## The codetermined firm in a Cournot duopoly: a stability analysis

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**Abstract** This paper aims to study the stability issue in a Cournot duopoly with codetermined firms. We show that when both firms codetermine employment together with decentralised employees' representatives, a rise in wages acts as an economic (de)stabiliser when the wage is fairly (high) low, while under profit maximisation a rise in wages always acts as a stabilising device because the parametric stability region monotonically increases with the wage in such a case. Moreover, a rise in the union's bargaining power has a de-stabilising effect, except when the wage is low and the firm power is already high. Therefore, under codetermination a change either in the wage or firm power in the Nash bargaining plays an ambiguous role on stability. We also show with numerical simulations that complex dynamics can also occur.

**Keywords** Bifurcation; Codetermination; Cournot; Duopoly; Employment

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## 1. Introduction

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A well known stylised fact about labour markets is the existence, especially in some important European countries such as Germany, of codetermination laws, according to which workers in large firms have nearly the same decision rights as capital owners. On the one hand, codetermination rights mainly concern employment, with wages being apart from the field of application of such laws. On the other hand, even if one abstracts from codetermination laws,[1] in several countries it is observed that: (1) a distinction is made, especially in Europe (for instance, Scandinavian countries and Austria) between centralised (e.g., national or economy-wide level) unions that set the wage for an entire industry in a country, and decentralised (e.g., firm or district level) unions that negotiate over employment alone, and (2) de-centralised wage setting procedures, which however establish wage contracts of long lasting effectiveness (e.g., the three-year contracts often observed in the US), and local bargaining over employment of higher periodicity do exist. Both make the case of bargaining over employment relevant.

An interesting study that has tackled this issue out from a point of view of a static bargaining game in a Cournot duopoly, is Kraft (1998). The author interestingly shows that: (*i*) bargaining over employment alone is the dominant strategy with respect to profit maximisation if the union power is not too large (which seems to be the case under codetermination laws),[2] and (*ii*) "codetermination is welfare maximizing!" (see Kraft, 1998, p. 200). Therefore, given both the empirical relevance of decentralised bargaining on employment alone and the surprising theoretical features of such a process evidenced by the literature above mentioned, we observe that so far nobody has considered, at the best of our knowledge, the effects of codetermination on product market stability in a duopoly with quantity competition. However, this is not an irrelevant issue to be dealt with given the long lasting debate on pros and cons of union power in both decentralised and centralised bargaining. The present paper aims to fill this gap in the economic theoretical literature by extending the duopoly model by Kraft (1998) in a dynamic context. The out-of-equilibrium dynamics is based on the assumption of "bounded rational" firms as suggested, for instance, by Dixit (1986) and recently popularised by the literature on dynamic oligopolies (see, e.g., Puu 1998; Bischi and Kopel, 2001; Tramontana, 2010; Fanti and Gori, 2012; Naimzada and Tramontana, forthcoming).

We find that an exogenous increase in the labour costs under codetermination destabilises the Cournot-Nash equilibrium when the wage is low enough (while playing a stabilising role for further increases in the wage when it is already high, but only whether the unions' power in determining employment is fairly low), while under profit maximisation a rise in wages plays an unambiguous stabilising role. Moreover, raising the relative union's bargaining power tends essentially to destabilise the equilibrium (by increasing the parametric instability region), except when the wage is low and the firm's power is already high.

Therefore, in order to keep the Nash equilibrium in a Cournot duopoly stable, the union's power in determining employment should be as low as possible: it should at most be close to the near-parity when the wage is fixed at not too high a level. In a similar way, we observe that an increase in wages, which is beneficial for stability when unions are absent or when their power in fixing employment is low, tends to destabilise the market equilibrium when the power of unions is high, unless the wage is high enough. This leads to a counterintuitive

[1] According to co-determination laws "employment determination are handled by the supervisory board in codetermined firms. On the supervisory board employees have near-parity rights. In the iron and steel industry as well as in mining employees have explicit parity decision rights." (Kraft, 1998, p. 195).

[2] Indeed Kraft (p. 199) notices, at least when co-determination is regulated by law, that a situation in which workers have higher bargaining power than firm owners would rather be unrealistic. Indeed, he argues that "the assumption of $\beta > 0.25$ seems to be acceptable for codetermination in German firms, given the fact of near-parity representation of the employees."

remark: under codetermination, when the power of unions in fixing employment is high (i.e., higher than the power of firms), it is convenient for stability to reduce (increase) wages when they are already low (high).

The present study contributes to two growing strands of literature on: unionised oligopolies (see, e.g., Dowrick, 1989, 1990; Bughin, 1995, Kraft, 1998, Correa-Lopez and Naylor, 2004, Fanti and Meccheri, 2011), and dynamic oligopolies (see, e.g., Bischi et al., 2010), and provides a novel analysis on the dynamic effects of bargaining on employment without wage negotiation.

The rest of the paper is organised as follows. Section 2 builds on the model Sections 3 introduces expectations and analyses the local stability properties of the unique positive Cournot-Nash equilibrium, showing the local bifurcations and the emergence of complex dynamics with numerical simulations. Section 4 concludes.

## 2. A Cournot duopoly with codetermined firms

The model is outlined in accordance with Kraft (1998). Without loss of generality, we consider a normalised Cournot duopoly for a single homogenous product with a negatively sloped inverse demand given by $p = 1 - q_1 - q_2$, where $p$ denotes the price and $q_1$ ($q_2$) is the output produced by firm $1$ (firm $2$). The average and marginal costs for each single firm to provide one additional unit of output in the market are equal and constants at $0 < w < 1$, where $w$ represents the wage negotiated by unions at the economy-wide level, with employment $L_i$ being determined at ($i$th) firm-specific level ($i = \{1,2\}$). The hypothesis of constant average and marginal costs implies that firm $i$ produces through a production function with constant (marginal) returns to labour, that is $q_i = L_i$ (see, e.g., Dowrick, 1989, 1990; Bughin, 1995; Correa-López and Naylor, 2004).

The objective of every firm is to maximise profits $\Pi_i(w, L_i) = pq_i - wL_i$ with respect to employment, while the objective of unions is to maximise utility $U_i(w, L_i) = (w - w^{\circ})^{\theta} L_i$ with respect to employment, where $\theta > 0$ is the relative weight attached by unions to wages and $w^{\circ}$ is the reservation or competitive wage. Without loss of generality, we set $\theta = 1$ and $w^{\circ} = 0$ henceforth. We assume that both firms codetermine employment with firm-specific unions. Since the production function is $q_i = L_i$, the Nash bargaining between firms and unions takes the form:

$$V_i = \left[(1 - q_i - q_j - w)q_i\right]^{\beta} (wq_i)^{1-\beta}, \quad 0 \le \beta \le 1, \tag{1}$$

where the control variable is $q_i$ and $\beta$ ($1-\beta$) is the relative bargaining power of firms (unions). Therefore, the best reply function of the $i$th firm is determined by:

$$\frac{\partial V_i}{\partial q_i} = \frac{\left[(1 - q_i - q_j - w)q_i\right]^{\beta} (wq_i)^{1-\beta} \left[1 - q_i(1+\beta) - q_j - w\right]}{(1 - q_i - q_j - w)q_i} = 0 \Leftrightarrow q_i = \frac{1 - q_j - w}{1+\beta}. \tag{2}$$

## 3. Expectations, equilibrium and local stability

Let $q_i(t)$ be firm $i$'s quantity produced at time $t = 0,1,2,\ldots$. Then, $q_i(t+1)$ is obtained as:

$$q_i(t+1) = \arg\max_{q_{i(t)}} V_i\left(q_i(t), q^e{}_j(t+1)\right), \tag{3}$$

where $q^e{}_j(t+1)$ represents the quantity that the rival, i.e. firm $j$, today (time $t$) expects will be produced in the future (time $t+1$) by firm $i$. Assuming now heterogeneous (i.e., bounded rational[3] and Cournot-naïve[4]) expectations by each firm (see, e.g., Tramontana, 2010) about the quantity that the rival will produce in the future period, the two-dimensional system that characterises the dynamics of the economy is the following:[5]

$$T:\begin{cases} q_1(t+1)=q_1(t)+\alpha\, q_1(t)\dfrac{\partial V_1}{\partial q_1(t)} \\ q_2(t+1)=q_2(t) \end{cases}. \tag{4}$$

where $\alpha>0$ is a coefficient that tunes the speed of adjustment of player $1$'s quantity with respect to a marginal change in $V_1$ when $q_1(t)$ varies. Notice that the intensity of the reaction of the bounded rational firm is given by $\alpha\, q_1(t)$, which is proportional to the quantity produced by firm $1$. Therefore, through the use of Eqs. (2) and (4) we get:

$$T:\begin{cases} q_1(t+1)=q_1(t)+\dfrac{\alpha\left[(1-q_1(t)-q_2(t)-w)q_1(t)\right]^{\beta}(w q_1(t))^{1-\beta}\left[1-q_1(t)(1+\beta)-q_2(t)-w\right]}{1-q_1(t)-q_2(t)-w} \\ q_2(t+1)=\dfrac{1-q_1(t)-w}{1+\beta} \end{cases}. \tag{5}$$

From Eq. (5) it can be seen that a rise in $\beta$ has a threefold effect on the marginal value of the Nash product of player $1$ and, hence, on the intensity of the reaction, i.e. the quantity it will produce, in the future period. First, it increases the relative bargaining power of firm $1$. Second, it tends to reduce the reaction of player $1$ through a direct negative effect. Third, it reduces the quantity produced by rival (firm $2$) at time $t$ and then tends to increase the reaction of player $1$ through an indirect positive effect. As regards wages, an exogenous positive shock on $w$, by increasing production costs, tends to reduce firms' profits while also raising the utility of unions. Moreover, as a direct effect, a rise in $w$ plays an ambiguous role on the marginal value of the Nash product and then both the direction and intensity of the reaction of the bounded rational firm is ambiguous through this channel. Indeed, as an indirect effect, an increase in wages tends to reduce the output produced by the naïve firm at time $t$ and then it also increases the reaction of the bounded rational firm because the marginal value of the Nash product raises through this channel. Definitely, the effect of a rise either in $\beta$ or $w$ at time $t$ is potentially uncertain on the quantity produced by the bounded rational firm at time $t+1$. Therefore, under codetermination ($0\le\beta<1$), a rise in wages can have a different effect on the size of the quantity produced in the future by the bounded rational firm than under pure profit maximisation ($\beta=1$). This can have interesting implications as regards the local dynamic properties of the Nash equilibrium when the wage and/or the union power for some exogenous reasons vary. This is the subject of the analysis that follows.

Equilibrium implies $q_1(t+1)=q_1(t)=q_1$ and $q_2(t+1)=q_2(t)=q_2$. Then, from Eq. (5) we get:

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[3] In the standard dynamic Cournot duopoly with profit-maximising firms, each bounded rational player uses information on current profits to adjust (i.e., to increase or decrease the quantity produced at time $t+1$) depending on whether marginal profits are either positive or negative (see Dixit, 1986).

[4] Cournot (1838) was *de facto* the first author to use naïve expectations in an oligopoly model.

[5] It is important to note that we have chosen to present the model with heterogeneous (i.e., bounded rational and naïve expectations) for analytical tractability. Indeed, the results of the present study holds even when both (codetermined) firms have bounded rational expectations as well as when one firm is profit-maximising and the rival is a codetermined firm, and both are bounded rational players.

$$\begin{cases} \dfrac{\alpha\left[(1-q_1-q_2-w)q_1\right]^{\beta}(wq_1)^{1-\beta}\left[1-q_1(1+\beta)-q_2-w\right]}{1-q_1-q_2-w}=0 \\ \dfrac{1-q_1-w}{1+\beta}-q_2=0 \end{cases}, \tag{6}$$

and the unique interior fixed point $E(q^*_1,q^*_2)$ of the two dimensional system is therefore characterised by:

$$E=\left(\frac{1-w}{2+\beta},\frac{1-w}{2+\beta}\right), \tag{7}$$

where $q^*_1=q^*_2=q^*$, which is in accord with the result obtained by Kraft (1998). From Eq. (7) it is clear that a positive exogenous shock either in the wage or the firm power in the Nash bargaining reduces the output produced by both firms. *Ceteris paribus*, this alternatively implies that a rise in union power tends to increase output. Therefore, it is expected that under profit maximisation a rise in the production cost, through an increase in the wage rate, reduces the intensity of the reaction of the bounded rational firm and then it is expected to act as an economic stabiliser. On the contrary, under codetermination, a rise in the wage (alternatively, in the firm power) – *ceteris paribus* as regards the other parameters of the model – is potentially uncertain even if they both work for reducing the reaction of the bounded rational firm. This because there exist other effects that pass through the change in the (marginal value of the) Nash bargaining function.

In order to investigate the local stability properties of the Cournot-Nash equilibrium $E$ we build on the Jacobian matrix

$$J=\begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22}\end{pmatrix}=\begin{pmatrix} 1-\dfrac{\alpha w\left(\beta^2+3\beta+2\right)}{\beta(2+\beta)}\left[\dfrac{\beta(1-w)}{w(2+\beta)}\right]^{\beta} & \dfrac{-\alpha w}{\beta}\left[\dfrac{\beta(1-w)}{w(2+\beta)}\right]^{\beta} \\ \dfrac{-1}{1+\beta} & 0 \end{pmatrix}, \tag{8}$$

where partial derivatives $J_{ii}$ and $J_{ij}$ are evaluated at the equilibrium point defined by Eq. (7). Trace and determinant of $J$ are given by:

$$T:=Tr(J)=J_{11}+J_{22}=1-\frac{\alpha w\left(\beta^2+3\beta+2\right)}{\beta(2+\beta)}\left[\frac{\beta(1-w)}{w(2+\beta)}\right]^{\beta}, \tag{9}$$

$$D:=Det(J)=J_{11}J_{22}-J_{12}J_{21}=\frac{-\alpha w}{\beta(1+\beta)}\left[\frac{\beta(1-w)}{w(2+\beta)}\right]^{\beta}<0. \tag{10}$$

Therefore, the characteristic polynomial of (8) is the following:

$$F(\lambda)=\lambda^2-T\lambda+D, \tag{11}$$

For the system in two dimensions defined by Eq. (5), the stability conditions that ensure that both eigenvalues $\lambda_a$ and $\lambda_b$ of the characteristic polynomial (11) remain within the unit circle are the following:

$$\begin{cases} (i) \quad F=2-\alpha\dfrac{1+(1+\beta)^2}{\beta(1+\beta)}w^{1-\beta}\left[\dfrac{\beta(1-w)}{2+\beta}\right]^{\beta}>0 \\ (ii) \quad TC=\alpha\dfrac{2+\beta}{1+\beta}w^{1-\beta}\left[\dfrac{\beta(1-w)}{2+\beta}\right]^{\beta}>0 \\ (iii) \quad H=1+\dfrac{\alpha}{\beta(1+\beta)}w^{1-\beta}\left[\dfrac{\beta(1-w)}{2+\beta}\right]^{\beta}>0 \end{cases}. \tag{12}$$

The violation of any single inequality in (12), with the other two being simultaneously fulfilled leads to: (*i*) a flip bifurcation (a real eigenvalue that passes through $-1$) when $F = 0$; (*ii*) a fold or transcritical bifurcation (a real eigenvalue that passes through $+1$) when $TC = 0$; (*iii*) a Neimark-Sacker bifurcation (i.e., the modulus of a complex eigenvalue pair that passes through 1) when $H = 0$, namely $D = 1$ and $|T| < 2$. From Eq. (12) it is clear that conditions (*ii*) and (*iii*) are always fulfilled, while condition (*i*) can be violated. The following equation $\mathrm{B}(\alpha, \beta, w)$, i.e. the first condition in (12), represents a boundary at which the Nash equilibrium loses stability through a flip bifurcation ($F = 0$) when:

$$\mathrm{B}(\alpha, \beta, w) := 2 - \alpha \frac{1 + (1+\beta)^2}{\beta(1+\beta)} w^{1-\beta} \left[ \frac{\beta(1-w)}{2+\beta} \right]^{\beta} = 0 . \tag{13}$$

Now, define

$$\alpha^F(\beta, w) := \frac{2\beta(1+\beta)}{\left[1 + (1+\beta)^2\right] w^{1-\beta} \left[ \frac{\beta(1-w)}{2+\beta} \right]^{\beta}} . \tag{14}$$

as the (unique) flip bifurcation value of $\alpha$. Then, the following proposition holds.

**Proposition 1**. *[Local bifurcation]. Let* $0 < \alpha < \alpha^F(\beta, w)$ *hold. Then, the Cournot-Nash equilibrium* $E$ *of the two-dimensional system (5) is locally asymptotically stable. A flip bifurcation emerges if* $\alpha = \alpha^F(\beta, w)$. *Let* $\alpha > \alpha^F(\beta, w)$ *hold. Then, the Cournot-Nash equilibrium* $E$ *is locally unstable.*

**Proof**. Since $\mathrm{B}(\alpha, \beta, w) > 0$ for any $0 < \alpha < \alpha^F(\beta, w)$, $\mathrm{B}(\alpha, \beta, w) = 0$ if $\alpha = \alpha^F(\beta, w)$ and $\mathrm{B}(\alpha, \beta, w) < 0$ for any $\alpha > \alpha^F(\beta, w)$, then Proposition 1 follows. **Q.E.D.**

It is now of importance to study: (*i*) the effects of a rise in $w$ on stability when both firms bargain on employment together with firm-specific employees' representatives ($0 \le \beta < 1$), in contrast to the case of pure profit maximisation ($\beta = 1$), and (*ii*) how the flip bifurcation value $\alpha^F(\beta, w)$ varies when $\beta$ is continuously changed for any given value of the wage. The results of points (*i*) and (*ii*) are summarised in Proposition 2 and Proposition 3, respectively.

**Proposition 2**. *[Effects on stability of a change in* $w$ *for any given value of* $\beta$*]. Let* $0 \le \beta < 1$ *(Codetermination). Then, an increase in wages acts as an economic de-stabiliser (stabiliser) if, and only if,* $w < 1 - \beta$ *(*$w > 1 - \beta$*). Let* $\beta = 1$ *(Profit-maximisation). Then, an increase in wages always acts as an economic stabiliser.*

**Proof**. Since

$$\frac{\partial \alpha^F(\beta, w)}{\partial w} = \frac{2\beta(1+\beta)(w - 1 + \beta)}{w^2(1-w)\left[1 + (1+\beta)^2\right] \left[ \frac{\beta(1-w)}{w(2+\beta)} \right]^{\beta}} , \tag{15}$$

then, for any $0 \le \beta < 1$, $\frac{\partial \alpha^F(\beta, w)}{\partial w} < 0$ $(> 0)$ if, and only, if $w < 1 - \beta$ $(w > 1 - \beta)$. In the particular case $\beta = 1$, $\frac{\partial \alpha^F(1, w)}{\partial w} = \frac{12}{5(1-w)^2} > 0$. **Q.E.D.**

Now, define

$$\mathrm{A} := 2\beta^4 + 5\beta^3 + 2\beta^2 - 6\beta - 4 < 0, \tag{16}$$

$$\mathrm{P} := \beta\left(\beta^4 + 5\beta^3 + 10\beta^2 + 10\beta + 4\right) > 0, \tag{17}$$

$$f(\beta) := -\frac{\mathrm{A}}{\mathrm{P}}, \tag{18}$$

to simplify notation, and

$$\overline{w}(\beta) := \frac{\beta}{\beta + (2+\beta)e^{f(\beta)}} < 1, \tag{19}$$

as a threshold value of the wage as a function of $\beta$, where $\frac{\partial \overline{w}(\beta)}{\partial \beta} = \frac{\left(6\beta^7 + 47\beta^6 + 168\beta^5 + 332\beta^4 + 384\beta^3 + 260\beta^2 + 96\beta + 16\right)e^{f(\beta)}}{\beta(2+\beta)(1+\beta)^2\left[1+(1+\beta)^2\right]^2\left[\beta + (2+\beta)e^{f(\beta)}\right]^2} > 0$, so that

$$\overline{w}(1) := \frac{1}{1+3e^{f(1)}} \cong 0.243, \tag{20}$$

for $\beta = 1$ (profit maximisation). Moreover, define $0 < \overline{\beta} < 1$ as a solution for $\beta$ to $\overline{w}(\beta) = w$. Then, the following proposition holds.

**Proposition 3**. *[Effects on stability of a change in $\beta$ for any given value of $w$]. Let $0 < w < \overline{w}(1)$ hold. Then, a rise in $\beta$ acts as an economic stabiliser (de-stabiliser) if, and only if, $\beta < \overline{\beta}$ ($\beta > \overline{\beta}$). Let $\overline{w}(1) < w < 1$ hold. Then, a rise in $\beta$ always acts as an economic stabiliser.*

**Proof**. Since

$$\frac{\partial \alpha^F(\beta, w)}{\partial \beta} = \frac{-2\left\{\mathrm{A} + \mathrm{P}\ln\left[\frac{\beta(1-w)}{(2+\beta)w}\right]\right\}}{(2+\beta)\left[1+(1+\beta)^2\right]w^{1-\beta}\left[\frac{\beta(1-w)}{2+\beta}\right]^{\beta}}, \tag{21}$$

then $\frac{\partial \alpha^F(\beta, w)}{\partial \beta} = 0$ if, and only if

$$\mathrm{A} + \mathrm{P}\ln\left[\frac{\beta(1-w)}{(2+\beta)w}\right] = 0. \tag{22}$$

Solving Eq. (22) for $w$ gives the threshold value $\overline{w}(\beta)$ that makes $\frac{\partial \alpha^F(\beta, w)}{\partial \beta} = 0$ as expressed by Eq. (19). For $\beta = 1$, such a threshold reduces to $\overline{w}(1) \cong 0.243$ (see Eq. 20). Therefore, (1) for $0 < w < \overline{w}(1)$, $\frac{\partial \alpha^F(\beta, w)}{\partial \beta} > 0$ ($< 0$) for any $0 < \beta < \overline{\beta}$ ($\overline{\beta} < \beta < 1$), and (2) for $\overline{w}(1) < w < 1$, $\frac{\partial \alpha^F(\beta, w)}{\partial \beta} > 0$ for any $0 < \beta < 1$. **Q.E.D.**

Proposition 2 shows that under codetermination an exogenous shock in wages (production cost) causes an ambiguous role on stability of equilibrium outcomes. Indeed, while a rise in wages under profit maximisation monotonically tends to stabilise the market equilibrium, because the direct negative effect of it on the reaction of the bounded rational firm is stronger than the positive effect due to the reduction in output produced by the naïve firm, a positive exogenous shock in wages under codetermination (due for instance to an increase in the bargaining power of unions at the economy-wide level), acts as an economic de-stabiliser (stabiliser) when the wage is fairly low (already high). As a consequence, when firms codetermine employment together with unions, the relative size of the wage matters for stability. Moreover, the higher the bargaining power of firms in determining employment, the lower the threshold value of the wage beyond which an exogenous positive shock in wages tends to stabilise the market equilibrium by increasing the flip bifurcation value $\alpha^F(\beta, w)$, and the wider the range of values of $w$ within which stability is guaranteed.

Proposition 3 symmetrically shows the effects of a change in the bargaining power $\beta$ on the stability-instability regions for any given value of the exogenously fixed wage. Indeed, it reveals that when the wage is low enough, the relative size of the union power in the Nash bargaining matters for stability, while playing an unambiguous destabilising role when the wage becomes higher. In the former case ($0 < w < \overline{w}(1)$), in fact, the relative weight of the utility of unions in the Nash bargaining is fairly low because the exogenously fixed wage is low. Then, a reduction in $\beta$ increases the union power and tends to destabilise through this channel. However, since the weight of union is not too large, such a destabilising effect dominates only whether $\beta$ becomes low enough. In fact, a reduction in $\beta$ also increases the output produced by naive firm and tends to reduce the reaction of the bounded rational player thus exerting a stabilising role through this channel. This positive effect dominates when $\beta$ is large enough and the firm power in the Nash bargaining is high.

In the latter case ($\overline{w}(1) < w < 1$), instead, a reduction in $\beta$ (by increasing the intensity of the reaction of the bounded firm), increases the relative weight of the utility of unions in the Nash bargaining (which is already high because the exogenously fixed wage is high), and then it tends to destabilise the market equilibrium through this channel. Moreover, the lower $\beta$, the higher the quantity produced by the naïve firm, and the lower the reaction of the bounded rational player. Therefore, a reduction in $\beta$ tends to stabilise the market equilibrium through this channel. However, the negative (destabilising) effect always prevails because the relative weight of the utility of unions in the Nash bargaining is high because of two reasons: both the exogenously fixed wage and the union power are high.

The hypothesis of normalised duopoly also allows us to completely characterise the flip bifurcation value $\alpha^F(\beta, w)$ for any couple $(\beta, w)$, and then to exemplify the effects on stability of different values of wage $w$ and firm's bargaining firm power $\beta$, as shown in Propositions 2 and 3, respectively. To this purpose, Table 1 shows how $\alpha^F(\beta, w)$ reacts to: (*i*) a change in $\beta$ for different values of $w$ (Rows), and (*ii*) a change in $w$ for different values of $\beta$ (Columns). We recall that a reduction in $\beta$ represents an increase (reduction) in the power of unions (firms) in the decentralised Nash bargaining.

**Table 1**. Flip bifurcation values $\alpha^F(\beta, w)$ for any couple $(\beta, w)$.

| $\alpha^F(\beta, w)$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\beta = 1$ | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 | 0.4 | 0.3 | 0.2 | 0.1 |
| $w = 0.1$ | 2.666 | 2.943 | 3.19 | 3.38 | 3.478 | 3.44 | 3.217 | 2.763 | 2.047 | 1.083 |
| 0.2 | 3 | 3.053 | 3.052 | 2.982 | 2.829 | 2.58 | 2.225 | 1.762 | 1.204 | 0.587 |
| 0.3 | 3.428 | 3.306 | 3.131 | 2.899 | 2.606 | 2.252 | 1.84 | 1.381 | 0.894 | 0.413 |
| 0.4 | 4 | 3.69 | 3.344 | 2.962 | 2.548 | 2.106 | 1.646 | 1.182 | 0.732 | 0.32 |
| 0.5 | 4.8 | 4.252 | 3.7 | 3.148 | 2.6 | 2.064 | 1.549 | 1.068 | 0.635 | 0.269 |
| 0.6 | 6 | 5.105 | 4.265 | 3.484 | 2.763 | 2.106 | 1.518 | 1.005 | 0.574 | 0.234 |
| 0.7 | 8 | 6.512 | 5.206 | 4.069 | 3.087 | 2.252 | 1.553 | 0.984 | 0.537 | 0.209 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.8 | 12 | 9.256 | 7.011 | 5.192 | 3.733 | 2.58 | 1.686 | 1.012 | 0.524 | 0.193 |
| 0.9 | 24 | 17.07 | 11.924 | 8.142 | 5.398 | 3.44 | 2.073 | 1.147 | 0.547 | 0.186 |

Table 1 clearly shows that a reduction in $\beta$ has two different effects on stability depending on the relative size of wages. If $w$ is fairly low ($w = 0.1$ and $w = 0.2$), a reduction in $\beta$ first acts as an economic stabiliser by increasing the flip bifurcation value $\alpha^F(\beta, w)$, and then acts as an economic de-stabiliser when $\beta$ further decreases (i.e., the power of unions in the Nash objective becomes large). If $w$ becomes larger, a reduction in $\beta$ monotonically reduces the flip bifurcation value $\alpha^F(\beta, w)$ and then it unambiguously acts as an economic de-stabiliser.

To sum up, as regards the problem of market stability, we observe two different effects on $\alpha^F(\beta, w)$ when $\beta$ reduces depending on whether the wage is low or high: (*i*) when the exogenously determined wage is low or, alternatively the price-cost margin is high (e.g., the case of industries with low-skilled manpower, or, alternatively, with an exogenously given high profitability), a reduction in the firm power first act an economic stabiliser, by increasing the flip bifurcation value $\alpha^F(\beta, w)$, and then it acts as a destabilising device because $\alpha^F(\beta, w)$ becomes lower. It is important to note that when the wage is fixed at not too high a level ($w = 0.1$), the stability region increases when the firm power reduces almost up to near-parity representation between parties ($\beta = 0.5$), as neatly shown in the first row of Table 1); (*ii*) when the exogenously determined wage becomes larger (and a fortiori the price-cost margin is low), the flip bifurcation value $\alpha^F(\beta, w)$ monotonically reduces when the union power in the Nash bargaining is continuously increased for some exogenous reasons.

Our findings also constitute a policy warning (e.g., for centralised wage setters) about the peril for market stability of raising the wage (unless they are fairly low) when the power of unions within the supervisory board that co-determine employment is high. The lesson drawn by these results is that in contrast with the case in which employment is not co-determined and unions only care about wages – indeed, in the latter case an increase in wages always stabilises the equilibrium (i.e. wage-interested unions are beneficial for stability) –, when codetermination laws do exist or under separation between a centralised wage setting and de-centralised employment bargaining, a high firm-specific unions' power (often included even in the case of near-parity) to determine employment is harmful for stability because it tends to reduce the flip bifurcation value of the speed of adjustment $\alpha$ (except when the exogenously determined wage is low).

### *3.1. A numerical example of dynamic outcomes under profit maximisation and codetermination*[6]

As a simple numerical illustration we now show the different dynamic events that can be observed depending on whether firms are profit-maximising or, alternatively, they are subject to codetermination laws. To this purpose, Figures 1 and 2 depict the bifurcation diagrams for $\alpha$ and portrait the limit point of $q^*$ when $w = 0.5$ and the initial conditions are $q_1(0) = 0.03$ and $q_2(0) = 0.01$. Figure 1 clearly shows that under profit maximisation ($\beta = 1$), the Cournot-Nash equilibrium is locally asymptotically stable for any $0 < \alpha < 4.8$. Then, a flip bifurcation occurs at $\alpha^F(1, 0.5) = 4.8$. Beyond such a value, a two-period cycle emerges and it is followed by four-period cycles broken off when $\alpha \cong 7$. Then, eight-period cycles followed by high periodicity and a cascade of flip bifurcations that ultimately lead to chaotic behaviours

[6] It is worth noting that we do not enter into details of the analysis of complex dynamics, as this is not the focus of the present study. It could however be interesting to advise the reader of the possible complicated dynamic events that can be observed in this simple model.

emerge. Figure 2 instead depicts the case of near-parity codetermination ( $\beta = 0.5$ ), showing that the equilibrium is locally asymptotically stable when $0 < \alpha < 2.064$. A flip bifurcation occurs at $\alpha^F(0.5, 0.5) = 2.064$. Then, a two-period cycle broken off at $\alpha \cong 3.25$ is observed. Then, under pure profit maximisation, the Nash equilibrium is more likely to be stable than under codetermination, as the flip bifurcation in the former case occurs at a higher value of $\alpha$ than with codetermined firms, and the range of values of $\alpha$ for which trajectories are not divergent is higher in the former case than in the latter one.

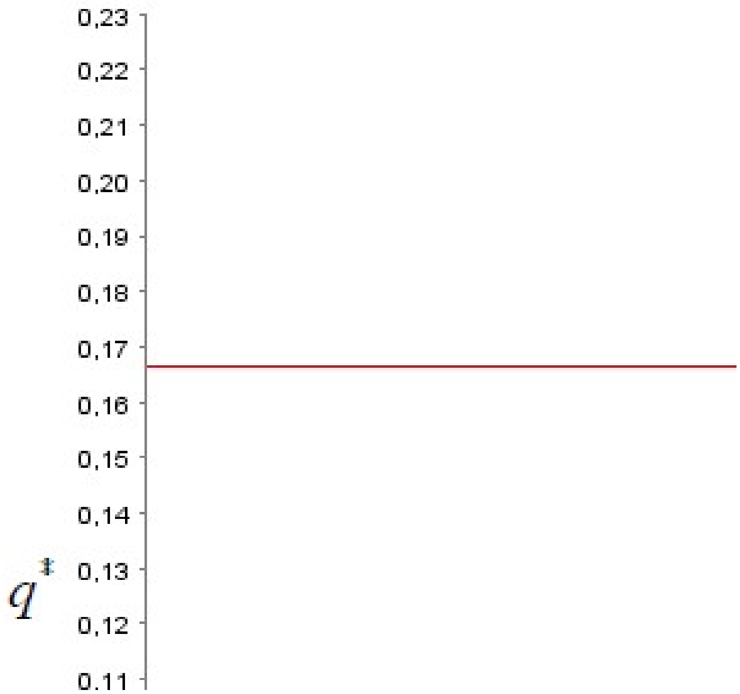


**Figure 1**. Profit-maximising firms ( $\beta = 1$ ). Bifurcation diagram for $\alpha$ .

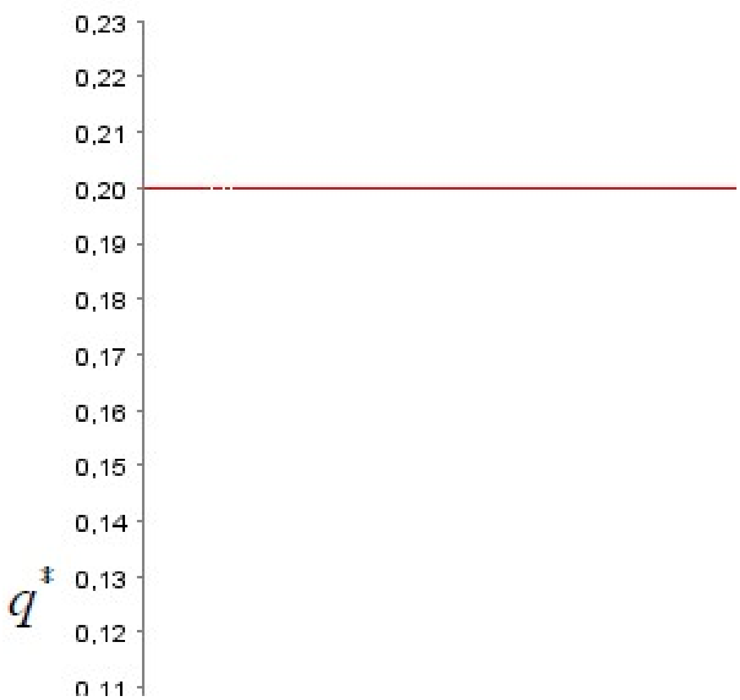

**Figure 2**. Codetermined firms ( $\beta = 0.5$ ). Bifurcation diagram for $\alpha$ .

## 4. Conclusions

While traditional economic theories assume that the single aim of competing firms is profit maximisation, in some important countries, such as Germany, workers in large firms have nearly the same decision rights as capital owners, due to the existence of codetermination laws as regards employment setting at the firm level. More in general, a bargaining over employment without considering wages is widely observed, especially in Europe.

The present study analysed the dynamics of a nonlinear Cournot duopoly in the case in which both firms (one of which is "bounded rational" and the other has the standard Cournot-naïve expectations) codetermine employment together with firm-specific workers' representatives, and compared the results with the standard case of profit-maximising firms. We found that a rise in wages under codetermination acts as an economic de-stabiliser (stabiliser) when the extent of the exogenously determined wage is still fairly low (already high), while under profit maximisation an increase in the labour cost always acts as a stabilising device. This means that under codetermination, the relative size of the wage matters for stability. Interestingly, we also found that when the wage is low enough, the relative size of the union power in the Nash bargaining matters for stability.

Therefore, on the one hand the existing literature (Kraft, 1998) established that, in equilibrium, co-determination may be preferred with respect to profit maximisation, on the other hand we found that, out-of-equilibrium, the bargaining power of firms should be fairly high in order to ensure market stability, except when the wage is low: in such a case in fact near-parity decision rights would be better for stability because tend to increase the parametric stability region.

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