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A Distributed, Passivity-Based Control of Autonomous Mobile Sensors in an Underwater Acoustic Network

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Abstract: This paper presents a cooperative and distributed control law for multiple Autonomous Underwater Vehicles (AUVs) executing a mission while meeting mutual communication constraints. Virtual couplings define interaction control forces between neighbouring vehicles. Moreover, the couplings are designed to enforce a desired vehicle-vehicle and vehicle-target spacing. The whole network is modelled in the passive, energy-based, port-Hamiltonian framework. Such framework allows to prove closed-loop stability using the whole system kinetic and virtual potential energy by constructing a suitable Lyapunov function. Furthermore, the robustness to communication delays is also demonstrated. Simulation results are given to illustrate the effectiveness of the proposed approach.

Keywords: Autonomous vehicles, Co-operation, Co-ordination, Distributed control, Passivity, Port-Hamiltonian.

1. INTRODUCTION

Recent advances in robotics have made AUVs more reliable and affordable allowing the execution of tasks that were previously dangerous, expensive and time consuming when performed by humans. The simultaneous use of a team of robots can provide significant benefits in several applications ranging from surveillance, patrolling to seabed search and mapping (Curtin et al. (1993)). All these application scenarios involve communication among multiple agents. In the underwater domain, due to the well known limitations of the acoustic channel (Caiti et al. (2013b); Stojanovic (2007)), it is of paramount importance to maintain desired communication performances in order to achieve the mission objectives.

Starting from the above mentioned considerations, this work focuses on the study and the implementation of a distributed control algorithm for the group coordination of a team of agents, applied to an AUV swarm. In our previous works, we proposed cooperative control algorithms based on the *behavioural approach paradigm* Caiti et al. (2012) and its adaptation as *potential game* Caiti et al. (2013a), to maintain desired communication performances and fulfil each agent task. These algorithm were able to guarantee the local stability of equilibria points. In this paper we propose a distributed control framework based on the definition of artificial potentials (Leonard and Fiorelli (2001)); furthermore, the *passivity* theory is exploited for guaranteeing the stability in the large even in presence of communication delays. Moreover, the degrees of freedom offered by the passivity based approach allow to tune the desired motion of the group in terms of transient behaviour and reached equilibria. Passivity techniques have been widely studied in the domain of bilateral teleoperations for the control of a traditional single-master/single-slave system (Hokayem and Spong (2006); Secchi et al. (2008)), or for a more complex single-master/multiple-slaves system (Franchi et al. (2011)). In spite of take advantage of the operator's intelligence for solving complex tasks as in bilateral teleoperations, the proposed framework seeks to provide full autonomy to the agents in order to accomplish the cooperative mission. The multi-agent problem tackled is the following: each AUV within a team of l vehicles has to accomplish its own task (or tasks), while keeping the communication connectivity with the other team members. The fulfilment of the communication constraints among the agents is included as a fundamental requirement for the success of the collective mission. As a matter of fact, the loss of communication, because the agents are too far away from each other, implies the degradation of the performance, or even the failure, of the overall task.

The paper is organized as follows: Sec. 2 presents the essentials mathematical and theoretical tools implied in the framework; Sec. 3 outlines the implementation details of the cooperative algorithm and demonstrates stability of the solution including delays on communication links, with Sec. 4 illustrating the effectiveness of the proposed

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framework in different application scenarios. Finally, Sec. 5 summarizes the work and draws the main conclusions.

2. BACKGROUND AND PRELIMINARIES

This section provides the essentials from the main analytical and theoretical tools required for the upcoming mathematical treatment. In particular, some reminders to graph theory, passivity and port-Hamiltonian systems will be very useful for a proper understanding.

2.1 Graph theory

A graph $\mathcal{G} = (V, E)$ is formally defined by a finite set of *nodes* (or *vertices*) V and a set of *edges* $E \subset V \times V$, connecting pairs of nodes. The node set $V = \{v_1, v_2, \ldots, v_l\}$ has $l = |V|$ elements, while the edge set $E = \{e_1, e_2, \ldots, e_m\}$ contains $m = |E|$ elements. Given $e_j \in E$, then there exist a pair $v_i, v_j \in V$ such that $e_j = (v_i, v_j)$; in this way, v_i and v_j are said to be *adjacent*, while (v_i, v_i) is called a self-loop. If the edges in graphs are to be interpreted as enabling information to flow between the vertices on the corresponding edge, these flows can be directed as well as undirected. Hence, *direct* and *indirect* graph can be distinguished. In the first case, edges have a fixed direction (i.e. the *tail* and the *head* of the edge are setted), while in the second case, if (v_i, v_j) belongs to E, then (v_j, v_i) belongs to E too. However, for indirect graph, one can arbitrarily assign an orientation to each edge.

Any key feature of a graph can be well described by means of different matrices. In particular, the *incidence matrix* $B(\mathcal{G})$ is a $l \times m$ matrix defined as follows:

$$[B(\mathcal{G})]_{ij} = b_{ij} = \begin{cases} -1 & \text{if } v_i \text{ is the tail of } e_j, \\ 1 & \text{if } v_i \text{ is the head of } e_j, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

The l rows of $B(\mathcal{G})$ correspond to the nodes of $\mathcal{G}$, while the m columns denotes the edges of such graph. For further details on the graph theory, refer to Mesbahi and Egerstedt (2010).

2.2 Port-Hamiltonian systems and Passivity

The port-Hamiltonian framework, introduced for the first time in Maschke and Van der Schaft (1993), allows to model complex (non-linear) systems as energy storing and energy dissipating components, connected via ports to power conserving transmissions and conversions. Basically, it is an energy-based framework in which each element interacts with the system via a port, that consists of a couple of dual effort and flow quantities, whose product gives the power flow in and out of the component. As well described in Van der Schaft (2006), let $x \in \mathbb{R}^n$ denotes the local coordinates for an n-dimensional state space manifold $\mathcal{X}$, $u \in \mathbb{R}^m$ the control input and $y \in \mathbb{R}^m$ the output of the system. The generalized input-state-output dynamics expressed in terms of port-Hamiltonian framework is given by:

$$\begin{cases} \dot{x} = [J(x) - R(x)] \left(\frac{\partial H(x)}{\partial x}\right)^T + g(x)u, \\ y = g^T(x) \left(\frac{\partial H(x)}{\partial x}\right)^T, \end{cases} \quad (2)$$

where $J(\cdot) \in \mathbb{R}^{n\times n}$ is a skew-symmetric structure matrix, $g(\cdot) \in \mathbb{R}^{n\times m}$ is also a structure function, $H(\cdot)$ is the Hamiltonian that represents the whole energy stored in the system and $R(\cdot) \in \mathbb{R}^{n\times n}$ a symmetric, positive semi-definite dissipation matrix. The entries of both matrices $J(\cdot)$ and $R(\cdot)$ depend smoothly on x. Modelling dynamical system as (2) provides several benefits: in particular, a basic property of a port-Hamiltonian system is related to its energy balance, tightly coupled with the notion of passivity. In fact, any port-Hamiltonian system is passive w.r.t. the supply rate and storage function $H(\cdot)$ if $H(\cdot)$ is bounded from below.

Definition 1. (from Duindam et al. (2009)). Let us consider a generic, affine, non-linear system

$$\begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x), \end{cases} \quad (3)$$

with the state vector $x \in \mathbb{R}^n$, a control vector $u \in \mathbb{R}^m$ and an output vector $y \in \mathbb{R}^m$. $f(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$, $g(\cdot) : \mathbb{R}^n \to \mathbb{R}^m$ and $h(\cdot) : \mathbb{R}^n \to \mathbb{R}^m$ are non-linear functions of the state. The above system is *passive* if there exists a continuous and differentiable lower bounded function of the state $V(\cdot) : \mathbb{R}^n \to \mathbb{R}$ (called storage function) such that:

$$\dot{V}(x) \leq u^T y \iff V(x(t)) - V(x(0)) \leq \int_0^t u^T(\tau) y(\tau)\, d\tau \quad (4)$$

$V(\cdot)$ represents the internal stored energy and $u^T y$ the energy flow exchanged with the external world (i.e. the *supply rate*). The pair (u, y) is called power port, where u and y are power variables: these latter allow to control and interconnect passive systems.

Back to the port-Hamiltonian theory, due to the fact that $H(x) \geq 0$, the passivity is always guaranteed and it is easy to show that $\dot{H}(x) \leq u^T y$. In this way, port-Hamiltonian framework provides a powerful tool for the stability analysis of dynamical systems, in order to achieve a feasible, stable and robust control.

3. COOPERATIVE ALGORITHM

The cooperative multi-agent problem tackled is the following: each AUV within a team has to accomplish its own task (or tasks), while keeping the communication connectivity with the other team members. In particular, modelling the agents/targets network as an indirect graph without any self-loop, the aim is to design a control law that allows to maintain the predefined graph during the development of the whole mission. The starting hypothesis to fulfil the communication link maintenance is that the connectivity is guaranteed if the agents lie within a fixed relative range. Moreover, each vehicle is supposed to be equipped with an acoustic positioning system, i.e. an Ultra-Short Base Line (USBL), capable of measuring the relative position of another vehicle with respect to itself. Due to the limitation of acoustic propagation, the presence of delays on the connectivity links is included and conclusions about the stability are treated separately.

Consider the generic network configuration in Fig. 1, with l-vehicles and m-targets. Each link, which is a communication connection or not, is represented by a pair of spring-damper element (*virtual coupling*). The graph $\mathcal{G} = (V, E)$ associated to the concerned network is defined by:

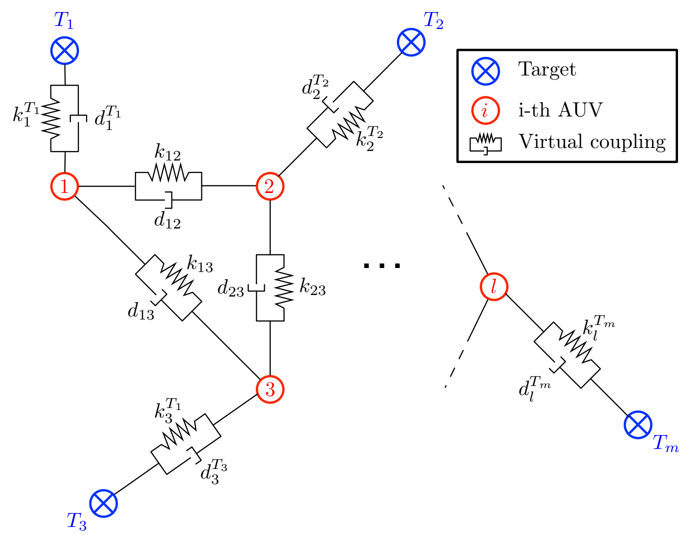

Fig. 1. A team of l-AUVs (numbered red circles) in a generic distribution in $\mathbb{R}^n$. The blue circled crosses suggest the target of the agents, while the spring-damper couples represent the virtual couplings between agents and/or targets.

$$\begin{aligned} V &= \{1, 2, \ldots, l, T_1, T_2, \ldots, T_m\} \rightarrow |V| = l + m, \\ E &= \{(1,2), (1,3), \ldots, (1, T_1), \ldots\} \rightarrow |E| = a + t. \end{aligned} \tag{5}$$

Note the separation between agent-agent and agent-target edges, with cardinality a and t, respectively. Considering the (i, j) edge, the vertex i is always seen as the tail and j as the head of the link. Then, the incidence matrix $B \in \mathbb{R}^{(l+m)\times(a+t)}$ is given by:

$$B = \left[\begin{array}{ccc:cc} -1 & -1 & \cdots & -1 & \cdots \\ 1 & 0 & \cdots & 0 & \cdots \\ 0 & 1 & \cdots & 0 & \cdots \\ \vdots & \vdots & & \vdots & \\ \hdashline 0 & 0 & \ldots & 1 & \ldots \\ 0 & 0 & \ldots & 0 & \ldots \\ \vdots & \vdots & & \vdots & \ldots \\ 0 & 0 & \ldots & 0 & \ldots \end{array}\right] \tag{6}$$

Conventionally, the rows of B are sorted as the elements in V, while the columns as those in E. In this way, the first a columns represent the links between agents and the second t stand for the agent-target connections. Each node has a double-integrator dynamics. Consider the i-th AUV and j-th target, then:

$$m_i \ddot{q}_i = -d\, \dot{q}_i + u_i, \quad m_{T_j} \ddot{q}_{T_j} = -d\, \dot{q}_{T_j} + u_{T_j}, \tag{7}$$

where m_i, m_{T_j} are the respective masses, q_i, $q_{T_j} \in \mathbb{R}^n$ are their positions (generalized coordinates) and u_i, $u_{T_j} \in \mathbb{R}^n$ are the control vectors. The d elements model a viscous friction acting on targets and agents. The corresponding linear momenta p_i, $p_{T_j} \in \mathbb{R}^n$ are given by:

$$p_i = \bar{m}_i \dot{q}_i \quad p_{T_j} = \bar{m}_{T_j} \dot{q}_{T_j}, \tag{8}$$

with $\bar{m}_i = I^n m_i$, $\bar{m}_{T_j} = I^n m_{T_j}$ and I^n is the $n \times n$ identity matrix.

Now, consider the group of $l + m$ vertices. To compactly denote the agent dynamics, the vector form is introduced. Thus, $q = [q_1^T, q_2^T, \ldots, q_{T_1}^T, \ldots]^T \in \mathbb{R}^{n(l+m)}$ stands for the generalized coordinates vector, $D^A = \mathrm{diag}(I^n\, d, \ldots, I^n\, d) \in \mathbb{R}^{n(l+m)\times n(l+m)}$ is the damping matrix and $u = [u_1^T, u_2^T, \ldots u_{T_1}^T, \ldots]^T \in \mathbb{R}^{n(l+m)}$ is the control vector. The linear momenta vector $p \in \mathbb{R}^{n(l+m)}$ is:

$$p = M\dot{q}, \tag{9}$$

where $M = \mathrm{diag}(\bar{m}_1, \ldots, \bar{m}_{T_m}) \in \mathbb{R}^{n(l+m)\times n(l+m)}$ is the diagonal, positive definite matrix of masses. Basing on the communication links, the relative distances vector $z \in \mathbb{R}^{n(a+t)}$ is defined as:

$$z = (B^T \otimes I^n) q, \tag{10}$$

where the operator $\otimes$ identifies the Kronecker product.

Including the virtual couplings, the energetic behaviour of the network is analysed. The global Hamiltonian function

$$H(p, z) = H^k(p) + H^t(z) + H^a(z) \tag{11}$$

is divided as the sum of three different terms, in which the potential contribution of the spring is split between AUV-target and AUV-AUV contributions. The first term is given by:

$$H^k(p) = \frac{1}{2} p^T M^{-1} p. \tag{12}$$

Consider the agent-target links. Then:

$$H^t(z) = \frac{1}{2} z^T K_T z, \tag{13}$$

with $K_T = \mathrm{diag}(0^{na\times na}, \bar{K}_T) \in \mathbb{R}^{n(a+t)\times n(a+t)}$, $\bar{K}_T = \mathrm{diag}(\bar{k}_1, \ldots, \bar{k}_t)$, $\bar{k}_i = I^n k_i^t$ and k_i^t is the elastic constant of the i-th vehicle-target coupling in E (for example, $k_1^t = k_1^{T_1}$ and $k_t^t = k_l^{T_m}$ in Fig. 1, for $i = 1, \ldots, t$). Defining R_c as the maximum distance at which the AUV can still communicate with each other, and $R_d \leq R_c$ as the desired distance to be maintained between two vehicles, the last term of $H(p, z)$ is given by:

$$H^a(z) = \frac{1}{2} \sum_{i=1}^{a} k_i^a(||z_i||) \cdot (||z_i|| - R_d)^2. \tag{14}$$

where the elastic constant $k_i^a(\cdot)$ models a non-linear spring over each agent-agent link and $||\cdot||$ denotes the Euclidean norm. Now, it is possible to define the force vector $f \in \mathbb{R}^{n(a+t)}$ acting on the $a + t$ links as:

$$f = \left(\frac{\partial H}{\partial z}\right)^T + D^C (B^T \otimes I^n) \left(\frac{\partial H}{\partial p}\right)^T, \tag{15}$$

where $D^C = \mathrm{diag}(\bar{d}_1, \ldots, \bar{d}_a, \ldots, \bar{d}_t) \in \mathbb{R}^{n(a+t)\times n(a+t)}$ is the diagonal, positive definite matrix that contains the mutual damping elements $\bar{d}_i = I^n d_i$ in E.

Assuming (p, z) as the state variables, the input-state representation of the multi-agents open loop system in the port-Hamiltonian framework is:

$$\begin{aligned} \begin{bmatrix} \dot{p} \\ \dot{z} \end{bmatrix} &= \left[\begin{array}{c:c} -D^A & 0^{n(l+m)\times n(a+t)} \\ \hdashline B^T \otimes I^n & 0^{n(a+t)\times n(a+t)} \end{array}\right] \begin{bmatrix} \left(\frac{\partial H}{\partial p}\right)^T \\ \left(\frac{\partial H}{\partial z}\right)^T \end{bmatrix} + \\ &+ \left[\begin{array}{c} I^{n(l+m)\times n(l+m)} \\ \hdashline 0^{n(a+t)\times n(l+m)} \end{array}\right] u. \end{aligned} \tag{16}$$

So far the targets are treated as agents. Now, Pfaffian constraints are introduced to consider fixed targets. In this way:

$$\underbrace{\left[\begin{array}{c:c} 0^{nm\times nl} & I^{nm\times nm} \end{array}\right]}_{A^T \in \mathbb{R}^{nm\times n(l+m)}} \frac{\partial H}{\partial p}^T = 0^{nm}, \tag{17}$$

and a sort of selection matrix $S \in \mathbb{R}^{n(l+m)\times n(l+m)}$ such that $A^T S = 0^{nm\times n(l+m)}$ is computed:

$$S = \left[\begin{array}{c:c} I^{nl\times nl} & 0^{nl\times nm} \\ \hdashline 0^{nm\times nl} & 0^{nm\times nm} \end{array}\right]. \tag{18}$$

Then, the open loop system in (16) assumes the following form:

$$\begin{bmatrix} \dot{p} \\ \dot{z} \end{bmatrix} = \left[\begin{array}{c:c} -S^T D^A S & 0^{n(l+m)\times n(a+t)} \\ \hdashline (B^T \otimes I^n)S & 0^{n(a+t)\times n(a+t)} \end{array}\right] \begin{bmatrix} \left(\frac{\partial H}{\partial p}\right)^T \\ \left(\frac{\partial H}{\partial z}\right)^T \end{bmatrix} + \\ + \left[\begin{array}{c} S \\ \hdashline 0^{n(a+t)\times n(l+m)} \end{array}\right] u. \tag{19}$$

3.1 Distributed control law without communication lags

First of all, only the dampers between agents and targets are considered, neglecting those between agents (i.e. $\bar{d}_1 = \ldots = \bar{d}_a = 0^{n\times n}$). In order to synthesize a stable control that fulfil the communication constraints, the Hamiltonian $H(p, z)$ is chosen as the Lyapunov candidate $V(p, z)$ of the multi-agents multi-targets network. Indeed, differentiating with respect to time:

$$\dot{V}(p, z) = \frac{\partial H^k}{\partial p}\dot{p} + \frac{\partial}{\partial z}(H^t + H^a)\dot{z}, \tag{20}$$

and replacing the dynamic (19) in (20), hence:

$$\dot{V}(p, z) = \frac{\partial H}{\partial p} S u - \frac{\partial H}{\partial p} S^T D^A S \left(\frac{\partial H}{\partial p}\right)^T + \\ + \frac{\partial H}{\partial p} S^T (B^T \otimes I^n) \left(\frac{\partial H}{\partial z}\right)^T. \tag{21}$$

Defining $\bar{S} = (B^T \otimes I^n)S$ and choosing the control u as:

$$S u = -\bar{S}^T \left(\frac{\partial H}{\partial z}\right)^T - \bar{S}^T D^C \bar{S} \left(\frac{\partial H}{\partial p}\right)^T, \tag{22}$$

the autonomous closed loop dynamic becomes:

$$\begin{bmatrix} \dot{p} \\ \dot{z} \end{bmatrix} = \left[\begin{array}{c:c} -(S^T D^A S + \bar{S}^T D^C \bar{S}) & -\bar{S}^T \\ \hdashline \bar{S} & 0^{n(a+t)\times n(a+t)} \end{array}\right] \begin{bmatrix} \left(\frac{\partial H}{\partial p}\right)^T \\ \left(\frac{\partial H}{\partial z}\right)^T \end{bmatrix}. \tag{23}$$

Invoking La Salle principle, applied to the obtained negative semi-definite $\dot{V}(p, z)$, the trajectories of (23) converge to the largest invariant set where $p = 0$, that is:

$$-\bar{S}^T \left(\frac{\partial H}{\partial z}\right)^T = 0. \tag{24}$$

When $(\partial H/\partial z)^T \in \ker\{\bar{S}^T\}$, the previous equation suggests that the proposed control stabilizes the closed loop system at the equilibrium point between the elastic forces generated to the agent-agent, agent-target links. Note that the control law in (22) is distributed. In fact, assuming the position of the targets known beforehand, each AUV can compute its control input knowing only the relative position of the agents connected with itself which is measurable directly by means of the on-board acoustic positioning system.

3.2 Including communication delays

The introduction of the agent-agent contributions require that each vehicle has access to motion information not directly measurable by means of on-board equipment (i.e. the velocity of the other team members). This information is affected by a communication delay on the connectivity links; in the following we use the same framework as in Pasumarthy and Kao (2009). The control policy synthesized above can be restated as:

$$S u = -\bar{S}^T \left(\frac{\partial H}{\partial z}\right)^T - \left[\bar{D}\left(\frac{\partial H}{\partial p}\right)^T - \bar{T}\left(\frac{\partial H}{\partial p}\right)^T_{\tau(t)}\right], \tag{25}$$

where $\bar{D}$ and $\bar{T}$ contain the diagonal and the out of diagonal elements of $\bar{S}^T D^C \bar{S}$, respectively (in this case, $\bar{d}_1 = \ldots = \bar{d}_a$ are non-zero matrices). The term $(\partial H/\partial p)_{\tau(t)}$ represents the partial derivative of the Hamiltonian function w.r.t. the linear momenta, computed in according to the time varying delay vector $\tau(t) = [\tau_1(t), \ldots, \tau_a(t), \ldots, \tau_t(t)]$. Assuming that each $\tau_i(t)$ is a non-negative, bounded above, Lipschitz function, i.e. $0 \leq \dot{\tau}_i(t) \leq \tilde{d} < 1$, the proposed distributed control law in (25) assures the stability of the networked system with communication delays. In fact, omitting the time dependence of τ and defining:

$$\nabla H^T = \begin{bmatrix} \left(\frac{\partial H}{\partial p}\right)^T \\ \left(\frac{\partial H}{\partial z}\right)^T \end{bmatrix}, \quad \nabla H_\tau^T = \begin{bmatrix} \left(\frac{\partial H}{\partial p}\right)^T_\tau \\ \left(\frac{\partial H}{\partial z}\right)^T_\tau \end{bmatrix}, \tag{26}$$

the Lyapunov candidate is now chosen as:

$$V(p, z) = H(p, z) + \int_{t-\tau}^{t} \nabla H_s P \nabla H_s^T \, ds, \tag{27}$$

with P symmetric, positive semi-definite matrix. Then, the derivative function is:

$$\dot{V}(p, z) = \frac{d}{dt} H(p, z) + [\nabla H P \nabla H^T - (1 - \dot{\tau}) \nabla H_\tau P \nabla H_\tau^T]. \tag{28}$$

Noting that only the term $(\partial H/\partial p)_\tau$ of ∇H_τ is really affected by lag, hence $P = \mathrm{diag}(\bar{P}, 0^{n(a+t)\times n(a+t)})$. After some manipulations, we get a quadratic form:

$$\dot{V}(p, z) \leq \begin{bmatrix} \left(\frac{\partial H}{\partial p}\right)^T \\ \left(\frac{\partial H}{\partial p}\right)^T_\tau \end{bmatrix}^T W \begin{bmatrix} \left(\frac{\partial H}{\partial p}\right)^T \\ \left(\frac{\partial H}{\partial p}\right)^T_\tau \end{bmatrix}, \tag{29}$$

that is negative definite iff:

$$W = \left[\begin{array}{c:c} -S^T D^A S - \bar{D} + \bar{P} & \bar{T} \\ \hdashline 0 & -(1 - \tilde{d})\bar{P} \end{array}\right] < 0. \tag{30}$$

In this way, if a matrix P exists that allows to solve the Linear Matrix Inequalities (LMI) problem in (30), the previous conclusions about stability of the closed loop system described in (24) continue to apply.

3.3 A trivial example

The discussion just presented is now applied to the trivial case of 2 agents 2 targets acting in $\mathbb{R}^2$ (Fig. 2). The aim is to provide a clearer interpretation of the invariant set in (24) that contains the trajectories of the controlled system at the equilibrium.

So, the set $E = \{(1, 2), (1, T_1), (2, T_2)\}$ contains the different connectivity links, the selection matrix $S = \mathrm{diag}(I^{4\times 4}, 0^{4\times 4})$ and the incidence matrix is:

$$B = \left[\begin{array}{c:cc} -1 & -1 & 0 \\ 1 & 0 & -1 \\ \hdashline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]. \tag{31}$$

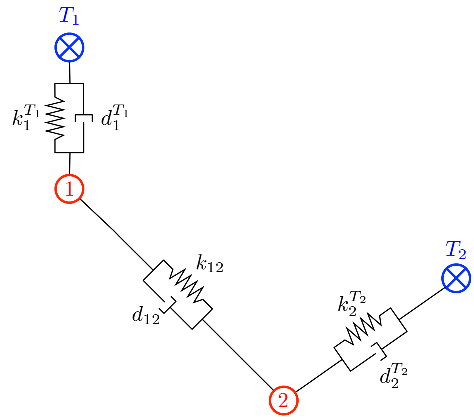

Fig. 2. A generic distribution in $\mathbb{R}^2$ for the 2 AUVs, 2 targets case.

Choosing a linear spring to model the agent-agent virtual coupling (i.e. $k_1^a(||z_1||) = k_1^a$), the system (23) is stabilized (with $p = 0$) to the set of z that fulfil the following system of equations:

$$\begin{cases} k_1^a\left(1 - \dfrac{R_d}{||z_{1,2}||}\right) z_{1,2_x} + k_1^t z_{1,T_{1x}} = 0 \\ k_1^a\left(1 - \dfrac{R_d}{||z_{1,2}||}\right) z_{1,2_y} + k_1^t z_{1,T_{1y}} = 0 \\ -k_1^a\left(1 - \dfrac{R_d}{||z_{1,2}||}\right) z_{1,2_x} + k_2^t z_{2,T_{2x}} = 0 \\ -k_1^a\left(1 - \dfrac{R_d}{||z_{1,2}||}\right) z_{1,2_y} + k_2^t z_{2,T_{2y}} = 0, \end{cases} \tag{32}$$

where $z_{1,2}$, z_{1,T_1} and z_{2,T_2} represent the relative distances ordered in E, while k_1^a, k_1^t and k_2^t the respective elastic constants. As regards the choice of the control gains, the proposed control law offers several degrees of freedom, as well as strong stability guarantees. In fact, selecting different elastic constants, is possible to determine in advance which task has higher priority than the other without any consequence about stability.

4. SIMULATIONS

The framework described in Section 3 can be applied to a large variety of cooperative tasks in the marine environment by changing properly the graph configuration (nodes and edges).

The maintenance of the communication link among all the vehicles of the team can be guaranteed by defining a complete sub-graph between the agents. After choosing a proper initial incidence matrix, the non-linear spring associated to each agent-agent connection is modelled so that the corresponding elastic potential is given by:

$$H^{a_i}(z_i) = \begin{cases} K_1\left(\|z_i\| - R_d\right)^2 & \|z_i\| \leq R_d \\ K_2\dfrac{\left(\|z_i\| - R_d\right)^2}{R_c - \|z_i\|} & \text{otherwise} \end{cases}$$

where $K_1, K_2 > 0$ are design parameters. As an example, Fig. 3 shows the above function with a typical value for the maximum communication range of a medium frequency acoustic modem, e.g. $R_c = 3000$ m, and $R_d = 2000$ m. Such a choice ensures that if two agents start within the communication range, they remain connected during the whole

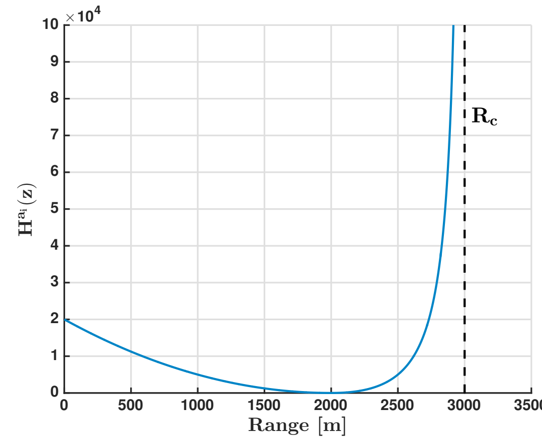

Fig. 3. Elastic potential associated to an agent-agent connection, with $R_c = 3000$ m and $R_d = 2000$ m.

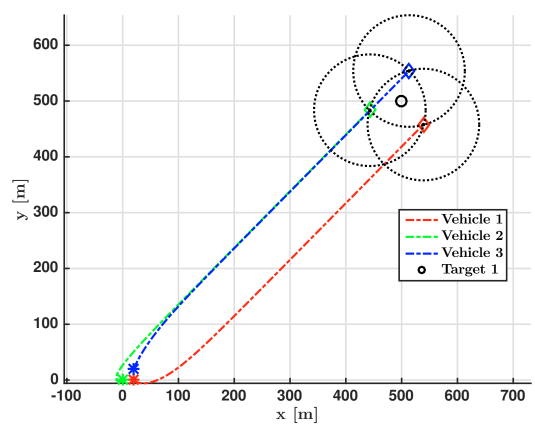

Fig. 4. Cooperative surveillance task. The vehicles start from the positions indicated by the coloured stars. At the end, the vehicles (coloured diamonds) are disposed around the asset (black circle) while satisfying the distance constraints, represented with the dotted circles.

task, converging if possible to the desired distance R_d, also maintaining the structure of the initial incidence matrix. Note that the assumption of an existing communication link among the vehicles at the beginning of the mission is realistic in practice. In fact, the deployment of the team vehicles during sea experimentations is commonly operated by either a support ship or a docking station within a restricted area.

On the other hand, the agent-target connections are strictly dependent on the specific task to be accomplished. In the following, we will present the simulation results obtained with two types of tasks, inspired on our previous works (Caiti et al., 2012, 2013a). Velocities are supposed to be exchanged between the vehicles at a rate of 1 Hz. Moreover, all the simulations are performed considering a fixed communication delay of 1 s.

4.1 Surveillance task

In a surveillance or patrolling task, the vehicles of the team have to cover a certain area around an asset to be

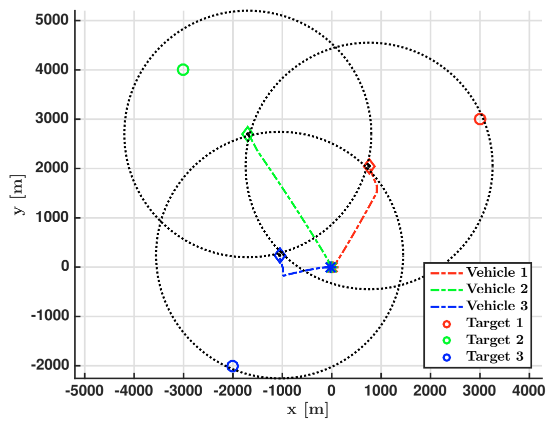

Fig. 5. Cooperative coverage task. The vehicles start to approach to the assigned targets until the communication limit (dotted circles) is attained.

defended. The only target in this scenario is represented by the asset itself ($m = 1$). Hence, the agent-target connections are modelled by a spring-damper couple from each vehicle to the asset. Fig. 4 shows the results obtained with three vehicles ($l = 3$), setting the desired intra-agent range $R_d = 100$ m. As can be noticed, the team reaches a stable configuration around the asset to be defended. Furthermore, each vehicle remains within the desired distance with respect to all the others.

4.2 Coverage task

In this case, each vehicle is assigned to a specific target ($l = m$) with the overall objective of covering the maximum area in the targets' neighbourhood. Each coupling between an agent and the associated target is modelled by a spring-damper connection. Fig. 5 shows the trajectories followed by the team members in a scenario with three vehicles ($l = m = 3$) and a relative distance among the agents $R_d = 2500$ m. In this case, the team reaches a stable configuration in which no vehicle can move closer to the corresponding target without breaking the imposed constraints due to the communication maintenance.

5. CONCLUSIONS AND FUTURE WORKS

The proposed distance-based distributed control law allows to fulfil communication constraints between interconnected marine vehicles during the execution of several tasks. It also offers a lot of degrees of freedom to be exploited upstream of the mission. The stability of the networked system is guaranteed even in presence of delays on the connectivity links, which are very common in marine environment. The port-Hamiltonian framework, tightly coupled with the passivity theory, provides a powerful tool to model a swarm of AUVs performing several tasks. Moreover, it suggests an easy way to draw quick conclusions about the soundness and stability of the whole system. However, there are some aspects that are to be explored. In fact, the fixed targets assumption, as well as the complete connectivity among the agents, can be quite easily relaxed. Future works will investigate these topics, including several considerations about network performance degradations due to the presence of communication delays.

REFERENCES

Caiti, A., Calabrò, V., Dini, G., Lo Duca, A., and Munafò, A. (2012). Secure cooperation of autonomous mobile sensors using an underwater acoustic network. *Sensors*, 12(2), 1967–1989.

Caiti, A., Fabbri, T., Fenucci, D., and Munafò, A. (2013a). Potential games and AUVs cooperation: First results from the THESAURUS project. In *OCEANS - Bergen, 2013 MTS/IEEE*, 1–6.

Caiti, A., Grythe, K., Hovem, J.M., Jesus, S.M., Lie, A., Munafò, A., Reinen, T.A., Silva, A., and Zabel, F. (2013b). Linking acoustic communications and network performance: integration and experimentation of an underwater acoustic network. *IEEE Journal of Oceanic Engineering*, 38(4), 758–771.

Curtin, T.B., Bellingham, J.G., Catipovic, J., and Webb, D. (1993). Autonomous oceanographic sampling networks. *Oceanography*, 6(3), 86–94.

Duindam, V., Macchelli, A., Stramigioli, S., and Bruyninckx, H. (2009). *Modeling and control of complex physical systems: the port-Hamiltonian approach.* Springer Science & Business Media.

Franchi, A., Giordano, P.R., Secchi, C., Son, H.I., and Blthoff, H.H. (2011). A passivity-based decentralized approach for the bilateral teleoperation of a group of uavs with switching topology. In *Robotics and Automation (ICRA), 2011 IEEE International Conference on*, 898–905.

Hokayem, P.F. and Spong, M.W. (2006). Bilateral teleoperation: An historical survey. *Automatica*, 42(12), 2035–2057.

Leonard, N.E. and Fiorelli, E. (2001). Virtual leaders, artificial potentials and coordinated control of groups. In *Decision and Control, 2001. Proceedings of the 40th IEEE Conference on*, volume 3, 2968–2973.

Maschke, B. and Van der Schaft, A. (1993). Port-controlled Hamiltonian systems: modelling origins and system theoretic properties. In M. Fliess (ed.), *Nonlinear Control Systems Design 1992*, IFAC Symposia Series, 359 – 365. Pergamon, Oxford.

Mesbahi, M. and Egerstedt, M. (2010). *Graph theoretic methods in multiagent networks.* Princeton University Press.

Pasumarthy, R. and Kao, C.Y. (2009). On stability of time delay hamiltonian systems. In *American Control Conference, 2009. ACC'09.*, 4909–4914. IEEE.

Secchi, C., Stramigioli, S., and Fantuzzi, C. (2008). Transparency in port-hamiltonian-based telemanipulation. *IEEE Transactions on Robotics*, 24(4), 903–910.

Stojanovic, M. (2007). On the relationship between capacity and distance in an underwater acoustic communication channel. *SIGMOBILE Mob. Comput. Commun. Rev.*, 11(4), 34–43.

Van der Schaft, A. (2006). *L2-gain and passivity techniques in nonlinear control.* Springer Science & Business Media.