# IMPLEMENTATION OF AN ASYMMETRIC NETWORK EQUILIBRIUM PROBLEM WITH DETAILED REPRESENTATION OF UNSIGNALIZED AND SIGNALIZED URBAN INTERSECTIONS.

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**Abstract.** This paper discusses the implementation of an asymmetric network equilibrium model with detailed representation of unsignalized and signalized urban intersections. A software has been developed to solve the deterministic user equilibrium (DUE) problem which takes into account real urban intersections in their detailed configurations. During the first phase this software was tested on a "toy" network and then on the real network of Villafranca (a town near Verona Italy). The comparison between the equilibrium flow patterns resulting from the model and some traffic counts on the Villafranca network confirms that the model is good.

## 1. Introduction

This paper describes the implementation of an asymmetric user equilibrium route choice model with non-separable cost functions. The study is focused on a detailed representation of unsignalized and signalized intersections in the network. This kind of representation can highlight in the best way the complex interactions between different traffic streams competing for the use of limited road capacities and allows the best possible determination of the link average delays of the flows approaching the intersection; the analysis procedures have followed the gap acceptance theory. The interactions among different traffic streams sharing some lanes on the same approach, and, above all, among conflicting traffic streams using different approaches, require the use of non-separable cost functions. One of the distinctive features of this paper is related to the computation of average delay for unsignalized intersections and, particularly, for modern roundabouts. Such computation is carried out in a detailed way which is not usual in ordinary assignment software. The equilibrium assignment problem needs to be formulated and solved as an asymmetric equilibrium problem. Our first target was the development of an asymmetric equilibrium assignment software for studying real urban intersections, both signalized and unsignalized, and particularly roundabouts in their detailed configurations. During the first phase of our

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work the software was tested on a “toy” network and subsequently on the real network of Villafranca near Verona.

## 2. Detailed cost functions for unsignalized intersections in assignment problems

During this study particular attention was devoted to modelling the delay of unsignalized intersections in urban network assignment problem. An example of unsignalized T-intersection is represented in Fig. 1.

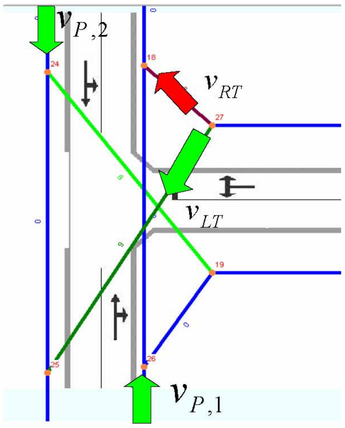

**Figure 1. Right-turn manoeuvre from a minor street in a shared lane.**

The average control delay *d* (sec/veh) for the right-turn stream from a minor street can be evaluated by means of the following expression:

$$d = d_q + d_s + 5 \tag{1}$$

where: $d_q$ (sec/veh) is the average queue waiting time (the time elapsed since joining the queue to the time when the vehicle is at the stop line ready to carry out its manoeuvre); $d_s$ (sec/veh) is the average service time (in this case it represents the average time necessary to carry out its manoeuvre as the vehicle is at the stop line). The constant value of 5 sec/veh is included in equation (1) to account for the deceleration of vehicles from free-flow speed to the speed of vehicles in a queue and the acceleration of vehicles from the stop line to free-flow speed (HCM, 2000).

To give an example of the method used, the determination of the average delay for a minor street right-turn stream, in a shared lane first and then in an exclusive lane are reported.

In the case of the shared lane, $d_q$ could be determined by means of the equation (HCM, 2000):

$$d_q = 900T\left[\frac{v_{RT}+v_{LT}}{c_{SH}}-1+\sqrt{\left(\frac{v_{RT}+v_{LT}}{c_{SH}}-1\right)^2+\frac{\left(\frac{3600}{c_{SH}}\right)\left(\frac{v_{RT}+v_{LT}}{c_{SH}}\right)}{450T}}\right] \tag{2}$$

where: $v_{RT}$ is the minor street right-turn stream flow rate in shared lane (veh/hr); $v_{LT}$ is the minor street left-turn stream flow rate in shared lane (veh/hr); $T$ (hr) is the analysis time period; $c_{SH}$ is the capacity of the shared lane (veh/hr):

$$c_{SH} = \frac{v_{RT}+v_{LT}}{\frac{v_{RT}}{c_{m,RT}}+\frac{v_{LT}}{c_{m,LT}}} \tag{3}$$

where: $c_{m,RT}$ is the movement capacity for minor street right-turn stream as if it had its own separate lane (veh/hr); $c_{m,LT}$ is the movement capacity for minor street left-turn stream as if it had its own separate lane (veh/h). The movement capacity $c_{m,x}$ for a given stream $x$ can be calculated correcting the potential capacity $c_{p,x}$ for the same stream with an adjustment factor (according to the suggestions in HCM 2000). The classic Harders' formula has been used to determine the potential capacity $c_{p,x}$ for a general minor street stream $x$:

$$c_{p,x} = v_{c,x}\frac{e^{-v_{c,x}t_{c,x}/3600}}{1-e^{-v_{c,x}t_{f,x}/3600}} \tag{4}$$

where $v_{c,x}$ is the conflicting flow rate (veh/hr) for minor street stream $x$; $t_c$ (sec) is the critical gap for minor street stream $x$; $t_f$ (sec) is the follow-up time for minor street stream $x$. In equation (4) $v_{c,RT}$ is equal to $v_{p1}$ and $v_{c,LT}$ is equal to $v_{p1}+v_{p2}$ *(Fig.1).*

The service time $d_s$ (sec/veh) can be calculated by means of the following equation:

$$d_s = \frac{3600}{c_{m,RT}} \tag{5}$$

where $c_{m,RT}$ is the minor street right-turn stream movement capacity which can be determined again correcting the minor street right-turn stream potential capacity with an adjustment factor; the conflicting volume $v_{c,RT} = v_{P1}$ must be included in equation (4).

It should be noted that the average control delay $d$ (equation (1)) for the minor street right-turn stream in a shared lane has been subdivided in the calculus of the average queue waiting time $d_q$ and in the calculus of the average service time $d_s$. The cost function for $d_q$ is a non-separable cost function which depends on the minor street right-turn flow rate but also on the minor street left-turn flow rate (equation (2)), as the lane is shared. But $d_q$ is also dependent on $v_{P1}$ and $v_{p2}$ because in equation (2) there is the shared lane capacity $c_{SH}$ which depends on $c_{m,RT}$ and on $c_{m,LT}$ (equation (3)). These variables depend, in their turn, on $v_{P1}$ and $v_{P1}+v_{P2,}$ respectively. From the point of view of the conflicting volume, the average service time $d_s$, instead, depends only on $v_{p1}$ (Fig. 1). In the end the minor street right-turn average delay in a shared lane is expressed by a non-separable cost function in which the independent variables are the left-turn flow rate and the flow rates of the major street in both directions $v_{p1}$ and $v_{p2}$, in addition to the right-turn flow rate.

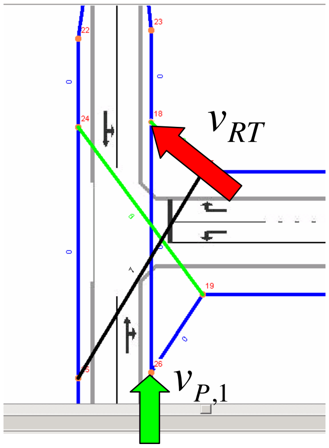

**Figure 2. Right-turn manoeuvre from a minor street in an exclusive lane.**

In the case of the right-turn stream from a minor street in an exclusive lane, $d_q$ could be determined by means of an equation being similar to (2):

$$d_q = 900T\left[\frac{v_{RT}}{c_{mRT}} - 1 + \sqrt{\left(\frac{v_{RT}}{c_{mRT}} - 1\right)^2 + \frac{\left(\frac{3600}{c_{mRT}}\right)\left(\frac{v_{RT}}{c_{mRT}}\right)}{450T}}\right] \tag{6}$$

the meanings of the terms in (6) are the same as in (2) and (3). Potential capacity $c_{p,RT}$ could be determined by means of Harders' formula:

$$c_{p,RT} = v_{c,RT}\frac{e^{-v_{c,RT}t_{c,RT}/3600}}{1 - e^{-v_{c,RT}t_{f,RT}/3600}} \tag{7}$$

in which $v_{c,RT}$, is equal to the volume $v_{p1}$ only (Fig.2).The movement capacity $c_{m,RT}$ can be calculated, again, correcting the potential capacity $c_{pRT}$ (7) with an adjustment factor (HCM 2000).

The service time $d_s$ (sec/veh) can be calculated by means of equation (5).

The cost function for $d_q$ is a non-separable cost function which depends only on the minor street right-turn flow rate and not also on the minor street left-turn flow rate (as in equation (2)) as the lane is exclusive. But $d_q$ is also dependent on $v_{P1}$ because in equation (6) there is the movement capacity $c_{m,RT}$ for minor-street right-turn stream which depends on $v_{P1}$. From the point of view of the conflicting volume, the average service time $d_s$, instead, depends only on $v_{p1}$ (Fig. 2). In the end the minor street right-turn average delay is expressed by a non-separable cost function in which the independent variable is the flow rate of the major street $v_{p1}$, in addition to the right-turn flow rate $v_{RT}$.

## 3. Application of the model

During the first phase, the algorithm was tested on a "toy" network formed by 40 links, 31 nodes and 3 centroids (nodes at which trips originate and/or terminate). For this network we have considered one unsignalized T-intersection, one modern roundabout and one signalized intersection in a detailed representation. The simulations developed on the "toy" network were focused on solving the computational problems occurring in the implementation of the diagonalization algorithm with the detailed representation of the intersections and the detailed cost functions used. In order to study the effectiveness of the flow assignment when the volume to capacity ratio is close to 1, we have considered a set of 5 OD demand matrixes, the elements of which were gradually increased. In the second phase the algorithm was applied to the real road network of Villafranca characterized by 55 intersections, 37 of which are unsignalized and, among these latter ones, 6 are modern roundabouts. The O/D matrix, which has been provided by the Municipality of Villafranca (Verona), corresponds to the private transport demand of the morning peak hour (8:00 – 9:00). This matrix was initially estimated basing on 1991 census data and a sample of interviews; then the O/D matrix was improved using traffic counts carried out over the years 1999-2001.

For each link of the network the flow rate in terms of veh/h, the flow vs capacity ratio and the control average delays were calculated. Some factors of the cost functions, such as critical gap, follow-up time and impedance factors, were calculated basing on field observations.

The simulated flow rates were compared with the traffic counts made on 68 links (Figure 3) using Hi-Star portable traffic analyzer.

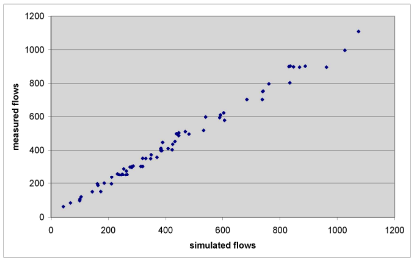

**Figure 3. Distribution of measured and simulated flow rates**

This comparison with the aim at verifying the effectiveness of the model was developed by calculating the mean square error (MSE) and the root mean square error (RMSE%) of the measured link flow vector $f^*$ and the simulated link flow vector $\bar{f}$:

$$MSE(f^*,\bar{f}) = \frac{1}{N}\sum_i (f_i^* - \bar{f}_i)^2 = 881,89 \tag{6}$$

$$RMSE\% = \frac{\left(MSE(f^*,f)\right)^{0.5}}{\frac{\sum_i f_i}{N}} = 6.59\% \tag{7}$$

where $N$ is the number of the counted links. The results of the comparison (Fig.2, equations (6) and (7)) highlight that the proposed model fits well the measured link flow vector as shown by the low values of MSE and RMSE%. The effectiveness of the model was also validated by comparing the average control delays $d^*$, calculated using HCS 2000 and referring to the measured link flow rates, with the average control delays $\overline{d}$, calculated using detailed non-separable cost functions, but referring to the simulated link flow rates. This comparison, made on 4 unsignalized intersections with the higher flow rates, shows that $\overline{d}$ fits well $d^*$.

## 4. Conclusions

This paper describes the implementation of an asymmetric user equilibrium route choice model with non-separable cost functions. We have developed an asymmetric equilibrium software in Visual Fortran code for studying real urban intersections in their detailed configurations and particularly unsignalized intersections. The procedures adopted for modelling the average delay of each link at an unsignalized intersection consider the delay as a sum of two terms: the average queueing delay and the average service delay. The former one is the average delay related to the time necessary to reach the stop line after joining the end of the queue; the latter one is the average time necessary to carry out the manoeuvre. In particular, we have considered different conflicting flows for queueing delay and for service delay, according to the right of way priority of each traffic stream at the intersection. We have also determined the potential capacity, the movement capacity and the shared lane capacity for each link at an intersection according to the methodology outlined in the Highway Capacity Manual 2000, after setting the critical gap, the follow-up time, and the impedance factor. The developed software was tested first on a “toy” network and then on the real network of Villafranca near Verona. The computational results of assignments were compared with the real flows recorded in 68 links of the Villafranca network. These comparisons highlighted the fact that the model framework fits well the real one. Moreover, the effectiveness of the model was supported by the comparison of the average control delays, calculated by means of the Highway Capacity Software 2000 for unsignalized intersections, and basing on the real flows measured, with the simulated average control delays.

## References

[1] E. Cascetta, Transportation Systems Engineering: Theory and Methods, Kluwer Academic Publishers, 2001.

[2] P. Ferrari, Fondamenti di Pianificazione dei trasporti, Pitagora Editrice Bologna, 2001.

[3] Kimber, Hollis, *Traffic queues and delays at road junctions*. TRRL Laboratory Report 909. Transport and Road Research Laboratory, 1979.

[4] Lupi, *Determinazione del livello di servizio ad una intersezione semaforizzata: alcune osservazioni sulla modellazione del ritardo,* Collana Trasporti, volume 1797.12, Franco Angeli, 1998.

[5] Meneguzzer, *An equilibrium route choice model with explicit treatment of the effect of intersections,* Transportation Research Vol.29-B n°5 pagg. 329-356, 1995.

[6] *Highway Capacity Manual,* Transportation Research Board, 2000.

[7] Webster, *Traffic signals settings,* Road Research Technical Paper, 39, Her Majesty's Stationary Office, London, 1958.

[8] Kendall, *Some problems in the theory of queues,* J.Roy. Statist. Soc. series B (Methodological), 13 (2), 1951.

[9] Hurdle, *Signalized intersection delay model- A primer for the uninitiated,* Transportation Research Record, 971, 1984.

[10] Robertson, *Traffic models and optimum strategies of control: a review,* Proceedings of the International Symposium on Traffic Control Systems, University of California, Berkeley*,* 1979.

[11] Akcelik, *The highway capacity manual delay formula for signalized intersections*, ITE Journal, 1988.

[12] Troutbeck, Brilon, *Unsignalized intersection theory*, in Traffic Flow Theory A state of the Art Report, 4, 1999.