquad\qquad \frac{2}{7} \stackrel{?}{=} \frac{3}{9}$$

3 x 8 = 4 x 6   2 x 9 ≠ 7 x 3 (fractions are
                                     not equal)

This test for equality, suggests May, can also be used for
inequality. Is 2/5 equal to, greater than, or less than 3/8?

$$2/5 > 3/8 \qquad\qquad \begin{array}{l} 2 \times 8 = 16 \\ 5 \times 3 = 15 \\ 16 \quad > 15 \end{array}$$

Some students will want to know why this technique works, so
now you can go back to equivalent fractions. Find the fraction in
the 40 family that names the same number as 2/5. Find the fraction

in the 40 family that names the same number as 3/8.

$2/5 \times 8/8 = 16/40$ $3/8 \times 5/5 = 15/40$ $16/40 > 15/40$

Thus cross multiplication is a short way of comparing fractions without having to find equivalent fractions each time.

Activities

3-1. Fraction Equivalence Devices (5).

    a. Make a frame from a piece of plywood on which narrow strips of wood are nailed or glued. Now make strips of stiff paper of a size to fit between the wooden strips. Mark each strip into fractional parts and cut the parts so that children can place them into position on the board and make their own comparisons.
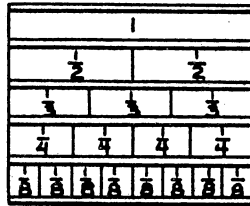
    b. Fractional parts of circles cut so that pupils can manipulate them are useful. If a small piece of coarse sandpaper (or felt) is glued to the back, the pieces will cling to a flannel board.

3-2. Equivalent Rummy (5).

Purpose: to provide practice in recognizing equivalent fractions.

Number of Players: 2 to 4.

Materials: A pack of cards each marked with a fraction. For each number, there must be three cards each with the number shown in a different form, as, 1/2, 2/4, 6/12. There should be 18 to 20 such sets.

Procedure: Players draw for dealer; largest number indicates dealer. Deal five cards to each player. Place balance of deck face down with top card turned up. Player to left of dealer begins. He/she may take either the turned up card or the top card from the "blind" deck. If he/she has three cards showing equivalent fractions, he/she lays them before him/her. When he/she has finished, he/she discards one card face up. Play then proceeds to the left. When one player has no cards, the game ends and the one with the most sets wins.

3-3. Fracto (27).

Purpose: to provide practice in naming equivalent fractions.

Number of Players: Small groups.

Materials: A 3 x 3 playing mat with 9 calls for each player, containing the notation for frequently used fractions; some call cards, containing different names for the fractional numbers on the playing mats; and discs.

Procedure: The game is played like Bingo. A caller is chosen, and holds up a card from the call pile. Any player with an equivalent fraction covers the corresponding call (or calls) with a disc. The first player to obtain three discs in a row

(vertically, horizontally, or diagonally) is the winner.

Mixed Numbers and Improper Fractions

Through models you can lead naturally into mixed numbers and improper fractions.

Picture A
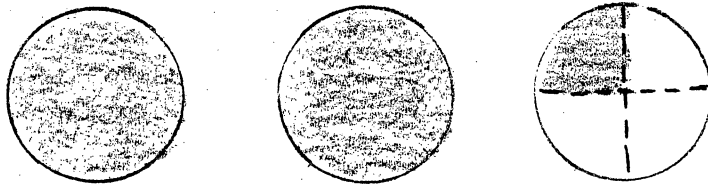
Picture B
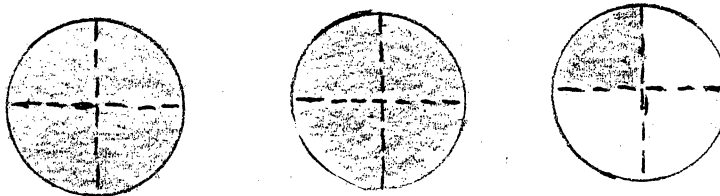
The most natural description of picture A is 2 1/4. Drawing in the fourths as in picture B helps us see the improper fraction representation of nine-fourths.

After becoming familiar with both improper fractions and mixed numbers, students need to be able to change from one form to the other without the use of models. However, familiarity with the models should help in the process.

Reducing a Fraction to Its Simplest Terms

Reducing fractions using a greatest common factor strategy would be presented during late fourth grade. Students are taught to reduce a fraction to its simplest terms by pulling out the greatest common factor of the numerator and denominator.

Preskill

Teaching students to find the greatest common factor of two numbers is the critical preskill for reducing fractions. The greatest common factor of two numbers is the largest number that can be multiplied by whole numbers to end with two target numbers. For example, the greatest common factor of 12 and 18 is 6. Six can be multiplied by whole numbers to end with 12 and 18.

The first step in teaching students to find the greatest common factor of two numbers is to teach them to determine all possible factors for a given number. For example, the numbers 1, 2, 3, 4, 6, and 12 are all factors of 12, since they can all be multiplied by another whole number to end with 12. The teacher defines the phrase, greatest common factor, as the largest number which is a factor of both target numbers. The teacher then leads the students through finding the greatest common factor. First the teacher asks students what the largest factor of the smaller target number is and if that factor is also a factor of the other target number. For example, assuming that 8 and 20 are the target numbers, the teacher asks what the largest factor of 8 is. The students reply, "8 is the largest factor of 8." The teacher then asks, "Is 8 a factor of 20?" Since the answer is no, the teacher asks the students to tell him/her the next largest factor of 8. "What is the next biggest factor of 8?" After the students answer 4, the teacher asks, "Is 4 a factor of 20?...So, 4 is the greatest common factor of 8 and 20" (29).

Reducing Fractions

The format for reducing fractions would be introduced when students are able to determine the greatest common factor of two target numbers. The teacher writes a fraction on the board with an equal sign next to it. Next to the equal sign are parentheses and a fraction bar for the reduced fraction:

$$\frac{12}{16} = (\quad) \underline{\hspace{3cm}}$$

The fraction in which the numerator and denominator are the greatest common factor of the two target numbers will be written inside the parentheses. For example, the greatest common factor of 12 and 16 is 4. Thus, the fraction in the parentheses will be 4/4, which equals 1. The teacher then asks, "12 equals 4 times what number?" The answer is 3, which is the numerator of the reduced fraction. The teacher then asks, "16 equals 4 times what number?" The answer is 4, which is the denominator of the reduced fraction. Since multiplying by 1 does not change the value of the fraction, 4/4 can be crossed out. Crossing out the fraction equal to 1 leaves the reduced fraction:

$$\frac{12}{16} = \left(\frac{\cancel{4}}{\cancel{4}}\right) \qquad \frac{3}{4}$$

There are three selection guidelines for exercises on reducing fractions. At first both the numerator and denominator should be below 25. Second, a third of the fractions should have the greatest common factor as the numerator. Third, about a third of the

fractions should already be expressed in their simplest terms.
Including several fractions already expressed in their simplest
terms provides the students with the knowledge that not all
fractions need to be reduced (29).

Adding and Subtracting Fractions with Like Denominators

Now students are ready to learn to add and subtract fractional
numbers. May (16) suggests that after the middle of the second
grade most pupils can begin to learn to add and subtract like
fractions. She feels that adding and subtracting fractional numbers
is based on adding and subtracting whole numbers. In adding and
subtracting whole numbers students learned that in our number system
only like units can be added or subtracted. In fractional numbers
the denominators play the same role as place value of the digits.
Only like fractions can be added or subtracted. Like fractions are
fractions with the same denominator. The denominator indicates the
unit being used and the units are added and subtracted with the
numbers in the numerator.

The first activities in helping children understand the
addition and subtraction of common fractions need to be concrete.
Each student can use a rectangular piece of paper divided into four
congruent parts, first by folding, and then by cutting along the
folds. The parts need to each be labeled one-fourth. Now pupils
are asked to place two-fourths on their desk, and then place another
one-fourth on the desk beside the two-fourths. How many fourths in

all? Then write on the chalkboard:

$$\frac{2}{4} + \frac{1}{4} \overset{?}{=} \frac{3}{4} \qquad\qquad \frac{2}{4} + \frac{1}{4} \overset{?}{=} \frac{3}{8}$$

Which of these equations reflects the activity just performed?
Point out that the parts are all fourths and the sum is in fourths.
Never add the denominators, because they indicate the unit being
used. Adding the denominators changes the unit.

Children gain in their understanding of fractions when they are
taught through physical operations on concrete media.

Similar activities on the number line should be practiced.
This is a more abstract exercise than using the geometric shapes.
With the number line, pupils will apply what they learned in adding
whole numbers to the addition of fractional numbers. On the number
line only the points 0 and 1 should be labeled to begin. Pupils
should divide the unit into seven congruent parts and label each of
the parts.

$$\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$$
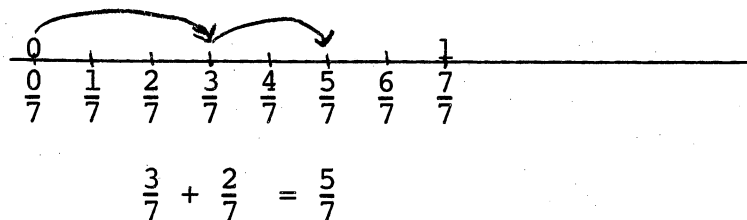
Subtraction of like fractions also should begin with concrete
experiences. The next stage is the more abstract method of seeing

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the subtraction of fractional numbers on a number line.

$$\frac{7}{9} - \frac{5}{9} = \boxed{}$$
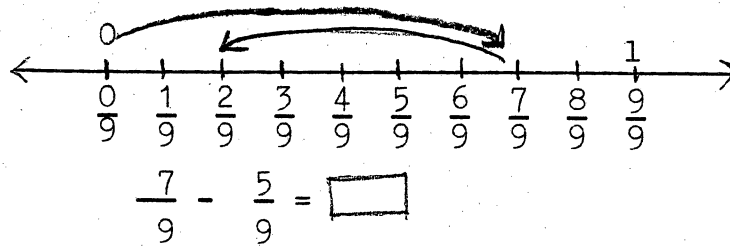
When assigning practice exercises in subtracting and adding
like fractions, May feels that some problems should be set in
vertical form and some in horizontal form. In doing the exercises,
some of the fractions will represent numbers larger than one. This
means that the numerator will be larger than the denominator. In
some cases pupils might be asked to write their answers as mixed
numerals when the number is greater than one, and as a whole number
when the numerator is a multiple of the denominator.

Thus, instead of beginning with a symbolic sentence such as 2/3
+ 1/4, the teacher should begin with joining and separating
situations. The problem needs to be solved using the pictorial
model; then a sentence will be written to describe the situation.
The purposes of this procedure are (1) to help children see that
adding and subtracting of fractions solves problems similar to those
with whole numbers, (2) to give them an idea of what a reasonable
answer will be, and (3) to help them see why a "common denominator"
is necessary when adding or subtracting (29).

Adding and subtracting like denominators should not take long
if the foundation of fractions has been built.

Story Problems

The basic guideline in introducing fraction story problems is that a new type of problem should be integrated into story problem exercises as soon as the students can accurately compute problems of that type. Story problems involving adding and subtracting fractions with like denominators should be introduced after students work such problems independently. Story problems with unlike denominators should be introduced only after students have mastered the strategy for adding/subtracting that type of problem. Story problems need variety in the types used, e.g., classification, action, comparison.

Activities

4-1. Arithmetic Race (5).

Purpose: to provide practice in problem solving.

Number of Players: 4 or more.

Materials: Problem solving sheets of paper for each team; items should be either identical or similar in difficulty. There should be as many problems as there are players on each team.

Procedure: Divide players into as many relay teams as desired; arrange pupils in rows or designate order if children are at tables. Distribute papers face down, one to the first player of each team. At a signal each works the first problem and then passes the paper on to the second player who must check the work of the first one; if the second player is satisfied that it is correct, he then works the second one. The third player checks the work of the one preceding him and then works the third problem and so on. The last player returns the sheet to the first player for checking. The team first completing all problems correctly is the winner.

4-2. Eraser Relay (5).

Purpose: to provide practice in problem solving.

Number of Players: 2 or more.

Materials: Chalk erasers and chalkboards; cards with problem solving situations.

Procedure: Divide players into two or more teams depending on the size of the group. Shuffle cards and divide them into as many stacks as there are teams; place face down at the end of the room opposite the chalkboard. At a signal, the first player of each team places an eraser on his head and goes to a given stack of cards, draws the top one, and proceeds to the chalkboard where he must do the problem; he then writes the answer in a designated place, erases his work, lays his card in the chalktray, and gives his eraser to the next player on the team. If a player's eraser falls from his head, he must pick it up and replace it before proceeding. Play continues until all have had a turn. Answers are checked and the team which has most correct answers is the winner; in case of a tie, the team finishing first wins.

Adding and Subtracting Mixed Numbers with Like Denominators

Adding and subtracting mixed numbers in which the fractions have like denominators can be introduced when students can read and write mixed numbers and can add and subtract fractions with like denominators. The child must be able to see why it is best to add the fractions first and then the whole numbers. A skill that students will use at this step is changing answers like 7 17/12 to 8 5/12. While children may have encountered this when changing from improper fractions to mixed numbers, usually they have concentrated on changing an improper fraction such as 25/8 to a mixed number. They have not changed a mixed number whose fraction part is an improper fraction to another mixed number.

On the other hand, subtracting is often more difficult, partly because they have not changed a mixed number to another mixed number and also because they lack understanding of regrouping.

$$6 \; \frac{3}{8} \; = \; 5 \; \frac{11}{8}$$

$$-2 \; \frac{4}{8} \; = \; 2 \; \frac{4}{8}$$

$$3 \; \frac{7}{8}$$

Children must understand in regrouping 6 3/8 to 5 11/8 that 1 is
8/8, and 6 is 5 + 1 or 5 + 8/8. Hence, 6 3/8 is 5 + 8/8 + 3/8 or 5
11/8.
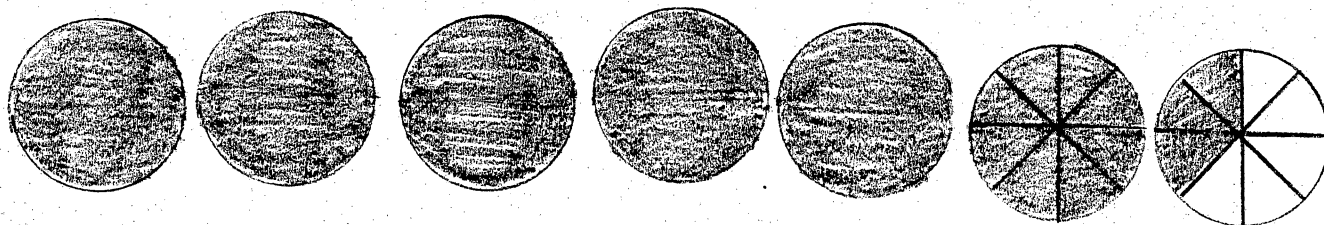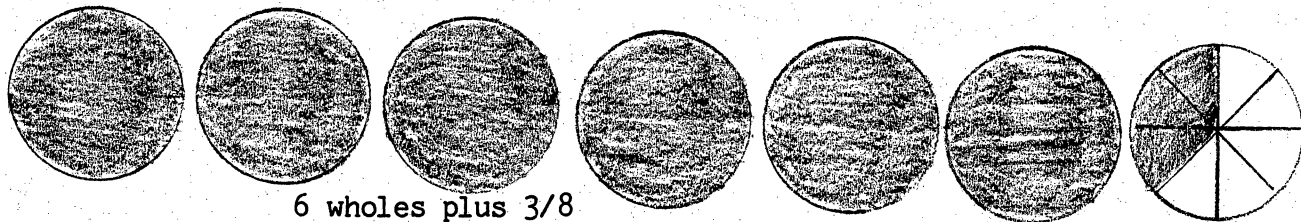
6 wholes plus 3/8

5 wholes plus 8/8 plus 3/8 = 5 11/8

A difficult problem type involving mixed numbers is:

$$8$$
$$-3 \; \frac{2}{4}$$
$$\overline{\phantom{-3 \frac{2}{4}}}$$

It is a subtraction problem involving renaming. The student must
rewrite the 8 as 7 and 4/4 to work the problem. Prior to
introducing such a problem, the teacher would present an exercise in
which the student must rewrite a whole number as a whole number and
a fraction equivalent to 1 (e.g. 6 + 5 + 4/4). In leading students
through this preskill exercise, the teacher points out that they
have to take one whole away from the original whole number and

rewrite that one whole as a fraction. Once this preskill is taught,
students should have little difficulty with problems that involve
renaming.

Adding and Subtracting Fractions with Unlike Denominators

Adding and subtracting fractions with unlike denominators is
usually introduced during fourth grade.

Preskill

Before adding and subtracting fractions with unlike
denominators is introduced, students must master finding the least
common multiple of two numbers. The least common multiple of two
numbers is the smallest number that has both numbers as factors.
For example, the least common multiple (LCM) of the numbers 6 and 8
is 24 since 24 is the smallest number that has both 6 and 8 as
factors. Likewise, the LCM of 6 and 9 is 18 since 18 is the
smallest number that has 6 and 9 as factors.

The teacher writes count-by series of two numbers on the board
so that students can visually find the lowest common multiple. An
example for finding the least common multiple of 3 and 5 would be:

   3   6   9   12   15   18
   5   10   15   20   25   30

Children will see that the smallest multiple common to 3 and 5 is
15. Thus the least common multiple of 3 and 5 is 15.

Unlike Denominators

May (16) suggests that when teaching the addition of unlike

fractional numbers, the teacher should write a problem such as 5/8 + 1/12 on the chalkboard. Then ask, "Can you add these two numbers with the names they now have?" In the discussion following this question, May says that it is important to point out that only fractional numbers with like denominators can be added. Teachers should be careful to avoid saying that they cannot add the numbers 5/8 and 1/12. This would be a false statement, for the numbers can be added but the difficulty is that the fractions need other names before the adding can be done.

According to May, a mistake that many teachers make in introducing the unit on adding unlike fractions is to start with problems that are so easy, that students can find the least common denominator by inspection. This gives the students the false impression that all least common denominators should be found in this way. May presents an alternative method of adding and subtracting unlike fractional numbers. She says that it can be taught to students in the middle grades who have difficulty in finding the least common denominator. The main prerequisite of this method is being able to add whole numbers.

Practice is needed in making display fractions before the alternative method can be taught. A display fraction is where you are given a fraction and you write it several times. There are always as many numbers in the numerator as there are in the denominator. An example of this would be: $\frac{2}{7} = \frac{2+2}{7+7}$. When

students can complete problems like these, they are ready to learn the new method of adding and subtracting unlike fractional numbers.

It is important that students be involved in doing each step. Therefore, students should write down each step on their paper. The model is established by doing the work (16).

In the alternative method for adding and subtracting unlike fractional numbers, the example I will use is to add 3/4 and 2/3. The first step is to write the denominator of each fraction. Next, the student will "give to the poor." A 3 is less than a 4, so a 3 is added to the denominator. Now 4 is less than 3 + 3, so you give to the poor.

Step 1   Step 2

$$\frac{3}{4} = \frac{}{4}$$ $$\frac{3}{4} = \frac{}{4}$$

$$\frac{+2}{3} = \frac{}{3}$$ $$\frac{+2}{3} = \frac{}{3+3}$$

A 4 is added to the denominator. Now 3 + 3 is less than 4 + 4, so you give to the poor again. This time add another 3 to the denominator.

Step 3   Step 4

$$\frac{3}{4} = \frac{}{4+4}$$ $$\frac{3}{4} = \frac{}{4+4}$$

$$\frac{+2}{3} = \frac{}{3+3}$$ $$\frac{+2}{3} = \frac{}{3+3+3}$$

Now 3 + 3 + 3 is greater than 4 + 4, so a 4 is added to the poor. The denominator 4 + 4 + 4 is greater than 3 + 3 + 3, so another 3 is added to the poor. Now the sum of each denominator is the same.

Step 5 $\quad \dfrac{3}{4} = \dfrac{}{4+4+4}$

$\quad\quad\quad \dfrac{+2}{3} = \dfrac{}{3+3+3}$

Step 6 $\quad \dfrac{3}{4} = \dfrac{}{4+4+4}$

$\quad\quad\quad \dfrac{+2}{3} = \dfrac{}{3+3+3+3}$

Now stop and make display fractions. "Every bottom has to have a top and every top has to have a bottom. NO TOPLESS BOTTOMS ALLOWED" (16, p. 226).

Step 7 $\quad \dfrac{3}{4} = \dfrac{3+3+3}{4+4+4}$ $\quad\quad\quad$ Step 8 $\quad \dfrac{9}{12}$

$\quad\quad\quad \dfrac{+2}{3} = \dfrac{2+2+2+2}{3+3+3+3}$ $\quad\quad\quad\quad +\dfrac{8}{12}$

Next add the numerators and the denominators. The last step is to add the like fractions.

According to May, the method always works and the common denominator is always the least common denominator. After practicing the method for several days the students will find they can add and subtract unlike fractional numbers. They have a method

that works for them and they are successful.

The main idea to be developed when adding or subtracting fractions with unlike denominators is that we cannot add or subtract them symbolically without first changing them to like denominators (29).

Activities

5-1. Eraser Bowl (5).

Purpose: to provide practice in the addition of fractional numbers.

Number of Players: Pairs, small teams.

Materials: A bowling alley marked out on the floor, with a 1-meter circle or oval near one end; and nine chalkboard erasers, with fractional numerals written on six of them.
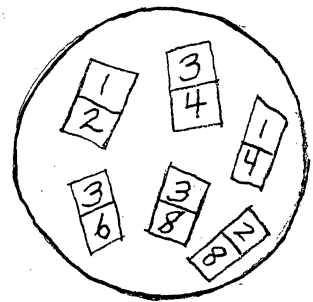
Procedure: Divide the group into two or more teams; place the six numbered erasers in the circle. The first player on one team is allowed three attempts at knocking the erasers outside of the circle, using an unmarked eraser. The total of the numbers named on the erasers that are knocked out of the circle is that player's score. The erasers are

then put back and play goes to the first player on the next team. At the end of play, each team totals its scores to see which team has the most points.

5-2. Common Fraction Addition (12).

Purpose: to name the sum of two fractional numbers smaller than 1.

Number of Players: 2

Materials: Make a game board 22" x 28". Use photo corners

RED		BLUE	
☐ 1/2 ☐	☐ 7/8 ☐	☐ 3/7 ☐	☐ 3/4 ☐
☐ 2/3 ☐	☐ 3/5 ☐	☐ 4/8 ☐	☐ 4/5 ☐
☐ 3/4 ☐	☐ 2/7 ☐	☐ 1/2 ☐	☐ 6/7 ☐
☐ 5/8 ☐	☐ 6/7 ☐	☐ 3/8 ☐	☐ 4/7 ☐
☐ 7/6 ☐	☐ 6/8 ☐	☐ 1/4 ☐	☐ 2/3 ☐
☐ 2/5 ☐	☐ 2/4 ☐	☐ 6/7 ☐	☐ 2/8 ☐
☐ 1/4 ☐	☐ 5/7 ☐	☐ 7/6 ☐	☐ 3/7 ☐
☐ 6/7 ☐	☐ 4/8 ☐	☐ 2/5 ☐	☐ 5/8 ☐
☐ 5/6 ☐	☐ 4/5 ☐	☐ 3/8 ☐	☐ 4/7 ☐
☐ 3/5 ☐	☐ 5/8 ☐	☐ 6/7 ☐	☐ 2/5 ☐
☐ 2/8 ☐	☐ 2/7 ☐	☐ 6/8 ☐	☐ 2/4 ☐
☐ 3/5 ☐	☐ 2/3 ☐	☐ 5/7 ☐	☐ 7/8 ☐

| 1/4 + 1/4 | 1/3 + 1/3 | 1/2 + 1/4 | 1/4 + 3/8 |

where the small squares are in the diagram. The photo corners will hold the sentence cards. Make two identical sets of sentence cards, one on red and the other on blue pieces of 2" x 4 1/2" tagboard, which indicate addition of common fractions. A die is needed. Make a key so players can check answers as they play.

Procedure:

1. Each player has a set of sentence cards.

2. The first player rolls the die. The number he rolls tells how many sentence cards he can put on the game board. He selects any of his cards and puts them up on his side of the board next to the appropriate sum.

3. An opponent may challenge any play. If a player has made an incorrect play, the challenger gets to play as many of his cards as the other player had left to play. If a challenger is wrong, the player gets to play as many extra cards as the number he had left to play.

4. Play continues with players alternating turns until one player is rid of his cards.

5. The winner is the player who is rid of his cards first.

5-3. Add to Make One (9).

Purpose: to name the sum of two fractional numbers smaller than 1.

Number of Players: 2 to 4.

Materials: A pair of dice - one red and one green - and paper and pencil for recording scores.

Procedure:

1. The player who has been chosen leader rolls the dice. The red die names a numerator while the green die names a denominator.

2. The first player names the fraction indicated by the dice. Then he names another fraction that can be added to it to give a sum of 1. If he is correct, he scores one point.

3. Play continues with each player taking his turns in order.

4. The winner is the player who has the most points after a given number of rounds have been played or a given period of time has passed.

Adding and Subtracting Mixed Numerals with Unlike Denominators

Children must be able to see why it is best to add and subtract the fractions first and then the whole numbers. Then the students would follow the procedures presented for working with fractions with unlike denominators. This would then be integrated with their previous knowledge of adding and subtracting mixed numbers.
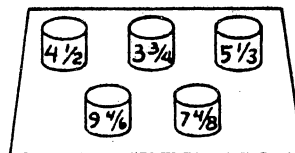
Activities

6-1. Canned Fractions (3).

Purpose: to provide practice in adding mixed numbers.

Number of Players: 1 to 10.

Materials: Five low cans, such as size 1/2 flat, nailed to a board, each with a paper taped to it showing a fraction numeral, as 4 1/2, 9 4/6, 3 3/4, 7 4/8, 5 1/3. Three small bean bags.

Procedure: Place board with cans on the floor and mark a line ten feet away. Players stand behind line and take three tosses each turn, aiming to get a bean bag into a can. When each has had three turns, he must total his own score. The player with the highest total wins. From time to time, vary the
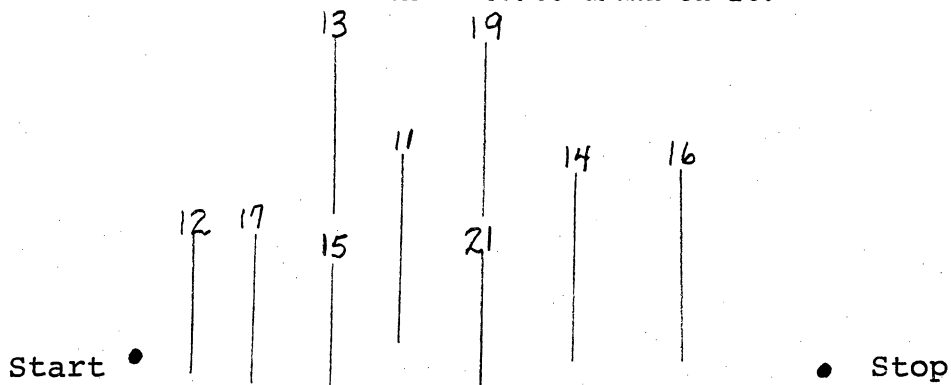
fractions on the cans to give practice as needed.

6-2. Croquet.

Purpose: to provide practice in subtracting mixed numbers.

Number of Players: 2 to 10.

Materials: Tagboard circles to represent balls with mixed
numbers less than 10 written on them. Tagboard
with a course drawn on it.

13   19

11

14   16

12 17   15   21

Start •   • Stop

Procedure: Each student selects a ball. The student must
then subtract the number on the ball from the
whole numbers on the wickets to go through them.
Give a penalty stroke for each error. The student
who reaches stop with the fewest penalties wins.

Multiplying Fractions

The algorithm for multiplication of fractions is one of the simplest: multiply numerators to find the numerator, multiply denominators to find the denominator. This method of teaching the algorithm does not provide background into why it works or when to use it. Sure, it can be taught in minutes, to be forgotten in seconds.

There are three types of multiplication problems. The first type involves multiplying two proper fractions. The second type involves multiplying a fraction and a whole number. (This type occurs often in story problems.) The third type of problem involves multiplying one or more mixed numbers.

In beginning to solve the first type of fraction multiplication problem involving multiplying two proper fractions, the teacher can draw a picture as shown below.

$$\frac{3}{4} \times \frac{5}{6} = \frac{15}{24}$$
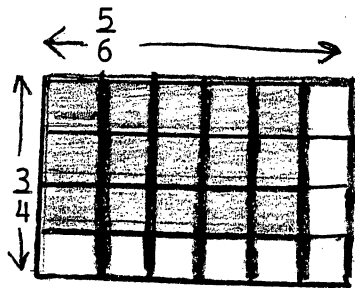
This model will need to be developed slowly. After doing this, find out whether any children can see a shorter way to find the product. Make a list of multiplication exercises and products (do not reduce the answers). See if the children notice the pattern (multiply the

numerators, multiply the denominators). You can refer to the picture to see why this works.

The second type of fraction multiplication problem, a fraction times a whole number, is important because it has many real-life applications. Fraction times whole number problems can be introduced when students have mastered multiplying proper fractions and converting an improper fraction to a mixed number.

Picture demonstrations of what takes place when multiplying a fraction by a whole number would look like this:

2/3 x 12 =

1111   1111   1111

The teacher explains that the bottom number tells how many groups to form and the top number tells how many groups are used. After drawing 12 lines in three groups, the teacher circles two groups:

(1111)   (1111)   1111

The lines within the circles are counted and we get 8: 2/3 x 12 = 8. The format to follow this picture demonstration would be:

$$\frac{2}{3} \times 12$$

Students make the whole number into a fraction.

$$\frac{2}{3} \times \frac{12}{1}$$

Students multiply the numerator and the denominator.

$$\frac{2}{3} \times \frac{12}{1} = \frac{24}{3}$$

Students convert the product into a whole number or mixed number.

$$\frac{2}{3} \times \frac{12}{1} = \frac{24}{3} = 8$$

Any whole number may be converted to a fraction by putting it over a denominator of 1.

The third type of fraction multiplication involving problems with a mixed number are an important component skill for advanced map reading. For example, if 1 inch equals 50 miles, how many miles will 3 1/2 inches equal?

$$50 \times 3 \ 1/2 = 175$$

The students convert a mixed number into an improper fraction before working the problem.

$$5 \ 1/2 \quad \times \quad 3 \ 2/4 \quad =$$

$$11/2 \quad \times \quad 14/4 \quad =$$

$$11/2 \quad \times \quad 14/4 \quad = \quad 154/8$$

$$154/8 \quad = \quad 8 \overline{\smash{)}\,154}^{\ 19 \ 2/8} = \quad 19 \ 1/4$$
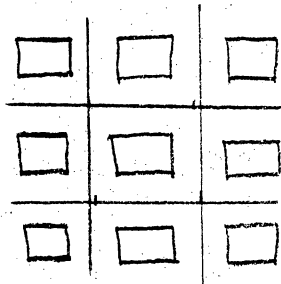
Activities

7-1.   Tic-Tac-Toe.

   Purpose: to practice multiplying with fractions or mixed
      numbers.

   Number of Players: 2, or 2 teams.

   Materials: Chalkboard, 3 x 5 cards on which problems have
      been written.

   Procedure: Draw a Tic-Tac-Toe grid on the board. Tape a
      problem card, face down, to each space.

The first student chooses where he/she would like to try placing
his/her X or O, then attempts to do so by correctly answering the
problem written on the card corresponding to the chosen space.
Students take turns in order. The winner is the first student (or
team) to place three X's or three O's in a horizontal, vertical, or
diagonal line. One point is awarded for each correct answer and 1
point for making Tic-Tac-Toe.

7-2.   Flash Card Baseball.

   Purpose: to practice multiplication of fractions.

Number of Players: 2 teams each with the same number of
                    players.

Materials: Index cards on which problems have been written,
            chalkboard.

Procedure: The index cards are sorted into four piles. A
            baseball diamond is drawn on the chalkboard.
            Students then decide whether they want to try for
            a single, double, triple, or homer. The "single"
            pile consists of easy problems; the "double" pile,
            average problems; the "triple" pile, difficult
            problems; and the "homer" pile consists of
            problems which represent an extension of what
            children have studied up to the present time. An
            incorrect answer is an out. There are no balls or
            strikes. One point is scored for each run.

Reciprocal Property

Two numbers whose product is 1 are each the reciprocal of the other. Two thirds is the reciprocal of three halves and three halves is the reciprocal of two thirds. The inverses are numbers that make the product 1. It is important for students to understand this reciprocal property because it is used to explain division of fractional numbers. In providing exercises to practice finding the reciprocal, all kinds of numbers should be included, such as whole numbers, fractional numbers, and decimals.

$$4/3 \times ? = 1 \quad ? \times .6 = 1 \quad 4\ 2/3 \times ? = 1 \quad ? \times 8 = 1$$

When a number is written in mixed form, such as 4 2/3, students must think of the number in fractional form in order to name the reciprocal.

Dividing Fractions

Models showing division situations get rather complicated. One model will be shown here. However, if you are presenting the division algorithm to children who are not ready for a symbolic treatment, then you should find other models for other situations.

Division of fractions is a highly complex operation. Much time and extensive visual teaching aids may be necessary with most children before they understand the meaning behind it.

"Suppose you have 3/4 of a rectangular pizza and you want to share it equally among 5 people. How much of the pizza would each person receive?"

Divide among 5 people.
Each person's share.

3/4 of a
pizza
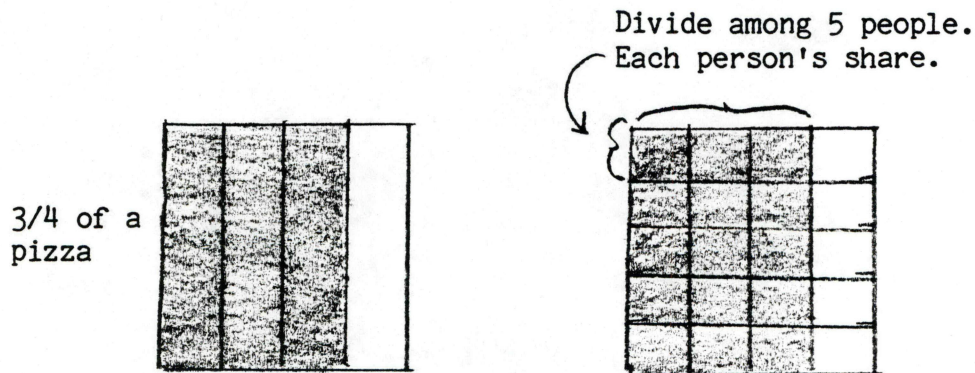
Each person would get 3 out of 20 pieces, or 3/4 ÷ 5 = 3/20. It is hoped that the students will recognize this as the same picture we used to show 1/5 x 3/4 or 3/4 x 1/5. This begins to develop the rule that 3/4 ÷ 5 = 3/4 x 1/5.

The division of fractions using the inversion method is based

on the property of multiplication, the property of one, and the reciprocal property. The first step is to remove the denominator.

$$\frac{2}{3} \div \frac{5}{7} = \frac{\frac{2}{3} \times \left(\frac{7}{5}\right)}{\frac{5}{7} \times \left(\frac{7}{5}\right)} = \frac{\frac{14}{15}}{1}, \text{ or } \frac{14}{15}$$

The next step is to rename the denominator as 1. Any whole number can be written with a denominator of 1 without changing the number. The reciprocal of five sevenths is seven fifths, so multiply by seven fifths to rename the denominator 1. Then the numerator must be multiplied by seven fifths because of the property of one. Now multiply and the answer is 14/15.

Now when we tell students that to divide two fractions, you must invert the divisor and multiply, the "rule" has meaning because invert means using the reciprocal property.

Activities

8-1. Flash Card Basketball.

Purpose: to provide practice in dividing with fractions.

Number of Players: Classroom of students.

Materials: Chalkboard, index cards with division of fraction problems.

Procedure: Divide the class into teams. The teams take turns answering problems from index cards. Score two points for each correct answer. (This is similar to scoring a basket.) If a student gives an

incorrect answer, the opposite team gets a foul shot (worth one point if it is made) before their turn. The team with the most points wins (determine amount of time to be used ahead of time).
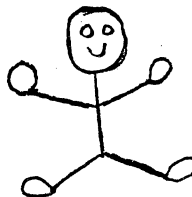
8-2. Stop the Magician.

Purpose: to provide practice in division of fractions.

Number of Players: 2 or 3 teams.

Materials: Chalkboard, 3 x 5 cards on which appropriate problems have been written.

Procedure: Start with a stick person.

Alternating teams and rotating among players, the magician (teacher or student) shows the problem to be solved. For each error, erase a part of the body (hand, foot, etc.) of the stick person. The object of the game is to stop the magician from making the stick person invisible. (Each team has its own picture.) After all the problems are answered, the team with the most complete stick person wins.

Story Problems - Multiplication and Division

Multiplication and division story problems can be introduced when students can solve the respective problem types. Multiplication story problems with fractions usually involve figuring out what a fractional part of a specified group equals. Here is a typical problem:

There are 20 children in our class; 3/4 of the children are girls. How many girls are in the class? Students would be taught that _of_ in this problem can be translated to times. The problem of 3/4 of 12 would be converted to 3/4 x 12 and worked.

The most common type of division story problem involves dividing a fraction by a whole number. Here is an example of this type of problem:

John has 3/4 pound of candy. He wants to divide the candy equally among 3 friends. How much candy should he give to each friend?

Structured teacher demonstrations are particularly important when introducing fractional strategies. This explicit teaching is very important if lower-performing students are to succeed. The amount of practice provided in commercial programs (state adopted textbooks) must be significantly supplemented if the students are to develop mastery. Teachers must not go on to a new skill until students have developed accuracy in its preskills.

Commercial games that reinforce understanding of fractional

numbers and ways they are represented:

One

Recognizing Fractional Parts

  Creative Publications

  P. O. Box 10328

  Palo Alto, CA 94303

IMOUT - a Game of Fractions

  IMOUT Arithmetic Drill Games

  706 Williamson Building

  Cleveland, Ohio 44114

Fractions Are As Easy As Pie

  Marie's Educational Materials

  P. O. Box 694

  Sunnyvale, CA 94086

Frac Pac

  A. R. Davis and Co.

  P. O. Box 24424

  San Jose, CA 95154

Information about Cuisenaire rods may be obatined from:

Cuisenaire Company of America, Inc.

12 Church Street

New Rochelle, New York 10805

REFERENCES

1. Behr, Merlyn J.; Wachsmuth, Ipke; Post, Thomas R.; and Lesh, Richard. "Order and Equivalence of Rational Numbers: A Clinical Teaching Experiment." Journal for Research in Mathematics Education XV, n. 5, 1984, pp. 323-339.

2. Behr, Merlyn J.; Wachsmuth, Ipke; and Post, Thomas R. "Construct A Sum: A Measure of Children's Understanding of Fraction Size." Journal for Research in Mathematics Education XVI, n. 1, 1985, pp. 120-131.

3. Clark, John R. and Eads, Laura K. Guiding Arithmetic Learning. New York: World Book Company, 1954, pp. 135-179.

4. Corle, Clyde G. Teaching Mathematics in the Elementary School. New York: The Ronald Press Company, 1964, p. 166.

5. Dumas, Enoch. Math Activities for Child Involvement. Boston: Allyn and Bacon, Inc., 1971, pp. 213-230.

6. Dutton, Wilbur H.; Petrie, Colin C.; and Adams, L.J. Arithmetic for Teachers. New Jersey: Prentice-Hall, Inc., 1970, pp. 164-194.

7. Gagne, R. M. The Conditions of Learning. New York: Holt, Rinehart, & Winston, 1965.

8. Hector, Judith H. and Frandsen, Henry. "Calculator Algorithms For Fractions With Community College Students." Journal for Research in Mathematics Education, November 1981, pp. 349-355.

9. Hickerson, J. Allen. Guiding Children's Arithmetic Experiences: The Experience-Language Approach to Numbers. New Jersey: Prentice-Hall, Inc., 1952.

10. Hunting, Robert P. "Alan: A Case Study of Knowledge of Units and Performance with Fractions." Journal for Research in Mathematics Education XIV, n. 2, 1983, pp. 182-197.

11. Kennedy, Leonard M. Experiences for Teaching Children Mathematics. Belmont, California: Wadsworth Publishing Company, Inc., 1973, pp. 77-119.

12. Kennedy, Leonard M. and Michon, Ruth L. *Games for Individualizing Mathematics Learning*. Columbus, Ohio: Charles E. Merrill Publishing Company, 1973, pp. 79-95.

13. Kramer, Klaas. *Teaching Elementary School Mathematics*. Boston: Allyn and Bacon, Inc., 1975, pp. 267-296.

14. Krich, Percy. "Meaningful vs. Mechanical Method, Teaching Division of Fractions by Fractions." *School Science and Mathematics*, October 1964, pp. 697-708.

15. Larson, Carol Novillis. "Locating Proper Fractions on Number Lines: Effect of Length and Equivalence." *School Science and Mathematics*, 1980, pp. 423-428.

16. May, Lola June. *Teaching Mathematics in the Elementary School*. New York: The Free Press, 1974, pp. 182-255.

17. Miller, Jack W. "An Experimental Comparison of Two Approaches to Teaching Multiplication of Fractions." *The Journal of Educational Research*, May/June 1964, pp. 468-470.

18. Novillis, Carol F. "An Analysis of the Fraction Concept into a Hierarchy of Selected Subconcepts and the Testing of the Hierarchical Dependencies." *Journal for Research in Mathematics Education*, May 1976, pp. 121-144.

19. Olson, Waldemar. *Methods of Teaching Elementary School Mathematics*. Minneapolis, Minnesota: Burgess Publishing Company, 1968.

20. Peck, Donald M. and Jencks, Stanley M. "Are Common Fractions Out-of-date?" *The Elementary School Journal*, September 1979, pp. 19-23.

21. Peck, Donald M. and Jencks, Stanley M. "Conceptual Issues in the Teaching and Learning of Fractions." *Journal for Research in Mathematics Education*, 1981, pp. 339-348.

22. Post, Thomas R.; Wachsmuth, Ipke; Lesh, Richard; and Behr, Merlyn J. "Order and Equivalence of Rational Numbers: A Cognitive Analysis." *Journal for Research in Mathematics Education*, 1985, pp. 18-35.

23. Pothier, Yvonne and Sawada, Daiyo. "Partitioning: The Emergence of Rational Number Ideas in Young Children." *Journal for Research in Mathematics Education*, 1983, pp. 307-317.

24. Reys, Robert E.; Suydam, Marilyn N.; and Lindquist, Mary M. Helping Children Learn Mathematics. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1984, pp. 161-178.

25. Riedesel, C. Alan. Teaching Elementary School Mathematics. New Jersey: Prentice-Hall, Inc., 1980.

26. Schminke, C.W.; Maertens, Norbert; and Arnold, William R. Teaching the Child Mathematics. Hinsdale, Illinois: The Dryden Press, Inc., 1973.

27. Schminke, C.W. and Dumas, Enoch. Math Activities for Child Involvement. Boston: Allyn and Bacon, Inc., 1981, pp. 165-202.

28. Scott, Lloyd. "Children's Concept of Scale and the Subtraction of Fractions." The Arithmetic Teacher, March 1962, pp. 115-118.

29. Silbert, Jerry; Carnine, Douglas; and Stein, Marcy. Direct Instruction Mathematics. Columbus, Ohio: Charles E. Merrill Publishing Co., 1981.

30. Smith, Seaton E. and Backman, Carl A. Teacher-made Aids for Elementary School Mathematics. Reston, Virginia: The National Council of Teachers of Mathematics, Inc., 1974, pp. 93-124.

31. Souviney, Randall J.; Keyser, Tamara; and Sarver, Alan. Mathmatters. Santa Monica, California: Goodyear Publishing Company, Inc., 1978, pp. 184-225.

32. Swenson, Esther J. Teaching Arithmetic to Children. New York: The MacMillan Company, 1964.

33. The School Mathematics Project. Book A. Cambridge: University Press, 1971.

34. Westcott, Alvin M. and Smith, James A. Creative Teaching of Mathematics in the Elementary School. Boston: Allyn and Bacon, Inc., 1967.

35. Wheeler, Ruric E. Modern Mathematics: An Elementary Approach. Monterey, California: Brooks/Cole Publishing Company, 1973.