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CHANNEL PARAMETERS ESTIMATION FOR COGNITIVE RADAR SYSTEMS

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ABSTRACT

This paper deals with the problem of a cognitive radar system that shares the same frequency band with a communication system, supposed to be the primary user of channel. A cognitive algorithm is proposed to estimate the channel parameters that describe the behaviour of the primary user and how to exploit these estimates to minimize the interference between the radar and the communication system. The performance of the proposed algorithm are assessed in terms of probability of collision, that is the probability that the radar transmits when the primary user already occupies the channel, and probability to lose a spectrum opportunity, that is the probability that the radar does not transmit when the channel is free.

***Index Terms*—**Cognitive Radar, Spectrum Opportunity, Hidden Markov Model, Channel Parameters Estimation.

1. INTRODUCTION

In recent years, the increasing demand of communication services and applications is pushing towards the redefinition of international regulations, to allocate these applications in the bands previously used by radar systems [1]-[8]. Consequently, the availability of frequency spectrum for radar sensors is going to be severely reduced. On the other hand, owing to the introduction of new requirements, in particular in the field of imaging, future radar sensors should be able to work on a wider frequency range than in the past.

In order to cope with the above two conflicting requirements, future radars should be able to coexist with other radio frequency systems and therefore they require the ability to recognize and react to the behaviour of other users radiating in the same operational environment. This, in turn, leads to the need of new methodologies and techniques, based upon cognition as enabling technology.

The cognitive methodology proposed here to reduce mutual interference between the radar and the other radiating elements is based on two main concepts: Spectrum Sensing and Spectrum Sharing. Spectrum Sensing has the goal to recognize the frequency bands used by other systems using the same spectrum in real time, while Spectrum Sharing has the goal to limit interference from the radar to other services and vice versa.

Through these functions, a cognitive radar can obtain necessary observations about the radio frequency channel, such as the presence of other users and the appearance of spectrum opportunities, i.e. spectrum holes where it is possible to transmit without interfering with other users of the channel. After using this information, a cognitive radar is able to adapt its transmitting and receiving parameters, such as the transmission power and the operating frequency, in order to achieve efficient spectrum utilization.

In this work, we focus on the problem of a radar system that shares the same frequency band with a communications system, assumed to be the primary user of the channel. In cognitive radio terminology, primary users can be defined as the users who have higher priority or legacy rights on the usage of a specific part of the spectrum. On the other hand, the cognitive radar, assumed to be the secondary user, having lower priority, exploits the spectrum in such a way that it does not cause interference to primary users. In particular, to avoid interference, the cognitive radar must avoid to transmit in the frequency channels occupied by the primary user.

As an illustrative example, Figure 1 shows the spectrum opportunities in the frequency channels. As apparent, the available spectrum is divided into narrower chunks of bands. Spectrum opportunity in this dimension means that not all the bands are used simultaneously; therefore, some bands might be available for opportunistic usage.

In the problem analyzed here, the Pulse Repetition Interval (PRI) of the radar system and the time slot duration of the communication system are assumed to be of the same time duration, therefore for each frequency channel at the time of transmitting (i.e. at the beginning of each PRI), the radar is not able to measure if the frequency channel is effectively occupied by the communication system.

In any case, analyzing the behaviour of the primary user in the frequency channel and exploiting the time history of the channel occupancy, the cognitive radar system can evaluate the probability to have a spectrum opportunity, i.e. the probability that the monitored frequency channel is free at the time of transmitting.

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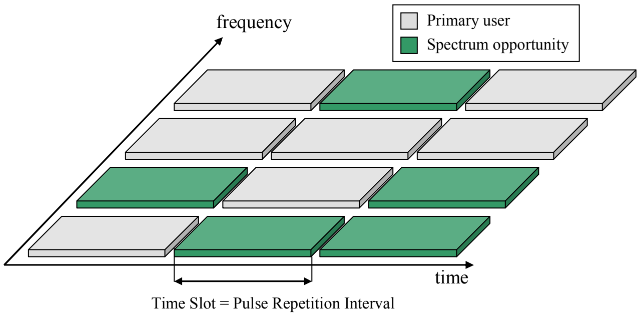

Figure 1 – Spectrum Opportunities.

Evaluating this probability, the cognitive radar can decide if it is possible or not to transmit in the monitored frequency channel in the current time slot at the beginning of each PRI.

The rest of this paper is organized as follows. Section 2 introduces a model to describe the primary user channel usage based on HMM (Hidden Markov Model). Section 3 describes how to estimate the main parameters of the channel model, while Section 4 describes how to evaluate the probability to have a spectrum opportunity using the parameter estimates. Some simulation results are reported in Section 5, that show how using the proposed cognitive methodology it is possible to improve the performance with respect to a conventional radar system. Section 6 reports some conclusions and final remarks.

2. CHANNEL MODEL

For minimizing interference to primary users while making the most out of the opportunities, the cognitive radar should keep track of variations in spectrum availability and, exploiting the history of the spectrum usage information, should make predictions of the future profile of the spectrum. In order to model the channel usage patterns of primary users, HMMs are proposed in [9]-[11]. HMMs have been used in several contexts, such as speech recognition, characterization of errors in digital radio channels, and statistical analysis of Internet communication channels.

In the context of dynamic spectrum access networks, HMMs are used to model the primary user occupancy of the channel. HMMs represent a useful tool for this problem since true occupancy states are not always known to the cognitive radar after the Spectrum Sensing process.

As discussed, the cognitive radar periodically scans and senses each frequency channel and exploits the received signal to evaluate if the channel is busy or free. In the open literature there are several Spectrum Sensing techniques, such as the energy detector, feature detector or matched filtering detection techniques [12], to recognize if the channel is occupied by the primary user. In particular, at each time slot, the cognitive radar records an observation symbol O_n depending upon the channel status, as follows

$$\begin{cases} O_n = 0, & \text{channel free,} \\ O_n = 1, & \text{channel busy.} \end{cases} \tag{1}$$

The radar periodically makes the observations and records an observation sequence $\boldsymbol{O}=O_1\ldots O_N$ over a period of N time slots. In this work, the sequence $\boldsymbol{O}$ of symbols “0”s and “1”s is modelled as an HMM.

In general, a HHM is comprised of a set S_n of possible states and a set O_n of possible emissions. In our case, the possible states represent the real activity of the primary user. If the primary user is transmitting at time slot n, the state is $S_n=1$, otherwise, if the channel is free, the state is $S_n=0$. However, due to the noise in the channel, a free channel can be classified as busy and a busy channel classified as free. Therefore, there are also two possible emissions, which are represented by the observation O_n at the output of the spectrum sensing detector.

Two statistical parameters describe the HMM. The first parameter **A** is the 2×2 state transition probability matrix representing the probabilities associated with changing from one state to another and it is given by

$$[\mathbf{A}]_{ij} = a_{ij} = \Pr\left[S_n = i \mid S_{n-1} = j\right], \quad i,j=0,1. \tag{2}$$

Similarly, the second parameter **B** is a 2×2 emission probability matrix, which represents the probabilities associated with obtaining a certain output given that the model is currently in a true state s_n and it is given by

$$[\mathbf{B}]_{ik} = b_i(k) = \Pr\left[O_n = k \mid S_n = i\right], \quad i,k=0,1. \tag{3}$$

In our context, the emission probability matrix **B** is related to the Receiver Operating Characteristic of the Spectrum Sensing detector. As a matter of fact, $b_0(1)$ is the probability of false alarm, that is the probability to classify a free channel as busy, while $b_1(0)$ is the probability of miss detection, that is the probability to classify a busy channel as free. Clearly, $b_0(0)=1-b_0(1)$ and $b_1(1)=1-b_1(0)$. These probabilities depend on the channel noise, the kind of signal emitted by the primary user and the spectrum sensing detector used at the cognitive radar receiver, that is on the specific characteristics of the systems that share the same channel. Knowing these characteristics, the elements of **B** can be calculated or evaluated through Monte Carlo simulations, therefore, without loss of generality, hereafter we assume that matrix **B** is known. The HMM model for channel occupancy is illustrated in Figure 2.

3. CHANNEL PARAMETERS ESTIMATION

As discussed, the statistical parameters that describe each frequency channel are the state transition probability matrix **A**, the emission probability matrix **B,** and the initial state distribution $\boldsymbol{\pi}=\{\pi_i\}$, defined as

$$\pi_i = \Pr\left[s_1 = S_i\right], \quad i=0,1. \tag{4}$$

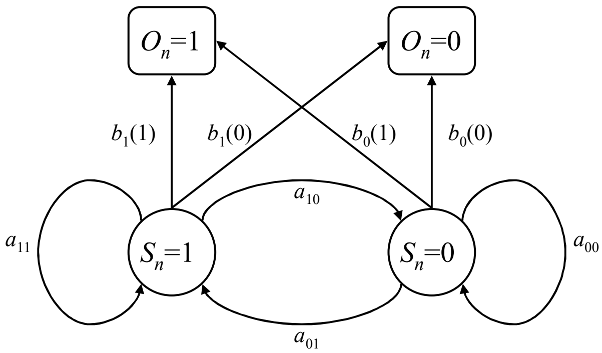

Figure 2 - HMM representation of spectrum occupancy.

In this work, matrix **B** is assumed to be a-priori known, so the problem of channel parameter estimation is to determine a method to estimate the model parameters **A** and $\boldsymbol{\pi}$ using a finite observation sequence $\boldsymbol{O}=O_1\ldots O_N$ of N elements. The observation sequence used to adjust the model parameters is called training sequence since it is used to "train" the HMM. There is no way to solve analytically this problem [13]. In fact, given any finite observation sequence as training data, there is no optimal way of estimating the model parameters.

However, the most widely adopted iterative procedure is the Baum-Welch method (closely related to the Expectation-Maximization method) [10],[11],[13],[14], in which the parameters **A** and $\boldsymbol{\pi}$ are chosen such that $\Pr[\boldsymbol{O}|\mathbf{A},\boldsymbol{\pi}]$ is locally maximized.

In order to describe the iterative procedure for estimation of the HMM parameters, first we must define some useful variables. First consider the forward variable $\alpha_n(i)$ defined as

$$\alpha_n(i) = \Pr\left[O_1O_2...O_n, s_n = S_i \mid \mathbf{A}, \boldsymbol{\pi}\right] \qquad (5)$$

That is the probability of the partial observation sequence $O_1\ldots O_n$ and state S_i at time n, given the channel parameters **A** and $\boldsymbol{\pi}$. The forward variable can be inductively calculated initializing

$$\alpha_1(i) = \pi_i b_i(O_1), \quad i=0,1 \qquad (6)$$

and iterating

$$\alpha_{n+1}(j) = \left[\sum_{i=0}^{1} \alpha_n(i) a_{ij}\right] b_j(O_{n+1}), \quad 1\leq n\leq N\text{-}1, j=0,1. \quad (7)$$

In a similar manner, the backward variable $\beta_n(i)$ is defined as

$$\beta_n(i) = \Pr\left[O_{n+1}O_{n+2}...O_N \mid s_n = S_i, \mathbf{A}, \boldsymbol{\pi}\right], \qquad (8)$$

that is the probability of the partial observation sequence from n+1 to N, given state S_i at time n and the channel parameters **A** and $\boldsymbol{\pi}$.

Similarly, $\beta_n(i)$ can be solved inductively initializing

$$\beta_N(i) = 1, \quad i=0,1 \qquad (9)$$

and iterating

$$\beta_n(i) = \sum_{j=0}^{1} a_{ij} b_j(O_{n+1}) \beta_{n+1}(j), \quad n=N\text{-}1,\ldots,1;\ i=0,1. \quad (10)$$

Another important variable is the probability

$$\gamma_n(i) = \Pr\left[s_n = S_i \mid \boldsymbol{O}, \mathbf{A}, \boldsymbol{\pi}\right], \qquad (11)$$

that is the probability of being in state S_i at time n, given the observation sequence $\boldsymbol{O}$ and the channel parameters **A** and $\boldsymbol{\pi}$. This probability can be expressed simply in terms of the forward-backward variables

$$\gamma_n(i) = \frac{\alpha_n(i)\beta_n(i)}{\sum_{j=0}^{1} \alpha_n(j)\beta_n(j)}, \quad i=0,1. \qquad (12)$$

Concluding, for the iterative estimation of the HMM parameter we must define the probability of being in state S_i at time n and state S_j at time n+1, given the observation sequence $\boldsymbol{O}$ and the channel parameters **A** and $\boldsymbol{\pi}$

$$\xi_n(i,j) = \Pr\left[s_n = S_i, s_{n+1} = S_j \mid \boldsymbol{O}, \mathbf{A}, \boldsymbol{\pi}\right], \quad i,j=0,1. \quad (13)$$

From the definitions of the forward and backward variables, we can write (13) in the form [13]

$$\xi_n(i,j) = \frac{\alpha_n(i) a_{ij} b_j(O_{n+1}) \beta_{n+1}(j)}{\sum_{i=0}^{1}\sum_{j=0}^{1} \alpha_n(i) a_{ij} b_j(O_{n+1}) \beta_{n+1}(j)}, \quad i,j=0,1. \quad (14)$$

It is easy to verify that, using (13), the probability in (11) is given by

$$\gamma_n(i) = \sum_{j=0}^{1} \xi_n(i,j), \quad i=0,1. \qquad (15)$$

If we sum $\gamma_n(i)$ over the time index n, we get a quantity which can be interpreted as the expected (over time) number of times that state S_i is visited, or equivalently, the expected number of transitions made from state S_i. Similarly, summation of $\xi_n(i,j)$ over n (from n=1 to n=N-1) can be interpreted as the expected number of transitions from state S_i to state S_j.

Using (12) and (14) with the concept of counting event occurrences, it is possible to define a method to iteratively estimate the parameters of an HMM.

Considering that the ij–th element of the state transition probability matrix **A** can be considered as the ratio of the expected number of transitions from state S_i to state S_j and the expected number of transitions made from state S_i, it is possible to estimate the elements of **A** by using the following equation

$$\hat{a}_{ij} = \frac{\sum_{n=1}^{N-1} \xi_n(i,j)}{\sum_{n=1}^{N-1} \gamma_n(i)}, \quad i,j=0,1. \tag{16}$$

Similarly, the initial state distribution π_i can be considered as the expected number of times in state S_i at time n=1, therefore we can estimate $\boldsymbol{\pi}$ using

$$\hat{\pi}_i = \gamma_1(i), \quad i=0,1. \tag{17}$$

If we define the current channel parameters **A** and $\boldsymbol{\pi}$ and we use them to compute (12) and (14), and we define the re-estimated channel parameters as $\hat{\mathbf{A}}$ and $\hat{\boldsymbol{\pi}}$, determined from (16) and (17), then it has been proven in [15] and [16] that the model described by $\hat{\mathbf{A}}$ and $\hat{\boldsymbol{\pi}}$ is more likely than the model described by **A** and $\boldsymbol{\pi}$, in the sense that $\Pr[\boldsymbol{O}|\hat{\mathbf{A}},\hat{\boldsymbol{\pi}}]>\Pr[\boldsymbol{O}|\mathbf{A},\boldsymbol{\pi}]$, i.e., we have found a new set of channel parameters from which the observation sequence is more likely to have been produced.

Based on the above procedure, if we iteratively use $\hat{\mathbf{A}}$ and $\hat{\boldsymbol{\pi}}$ in place of **A** and $\boldsymbol{\pi}$ and repeat the re-estimation, we can improve the probability of $\boldsymbol{O}$ being observed from the model until some limiting point is reached.

The final result of this procedure is a maximum likelihood (ML) estimate of the HMM [13].

This procedure is called Baum-Welch method and it is summarized in Table 1.

By Monte Carlo simulation we evaluated that using 30 iterations the algorithm converges to a stable estimate of **A** and $\boldsymbol{\pi}$. Fig. 3 shows the Root Mean Square Error (RMSE) of the estimation of the elements of **A** as a function of the number of elements of the observation sequence N.

These results have been obtained through 10^3 Monte Carlo runs by random generating a_{00} and a_{11} as independent and identically distributed (IID) random variables, uniformly distributed in [0,1]. Considering that (12) and (14) measure the expected number of transitions from one state to the other, it is clear that in order to have a good estimate of **A**, we need an high value of N, when the number of elements of the observation sequence is too low the estimate of **A** is biased.

Input: observation sequence $\boldsymbol{O}=O_1\ldots O_N$ initialize **A** and $\boldsymbol{\pi}$ **for** k=1:MaxIter calculate $\gamma_n(i)$ and $\xi_n(i,j)$ from **A** and $\boldsymbol{\pi}$ estimate $\hat{\mathbf{A}}$ and $\hat{\boldsymbol{\pi}}$ from $\gamma_n(i)$ and $\xi_n(i,j)$ substitute **A** and $\boldsymbol{\pi}$ with $\hat{\mathbf{A}}$ and $\hat{\boldsymbol{\pi}}$. **end** **Output**: estimate of **A** and $\gamma_n(i)$, n=1,…N; i=0,1.

Table 1 – Baum-Welch procedure.

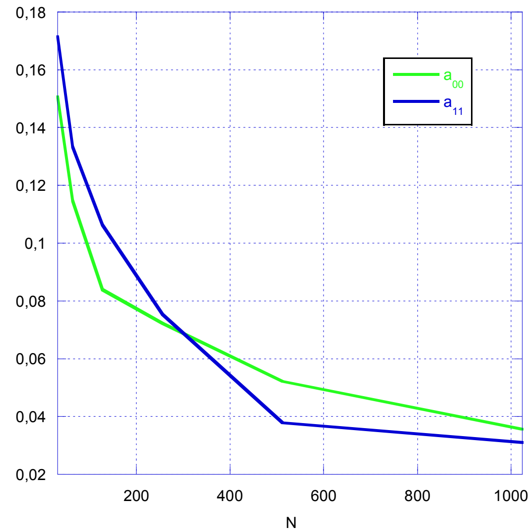

Figure 3 – RMSE in the estimation of **A** as a function of the number of elements of the observation sequence.

4. PERFORMANCE ANALYSIS

In the previous section, we showed how to estimate the channel parameters using a finite observation sequence. In this section, we show how the cognitive radar at the beginning of each PRI exploits these estimates to decide whether to transmit or not, to avoid interference with the primary user. We also show some simulation results that highlight how the proposed methodology can provide good radar performance in the presence of the user and low impact on the performance of the primary user by the presence of the radar. As discussed in previous sections, in the analysed scenario, at the beginning of each PRI the cognitive radar is not able to evaluate if the operating channel is free or busy. However, using the channel parameter estimates obtained from the last N channel observations, the cognitive radar can calculate the probability that in the current PRI the channel is free, i.e. the probability to have a spectrum opportunity; if this probability is sufficiently high, the cognitive radar transmits, and otherwise the radar does not transmit.

Figure 4 shows how the radar processes the continuous sequence of observations at the output of the spectrum-sensing detector. As mentioned, each observation symbol 0/1 is received each PRI. Since the estimation of **A** and $\gamma_n(i)$ is time consuming, the radar receiver performs these estimates using non-overlapping blocks of N elements, in each block the initialization is performed using the channel parameter estimates of the previous block. As showed in Figure 4, the channel parameter estimates performed in each block are used to evaluate the probability to have a spectrum opportunity using a sliding window of dimension N. The sliding window collects the last N observations received in the N previous PRIs to the current PRI, where the radar must decide if he can occupy the frequency channel.

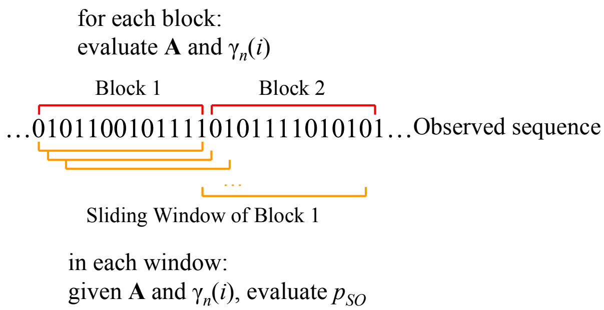

Figure 4 – How to process the observed sequence.

There are N sliding windows for each block, in particular in the k-th sliding window, using the estimate of $\mathbf{A}$ and fixing $\pi_i = \gamma_k(i)$, the signal processor of the radar evaluates the forward and the backwards variables using (6)-(10). Therefore, similarly to (12), evaluates the probability that the last observation in the sliding window corresponds to the channel state S_i, that is

$$\gamma(i) = \frac{\alpha_N(i)\beta_N(i)}{\sum_{j=0}^{1}\alpha_N(j)\beta_N(j)}, \quad i=0,1. \tag{18}$$

This probability is used to evaluate the probability to have a spectrum opportunity

$$p_{SO} = \gamma(0)a_{00} + \gamma(1)a_{01}, \tag{19}$$

i.e. the probability that in the previous PRI the channel was free and in the current PRI it remains free plus the probability that in the previous PRI the channel was busy and in the current PRI it becomes free. The signal processor compares the probability to have a spectrum opportunity with a threshold λ, and transmits only if the probability is greater than λ.

There are two kinds of errors. The first one e_0 is the event in which the cognitive radar does not transmit and the channel is free, i.e. the probability to lose a spectrum opportunity. The other kind of error e_1 is the case in which the radar transmit and the channel is occupied by the primary user, i.e. the probability to have a collision.

Figure 5 shows the probability of these two errors as a function of the threshold λ, this graph can be used to tune the cognitive radar to the desired performance. These results have been obtained through 10^3 Monte Carlo runs by random generating a_{00} and a_{11} as independent random variables uniformly distributed in the range [0,1].

It is clear that when threshold λ is zero, the radar is always transmitting, therefore the probability of e_1 coincides with the probability that the channel is busy, that for the matrix $\mathbf{A}$ that we used in our simulation, is 0.5. Similarly, when the threshold λ is one, the radar never transmits and the probability of e_0 coincides with the probability that the channel is free, that in our particular case, is 0.5.

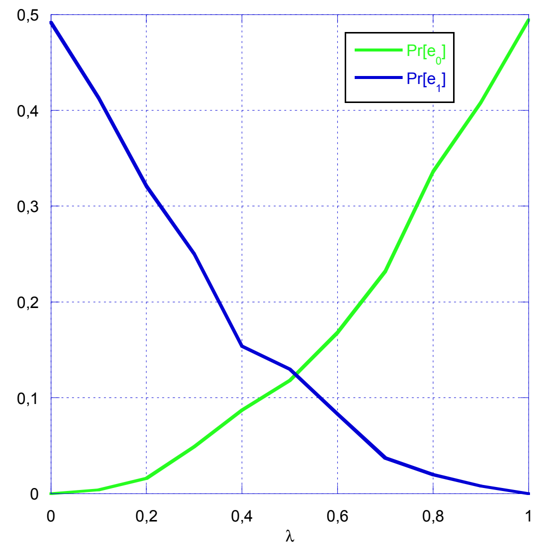

Figure 5 – Probabilities of e_0 and e_1 as a function of λ.

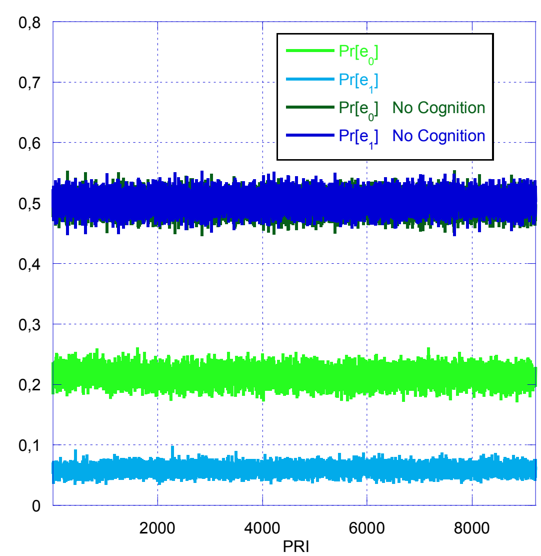

Figure 6 - Probabilities of e_0 and e_1 as a function of time.

Figure 6 shows the probability to lose a spectrum opportunity and the probability to have a collision as a function of time, observing the performance of the system for 9246 PRIs (i.e. 9 blocks of 1024 elements). These results have been obtained through 10^3 Monte Carlo runs, generating a_{00} and a_{11} as IID random variables uniformly distributed in [0,1] and fixing the threshold λ to 0.65.

The simulation results show how the performance of a cognitive radar that adopts the proposed methodology are constant during the time and much better than the performance of the non cognitive radar that always transmits ignoring the presence of the primary user and than the radar that never transmits to avoid interference with the primary user of the channel.

5. CONCLUSIONS

Since the availability of frequency spectrum for radar sensors is continuously diminished and fragmented, next generation radar systems should be able to operate in spectrally dense environments, coexisting with other systems operating in the same frequency channel. For this reason, an important system requirement is the ability to recognize and react to the behaviour of other users radiating in the same operational environment that, in turn, leads to the need of new methodologies and techniques, based upon cognition as enabling technology.

This paper describes a technique to estimate the channel parameters that model the behaviour of the primary user of the channel, and propose a cognitive method that, exploiting these estimates, enables a radar to operate in a spectrally dense environment. The performance of the cognitive radar are evaluated in terms of probability to lose a spectrum opportunity and probability to have a collision with the primary user of the channel. The numerical results suggest that the proposed cognitive algorithm lowers the mutual interference between the radar and the primary users.

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