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JOHN VON NEUMANN'S PANMATHEMATICAL VIEW

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A review essay on Israel Giorgio and Millán Gasca Ana, *The World as a Mathematical Game. John von Neumann and Twentieth Century Science*, Basel-Boston-Berlin, Birkhäuser, 2009, pp. xii+208 [English translation of *Il mondo come gioco matematico. La vita e le idee di John von Neumann*, Torino, Bollati Boringhieri, 2008, pp. 274].

There is no doubt that John von Neumann deserves a place among the giants of 20th-century science. An exceptional mathematician, his incredibly versatile mind lies behind the development, and often the sheer invention, of fields as diverse as set theory, topology, quantum mechanics, atomic energy, meteorology, operations research, computing and automata, and many others. As far as economics is concerned, he is renowned for two great achievements: the birth of modern game theory and the development of the first rigorous general equilibrium model. Yet, surprisingly enough, until now it was hard to find in the literature a truly complete biography of such an outstanding figure. Only partial reconstructions existed, either focusing on one aspect or the other of his extraordinary scientific contributions[1] or, in the worst cases, failing to rise above the level of ideologically-oriented gossip.[2] If only for this reason, this new biography by Giorgio Israel and Ana Millán Gasca (i-mg henceforth), should deserve high praise.[3]

Indeed, the book recommends itself for many other reasons. Through their reconstruction of von Neumann's life and career i-mg

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[1] See Heims 1980 on atomic energy, Aspray 1990 on computing and automata, Mirowski 2002, ch. 3 on economics. See also the various review articles published after von Neumann's death in the major mathematics journals of the time, which for obvious reasons focused on his purely analytical contributions (*e.g.*, Kuhn and Tucker 1958).

[2] Exemplary in this respect is Poundstone 1992, not to mention the nonsensical statements – fortunately only available in Italian – by a self-proclaimed historian of mathematics (on which see the scathing critiques in Israel 2008, 273-286).

[3] And prize as well! The Italian edition of the book has been awarded the 2008 Peano Prize for books contributing to the public understanding of mathematics.

lead the reader to a fascinating journey inside the phantasmagorical world of high-brow science during the first half of the last century, a period which – with all its incredible inventions and discoveries (from quantum physics and relativity theory to the computer, from the civil and military use of atomic energy and the management of 'complex' systems to genetics and neuroscience) – has been absolutely crucial in shaping the character of contemporary science, with all its merits and defects, and in raising the decisive ethical and social questions which surround its use in the present days.

It is far beyond this reviewer's competence to be able to faithfully render (let alone discuss!) the essence of von Neumann's multi-faceted activity in so many scientific realms, as narrated in I-MG's volume. What I aim to do in the rest of this review is to summarize the authors' main thesis as far as von Neumann's overall contribution to modern science is concerned and then focus on the part of his work which, either directly or indirectly, has influenced modern economics.

1.

In the last chapter of the book the authors use the words «fragile science» to characterize the status of science after the epistemological crisis it suffered at the turn of the 20th century – a crisis which seriously damaged its basic pillars, like determinism, mechanical reductionism and the internal consistency of mathematics. Still, this fragile science, albeit deprived of its traditional foundations, was to enjoy what I-MG call a «paradoxical triumph» in that «the scientific conception of the world invaded all sectors of knowledge, in particular the human and social sciences» (p. 168), as well as technology, which abandoned its character of 'art' and became itself a form of theoretical knowledge built upon scientific notions and methods. During this process the goal of science changed too. 'Truth' – now understood as fundamentally unattainable – was not anymore the yardstick for evaluating science and was progressively replaced by «effective» or «useful». Describing and controlling phenomena, rather than the old ambition of explaining them, became the main motivation behind any scientific efforts, with a clear pragmatic penchant towards the applicability of results. Even mathematics was not immune of this revolution: what the historian of mathematics Morris Kline has called the loss of certainty (Kline 1980) meant the shattering of the old Galilean dream of mathematics as «the true language of nature» and its new characterization as a mere, though extremely powerful and versatile, reservoir of useful models and techniques.

What was von Neumann's position with respect to these epoch-making events? I-MG very effectively explain that he was, at the same time, one of the main initiators and actors of the process (just think of his contribution to quantum mechanics or of his strenuous efforts to effectively apply science to all aspects of human life), but also one of the last defenders of the ideal of scientific reductionism – an ideal so typical of 18th- and 19th-century scientists which however was to take a very different meaning in von Neumann. Accordingly, the authors speak of a «tension» (p. 169) accompanying von Neumann's entire career and originating from his awareness of (and effective contribution to) the above-mentioned paradox of a fragile, yet triumphant, science.

At the centre of von Neumann's world we find axiomatics and formal logic, which in his hands became the ultimate weapons capable of reconciling the opposite poles of this tension. On the one side, the logic-axiomatic approach was a very flexible and powerful method which could be applied to investigate every kind of scientific problem and every aspect of human life – a method wholly consistent with the belief that, far from explaining phenomena, what scientists actually do is to build models for representing and, possibly, controlling them. On the other, logic and axiomatics constituted the unifying language which allowed von Neumann to give a concrete content to the Vienna Circle's scientific conception of the world he had became acquainted with during the early phase of his career (see pp. 50-51 and 124 ff.), that is to say, the idea that science and logic were the only possible instruments for dealing with real world phenomena.

Armed with these weapons von Neumann could elaborate and promote what I-MG effectively call a panmathematical view of the world (p. 50), capable of reducing to unity all the different threads among which science had been divided following the crisis in its foundations. Hence, von Neumann's reductionism was methodological, rather than ontological: mathematics in general, and the logic-axiomatic approach in particular, provided the only true unifying element for every branch of science.[1]

2.

Let me now move on to examine how I-MG deal with von Neumann's contribution to economics. Having written extensively on the subject,[2] my expectations were almost inevitably not so high, given the obvious impossibility, in a general biography, to delve into the details

[1] Von Neumann's extraordinary career is also used by I-MG to cast light on two other crucial issues of 20th-century science, namely, its relation with technology and with the economic, political and military establishment (see chs 4-5). Fascinating, and wonderfully narrated, as it is, this part of the story goes far beyond the limits of the present review.

[2] See Giocoli 2003, esp. chs 4-5; Giocoli 2003a; Giocoli 2006.

of, say, game theory or general equilibrium. Yet, I must admit the authors have managed to highlight the relevant aspects of von Neumann's work in the field and to give ample justification for his placement in the Hall of Fame of 20th-century economics, as well as in that of mathematics, physics, etc. More than that, I do believe that I-MG always got it right when faced with the trickiest points of von Neumann's economics (say, by correctly interpreting his very peculiar notion of rationality-as-prudent-behaviour)[1] and, above all, when called to single out where his truly fundamental achievement in the field actually lay (say, in conquering economics to the logic-axiomatic method and to topological techniques).[2]

Four general themes may be identified under the headline «von Neumann's contributions to economics». The first is, of course, game and general equilibrium theory, with the attached notion of strategic rationality. Second, the provision of a new powerful tool box for mathematical economics. Third, the diffusion of the axiomatic method. Fourth, systems and control theory, which is strictly related to the alleged existence of a break in von Neumann's intellectual life. Let me now briefly examine them in inverse order.

In his *Machine Dreams* Philip Mirowski maintains that *two* von Neumanns actually existed, the boundary between them being constituted by Kurt Gödel's 1931 impossibility result (see Mirowski 2002, ch. 3). According to this thesis, the first von Neumann was a pure theorist and a keen supporter of David Hilbert's formalist program of unification of mathematics,[3] while the second was just a pragmatic scientist, devoid of any unitary research project and ready to sell his immense capabilities to those who happened to hire him, particularly when the request came from the industrial and military establishment. This dichotomy would explain von Neumann's late interest in more applied themes, such as, crucial to Mirowski's reconstruction, those related to the management of complex systems or to the control and programming of automata. The thesis is especially relevant when applied to von Neumann's economic contributions in that game theory would lie on both sides of the boundary: the 1928 pioneering paper would belong to the pre-Gödel, formalist von Neumann, while the 1944 *Theory of Games and Economic Behavior* (TGEB) to the post-Gödel, pragmatic one.

This journal has already published a review of mine of Mirowski's volume, where I explain, among other things, why I consider the thesis of the two von Neumanns totally unfounded (Giocoli 2003b). Now I-MG

[1] Pp. 42 ff. of the volume. On this also see Giocoli 2003, ch. 4; Giocoli 2006.

[2] On this also see Giocoli 2003a.

[3] On which see this book, pp. 22-31, and, more extensively, Corry 1996, 1997.

give further reasons to definitely reject Mirowski's view. As they explain time and again in the book (see, *e.g.*, pp. 49-53), there has always been only one von Neumann, that is to say, a scientist who always consistently believed in a panmathematical view of the world. Yes, Gödel's result was very significant in von Neumann's career (he himself admitted that), but only in the limited sense of leading him to abandon the strong version of Hilbert's formalist program,[1] and not in that of abandoning or weakening his faith in the power of logic and axiomatics. On the contrary, the result contributed to convince him that a real possibility of unification of mathematics, and eventually of all science, did exist and consisted in the adoption of a common method of analysis, the logic-axiomatic method. Freed from the straitjacket of Hilbert's program, axiomatics became the foundation upon which von Neumann contributed to rebuild the mathematicians' faith in the practical power of their discipline (p. 30). Thus, von Neumann's late interest in those highly applied fields, like systems theory or operations research or automata, upon which Mirowski places so much emphasis, was nothing but the extension of one and the same method of analysis to new research areas, in full compliance with his old master's battle cry.[2]

To confirm this, I-MG explain quite effectively that the outcome of von Neumann's incursions in any scientific or applied field was always the same, namely, a quick conversion of the discipline and its best practitioners from a piecemeal, or merely technical, approach to a rigorously axiomatic, model-based analysis. Exemplary in this respect are the pages that I-MG dedicate in chapter 4 to von Neumann's role in the planning of the US anti-air attack system of national defence (SAGE) and in the intercontinental ballistic missile project (Atlas ICBM). Let me quote them in full:

> A new theoretical approach to management problem was put forward, which included two fundamental elements. On the one hand, the starting point was a global approach to system design, engineering and management marked by the technological conception of "control". On the other, the emphasis was placed, for each individual problem, on identification of the underlying logical and mathematical structures, and so a method of addressing the concrete material reality was chosen which involved the use of an abstract theoretical screen.
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> (p. 112)

3.

Consistently with his panmathematical view, economics was one of the fields where von Neumann tried more forcefully, and more successful-

[1] Namely, the dream of being able to formally demonstrate the internal consistency of the whole of mathematics. On the existence of two versions of Hilbert's program see CORRY 1997 and WEINTRAUB 2002, ch. 3.

[2] See HILBERT 1996 [1918].

ly, to export the axiomatic method. This, as I said before, was possibly his greatest contribution to economics in that he left an indelible sign on the way economists had since then conceived of the proper way to do research. The idea itself of building a formal model every time we endeavour to explain an economic phenomena must be traced back to von Neumann's foray in the field.

Under this respect, the exemplary item is his much celebrated 1937 general equilibrium model. Hardly the first mathematical model on this theme, nor the approach later neoclassical economists elected to follow, von Neumann's GE model provided the blueprint for generations of economists on how a model should be built, how axioms should be formulated, how results should be derived from the axioms. Similarly, the axiomatic derivation of expected utility theory (EUT) in the second, 1947 edition of the TGEB provided another major instance of how decision theory problems should be tackled.[1]

Strictly related to the issue of modelling phenomena through the axiomatic method is the other topic of the analytical tools which, thanks almost entirely to von Neumann, were spread among mathematical economists. Again, both the 1937 paper and the TGEB were literal goldmines of new techniques which were for the first time applied to economics and which were soon to be mastered by an ever increasing number of scholars. To make just an example, the fixed point technique was, in its various versions, transmitted by von Neumann to John Nash, then from Nash to the Cowles Commission, and in particular to Arrow and Debreu's 1954 classic existence proof, and from there to the rest of the economists' community.[2] Comparable stories could be told for other key techniques, like convex analysis, matrix algebra or the simplex – all of which carrying the von Neumann's label. More generally, we can fully credit to von Neumann the replacement in so many areas of economic theory of the mathematics of time, *i.e.*, Newtonian differential calculus, with the static, atemporal mathematics of algebraic and topological structures. In doing that, economics conformed to a pattern shared by many scientific disciplines of the time.

I-MG explain quite effectively von Neumann's contribution in spreading both the axiomatic method and the new mathematical tools among economists. Yet, if a critique is to be made to the authors, is that they should have tried to be a bit more specific in detailing the channels through which von Neumann's approach and ideas actually entered the

[1] Note that, as the authors correctly observe (p. 65), the kind of axiomatics employed by von Neumann was never that of the Bourbakists (namely, a system of purely formal statements, completely detached from any physical interpretation), but rather the classic, Hilbertian one (namely, one where the axiom system was never totally separated from an underlying concrete reality).

[2] I have reconstructed this story in Giocoli 2003a.

economic literature (say, EUT was intensively studied at the Cowles Commission, much more than game theory itself) and, possibly, also to say something more on those parts of his works which, on the contrary, failed to raise the economists' interest (say, the idea of studying market interactions progressively, starting from 2-agent situations, then moving on to 3-agent, 4-agent, etc., rather than immediately beginning with n-agent models). Indeed, I-MG do that in some cases (for example, they tell the story of von Neumann's contribution to the development of Dantzig's linear programming method: pp. 136-137), but not as often as it might be expected or desidered.

4.

Finally, the specific contributions for which von Neumann's name is most immediately associated to economics, namely, game theory, the characterization of rationality and the general equilibrium model. Here my evaluation of the book is mixed. On the one side, I can single out no major omission or disagreement in the way I-MG deal with these issues.[1] On the other, I must confess that I would have liked the authors to dig more deeply some crucial historiographic puzzles.

Take for example von Neumann's characterization of rational strategic behavior. Here I-MG do an excellent job in carefully distinguishing von Neumann's rationalistic view from Émile Borel's psychological approach and especially in highlighting (pp. 44-45) what I do consider the two real cornerstones of von Neumann's game theory: his normative goal and his explicit willingness to «escape from psychology» (*i.e.*, of characterizing strategic rationality without having to refer to the players' expectations and beliefs).[2] This is not usually done and credit must be given to the authors for having endorsed this interpretation. Yet, they fail make a further step and compare von Neumann's view of rationality with the mainstream one in neoclassical economics. That such a comparison might be really interesting is testified by the obvious re-

[1] This apart from a few minor quibbles, like: i. having forgotten at p. 44 the most modern interpretation of mixed strategy as an expression of the rival's ignorance about a player's choice (so-called Harsanyi-Aumann interpretation), or ii. having failed to mention at p. 127 that, in his later work in statistics, Abraham Wald applied von Neumann's minimax as the rational criterion for choice under uncertainty in what came to be dubbed 'games against nature', or (and this is frankly surprising, given the notion's perfect fit with the authors' overall characterization of von Neumann's goals) *iii*) having omitted von Neumann's most favoured solution concept in the TGEB, namely, that very notion of stable set which, analytically complicated as it was, had a clear institutionalist flavour, particularly appealing for someone who believed that «Social and economic phenomena could be treated and managed using rigorous scientific methods» (p. 46; on stable sets in the TGEB see GIOCOLI 2003, ch. 4).

[2] See GIOCOLI 2003, ch. 4, and 2006, where these two aspects of von Neumann's game theory are amply explained.

mark that the orthodox characterization of rational behaviour as the maximization of an agent's objective or subjective expected utility is itself a direct offspring of von Neumann's TGEB. Thus, a deeper investigation of this issue would have cast new light on the exact extent of von Neumann's influence on modern economics, as well as on the actual conformity of this influence with his goals and expectations.

Or take the famous general equilibrium model. Again, the authors give the proper tribute to von Neumann and especially stress the importance of his introducing the fixed point technique in economics (p. 47). Yet, it is this reviewer's opinion that what has been authoritatively praised as «the single most important article in mathematical economics» (see Weintraub 1985, 74) would have deserved ampler space and a more careful analysis.[1] To mention just one missing question: why did von Neumann adopt in the 1937 paper a purely non-constructive demonstration technique while only a few years later in the TGEB he will employ both a non-constructive and a constructive proof of the minimax theorem? The issue here is clearly to understand whether there was any deeper reason (deeper, I mean, than sheer analytical convenience) behind von Neumann's choice between constructive and non-constructive techniques in his economic works. One could ask for instance whether the choice was dictated by the dichotomy between a normative and a descriptive interpretation of the different models.[2]

5.

In conclusion, while I surmise that those historians of economics who are specifically interested in delving deep inside von Neumann's explicit contributions to the discipline would have better also refer to other, more specialized literature, this should in no way diminish the importance of I-MG's achievement. As I said before, they have managed to write the first complete biography of the Hungarian genius where the fundamental unity of his thought – the panmathematical view of the world – is explained to the reader in the most rigorous, exhaustive and also readable way. After this biography, it would be extremely hard work for those wishing to deny such a unity or, worse, to depict von Neumann as a cartoonish, Nazi-style figure, ready to trigger a new World War if only his well-paying military patrons would have asked him to.

If we consider that, as I have tried to explain in this review, all the most important features of von Neumann's impact on 20th-century economics – from the spread of the axiomatic method to the populariza-

[1] Admittedly, one of the authors has already done that in INGRAO and ISRAEL 1990, ch. 7. On von Neumann's paper also see WEINTRAUB 1985, ch. 6.

[2] On this specific issue, see GIOCOLI 2003a.

tion of several analytical techniques to the idea itself of 'modelization' as the only proper way to investigate economic phenomena – descend precisely from the application of his panmathematical view to social sciences, it follows that this book is a real boon, as well as a must-read, for every historian or economist wishing to understand how, and by whom, modern neoclassical economics has been shaped in the first two or three decades after World War II.

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