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Form-finding and buckling optimisation of gridshells using genetic algorithms

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Summary: We present a strategy for the design of gridshells where form-found structures are optimised for buckling resistance. A genetic algorithm is employed for the initialisation of pre-stress forces required in form-finding using dynamic relaxation. Dynamic relaxation takes this initial pre-stress, a flat grid, as well as self-weight and nodal loads to calculate a static equilibrium. The structure is then analysed for the estimation of the critical buckling load. Different boundary conditions, structural parameters and typology of connections are compared, including a gridshells with triangular and quadrangular patterns. Optimised structures are measured against the trivial solution, which is a structure where dynamic relaxation is initialised with uniform pre-stress. Our results show the proposed strategy can successfully form find gridshells with improved buckling performance.

Keywords: *form-finding, non-linear buckling, optimisation, genetic algorithms, fitness function*

1. INTRODUCTION

Gridshells are lightweight and elegant structures for large-span roof designs. Form-finding is the key design step to realise a light and efficient gridshells. Historically, early gridshells were based on techniques developed for other type of structures, such as concrete shells and tensile structures, like the pioneer shell works of Heinz Isler or Frei Otto's tensile and membrane structures. One basic design assumption in those earlier works was the minimal surface solution, computed using physical models like soap films, resulting in homogeneous distribution of stresses, ie equitensional or minimal surfaces. For instance, Isler's form-finding process allowed a coherent utilisation of isotropic material, such as reinforced concrete, and at the same time minimised the surface thickness and bending moments. The same concept was developed for tensile structures, like the ones done by Otto, where equitensional stress distributions were sought as equilibrium solutions of soap films. For simple roofs, the minimal surface with homogeneous distribution of stresses is probably the simplest solution, as described with soap films.

Here we drop the constraint of minimal surface solutions to look for structures with enhanced structural characteristics. There is a wealth of literature on structural optimisation, eg [1-9], with typical parameters to improve like the minimisation of bending moments, stresses, deflection, weight, or the maximisation of buckling load. In this paper, we optimise gridshells, form-found beyond the minimal surface constraint, where the critical buckling load is maximised. The study is done using a genetic algorithm, with form-finding and buckling load calculations in the inner loop. It is thus required a fast but reliable estimation of the buckling load factor, hence the motivation for this research.

After briefly introducing the evolution of gridshells in section 2, we then give an overview of relevant form-finding aspects of our work in section 3. Sections 4 and 5 present the optimisation strategy, while section 6 describes several test cases. Results and discussions are shown in sections 7 and 8, respectively. Finally, conclusions appear in section 9.

2. EVOLUTION IN GRIDSHELLS

Gridshell designs move beyond the minimal surface approach especially when escaping the traditional triangular grid, like the glass roof at the British Museum, London and the quadrilateral grids with bracings like Schlaich's designs (eg Museum for Hamburg History). The former are clearly a discretised version of a continuous surface; while the latter has orthotropic in-plane wiring which also behaves not much different from the triangular discretisation of a continuous surface.

Other type of gridshell can be designed. The current steel methodology allows grids having different member sizes and patterns other than the triangular or quadrangular ones. For instance, in the glass roof at Neumünster Abbey [10] in Luxemburg, see Figure 1a, the triangular grid is realised using single oriented arcs and pre-stressed cables in the other two directions. This reduction in the element size enhanced the roof transparency. Moreover, a theoretical study, based on the same site (called Neumünster II, Figure 1b), implemented another strategy. The grid is still based on elements of different size and connectivity, and organised not in a regular way but according to a viable geometry based on single curvature (developable) panels [10,20].

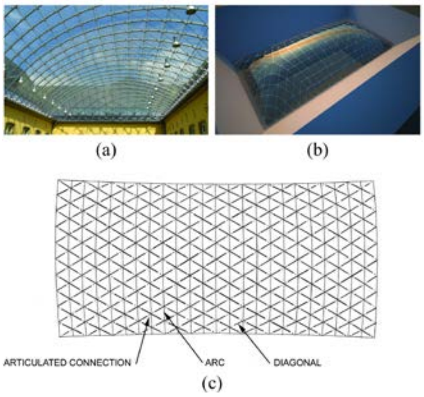

Fig. 1: (a) Plan of Neumünster I. The glass roof of the main courtyard of the Neumunster Abbey, the gridshell is composed by inclined arcs, in plan, and two cable line running longitudinally and transversally; (b) Neumünster II. The gridshell is composed by continuous arcs linked together by struts with cable bracings, the quadrilateral grid deforms to changing of direction at corners; (c) Connectivity of Toulouse's Jean Jaures metro station. The triangular grid is composed by a series of continuous arcs connected by two discontinuous lines compose of elements alternatively clamped or articulated at their extremity.

The play on element connectivity can be extended to more complex configurations, like the un-build gridshell for the Jean Jaures metro station in Toulouse [9]. Such in-plane organisation of the structure and mixture of steel members of different sizes have an influence in the distribution of the structural element forces, and diverging from the concept of minimal/equitensional surface principle. Due to those novel structural approaches, new form-finding strategies are required to fully cover the complexity of free-form of gridshells.

3. FORM-FINDING

Force-density and dynamic relaxation algorithms are perhaps the two most popular form-finding methods. Here GsRelax, a dynamic relaxation algorithm available in Oasys GSA [12] was employed. It is a fast and versatile solver for structures in the large displacement regime and hypostatic structures. The form-finding with dynamic relaxation requires an initialisation step where pre-stress forces and material properties are assigned to each member in the grid. We assign zero Young's modulus and certain pre-stress forces to every element of the initially-flat grid. External nodal forces are applied and the system is then run to converge to configurations where nodal loads and self-weight balance the internal pre-stress forces. The real material properties are then assigned to the resulting equilibrium geometry, while keeping the boundary conditions and connectivity. This is the base for static and buckling calculations, as well as further engineering work.

The selection of a suitable pre-stress initialisation, and the understanding on how it impacts the final geometry, is far from obvious. In engineering practice, e.g. at RFR and elsewhere [17, 19], a selection of homogeneous, initial pre-stress forces is the first choice. The shape can be refined by adjusting the initial pre-stresses in empirical ways (eg Neumünster II, in Figure 1b). In some particular cases, like the force density method for hyperbolic paraboloids and some tensegrity structures, it is indeed possible to calculate the pre-stress for static equilibrium in a precise manner [17-18], but there is no clear way to couple the form-finding and buckling optimisation. In dynamic relaxation, nothing prevents the designer to initialise the algorithm with non-homogeneous pre-stresses. It might be a time-consuming and non-intuitive task, especially if buckling load or other structural performance is a relevant parameter to optimise.

In the next section we present an optimisation strategy for the generation of initial pre-stress forces in form-finding of grid shells such that the elastic buckling load is maximised.

4. OPTIMISATION

Structural optimisation, mono or multi-objective, has a long tradition in computational mechanics, eg [1-8]. However, the combination of structural optimisation and form-finding, a key issue in the design of real-life gridshell structures, has not received the attention it deserves. Recent work [4] has focused more on the stiffness optimisation of structures which concentrates on the modification of the in-plane structural arrangement (in-plane anisotropy and angles between rods) rather than modifying the reference surface. Other works, eg [11], deal with the optimisation of stiffness in continuous shells modelled with Bezier surfaces. Although stiffness is related to buckling, it is the latter the more important one for engineering work of large roof structures.

One method based on dynamic relaxation [9], maps a flat grid composed of continuous elastic rods to a fixed geometry, and finds structures with minimal strain energy. The optimisation presented in [7] search, instead, a structure without bending moment. The base geometry to optimise is however described by a given reference geometrical function. Other works [1-2] employing genetic algorithms optimise member properties, eg cross-sectional area, to improve buckling. It is thus a given structure which is then optimised for buckling resistance. We do not assume a given structure; the 3D shape itself is an unknown. The procedure here presented generates both the optimised 3D shape with enhanced buckling resistance at the *same* time.

Another major difference between those works and our present study lie in the type of surface being sought. We seek optimised structures arising from the form-finding process itself. When topology is missing, like in topology optimisation problems [14], the design space is typically a block of material, with a selection field of 1/0 depending on the presence or absence of material. The optimised discrete selection field is still required to undergo a further step, in which 1/0 blocks are then grouped into real 1D-elements. Unfortunately, designs of that nature are difficult to extrapolate to real-life 3D structures comprising hundreds of basically known elements (bars, beams, cables), and being at the risk of converging to structures too complex to manufacture.

Here the optimised form arises differently; it is not a discretisation of continuous surfaces or selection fields, but comes directly from a form-finding step based on the inversion of a funicular geometry done by a dynamic relaxation algorithm. By wrapping the form-finding and buckling analysis into a genetic algorithm, the resulting structure is optimised both from the static and buckling point of view.

5. FORM-FINDING AND BUCKLING OPTIMISATION

Genetic algorithms were employed due to their versatility and well-known capabilities to deal with non-linear problems, including the design of space structures [3,5,8,15]. The main difference from those works is that we do not assume a given shape to be optimised; the optimisation procedure has a form-finding within. Although similar works [5-6] have a form-finding method and genetic optimisation together, no initial shape is given. Moreover, we start from a well-defined connectivity and avoid computing a Chebyshev mesh [6] or Voronoi diagrams for discretisation [6].

There are many variations in genetic optimisation, but the general concept [15] is the creation of a pool of solutions, in which its individuals are then selected and mixed up (crossover) to improve the population at each generation. Individuals are also allowed to change an entry in their representation, here a vector of pre-stresses, in a way that the so-called “mutations” improve the individual’s fitness. The fitness of a given individual is a number, or something easily quantified, such that it can be either maximised or minimised, depending on the problem at hand. Particularly speaking, we used a proportionate roulette wheel selection process, with uniform crossover and Gaussian mutation. We selected the buckling load as fitness function which is maximised over the generations. We employed populations of 10 individuals and a cut-off of maximum 50 generations, all ran on a standard quad-core PC with 8G of memory. Figure 2 shows the overall optimisation strategy. Next sections will focus on its principal steps.

5.1 Optimisation procedure

The optimisation procedure is based on the grid shell design process shown in Figure 2, where the main calculations that a single structure undergoes can be summarised as follows:

- a flat grid is initialised with certain pre-stress and zero Young’s modulus,
- form-finding is performed,
- real material proprieties and boundary conditions are introduced,
- static analysis is performed,
- buckling calculation is performed.

The initialisation phase requires each of the *n* elements (bars, beams and cables) be uniquely assigned into $1 \leq m \leq n$ groups. A worst case scenario, in which each element is itself a design variable, $\mathbf{p}=(g_1,\ldots, g_n)$, lead to very slow convergence, with irregular and not aesthetically pleasing final structures. Groups, therefore, should be created based on symmetry, see Figure 4 for two sample groups assigned to triangular and quadrangular patterns. Section 6 has more details on how those groups were generated. It is important to remark that our optimisation procedure seeks a structure of certain height. The vector of pre-stresses is scale up or down depending on the target height, and the precise value is computed with a simple bisection algorithm. This procedure allow us to find the correct rise with a considerable saving of iterations and computational time.

The genetic optimisation generates new structures and evaluates their fitness. If the initial pre-stress force for the dynamic relaxation algorithm is the vector $\mathbf{p}=(g_1,\ldots,g_n)$, for *m* groups of structural elements, and form-found shape s, the optimisation evolves solutions of **p** maximising

$$\max b(s),$$

$$\text{where } s = shape(\mathbf{p}),$$

$$\text{subject to } g_i^{min} \leq g_i \leq g_i^{max}.$$

Here, the fitness function $b(s)$ is the first buckling mode computed from an eigenvalue buckling analysis of a structure, *shape*(**p**), previously form-found with initial pre-stress vector **p**, subject to individual constraints per group of elements. Constraints on $\mathbf{p}_i$ allow a fast convergence. See Figure 2 for the workflow and fitness function of this genetic algorithm. Notice the topology and boundary conditions are kept fixed during the optimisation, as well as the target height. The genetic optimisation can, of course, be extended to other types of form-finding and fitness functions [3,5-6,8].

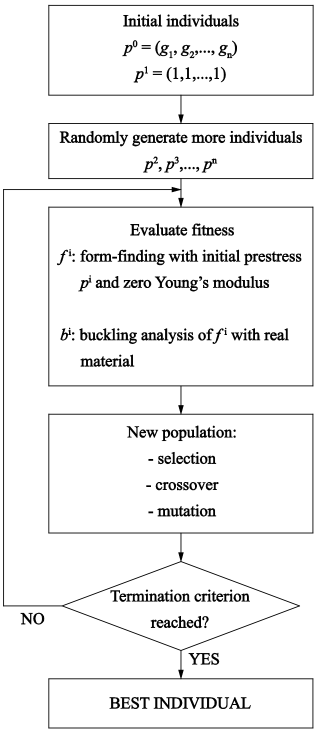

Fig. 2: Flow chart of the proposed optimisation procedure.

We took advantage of the existing API in GSA to script the form-finding, the static analysis and buckling calculations. The project is developed in Python. The form-finding and structural calculations were performed in Oasys GSA 8.5, while the genetic algorithm was PyEvolve, a free library [16]. Different geometries, generated by the genetic algorithm, were found by initialising the dynamic relaxation algorithm with various pre-stresses, **p**, including the trivial solution of unitary pre-stress in all elements. The second main component in the genetic algorithm is the fitness, *b*(s), of those *shape*(**p**) structures.

5.2. Fitness function

Design experience has highlighted the importance of the buckling performance as the main driving parameter [5-6,8]. Gridshells refer to large displacement structures and therefore a plain modal analysis based on the elastic stiffness matrix should be handled with care, especially if non-linear elements like cables are combined with bars and beams. The member linearisation in the stiffness matrix may under-estimate the real buckling load. Modal analysis is not really appropriate for structures in the large displacement regime, where structures do not show a bifurcation point. In this case, a full non-linear analysis is appropriate to determinate the correct buckling behaviour but at higher computing time. For instance, a P-Delta curve of a sample structure, shown in Figure 3a, takes several minutes to calculate and eventually slows down the entire genetic algorithm. Since this step is part of the fitness function, it will potentially be called thousand times. We therefore need a compromise between a modal analysis and a full P-Delta curve.

Among the many ways to compute the buckling load [5-6,8], we estimate it over a deflected configuration (both in term of displacement and forces) near the yielding point and then run a modal analysis on top of the deflected one (so called a "restart file" approach [13]). The "restart file" approach is perhaps little known, but it is a fast way to get a rough, conservative approximation of a deflected geometry before yield, from which a more detailed analysis is carried out. It is also claimed [13] that this solution is more precise, ie being independent from fine tuning of converging parameters.

This multi-stage or restart file approach is performed in three steps. Firstly, a linear buckling analysis is performed to compute the first eigenvalue of the grid shell. Secondly, a non-linear static analysis is performed up to 95% of the linear buckling, which is indeed a conservative estimate. In the final step, the first buckling mode is computed via eigenvalue analysis on the base of the deformed structure (both having displacements and internal forces) found in the previous step (hence the "restart" point). In this way, the load factor found is not related the initial geometry but the loaded and deformed one. It is a loaded and deformed structure close to the real, critical buckling load. The final load factor, our fitness function for a form-found shape(**p**), is calculated as:

$$b(s) = (0.95\lambda_1^1) \cdot \lambda_1^2$$

where λ^1_1 is the first eigenvalue for the form-found shape, shape(**p**), and λ^2_1 is the first buckling mode for the structure loaded with 95% of λ^1_1. A unitary nodal load is used to carry out the buckling analysis. In the rare event when this analysis does not converge, eg when a gridshell is near to the collapsing point already, the procedure is then repeated by placing the 80% of form-found structure's load factor.

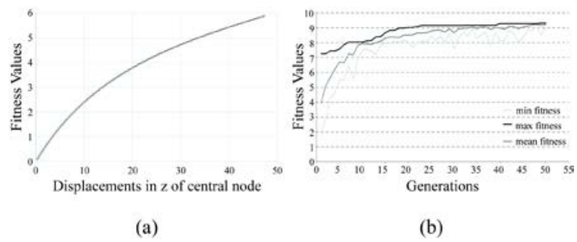

Fig. 3: (a) Example of a P-Delta curve, load vs the displacement of the central node in the gridshell; (b) convergence of the genetic algorithm. It reaches good fitness values rapidly and converges in 1:30 hours.

For comparison purposes, Table 1 shows the load factor found with this procedure (restart file). It is close to the non-linear buckling load, but faster to calculate and easier to automate than other techniques, including elasto-plastic approaches [8] or trimodal analysis [2], which makes it an ideal choice for a genetic algorithm.

Table 1: A sample analysis showing the time and quality of solutions for the critical buckling load.

Analysis	**Load factor**	**Time (seconds)**
Non-linear buckling	5.88	85
Restart file	5.36	2
Modal analysis	4.9	0.5

As Table 1 shows, the difference is merely 0.52 in the load factor between the restart approach and non-linear one, but estimated much faster, while modal analysis is more conservative. We took the restart file approach for the fitness function of the genetic algorithm.

Overall, the genetic algorithm is stopped when it reaches the maximum number of generations or when the fitness function does not improve much through a few generations. Figure 3b shows a typical convergence of the proposed optimisation procedure. A few iterations bring the fitness function to acceptable buckling loads.

6. TEST GRIDSHELLS CONFIGURATIONS

6.1 Topologies

The proposed genetic optimisation has been applied to different topologies of gridshells to study the influence of restraints, element types, cross-sectional areas and different nodal connectivity. Here we have selected a number of parameters representative of interesting gridshell structures as mentioned in section 2.

Instead of assigning a design variable to each element in the structure (cables, bars or beams), the properties are assigned to pre-defined groups of elements based on symmetry being the structure symmetrical. We present two reference configurations (Figure 4), described below.

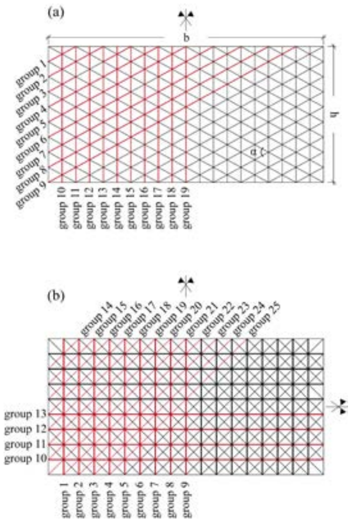

Fig. 4: (a) the triangular and (b) quadrangular configurations tested in this study. The groups are arranged by central symmetry, and are described in Table 2 and 3 respectively.

The reference plan dimensions are 15x31 metres, the ones employed in Neumünster [9], see Figure 1, and the maximum height equal to three meters. The two sample gridshells consist of:

- triangular pattern (described in section 6.1),
- quadrangular pattern (described in section 6.2).

For each of them, the following parameters have been studied:

- boundary conditions: clamped or articulated end fixing,
- members' sections: all equals; primary arches bigger than the transverse elements and arcs smaller than transverse elements,
- internal connectivity: all members are clamped; secondary members are articulated.

The form-finding process has been done initialising the flat grid using two ways:

- pre-stress in all direction,
- pre-stress in only two directions.

The former case, pre-stress in all directions, is the conventional one; while the latter test case represents a privileged direction of forces thus controlling the forces path. For the sake of completeness, we should mention that a pre-stress initialisation in only one direction gives structures with wrinkles and therefore is not part of this study. As show in Figure 5, this initialisation does not produce valid gridshells structures. It converges to invalid corrugated roofs.

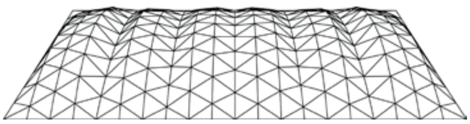

Fig. 5: Corrugate roof resulted from a pre-stress initialisation in only one direction.

Beam elements (clamped at the ends nodes) had a circular section with diameter equal to 40 mm; to simulate a structure braced with cables, bar elements (pinned at both ends) have cross-section areas equal to 1/10 of the beam elements, i.e. a diameter of 13 mm.

6.2 Triangular pattern

Let Direction 1 be the direction corresponding to the groups from 1 to 9 and Direction 2 the direction from 10 to 19, see Figure 4a. The parameters which have been varied in the study are:

- The pre-stress initialisation. It can be done with respect to two or three directions; see Figure 6 for a diagram.
- The internal connectivity, see Figure 6b. It has all elements clamped or articulated according Direction 1 or Direction 2.
- The section. They equals or according to Direction 1 or Direction 2, see Figure 6c.

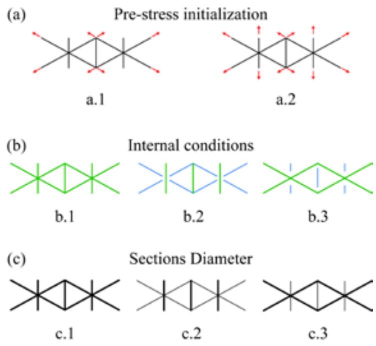

Fig. 6: (a) Pre-stress initialisation: a1 Direction 2; a2 both directions; (b) Elements' type: b1 Beam in both direction;, b2 Direction 1: Beam, Direction 2: Bar; b3: Direction 1: Bar, Direction 2: Beam; (c) Section diameter: c1: 40 mm all elements; c2 Direction 1: 40 mm, Direction 2: 13 mm; c.3 Direction 1: 13 mm, Direction 2: 40 mm.

Table 2 shows test cases across a range of initialisations of the pre-stress, the elements' sections and the internal and boundary conditions. For each different distribution of pre-stress, form-found geometries changed boundary conditions (fixed joints or hinges) and the type of the elements, i.e. the internal conditions as well as cross section area. On the base of the parameters identified in Figure 6, all tested combinations are summarised in Table 2.

Table 2: Triangular pattern with 220 nodes and 541 elements, see Fig. 6.

Test	Pre-stress Initialisation	Boundary Conditions	Elements Type	Sections Diameter
1.1	a2	Hinges	b1	c1
1.2	a2	Fixed	b1	c1
2.1	a2	Hinges	b2	c2
2.2	a2	Fixed	b2	c2
3.1	a1	Hinges	b2	c2
3.2	a1	Fixed	b2	c2
4.1	a2	Hinges	b3	c3
4.2	a2	Fixed	b3	c3
5.1	a1	Hinges	b3	c3
5.2	a1	Fixed	b3	c3

6.3 Quadrangular pattern

The most promising conditions were replicated in quadrangular gridshells. For this grid we have done tests changing the condition of the external constraint, the direction of the pre-stress and the internal condition, shown in Figure 7, and listed below.

- The pre-stress is assigned to all the elements or only to the orthogonal elements (groups 1-9 and groups 10-13 in Figure 4b).
- The connectivity consists of clamped condition in the two orthogonal directions and pinned for the diagonal direction (groups 14-25 in Figure 4b).
- Bigger sections correspond to the clamped elements, while the smaller sections correspond to the pinned ones.

Table 3 summarises the characteristics of the test cases.

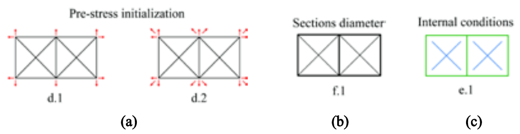

Fig. 7: (d) Pre-stress initialisation: d1 main structural elements; d2 all elements; (e) Elements' type: e1 main structural elements: beam, bracings: bar; (f) Section diameter: f1. Main structural elements: 40 mm, bracings 13 mm.

Table 3: Geometrical features of quadrangular gridshells, see Figure 7.

Test	Pre-stress Initialisation	Boundary Conditions	Elements Type	Sections Diameter
6.1	d1	Hinges	e1	f1
6.2	d1	Fixed	e1	f1
7.1	d2	Hinges	e2	f1
7.2	d2	Fixed	e2	f1

6.4 Inspiration for test cases

Test cases 6.1 and 6.2 with bracing cables appear in a number of designs, like the one in the roof of the Museum for Hamburg History; test cases 2.1 and 2.2 are based on Neumünster I; finally, test cases 3.1 and 3.2 refer to the gridshell in Toulouse's Jean Jaures metro station.

7. RESULTS

7.1 Triangular pattern

Results of the triangular pattern are shown in Table 4.

Table 4: Results for the triangular pattern.

Test	Load Factor	Trivial Load Factor	Gain
1.1	23.33	18.04	29.30%
1.2	23.83	20.99	13.50%
2.1	6.65	2.20	202%
2.2	5.88	2.36	149%
3.1	6.79	5.33	27.3%
3.2	6.90	5.41	27.5%
4.1	8.88	7.51	18%
4.2	9.23	8.43	9.5%
5.1	9.51	7.85	21%
5.2	12.91	8.43	53%

7.2 Quadrangular pattern

As for the triangular grid, the conditions of external constraints did not significantly affect the load factor. Geometries initialised with pre-stress for all elements were better than the ones only in the main elements.

Table 5: Results for the quadrangular pattern. Gridshells have bars in direction 1 and beams in direction 2.

Test	Load Factor	Trivial Load Factor	Gain
6.1	12.00	8.32	44.2%
6.2	13.01	11.15	16.7%
7.1	14.03	8.28	69.4%
7.2	16.90	11.10	52.3%

7.3 Overall results

The results show that we have always an improvement in buckling loads, and Figure 8 shows a typical example. Although the trivial solution (Figure 8a) and the optimal one (Figure 8b) are seemingly equal, there is a difference in the position of nodes with a minimum displacement of -4.0 cm and a maximum of about 20 cm (Figure 8c). It improves the buckling load from 8.43 to 12.91. Finally, an example of the optimised pre-stresses is shown in Figure 8d.

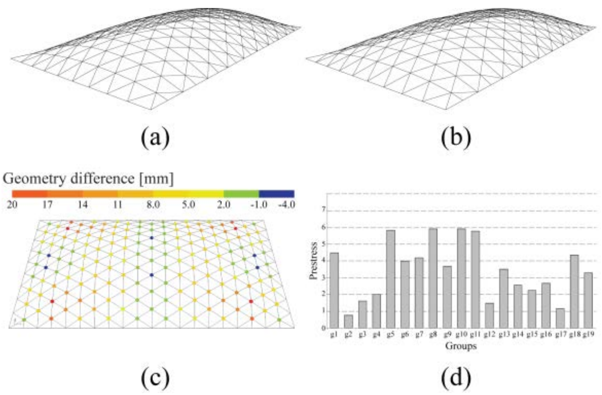

Fig. 8: Example of optimised gridshells. (a) Trivial solution computed with a uniform initialisation of pre-stresses in dynamic relaxation; (b) solution found by the genetic algorithm; (c) superposition of (a) and (b). Although the solutions are alike, there are significant differences, improving the buckling load; (d) typical histogram showing the optimised pre-stresses in 19 groups

With the exception of test 2.1 and 2.2, the average improvement in buckling load is 31.8%, a minimum of 9.5% and maximum of 69.4%. Moreover analysing the above result for both the triangular and quadrilateral mesh we see that there are recurring patterns:

(i) Boundary conditions (either clamped or pinned) do not significantly affect the load factor of optimised structures;

(ii) The internal connectivity has a strong influence on buckling loads;

(iii) Buckling loads for structures having member with all the same sections are higher than configurations where the gridshell presents two types of sections;

(iv) An initialisation of the pre-stress for all elements of the structural grid, ie three directions, produces geometries that have a higher buckling load factor with respect to the one that have an initialisation of the pre-stress in only two directions;

(v) The initialisation with pre-stress in only one direction proved to converge to shapes which cannot be defined as gridshells, and those structures were not considered valid ones. It has to be noted that, even in this case, the genetic algorithm was able to identify configurations having good buckling loads.

8. DISCUSSION

These tests confirm the optimisation process is able to find geometries with buckling resistance better than those found with a trivial initialisation of pre-stress, i.e. uniform pre-stress. It is worth noting that the optimisation always improved the buckling, showing that the results are independently of the gridshell type and are inherent to the optimisation method. This point is reinforced by the methodology of the work since the gridshell have been chosen with the aim of covering a spectrum of typologies, including the state-of-the-art and the current designs trends in terms of pure mechanical efficiency, construction technology and aesthetics. The results are four-fold and are consistent with engineering logic:

(i) Buckling loads of a gridshells with clamped arcs are slightly better than the case with pinned end-conditions. The gain is small since the buckling length of the arcs is reduces by fixing the end-condition; however, at the same time its influence on the overall structural behaviour is minimal since the grid shell resistance comes from the 3D and not the 2D behaviour.

(ii) Part of the resistance of grid shell comes out from its capacity to resist second-order bending moments. The articulations created along one or more structural directions weaken the out-of-plane bending stiffness and consequently the buckling load is reduced. It has to be stressed out that, such less-efficient structural schemes have the practical advantage of being easier to manufacture and built.

(iii) Variations of cross sections in certain directions reduces the overall axial stiffens. What is even more interesting to note is that the reduction of stiffness in one direction re-orients the loading path towards the stiffer path, but this re-orientation is not capable to compensate for the global loss of stiffness. A loss in structural performance is compensated in architectural terms by a visual effect of lightness, especially when the standard bars are replaced with pre-stressed cables, eg Neumünster I.

(iv) Initialisations in two directions or in only one also re-orient the load path. Initialisations in one direction crate solutions out of the field of the grid shell (corrugated structure) and are not of interest in this paper. Buckling factors in gridshells initialised with pre-stress in three directions are always higher than the one initialised in two directions. This means that the re-orientation of the loading path is not effective. When gridshell are initialised in only two directions, bigger gains in the buckling load are observed with respect to trivial solutions.

9. CONCLUSIONS

We have here presented an optimisation procedure with genetic algorithms that allow us to estimate the initialisation the pre-stress forces, in each element of a gridshell, such that the critical buckling load is maximised. The fitness function, critical for overall running time, is tuned for speed. Results are encouraging since a fast convergence is reached with minimal manual tuning, proving that the algorithm is effective and robust.

Results open new ways for the optimisation of gridshells and further work should be carried out in developing new strategies to cope with more complex pre-stress initialisations, such as the estimation of pre-stress groups as part of the optimisation procedure itself. It also could be interesting to use other form-finding methods including the possibility of testing geometries not perfectly funicular, eg presenting a residual bending moment after form-finding.

We think that our proposed strategy can be successfully applied to the form-finding and structural optimisation of gridshells and similar structures, by directly addressing the buckling resistance as the most critical design parameter. The proposed strategy can be a valuable tool for the structural design of lighter and more transparent gridshells.

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