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Design of Piezo-Actuated MEMS Flexural Phononic Crystals for Mass Sensing Applications

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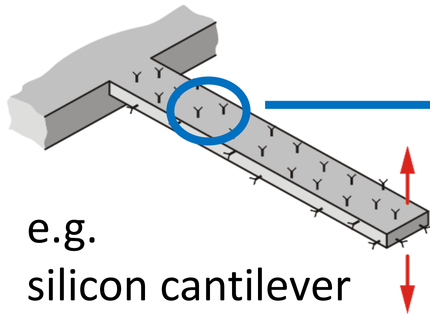
Outline

- MEMS resonant mass sensors
  - working principle
  - design approach
- Flexural phononic crystals as mass sensors
  - proposed sensing mechanism
  - proposed structure
  - design parameters, strategy
  - modeling
  - results
- Conclusions and developments

MEMS Resonant Mass Sensors

mechanical resonator

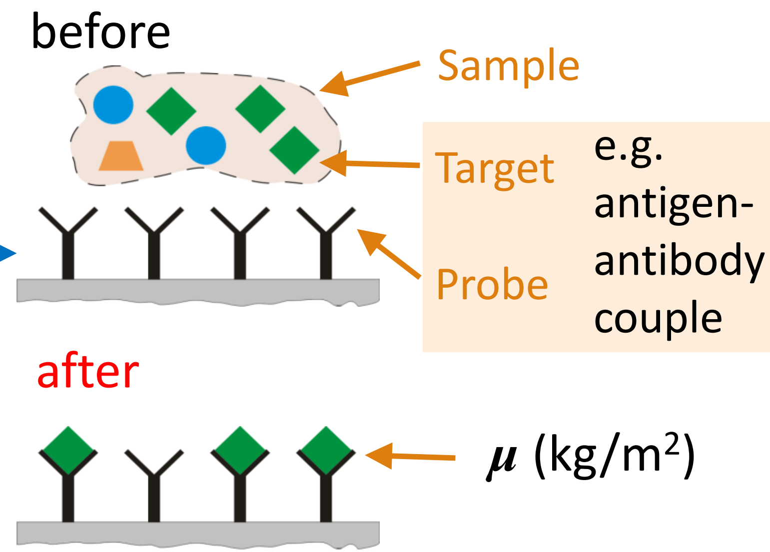
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



e.g.
silicon cantilever

equivalent lumped parameters *k*, *m*

functionalized surface

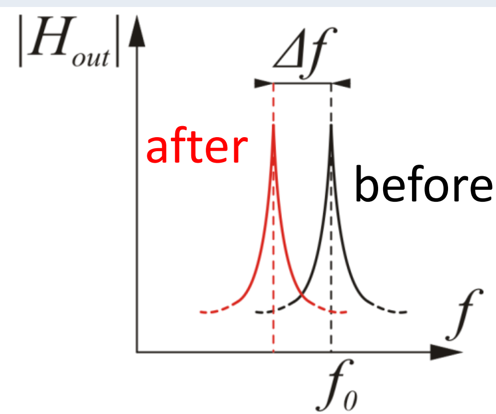


working principle

Added mass detected by measuring Δf

$$\frac{\Delta f}{f_0} = -\frac{1}{2}\frac{A}{M}\mu$$

Sensitivity (*S*)



Design approach for high sensitivity

"Standard" approach

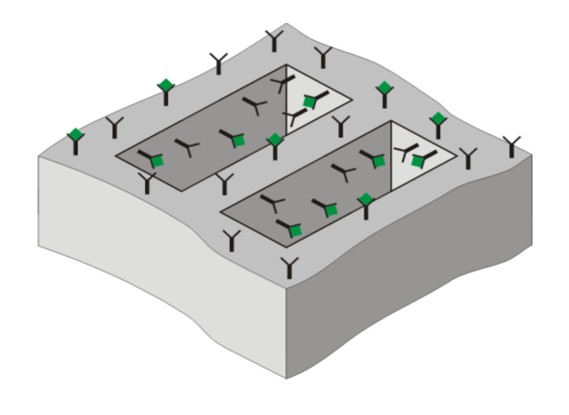
- resonator scaling (nanoresonators)
- Sub-micrometer thickness
- Lightweight structural material

$$S = \frac{1}{2}\frac{A}{M}$$

This work

thick structures with periodic perforations (which are often required anyway)

- decrease mass
- increase active surface



Phononic bandgaps can be introduced

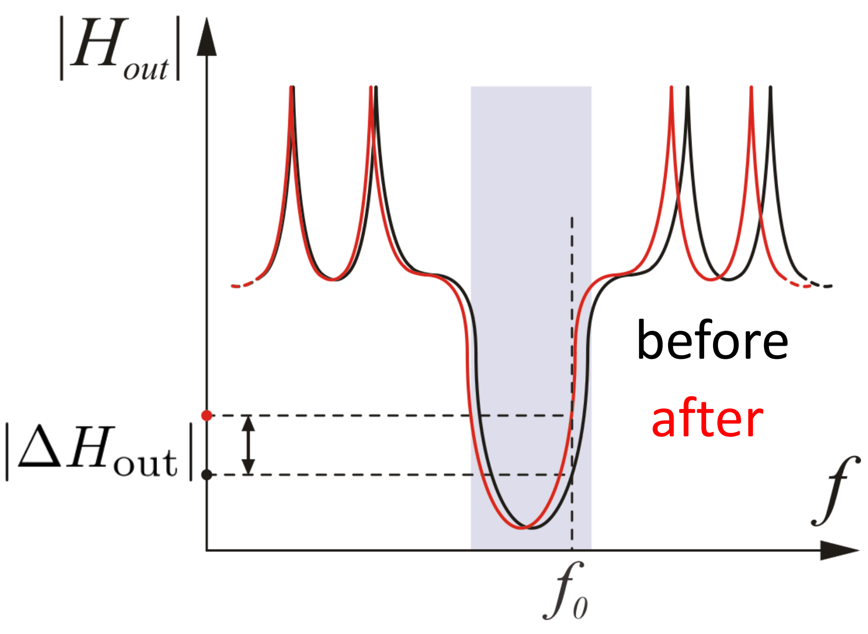
- The perforated resonator is also a phononic crystal
- Explore the possibility of exploiting the band gaps in the sensing process

Proposed sensing mechanism

The output spectrum scales down as $\Delta f = f_0 S \mu$ upon mass loading:

$$|H_{\text{out}}(f_0, \mu)| \approx |H_{\text{out}}(f_0(1 + S \cdot \mu), 0)|$$

The shift is detected by exciting at constant frequency f_0 on the edge of the band gap before and after analyte interaction



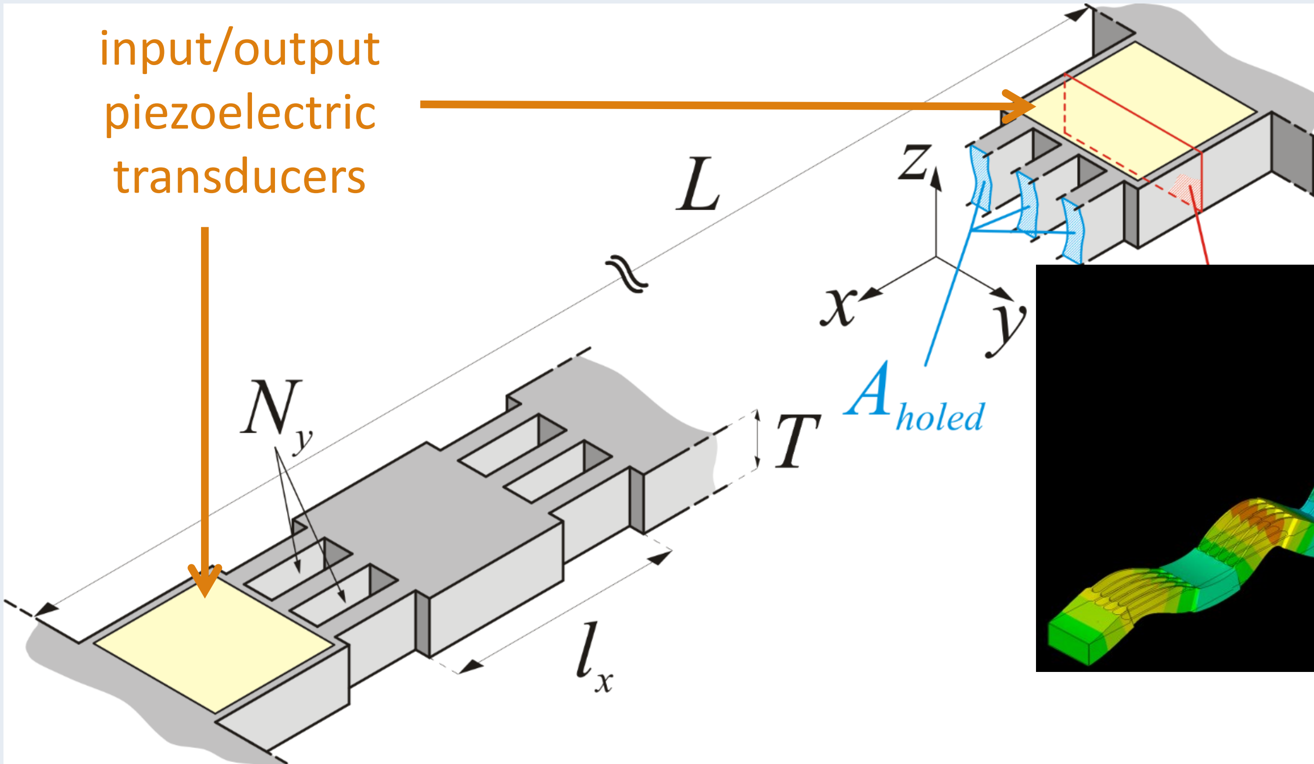
$$|\Delta H_{\text{out}}| \approx f_0 S \mu \cdot \frac{\partial |H_{\text{out}}|}{\partial f}$$

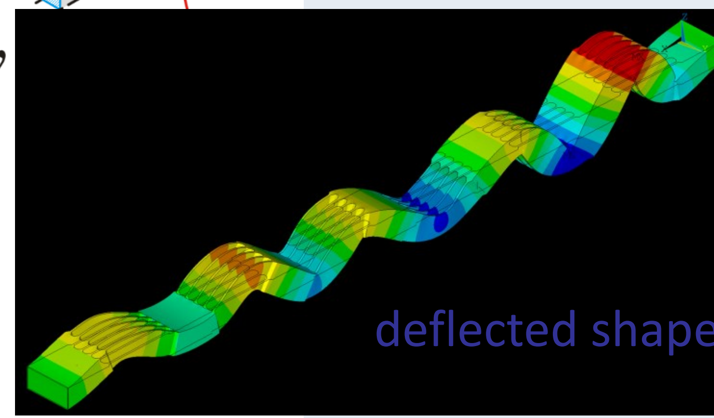
Spectrum shift at f_0

new aspect: band gap engineering

Proposed device

Clamped-clamped beam with periodic cross-section





Design parameters

L total length

T thickness

$\boldsymbol{l_x}$ spatial period

$\boldsymbol{N_y}$ holes along *y*

$\boldsymbol{\alpha_y}$ (A_{holed} / A_{full}) perforation ratio along *y*

Design strategy – one

Goal: maximize the output signal

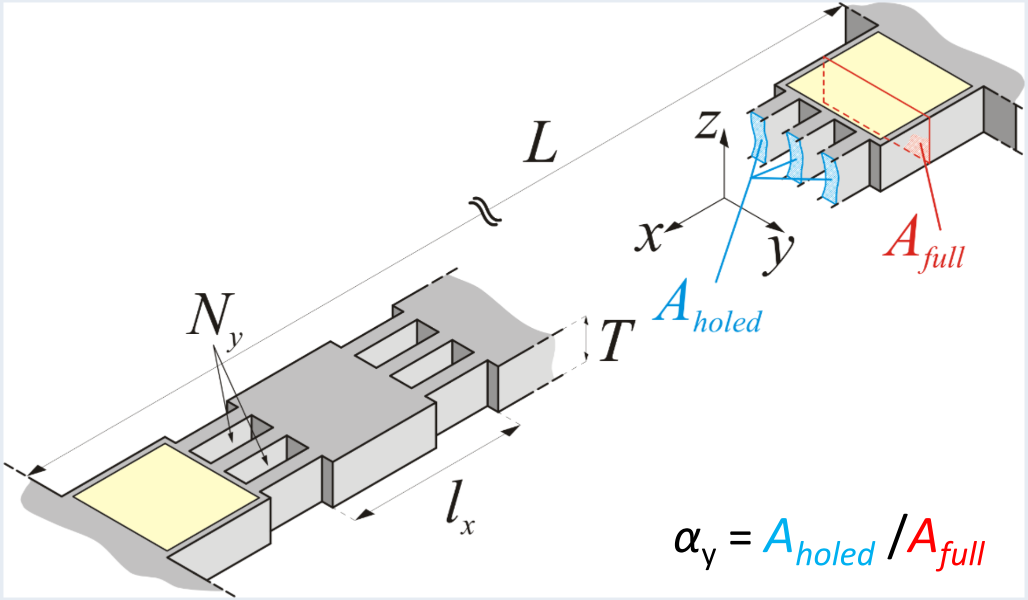
$$|\Delta H_{\mathrm{out}}| \approx f_0 S \mu \cdot \frac{\partial |H_{\mathrm{out}}|}{\partial f}$$

Spectrum shift

Dense and **large** holes
(i.e. small pitch) maximize *S*

How to do it:

- small α_y (larger holes along *y*)
- small l_x (many *x* repetitions)
- large N_y (many *y* repetitions)



Design strategy – two

Goal: maximize the output signal

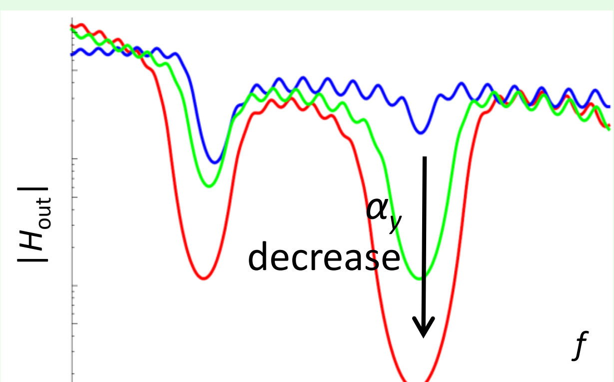
$$|\Delta H_{out}| \approx f_0 S \mu \cdot \boxed{\frac{\partial |H_{out}|}{\partial f}}$$

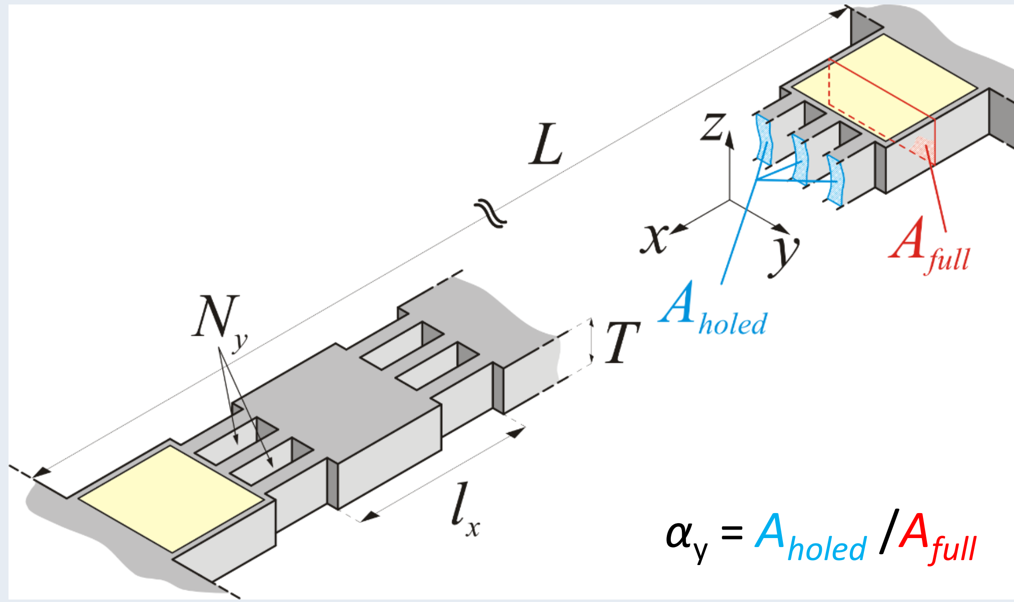
Band gap slope

Strong discontinuity increases wave reflection in the band gap

How to do it:

- small α_y (larger holes along y)





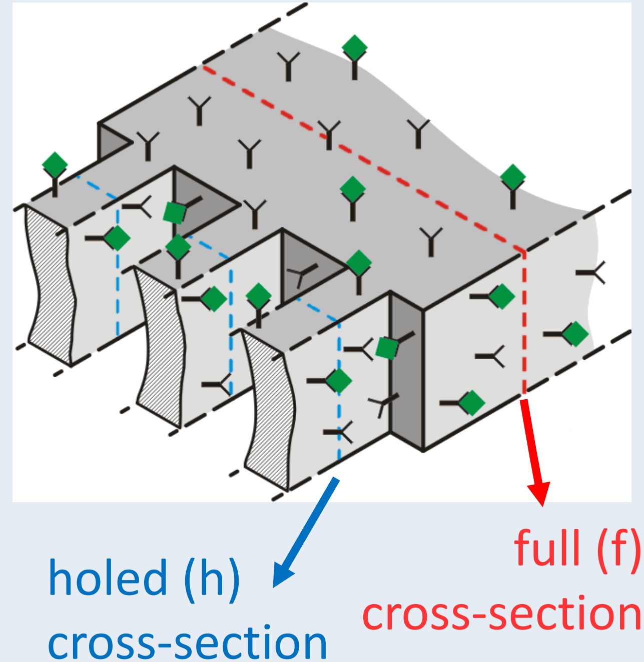
There is a synergy between the two goals

Large spectrum shift and steep band gap slope both require small α_y

Modelling – one

Full **Timoshenko beam model** used

analyte interaction (i.e. $\mu > 0$) increases both translational and rotational inertia



4 parameters describe the cross-sections:

	full (f)	holed (h)
flexural stiffness	EI	$\alpha_y EI$
shear stiffness	$k'AG$	$\alpha_y k'AG$
translational inertia	$\rho A + K_{\mathrm{tf}}\mu$	$\alpha_y \rho A + K_{\mathrm{th}}\mu$
rotational inertia	$\rho I + K_{\mathrm{rf}}\mu$	$\alpha_y \rho I + K_{\mathrm{rh}}\mu$

K_{tf} , K_{rf} , K_{th} , K_{rh} are obtained from straightforward geometric considerations

E Young's modulus
G shear modulus
ρ mass density
I section moment of inertia
A section area
k' shear coefficient

Modelling – two

A transmission matrix approach* models the propagation of **flexural waves**:

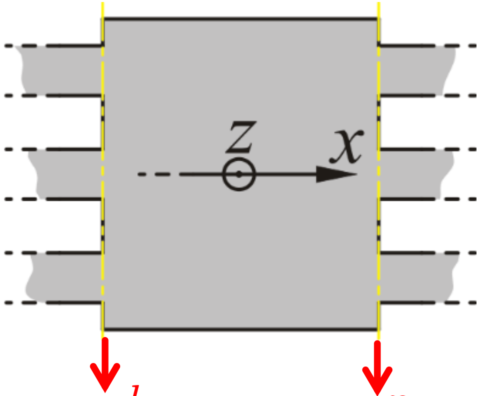
Timoshenko beam theory

- State 4-vector:

$$\mathbf{v}_\mathrm{F} = [U_z,\, \theta,\, M,\, T]^T$$

- Two 4x4 transmission matrices for full and holed segments:

$$\mathbf{M}_\mathrm{F}^h,\ \mathbf{M}_\mathrm{F}^f$$



$\mathbf{v}_\mathrm{F}^l$ $\mathbf{v}_\mathrm{F}^r$

at each segment: left (*l*) and right (*r*) state vectors

$$\mathbf{v}_\mathrm{F}^r = \mathbf{M}_\mathrm{F}^f \mathbf{v}_\mathrm{F}^l \qquad (\mathbf{v}_\mathrm{F}^r = \mathbf{M}_\mathrm{F}^h \mathbf{v}_\mathrm{F}^l)$$

Extensions:

- damping
- boundary conditions (clamped-clamped: $U_z(0) = U_z(L) = 0$, $\Theta(0) = \Theta(L) = 0$)
- piezoelectric actuation/sensing

*Adapted from L. Liu and M.I. Hussein, *J. Appl. Mech.* **79**, 011003 (2012)

Modelling - three

Piezoelectric model:

- **forcing:** shear force τ_{in} at the input piezo-PnC interface
- **sensing:** average longitudinal ε_x strain at the output piezo-PnC interface

$$H_{\text{out}} = \frac{\varepsilon_{\text{x}}}{\tau_{\text{in}}} \quad (\text{Pa}^{-1})$$

due to the nature of this excitation there is a **longitudinal load**

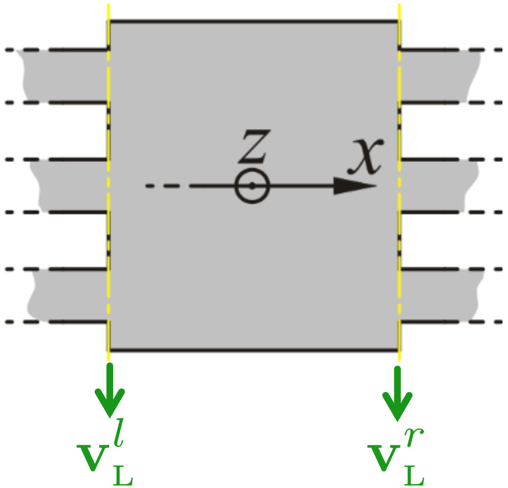
Longitudinal wave propagation

- State 2-vector:

$$\mathbf{v}_{\text{L}} = [U_x,\, F_x]^T$$

- Two 2x2 transmission matrices for full and holed segments:

$$\mathbf{M}_{\text{L}}^h,\ \mathbf{M}_{\text{L}}^f$$



$$\mathbf{v}_{\text{L}}^r = \mathbf{M}_{\text{L}}^f \mathbf{v}_{\text{L}}^l \qquad (\mathbf{v}_{\text{L}}^r = \mathbf{M}_{\text{L}}^h \mathbf{v}_{\text{L}}^l)$$

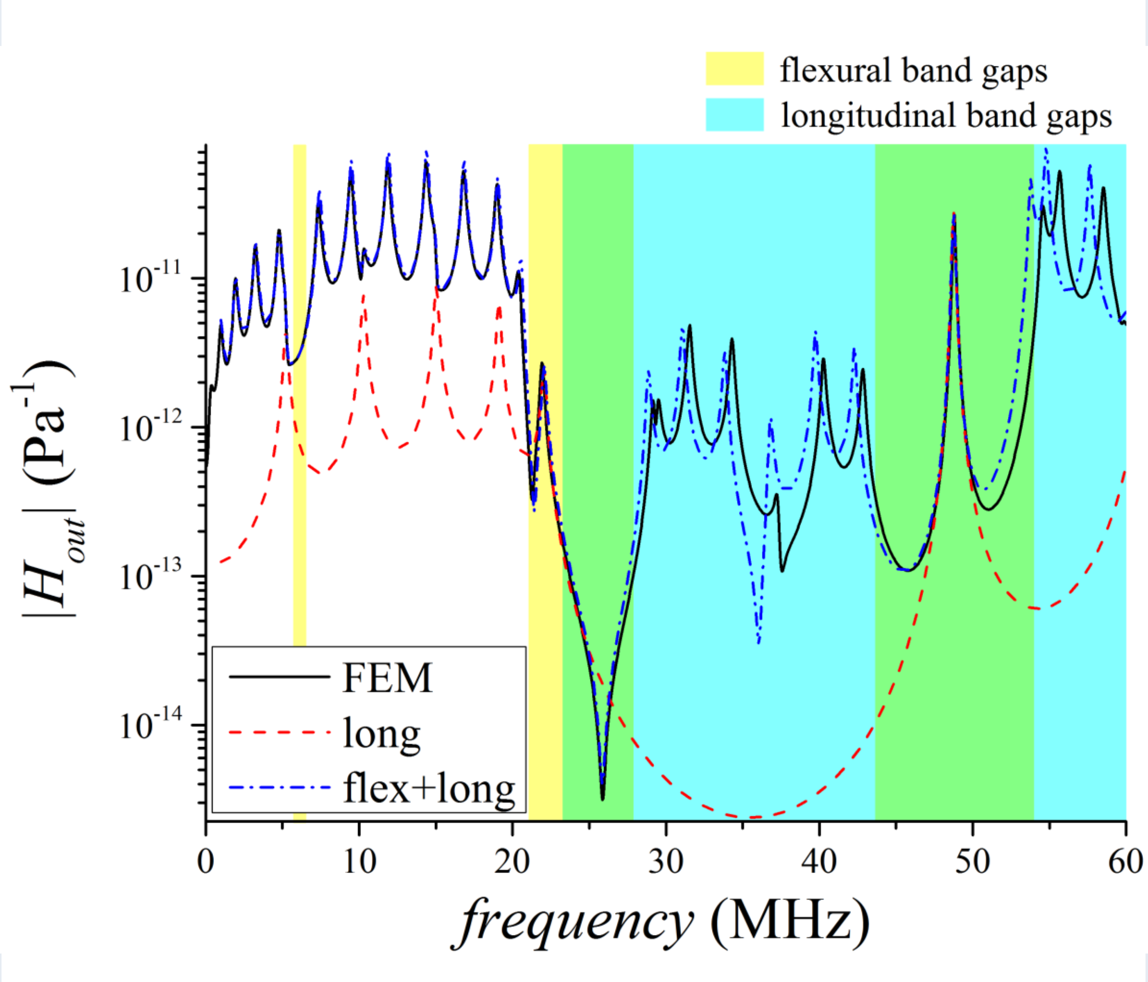
Results - one

Simulated geometry: L = 640 μm, l_x = 100 μm, T = 20 μm, N_y = 4, α_y = 0.2

Material:
monocrystalline silicon

Excellent agreement with ANSYS FEM simulations

Some features can only be caught if longitudinal effects are included



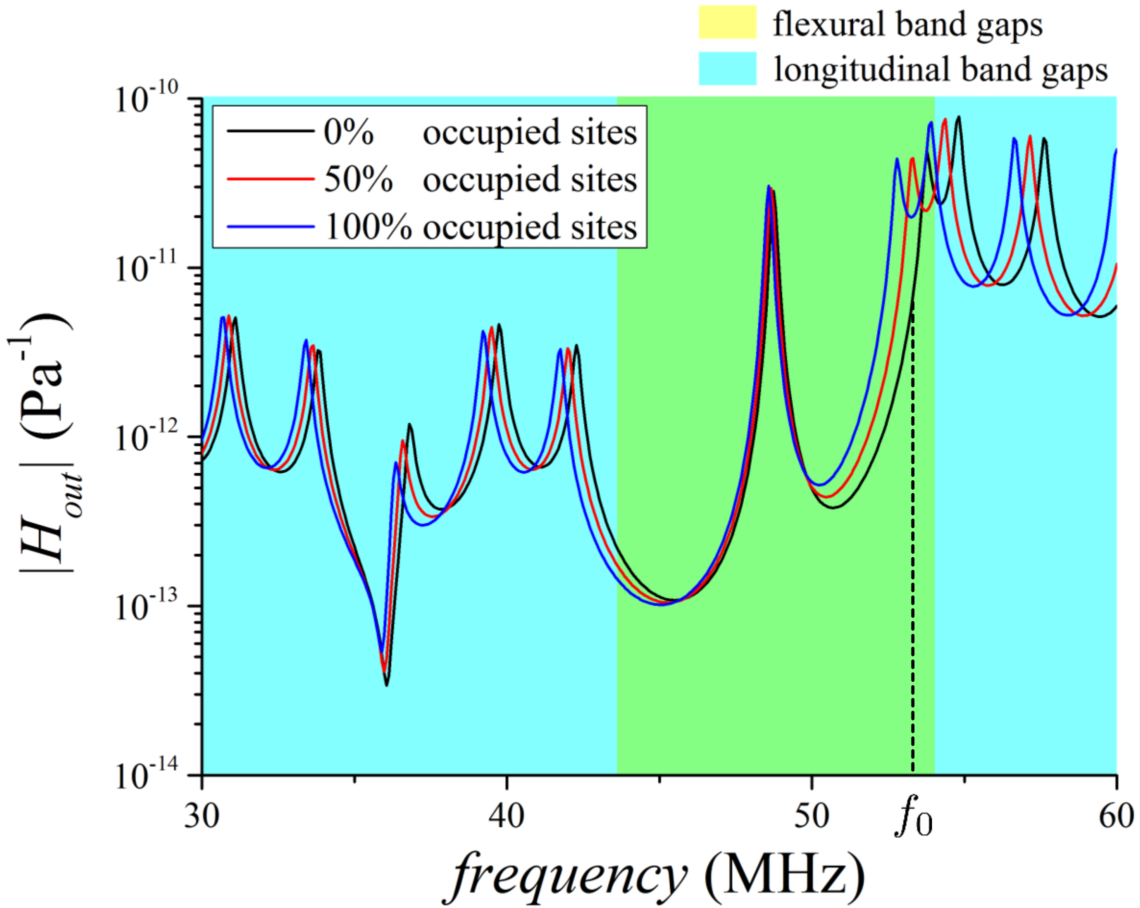
Results – two

Estimated analyte interaction (PSA prostate cancer marker)

PSA mass ≈ 33 kDa
PSA diameter ≈ 4 nm
10^4 binding sites/µm^2

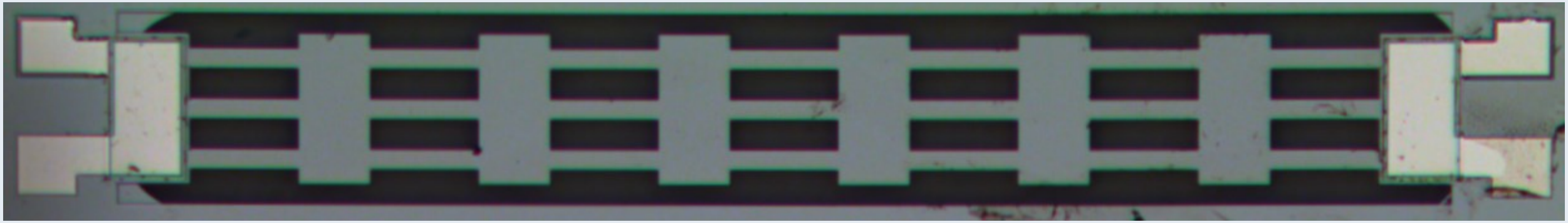
Test on a flexural bandgap border (f_0 = 53.5MHz)

$$\frac{\Delta H_{\mathrm{out}}}{H_{\mathrm{out}}} \approx 8.6\%$$



Effect of mass magnified (50x) for clarity

Conclusions and developments



- A new MEMS resonant mass sensor based on flexural PnCs has been proposed
- The device was modelled with a fast, transmission matrix-based approach in excellent agreement with FEM simulations
- Devices were fabricated in an Silicon-on-Insulator piezo-MEMS technology
- Developments:
  - Modelling of electrical transduction
  - Frequency characterization

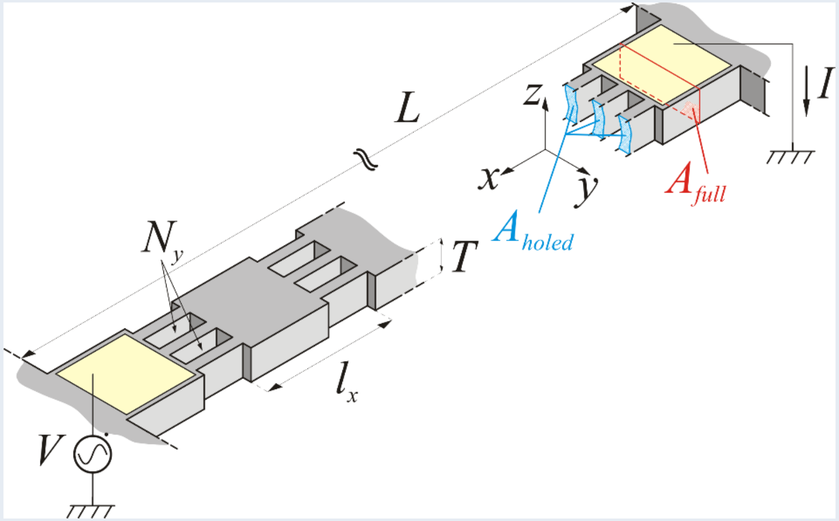
Thank you!

More on the piezo model

- The measurable frequency response is the transconductance *Y*:

$$Y \triangleq \frac{I}{V} \ (\Omega^{-1})$$

$$H_{out} \triangleq \frac{\bar{\varepsilon}_x}{\tau_{in}} (Pa^{-1})$$



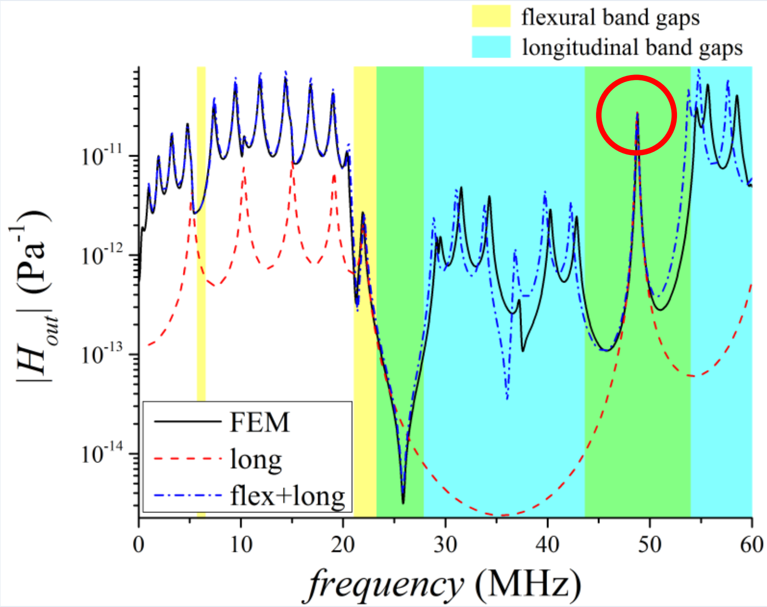
- given the piezo properties a simplified model can be built:

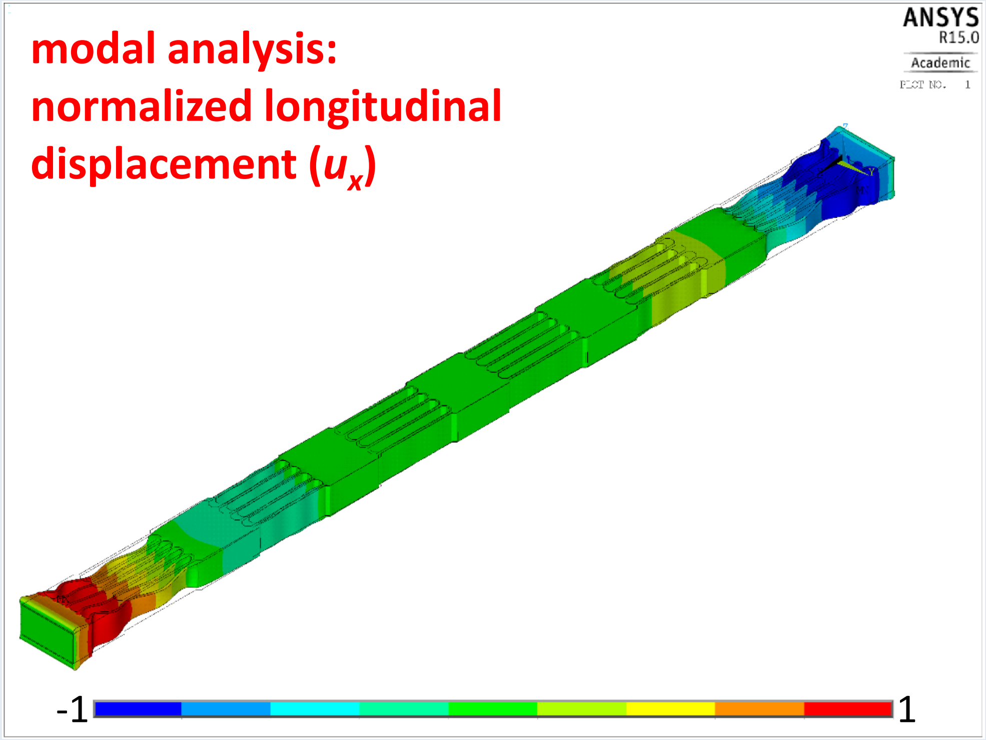
$$\tau_{in} = K_V V \qquad I = K_I \bar{\varepsilon}_x \qquad Y \approx H_{out} K_V K_I$$

- comparison with FEM with piezo-layer (1 µm thick PZT-5H):
  - good accordance on shape
  - understimates the amplitude (FEM */Y/* ≈ 60 µS @ f_0)
  - development of a more accurate model ongoing

Localized state

- the resonance peak at $f \approx 48.8$ MHz is a longitudinal peak located in the middle of longitudinal and flexural band gaps
- the simulated (FEM) mode shape shows that the energy is located at the anchors





Effects of Q

- high slopes are reached in the proximities of resonance peaks, but the frequency response in these regions is strongly dependent from the losses (*Q* factor)
- the spectrum inside the band gaps depends only on the geometrical properties