A Nonlinear Complementary Filter for Underwater Navigation using Inertial Measurements

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***Abstract*—This paper describes a nonlinear complementary filter capable of estimating the course motion variables namely the position, velocity, heading and accelerometers bias of an agile, over-actuated AUV during underwater operations, using the inertial sensors (IMU), the DVL, the depth sensor and the compass. The proposed work is within the framework of the V-Fides project, co-funded by Tuscany Region (Italy) and developed by a team lead by WASS S.p.A. (Whitehead Sistemi Subacquei, Livorno). The aim of the project was to develop and evaluate an high-depth, over-actuated, long endurance Autonomous Underwater Vehicle (AUV). The paper proposes the mathematical development of the observer, together with some experimental results, able to demonstrate the capabilities of the estimation scheme, compared with the estimations obtained via a standard Kalman Filter.**

I. Introduction

Underwater robotics and specifically Autonomous Underwater Vehicles (AUVs) have received nowadays a growing interest of the scientific community given their high flexibility and adaptability to a wide range of applications. In the most recent years, the attention of the research focused on increasing the endurance of the AUVs, thus making the problem of robust and reliable autonomous navigation [1] a key aspect toward the extensive application of underwater robotics. A great heritage of aided-inertial navigation techniques are available for improving navigation accuracy, see for instance [2], [3] and references therein. The approaches classically adopted employ the fusion, optimal in some sense, of inertial measurements [4] with some aiding devices, mainly DVLs, depth sensors, GPS and relative acoustic positioning devices. The work proposed in this paper is in the framework of the V-Fides project [5], [6], [7], which main focus, following this general impulse, was to innovate with respect to vehicle's structure, components and payloads, by pushing forward the vehicles design in terms of pressure resistant shells, high-energy Li-Po batteries, open architecture control systems and integrated payloads. The project was co-funded by Tuscany Region (Italy) and developed by a team lead by WASS S.p.A. (Whitehead Sistemi Subacquei, Livorno) with the participation of SMEs and two research centers of the University of Pisa. The vehicle (Figure 1) is a general purpose, 3000m depth rated underwater vehicle with highly maneuverability capabilities, able to operate as AUV and ROV. The vehicle is equipped with a sensors payload for autonomous navigation, composed by an Inertial Measurement Unit (IMU), a Doppler Velocity Logger (DVL), a depth sensor, a magnetic compass and an acoustic modem for underwater communication and localization. This paper concentrates on the description of a nonlinear observer, capable of estimating the course motion variables – namely the position, velocity, heading and accelerometers bias – of the AUV, using the inertial sensors (IMU), the DVL, the depth sensor and the compass. The estimation scheme is formulated in a complementary filtering framework [8], [9]. Previous applications of estimation schemes of this class to the case of underwater vehicles navigation can be found for example in [10], [11], [12]; usually these schemes are used in conjunction with a dynamic model of the vehicle.

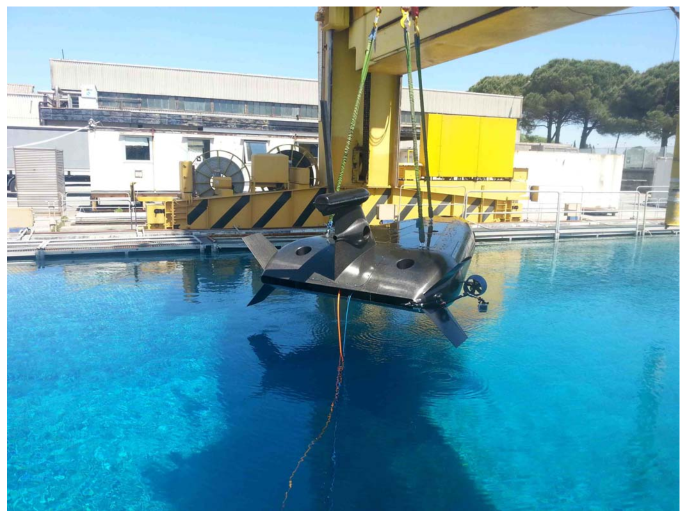

Fig. 1. The V-Fides AUV during the deployment in the WASS pool for the preliminary communication and control allocation tests.

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The peculiarity of this filtering approach lies on the ease of design and tuning phase, obtained via frequency techniques which allow designing the expected filtering capabilities of the estimation scheme. In fact, the idea behind the complementary filtering techniques is that [8] the estimation of a given variable is obtained as the frequency-weighted sum of two noisy signals, having complementary characteristics in the frequency domain and measure, respectively, the high and low frequency characteristics of the variable. The main assumption made in this work is that an inner control law does exist able to regulate the attitude to zero (with faster dynamics than the one of the vehicle course motion), or, at very least, the vehicle is metacentric stable with respect to the roll and pitch degrees of freedom. The small deviations of the attitude of the vehicle from the null value implies the possibility to easily remove the gravitational bias from the sensed accelerations and extract the dynamic accelerations of the vehicle, due to the applied forces and external disturbances. The nonlinear filtering algorithm employs a second order kinematic model of the vehicle motion in the surge, sway and heave degrees of freedom and a first order kinematic model for the heading estimation loop. Such structure allows embedding the linear acceleration measurements into the filtering task, together with the measured linear and angular yaw velocity in the body frame, the depth and the compass. The work describes a basic estimation architectures, giving the mathematical details. Moreover, the filter performances are demonstrated by processing offline the data collected during experimental campaigns and compared with the estimations obtained with a standard Extended Kalman Filter.

II. Navigation Estimation via Complementary Filtering

The basic characteristic of the complementary filtering schemes is to obtain the estimation of a given variable $x(t)$ as the frequency-weighted sum of two noisy signals, $\widetilde{y}_h(t)$ and $\widetilde{y}_l(t)$, which can be considered *complementary* in the frequency domain and contain, respectively, the high and low frequency most informative features of the variable x. For instance, an ideal model for the $\widetilde{y}_i$ signals, useful to describe the basic idea, can be the following:

$$\widetilde{y}_l(t) = x(t) + \nu_x(t) \quad (1)$$
$$\widetilde{y}_h(t) = \delta(t) + x(t) \quad (2)$$

where $\nu(t)$ is a zero-mean white noise with proper covariance while $\delta(t)$ is a varying signal (for the purposes of this work, at most linearly diverging in time) which spectral components are predominant at low frequencies. With these hypotheses, the filtering of x can thus be obtained as:

$$\hat{x}(s) = G(s)\,\widetilde{y}_h(s) + (1 - G(s))\,\widetilde{y}_l(s) \quad (3)$$

where $G(s)$ is a n-th order low-pass filter with unit gain at low frequencies and a proper cut-off frequency, specifically tuned depending on the characteristics of the measuring signals.

We consider the kinematic model of the AUV in four degrees of freedom, represented by movements in the surge, sway, heave and yaw space. The equations describing the motion of the vehicle are written with respect to a reference frame in NED configuration, assumed fixed in a given position on the Earth surface and always aligned with the North, East and Down directions. This is a realistic assumption in the framework of the proposed work, since the motion of the vehicle was assumed to be enclosed inside a *small enough* area. The reference frame fixed on the vehicle and moving with it is considered to be placed on the principal axes. The vehicle is assumed to rotate principally along the yaw axis only. In other words, we assume that an inner control law does exist able to regulate the attitude to zero (with faster dynamics than the one of the vehicle course motion), or, at very least, the vehicle is metacentric stable with respect to the roll and pitch degrees of freedom. This means that we are able to remove the most of the gravitational contribution from the acceleration measurements, in order to extract the dynamic acceleration contributions due to the thrusters and external forces. However, we take into account the non-ideal cancellation of the gravity measurement, due to limited attitude variations around zero, by including a bias term in the motion model. With the above hypotheses, the continuous-time local motion model can be written as:

$$\dot{T}_n = v_n \quad (4)$$
$$\dot{v}_n = R(\psi)(a_b - b_a) \quad (5)$$
$$\dot{\psi} = \omega_z \quad (6)$$
$$\dot{b}_a = 0 \quad (7)$$

where T_n and v_n denote respectively the position and velocity of the vehicle in the navigation (NED) frame. Note that we did not include a bias term compensation on the gyro measurement ω_z, since we take advantage of the high accuracy of the IMU, which allows to neglect such term. Therefore, we actually work with a reduced model in which the gyro bias is removed. The rotation matrix $R(\psi)$ is used to transform the variables referred to the vehicle frame into the NED frame. The velocity dynamics are obtained via integration of the body accelerations a_b measured by the accelerometers, depurated by the bias term b_a (due to non ideal cancellation of the gravity) and then converted in the local navigation frame, by using the yaw angle.

In this work we rely on measurements coming from a DVL, a depth sensor and a magnetic compass. Further developments of the proposed approach toward the integration of measurements from the USBL and GPS are currently subject of research by the same authors. The output model, used by the estimation filter, is formulated as:

$$y_v = R_{vb}^T R(\psi)^T v_n \quad (8)$$
$$y_d = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} T_n \quad (9)$$
$$y_\psi = \psi \quad (10)$$

where the term R_{vb} represents the (known) constant rotational calibration map from the DVL reference frame to the body fixed axes.

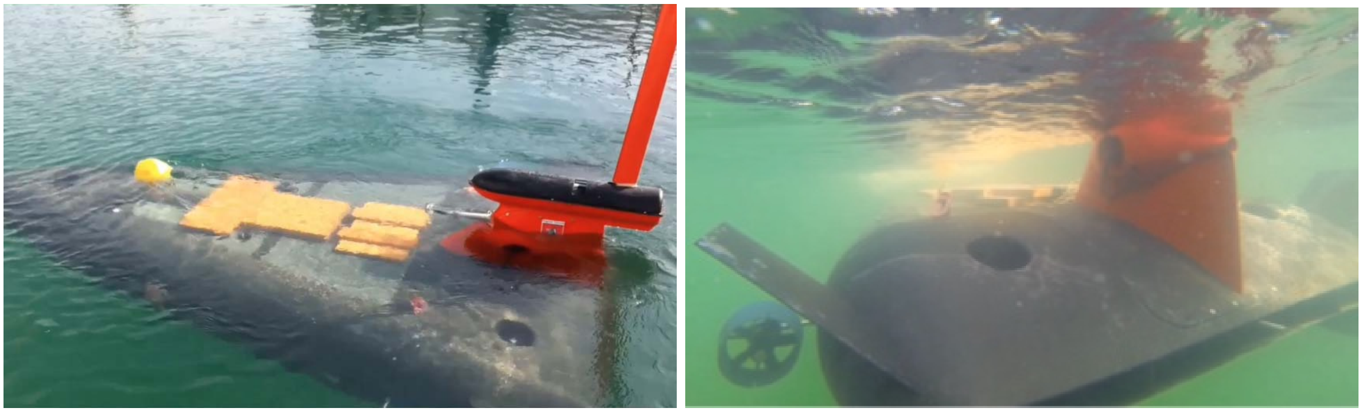

Fig. 2. The V-Fides AUV during some experimental tests in water c/o WASS facilities. Right: GNC scheme validation tests in pool, October 2014; Left: Underwater snapshot taken during experimental tests at sea.

A. Nonlinear Observer Design

The equations of the nonlinear state observer proposed in this work can be written as:

$$\dot{\hat{T}}_n = \hat{v}_n + K_{11}\widetilde{y}_d \tag{11}$$

$$\dot{\hat{v}}_n = R\left(\hat{\psi}\right)\left(a_b - \hat{b}_a\right) + K_{21}\widetilde{y}_d + K_{22}R\left(\hat{\psi}\right)R_{vb}\widetilde{y}_v \tag{12}$$

$$\dot{\hat{\psi}} = \omega_z + k_{33}\widetilde{y}_\psi \tag{13}$$

$$\dot{\hat{b}}_a = -K_{42}R_{vb}\widetilde{y}_v \tag{14}$$

$$\tag{15}$$

Note that the observer model is nonlinear with respect to the yaw state variable. The terms with the *hat* represent the estimated states and the ones with the *tilde* represent the errors between measured and estimated outputs, that is:

$$\widetilde{y}_d = y_d - \hat{T}_{n,3} \tag{16}$$

$$\widetilde{y}_v = y_v - R_{vb}^T R\left(\hat{\psi}\right)^T \hat{v}_n \tag{17}$$

$$\widetilde{y}_\psi = y_\psi - \hat{\psi} \tag{18}$$

The observer feedback gains K_{22} and K_{42} are positive definite matrices, which are taken diagonal to reduce the complexity of the tuning phase. The gain $k_{33} > 0$ is scalar, being applied on the heading channel. The gain vectors K_{11} and K_{21} take the form $K_{i1} = \begin{pmatrix} 0 & 0 & k_{i1} \end{pmatrix}^T$, with $k_{i1} > 0$, given that they feed back the depth error into the third component of the position and velocity estimations, respectively. The remaining 2 components – that is the North-East positions and velocities – are not observable given a direct measurement of the depth only, thus we must keep the consistency of the estimation by forcing to zero the correction on these channels. Note that the velocity estimation error (natively expressed in the DVL reference frame) is first transformed in the navigation frame and then fed to the velocity state estimation; in a similar manner, the same error is transformed in the body axes, before being passed to the bias state variables, which are defined in the body reference frame. Figure 3 shows a logical schematic of the designed observer; the heading estimation loop was omitted.

With the above hypothesis of diagonal observer gains, the complementary filtering characteristic of the observer can be shown. By expanding the observer equations for the position, velocity and heading we can write:

$$\dot{\hat{T}}_n = -K_{11}\hat{T}_{n,3} + \hat{v}_n + K_{11}y_d \tag{19}$$

$$\begin{aligned}\dot{\hat{v}}_n = &-K_{21}\hat{T}_{n,3} - K_{22}v_n + R\left(\hat{\psi}\right)\left(a_b - \hat{b}_a\right) \\ &+ K_{21}y_d + K_{22}R\left(\hat{\psi}\right)R_{vb}y_v\end{aligned} \tag{20}$$

$$\dot{\hat{\psi}} = -k_{33}\hat{\psi} + \omega_z + k_{33}y_\psi \tag{21}$$

It can be easily shown that the estimated depth, velocity and heading are obtained by properly weighting in frequency the inertial and auxiliary measurements, typical of the complementary filtering schemes. For instance, it is straightforward to show that the estimation of the heading variable employs a first order complementary filtering scheme; that is, by solving Equation (13) for $\hat{\psi}$, we have:

$$\hat{\psi} = \frac{1}{s+k_{33}}\omega_z + \frac{k_{33}}{s+k_{33}}y_\psi \tag{22}$$

$$= \frac{s}{s+k_{33}}\frac{\omega_z}{s} + \frac{k_{33}}{s+k_{33}}y_\psi \tag{23}$$

Therefore, $G(s) \doteq \frac{k_{33}}{s+k_{33}}$ and $1 - G(s) \doteq \frac{s}{s+k_{33}}$. The two signals y_ψ and $\frac{\omega_z}{s}$ represent complementary (spectral) features of the variable $\hat{\psi}$; for instance the compass measurement can be represented with acceptable approximation by Equation (1), that is a noisy, non diverging, direct measurement of the heading variable. On the other hand, the integration of the gyroscope measurement can be properly represented by Equation (2), where the low frequency signal $\delta(t)$ is the slowly varying disturbance induced by the integration of the gyroscope measurement, mainly due to the noise. The foregoing relationship clearly shows the effect of tuning the value of the gain k_{33}, which imposes the cut-off frequency where the noise filtering on the heading measurement becomes significant. At the same time, the gain determines the frequency from which the static gain of the transfer function $1 - G(s)$ (which is a high-pass filter) tends to one.

Showing the complementary nature of the remaining estimation loops is straightforward by introducing some simplifying assumptions. First, we assume to perform the heading estimation *before* evaluating the value of the body-to-navigation rotation matrix $R_{bn}\left(\hat{\psi}\right)$. This is motivated by the fact that we want to compute the estimation of the remaining variables by employing the *best* estimation of the heading. In this way, the observer becomes linear (although time-varying) in the state variables, the acceleration input and the measurements. Second, since the acceleration bias estimation is obtained via integration of the velocity error, we assume that its dynamic is sufficiently slower than the one of the velocity itself. With the above assumptions, we can evaluate the frequency characteristic of the i-th component of the velocity estimation on the channel $\{a_b, y_v\} \rightarrow \hat{v}_n$, by eliminating the depth error feedback. Thus:

$$\hat{v}_{n|i} = \frac{s}{s+k_{22,i}}\left(\frac{a_{n|i}}{s} - \frac{b_{n|i}}{s}\right) + \frac{k_{22,i}}{s+k_{22,i}} y_{v,n|i} \qquad (24)$$

where $a_{n|i}, b_{n|i}$ and $y_{v,n|i}$ are the i-th elements of, respectively: $R\left(\hat{\psi}\right)a_b$, $R\left(\hat{\psi}\right)\hat{b}_a$ and $R\left(\hat{\psi}\right)R_{vb}y_v$. Note that the sole feedback of the velocity measurement on the velocity estimation induces a first-order filtering characteristic between the velocity and acceleration measurements. Thus, the same discussions made for the case of heading filtering does apply to the velocity estimation also. In particular, the gains $k_{22,i}$ determine the cut-off frequency and allow to filter out (or, at least, attenuate) the residuals between the acceleration input and the estimated bias. In a similar manner, by considering the effect of the sole depth error feedback on the down velocity and depth estimation (i.e. the upper loop in Figure 3), we have:

$$\hat{v}_{n|3} = \frac{1}{s}\left(\left(a_{n|3} - b_{n|3}\right) + k_{21}\widetilde{y}_d\right) \qquad (25)$$

$$\hat{T}_{n|3} = \frac{1}{s}\left(\hat{v}_{n|3} + k_{11}\widetilde{y}_d\right) \qquad (26)$$

which, solving for $\hat{T}_{n|3}$ and $\hat{v}_{n|3}$ leads to:

$$\hat{v}_{n|3} = \frac{s+k_{11}}{s^2+k_{11}s+k_{21}}\left(a_{n|3} - b_{n|3}\right) + \frac{k_{21}s}{s^2+k_{11}s+k_{21}} y_d \qquad (27)$$

$$= \frac{s^2+k_{11}s}{s^2+k_{11}s+k_{21}}\left(\frac{a_{n|3}}{s} - \frac{b_{n|3}}{s}\right) + \frac{k_{21}}{s^2+k_{11}s+k_{21}}\left(sy_d\right) \qquad (28)$$

$$\hat{T}_{n|3} = \frac{1}{s^2+k_{11}s+k_{21}}\left(a_{n|3} - b_{n|3}\right) + \frac{k_{11}s+k_{21}}{s^2+k_{11}s+k_{21}} y_d \qquad (29)$$

$$= \frac{s^2}{s^2+k_{11}s+k_{21}}\left(\frac{a_{n|3}}{s^2} - \frac{b_{n|3}}{s^2}\right) + \frac{k_{11}s+k_{21}}{s^2+k_{11}s+k_{21}} y_d \qquad (30)$$

Note that the transfer function (in equation (27)) mapping the debiased acceleration into the velocity estimation has a static gain equal to $\frac{k_{11}}{k_{22}}$, which is less than one when $k_{22} > k_{11}$. This characteristic, again, helps attenuating the destabilizing effects that a residual between the true and estimated bias has on the velocity (and position) estimation. However, we remark that when $k_{22} > \frac{k_{11}^2}{4}$ the poles of the transfer functions in the previous equations become complex, and the more k_{22} grows compared with k_{11}, the more the damping factor decreases. This should be kept in consideration when tuning the filter gains.

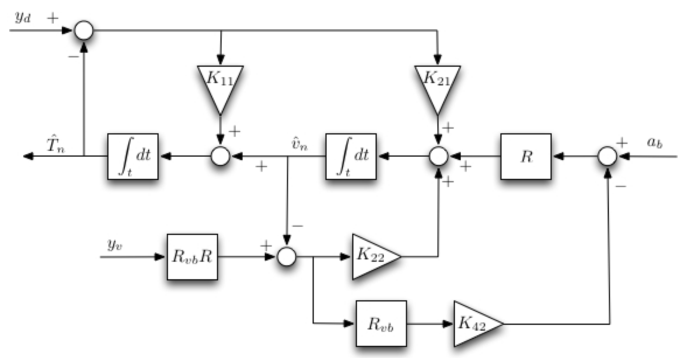

Fig. 3. Logical realization of the nonlinear observer. The accelerometers measurements (modulo the gravity and the bias term) are rotated using the estimated heading, then accelerations and velocities are updated by using the DVL and depth measurements, when available. The heading estimation loop is not shown. R represents the rotation matrix, obtained using the estimated yaw angle.

III. Comparative Study with EKF

The proposed observer was tested with real data sets and compared with the results obtained with a standard Extended Kalman Filter, designed over the same kinematic model and using the same measurements. For the purposes of numerical implementation, both the algorithms, the non-linear observer and the EKF, were time discretized by using the Forward Euler with the sample time coincident with the IMU data-frame. The data were collected during an extensive experimental campaign, conducted in the framework of the project V-Fides. The goals of the experimental tests was to validate the Guidance, Navigation and Control systems designed. During the tests, a separated attitude estimation algorithm was running in real time, which duty was to feed continuously the current attitude value to the inner regulator. This latter system was designed to regulate the attitude to zero and keep the *course maneuverability*. Finally the data were processed offline to test the capabilities of the observer presented in this work. Some representative results are proposed in the following Figures. In particular Figure 5 shows the comparison between the estimated bias in the case of EKF and the proposed observer. Note that the feedback gains were kept small to track the lower frequency behavior of the bias terms. In fact, the main difference with the Kalman Filter is that the observer gains are not adjusted depending on the covariance of the estimated variables, thus the tuning has to meet a trade off between the speed of convergence and the smoothness of the estimation. However, the proposed scheme was able to countermeasure the complexity of the Kalman Filter estimation scheme in a couple of ways: first, it does not need the local linearization of the kinematic model around the current heading prediction; second, the observer tuning is simpler and more intuitive, since the parameters take a physical meaning, that is the poles of the filtering transfer functions. Figures 6 and 7 show the estimated down component of the velocity and the depth estimations respectively using the proposed observer and the EKF, compared with the measurements. Note the capability of both the algorithm to filter out the noise on the measured variables, by means of the low pass characteristic introduced by the feedback. In fact, it can be shown that the EKF itself has complementary filtering characteristic in frequency, improved

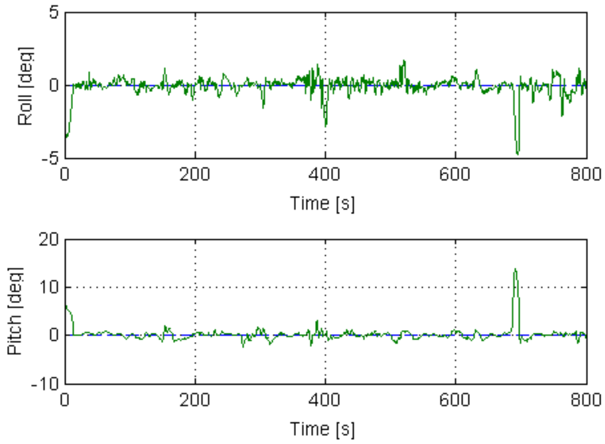

Fig. 4. Representative result of the attitude regulation performed by the inner control during some experimental tests in pool. The instants of larger attitude values are coincident with controller disabling/enabling tests.

by the fact that the feedback gains are adapted depending on the covariance of the estimation error.

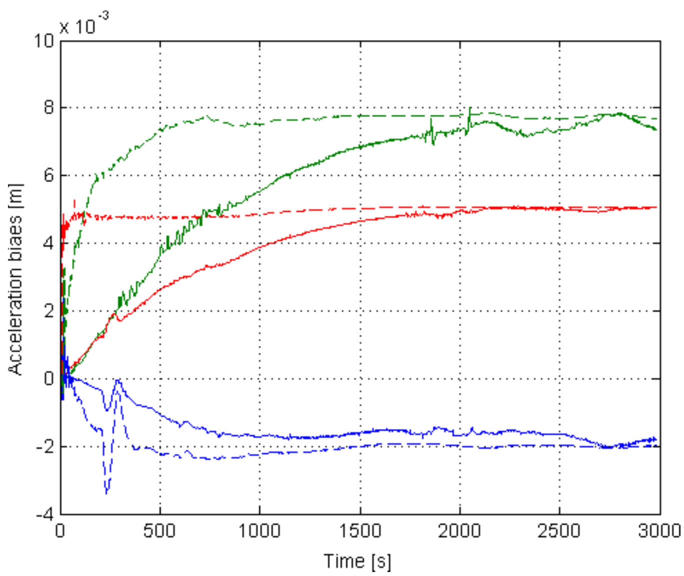

Fig. 5. Comparison of the bias estimation obtained via the proposed complementary filtering approach and via a standard Extended Kalman Filter (x axis (blue), y axis (green) z axis (red). Dashed line, EKF; solid line, Complementary Filter.

IV. Conclusion

The paper presented a non linear observer for autonomous navigation of an underwater vehicle, designed using the general idea of complementary filtering schemes. So far, the estimation scheme was designed on the kinematic description of the vehicle motion, by neglecting the correction of the relative acoustic positioning system. Further developments will foresee the inclusion of a system dynamical model and of the USBL measurements. Some experimental results were presented to demonstrate the performances of the described algorithm.

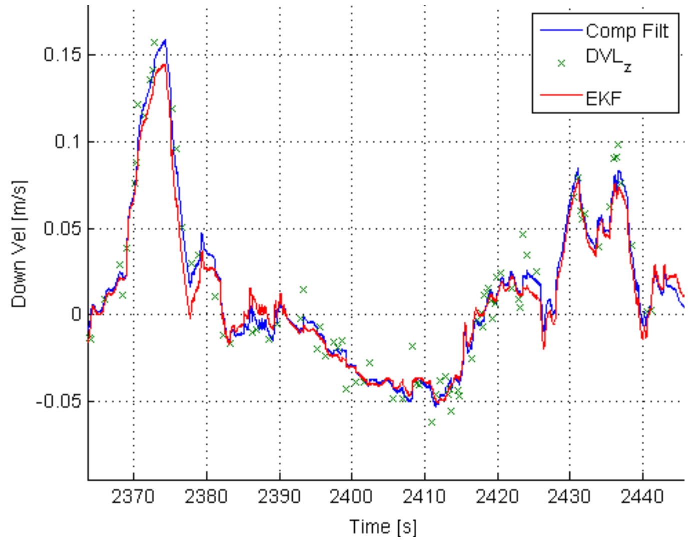

Fig. 6. Comparison of the down velocity estimation obtained via the proposed complementary filtering approach and via a standard Extended Kalman Filter. The crosses represent the DVL measurements.

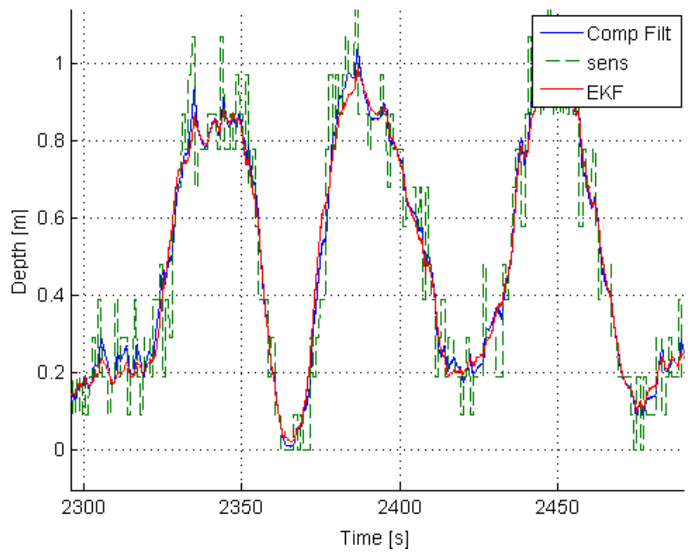

Fig. 7. Comparison of the estimated depth, obtained via the proposed complementary filtering approach and via a standard Extended Kalman Filter. The dashed line is the measurement from the depth sensor.

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