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Non-permutation Flow Line Scheduling by Ant Colony Optimization

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Abstract: A flow line is a conventional manufacturing system where all jobs must be processed on all machines with the same operation sequence. Line buffers allow non-permutation flowshop scheduling (NPFS) and job sequences to be changed on different machines. A mixed-integer linear programming model for non-permutation flowshop scheduling and the buffer requirement along with manufacturing implication is proposed. Ant Colony Optimization (ACO) based heuristic is evaluated against Taillard's (1993) well-known flowshop benchmark instances, with 20 to 500 jobs to be processed on 5 to 20 machines (stages). Computation experiments show that the proposed algorithm is incumbent to the state-of-the-art ACO for flowshop with higher job to machine ratios, using the makespan as the optimization criterion.

Keywords*: manufacturing system; scheduling; non-permutation flowshop (NPFS); ant colony system (ACS); benchmark problems.*

Introduction

A flow line is a conventional manufacturing system where all jobs must be processed on all machines with the same operation sequence (Figure 1). Jobs are processed only once by each machine, as opposed to reentrant flow lines.

Figure 1. Two flow lines, with and without buffers. Permutation (PFS) and non-permutation flowshop (NPFS) are compared. In both cases, jobs see machines (routing) in the same sequence (flowshop). In non-permutation flowshop, buffers allow changes (permutations) of job sequences on subsequent machines.

Figure 1 about here

Examples of flow lines include transfer lines, assembly lines, chemical plants, logistics, and many more (Rossi et al., 2012); the problem is scalable in many senses: a job can be a part, the whole product or a batch; machines (or stages) can be a single operating unit, a cell, a line, or their combinations; time is measured by non dimensional units and can indicate seconds, hours, days etc.

Flow line is referred to as the physical layout; flowshop is the mathematical model, as defined in the next chapter.

The flowshop scheduling problem occurs whenever it is necessary to schedule a set of n jobs on m machines so that each job visits all machines in the same order.

In *non-permutation* flowshop (NPFS) scheduling, the most general flowshop case, which is examined here, the order in which all m machines are visited by the n jobs changes, allowing job sequences to be different on subsequent machines.

In a *permutation* flowshop (PFS) the sequence jobs visit machines (routing) is the same for all jobs, as for non-permutation.

The sequence of jobs on all machines is the same in permutation flowshop; instead in non-permutation flowshop the sequence of jobs can be different on subsequent machines.

Figure 2. Flow line (clockwise from top left) with *m* machines (or stages) M (bright red) and different examples of buffer configurations (dark blue) to allow job sequence permutation between machines.

Figure 2 about here

To allow non-permutation, buffers between, on board or shared among machines are necessary. Examples of buffers are shown in the U-shaped flow line of Figure 2: input and output buffers at the two ends and between machines, cells, lines or plants; they can be shared, in the form of an automatic warehouse or an open space. To allow permutations, jobs travel through buffers between machines. The flow line in the pictorial example itself is made of flow lines: a transfer line and a flexible cell.

The buffer requirement has also been formally included in the proposed model.

If buffers are not present, either the *blocking* or the *no-wait* condition should be applied to the algorithm to achieve a feasible schedule. In the former case, a job completed on one machine may block that machine until the next downstream machine is free; in the latter case, the next machine must be available before a job leaves the previous one.

As for the problem complexity, there are $(n!)^m$ different schedules for ordering jobs on machines in non-permutation flowshop; the number of schedules for permutation flowshop reduces to $n!$.

In this work transport and setup times are neglected. This hypothesis often applies when pallet changing systems on machines and fast transport and buffer loading/unloading devices are present. The processing time can be increased by standard transport and/or setup time, if it is relatively small with respect to the processing time; a transport time to and from the

warehouse of hours can be considered negligible if the processing time is in the order of days, like in the case of welding, heat treatments, painting and inspection of large and bulky parts. Operations and transport can be automated, like in computer integrated manufacturing, or manual.

Manual operations can also be represented, using standard times.

Examples of scheduling optimization targets are: minimizing total completion time (makespan) or weighted tardiness, balancing mean flow time, and meeting due date. The problem examined here is referred to as $F_m|B_i=+\infty|C_{\max}$ using Graham's notation, where F_m stands for flowshop with m machines, $B_i=+\infty$ denotes that buffers with infinite capacity are present, allowing non-permutation schedules, and $C_{\max}$ denotes the makespan minimization as the optimization criterion. Minimizing the makespan is one of the most common criteria in the literature: lower total completion time is associated with less idle time, higher machine utilization and efficiency.

Some authors generate random problems or use data taken from realistic cases to test the performance of their proposed algorithms. Demanding benchmark problems allow comparing objectively and quantitatively the performance of different algorithms, also belonging to different classes, e.g. heuristics and metaheuristics. Among the most used flowshop benchmarks is the set by Taillard (1993) considered in this work, which includes small, medium and large sets as opposed to Demirkol, whose dataset is limited to medium size. Non-permutation bounds from several authors are available in http://www.mathematik.uni-osnabrueck.de/research/OR/fsbuffer/taillard2.txt, mirrored in http://www.ing.unipi.it/lanzetta/flowshop/taillard2.txt and have been included in current analysis.

Biologically inspired general-purpose optimization algorithms are capable to deal with large job-size problems and with the exponential increase in the solution search space with the

number of machines and jobs. Examples of metaheuristics include taboo search, simulated annealing, genetic algorithms (Elbeltagi et al., 2005) and memetic algorithms (Amaya et al., 2012). Despite their successful performance, in the extensive reviews by Ruiz and Maroto (2005) and by Ribas et al. (2010) ant colony or pheromone-based systems are not present. Ant colony systems, a subset class of Ant Colony Optimization (ACO), use artificial or swarm intelligence by exploiting the experience of an ant colony as a model of self-organization in co-operative food retrieval (Wang et al., 2003).

ACO has been selected among metaheuristics because of its ability to build constructively arbitrary permutations of job sequences (NPFS schedules) by two inverse mechanisms: negative and positive pheromone deposition, respectively through the local update rule and off-line pheromone update rule, detailed in the ACO description. Diversification by the local update rule pushes towards permutated schedules and is the core mechanism to generate natively non-permutation solutions.

Standard ACO by Bonabeau et al. (1999) and disjunctive graph model inspired by Rossi and Dini (2007) are combined in this paper. It seems that the only Ant Colony algorithm applied to non-permutation flowshop scheduling is by Sadjadi et al. (2008), which provides the relative average performance on the Taillard's benchmarks. Other non-permutation flowshop benchmarks from Demirkol have been considered by Ying and Lin (2007) and by the authors (Rossi & Lanzetta, 2013a, 2013b).

Sadjadi et al. applied the standard ACO specifications from Bonabeau et al., except for the diversification mechanism. The other main difference is on the selection of the initial population, which is determined by improving a permutation solution found by heuristics using local search.

Other approaches to the permutation flowshop problem tested on benchmarks based on Ant Colony Systems by Rajendran and Ziegler (2004), Min-Max Ant Systems by Stuetzle (1998),

the state-of-the-art based on Tabu Search by Brucker et al. (2003), and Genetic Algorithms by Färber and Coves Moreno (2006) are also compared with the proposed ACO.

Non-permutation flowshop scheduling (NPFS) problem

The mixed-integer linear programming (MILP) model for the NPFS problem is the following:

Parameters

p_{ij} = processing time of job i on machine j

$BigM$ = a sufficiently large positive value

Decision variables

Z_{ilj} = 1, if job i is assigned to sequence position l on machine j; 0 otherwise

Dependent variables

S_{lj} = starting time of job in sequence position l on machine j

The subscript symbols are: i and i' for jobs, i, i' = 1,2,. . .,n; l and l' for the sequence positions, l, l' = 1,2,. . .,n; j for machines, j = 1,2,. . .,m; n and m are the number of jobs and machines, respectively.

Objective function

$$\text{Min } C_{\max}$$

Subject to the following constraints:

$$\sum_{i=1}^{n} Z_{i l j} = 1 \qquad \begin{matrix} l = 1,..,n \\ j = 1,..,m \end{matrix} \quad (1)$$

$$\sum_{l=1}^{n} Z_{i l j} = 1 \qquad \begin{matrix} i = 1,..,n \\ j = 1,..,m \end{matrix} \quad (2)$$

$$S_{1j} + \sum_{i=1}^{n} p_{ij} Z_{i1j} \le S_{1(j+1)} \qquad j = 1,..,m-1 \quad (3)$$

$$S_{lj} + \sum_{i=1}^{n} p_{ij} \cdot Z_{ilj} \le S_{l(j+1)} \qquad \begin{matrix} i = 1,..,n-1 \\ j = 1,..,m \end{matrix} \quad (4)$$

$$BigM(2 - Z_{il\,j} - Z_{i\,l'(j+1)}) \ge S_{l\,j} + p_{l\,j} + S_{l'(j+1)} \qquad \begin{matrix} i,l,l' = 1,..,n \\ j = 1,..,m-1 \end{matrix} \quad (5)$$

$$\left| \langle i,\ i = 1,..,n,\ i \ne i' :\ S_{1\,j} < S_{l\,j} \cdot Z_{il\,j} \le S_{1\,j} + p_{i'\,j} Z_{i'l\,j},\right.$$
$$\left. l = 2,...,m \rangle \right| \le n-2 \qquad j = 2,..,m \quad (6)$$

Constraint (1) ensures that each job is assigned to exactly one position of the job sequence on every machine. Constraint (2) states that each position of the job sequence processes exactly one job on every machine. Constraint (3) denotes the starting times of the first job on every machine. Constraint (4) insures that the the $(l + 1)^{th}$ job in the sequence of machine j does not start on machine j until the l^{th} job in the sequence of machine j has completed. Constraint (5) insures that the starting time of job i which is assigned to position l in the sequence on machine $j + 1$ is not earlier than its finish on machine j. Constraint (6) ensures that the buffer size is subject to:

Lemma

 The flowshop scheduling with n jobs and m machines is B_i=+∝ if and only if the interoperational buffer size for machine j ($2 \le j \le m$) is at least (n-2).

The buffer size for machine j=1 and j=m+1 is n (i.e. the input and output buffers contain up to n jobs).

■ In the worst case, only one blocking with (n-1) jobs waiting occurs. Let j ($2 \leq j \leq m$) be the blocked machine. If the last job on machine (j-1) is completed, no blocking occurs because (n-1) jobs have been already processed on machine (j-1). Hence (n-2) jobs wait in the interoperational buffer between machines (j-1) and j ■

The optimization problem (1)-(6) can also be represented by a disjunctive graph (Figure 3):

$$DG = (N, A, E_j, W) \qquad (7)$$

where N is the set of operations, plus the dummy start and finishing operations represented by the symbols 0 and *; A is the set of conjunctive arcs (directed arrows) between every pair of operations on a job routing; E_j is the set of disjunctive arcs between pairs of operations at stage j; W is the set of weights (processing times) on nodes.

Figure 3. Disjunctive graph (digraph) for flowshop scheduling, with processing times p_{ij} at nodes O_{ij} for n jobs on m machines.

Figure 3 about here

Ant Colony Optimization for NPFS

The pheromone trail is the basic mechanism of communication among real ants. It is mimicked by ACO by an iterative method (in epochs) able of finding the shortest path

connecting source 0 (nest) and destination * (food) on a weighted graph (Figure 3), which represents the optimization problem.

The ant runs the nest-food path by a probabilistic selection of nodes according to the following mechanisms: i) *intensification* to select a node in the vicinity of the current best paths; ii) *diversification* in order to produce promising alternative paths.

The proposed ACO follows the standard recommendation for applications to scheduling problems, as opposed to the other implementations available in the literature for the flowshop problem introduced above.

The proposed digraph approach builds *natively* non-permutation sequences by the *path generation* mechanisms. In this stochastic process, each artificial ant selects probabilistically the next node (move selection) according to the amount of pheromone on the connecting arc (*learned desirability*).

The path associated with each ant starts from 0, follows routing arcs, directs disjunctive arcs and ends in *. By design, non-permutation schedules are achieved by directing arcs differently at each stage. C_{max} is evaluated from W (7). At each epoch, as soon as all the paths of the ants in the colony are generated, the best ant (lowest C_{max}) deposits on its arcs an amount of pheromone proportional to the path length (*pheromone updating*). A pheromone decay routine is also performed to prevent stagnation in local optima solutions (*evaporation* ρ=0.12).

The two inverse mechanisms are achieved by negative and positive pheromone deposition, respectively through the local update rule and off-line pheromone update rule. Diversification by the local update rule pushes towards permutated schedules and is the core mechanism to generate natively non-permutation solutions.

This is a constructive way to generate a schedule. A complete solution is generated forward by a partial solution using the stigmergy of the colony, i.e. the selection of the more

promising disjunctive arcs where a higher amount of pheromone is laid. The main goal of the ACO mechanism is to generate optimal solutions by constructive schedules. The concept is similar to “divide et impera”, because the stigmergy progressively concentrates the search in a low number of very small promising regions. Differently to local search, this fact makes the algorithm intrinsically parallel and may take advantage of modern processors.

Path generation

By the pheromone mechanism ants may select arbitrary path, consequently the resulting scheduling sequences (ant tours) are different permutations (non-permutation approach). Random initial solution are generated and iteratively improved at each epoch by the ant behavior. By this natively constructive approach we are able to assess the net performance of the algorithm.

An ant *a* to generate an acyclic conjunctive graph with weights on the conjunctive arcs, i.e. feasible schedule S_a, visit every operation on the pheromone-learning model *DG* (7) one and only one time with a complexity of O(*m*·*n*) in order to transform the digraph in a feasible schedule. Path generation is a stochastic process where an ant starts from the dummy 0 and selects the next node from the set of allowed operations. It uses the following *transition probability* rule as a function of both the heuristic function of desirability, η (termed *visibility function*), and the amount of pheromone τ on the edge (O_{ij} , *J*), with $J \in AL$, of the pheromone trail:

$$z = \begin{cases} \arg\min_{O_{ij} \in AL} \left\{ \tau(o_{ij}, o_{ij})^{\alpha} \eta(o_{ij}, o_{ij})^{\beta} \right\}, & \text{if } q \le q_0 \\ J, & \text{if } q > q_0 \end{cases} \qquad (8)$$

The non-negative parameters α and β represent the intensity of respectively, the amount of pheromone and the visibility included in the transition probability function. The non-negative

parameter q_0 is the *cutting exploration*, a mechanism that restricts the selection of the next operation from the candidate list *AL*. If a random number q is higher than the cutting exploration parameter q_0 ($0 \leq q_0 \leq 1$), the candidate operation is selected by examining the probability of all candidate operations that are as much desirable as higher visibility and pheromone amount are; otherwise the most desirable operation is selected, i.e. the arc with the highest amount of pheromone and the highest visibility.

The role of cutting exploration is that of explicitly split the search space in order to achieve a compromise between the probabilistic mechanism adopted for $q \leq q_0$ or the further intensification mechanism of exploring near the best path so far, which corresponds to an exploitation of the knowledge available about the problem. Cutting exploration by tuning parameter q_0 near 1 allows the activity of the system to concentrate on the best solutions (*exploitation activity*) instead of letting it explore constantly (*exploration activity*, achieved by tuning parameter q_0 near 0). In fact, when q_0 is close to 0, all the candidate solutions are examined in probability, whereas when q_0 is close to 1, only the local optimal solution is selected by equation (8). In this paper, a *freezing function* is considered, which is similar to the one proposed by Kumar *et al.* (2003). This function progressively freezes the system by tuning q_0 from 0 to 1, in order to favor exploration in the initial part of the algorithm and then favor exploitation by means of the following expression:

$$q_0 = \frac{\ln(epoch)}{\ln(n\_epochs)} \tag{9}$$

where *epoch* is the current iteration and *n_epochs* is the total number of iterations of the ant colony system.

The heuristic function of desirability η is a very critical component of ant colony systems. Generally it is implemented by dispatching rules. A comparison among a number of dispatching rules to implement the visibility function has been performed by Blum and Sampels (2004). In this paper the earliest starting time (EST) rule is used, the best one according to Blum and Sampels.

Local update rule

The local update rule is applied to favor the exploration of not visited nodes by other ants of the colony. This rule imposes to the ant that has selected a candidate operation J, of laying on the connecting arc $(O_{i'j}, J)$ the following negative amount of pheromone:

$$\tau(O_{i'j}, J) = (1-\rho) \cdot \tau(O_{i'j}, J) + \rho \cdot \tau_0 \qquad (10)$$

The local update rule is a convex combination of parameters equal to the evaporation coefficient; in this case the convex combination has points $\tau(O_{ij}, J)$ and τ_0. The amount of pheromone that remains on a selected edge diminishes because it ranges between the previous value $\tau(O_{i'j}, J)$ and the initial value τ_0. As a consequence, the effect of this rule is making nodes less and less attractive as they are visited by ants, indirectly favoring the exploration of not visited nodes. This is a basic diversification mechanism because it pushes the next ants to generate alternative paths.

Off-line pheromone update rule

This feature arises when a positive amount of pheromone has to be deposited. The ant that detects the best path at each epoch is termed best-epoch ant (S_{be}). In order to direct the exploration of the best nest-food path by the entire colony, an *off-line update rule* of pheromone is performed. At the end of each epoch, the best-epoch ant S_{be} deposits on all paths of the acyclic graph generated a further amount of pheromone, proportional to the following convex combinations of points $\tau(O_{ij}, J)$ and $makespan(S_{be})^{-1}$. This produces a search intensification by other ants of the colony in the vicinity of the best solution:

$$\tau'(O_{ij}, J) = (1-\rho) \cdot \tau(O_{ij}, J) + \rho \cdot makespan(S_{be})^{-1}, \qquad (O_{ij}, J) \in S_{be} \qquad (11)$$

$$= (1-\rho) \cdot \tau(O_{ij}, J), \qquad \text{otherwise}$$

As for the local update rule, the amount of pheromone $\tau'(O_{ij}, J)$ that remains on the selected edge ranges between the previous value, $\tau(O_{ij}, J)$, and a value closer to the optimum: $makespan(S_{be})^{-1}$. A routine of pheromone decay on pheromone trails is performed on other arcs of the digraph, thus indicating that a path rarely used probably does not lead to optimal solutions.

Pseudo code

The algorithm has been implemented in C++ according to the following scheme.

Algorithm. High-level description of Ant Colony System for Native Non-Permutation Flowshop Scheduling

Input: a weighted digraph $WDG=(N, A, E_j, W_N, W_E)$

// **Initialization**

for each disjunctive arc $(O_{i'j'},O_{ij})$ of E_A deposit a small constant amount of pheromone

$W_E(O_{i'j'},O_{ij}) = (\tau_0, \tau_0)$ where $\tau_0 = \left[n \cdot m \cdot \max_{j=1,..,m} \sum_{i=1}^{n} t(O_{ij}) \right]^{-1}$

epoch ← 1; *not_improve* ← 0;

// Main Loop

while(*not_improve* < *stability_condition*) **do**

// Epoch Loop

for each ant *a*, *a*=1 **to** *population size* **do**

// Path Generation

$S_a \leftarrow \varnothing$;

1. $\boldsymbol{O} \leftarrow \{O_{ij} \mid i=1,..,n, j=1,..,m\}$;
2. *Initialization of Candidate Nodes:* $AL_w \leftarrow \boldsymbol{O}$*;*

for each *w* =1 **to** $n \cdot m$ **do**

3. *Initialization of Feasible Moves* (i.e. the disjunctive arcs connected to operation of AL_w)*;*
4. *Move Selection*: select a feasible move $(O_{i'j}, O_{ij})$ of E_A where $O_{i'j}$ is the last operation in the queue of machine *m* ($O_{i'j}$ = dummy 0, if *m* =1) by means of the transition probability rules (8); directing the related disjunctive arc ($O_{i'j}$ =dummy 0, if *m* =1);
5. *Arc Removing:* remove all the disjunctive connected to $O_{i'j}$ (i.e. no other operation can be immediately subsequent to $O_{i'j}$ in the machine sequence);

6. *Computing length:* move $t(O_{ij}) \in W_N$ from the selected node to the directed; also, move $t(O_{ij})$ on $(O_{i(j-1)}, O_{ij}) \in A$;
7. *Path length evaluation:* the longest path between that one connected to the directed arc and that one connected to the arc of the job routing is placed as a mark of the scheduled operation;
8. *Local Updating*: apply the local update rule (10) to the arcs $(O_{i'j'}, O_{ij}) \in W_E$;
9. *Update Allowed:* remove the scheduled operation to the allowed list, $AL_w \leftarrow AL_w / \{O_{ij}\}$;

end for

10. Directing the remaining disjunctive arcs. These arcs are connected to dummy *.

11. *Local Search*: Apply local search with neighbor structure of Nowicki and Smutnicki (1996) to S_a;

12. *Best Evaluation:* **if** ($makespan(S_a) < makespan(S_{be})$)

then ($makespan(S_{be}) \leftarrow makespan(S_a)$ **and** $S_{be} \leftarrow S_a$)

end if

end for

Global Updating: Apply the global update rule (11);

Best Ant Evaluation: **if** ($makespan(S_{be}) < makespan(S^*)$)

then (($makespan(S^*) \leftarrow makespan(S_{be})$; $S^* \leftarrow S_{be}$ **and**

$epoch \leftarrow 0$) **and** $not\_improve \leftarrow 0$;

else $epoch$++ **and** $not\_improve$++;

end if

end while

Output: S^*

Figure 4. Performance of ACO systems in non-permutation and in permutation (PFS) configuration on the Taillard's benchmarks with respect to *permutation* upper bounds from Stuetzle (1998) [S], Rajendran and Ziegler (2004) [RZ] and Sadjadi et al. (2008) [SBZ].

Figure 4 about here

Computation experiments

Benchmark instances are arrays $b_{n\times m}$. The $n\times m$ operations of each job on all m machines are represented by their processing times ordered by routing. Taillard's benchmarks include 12 sets of 10 instances for job numbers i=20, 50, 100, 200, 500 and machine numbers j=5, 10, 20. Each benchmark instance k includes a non trivial lower (LB_{ijk}) and upper bound (UB_{ijk}). The lower (upper) bound is the maximum (minimum) known theoretical minimum (maximum) attainable makespan. The upper bound can be reduced by new improved solutions. If it coincides with the lower bound, the optimum for benchmark b_{ijk} has been reached.

Metrics for algorithm performance are the individual relative distances from the upper bound of benchmark instances $b_{n\times m}$ or the mean relative error in each set (i,j):

$$MRE_{ij}^{best} = \frac{1}{10}\sum_{k=1}^{10}\frac{C_{best_{ijk}} - UB_{ijk}}{UB_{ijk}} \tag{12}$$

$$MRE_{ij}^{avg} = \frac{1}{10}\sum_{k=1}^{10}\frac{C_{avg_{ijk}} - UB_{ijk}}{UB_{ijk}} \tag{13}$$

where the best and the average solutions for each set (i,j) of 10 benchmark instances k=1,…,10 are respectively $C_{best_{ijk}}$ and $C_{avg_{ijk}}$.

The proposed NNP-ACO has been run 10 times with the (selected) parameters in Table 1 on 3 GHz 32 bit Intel® Pentium® IV based PCs with 2 GB RAM.

Table 1. Preliminarily tested and selected parameters for the proposed NNP-ACO.

Table 1 about here

The main ACO parameters described are summarized in Table 1, have been derived from the job shop application in Rossi and Dini (2007) and have been explored in preliminary tests with the values indicated for *population_size*, α, β, ρ and η.

As for the population size, fewer ants have been used vs. Rajendran and Ziegler (40 ants) and vs. Sadjadi et al. (1000 ants) in order to reduce the processing time. Consequently, the evaporation rate has been reduced vs. Sadjadi et al. (ρ=0.9) to reduce the effect of random search.

The stop criterion from Sadjadi et al. is a fixed computation time, instead we use a stability condition, corresponding to 3000 epochs with error reduction of at least one processing time unit.

Results

The average performance (MRE^{avg}) of the proposed ACO are compared in Figure **4** within the same class of problems with Sadjadi et al., Rajendran and Ziegler and Stuetzle, which do not provide results for datasets of 200 and 500 jobs.

Although results are discrete, a graphical representation with connecting lines has been preferred to show the separation among the performance of different algorithms.

The differences of makespan of the proposed algorithms of the respective authors have been calculated from the upper bound of the *permutation* flowshop benchmark. Because the detailed values are not available, the proposed algorithm has been compared with the (slightly higher) permutation upper bound for performance assessment.

A lower value of MRE^{avg} means a better performance (lower makespan) of the proposed algorithm compared to the state-of-the-art. A negative value represents a new (lower) upper bound.

For comparison within the same class of algorithms (ACO), MRE^{avg} has been conservatively calculated with respect to the original permutation upper bounds from Taillard, because most available results are for permutation flowshop, except Sadjadi et al.

The MRE^{avg} of the proposed ACO ranges between +0.035 and +0.159, while Sadjadi et al. is between –0.075 and +1.12, Rajendran and Ziegler is between +0.72 and +1.86 and Stuetzle is between +0.196 and +2.475 (not shown). This also means that the MRE^{avg} of the proposed ACO is upper limited to 16% as opposed to 112% from Sadjadi et al. The algorithms in Rajendran and Ziegler, and Stuetzle show the worst performance overall. Out of scale MRE^{avg} values (available on the respective articles) have not been represented to achieve a higher visualization detail on the best results. The non-permutation algorithm from Sadjadi et al. behaves clearly better than with the permutation (PFS) constraint. The algorithm from Sadjadi et al. has the best performance with 20 jobs or with 5 machines (small problems). Although the performance on large instances (200 and 500 jobs) are not available from these authors, a degradation of performance with benchmark size (job and machine number) is clearly visible on medium instances. This is enhanced by the steeper trend line for the better (non-permutation) algorithm from Sadjadi et al.

The upper bounds for the makespan of all 12 sets of 10 benchmark instances in *non-permutation* flowshop configuration from various authors and methods, averaged are reported in Table 2. The MRE^{avg} found by the state-of-the-art from Brucker et al. and Färber and Coves Moreno, based on Tabu Search and Genetic Algorithms respectively is compared with the proposed ACO. The better results (highlighted) have been obtained for higher machine numbers and job numbers. Results are not available from Brucker et al. and Färber and Coves Moreno for the 40 largest instances, where the proposed ACO becomes the best known solution.

Here the non-permutation upper bounds have been used for comparison between the proposed NNP-ACO and the state-of-the-art of metaheuristics in general, using Taillard's benchmarks.

Table 2. Performance assessment in non-permutation (NPFS) configuration. $C_{best_{i\,j\,k}}$ is the best makespan obtained by the proposed ACO in a single run or otherwise defined by Brucker et al. [B] and Färber and Coves Moreno [FCM].

* 300 epochs.

** from http://www.mathematik.uni-osnabrueck.de/research/OR/fsbuffer/taillard2.txt

Table 2 about here

The last 4 sets of instances are one order of magnitude more time consuming compared to the other instances (200 vs. 10 min.) because of their large size. Other methods use a stop criterion based on a fixed number of epochs or computation time. Instead we use a stability condition (of 3000 epochs with an improvement of at least one processing time unit), which has been reduced by one order of magnitude and still results in a processing time one order of magnitude higher. By the stability condition instead of a stop criterion, convergence is assured regardless of the epoch number.

Discussion

As shown, the proposed algorithm becomes the state-of-the-art on the benchmarks used, with the ACO approach, particularly on larger instances.

Possible reasons of the better performance compared to Sadjadi et al. as a function of the benchmark size are inferred:

1. the ACO implementation by Sadjadi et al. has a higher colony size and lower epoch number, which do not allow sufficient differentiation despite the high evaporation, particularly on larger instances;
2. Sadjadi et al. find an initial permutation schedule (pheromone trails) by the NEH heuristic (Nawaz et al., 1983). Initial good solutions provide good final solution for small sized benchmarks. For larger benchmarks the NEH heuristic suffers some performance decrease. Consequently, ACO search can be trapped in local optima;
3. Sadjadi et al. start from a solution of the permutation problem and find an NPFS solution by a local search, which causes a further performance decrease.

The proposed ACO has also been compared with permutation upper bounds from Rajendran and Ziegler and still provides better performance, despite the higher problem complexity of NPFS ($n!^m$ compared to $n!$).

A regression analysis has been carried out to assess the effect of the machine and job number on the makespan of the best scheduling found by the proposed ACO. A correlation has been found between MRE^{avg} and machine number at constant job number. This is also qualitatively shown by the periodic MRE^{avg} increase in Figure **4**. The same trend also shows the relative independence of the algorithm performance on the job number.

A stronger correlation has been found between computation time and both job number and machine number. The computation time with the proposed ACO, which has not been optimized in this work, is one order of magnitude higher than Sadjadi.

Compared to non-permutation metaheuristics, new upper bounds have been proposed on larger instances and there is still margin of improvement, by parameters optimization, on others.

A summary of benefits and drawbacks of the proposed approach is available in Table 3

Table 3. Summary of benefits and drawbacks of the proposed approach.

Table 3 about here

Conclusions

A mathematical model of the flow line scheduling problem with unlimited buffers has been proposed. The few existing approaches have been compared using well-known benchmarks on a wide size spectrum available from Taillard (1993).

The NP-hardness has been tackled by metaheuristics and ACO have been selected. The proposed ACO is natively non-permutation as opposed to other authors who apply a local search to permutation solutions. Natively means that initial ant paths are selected arbitrarily and the pheromone mechanism stimulates differentiation among permutated schedules (non-permutation scheduling).

The proposed approach shows the best performance in non-permutation flowshop configuration, particularly on larger instances and is very close to the state-of-the-art metaheuristics.

Based on computation experiments, it can be concluded that such general-purpose optimization tool has high potential in non-permutation flowshop scheduling and can provide good solutions, regardless of the problem complexity increase in the examined range.

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Tables

Table 1

Parameter	NNP-ACO (tested)	NNP-ACO (selected)
population_size	5, 10, 20	5
τ_0	$\left[n \cdot m \cdot \max_{j=1,..,m} \sum_{i=1}^{n} t(O_{ij}) \right]^{-1}$	$\left[n \cdot m \cdot \max_{j=1,..,m} \sum_{i=1}^{n} t(O_{ij}) \right]^{-1}$
α	0.1, 0.2, 0.5, 1, 1.5, 2	2
β	$(0.1 \cdot i)$, i=1,…,8	0.3
stop criterion	*not_improve* < *stability_condition*	*not_improve* < *stability_condition*
stability_condition	3000	3000
q_0	$\frac{\ln(not\_improve + 1)}{\ln(stability\_condition)}$	$\frac{\ln(not\_improve + 1)}{\ln(stability\_condition)}$
ρ	$(0.04 \cdot i)$, i=1,…,9	0.12
local search	steepest descent (Nowicki and Smutnicki, 1996)	steepest descent (Nowicki and Smutnicki, 1996)
η	EST, PAST (Rossi and Dini, 2007)	Earliest Starting Time (EST)

Table 2

Jobs	I	J	Mean NPFS upper bounds**	MRE^{avg} (12)	MRE^{best} (13)
Instances			Taillard's Benchmarks	State-of-the-art [reference]	Proposed NNP-ACO
	20	5	1217.1	0.000 [B]	0.057	0.023
	20	10	1494.0	0.013 [FCM]	0.107	0.079
	20	20	2228.8	0.130 [B]	**0.096**	0.072
	50	5	2731.9	0.001 [FCM]	0.048	0.026
	50	10	2979.1	0.020 FCM]	0.136	0.119
	50	20	3717.1	0.290 [B]	**0.163**	0.143
	100	5	5237.3	0.020 [B]	0.036	0.021
	100	10	5618.6	0.130 [B]	**0.107**	0.081
	100	20	6312.4	--	**0.165**	0.141
	200	10	10663.1	--	**0.084**	0.064
	200	20	11272.8	--	**0.160***	0.149*
	500	20	26362.8	--	**0.120***	0.116*

Table 3. Summary of benefits and drawbacks of the proposed approach.

Benefits	drawbacks
general purpose optimization algorithm	parameters need to be selected (and optimized) by preliminary tests
constructive solutions from random initialization: net performance can be assessed	local optima are found (no global optima)
relative invariance of performance with problem size/complexity	further research is required to match the performance of other metaheuristic approaches

Captions of Figures

Figure 1. Two flow lines, with and without buffers. Permutation (PFS) and non-permutation flowshop (NPFS) are compared. In both cases, jobs see machines (routing) in the same sequence (flowshop). In non-permutation flowshop, buffers allow changes (permutations) of job sequences on subsequent machines.

Figure 2. Flow line (clockwise from top left) with *m* machines (or stages) M (bright red) and different examples of buffer configurations (dark blue) to allow job sequence permutation between machines.

Figure 3. Disjunctive graph (digraph) for flowshop scheduling, with processing times p_{ij} at nodes O_{ij} for *n* jobs on *m* machines.

Figure **4**. Performance of ACO systems in non-permutation and in permutation (PFS) configuration on the Taillard's benchmarks with respect to *permutation* upper bounds from Stuetzle (1998) [S], Rajendran and Ziegler (2004) [RZ] and Sadjadi et al. (2008) [SBZ].

Captions of Tables

Table 1. Preliminarily tested and selected parameters for the proposed NNP-ACO.

Table 2. Performance assessment in non-permutation (NPFS) configuration. $C_{best_{i\,j\,k}}$ is the best makespan obtained by the proposed ACO in a single run or otherwise defined by Brucker et al. [B] and Färber and Coves Moreno [FCM].

* 300 epochs.

** from http://www.mathematik.uni-osnabrueck.de/research/OR/fsbuffer/taillard2.txt

about here

Table 3. Summary of benefits and drawbacks of the proposed approach.

. Summary of benefits and drawbacks of the proposed approach.

Figure 1 Color online

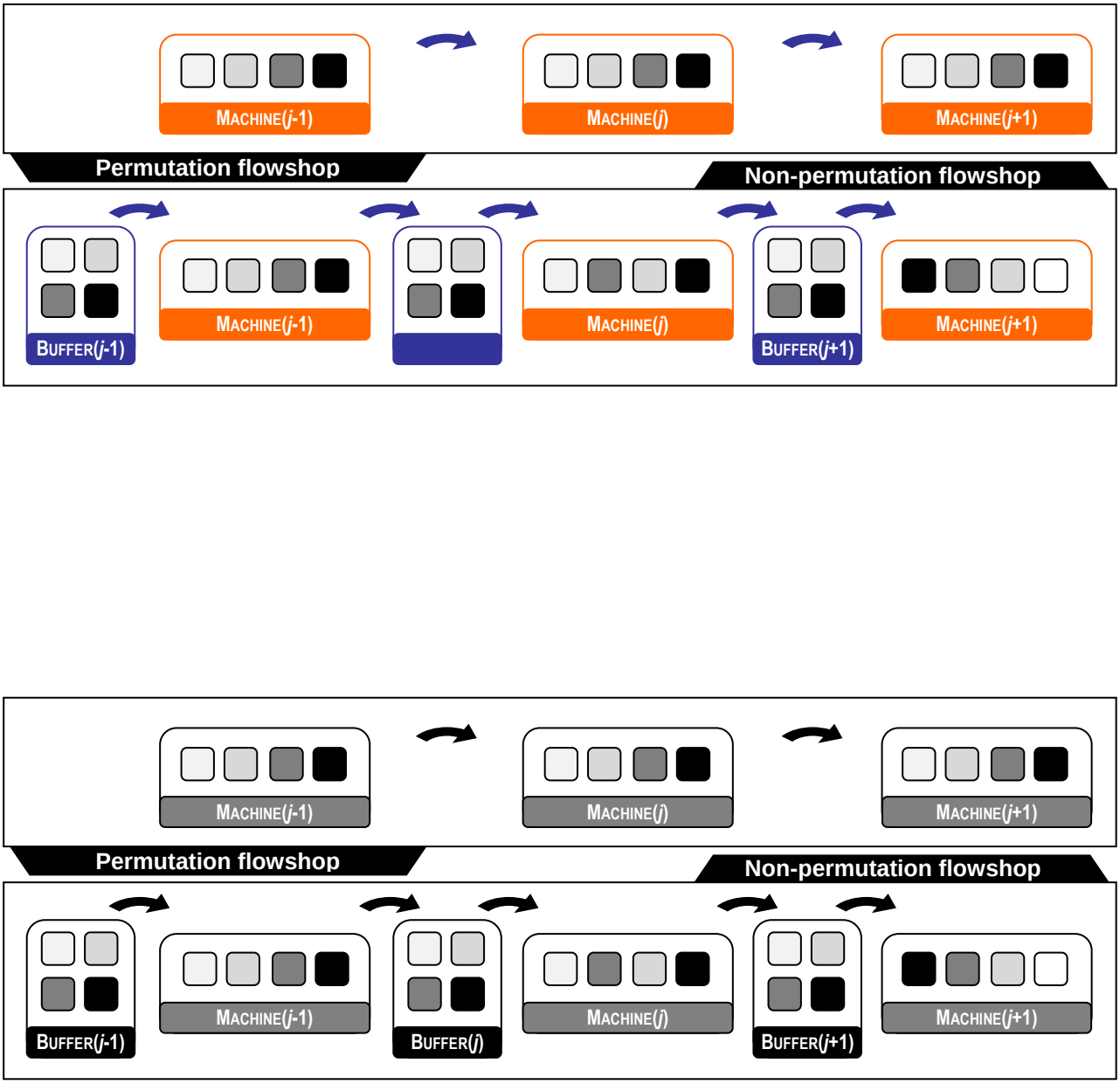
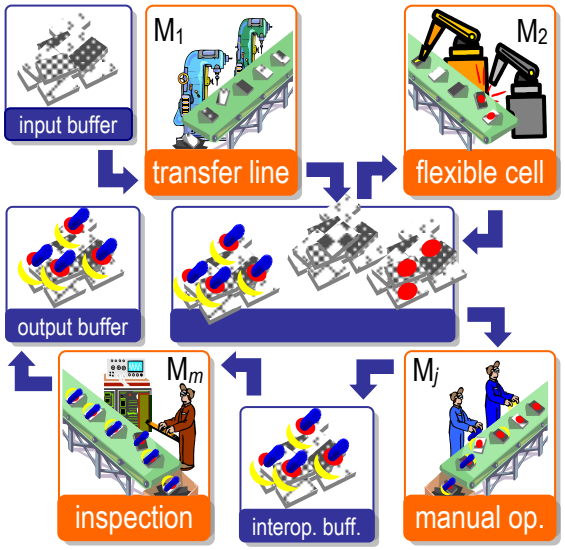


Figure 2 Color online



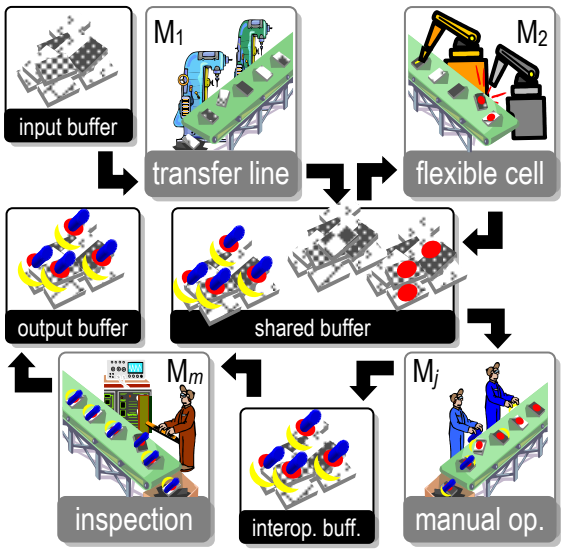


Figure 3

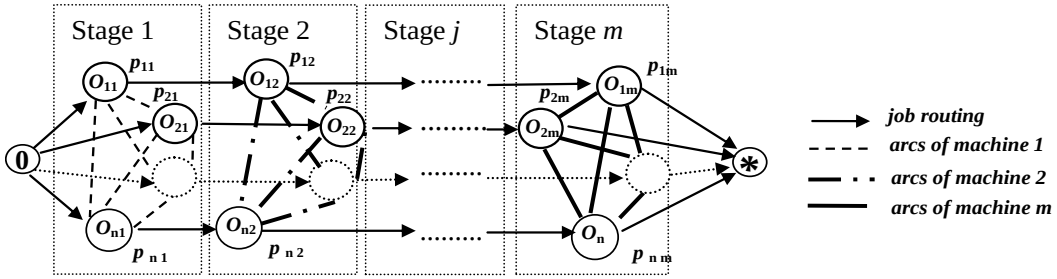

Stage 1
Stage 2
Stage j
Stage m
0
*
job routing
arcs of machine 1
arcs of machine 2
arcs of machine m

Figure 4 Color online