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A hybrid time-frequency domain approach for numerical modeling of reciprocating compressors

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Abstract

In the reciprocating compressor field, strong attention is paid to the study of pressure wave propagation in the discharge and suction pipelines. Oscillating pressure waves may lead to mechanical vibrations and failures and affect the machine performance. For this reason, an accurate analysis of the acoustic response of suction and discharge pipelines in a reciprocating compressor plant is of great interest.

By solving a linear system of equations, the acoustic domain of a piping system can be easily determined. Usually, the acoustic pulsation analysis of the pipelines is carried out without considering the interaction between the machinery and the pipelines. Consequently, the reciprocal interaction between the compressor and the pipelines can not be considered.

The aim of this work is to perform a fluid-dynamic analysis of the full compressor-pipelines system. For this purpose, a hybrid time-frequency domain approach is adopted. The reciprocating compressor thermodynamic cycle is simulated with a 0D time-domain model, while the pressure wave propagation in the pipelines is modelled by mean of a transfer matrix approach in the frequency domain. This analysis allows one to take into account the mutual interaction between the compressor and its pipelines by using the FFT and the Inverse FFT alternatively.

The methodology was assessed by comparing the results of the simulation of a test case performed with both the hybrid approach and a commercial 1D code.

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Nomenclature										
Av	valve section	[m2]	s	piston displacement	[m]					
A _{v,max}	maximum valve section	[m2]	$\mathbf{S}_{\mathbf{r}}$	reeds surface	[m2]					
b	damping	[Ns/m]	t	time	[s]					
CD	drag coefficient	-	u	specific internal energy	[J/kg]					
dt	time step	[s]	V_{f}	flow velocity	[m/s]					
dV	volume variation	[m3]	V_r	reeds velocity	[m/s]					
F	force	[N]	W_{hyb}	hybrid model work	[J/cycle]					
k	spring stiffness	[N/m]	W _{time}	time-domain model work	[J/cycle]					
Ks	flow coefficient	-	х	displacement	[m]					
1	rod length	[m]	x _{max}	maximum displacement	[m]					
М	mass of gas inside the cylinder	[kg]	ż	velocity	[m/s]					
m	mass	[kg]	ÿ	acceleration	[m/s2]					
ṁ	mass flow rate	[kg/s]	Symbol.	S						
р	pressure	[Pa]	Δ	difference	-					
p ₀₁	upstream total pressure	[Pa]	γ	specific heat ratio	-					
p_2	downstream pressure	[Pa]	ρ	density	[kg/m3]					
r	crank radius	[m]	ω	angular velocity	[rad/s]					

1. Introduction

In the design of reciprocating compressor plants, numerical simulations have an important role in the preliminary phases. Up to now, the development of reciprocating compressors was mainly focused on reliability and effectiveness. Nowadays particular attention is paid to efficiency increase, noise reduction and vibration control. In order to face the above-mentioned issues, the design of reciprocating compressors needs a deep preliminary investigation based on numerical modelling.

For this purpose, several simulation tools can be adopted, according to the details of the analysis that need to be carried out. For example, the 0D models give results quickly and allow a global prediction of the compressor performance [1]. On the other hand, the 3D Computational Fluid-Dynamics (CFD) analysis gives more detailed results but leads to an increase of computational time [2].

A simplified model focusing on the performance evaluation with improved physical phenomena details is presented by R. Aigner et al. [3]. This work deals with the study of the internal flow in the reciprocating compressor and the analysis of the automatic valves motion. R. Aigner et al. performed a one-dimensional and two-dimensional numerical model for the prediction of the valve motion and wave propagation inside the cylinder. The simulation results are compared with experimental measurements. It is shown that a more complex modelling increases the computational time, but does not give a more detailed information in terms of performance prediction.

In the works by E.Winandy et al. [4] and M. Elhaj et al. [5] simplified tools for the estimation of the compressor performances are described and compared with experimental data. Both these works confirm the good prediction of the simplified numerical model.

The coupling of the reciprocating machine to the line is a recurrent topic [6] in industrial applications. The interaction between the machine and the pipelines can bring to noise issues, vibrations and mechanical failures. An exemplifying work examining in depth these phenomena is the report by M. G. Nored et al. [7] with a detailed analysis of the pressure wave propagation in the pipelines. This research shows in detail the issue of the pressure wave propagation in the pipeline on the mechanical structure.

In most cases both the compressor and the pipelines analysis is performed without considering the reciprocal interaction. Therefore, the influence of the pipelines acoustic response on the compressor thermodynamic cycle is neglected.

A method to numerically investigate the interaction between the source of the noise and the source-connected system is to realise a hybrid model in which the compressor (time-domain model) directly acts on the pipelines (frequency-domain model). Some examples of the topic in the literature, are the work by P. Davies et al. [8] dealing with the control of the acoustic performance of the intake/exhaust engine systems, or the work by Y. Sathyanarayana et al. [9] on the acoustic analysis. In the field of the reciprocating compressors is of particular interest the study by A. Kwang-Hyup [1] on the analysis of the compressor-line coupling, which considers the interaction of the machine with the pipelines. In all these cases, the authors do not estimate the influence of the pipelines acoustic response on the thermodynamic cycle.

In this scenario, the authors have devised and defined a numerical model for the analysis of the reciprocating compressor which takes into account the reciprocal interaction between the machine and the piping system. The hybrid model has the task to predict the machine-pipelines interaction and the influence of the pipelines acoustic response on the compressor performances. The input to investigate this phenomenon came from both the oil&gas and the refrigeration-systems industries collaborating with the Department of Industrial Engineering of Florence University (D.I.E.F). To meet the industrial partners' requests, a fast and flexible numerical model able to support the design phase of the reciprocating compressors was realized. According to the predetermined aim, a 0D-1D code has been realized in MATLAB®, for the simulation of both the reciprocating compressor performance and the pressure wave propagation in the pipelines.

The numerical model is composed by two main sub-models: one is the thermodynamic model of the compressor and the other is the acoustic model of the compressor piping system. The compressor simulation is based on a quasistationary modelling. The thermodynamic parameters of the processed gas (i.e. pressure, temperature, density etc.) are analytically computed in subsequent time-steps. During the simulation, the gas flow inside the cylinder is regulated by self-acting valves, whose dynamics is computed in parallel to the thermodynamic cycle. The other sub-model is that focused on compressor pipeline (e.g. ducts, plenum, orifices, etc.,). This sub-model follows the mono-dimensional acoustic approach of transfer matrix method. This approach allows one to calculate the pressure wave propagation through the elements of a defined geometry with a mono-dimensional approach. A system of linear equations describe the pipeline geometry. Moreover, this method allows one to simulate geometrically complex elements after a previous experimental or 3D-numerical acoustic characterization. The reciprocal interaction of the compressor and pipelines is achieved through the sub-models coupling.

In order to assess the proposed methodology, the authors have performed a comparison of the model results with a time-domain commercial 1D code. At first, a test case configuration of a compressor with a simple configuration of suction and discharge pipelines was tested. The comparison proved the effectiveness of the hybrid model. Then a sensitivity analysis of the compressor and pipelines interaction was carried out on the same configuration, by varying geometrical parameters of the suction and discharge pipelines.

2. The hybrid numerical model

A numerical model in MATLAB® was developed to estimate the reciprocating compressor performance, to carry out an acoustic pulsation analysis in the pipelines and to take into account the machine-pipelines reciprocal interaction,. The compressor sub-model performs in the time-domain, while the pipelines sub-model performs in the frequency-domain. The hybrid model couples these sub-models using the FFT algorithm and its inverse. The algorithm links the compressor and pipelines sub-models and accomplishes the reciprocal interaction. The procedure is iterative and is repeated until the convergence condition of the thermodynamic cycle is reached.

In the present work, the authors developed a 0D quasi-steady numerical sub-model of the reciprocating compressor named Re.Co.A. (Reciprocating Compressor Analysis) by following [1, 3, 4]. For the pipelines, an acoustic 1D sub-model based on the transfer matrix method [10] was developed. Above all, the coupling of these sub-models is obtained using the hybrid model algorithm.

The hybrid model joins the advantages of the quasi-steady time-domain model with the advantages of the acoustic transfer matrix method, resulting both in a fast simulation tool for reciprocating compressor performance evaluation and pipelines pulsation analysis.

2.1. Hybrid model algorithm

The hybrid model, which couples the Re.Co.A. and transfer matrix sub-models, is based on the reciprocal interaction between time-domain and frequency-domain approaches. The use of FFT algorithm [9] and its inverse allows one to work in both the physical domains (Figure 1).



Fig. 1. FFT-IFFT analysis: compressor (time domain) and pipelines (frequency domain) interaction. $\dot{m}(t)$ is the time-domain mass flow profile in time-domain, and $\dot{M}(f)$ is the same profile in the frequency-domain. Similarly, p(t) and P(f) are the pressure profiles in the two domains.

The first step of the hybrid algorithm is the computation of the compressor thermodynamic cycle. Constant pressure profiles for both suction and discharge boundary conditions are imposed. Subsequently, the mass flow profiles of the thermodynamic cycle are transformed in the frequency domain by the FFT algorithm. The time-step of the compressor sub-model defines the resolution of the processed signal (i.e. mass flow), and has a duration equal to a thermodynamic cycle (i.e. 360 degree of crank-angle).

Then the transformed mass flow profiles are the inputs of the pipeline sub-models. The pipeline sub-models compute the acoustic response in terms of pressure at the compressor interface. The suction and discharge pressure profiles, expressed in the frequency-domain, are transformed by the IFFT (Inverse Fast Fourier Transform) in the time-domain pressure profiles over a compressor cycle. These profiles are the new boundary conditions imposed to the compressor sub-model that computes the new thermodynamic cycle.

These steps are iterated until the convergence condition on the compressor thermodynamic cycle is reached.

2.2. Re.Co.A. sub-model

The Re.Co.A. sub-model was developed in DIEF (Department of Industrial Engineering of Florence) for industrial research. Re.Co.A. is a 0D quasi-steady numerical model that solves the energy and mass continuity equations in the fluid volume inside the cylinder for subsequent time-steps (namely *i-th* time step).

The motion of the piston controls the thermodynamic cycle phases. This implies the variation of volume and pressure inside the cylinder and, as a consequence, the opening or the closing of the self-acting valves.

For each time-step of simulation, the piston displacement is calculated using the cinematic equation of the crankshaft mechanism:

$$s = \sqrt{l^2 - (r \cdot \sin(\omega \cdot t))^2} + r \cdot \cos(\omega \cdot t)$$
⁽¹⁾

The sub-model follows a two-step numerical computation. At a first time, a set of preliminary thermodynamic conditions inside the cylinder is calculated solving the mass continuity and energy equations for the new in-cylinder volume value. Then, depending on the preliminary in-cylinder pressure, the compressor model determines in which phase (suction, compression, discharge or expansion) the cycle is (Figure 3(a)). Finally, the mass continuity and energy equations are solved again to compute the effective thermodynamic conditions inside the cylinder.

$$M^{i} = M^{i-1} + \sum_{k} \dot{m}^{i}_{k} \cdot dt \tag{2}$$

$$u^{i} = \frac{u^{i-1} \cdot M^{i-1} - dV^{i} \cdot p^{i-1} + \sum_{k} \dot{m}^{i}_{k} \cdot T^{i} \cdot c_{p}^{i}_{k}}{M^{i-1}}$$
(3)

During the suction/discharge phases the valves dynamic is simulated, and the mass continuity and energy equations take into account the mass flow incoming/outgoing the cylinder. Otherwise, for compression/expansion phases, the new thermodynamic state is computed solving polytropic transformations:

$$p^{i} = p^{i-1} \cdot \left(\frac{\rho^{i}}{\rho^{i-1}}\right)^{\gamma} \tag{4}$$

The valve dynamic [10-12] is implemented in the reciprocating compressor sub-model and is calculated at each time step. The valve dynamics calculation starts when the pressure inside the cylinder is higher than the pressure of the discharge ambient. On the other hand, the suction valve dynamics calculation starts when the cylinder pressure is lower than the suction ambient. The valve dynamic, represented by a mass-spring-damper model, is mathematically described by the second order differential equation:

$$m\ddot{x} + b\dot{x} + kx = F(t) \tag{5}$$

Fig. 2. Re.Co.A. computational domain and crank-shaft mechanism scheme.





rebound on the valve seats. The same behavior is simulated when a displacement lower than the minimum allowable value is reached. In order to model this condition the rebound coefficient is introduced.

The driving force F(t) is represented by the pressure that acts on the reeds surface. If the value at the previous timestep of computation is closed, the pressure acting on the reeds is equal to the difference between the ambient pressure and the in-cylinder pressure. Then the driving force is the product of the pressure drop across the value and the reeds surface.

$$F(t) = \Delta p \cdot S_r \tag{6}$$

When the valve opens, the driving force of the system is the product of the stagnation pressure on the reeds surface and the reeds surface area.

$$F(t) = \frac{1}{2} \cdot C_D \cdot \rho \cdot (V_f - V_r)^2 \cdot S_r$$
⁽⁷⁾

The geometrical section of the valve is computed following a proportional scaling law function of the valve reeds displacement.

$$A_{\nu} = A_{\nu,\max} \cdot \frac{X_{\nu}}{X_{\nu,\max}}$$
(8)

Once the valve is opened, the mass flow is computed using the nozzle isentropic flow formulation. The control logic of the valve dynamic is depicted through the flow chart in Figure 3(b)

$$\dot{m} = K_{s} \cdot A_{v} \cdot \sqrt{\left[\frac{2\gamma}{\gamma - 1}\right]} \left\{ \rho_{1} \cdot p_{01} \left[\left(\frac{p_{2}}{p_{01}}\right)^{2/\gamma} - \left(\frac{p_{2}}{p_{01}}\right)^{1 + \frac{1}{\gamma}} \right] \right\}$$
(9)

The non-isentropic behavior of the mass flow is taken into account using the flow coefficient (i.e. Ks). The whole compressor model logic is summarized in Figure 3(a).

2.3. The transfer matrix method sub-model

The pipelines of the reciprocating compressor are modelled in the frequency-domain using the transfer matrix method [13]. This method allows one to correlate the upstream and downstream state variables (pressure and acoustic velocity) of an acoustic element using a simple matrix formulation. The physical domain is described by a system of linear equations.

$$\begin{bmatrix} p_u \\ v_u \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_d \\ v_d \end{bmatrix}$$
(10)

By following this approach, a pipeline system can be modelled as a composition of adjacent simple elements. This results in a single four pole matrix that is the product of the single matrices.

$$\begin{bmatrix} p_n \\ v_n \end{bmatrix} = T_n \cdot T_{n-1} \cdot \ldots \cdot T_1 \cdot T_0 \begin{bmatrix} p_0 \\ v_0 \end{bmatrix}$$
(11)

Each matrix depends on the geometrical parameters of the element, the thermodynamic conditions of the fluid and the investigated frequency.



Fig. 3. (a) Re.Co.A. computational logic flow-chart; (b) Valve dynamics computational logic flow chart.

3. Hybrid model simulation results

In order to assess the proposed methodology, the authors have performed a test case simulation with both the hybrid model and a commercial full time-domain 1D code. The tested configuration (Figure 4) is a single-effect reciprocating compressor for refrigeration applications.



Fig. 4. Scheme of the hybrid model tested configuration

The suction and discharge pipelines are composed by a volume (representing the compressor plenum) and a duct which is connected to an ambient at constant thermodynamic conditions (i.e. pressure and temperature). In both the

simulations, the viscous effects, the frictions of the flow and the heat exchange in both the pipelines and the cylinder are neglected. The main simulation data are reported in Table 1.

Table 1. Technical data of the numerical tested configuration.

Compressor			Suction		
Rotating speed	[rpm]	1440	Pressure	[bar]	2.06
Fluid	-	CO_2	Temperature	[°C]	21.6
Bore	[mm]	61	Di	Discharge	
Stroke	[mm]	52	Pressure	[bar]	18.9
Rod length	[mm]	98	Temperature	[°C]	118
Suction valve diameter	[mm]	15	Pipelin	Pipelines geometry	
Discharge valve diameter	[mm]	9.5 volume		[dm ³]	0.3
			Duct length	[m]	1.0
			Duct diameter	[mm]	30

As shown in Figure 5, the results of the hybrid model agree with the results of the time domain model on the whole thermodynamic cycle, both in terms of in-cylinder pressure and in-cylinder mass of gas.



Fig. 5. (a) In-cylinder pressures, comparison between the hybrid model and the time-domain 1D model; (b) In-cylinder mass of gas, comparison between the hybrid model and the time-domain 1D model.



Fig. 6. (a) Suction pressure profiles, comparison between the hybrid model and the time-domain 1D model; (b) Discharge pressure profiles, comparison between the hybrid model and the time-domain 1D model.

In Figure 6, the results in terms of suction and discharge pressure profiles are shown. These pressure profiles directly depend on the compressor-pipelines coupling. The agreement between the above data highlights the good prediction capability of the hybrid model compared to the full time-domain model.

3.1. Sensitivity analysis

A further analysis of the compressor and the pipelines configuration has been done by varying both the volume dimensions and the duct lengths (Table 2). The aim of this sensitivity analysis was to investigate the prediction capability of the hybrid model.

For sake of simplicity, the work absorbed during the cycle was used as reference parameter for the comparison. The computed absorbed work and the work percentage differences of the two models are shown in Figure 7.

$$\Delta\% = \frac{W_{hyb} - W_{time}}{W_{hyb}} \tag{12}$$

In the most of configurations tested, the hybrid model shows a good agreement with the time domain model, with a percentage difference of work under the 4%. Moreover, the comparison among the configurations underlines the influence of mutual interaction of the reciprocating compressor with the pipeline systems.

Among the tested configurations, the hybrid model showed some convergence problems in the case with volume dimension of 0.3 dm³ and duct length of 0.5 m. A further analysis of the equivalent impedance of the pipelines highlighted that in this case the compressor excites the resonance frequencies of both suction and discharge pipelines. The excitation of pipeline resonant frequencies brings to high amplitude oscillations in pressure profiles. In Figure 8(a) the module of the suction pipeline impedance is shown. The second harmonic of the reciprocating compressor rotation speed (i.e. 48 Hz) overlaps the resonance frequency of the suction pipeline.

ID	Volume [dm ³]	Duct length [m]
01	0.2	1.0
02	0.3	1.0
03	0.5	1.0
04	1.0	1.0
05	1.5	1.0
06	0.3	0.2
07	0.3	0.5
08	0.3	0.8
09	0.3	1.0
10	0.3	1.5

The same impedance analysis was carried out for the configurations with a percentage difference in absorbed work higher than 4%. For such cases the resonances of the pipelines systems was close to the harmonics of the reciprocating compressor rotation speed. For example, in Figure 8(b) the impedance module of the suction pipeline with a volume dimension of 0.3 dm³ and a duct length of 1.5 m is show. The first resonance of the suction pipeline is close to the first harmonic of the reciprocating compressor rotation speed. The thermodynamic cycle convergence is achieved but there is a not-perfect matching of the hybrid model results with the time-domain model results in terms of pressure profiles and thermodynamic cycle parameters.

In all the other configurations, the resonance frequencies of suction and discharge pipelines are far from the harmonics of the compressor rotation speed. For these configurations, a good matching between the hybrid model and the time-domain model results is assessed.



Fig. 7. (a) Work absorbed per cycle for duct length 1 m and volume dimension variation; (b) Work absorbed per cycle for volume dimension 0.3 dm³ and duct length variation.



Fig. 8. (a) Impedance module of suction pipeline of the configuration with volume dimension 0.3 dm³ and duct length 0.5 m. On the abscissa the compressor rotating speed harmonics are reported; (b) Impedance module of suction pipeline of the configuration with volume dimension 0.3 dm³ and duct length 1.5 m. On the abscissa the compressor rotating speed harmonics are reported.

4. Conclusions

In the present study, the authors developed a numerical model for the analysis of the reciprocating compressor coupled with its pipelines. The main feature of the model is to couple the time-domain computation of the thermodynamic cycle of the reciprocating compressor with the frequency domain modelling of the pipeline systems. The advantage of this multi-domain interaction is in the possibility of modelling the compressor with a quasi-steady time-domain approach and the pipelines with an acoustic approach. The former gives a good prediction of the compressor performance with low computational resources, while the latter allows the modeling of complex pipelines configurations with linear equation systems (i.e. transfer matrix method). This results in a fast and reliable computational tool for the thermodynamic and acoustic analysis of reciprocating compressor plants considering the mutual interaction between the compressor and its pipelines.

The hybrid model was tested with a simple compressor configuration. Results are compared with a full time-domain 1D commercial code as reference. For the case tested, the hybrid model shows a good agreement with the reference 1D model in terms of thermodynamic parameters (e.g. pressures, mass flow, etc.).

A further analysis of the hybrid model for different pipelines configurations underlined a high sensitivity of this modelling to the pipeline resonance frequencies. The impedance analysis of these pipeline configurations confirmed

the presence of resonance frequencies in correspondence of the compressor rotation speed harmonics. For some of them the hybrid model could not reach the convergence due to the resonant response of the piping systems.

In the future developments, the above-mentioned issue will be faced improving the acoustic modelling of the pipelines with the aim to compute the physical response of the compressor plant even in presence of singularities (i.e. resonances). Moreover, a 3D complex configuration of pipelines will be tested in order to determine whether the hybrid-model allows overcoming the typical time-domain 1D model limits.

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