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Some Remarks on Negation and Quantification in Leibniz's Logic

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ABSTRACT. We argue, with special reference to various passages from Leibniz's *Generales Inquisitiones*, that the simultaneous presence of two switches concerning the negation operator is the main source of the difficulties raised by many people against his logical investigations. The former switch carries negation from outside to inside of the proposition to which it is applied; the latter is the so-called *obversion* law (which asserts the equivalence of "non est" and "est non"). Applied in sequence, the two switches transform an outside (propositional or *de dicto*) negation into a conceptual (predicative or *de re*) negation. We also investigate the further difficulties which arise from the interplay between these switches and Leibniz's use of indefinite letters.

KEYWORDS: Leibniz's logic; negation; obversion law; quantification.

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1. Introduction

The critical edition of *Generales Inquisitiones* (*G. I.* for shorthand), published in 1999 by the Berlin Akademie Verlag in the Vol. A VI 4A of the *Sämtliche Schriften und Briefe*, promoted new interests in Leibniz's logic, as witnessed by new translations and many both specific and general papers devoted to these topics[1]. Often scholars stressed some mistakes made by Leibniz, notably the assertion of the false equivalence, made in § 82 of *G. I.*, between "B non est A" and "B est non-A" together with all consequences which can be drawn from it.

It is well known that logical matters, especially those concerning negation and its interplay with quantification (which we are dwelling upon here), are intertwined with many other more general features of Leibniz's investigation, metaphysical, theological, ethical, etc. Here, however, we will restrict ourselves to the logical ones, trying to find a rationale for Leibniz's "mistakes".

2. The obversion law

It is worth reminding that the previously mentioned equivalence of § 82 (the so called *Obversion Law*, henceforth O. L.) was rejected within the Aristotelian tradition because the truth conditions of "B est non A" include non-emptiness of the subject B, whereas "B non est A" is always true if no B does exist. In order to rightly understand Leibniz's way to O. L., we judge opportune to take in account § 80 of *G. I.*, where he envisages the possibility of eliminating infinite terms (like, for instance, "non-A"), assuming that "non-A" means "is qui non est A". This is reminiscent of Ockham's definition of the infinite term "non-homo" as "aliquid quod non est homo", so that a proposition as "Asinus est non homo" is paraphrased (by *expositio*) as "Asinus est aliquid et asinus non est homo" (cf. *S. t. L.* pars II, cap 12). Since Leibniz is not bothered about non denoting subjects, he drops the existential component and "Asinus est non homo" becomes simply "Asinus non est homo".

However, in spite of the clear endorsement of O. L. made in § 82, subsequently, in § 92 Leibniz seems to argue against what he previously stated, as-

[1] We limit ourselves to record the Italian translation of *Generales Inquisitiones*, with an important commentary, made by M. Mugnai in 2008 (see Mugnai (ed.) (2008)), and Lenzen (1995) (both containing many references to secondary literature).

serting that the step from "A non est non-B" to "A est B" does *not* constitute a *bona consequentia*. He offers evidence for his thesis providing the following counterexample: "All animals are men" does not follow from the falsehood of "All animals are not-men". In order to understand why and how this counterexample works we have to introduce a second switch concerning negation (after the switch involved in O.L.), this time from outside to inside the proposition. In fact, the *consequentia* labelled as *non bona* by Leibniz can be so detailed:

Assume that:

1) It is **false** (omne animal esse non-homo)

From (1), by the "outside-inside" switch of negation, we get:

2) Omne animal **non** est non-homo.

Applying the O.L., (2) is transformed into:

3) Omne animal est **non**-non-homo.

Lastly, by an application of the double negation law, we get:

4) Omne animal est homo.

Keeping aside the unquestionable double negation law, applied in the last step, what compels Leibniz to reject the above argument is the *combination* of the two switches concerning negation (both separately accepted by Leibniz). It is worth stressing that by putting the two switches in sequence from a propositional negation we get a conceptual negation: a passage which strongly resembles the *de dicto-de re* transformation. In both cases the equivalence between what we started from and what we get is absolutely not guaranteed.

At times, however, Leibniz does not pay much attention to unwelcome consequences of applying in sequence the two switches. In fact, in spite of the more or less explicit refusal of applying both switches of negation contained in the previous counterexample, they seem to be still at work in the argument offered in § 94. In this paragraph he provides a proof that "A est B" follows from "non-B est non-A". Reasoning by the contraposition rule, he shows that "Non (non-B est non-A)" follows from "non (A est B)":

non (A est B)	→ A non est B	switch of negation
	→ A est non B	by O. L.

$\rightarrow$ Quoddam non-B est A by the conversion law
$\rightarrow$ Non (Quoddam non-B est non-A)
$\rightarrow$ Non (non-B est non-A)

In this derivation various steps deserve some remarks. Firstly, the second line states that universal negative propositions follow from particular ones, which is obviously false. In fact, since Leibniz intends "A est B" as an abridged form for "Omne A est B", its negation should produce a particular proposition. In this way, however, it would become impossible to apply the conversion law in order to obtain "Quoddam non-B est A". Secondly, in order to justify the third step, from "Quoddam non-B est A" to "Non (Quoddam non-B est non-A)", Leibniz firstly exploits what he previously stated in § 91 and obtains:

Quoddam non-B est A $\rightarrow$ Quoddam non-B non est non-A;

and then, by the "inside-outside" switch of the negation, he can conclude:

(*) Quoddam non-B non est non-A $\rightarrow$ Non (Quoddam non-B est non-A).

Finally, the completion of the alleged proof is obtained by an *a fortiori* argument: the falsity of a particular proposition entails the falsity of the corresponding universal. Moreover, it is worth stressing that, besides exploiting the switches of negation, the starred step relies on a peculiar use of the quantified term "Quoddam non-B". In fact, this step makes sense only if "Quoddam non-B" is considered as a *singular* term.

2. Indefinite letters and quantification

It is now good time to add some more details about the relationship between negation and quantified terms. From a general point of view, O.L. is justified only within an extensional framework, whereas the "inside-outside" switch of negation holds only within an intensional framework. In particular, if the subject of a proposition is a non-quantified universal term (it is the case in which Aristotle speaks of propositions that, while having a universal term for subject, are not universal[2]) from an intensional point of view only the "outside-inside (and vice versa)" switch holds, whereas from an extensional point of view only O. L. is valid. On the contrary, if the subject A of a proposition is

[2] Cf. *De Int*. 7, 17b7-12.

quantified, only O.L. holds, no matter whether the proposition in which A occurs is interpreted either intensionally or extensionally. For instance, “Homo non est albus” intensionally considered means that the property to be white does not occur in the notion of man; and this is the same meaning conveyed by “It is not true that Homo est albus”. On the contrary, “Homo est non-albus” means that the property not to be white occurs in the notion of man, which asserts something different from the previous proposition. Moreover, from an extensional point of view “Omnis homo non est albus” and “Omnis homo est non-albus” express the same meaning.

Another relevant matter we have to dwell upon is Leibniz's use of indefinite letters. Sometimes Leibniz tries to express the propositions of the Aristotelian square by identities in which capital Latin letters X, Y, Z united with a term A serve to denote *a certain* A, whereas the same letters, when underlined[3], stand for *any* A. This different use of indefinite letters is drawn by Leibniz in § 81: “$\underline{Y}$ seu Y indefinita cum lineola mihi significat quilibet , Y est unum incertum, $\underline{Y}$ est quodlibet”. Later on, in § 112, Leibniz will have to face notable difficulties in his attempt to express the square of oppositions by exploiting indefinite letters. Granted that “AY est B” expresses the I-proposition (*Quoddam A est B*), we gather from his remarks concerning a negative context that “A$\underline{Y}$ est B” expresses the A-proposition (*Quodcumque A est B*). So far so good. Troubles come in when Leibniz tries to formalize negative propositions. The latter should be built up by negating affirmative propositions of the opposite quantity. On the contrary, according to Leibniz, negating “AY est B”, i. e. asserting “Non (AY est B)”, is equivalent to assert “AY non est B”, which is again a particular proposition. In a similar vein negating “A$\underline{Y}$ est B”, i.e. asserting “Non (A$\underline{Y}$ est B)”, is equivalent to assert “A$\underline{Y}$ non est B”, which is again a universal proposition. It seems evident that here a general Leibniz's disposition comes to surface, which we have already seen analysing § 94 and which consists in treating the expressions “AY” and “A$\underline{Y}$” like singular terms. This fact, we think, is able to explain why Leibniz is prone to apply in an unproblematic way the “outside-inside” switch of negation.

A possible, and, so to speak, more contingent explication of Leibniz's procedure is given by the fact that in this paragraph underlined letters occur in negative contexts, so that the distinction is between “*Quoddam A est B*” and

[3] Instead of Leibniz's superscript letters, for typographical reasons we will use underlined letters.

"[*nego*] *Ullum A esse* B". Here probably Latin language itself helps to get the mistake, since "[*nego*]*Ullum A esse* B" drives immediately, almost spontaneously, we could say, to "*Nullum A est B*", which means that by negating "A$\underline{Y}$ est B" we get again a universal proposition.

In general, Leibniz's use of indefinite letters can be considered his way to provide an algebraic treatment of the medieval theory of *distributio*. This theory, however, is at odds with the above mentioned "outside-inside" switch of negation. Many of the problems Leibniz meets stem from his applying this switch in propositions where quantified terms have been replaced by indefinite ones.

Let us consider Leibniz's use of indefinite terms within the general framework of the theory of distribution:

AY est…	Quoddam A est..	A is not distributed
A$\underline{Y}$ est…	Quodlibet A est..	A is distributed
Non (AY est…)	Quoddam A non est …	A is distributed
Non (A$\underline{Y}$ est…)	Quodlibet A non est…	A is not distributed

Some remarks are here in order. In the theory of distribution a negative sign turns every distributed term in its scope into a non distributed one, and viceversa. But, on the contrary, we have already seen that, according to Leibniz, YA and $\underline{Y}$A work nearly as individual terms: as a consequence, "non(YA est B)" is the same as "YA non est B" and "non($\underline{Y}$A est B)" is the same as "$\underline{Y}$A non est B".

2. Conclusion

In spite of the clear differences just stressed, we think that Leibniz's use of indefinite letters is much more close to the medieval theory of *suppositio* and *distributio* than to the post-fregean use of variables in quantification theory. Indeed, indefinite letters work still as modifiers of the term A they are applied to, producing either an arbitrary, though fixed, object enjoying A, or an undetermined and distributed object enjoying A (i.e., a *generic* A).

The previous remarks, which are of course in need of a development far more extended than it is possible to provide here, point of course against claims sometimes made (for instance by W. Lenzen[4]) to the aim of supporting the presence of a true *quantification* theory in Leibniz's logical works. In this context, we rate sufficient, in conclusion, to remark that one thing is to use the machinery of quantifiers as a useful heuristic tool[5], and a completely different thing is to claim that Leibniz's introduction of indefinite letters could be seen as an anticipation of modern quantification theory.

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[4] See Lenzen (1990).

[5] For instance, nothing prevents us from using "$\exists Y(A=BY)$" to explain the meaning of "A= BY", the identity between terms which Leibniz sometimes uses in order to express the A-proposition "A est B".