The relativity experiment of MORE: global full-cycle simulation and results

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***Abstract*—BepiColombo is a joint ESA/JAXA mission to Mercury with challenging objectives regarding geophysics, geodesy and fundamental physics. In particular, the Mercury Orbiter Radio science Experiment (MORE) intends, as one of its goals, to perform a test of General Relativity. This can be done by measuring and constraining the parametrized post-Newtonian (PPN) parameters to an accuracy significantly better than current one. In this work we perform a global numerical full-cycle simulation of the BepiColombo Radio Science Experiments (RSE) in a realistic scenario, focussing on the relativity experiment, solving simultaneously for all the parameters of interest for RSE in a global least squares fit within a constrained multiarc strategy. The results on the achievable accuracy for each PPN parameter will be presented and discussed, confirming the significant improvement to the actual knowledge of gravitation theory expected for the MORE relativity experiment. In particular, we will show that, including realistic systematic effects in the range observables, an accuracy of the order of 10^{-6} can still be achieved in the Eddington parameter β and in the parameter α_1, which accounts for preferred frame effects, while the only poorly determined parameter turns out to be ζ, which describes the temporal variations of the gravitational constant and the Sun mass.**

Keywords*—*Radio Science, General Relativity, Mercury, BepiColombo.

I. Introduction

BepiColombo is an ESA/JAXA mission for the exploration of planet Mercury [2], including two spacecrafts, the Mercury Planetary Orbiter (MPO) and the Mercury Magnetospheric Orbiter (MMO), to be put in orbit around Mercury. The launch is scheduled for 2017 and the orbit insertion around Mercury for 2024. The Mercury Orbiter Radio science Experiment (MORE) is one of the experiments on-board the MPO spacecraft, devised for improving our understanding of both planetary geophysics and fundamental physics. By top accuracy measurements of Mercury orbit, MORE will allow a precise test of General Relativity, constraining the parametrized Post-Newtonian (PPN) parameters and other related quantities. It will also measure the gravity field and the rotation state of Mercury, to constrain the size and physical state of the core and the planet internal structure. A Mercury planetary ephemerides improvement is also expected. To reach these goals, very accurate tracking from ground stations will be performed by using a multiple frequency radio link (in X and Ka bands) [7], allowing to eliminate the uncertainty in the refraction index due to plasma content along the radio waves path.

While from a conceptual point of view, the Radio Science Experiments (RSE) are a complex set of measurements and scientific goals and thus cannot be split in independent experiments, since each one depends in some way upon the others, in practice it can be convenient to distinguish between a *relativity experiment* [12], [13], with the goal of constraining PPN parameters and other quantities such as mass and dynamical oblateness of the Sun and a *gravimetry experiment* [11] together with a *rotation experiment* [4], with the goals of determining the gravity field of Mercury and inferring the rotation state of Mercury.

The plan of this work is as follows: in Section II the science background is outlined with a brief description of the parameters of interest and of the method used to sort them out. In Section III we analyze the results of a simulation carried out in a general contest, i.e. a global full-cycle simulation, including all the three experiments but focussing on results for relativity experiment parameters. Finally, in Section IV we draw conclusions about the expected results from the mission.

II. Science background

The scientific goals of a radio science experiment can be achieved first of all by performing an accurate orbit determination of the spacecraft and the planet. To this aim, the observable quantities to be used are the range and range-rate between one or more ground stations and the spacecraft, as measured by the on-board multiple frequency transponder with top accuracy [7] and the non gravitational perturbations acting on the spacecraft, as measured by the on-board Italian Spring Accelerometer (ISA) [6]. We perform the orbit determination together with the parameters estimate within a comprehensive software, ORBIT14, developed by the Celestial Mechanics Group of University of Pisa, under ASI contract. The software consists of two main programs: the *data simulator*, which generates the simulated observables (range, range-rate, accelerometer readings) and preliminary orbital elements (initial orbital elements of the spacecraft for each tracking interval), and the *differential corrector*, which solves for the parameters to be determined by a global least squares fit.

A. Least squares method and constrained multi-arc strategy

Following a classical approach [14], we define the residuals $\xi(\mathbf{u}) = \mathcal{O} - \mathcal{C}(\mathbf{u})$ as the difference between the observed quantities $\mathcal{O}$ and the predicted ones $\mathcal{C}(\mathbf{u})$ (computed following suitable models), where $\mathbf{u}$ represents the parameters to solve for. The non-linear least squares method consists in computing

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the set of parameters $\mathbf{u}^*$ which minimizes the target function:

$$\mathcal{Q}(\mathbf{u}) = \frac{1}{m}\xi^T(\mathbf{u})\mathbf{W}\xi(\mathbf{u}) = \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{m} w_{ij}\xi_i\xi_j\,,$$

where m is the number of observations and $\mathbf{W} = w_{ij}$ is the *weigtht matrix*, a symmetric matrix with non negative eigenvalues. The minimum $\mathbf{u}^*$ is then computed by an iterative *differential corrections method*, which is based on a modified Newton's method (see e.g. [14], Chap. 5):

$$\mathbf{u}_{k+1} = \mathbf{u}_k - \mathbf{C}^{-1}{\xi_{\mathbf{u}}}^T\mathbf{W}\xi\,,$$

where $\xi_{\mathbf{u}} = \partial\xi/\partial\mathbf{u}$ and $\mathbf{C} = {\xi_{\mathbf{u}}}^T\mathbf{W}\xi_{\mathbf{u}}$ is the *normal matrix*. In a probabilistic interpretation, $\mathbf{u}$ can be considered as a multivariate Gaussian distribution with mean $\mathbf{u}^*$ in the space of parameters and *covariance matrix* given by $\mathbf{\Gamma} = \mathbf{C}^{-1}$. The method is applied iteratively until the change of $\mathcal{Q}$ between two subsequent iterations or the increment $\Delta\mathbf{u} = \mathbf{u}_{k+1} - \mathbf{u}_k$ become smaller than a given tolerance.

To process the data with a least square fit we make use of a *constrained multi-arc strategy* [1]. Due to visibility conditions, tracking from ground stations is not continuous, thus observations are split in arcs: we call *observed arc* each set of range and range-rate tracking data (with a duration between 14 and 19 hours) and *extended arc* an observed arc broadened from half of the dark period before it to half the dark period after it. In a *multi-arc strategy* every single arc has its set of initial conditions, thus reducing with an overparametrization the errors in orbit propagation due to lack of knowledge in the non gravitational dynamical model. In particular, our strategy consists in assuming that at connection times between two subsequent extended arcs the orbits should coincide, i.e. we constrain by a given weight the difference in position and velocity of the orbits of two subsequent arcs at connection time.

B. Error models

The error models used to simulate the observable quantities must include a random and a systematic component (which particularly needs to be taken into account as it can significantly affect the true accuracy of the results). Concerning the range and range-rate, we use the following top-accuracy standard deviations for the random part [7]: $\sigma_{range} = 15$ cm at 300 s and $\sigma_{r-r} = 1.5 \times 10^{-4}$ cm/s at 1000 s for Ka-band tracking and a factor 10 larger for X-band tracking. These value are associated to range and range-rate observations which are conventionally defined as two-ways measurements divided by 2. For the systematic component, we included the effect of a slow accuracy degradation in range up to 20 cm over one year. In fact, systematic effects in range measurements turn out to be the main source of error for the relativity experiment [12].

For the accelerometer simulated data, the error model will be provided by ISA team. Currently, we make use of a simplified model described as follows. Within the accelerometer measuring bandwidth ($[3 \times 10^{-5} - 10^{-1}]$ Hz), we considered three main terms: two systematic components, with the periodicity of the spacecraft orbit around Mercury (2 hours 22 min) and of Mercury orbit around the Sun (88 days) with amplitudes of 1×10^{-7} cm/s^2 and 4×10^{-6} cm/s^2, respectively, and a random component, consisting of a random background around 10^{-6} cm/s^2/$\sqrt{\text{Hz}}$ in the $[10^{-4} - 10^{-3}]$ Hz band and a random rise from background at bandwidth edges (within the limits imposed by mission requirements [22]).

III. Relativity experiment results within a global full-cycle simulation

As outlined in Section I, although in data processing we will deal with a unique set of measurements, it is possible to conceptually separate gravimetry and rotation experiments on one side and a relativity experiment on the other. This possibility is assisted by the fact that gravimetry is mainly dependent on range-rate measurements (since they are more accurate over time scales of the order of one spececraft orbital period), while for the relativity experiment what matters are mainly the range measurements, which are more accurate over time scales of months [12], [14]. In the following, we will focus on a detailed analysis concerning the expected results for the relativity experiment.

A. Mathematical formulation in a relativistic contest

To perform the relativity experiment in a significant way, we need to solve an orbit determination problem with a full relativistic model. For details we refer to [12], [13], [18], [19]. We can separate the problem of a relativistic model for Mercury orbit from that of the determination of the Mercury-centric orbit of the spacecraft. In fact, concerning the Mercury-centric orbit of the spacecraft, from a relativistic point of view we assume that the Newtonian equation which describes the motion of the probe is valid if, as independent variable, the Mercury proper time is used. To this aim, we define a new time coordinate (Mercury Dynamic Time - TDM), containing terms of 1-PN order: in this way our model is consistent with the level of accuracy required by RSE (see [14], Chap.7).

For the relativistic model of Mercury orbit, we make use of the parametric post-Newtonian approach: the relativistic equation of motion is linearized with respect to the small parameters v_i^2/c^2 and $Gm_i/r_{ik}c^2$ (v_i: barycentric velocity for each body of mass m_i, G: gravitational constant, c: speed of light, r_{ik}: mutual distance appearing in the metric of the curved space-time). This approach can be formalized by adding to the Newtonian Lagrangian of the N-body problem some corrective terms of post-Newtonian order 1 in the small parameters [12]. The described formalism allows us to parametrize the equation of motion by means of some PPN parameters, which have fixed values in Einstein General Relativity (GR) theory: solving for their value within orbit determination procedure we can, hence, test for possible deviations from GR to an unprecedent level of accuracy[1].

The PPN parameters we deal with are briefly described in the following. We considered the Eddington parameter γ, which accounts for the velocity-dependent modification of the 2-body interaction ($\gamma = 1$ in GR) and gives informations about the space-time curvature through the Shapiro effect (see Section III-B), and the Eddington parameter β, linked to the

[1]This goal can be reached due to the very high tracking accuracy of BepiColombo RSE: the relativistic signal in range over 1-year mission has a peak-to-peak amplitude of roughly 900 km (see [14], Chap.6), thus it is accurately measurable.

non-linear 3-body interaction ($\beta = 1$ in GR). We included the parameter $\mu_\odot = GM_\odot$ and its time derivative $\zeta = d\mu_\odot/dt$, which accounts for the dynamical effect of variations in both Sun mass $M_\odot$ and G (in GR G is the only free parameter, with a constant value) and the zonal spherical harmonic of degree 2 of the Sun $J_{2\odot}$, which is not strictly a PPN parameter but, accounting for the dynamical effects of the solar gravity field oblateness, is a primary cause of uncertainty in Mercury perihelion modelling (hence it is intriguing to infer its possible variation from standard value 2×10^{-7}). Finally, we included the parameters α_1 and α_2, which describes phenomenologically the preferred frame effects to PN order 1[2] (both 0 in GR), and the parameter η, which describes the violation of Strong Equivalence Principle (SEP) ($\eta = 0$ in GR).

B. *Assumptions*

The simulation scenario consists of a 365 arcs long simulation, which corresponds to about one year, starting on April 10th, 2024 at 18:41:45 UTC (this is the actual estimate of spacecraft orbit insertion) until April 18th, 2025. The assumptions made are briefly described as follows:

- two ground stations are available for tracking, one at Goldstone Deep Space Communications Complex (California, USA) for the Ka-band and the other in Spain, at Cebreros station, for X-band; range measurements are taken every 120 s and range-rate every 30 s, both with the accuracies described in Section II-B;
- the gravity field spherical harmonics are simulated up to degree $l_{max} = 25$ assuming as nominal gravity field the one estimated by Messenger mission [5]; the Sun tidal effects are described by the Love number k_2, for which we assume in simulation the value $k_2 = 0.25$, as explained in [4] (to define the parameter k_2 we followed the description given for the Earth in [10]); a semi-empirical model for rotation is included (see details in [4]): the rotation parameters are 2 angles, δ_1 and δ_2, defining the *obliquity* of Mercury spin axis and the amplitude ε_1 of librations in longitude at Mercury orbital period;
- solar radiation pressure and indirect Mercury albedo radiation pressure are included in the simulation of the observables; non gravitational effects are assumed as read by the accelerometer which is always on;
- taking advantages of the fact that the PPN parameter γ appears both in the equations of motion for Mercury and Earth and in the equations for radio waves propagation, the possibility of performing one or more Superior Conjunction Experiments (SCE)[3] during cruise phase has been devised: in such a way it would be possible to independently determine the parameter γ, possibly with a higher accuracy than during orbit phase; hence, we assume a constraint on PPN parameter γ given by $\gamma = (1 \pm 2 \times 10^{-6})$, as a result of a cruise phase SCE simulation [12];
- we impose the Nordtvedt equation [15], i.e. we assume a metric theory to remove the approximate symmetry between β and $J_{2\odot}$ (see comments in Section III-C): $\eta = 4(\beta - 1) - (\gamma - 1) - \alpha_1 - \frac{2}{3}\alpha_2$.

C. *Results*

The MORE relativity experiment needs to solve an orbit determination problem, within the described full relativistic model, not for a generic space-time but for the one where we are now. As a consequence, the parameters to solve for are the following: the PPN and related parameters, expressing the violations of GR and other effects (γ, β, η, α_1, α_2, $J_{2\odot}$, μ, ζ) together with the initial conditions for Mercury barycenter ($x_M, y_M, z_M, \dot{x}_M, \dot{y}_M, \dot{z}_M$) and Earth-Moon barycenter - EMB ($x_{EMB}, y_{EMB}, z_{EMB}, \dot{x}_{EMB}, \dot{y}_{EMB}, \dot{z}_{EMB}$) with respect to the Solar System Barycenter (SSB) in the Ecliptic J2000 reference frame[4]

Together with these parameters, we need to determine also the initial conditions of the probe for each extended arc (as explained in Section II-A), which correspond to 6 parameters per arc (3 position and 3 velocity components). Moreover, the solve for parameters for the gravimetry and rotation experiments are: the gravity field harmonic coefficients up to degree $l = 25$, Love number k_2, two obliquity angles δ_1 and δ_2 and the amplitude ε_1 of Mercury librations in longitude at 88 days. Finally, since we included in the simulation scenario the accelerometer to measure the non gravitational perturbations, it is necessary to conveniently calibrate its effects which, if not considered, would lead to systematic errors in the dynamics. To this aim, we developed a calibration system based on $\mathcal{C}^1$ splines: we fit the systematic component with a hermite cubic spline and we solve for the coefficients of the polynomial for each extended arc, imposing that, at each time node, the polynomial value and its first derivative for the previous arc coincide with those of the following arc [4]. With this method we add to the solve for parameters list 6 parameters per arc for calibration.

We solve for all the parameters listed above in a global least square fit. We performed both an analysis based on formal statistics (standard deviations and correlations) as given from the formal covariance matrix $\mathbf{\Gamma} = \mathbf{C}^{-1}$, and an analysis based on "true errors", defined as the difference between the value of parameters at convergence and the simulated value. We perform a statistical analysis over different runs, each time varying the random generator of Gaussian distribution and we consider as final true error the systematic error obtained as the rms error from all the runs.

The results for PPN and related parameters are shown in Table I ($\mu_\odot$ is expressed in cm^3/s^2 and ζ in y^{-1}): for each parameter we include the estimated value at convergence, the formal uncertainty and the true error. We notice that true errors tend to be higher than formal ones for almost all the parameters. This is due mainly to the systematic effect in range.

[2]Since the observable effects linked to α_1 and α_2 depend on the choice of gravitationally preferred frame, we follow the standard assumption that this frame, being of cosmological origin, can be identified with the rest frame of cosmic background [20].

[3]A Superior Conjunction Experiment is based on the fact that the light propagation delay, related to PPN parameter γ via the Shapiro effect [17], is higher when the Sun-Mercury angle as seen from the Earth is small, with Mercury beyond the Sun (i.e. a *superior conjunction*).

[4]As will be explained at the end of this Section, we solve for only 8 of the 12 initial conditions.

TABLE I. PPN AND RELATED PARAMETERS

	Estimated value	Formal sigma	True error
β	1.00000086	3.8×10^{-7}	1.0×10^{-6}
γ	1.00000104	8.9×10^{-7}	2.2×10^{-6}
η	4.63×10^{-6}	2.0×10^{-6}	1.0×10^{-6}
α_1	2.94×10^{-6}	4.8×10^{-7}	2.9×10^{-6}
α_2	2.15×10^{-7}	6.9×10^{-8}	2.1×10^{-7}
$\mu_\odot$	1.33×10^{26}	4.0×10^{13}	1.4×10^{14}
$J_{2\odot}$	2.01831×10^{-7}	3.7×10^{-10}	1.9×10^{-9}
ζ	-1.51×10^{-13}	2.0×10^{-14}	1.5×10^{-13}

To verify this assertion we performed a simulation removing the non-gravitational perturbations (hence the accelerometer readings) from the simulation scenario; we found almost the same results for true errors in the PPN and related parameters, confirming that the accelerometer and expecially its error model does not affect significantly the determination of the relativity parameters, while the other source of systematic effects, i.e. range error model, plays a significant role.

Comparing true and formal uncertainties, the parameter which turns out to be the most sensitive to the effect of systematic error in range is ζ, with a true to formal error ratio of 7.5. The results obtained for the accuracy that BepiColombo mission can provide on ζ are critical: at present ζ has been inferred with an accuracy of 4.1×10^{-14} y^{-1} [16], which is still a factor 2 higher than the computed formal sigma for a 1-year mission, but significantly better than the true accuracy predicted by our simulations. Thus, the lack of accuracy due to systematic effects in range turns out to be particularly severe in this case. Concerning the other parameters of interest for the MORE relativity experiment, the results are highly encouraging, since, even if predicted true errors are higher than formal uncertainties, they remain in any case significantly lower than actual accuracies on each parameter. For example, the inferred value for β from Lunar Laser Ranging (LLR) has an accuracy of 1.1×10^{-4} [21], two orders of magnitude worse than our result (assuming the validity of Nordtvedt equation). Also the preferred frame effects can be estimated to a significantly better accuracy than previous estimates, which provide 6×10^{-6} for α_1 and 3.5×10^{-5} for α_2 [9] to be compared with our estimated accuracies of 2.9×10^{-6} and 2.1×10^{-7}, respectively.

Special considerations need to be done in the cases of η and γ. The actual accuracy on η from LLR is 4.5×10^{-4} [21], hence the results of our simulations show that MORE can provide a significant improvement. Nevertheless, a more accurate analysis needs to be performed in a more general scenario, since the signal of range measurements due to SEP violation could be, in principle, sensitive to factors not included in our model[5], resulting in an eventually downgraded accuracy on η parameter [19]. Regarding γ, the estimated error is strictly related to the apriori accuracy imposed considering a SCE experiment during the cruise phase; nevertheless the actual best estimate for γ from Cassini mission gives an error of 2.3×10^{-5} [3]: an improvement in accuracy is hence expected from MORE. We also notice that the determination of PPN parameters is improved by adding the constraint on γ as already measured at best during cruise phase. The assumed value of 2×10^{-6} in accuracy for SCE during cruise phase seems optimistic with respect to updated simulations (see [8]); nevertheless we verified that even worsening the assumed accuracy on γ by a factor 2 or 3, as long as γ is constrained at least at 10^{-5} level, the results on PPN parameters remain almost unchanged.

[5]For example, the determination of parameter η could be in principle sensitive to the level of knowledge of the positions and masses of the main Solar System bodies (we just determine the initial conditions of Mercury and EMB).

TABLE II. CORRELATIONS BETWEEN RELATIVITY PARAMETERS

$\rho(J_{2\odot}, \mu_\odot) = 0.87$	$\rho(\alpha_1, J_{2\odot}) = 0.84$
$\rho(\alpha_1, \mu_\odot) = 0.82$	$\rho(\beta, \eta) = 0.89$

In Table II we show the analysis of correlations from the covariance matrix at convergence (only correlation values ρ higher than 0.8 are shown). In particular, we want to underline that if we had not used Nordtvedt equation as a constraint between relativity parameters, we would have found a high correlation between β and $J_{2\odot}$. In fact, the main orbital effect of β is a precession of the argument of perihelion, that is a displacement taking place in the plane of the orbit of Mercury, while $J_{2\odot}$ affects the precession of the longitude of the node, meaning that it generates a displacement in the plane of the solar equator, which has an inclination of only 3.3° with respect to the plane of Mercury orbit. As a consequence, the correlation between the two parameters is high, thus degrading the marginal accuracy in the solution of both parameters, but this effect can be mitigated by using the Nordtvedt equation for a generic theory of gravitation under the assumption that it is a metric theory (we imposed that the equation is valid to an accuracy of 10^{-9}).

Finally, in Table III we show the results for the initial conditions of Mercury barycenter and EMB, which are determined at mission initial time: positions are in cm, velocities in cm/s. Both positions and velocities can be determined very well. Some of the velocity components are significantly affected by the systematic error in range, in particular the components of EMB velocity, $\dot{x}_{EMB}$ and $\dot{y}_{EMB}$, with a true to formal error ratio of 6.1 and 7.7, respectively, but the predicted true accuracies, of the order of 10^{-6}, are anyway fully adequate. We underline that the effect of systematic error in range, expecially for the determination of Mercury and EMB initial conditions, is highly related to the spatial configuration consequent to the scientific operation starting time and also to the choice of the functional form of the systematic effect in range, a sine function being only one suitable way to describe the growth of the noise over time.

We also underline that only moderate correlation can be

TABLE III. INITIAL CONDITIONS OF MERCURY AND EARTH-MOON BARYCENTER

	Estimated value	Formal sigma	True error
x_M	-5.88×10^{12}	0.40	0.69
y_M	-2.59×10^{12}	0.87	1.6
z_M	3.25×10^{11}	2.8	8.8
$\dot{x}_M$	9.52×10^{5}	4.6×10^{-7}	6.3×10^{-7}
$\dot{y}_M$	-4.25×10^{6}	5.0×10^{-7}	2.7×10^{-6}
$\dot{z}_M$	-4.34×10^{5}	1.6×10^{-6}	2.2×10^{-6}
$\dot{x}_{EMB}$	1.10×10^{6}	1.8×10^{-7}	1.1×10^{-6}
$\dot{y}_{EMB}$	-2.76×10^{6}	2.2×10^{-7}	1.7×10^{-6}

detected between initial conditions of Mercury and EMB. This condition can be reached only by solving for just 8 of the 12 initial conditions; in fact, it is not possible to solve for all the 12 conditions together without achieving a downgraded solution. This is due to an approximate rank deficiency of order 4 caused by the breaking of an exact simmetry between Sun, Earth and Mercury with respect to the rotation group $SO(3)$, and to an approximate simmetry for scale changes. The standard tecnique to remove rank deficiency (*descoping*) is described in [14], Chap. 6. In particular, we removed it by adding 4 constraints in solving for the 12 initial conditions, i.e. we assumed as known the three position components of EMB and the velocity component of EMB perpendicular to the ecliptic and we did not solve for them.

IV. Conclusion

In this work we show for the first time the results of a numerical full-cycle simulation for the relativity experiment of the BepiColombo RSE carried out in the up-to-date expected scenario. The simulations have been performed in the general contest of a simultaneous determination of all the parameters of interest for RSE using a global least squares fit within a constrained multi-arc strategy, including an updated model for non gravitational effects as read by the on-board accelerometer and a realistic systematic error model for the range observations.

Our simulations show that almost all the PPN and related parameters can be determined with a true error at convergence significantly better than the respective actual accuracies. This result is particularly encouraging since we included systematic effects in the range error model, which is the main source of uncertainty for the MORE relativity experiment. Moreover, we verified that the systematic errors due to the on-board accelerometer do not affect the estimated accuracy for the PPN and related parameters determination. At present, the only parameter that can be determined with an accuracy level lower than the state-of-the-art is ζ, which accounts for the simultaneous temporal variations of G and $M_\odot$; this effect is mainly due to systematic errors in range measurements.

Nevertheless, we can conclude that the results, although not definitive, are really encouraging since they prove on one side the feasibility of the MORE relativity experiment in a reasonably realistic scenario and the robustness of our approach, on the other the possibility of significantly improving the actual knowledge of gravitation theories.

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