

1 **Extended EDC local extinction model accounting finite-rate chemistry**
2 **for MILD combustion**

3
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8 **Abstract** An extended Eddy Dissipation Concept (EDC) local extinction model is proposed to take into
9 account the effects of finite-rate chemistry, normally occurred in Moderate to Intense Low oxygen Dilution
10 (MILD) combustion, on the extinction limits. Local extinction is predicted when the local fine structure
11 residence time is below a local critical value that is determined theoretically in the present study. The
12 proposed model has been evaluated against experimental data reported for CH₄/H₂ jet-in-hot and diluted
13 coflow flames. Comparison with the standard EDC extinction model is also presented. Results show that
14 prediction of extinction threshold in MILD conditions is attainable only through the application of the
15 extended EDC extinction model on a well resolved turbulence-chemistry interaction field. The effect of
16 penetrating of surrounding air into the reaction zone and subsequent flame cooling at downstream is also
17 captured by the proposed extinction model. Despite its simplicity, the extended EDC extinction model
18 describes many features of localized extinction under the MILD combustion as well as conventional
19 combustion conditions.

20 **Keywords:** *Extinction model; flameless combustion; turbulent jets; diffusion flames;*
21 *finite-rate chemistry effects; numerical analysis*

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25 **1 Introduction**

26 A good understanding of the phenomena governing the extinction of turbulent
27 diffusion flames is essential because of their widespread appearance in practical
28 combustion applications. A large amount of experiments in jet diffusion flames using a
29 variety of laser techniques revealed that local extinction may occur in two physical and
30 chemical stages [1]. In a coflow jet diffusion flame studied by Takahashi et al. [2], these
31 stages were found to be related to the unsteady transport effects of external or internal
32 vortices on chemical kinetics. Detailed experiments of Rolon et al. [3] on a counter flow
33 diffusion flame showed that the strong vortices were responsible of flame extinction and
34 subsequent blowout while re-ignition occurred after interaction with weak vortices. Kim
35 et al. performed a comprehensive study on the effect of strain rate and conductive heat
36 loss on the premixed and partially premixed syngas-air flames extinction [4]. They
37 investigated various mechanisms responsible for flame extinction and showed that the
38 lower and upper extinction boundaries as well as reaction zone thickness can become
39 narrower with increasing strain rate. By the analysis of flame structure near extinction on
40 CH₄, H₂, H₂/Ar, and CO/H₂/N₂ jet diffusion flames, Masri et al. [5, 6] revealed that
41 extinction is only slightly affected by turbulence and is mainly controlled by the width of
42 the reaction zone. For instance, addition of methane to fuel promoted local extinction as it
43 reduces the width of reaction zone due to scavenging reactive radical species like H and
44 OH. Based on the comprehensive set of measurements of the local flame structure
45 provided by Masri et al. [5, 6], Koutmos developed a local critical Damkohler number
46 criterion to determine extinction limits in turbulent methane jet diffusion flames [7].

47 As presented above, various studies are being attempted worldwide to investigate the
48 governing concepts of flame extinction under conventional premixed or diffusion
49 combustion regimes. However, for innovative combustion technologies such as the
50 Moderate or Intense Low-oxygen Dilution (MILD) combustion, phenomena which
51 control extinction may be different due to the strong differences in kinetics and flow
52 fields with respect to conventional combustion. In MILD combustion, the fuel is burnt
53 with a highly diluted oxidant supplied at a temperature higher than the reacting mixture
54 self-ignition temperature. Spontaneous ignition occurs and progresses with no visible or
55 audible signs of the flames usually associated with conventional combustion. In this
56 regime the diffusing and broaden reaction zone leads to almost uniform heat release and
57 smooth temperature field. These features results in a much more efficient combustion as
58 well as in a suppression of pollutant emissions [8]. This technology is also known as
59 flameless combustion due to its invisible flame front [9] and high temperature air
60 combustion (HiTAC) due to the common procedure of preheating the oxidizer to ensure
61 the mixture temperature to be higher than the self-ignition temperature [10].

62 A few works discussed the extinction behavior in MILD combustion, partly with the
63 aim to elucidate the role of governing parameters on the extinction limits. Mastorakos et
64 al. [11] investigated the effects of simultaneous dilution and preheating of reactants by
65 mixing with hot combustion products in terms of the stability of turbulent counter flow
66 flames. The air was heated from ambient temperature up to 1750 K while it was diluted
67 down to 0.02 mole fraction of oxygen. Extinction limits were measured by igniting the
68 flame under stable conditions and gradually increasing the bulk velocity or decreasing the
69 oxygen content until the flame was extinguished. They concluded that a temperature

70 increase of 100 K is necessary for every 0.02 mole fraction of oxygen loosed by dilution
71 to maintain stability. Flame instabilities were observed for air temperatures less than
72 1400 K. They reported that dilution level of vitiated air did not affect the extinction of
73 lean premixed flames. Its temperature, however, was indicated as the main parameter
74 affecting the transition from sudden extinction to no-extinction regime [10, 11].

75 Maruta et al. carried out an experimental study on a counter flow burner fed with N₂-
76 diluted methane and air with temperature between 300 to 800 K to study the combustion
77 limit and reaction zone structure in flameless combustion regime [12]. One-dimensional
78 computation with detailed chemistry was also performed to cover a wider air temperature
79 range. Similar to Mastorakos et al. they reported that when the air temperature was kept
80 higher than 1300 K, extinction limits disappeared. In this temperature range, combustion
81 continues even under extremely fuel-lean conditions such as 1% methane in nitrogen
82 since the energy that high temperature air brings into the reaction zone is high enough to
83 sustain a weak reaction zone [11, 12].

84 Kumar et al. proposed an empirical flame extinction model based on the competition
85 between mixing and chemical time-scales to predict extinction limits and flame lift-off
86 height in MILD combustion conditions [13]. The extinction model which accounted for
87 reactants dilution and preheating of the oxidizer was evaluated against flame lift-off
88 height for a variety of experiments reported in the literatures [13, 14].

89 Very recently, Lilleberg et al. [15] conducted a numerical study using the EDC
90 combustion model with a pre-calculated extinction database using single and two-step
91 chemical mechanisms on the Sandia/TNF Flame D and Flame E. For the local extinction
92 approach a database of chemical time scales for different inlet temperatures (300 to 865

93 K) and equivalence ratios (about zero to 5) was pre-calculated based on the well-stirred
94 reactor assumption. Similar to the Kumar's approach, Lilleberg et al. [15] assumed that if
95 the fine structure residence time was lower than the pre-calculated chemical time-scales
96 extinction occurs. They compared results of the local extinction approach with a fast-
97 chemistry EDC approach known as the Eddy Dissipation Model and a full detailed
98 chemical mechanism of the gas research institute (GRI-Mech 3.0) and showed that the
99 detailed chemistry approach gave the best predictions compared to the experiments with a
100 considerable higher calculation efforts compared to the fast chemistry and local
101 extinction approaches.

102 In MILD combustion, accurate treatment of turbulence-chemistry interaction (TCI)
103 through a proper combustion model plays a fundamental role for modeling and predicting
104 this regime. Most works have highlighted the superior performances of the Eddy
105 Dissipation Concept (EDC) [16-18] combustion model with respect to others, mainly
106 because such a model allows taking into account finite-rate chemistry effects and an
107 efficient implementation of detailed kinetics. However, some researchers showed that the
108 EDC combustion model usually over-predicts the flame temperature for MILD
109 conditions, so both Aminian et al. [19] and De et al. [20] suggested a revision of the
110 model for MILD conditions, based on an increase of the EDC fine structure residence
111 time constant. The importance of combustion model on capturing the interaction between
112 chemical oxidation and turbulence are also emphasized by Duwig and Dunn [21] for Jet-
113 in-Hot Coflow (JHC) with highly turbulent shear layers.

114 The present study is aimed at developing a new extinction model applicable for
115 MILD combustion conditions and at further characterizing the importance of accurate
116 treatment of the turbulence-chemistry interaction towards local extinction analysis.

117 **2 Experimental observations**

118 The Jet-in-Hot Coflow burner experimentally studied by Dally et al. [22] consists of a
119 central fuel jet (i.d. = 4.25 mm) which is surrounded coaxially by an annulus (i.d. = 82
120 mm) equipped with a secondary burner providing hot combustion products (Fig. 1). The
121 hot flue gases are premixed with air and nitrogen via two side-inlets at the bottom of the
122 annulus to vitiate the oxidizer and produce coflow streams with 9%, 6% and 3% oxygen
123 mass fraction denoted as HM3, HM2, and HM1 flames, respectively. The whole burner
124 was placed inside a wind tunnel, with room temperature air at the same velocity as the
125 hot coflow, to help the stabilization of the flames. In this research the HM3 and HM1
126 flames are interested as they mimic characterization of the diffusion-like flames as well
127 as the MILD conditions, respectively.

128 Using the scatter data for hydroxyl radical (OH), Dally et al. [22] reported that no
129 sign of localized extinction was observed at upstream ($z < 100$ mm) for all JHC flames.
130 However, based on the large scatter of OH around stoichiometric mixture fraction for
131 HM1 flame they concluded that only HM1 flame partly extinguished at the downstream
132 ($z = 120$ mm). The scatter plots of hydroxyl radical alone, while providing useful insights
133 into the flame structure does not give enough information to fully resolve issues
134 regarding reaction zone structure and local extinction effects. Such issues require
135 additional information; best provided by radical species, such as oxygen and
136 formaldehyde (CH₂O). The hydroxyl radical is, normally, used as a flame marker, while

137 the formaldehyde intermediate species is predominant at low temperatures typical of
138 those found in MILD combustion.

139 Medwell et al. [23] performed a more in-depth analysis of local extinction using the
140 instantaneous images of the OH, CH₂O and temperature on the similar experiment of the
141 JHC burner with slightly higher jet Reynolds number. They reported that at the
142 downstream the entrainment of surrounding air can lead to localized extinction of the
143 reaction zone by means of cooling. They stated that the extinction/re-ignition phenomena
144 in JHC flames occur in a consecutive manner. First, extinction occurs due to cooling by
145 the surrounding air. Then, breaking of the reaction zone leads to premixing of the fuel
146 and surrounding air. Finally, the premixed fuel-air ignites by the heated coflow which
147 acts as a pilot. Employing the OH and temperature images they showed that the
148 associated temperature drop from the surrounding air can lead to localized extinction at
149 downstream for the HM1 flame. However, the entrainment of surrounding air into the
150 HM3 flame with more intense initial reaction zone lead to weaken the reaction zone
151 rather than extinction. In addition, they have reported that physical strained induced
152 mechanisms have no effect on the extinction of JHC flames and the increased frequency
153 of extinction events with the increased Reynolds number was attributed to the increased
154 mixing and entraining more cooling air into the reaction zone. Based on the analysis of
155 the OH, CH₂O and temperature image set at $z = 125$ mm they showed that 11.9% of HM1
156 flame was extinguished while no extinction was observed for the HM3 flame [23].

157 In another study on the HM1 flame, Medwell et al. demonstrated that different
158 stabilization mechanisms are governed in the MILD conditions [24]. They showed that
159 lower amount of O₂ in the heated coflow lead to lower reaction rates and therefore lower

160 OH concentration which was defined as weakened reaction zone. The non-intense
161 weakened reaction zone subsequently allows more permeation of oxygen across the
162 reaction zone and leads to some partial premixing in the lift-off region. Their observation
163 suggests that molecular transport and finite-rate chemistry effects are essential in order to
164 capture the stability and structure of flames in MILD conditions.

165 The comprehensive set of detailed measurements of Dally et al. [22] and Medwell et
166 al. [23, 24] provides a thorough insight on the local structure of the HM1 and HM3
167 flames and help to identify a set of parameters that control their behavior close to
168 extinction. Based on these findings, a new local extinction criterion will be hereby
169 developed and presented following to the standard EDC extinction model.

170 **3 Computational and Physical models**

171 Due to the symmetry of the HM1 and HM3 flames, a 2D axisymmetric domain
172 starting from the burner exit was constructed (Fig. 2). A mesh independency task was
173 performed on four structured grids of 13, 20, 25 and 33k elements. Comparing the cold-
174 flow velocity profile along the burner center line the grid with 25k cells was found to be
175 the optimum grid.

176 The steady-state Reynolds-Averaged Navier-Stokes (RANS) equations are solved
177 with a finite volume scheme using the commercial CFD code FLUENT. The modified k-
178 ϵ turbulence model ($C_{\epsilon 1}$ is set to 1.6 instead of 1.44) was employed to compensate for the
179 round-jet/plane-jet anomaly [25]. The full KEE58 mechanism [26] consists of 17 species
180 (CH_4 , O_2 , CH_3 , CH_2 , CH , CH_2O , HCO , CO_2 , CO , H_2 , H , O , OH , H_2O , HO_2 , H_2O_2 , and
181 N_2) and 58 reversible reactions related to methane is employed in this study. Differential
182 diffusion was considered based the kinetic theory and a modification of the Chapman-

183 Enskog formula [27]. The discrete ordinate (DO) method together with the Weighted-
184 Sum-of-Gray-Gases (WSGG) model was employed to solve the radiative transfer
185 equation (RTE) in 16 different directions across the computational domain. Second-order
186 upwind scheme was applied for discretizing all transport equations and the SIMPLE
187 algorithm to handle velocity-pressure coupling. Table 1 shows the operating conditions of
188 all inlet streams for the HM1 and HM3 flames.

189 **4 Model descriptions**

190 The main focus of this paper is on developing a new extinction model for MILD
191 combustion conditions based on the classical EDC extinction model developed for the
192 conventional diffusion flames. However, every extinction model should be coupled with
193 a combustion model to enjoy the thermo-chemistry and turbulence properties calculated
194 by the combustion model. Therefore, at first the modified EDC combustion model which
195 is previously studied by the authors of this paper will be briefly discussed in section 4.1.
196 Then details of the EDC extinction model will be critically reviewed and an extended
197 extinction model for MILD combustion conditions will be presented in sections 4.2 and
198 4.3, respectively. Fig. 3 illustrates a conceptual scheme of the modeling strategy
199 performed in the previous studies (STEP 1 and STEP 2: modifying the EDC combustion
200 model constant for MILD conditions) and the overall procedure of developing a new
201 extinction model for MILD conditions in the present paper (STEP 3 to STEP 5).

202 **4.1 EDC combustion model**

203 The EDC combustion model provides an empirical expression for the mean reaction
204 rate based on the assumption that the chemical reactions occur in the regions of the flow
205 which represent only a fraction of the entire volume and where the dissipation of

206 turbulent kinetic energy takes place [28]. These regions are denoted as fine structures and
 207 they are believed to be vortex tubes, sheets or slabs, whose characteristic dimensions are
 208 of the same order of the Kolmogorov length scale. Gran and Magnussen [29] proposed an
 209 expression for the mean reaction rate of specie i in a fine structure as:

$$\bar{R}_i = \frac{\gamma^{*2/3}}{\tau^*} (Y_i^* - Y_i^0) \quad (1)$$

210 where, γ^* is the fine structure volume and τ^* is the fine structure residence time. Y_i^0 is
 211 the mass fraction of species i in the surrounding fluid and Y_i^* is the fine structure species
 212 mass fraction after reacting over the time τ^* . From the concept of step-wise turbulence
 213 energy cascade, characteristic scales of the fine structure have been introduced in the
 214 EDC combustion model as follows:

$$\gamma^* = \left(\frac{3C_{D_2}}{4C_{D_1}} \right)^{3/4} \left(\frac{\nu \varepsilon}{k^2} \right)^{3/4} = C_\gamma \left(\frac{\nu \varepsilon}{k^2} \right)^{3/4} \quad (2)$$

$$\tau^* = \left(\frac{C_{D_2}}{3} \right)^{1/2} \left(\frac{\nu}{\varepsilon} \right)^{1/2} = C_t \left(\frac{\nu}{\varepsilon} \right)^{1/2} \quad (3)$$

215 where, ν , k and ε are the kinematic viscosity, turbulent kinetic energy and its
 216 dissipation rate, respectively. C_{D_1} and C_{D_2} are the model constants set equal to 0.134 and
 217 0.5 leading to fine structure volume and residence time constants equal to $C_\gamma = 2.1637$
 218 and $C_t = 0.4083$.

219 In recent studies, an increase of the fine structure residence time constant in the EDC
 220 combustion model has been suggested for better prediction of the interaction between
 221 turbulence and chemistry (TCI) in MILD combustion conditions [19, 20]. Figs. 4 and 5
 222 illustrate the effect of increasing C_t from the default value (i.e., 0.4083) to 1.5 on the

223 prediction of temperature and hydroxyl profiles obtained with the KEE58 mechanism and
224 the modified k - ϵ turbulence model for the HM1 and HM3 flames [19].

225 As can be seen in Figs. 4 and 5, results of the standard EDC combustion model,
226 hereafter called *Std. EDC-TCI*, showed an intensified reaction zone at downstream.
227 However, the modified EDC combustion model (*Mod. EDC-TCI*) accounts for the
228 reaction zone weakening as it reduces the peak values of temperature and hydroxyl
229 radical makes it more reliable for MILD combustion conditions. Some under prediction
230 of OH radical at the upstream of HM1 and HM3 flames is attributed to the unattained
231 temperature fluctuations via the RANS approach [30]. The thermo-chemistry and
232 turbulence properties calculated by the standard and modified EDC combustion models
233 will be applied in the extinction models describing in sections 4.2 and 4.3.

234 **4.2 EDC extinction model**

235 It is well accepted that localized extinction occurs when the mixing time-scale
236 becomes smaller than a typical chemical time-scale in the combustion process. According
237 to this, the EDC combustion model cannot be employed directly to analyze local
238 extinction. Since, if the residence time in the fine structure is too short, not only
239 extinction occurs but also a fast chemistry will be approached according to Eq. (1).
240 Therefore, neither versions of the EDC combustion model can be employed directly to
241 predict local extinction by comparing of mixing and chemical time-scales.

242 In the EDC extinction model, however, a critical fine structure residence time is
243 introduced which must always satisfy the energy and mass balance equations for the fine
244 structure [31]. The fine structure mass balance is defined as:

$$Y_{fu} = \frac{R_{fu}^* \cdot \tau^*}{c_{fu}^0} \frac{\rho^0}{\rho^*} \quad (4)$$

245 where, R_{fu}^* is the rate of mass transfer between the fine structure and surrounding fluid
 246 and c_{fu}^0 is the local concentration of the fuel in surrounding fluid. The EDC extinction
 247 model assumes that the rate of reaction between fuel and oxidizer is infinitely fast [31].
 248 Hence, the rate of combustion is controlled by the mass transfer between the fine
 249 structure and surrounding fluid which is formulated as follows:

$$R_{fu}^* = \frac{\dot{m} \cdot \bar{c}_{\min}}{1 - \gamma^*} \quad \left[\frac{kg}{m^3 \cdot s} \right] \quad (5)$$

250 where, \bar{c}_{\min} is the smallest of local mean concentration of the fuel (\bar{c}_{fu}) and oxidizer
 251 (\bar{c}_{O_2}/r_{fu}) and r_{fu} is the stoichiometric O_2 requirement. \dot{m} is the transfer rate of unit mass
 252 of fluid between the fine structure and surrounding which has been obtained based on
 253 turbulence parameters [31] as follows:

$$m = 23.6 \left(\frac{\nu \cdot \varepsilon}{k^2} \right)^{\frac{1}{4}} \cdot \frac{\varepsilon}{k} \quad \left[\frac{1}{s} \right] \quad (6)$$

254 If fine structures are considered to be adiabatic, the fine structure energy balance can
 255 be defined as:

$$Y_{fu} = \frac{C_p (T^* - T^0) \rho^0}{\Delta H_R \cdot c_{fu}^0} \quad (7)$$

256 where, T^* and T^0 are the fine structure and surrounding fluid local temperatures,
 257 respectively. ΔH_R is, also, the heat of combustion generated in the fine structures. Thus,
 258 the critical fine structure residence time (τ_{cr}^*) which satisfies both heat and material
 259 balances can be derived from Eq. (4) and Eq. (7) as follows:

$$\tau_{cr}^* = \frac{C_p(T^* - T^0)\rho^*}{R_{fu}^* \cdot \Delta H_R} \quad [s] \quad (8)$$

260 If the fine structure residence time defined in Eq. (3) becomes smaller than the critical
 261 residence time obtained in Eq. (8) reactions will not complete [31]. In other words, in fine
 262 structures with residence time smaller than τ_{cr}^* one of the mass or energy balance
 263 equations has not be satisfied resulting in extinction of chemical reactions in the fine
 264 structure.

265 **4.3 Extended EDC extinction model**

266 It is well accepted that in MILD conditions the chemical time-scale is of the same
 267 order of turbulent or mixing time-scale, leading to conditions far from the fast-chemistry
 268 assumption [32]. The aim of this section is to extend the applicability of the EDC local
 269 extinction model to the MILD combustion conditions by incorporating the effect of
 270 finite-rate chemistry in the rate of combustion reactions. The theoretical basis of model
 271 extension, here, will be discussed using two different perspectives, one based on diffusive
 272 fluxes and the other based on rate of reactions.

273 **4.3.1 Diffusive perspective**

274 According to Eq. (5) the rate of combustion reactions in the EDC extinction model is
 275 assumed to be controlled by the rate of mass transfer (R_{fu}^*) from surrounding fluid to the
 276 fine structures [31]. Eq. (5) can be re-written using the molar flux of species i diffusing
 277 from surrounding fluid toward the fine structures as follows:

$$R_{fu}^* = \rho k_g (Y_i^0 - Y_i^1) \quad (9)$$

278 where, k_g is surrounding fluid mass transfer coefficient and Y_i^0 and Y_i^1 are the mass
 279 fraction of species i in surrounding fluid and at the fine structure entrance, respectively,

280 as schematically illustrated in Fig. 6a. Under the assumption of fast-chemistry, Y_i^1 can be
 281 assumed as Y_i^* resulting to development of the EDC extinction model [31] described in
 282 section 3.2. However, finite-rate effects may lead to deviation of Y_i^1 from Y_i^* (see Fig.
 283 6b). According to the EDC combustion model [29] the source term in the conservation
 284 equation for the mean species i is modeled using Eq. (1). $\bar{R}_i = \frac{\gamma^{*2/3}}{\tau^*} (Y_i^* - Y_i^0)$ As shown in
 285 Fig. 6b Y_i^0 at the fine structure entrance is denoted as Y_i^1 to consider the effect of slow
 286 chemistry inside the fine structure. Therefore, another combustion resistance for the slow
 287 chemistry inside the fine structures could be defined as follows:

$$\bar{R}_i = \rho k_g^* (Y_i^* - Y_i^1) \quad (10)$$

288 where, k_g^* is the mass transfer coefficient inside the fine structures. Therefore,
 289 eliminating Y_i^1 between Eq. (9) and Eq. (10) will result in:

$$\begin{aligned} R_{new}^* &= \rho k_g \left(Y_i^0 - Y_i^* + \frac{\bar{R}_i}{\rho k_g^*} \right) \\ &= \rho k_g (Y_i^0 - Y_i^*) + \frac{k_g}{k_g^*} \bar{R}_i \end{aligned} \quad (11)$$

290 Since, temperature of the fine structures is, only, slightly higher than the surrounding or
 291 local mean temperature it can be assumed that k_g and k_g^* are almost equal. The fine
 292 structure temperature and local mean temperature of the HM1 and HM3 flames are
 293 depicted in Fig. 7. As can be seen in the contours of Fig. 7 the local mean and fine
 294 structure temperature fields are almost similar in both inner and outer regions of the
 295 flames. A more accurate comparison is shown in the X-Y plots of Fig. 7 where radial
 296 profiles of local mean and fine structure temperatures of both flames at to axial locations

297 are illustrated. It is observed that both temperature fields at two axial locations are quite
 298 similar in fuel-rich and fuel-lean sides of the HM1 and HM3 flames. A minor deviation
 299 is, however, observed at the peak temperatures which correspond to the main reaction
 300 zones. Quantitatively speaking, the maximum values of local mean temperature at $z = 30$
 301 mm from the burner tip are 1376.3 and 1718.3 K for HM1 and HM3 flames, respectively.
 302 The corresponding maximum values for fine structure temperatures are 1433.5 and
 303 1811.7 K. Similarly, at $z = 120$ mm from the burner tip the maximum values of local
 304 mean temperatures are 1635.2 and 1874.8 K and the corresponding maximum fine
 305 structure temperatures are 1756.2 and 2003.2 K for HM1 and HM3 flames, respectively.
 306 These quantities reveal that deviation of local mean temperature from the fine structure
 307 temperature in the main reaction zone is about 4 to 7 percent at all situations mentioned
 308 above. As a consequence, the assumption of equal mass transfer coefficients at
 309 surrounding and fine structure temperature fields could be reasonable. Therefore,
 310 following to Eq. (11) a new definition for the rate of combustion reactions under the
 311 effects of finite-rate chemistry is proposed as follows:

$$R_{new}^* = R_{fu}^* + \bar{R}_i \quad (12)$$

312 where, R_{fu}^* is computed based on the Eq. (5) and \bar{R}_i is the mean reaction rate of species i
 313 in the fine structure computed using the EDC combustion model.

314 **4.3.2 Reactive perspective**

315 Again, according to Eq. (5) the rate of combustion reactions in the EDC extinction
 316 model is calculated based on a single-step mass transfer-controlled phenomenon leading
 317 to the full equilibrium of $Y_{i,in}^*$ and $Y_{i,out}^*$ inside the fine structure (Fig. 6a). However, the
 318 EDC combustion model can incorporate detailed chemistry into turbulent combustion to

319 account for finite rate effects. Based on Eq. (1) the concentration of specie i at the fine
 320 structure entrance $Y_{i,in}^*$ deviates from its outlet concentration $Y_{i,out}^*$ as they are not in
 321 equilibrium due to finite rate effects. In other words, in the EDC combustion model fine
 322 structures are assumed as partially stirred reactors which converts $Y_{i,in}^*$ to $Y_{i,out}^*$ with a
 323 finite rate of \bar{R}_i . To incorporate the effect of finite rate chemistry in the EDC extinction
 324 model we can start from the finite reaction rate available in the EDC combustion model
 325 and perform addition/subtraction of the inlet concentration:

$$\bar{R}_i = \frac{\gamma^{*2/3}}{\tau^*} (Y_{i,out}^* - Y_i^0) = \frac{\gamma^{*2/3}}{\tau^*} (Y_{i,out}^* - Y_i^0 - Y_{i,in}^* + Y_{i,in}^*) \quad (13)$$

326 Rearranging above formula we have:

$$\begin{aligned} \bar{R}_i &= \frac{\gamma^{*2/3}}{\tau^*} (-Y_i^0 + Y_{i,in}^*) + \frac{\gamma^{*2/3}}{\tau^*} (Y_{i,out}^* - Y_{i,in}^*) = -R_{fu}^* + R_{new}^* \\ \Rightarrow R_{new}^* &= R_{fu}^* + \bar{R}_i \end{aligned} \quad (14)$$

327 Eq. (14) is quite similar to Eq. (12) which was obtained based on the diffusive fluxes.
 328 Therefore, as shown in Fig. 6b, in the extended EDC extinction model proposed here
 329 based on diffusive and reactive viewpoints, besides the transfer rate of reactants from
 330 surrounding fluid to the fine structures (R_{fu}^*), the rate of chemical reactions inside the
 331 fine structures (\bar{R}_i) is incorporated to determine the overall rate of combustion. Hence,
 332 the new critical fine structure residence time is defined as follows:

$$\tau_{cr,new}^* = \frac{C_p (T^* - T^0) \rho^*}{R_{new}^* \cdot \Delta H_R} \quad [s] \quad (15)$$

333 Based on the above new definition, the final expression for occurrence of local
 334 extinction in MILD conditions can be formulated as:

$$\lambda = \frac{\tau^*}{\tau_{cr,new}^*} \leq 1 \quad (16)$$

335 In the proposed new extinction model the calculation of chemical time-scale is hidden
336 in the determination of the mean reaction rate of species i in Eqs. (12) and (14) using the
337 EDC combustion model. In addition, the extended EDC extinction model is applicable to
338 both classical diffusion flames with $\bar{R}_i = 0$ (fast-chemistry assumption) in Eqs. (12) and
339 (14), and newly developed MILD combustion flames. Last but not least, no ad-hoc
340 constant parameters are employed in the new extinction model, extending its applicability
341 to any kind of fuel.

342 **5 Results and discussion**

343 The implications of the extended EDC extinction model are now examined with the
344 aid of the experimental observations of Dally et al. [22] and Medwell et al. [23] on the
345 HM1 and HM3 flames of the JHC experiments reviewed in section 2. Moreover, the
346 proposed new extinction model will be compared with the standard EDC extinction
347 model on the prediction of extinction limits and position of the reaction zone. Here, we
348 have focused on the HM1 and HM3 flames of the JHC experiments [22] which represent
349 a MILD condition and flame-like behaviors, respectively. The flames conditions and their
350 local extinction positions are presented in Table 2.

351 As can be seen in Table 2, while the jet Reynolds numbers are slightly different, in
352 both experiments, the extinction was approached in HM1 flame at downstream. It arises
353 from the fact that the reduced oxygen content in the oxidizer coflow at downstream can
354 extinguish the weak HM1 flame. In addition, entraining more cooling air into the reaction
355 zone can accelerate flame extinction at downstream [23].

356 From the numerical results (thermo-chemistry and turbulence parameters) obtained
357 using the standard and modified EDC combustion models [19, 30], the local extinction
358 criterion is, here, evaluated for the HM3 (flame-like behavior) and HM1 (MILD
359 behavior) flames presented in Table 2. The resulting λ distributions are plotted in Figs. 8
360 and 9 against the mixture fraction calculated using the Bilger formula [26]. The local
361 extinction analysis presented in Figs. 8 and 9 includes two extinction models that are:

- 362 1. The standard EDC extinction model: $\lambda = \tau^* / \tau_{cr}^* \leq 1$, where τ^* and τ_{cr}^* are defined
363 based on Eq. (3) and Eq. (8), respectively.
- 364 2. The new extended EDC extinction model: $\lambda = \tau^* / \tau_{cr,new}^* \leq 1$, where τ^* and $\tau_{cr,new}^*$
365 are defined based on Eq. (3) and Eq. (15), respectively.

366 As mentioned above, to evaluate the role of turbulence-chemistry interaction model
367 on the extinction analysis the above extinction models are examined based on the
368 numerical results obtained using the standard and modified EDC combustion models
369 [19]. As suggested by Medwell et al. [24], the reaction zone thickness (δ) was considered
370 as the distance between the OH and CH₂O peaks which represent the fuel-lean and fuel-
371 rich sides of the reaction zone, respectively. In Figs. 8 and 9, due to the steep distribution
372 of λ across the main reaction zone, denoted by $\delta_{std.}$ and $\delta_{mod.}$ (obtained using the standard
373 and modified EDC combustion models), part or the whole of the reaction zone may reach
374 λ values below the limit as extinction approaches.

375 According to Table 2, no extinction has been observed for the HM3 flame at both
376 upstream and downstream locations. The standard EDC extinction model coupled with
377 the standard EDC combustion model completely failed to capture this behavior at
378 upstream ($z = 30$ mm) as displayed in Fig. 8a. The extended EDC extinction model on

379 the same combustion field provides slightly improved results, but still half of the flame
380 predicted to be extinguished. The problem is solved when either the standard or extended
381 EDC extinction models have applied on the numerical solution obtained based on the
382 modified EDC combustion field (Fig. 8b). In other words, for the flame-like behavior of
383 HM3 flame the role of turbulence-chemistry interaction treatment on the accurate
384 prediction of local extinction is much more important than the direct role of the extinction
385 model. This issue supports the importance of previously proposed modification on the
386 treatment of turbulence-chemistry interaction in JHC flames [19].

387 At downstream ($z = 120$ mm), no extinction has been observed for the HM3 flame as
388 reported in Table 2. While the standard EDC extinction model was developed for the
389 classical combustion regimes which benefit an intense reaction zone, coupled with the
390 standard EDC combustion model it could not provide reasonable predictions for the HM3
391 flame at downstream (Fig. 8c). Even with the extended EDC extinction model on the
392 same combustion field the λ values, still, below the cross-over line for the entire reaction
393 zone. This failure is resolved when either the standard or extended EDC extinction
394 models were applied on the modified EDC combustion field (Fig. 8d). While the
395 extended EDC extinction model reveals reasonable results like the standard version, the
396 accurate resolving of turbulence-chemistry interaction using the modified EDC
397 combustion model shows superior influence on the extinction prediction for the HM3
398 flame.

399 To further evaluate the extended extinction model similar analysis is performed for
400 the HM1 flame. As illustrated in Fig. 9a, results of the standard and extended EDC
401 extinction models obtained on the standard EDC combustion field reveal total extinction

402 of the HM1 flame at 30 mm from the burner tip. This is against the experimental
403 observations on the HM1 flame in this position. However, by applying both standard and
404 extended EDC extinction models on the modified EDC combustion field (Fig. 9b) λ
405 values lie totally above the cross-over line in agreement with the experimental findings
406 that no extinction occurred at upstream of the HM1 flame.

407 According to the experimental observations presented in Table 2, HM1 flame partly
408 encountered to extinction at downstream. As can be seen in Fig. 9c and d only the new
409 extended EDC extinction model when applying on the numerical results obtained with
410 the modified EDC combustion model can predict this phenomenon properly. In relation
411 to the standard EDC extinction model, complete extinction of the HM1 flame across the
412 full extent of the reaction zone is thought to occur even when employed on the modified
413 EDC combustion field.

414 In general, it can be concluded that prediction of extinction limits in MILD conditions
415 is attainable only through the application of the new extended EDC extinction model on a
416 well resolved turbulence-chemistry interaction field.

417 Development of the extended EDC extinction model enables one to investigate the
418 influence of the amount of cooling air entrained into the reaction zone on the chemical
419 reaction rates. Fig. 10 illustrates the net reaction rate of CH₄ (as the bottleneck for
420 combustion reactions comparing with H₂) together with the available O₂ content at
421 different axial locations for the HM3 and HM1 flames. In Fig. 10 the HM3 and HM1
422 flames show a quite opposite manner along the axial direction. In HM3 flame, the net
423 reaction rate of methane is high at upstream and it decreases along the axial direction.
424 Conversely, for the HM1 flame the net reaction rate of methane close to the burner is low

425 and it increases toward downstream. This antithetical manner may be attributed to the
426 competition between the oxygen penetrating from surrounding air to the reaction zone
427 and its cooling effects on the flame. In the HM3 flame which benefits higher oxygen
428 content in the hot coflow (9% by weight) a fixed amount of oxygen is available in the
429 reaction zone along the axial distance, as can be seen in Fig. 10a. Therefore, penetrating
430 surrounding air into the reaction zone at downstream, only, cools down the flame and
431 resulted to slower methane oxidation in comparison with that at upstream. However, for
432 the HM1 flame the oxygen deficient content in the hot coflow stream (3% by weight)
433 slows down the methane reaction rate and resulted to partial extinction at upstream as
434 previously captured by the proposed extended EDC extinction model. In addition,
435 entraining more oxygen to the reaction zone towards downstream has accelerated
436 methane oxidation as displayed in Fig. 10b. Interestingly, these specific behaviors of the
437 JHC flames are captured by the proposed extension on the EDC extinction model and as
438 shown previously resulted in accurate capturing of localized extinction for both HM3 and
439 HM1 flames at upstream and downstream locations.

440 Despite the success in describing local extinction behavior of the Adelaide JHC
441 flames, the extended EDC extinction model requires further tests and refinements to
442 verify its wider applicability for other geometries, conditions and fuels.

443 **6 Conclusions**

444 In this paper, localized extinction is studied in the context of turbulent jet flames
445 issuing into a heated and diluted coaxial oxidizer stream. A theoretical extension of the
446 EDC extinction model was proposed to account the finite-rate chemistry effects normally
447 occurs in MILD conditions, on the overall rate of combustion. According to this criterion

448 the flame is assumed to be locally extinguished, when the fine structure mixing time-
449 scale is lower than a critical mixing time-scale determined in the present study. The
450 proposed extinction model is evaluated against the experimental observations for JHC
451 flames with 9% (HM3) and 3% (HM1) oxygen mass fraction in the hot coflow. It is
452 found that, accurate prediction of flame extinction not only requires a reliable extinction
453 model but also strongly dependent on the well treating of turbulence-chemistry
454 interaction field. The presented local extinction model when applying on a well-resolved
455 turbulence-chemistry interaction field is demonstrated to be able of capturing the local
456 extinction behavior of purely MILD flames (HM1) as well as classical diffusion-like
457 flames (HM3). Further investigations are needed to assess the suitability of the model for
458 different types of flames.

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Table 1 Operating conditions of all three streams for the HM1 and HM3 flames [22]

Flame	Fuel jet				Oxidant coflow					Tunnel air		
	\dot{Q} (kg/s)	T (K)	CH ₄ (wt.%)	H ₂ (wt.%)	u (m/s)	T (K)	O ₂ (wt.%)	H ₂ O (wt.%)	CO ₂ (wt.%)	u (m/s)	T (K)	O ₂ (wt.%)
HM1	3.12e-4	305	88	11	3.2	1300	3	6.5	5.5	3.2	294	23.2
HM3	3.12e-4	305	88	11	3.2	1300	9	6.5	5.5	3.2	294	23.2

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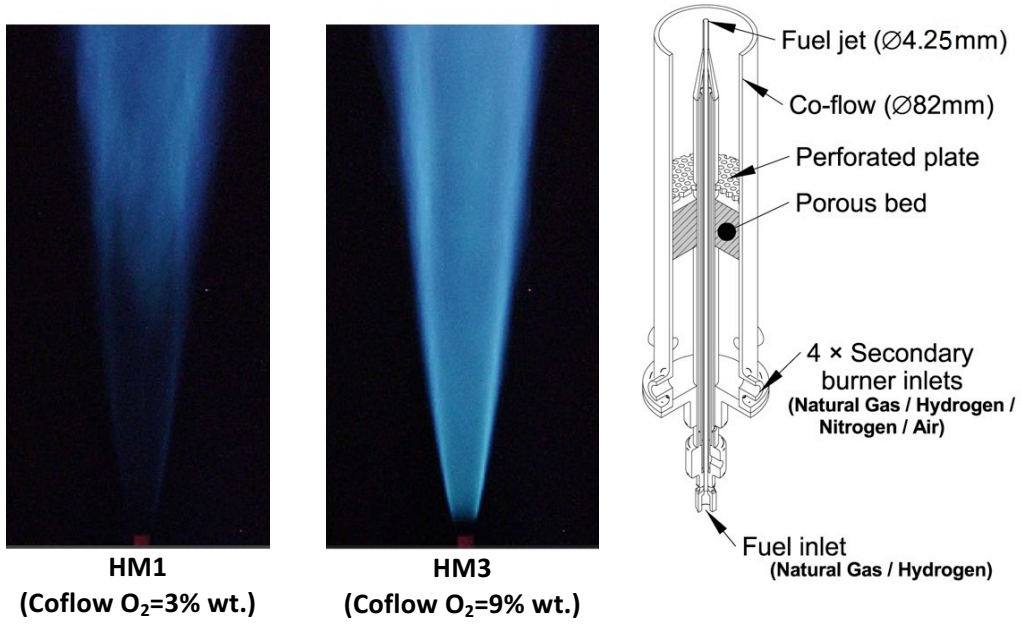
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Table 2 Flame conditions and the reported local extinctions

Flame	$Y_{O_2,coflow}$	T_{coflow}	Re_{jet}	Local extinction @ $z = 30$ mm	Local extinction @ $z = 120$ mm
HM1 ^a	0.03	1300	10000	No	Yes
HM1 ^b	0.03	1300	11000	No	Yes
HM3 ^a	0.09	1300	10000	No	No
HM3 ^b	0.09	1300	11000	No	No

^aDally et al. experiment [22], ^bMedwell et al. experiment [23]

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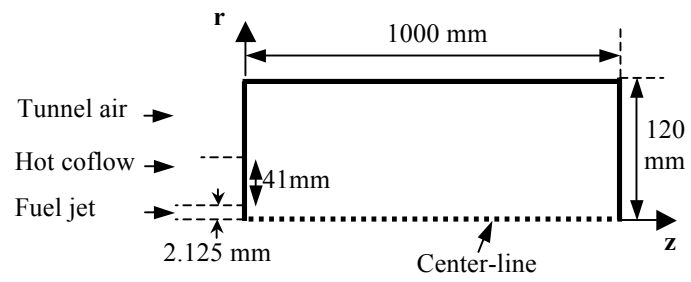
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Fig. 1 Jet-in-hot coflow burner configuration and hydrogen-methane flames [22]

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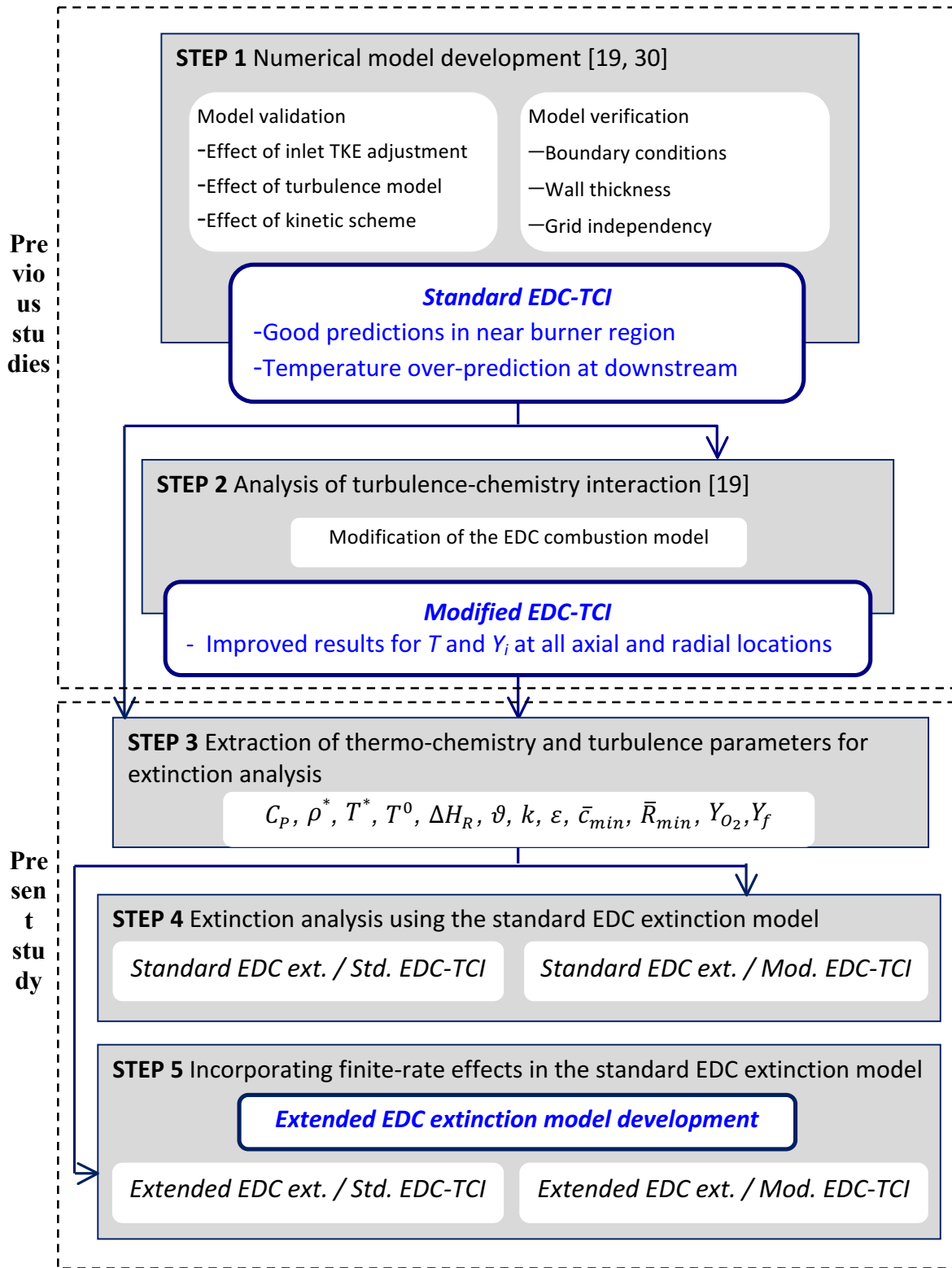


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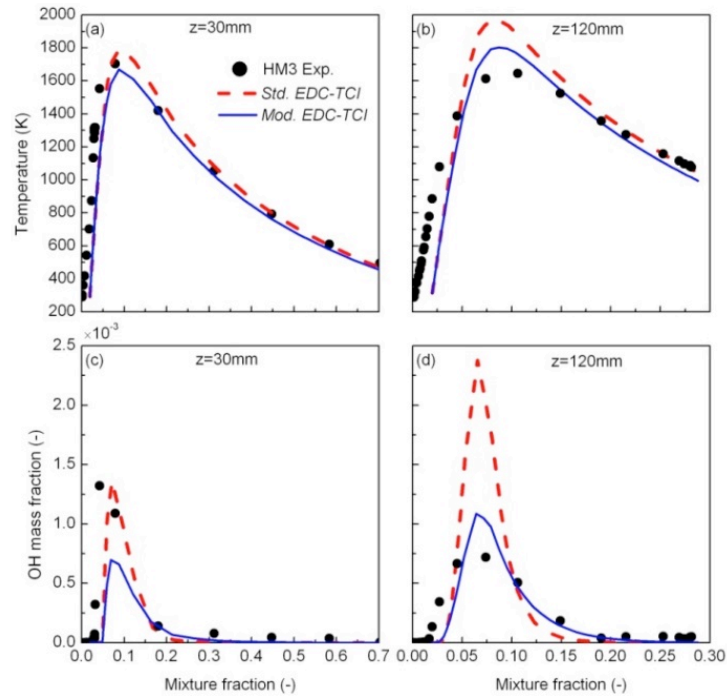
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Fig. 2 Computational domain with boundary conditions



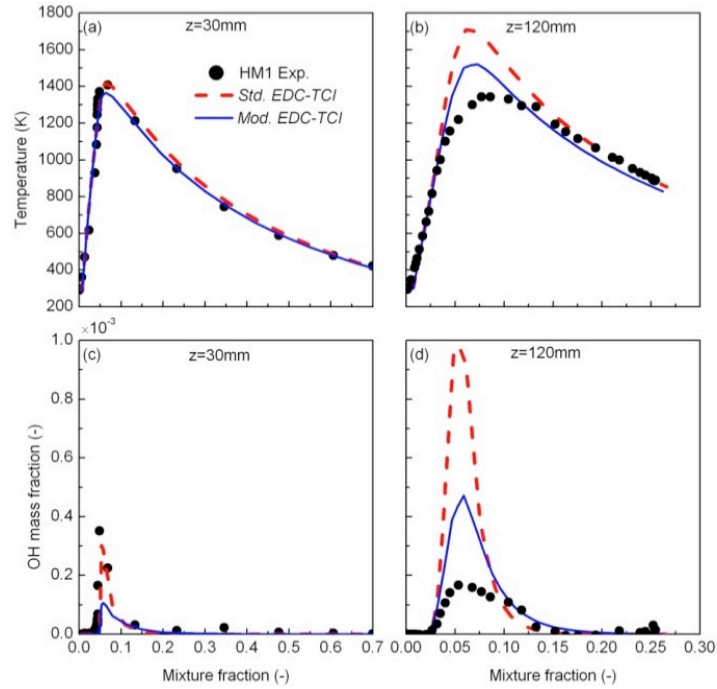
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 561 **Fig. 3** Conceptual scheme of the modeling strategy performed in previous studies (STEP 1 and STEP 2) for
 562 modifying the EDC combustion model constant and the overall procedure of developing the extended EDC
 563 extinction model in the present study (STEP 3 to STEP 5)

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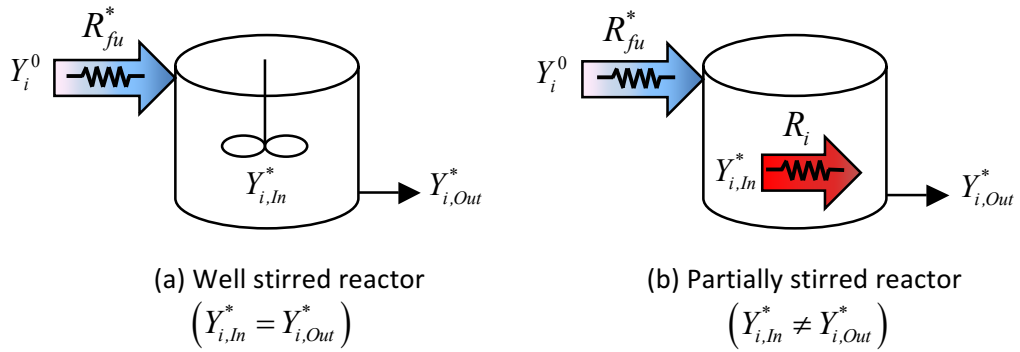
Fig. 4 Comparison of the standard (*Std.*) and modified (*Mod.*) EDC combustion models for prediction of temperature and hydroxyl profiles at two axial locations ($z = 30$ and 120 mm for the HM3 (9% O_2 mass fraction in the hot coflow stream) flame [19]



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Fig. 5 Comparison of the standard (*Std.*) and modified (*Mod.*) EDC combustion models for prediction of temperature and hydroxyl profiles at two axial locations ($z = 30$ and 120 mm) for the HM1 (3% O_2 mass fraction in the hot coflow stream) flame [19]

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Fig. 6 Schematic of combustion resistance(s) accounted for in a fine structure in the (left) standard EDC extinction model and (right) extended EDC extinction model

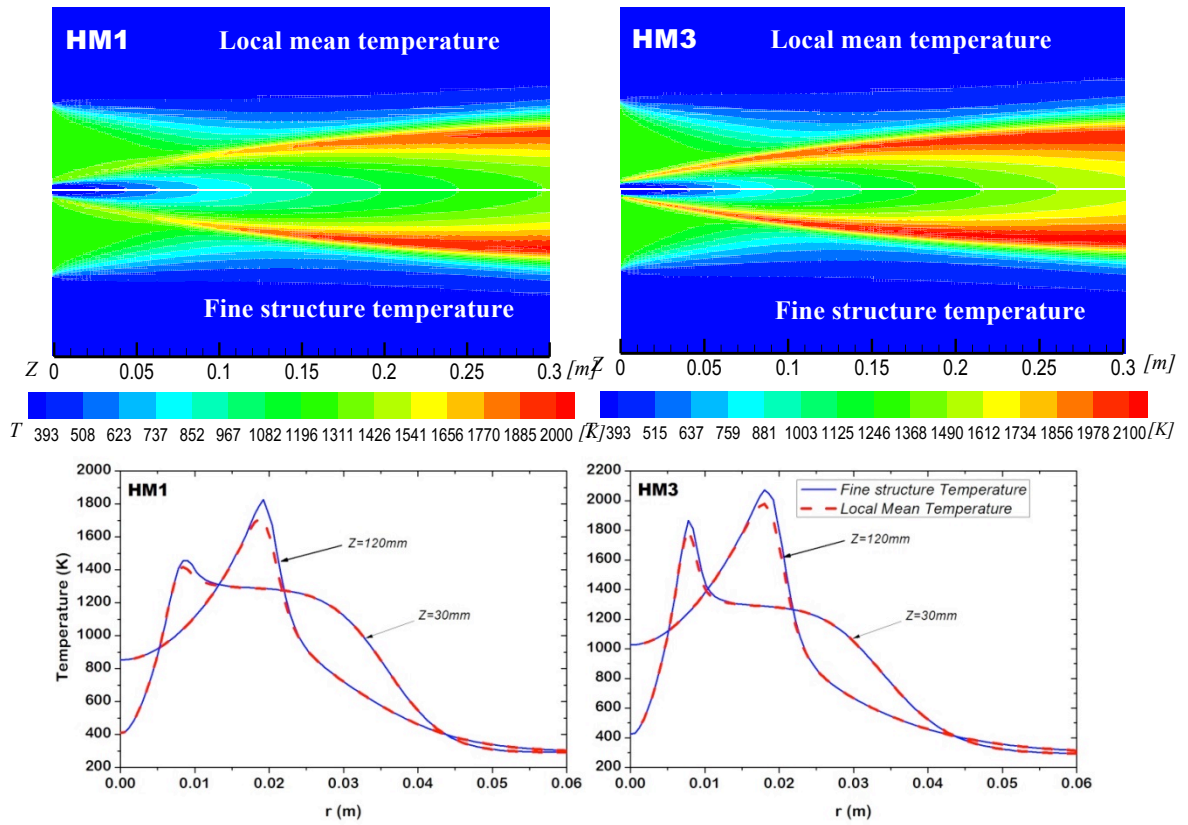
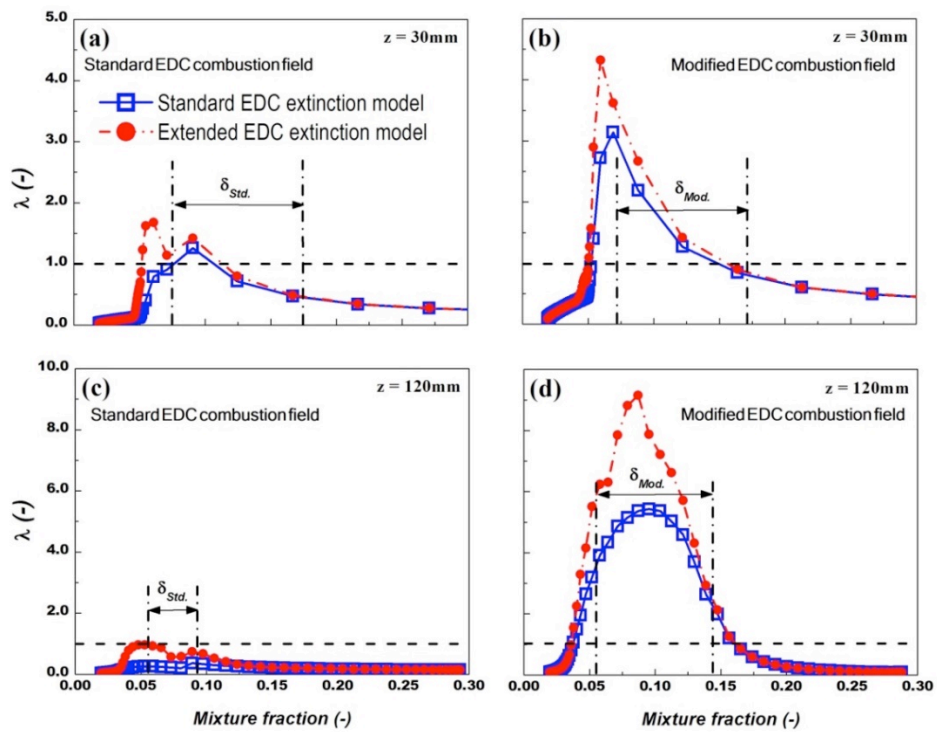


Fig. 7 Comparison of fine structure and local mean temperature distributions in HM1 and HM3 flames

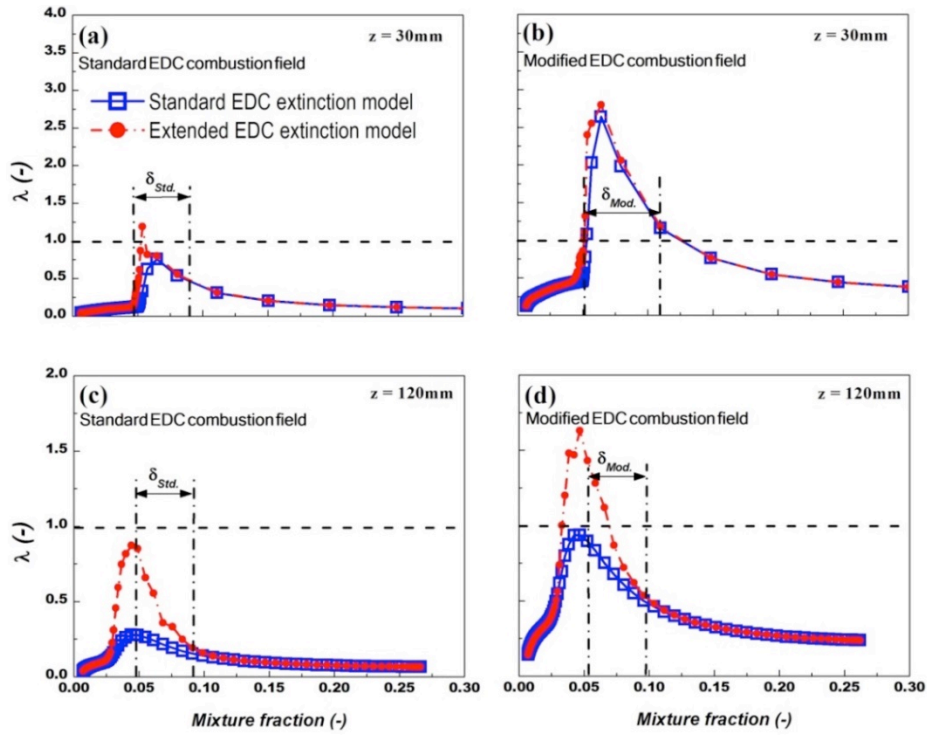
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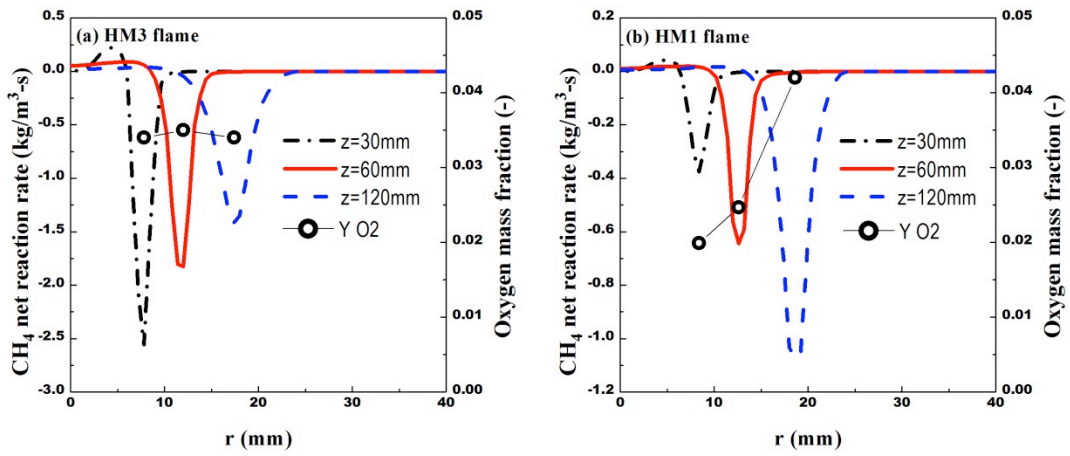
Fig. 8 Effect of extinction models combined with two versions of the EDC combustion model on local extinction analysis of the HM3 (9% O₂ mass fraction in the hot coflow stream) flame at two axial locations ($z = 30$ and 120 mm). The cross-over horizontal dashed line represents extinction limit in reaction zone below which extinction occurred.



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Fig. 9 Effect of extinction models combined with two versions of the EDC combustion model on local extinction analysis of the HM1 (3% O₂ mass fraction in the hot coflow stream) flame at two axial locations (z = 30 and 120 mm). The cross-over horizontal dashed line represents extinction limit in reaction zone below which extinction occurred.

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Fig. 10 Net reaction rate of methane and oxygen mass fraction in the reaction zone for HM3 and HM1 flames. First circle from left corresponds to axial location of $z = 30$ mm. Second circle corresponds to $z = 60$ mm and the third circle corresponds to axial location of $z = 120$ mm.