# Implication of the Proton-Deuteron Radiative Capture for Big Bang Nucleosynthesis

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The astrophysical $S$ factor for the radiative capture $d(p, \gamma)^3\text{He}$ in the energy range of interest for big bang nucleosynthesis (BBN) is calculated using an *ab initio* approach. The nuclear Hamiltonian retains both two- and three-nucleon interactions—the Argonne $v_{18}$ and the Urbana IX, respectively. Both one- and many-body contributions to the nuclear current operator are included. The former retain for the first time, besides the $1/m$ leading order contribution ($m$ is the nucleon mass), also the next-to-leading order term, proportional to $1/m^3$. The many-body currents are constructed in order to satisfy the current conservation relation with the adopted Hamiltonian model. The hyperspherical harmonics technique is applied to solve the $A = 3$ bound and scattering states. Particular attention is paid in this second case in order to obtain, in the energy range of BBN, an uncertainty on the astrophysical $S$ factor of the order or below ~1%. Then, in this energy range, the $S$ factor is found to be ~10% larger than the currently adopted values. Part of this increase (1%–3%) is due to the $1/m^3$ one-body operator, while the remaining is due to the new more accurate scattering wave functions. We have studied the implication of this new determination for the $d(p, \gamma)^3\text{He}$ $S$ factor on the deuterium primordial abundance. We find that the predicted theoretical value for $^2\text{H}/\text{H}$ is in excellent agreement with its experimental determination, using the most recent determination of the baryon density of the Planck experiment, and with a standard number of relativistic degrees of freedom $N_{\text{eff}} = 3.046$ during primordial nucleosynthesis. This calls for a more accurate measurement of the astrophysical $S$ factor in order to confirm the present predictions.

*Introduction.*—The radiative capture $d(p, \gamma)^3\text{He}$ is a relevant process in many astrophysical environments. For instance, it is the second step in the chain of nuclear reactions which, starting from the proton-proton weak capture, allows stars like our Sun to shine via $pp$ chain. Interest in this reaction is also present in the context of big bang nucleosynthesis (BBN), see, e.g., Ref. [1] and references therein, since it is one of the main processes through which deuterium can be destroyed and thus affects its eventual yield. BBN is a powerful method to test the validity of the cosmological model at the MeV energy scale. For a given set of values of leading nuclear reaction rates, primordial element abundances depend upon two key cosmological parameters, the energy density in baryons, $\Omega_b h^2$, and the energy density of relativistic species $\rho_{\text{rel}}$ or, equivalently, the effective neutrino number, $N_{\text{eff}}$, defined as $\rho_{\text{rel}} = \rho_\gamma (1 + \frac{7}{8}(\frac{4}{11})^{4/3} N_{\text{eff}})$ where $\rho_\gamma$ is the cosmic microwave background (CMB) photon energy density. The benchmark value for this parameter with only three active neutrinos contributing is $N_{\text{eff}} = 3.046$ [2].

The baryon energy density has been measured with a remarkable precision by CMB anisotropy experiments, so in the minimal $\Lambda$CDM scenario light nuclei abundances are basically fixed [3]. Using the most recent determination of the Planck satellite experiment, $\Omega_b h^2 = 0.02225 \pm 0.00016$ [3], and a modified version of the public BBN

code PArthENoPE [4], already used in Ref. [5], the primordial deuterium to hydrogen ratio is predicted to be $^2\text{H}/\text{H} = (2.61 \pm 0.07) \times 10^{-5}$ (68% C.L. error). (In the public version of PArthENoPE the uncertainty on the $d(p, \gamma)^3\text{He}$ rate is taken from the analysis of Ref. [6] and is almost a factor of 2 smaller than what we adopt in the present study where, as in Ref. [5], the more conservative 1-$\sigma$ error estimate of Ref. [7] is used. This is the reason for the larger uncertainty quoted on $^2\text{H}/\text{H}$. Central values for $^2\text{H}/\text{H}$ using the rate of Ref. [6], used in the public version of PArthENoPE, and the best fit of Ref. [7] agree at the second decimal digit). Notice that this determination is lower than before, due to the order 1% increase of $\Omega_b h^2$ with respect to the Planck 2013 results. For a given baryon density value the theoretical uncertainty quoted above is dominated (more than 90% of the nuclear rate error contribution summed in quadrature) by the present error on experimental determination of the $d(p, \gamma)^3\text{He}$ cross section; see Table I of Ref. [5].

This result is in agreement at 1-$\sigma$ with the recent determination of Ref. [8] where, through a new analysis of all known deuterium absorption-line systems, it was found $^2\text{H}/\text{H} = (2.53 \pm 0.04) \times 10^{-5}$ at 68% of C.L., but it is slightly larger. A possible way to get an even better agreement between the two values is to slightly decrease the effective neutrino number down to $N_{\text{eff}} \sim 2.84$. Another

way is to increase the value of the $d(p,\gamma)^3$He astrophysical $S$ factor. This possibility was first explored in Ref. [9] and then analyzed in detail in Ref. [5], using the Planck 2013 data release, and in Ref. [3]. The conclusion of these studies is that increasing the $d(p,\gamma)^3$He thermal rate in the BBN temperature range by a factor of order 10% leads to a very good agreement between CMB anisotropy results and primordial deuterium abundance. Thus, a better determination of this $S$ factor with a reduction of the corresponding uncertainty in the BBN energy range, $E \simeq 30$–$300$ keV, would be very important.

The astrophysical $S$ factor at low energy, around the solar Gamow peak $E_G \simeq 9$ keV, is well known, thanks to the results of the LUNA experiment [10]. However, for the BBN relevant energy range, the experimental situation is rather unclear, since the only available experimental data [11] are quite in disagreement with the polynomial best fit of $S(E)$ for $E \simeq 0$–$2$ MeV [7]. This gives rise to an uncertainty on the cross section at the level of 6%–10%. This is the main motivation of the experiment recently proposed by the LUNA Collaboration, with the goal of measuring the $d(p,\gamma)^3$He astrophysical $S$ factor in the BBN energy range with a 3% accuracy. A feasibility test has already been performed [12].

On the other hand, the $d(p,\gamma)^3$He astrophysical $S$ factor can be calculated using a microscopic *ab initio* approach. In fact, in Refs. [13,14] (see also references therein) the hyperspherical harmonics (HH) technique was used to solve for the $A = 3$ nuclear wave functions (WFs) using a realistic description of the nuclear interaction, which includes both two- and three-nucleon interactions. These are constructed to reproduce the $A = 2$ large body of experimental data with a $\chi^2$/datum $\sim 1$ (the Argonne $v_{18}$—AV18—model [15]), and the $A = 3$ binding energies (the Urbana IX—UIX—model [16]). Note that the HH method (see Ref. [17] for a review) is the only available one able to calculate the nuclear WF for the initial $p$-$d$ scattering state at low energies, as the ones of interest here, including the Coulomb interaction between the charged initial particles. In the latest study of Ref. [14], a realistic model for the nuclear current operator was used, retaining both one- and many-body contributions. The latter are necessary in order to maintain gauge invariance in the presence of a system of interacting particles, and in Ref. [14] all the effort was put to construct these contributions which exactly satisfy the current conservation relation in conjunction with the AV18/UIX potential. The former, instead, was simply obtained performing a $1/m$ expansion ($m$ is the nucleon mass) of the single-nucleon covariant operator, and retaining the leading order contribution. The many-body contributions were found already in Ref. [13] essential to reach an excellent agreement with the LUNA data [10] around the solar Gamow peak [7,14]. In the energy range of interest for BBN, on the other hand, the theoretical predictions of Ref. [14] were

found to be 2%–10% higher than the central value for the polynomial fit of Ref. [7]. In the present Letter, our starting point is the work of Ref. [14], which, although very accurate, should be considered incomplete for two reasons: (i) no estimate of the theoretical uncertainty was given, in particular, that one arising from the solution of the $p$-$d$ scattering problem with the HH method; (ii) the one-body terms beyond the leading order operator, of the order $1/m^3$, were found a few years later [18], essential in order to get a reasonable agreement between theory and experiment for a related process, the $d(n,\gamma)^3$H radiative capture. Given the similarities between the $p$-$d$ and $n$-$d$ radiative captures, it is to be expected that these $1/m^3$ one-body contributions might be important also for the process here under consideration. The goal of the present Letter is to address the two above-mentioned issues and to verify whether the new prediction for the $d(p,\gamma)^3$He astrophysical $S$ factor goes in the direction of improving the consistency of theoretical BBN deuterium abundance prediction, the new Planck results, and the experimental data of Ref. [8]. We do not consider here $^4$He primordial mass fraction $Y_p$, since it is insensitive to this reaction rate. For example, a change of the $d(p,\gamma)^3$He $S$ factor by a factor of 2 affects $Y_p$ at the level of 0.04%, too small to be appreciated with the present statistical and systematic uncertainties on its experimental determination; see, e.g., Ref. [19].

*The present calculation.*—We discuss here the two significant improvements in the calculation with respect to Ref. [14]. First of all, in the present work, we pay particular attention to the numerical determination of the $p$-$d$ WF. In particular, in each $L$, $S$, $J$ channel ($L$ is the $p$-$d$ orbital angular momentum, $S = 1/2$, $3/2$ is the $p$-$d$ total spin, and they are coupled so that $\mathbf{J} = \mathbf{L} + \mathbf{S}$) the WF $\Psi^{LSJ}$ has been tested calculating $\langle H \rangle \equiv \langle \Psi^{LSJ} | H | \Psi^{LSJ} \rangle$ in a box with a radius of 70 fm, using a Monte Carlo method (independently on our technique to determine $\Psi$) and verifying that the correct result $\langle H \rangle = E - B_d$ is obtained within the requested accuracy (here $E$ is the $p$-$d$ center-of-mass energy and $B_d$ the deuteron binding energy). Different grid points, dimensionality of the HH expansion basis, and values for the nonlinear parameter entering the polynomial expansion of the hyperradial functions (see Ref. [17]) have been checked in order to verify the above relation within 0.1%. This procedure was not used in Ref. [14], and therefore the pattern of convergence for the WF (not the scattering matrix, i.e., the $p$-$d$ phase shifts) was not verified. With this procedure we have been able to reduce the numerical uncertainty relative to the WFs in our astrophysical $S$ factor estimates of better than 1% for the whole energy range here under consideration (see Table I). Recall that the nuclear Hamiltonian used in the present study is the same as that of Ref. [14]; i.e., it retains the AV18/UIX potential model, which allows us to nicely reproduce, using the HH method, the $^3$He binding energy and many $A = 3$ scattering observables. In particular, in

TABLE I. The *p*-*d* astrophysical *S* factor (in eV b) for a representative set of energy values *E* (in keV). The theoretical percent uncertainty arising from the solution of the *p*-*d* scattering problem with the HH method is given in the second column ($\Delta S_{WF}$), while the additional contribution due to the one-body term of Eq. (1) ($\Delta \mathbf{j}^{(RC)}$) is given in the last column, also in percent. Note that for $E = 2$ MeV (last row), the value for $\Delta S_{WF}$ is below the permil level, and therefore not quoted.

| $E$ [keV] | $S(E)$ [eV b] | $\Delta S_{WF}$ [%] | $\Delta \mathbf{j}^{(RC)}$ [%] |
|---|---|---|---|
| 10 | 0.286 | 0.1 | +0.8 |
| 20 | 0.355 | 1.0 | +1.1 |
| 35 | 0.460 | 1.1 | +1.3 |
| 50 | 0.570 | 0.9 | +1.7 |
| 70 | 0.716 | 0.4 | +2.1 |
| 95 | 0.912 | 0.3 | +2.3 |
| 120 | 1.112 | 0.8 | +2.4 |
| 145 | 1.317 | 0.4 | +2.5 |
| 170 | 1.529 | 0.4 | +2.6 |
| 195 | 1.748 | 0.4 | +2.6 |
| 220 | 1.968 | 0.5 | +2.8 |
| 245 | 2.197 | 0.4 | +2.7 |
| 260 | 2.343 | 0.9 | +2.8 |
| 300 | 2.716 | 0.5 | +2.7 |
| 400 | 3.676 | 0.6 | +2.7 |
| 500 | 4.739 | 0.2 | +2.7 |
| 750 | 7.539 | 0.3 | +2.6 |
| 1000 | 10.685 | 0.4 | +2.7 |
| 2000 | 25.908 | ⋯ | +2.3 |

*p*-*d* elastic scattering, the total cross section available data are nicely reproduced. Some discrepancies appear only for few delicate polarization observables (see Fig. 16 of Ref. [20]). Furthermore, all partial waves with $J \leq 5/2$ and both parities have been retained.

The new term of the nuclear electromagnetic (EM) current here included is that arising from the $1/m$ expansion of the single-nucleon covariant current operator, and is a relativistic correction of the order $1/m^3$. It has been derived in Refs. [21,22] in the context of chiral effective field theory, and can be written as [21]

$$\mathbf{j}_i^{RC} = -\frac{ee_i}{8m^3}[2(K_i^2 + q^2/4)(2\mathbf{K}_i + i\boldsymbol{\sigma}_i \times \mathbf{q})$$
$$+ \mathbf{K}_i \cdot \mathbf{q}(\mathbf{q} + 2i\boldsymbol{\sigma}_i \times \mathbf{K}_i)] - \frac{ie\kappa_i}{8m^3}[\mathbf{K}_i \cdot \mathbf{q}(4\boldsymbol{\sigma}_i \times \mathbf{K}_i - i\mathbf{q})$$
$$- (2i\mathbf{K_i} - \boldsymbol{\sigma}_i \times \mathbf{q})q^2/2 + 2(\mathbf{K_i} \times \mathbf{q})\boldsymbol{\sigma}_i \times \mathbf{K}_i], \quad (1)$$

where $\mathbf{K}_i = (\mathbf{p}_i' + \mathbf{p}_i)/2$, $\mathbf{p}_i$ and $\mathbf{p}_i'$ being the initial and final momenta of the nucleon, $\mathbf{q}$ is the photon momentum, $e$ is the electron charge, $e_i = (1 + \tau_{i,z})/2$, the charge-projection isospin operator, $\kappa_i = (\kappa_S - \kappa_V \tau_{i,z})/2$, $\kappa_S = -0.12\mu_N$ ($\kappa_V = 3.706\mu_N$) being the isoscalar (isovector) combination of the anomalous magnetic moments of proton and neutron, and $\boldsymbol{\sigma}_i$ ($\boldsymbol{\tau}_i$) are the spin (isospin) Pauli matrices. It was found in Ref. [18] that $\mathbf{j}_i^{RC}$ reduces the

*n*-*d* total cross section at thermal energies of about 4%–5%, bringing the theoretical prediction in a much better agreement with the experimental datum (within 4%). In the *p*-*d* case, instead, we have found that the operator $\mathbf{j}_i^{RC}$ gives a positive contribution, increasing the astrophysical *S* factor of 1%–3% over the whole energy range considered here (see Table I).

The astrophysical *S* factor obtained in the present work is listed in Table I and plotted in Fig. 1, where it is compared with the previous calculation of Ref. [14], with the existing data of Refs. [10,11,23,24], and with the polynomial fit of Ref. [7]. To be noticed that the theoretical uncertainty arising from the solution of the *p*-*d* scattering problem with the HH method are not visible on the plot, although the corresponding symbols retain an error. The present results are systematically larger (about 8%–10%) than those of Ref. [14]. We have investigated the origin of such an increase, and we have found that only 1%–3%, depending on the energy value, is due to the one-body $1/m^3$ contribution. Therefore, the remaining 5%–8% is due to the new solutions of the $A = 3$ (scattering) problem. In fact,
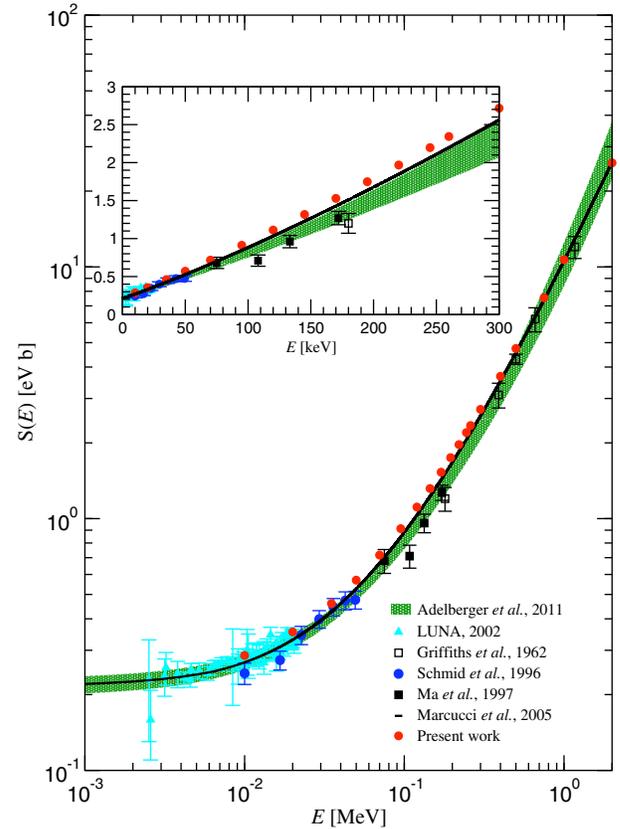


FIG. 1. The astrophysical *S* factor obtained in the present work (red points) is plotted together with the available experimental data of Refs. [10,11,23,24], the calculation of Ref. [14] (solid black line), and the quadratic best fit to the data of Ref. [7] (green band). The inset shows the astrophysical *S* factor in the 0–300 keV energy range, since the relevant BBN energy range is 30–300 keV.

the present WFs have been obtained with the same HH technique as in Ref. [14], but with the goal of reaching the required higher accuracy dictated by BBN, and thus they have been tested one by one, as explained above.

*Implications for BBN.*—To study the effect of the new *ab initio* determination of the $d(p,\gamma)^3$He $S$ factor on primordial deuterium produced during BBN we have computed the corresponding thermal rate using the best fit values reported in the second column of Table I and modified the numerical code PARtHENoPE [4] accordingly. The new rate in the BBN temperature range (0.01–0.1 MeV) is larger by a factor 1.12–1.15 with respect to what is found using the $S$ factor of Ref. [7]. The theoretical results for deuterium to hydrogen density ratio $^2$H/H$_{th}$ are then computed as a function of two parameters, the baryon density $\Omega_b h^2$ and $N_{eff}$, and compared with the experimental determination $^2$H/H$_{exp}$ of Ref. [8]. To obtain the best fit values and uncertainty on these parameters we then consider the likelihood function

$$\mathcal{L}(\Omega_b h^2, N_{eff}) = \mathcal{L}_{Planck}(\Omega_b h^2)$$
$$\times \exp\left(-\frac{(^2\text{H/H}_{th}(\Omega_b h^2, N_{eff}) - {}^2\text{H/H}_{exp})^2}{2(\sigma_{exp}^2 + \sigma_{th}^2)}\right),$$
$$(2)$$

where $\sigma_{exp} = 0.04$ as obtained in Ref. [8]. $\mathcal{L}_{Planck}(\Omega_b h^2)$ is a Gaussian prior corresponding to the Planck result of Ref. [3], while $\sigma_{th}$ is the propagated error on deuterium yield due to the present experimental uncertainty on other leading nuclear reactions relevant for deuterium production and destruction during BBN, namely, $d(d,n)^3$He and $d(d,p)^3$H. It also accounts for the $d(p,\gamma)^3$He $S$ factor theoretical uncertainty of Table I, and the small error on the $p(n,\gamma)d$ rate, both very subdominant with respect to the other error sources.

For the Planck 2015 value of $\Omega_b h^2$ and standard $N_{eff}$ we get $^2$H/H$_{th} = (2.49 \pm 0.03 \pm 0.03) \times 10^{-5}$, where the two errors are due to nuclear rate and $\Omega_b h^2$ uncertainties, respectively. In Fig. 2 we show the likelihood contours (68, 95, and 99% C.L.) in the $\Omega_b h^2 - N_{eff}$ plane. The Planck 2015 best fit results and 68% C.L. error bars are also shown. The agreement is very good, within 1-$\sigma$. Marginalizing over baryon energy density, we find $N_{eff} = 3.18 \pm 0.16(0.32)$, where the error is at 68% (95%) C.L., which is fully consistent with a standard radiation content during BBN. Notice that, once the Planck 2015 prior on $\Omega_b h^2$ is used, the uncertainty one gets on $N_{eff}$ from $^2$H/H alone is of the same order of magnitude obtained from CMB, once baryon acoustic oscillation data are also exploited. In this case, the Planck result is $N_{eff} = 3.04 \pm 0.18$ (68% C.L.) [3].

*Summary and outlook.*—The astrophysical $S$ factor for the $d(p,\gamma)^3$He reaction is crucial to determine the consistency of the BBN theoretical prediction for deuterium
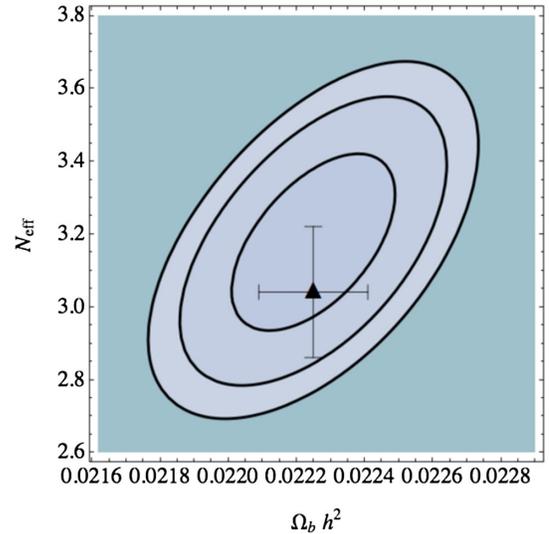


FIG. 2.    The likelihood contours (68, 95, and 99% C.L.) in the $\Omega_b h^2 - N_{eff}$ plane from $^2$H/H, with the Planck 2015 prior on $\Omega_b h^2$, a free $N_{eff}$, and using the experimental result of Ref. [8]. The triangle is the best fit value of Planck 2015 results for these parameters, with corresponding 68% C.L. error bars [3].

abundance, the new Planck results, and the most recent experimental determination of such abundance [8]. In the absence of an accurate experimental determination in the energy range of interest for BBN, 30–300 keV, we have performed a new theoretical *ab initio* calculation, using the most up-to-date techniques to calculate the WFs for the initial scattering and final bound states, with the realistic AV18/UIX potential model, as well as the most up-to-date realistic model for the nuclear current operator, which satisfies gauge invariance with the adopted Hamiltonian and retains the $1/m^3$ contribution in the one-body operator. We have found that the numerical uncertainty relative to the WF in the $S$ factor is lower than 1%, while the $1/m^3$ one-body contribution increases the $S$ factor by 1%–3% over the whole energy range. We have then investigated the effect of this new *ab initio* determination on the primordial deuterium abundance. We find that BBN predictions are in very good agreement with the Planck 2015 results and the experimental result of Ref. [8]. Also, the inferred value for $N_{eff}$ is fully consistent with a standard radiation content during BBN. Of course, our results ought to be confirmed by direct measurement of the $d(p,\gamma)^3$He $S$ factor, as it is planned at the Gran Sasso National Laboratories (Italy), by the LUNA Collaboration. Such a measurement will therefore turn out to be crucial in this context. Finally, it should be mentioned that a theoretical *ab initio* calculation of this process is at reach also within the so-called chiral effective field theory ($\chi$EFT) framework, which provides a direct connection between quantum chromodynamics and the strong and electroweak interactions in nuclei, and is a practical calculational scheme, which can be improved systematically. The first studies along this line have been

done for the EM structure of light nuclei [25,26]. At present, however, the consistency between the $\chi$EFT nuclear potentials and EM currents necessary to satisfy *exactly* gauge invariance (as in the case of the calculation presented here) has not been yet achieved, making the $\chi$EFT results not completely reliable at the accuracy level necessary for BBN predictions. Work on this issue, though, is currently underway.

*Note added in proof.*—After completing this work a reevaluation of the $d(p,\gamma)^3$He, $d(d,n)^3$He, and $d(d,p)^3$H reaction rates has been also presented in Ref. [27].

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