WHERE DOES FEAR OF MATHS COME FROM?

BEYOND THE PURELY EMOTIONAL

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Abstract: *Fear of mathematics is a widespread emotion that has many negative consequences in students: it is a possible factor of local failure (since it prevents the best use of one's competence and knowledge) but also a possible factor of global failure (since it might lead to students giving up any engagement with mathematics). Investigating the cognitive origin of this emotion is fundamental to prevent it and to overcome its negative consequences. In this study we try to understand this origin giving voice to the students, analysing students' narratives about their relationship with mathematics.*

Keywords: *fear of mathematics, cognitive origin of emotions, affect*

INTRODUCTION AND THEORETICAL FRAMEWORK

In the last two decades, the specific field of research on affect in mathematics education has grown recognizing four specific constructs: beliefs, emotions, attitudes (McLeod, 1992) and values (De Bellis & Goldin, 1999).

Scholars have underlined the role of the emotional component in mathematics learning processes (Zan et al., 2006) and in particular in problem solving processes (Adams & McLeod, 1989). The deep interplay between emotions and cognition is bidirectional: emotions affect cognitive processing in several ways, for example biasing attention and memory and activating action tendencies (Hannula, 2002); conversely the emotional experience is the result of a combination of cognitive analyses and physiological responses (Mandler, 1984). According to Mandler, it is not the experience itself that causes emotion, but rather the interpretation (influenced by individual's beliefs) that one gives to the experience. Understanding these interpretations is therefore central to answering the question of *where emotions come from*, and then to preventing and overcoming negative emotions towards mathematics (as hatred, anxiety and fear). These emotions may become very stable and have very negative influence on mathematical school performance, but more in general cause math-phobia also in adult age (Tobias, 1978; Buxton, 1981): Schlöglmann (2002) tells about stories of oppressive nightmares that are recalled even in adulthood.

In a longitudinal research study aimed to characterize students' attitude towards mathematics through the analysis of students' narratives (Di Martino & Zan, 2010) we suggested a model for attitude characterized by three dimensions: emotional disposition toward mathematics, vision of mathematics and perceived competence in mathematics. In this study we found recurrent patterns in the stories characterized by

failures: a particularly interesting case - because it may end up in a generalized rejection of mathematics - is the recurrent pattern with the emotional dimension involving *fear*.

Ortony Clore and Collins (1988), proposing a categorization of the basic emotions according to their cognitive source, classify fear as a valenced reaction to events, focusing on consequences for self, and "prospect-based". With the latter term the scholars mean emotions that are characterized as reactions to the prospect of an event, or to the confirmation (or disconfirmation) of the prospect of an event. In particular, fear arises when the individual is displeased about the prospect of an undesirable event. The variables affecting the intensity of fear are the degree to which the event is undesirable and the likelihood of the event. Then the cognitive component (belief) intervenes in the perception of the event, in the evaluation of the consequences for self (hence in the eliciting of the emotion itself), and in the evaluation of the likelihood of the event (hence in the intensity).

In this contribution we try to understand *where fear of math comes from*, analyzing how students narrate this emotion in their autobiographical essays.

METHODOLOGY

Scholars have used many different approaches to studying affect and in particular emotions: in our theoretical researches about affective constructs and their influence in the process of teaching and learning mathematics, we needed an instrument consistent with an interpretative approach, capable of capturing students' emotions, beliefs and attitudes towards mathematics, giving voice to the students through the possibility of talking about the aspects they considered relevant for their own experience with mathematics (Di Martino & Zan, 2010).

We recognized in the narrative approach the key to reach our methodological goals. The primary focus of a *narrative approach* is people's expressions of their experiences of life. These expressions emerge in particular from autobiographical narratives, where the narrator tends to paste fragments, introducing some causal links, not in a logical perspective but rather in a social, ethical and psychological one (Bruner, 1990). These links become a bridge between the writer's beliefs and emotions, highlighting the psychologically central role they play for the narrator.

We claim that in order to grasp the cognitive source of emotion, this pasting process is more important than an objective report of one's experience:

> There is as yet no known objective measures that can conclusively establish that a person is experiencing some particular emotion (…) In practice, however, this does not normally constitute a problem because we are willing to treat people's reports of their emotions as valid (…) we evaluate them as being appropriate or inappropriate, or justifiable or unjustifiable, not as being true or false. (Ortony, Clore, Collins 1988, p. 9)

Within a National Project in Italy, we asked schools across Italy to participate in a study about the students' attitude towards mathematics. In particular, we asked school directors to help us in collecting students' autobiographical essay "Me and mathematics: my relationship with maths up to now".

Essays were anonymous, assigned and collected in the class not by the class mathematics teacher. The study involved a large sample of students from different school levels: 874 from primary school (grade 1-5), 368 from middle school (grade 6-8), 420 from high school (grade 9- 13). The three different typologies of Italian high school (Technical Institute, Vocational School and Lyceum) were all represented.

In the field of affect the need for social and anthropological approaches, i.e. for studying affect in its natural contexts, is particularly stressed, motivating the use of non-traditional methods, such as narratives (see for example Kaasila et al., 2006).

As regards the analysis, we referred to Lieblich et al. (1998), who identify two main independent dimensions in the process of reading and analysing a narrative: holistic versus categorical, and content versus form. The combination of these two dimensions produces four modes of reading a narrative: holistic – content; holistic – form; categorical – content ('content analysis'); categorical – form.

Our analysis of students' narratives about fear used two of these modes of analysis in two subsequent steps: categorical - content mode to select the stories characterized by *fear of math*; holistic – content mode to analyze in the selected essays the view of math associated with fear, and the *causal* links introduced by pupils (for example through utterances such as 'I am afraid *because*...').

RESULTS AND DISCUSSION

In the collected essays students often report (almost one student out of six) fear of mathematics, using several tokens that refer to fear (the most recurrent are anxiety, terror, nightmare, distress, worry, fright).

Our data shows that fear associated with mathematics is nearly always fear of *not being able*. We might therefore claim that the undesirable event that triggers the emotion is failure in mathematics:

> "When Miss Sandra explains something new, I get goose bumps, because I'm afraid I won't be able to do the exercises" 4P.106[1] - grade 4
>
> "Problems are the thing I like less because I'm afraid I won't be able to solve them" 5P.135 - grade 5

[1] In the excerpts, the first number refers to the class level, the letter refers to the school level (Primary / Middle / High), the last number indicates the progressive numbering of the essay within the category.

If the undesirability of an event (in this case failure in mathematics) seems to be related to an individual's goals, the probability that this event takes place is based upon two elements: the subject identification of the factors needed to be successful in mathematics, which we call *factors of success*, and the evaluation of one's competence with reference to those factors. If failure in mathematics is viewed as a highly undesirable event, this subjective probability is, according to Ortony et al. (1988), what determines both the rising and the intensity of fear. In this framework, the individual's beliefs about the stability/modifiability of factors of success in mathematics and about the indispensability of such factors (i.e. his/her theories of success, Nicholls et al. 1990) come to be extremely relevant. As a matter of fact, a low perceived competence with respect to a certain factor of success considered unmodifiable and indispensable may induce a sort of *mathematical fatalism*. This status marks the renunciation to put all the available energies into the attempt to improve the situation (useless effort) and therefore may lead to the rejection of mathematical engagement. Moreover, in these cases, students feel virtually certain that the undesirable event (failure in math) will take place and they will then learn to associate fear to mathematics automatically and independent of the kind of activity they are given, the level of difficulty and any other consideration.

For example, amongst the factors of success that are most frequently associated with mathematics, in the essays we collected, there is intelligence:

> "In my opinion, to be good at mathematics one needs to be intelligent." 5P.120 - grade 5

In general, in the essays that describe unease in the relationship with mathematics and that talk about intelligence as a factor of success, a judgment on one's intelligence is given that appears definitive and unmodifiable:

> "I often think I am not that intelligent and I believe that when they tell me I am, and blame the fact I don't study, they make me feel even more stupid, thick and useless" 5H.2 - grade 13

In other words students seem to have what Dweck and Bempechat (1983) call an 'entity' theory of intelligence, i.e. a vision of intelligence as a rather stable global feature in opposition to a vision of intelligence as a continuously expanding repertoire of abilities and knowledge ('incremental' theory).

Recognizing intelligence as a necessary condition to be successful in math has extremely negative consequences on those who fail, because it implicitly carries the message that failing is a proof of scarce intelligence:

> "I must say that sometimes I was rather good at some kind of reasoning but then as soon as I got things wrong I convinced myself I was dull. I wouldn't like you to misunderstand the word I've written, I used to think I was obtuse, close-minded, unable to understand and reason. Maybe I had made up in my mind a little image of mathematics, a subject I hate." 2H.118 - grade 10

The idea that failure in mathematics is a proof of scarce intelligence probably bears on the high undesirability of failure in mathematics with respect to failure in others school subjects, eliciting - as we can grasp by the above excerpt - a strong negative emotional charge towards mathematics. Not surprisingly, this negative emotional charge may lead students to hate mathematics and to choose to avoid engaging with it, rather than repeatedly failing, thus getting a confirmation of being '*scarcely intelligent*'.

From the analysis of the students' narratives another recurrent aspect emerges: in many cases students seem to simply identify mathematics success with school achievement. Therefore the recognition of success or failure is completely delegated to another person (the teacher) that becomes the object of fear, since he/she is an agent of an undesirable event for the student:

> "Last year I did not get on well with the teacher because I was rather frightened by him" 2H.51 - grade 10

Sometimes it seems not clear for the narrator himself if fear is related to the teacher, to mathematics or to both:

> "My relationship with mathematics is not great, but this does not mean that I don't study it, it's that when I hear the word, even when I see the teacher, I'm terribly scared, and I forget everything. Maybe, my only fear is the teacher and not mathematics, I never understood why" 3M.38 - grade 8

If the student identifies success with school achievement, his/her beliefs about the behaviour rewarded by the teacher come to be determinant in the recognition of factors for success or failure.

What emerges from the analysis of the essays is that success in mathematics is often identified with being quick at solving mathematical tasks (mainly at early school levels) as well as with not making mistakes. Therefore, there is a sort of identification failure – "slowness and mistakes", which brings about two worrying phenomena.

On the one hand, being slow, sometimes with the additional perception of being slower than others, becomes a proof of inability, with heavy negative consequences on the emotional side:

> "While I am tackling a problem and I don't manage to solve it, after some or many minutes I hear the voices of my classmates shouting they have finished. The queue to get the problem corrected by the teacher becomes longer and longer and this makes me even more anxious" 5P.110 - grade 5

On the other hand, fear of mathematics becomes fear (or even terror) of making mistakes since the very beginning:

> “During tests I’m so afraid to make mistakes that I put lucky charms on the desk” 3P.46 - grade 3

> “I don’t like mathematics: it scares me a little bit and makes me anxious because I’m always terrified of making mistakes” 5P.77 - grade 5

Sometimes time and mistakes are explicitly linked to each other. On the one hand needing time to do things in mathematics becomes frustrating and may lead students to hurry up and force *distraction errors*:

> “What made me really sad was the fact I was able to solve most of the exercises in the tests, but because I was slow I run out of time and couldn’t finish the test. So sometimes I speeded up and then I made mistakes in the calculations” 5H.14 - grade 13

On the other hand, fear of making mistakes may lead students to slow down and does not allow the students to finish what they are doing:

> “One thing I really hate is mathematics assessment. I actually HATE it; I can’t stand it because I’m afraid to make mistakes and hence I do everything very slowly and then time finishes before I can complete my work” 5P.170 - grade 5

Such vision of failure in mathematics not only has negative consequences, but it is far from the real experience of anyone who has studied mathematics at higher level, which shows that one needs to take all the necessary time and that in new and non repetitive problems errors are possible and sometimes functional for the process.

In a sense, we might therefore define *epistemologically incorrect* this characterization of failure in mathematics linked to a way of considering errors as something to absolutely avoid, which is so widespread and transmitted in the classroom. But it is most of all a losing vision, since it may not support the effort which is necessary in problem solving, coming to generate the widespread phenomenon of math-anxiety:

> “Many times, when there are problems in the classroom, I get anxious, I’m afraid I can make mistakes, also when I am at home I’m afraid to get it wrong, because I don’t like to make mistakes” 5P.63 - grade 5

Moreover, fear of making mistakes has negative consequences from the point of view of the non-optimal exploitation of the cognitive resources (metacognitive aspects), and pupils themselves are aware of that, describing it as the main cause of their difficulties in mathematics:

> “My problem is not being unable to carry them out, but rather fear to make mistakes, as a matter of fact up to now in the oral tests I am always afraid of making mistakes, of giving the wrong answer, even though I know things” 2H.52 - grade 10

> “I just give up when, even in front of a simple division, only because I’m uncertain or I’m afraid to make mistakes” 3M.61 - grade 8

The influence of fear of making mistakes on the optimal management of one’s own resources, including cognitive ones, is felt by students themselves, to the point that some of them talk about their own fear of being scared:

> “When I do some mathematics tests, I am often, actually very often, scared and this causes problems in carrying them out, because I’m afraid that fear will make me make many mistakes, I get stuck because of that” 5P.97 - grade 5

Sometimes this fear of making mistakes works as an inhibitor (one prefers to stay silent rather than answering and risking to make a mistake):

> “I remember that when she asked me questions I was terrified not to know anything, and if she asked me something I was afraid to get it wrong, and I didn’t answer the questions” 2H.120 - grade 10

> “In the classroom, when we were doing exercises, I was always afraid that the teacher could ask me something I would not have been able to answer and, for fear of making a mistake, I used to staying silent” 5H.29 - grade 13

Fear of math caused by fear of making mistakes may have another negative consequence: it can elicit fear of the new, of anything that may cause difficulties, going beyond the mere repetitive exercise.

> “I remember that once, in grade 3, when we started to learn divisions, I was sort of scared of learning, scared of going ahead with the program” 5P.211 - grade 5

This fear of learning and of going ahead is obviously a big obstacle for learning itself (it is particularly dramatic that a very young pupil has this fear) and, moreover, mortifies the possibilities of mathematics education promoting the development of a taste for discovery and for investigating difficult problems.

> “When we deal with something new I always fear it is difficult” 5P.125 - grade 5

In the context of school mathematics, easy/difficult and like/dislike are often identified in a perverse way.

The weight of fear of mathematics, its oppressing role and therefore the importance of fighting against this feeling with purposeful teaching interventions, comes up in the stories that narrate how this emotion can be overcome:

> “During the past few years my mind changed, and fear of mathematics changed with it. In the first year of primary school, I thought mathematics was a monster, with difficult numbers, which went around in my head all at the same time, with no rest. In the following years, the monster of mathematics became smaller and smaller and I got bigger and bigger. That tiny little monster later disappeared; and from that moment onwards I started to love mathematics. I was no longer annoyed by all those numbers, on the contrary, it made me happy, I was content, anxious to

> see the next problem to be tackled. I was there, ready with paper and pencil, ready to rack my brains. I found something in mathematics I had never found before, due to fear which increased and increased and never stopped, until my strength managed to have control over it and push away from my thoughts." 5P.219 - grade 5

In the stories talking about the overcoming of one's fear of mathematics, the teacher is often the decisive figure to determine the change:

> "I remember that every time I had a mathematics test at school I felt a strong pain in my stomach and in my head, which made me feel not so ready, so that when the day of the test arrived, my uncertainty led me to make mistakes. Fortunately this is not happening nowadays, and I cannot explain this change, but with this teacher I feel more confident and free from my fears." 2H.119 - grade 10

CONCLUSION

The analysis we carried out shows that fear of mathematics, that it is often correlated to fear of failure, has a strong cognitive origin and at the same time, influences the possibility of managing one's own cognitive resources in the best possible way. Fear of math is a possible factor of local failure (since it does not allow for the best use of one's competence and knowledge) but also a possible factor of global failure (since it might lead to give up engagement with mathematics).

From the stories of reconciliation with mathematics emerges that getting over fear of mathematics is the primary and essential condition to be able to start a new and positive relationship with the discipline.

All these remarks charge the teacher with responsibilities, since he/she can convey but also modify a particular vision of mathematics: for example, the weight students attach to mistakes is influenced by the teacher's management and evaluation of mistakes.

In our study, a strong interplay between the rise and the intensity of fear of mathematics (viewed as a prospect based emotion) and the student's beliefs about success in mathematics (theories of success) and about self, emerges. Within a student's theories of success, the interaction among his beliefs about those we called 'factors of success', about his perceived competence related to these factors, and about their modifiability, appears particularly significant in order to interpret the evolution of fear toward fatalism. Consequently, in order to get over the students' fear of mathematics we have to work on both the vision of the discipline and one's own perceived competence, in particular sharing the modifiability of the *factors of success* in mathematics.

This action needs that the teachers are prepared to eventually modify their own beliefs towards mathematics and its teaching in the first place.

Moreover, another point of interest is the relationship between teachers emotions towards math and their students emotions towards math. This appears to be particularly in the case of primary teachers, that are not specialist in math, and often have had a troubled story with mathematics. Recent studies (Di Martino & Sabena, 2011) show as many pre-service primary teachers have fear of math and are terrorized by the idea of having to teach it.

These results confirms the need to go beyond the purely cognitive also in the research about teacher development, trying to provide teachers the theoretical instruments to interpret, analyze and counteract the students' fear of math, but also to overcome their own eventual fear of math.

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