Public expenditure on health and private old-age insurance in an OLG growth model with endogenous fertility: chaotic dynamics under perfect foresight

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Abstract This study analyses the dynamics of an economy with overlapping generations, endogenous population (fertility and adult mortality), logarithmic preferences and Cobb-Douglas technology. We show that the public provision of health investments and the existence of a private system of old-age insurance (i.e., transfers from children to parents) may cause the birth and death of multiple (three) steady states, deterministic chaos and bubbling phenomena when individuals have perfect foresight. Interestingly, however, we show that periodic dynamics (cycles) or complex dynamics (chaos) and global stability of the economy can endogenously be reconciled in the model, because the rise either in public health investments or transfers from young to old people can have the potential to smooth and ultimately suppress endogenous fluctuations in the cases of existence of both a single steady state or multiple steady states.

Keywords Chaos; Endogenous fertility and longevity; OLG model; Perfect foresight; Private old-age support; Public expenditure on health

JEL Classification C62; H55; I18; J14; J18

1. Introduction

A typical feature of all societies is that young people have often contributed to the support of the elderly, especially through voluntary intra-family transfers (see, e.g., Ehrlich and Lui,

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1991; Wigger, 1999). Currently, this occurs in underdeveloped and developing economies, where per capita income is low and children represent a partial substitute for other saving opportunities (see, .e.g., Neher, 1971; Cain, 1981, 1983), essentially because financial and securities markets are not highly developed, and public health and social security programmes are either slightly used or completely absent, with the consequence that population growth and mortality rates are still fairly high, in contrast with developed economies where per capita income is high and the opposite phenomenon is observed (see Fogel, 2004; Cervellati and Sunde, 2005, 2011; Galor, 2005; Livi-Bacci, 2006).[1]

The aim of the present paper is to study the interrelationship between private intra-family transfers (gifts) from young to old people and endogenous lifetime determined by public investments in health (see Chakraborty, 2004), on capital accumulation, economic growth and *stability* of an overlapping generations (OLG) economy with production (Diamond, 1965), extended with endogenous fertility. In the economic literature, there exist at least two main approaches to consider fertility as an endogenous variable: the former approach is based on the hypothesis that a child is a consumption (normal) good and enters the utility function of parents. This is the case of forward altruism[2] (i.e., from parents to children), see Eckstein and Wolpin (1985), Eckstein et al. (1988) and Galor and Weil (1996), which is in line with the view of the new home economics (Becker, 1960). The latter one is to hypothesise that an individual can give birth to children because of old age support motives (see, e.g., Raut and Srinivasan, 1994), a broader notion of which is those introduced by Azariadis and Drazen (1993), with respect to which having a child is valued by parents as the former can be both a source of labour and of inter-generational transfer when a parent . This approach can indeed well explain the reason why sometimes it has historically been observed that in several developed countries population growth rates are lower than in developing or underdeveloped ones, that is children can be viewed as a partial substitute for other savings opportunities, as it has been pointed out at the beginning of this section. Indeed, as clearly stated by Chakrabarti (1999, p. 395) "in developing countries, children often start contributing to family income while living at home prior to adulthood. It is also not unusual for them to migrate to the city and remit part of their income to their parents as an adult (Katz and Stark, 1986)." Moreover, in the old age support literature the transfer from children to parents can be assumed to be either exogenously given,[3] the size of which being determined, for instance, by social norms,[4] religious beliefs and cultural reasons, or endogenously determined,[5] by assuming that agents care about consumption of their parents when old, that is children are altruistic towards parents (backward altruism). In other words, in such a case the utility of a member of every generation is affected by material consumption of the old parents. By passing, we note that in either cases of exogenous and endogenous determination of the transfer, the issue of why it is in the interest of children to remit to their parents is raised: in

[1] See, e.g., Dahan and Tsiddon (1998) and Galor and Weil (1999, 2000) for an analysis of the Demographic Transition, and the recent paper by Gonzalez-Eiras and Niepelt (2012) that deals with the effects of demographic ageing on economic growth in an OLG context.

[2] This is also called weak (Zhang and Zhang, 1998) or *ad hoc* (see de la Croix and Michel, 2002, p. 3) altruism towards children, which contrasts the case of pure altruism with respect to which the utility of descendants enters the utility function of parents (see Barro, 1974; Becker and Barro, 1988; Barro and Becker, 1989; Becker et al., 1990).

[3] In particular, in the present study we assume that the young voluntary transfer a constant fraction of their income to parents in order to support them when old (see, amongst many others, Bental, 1989; Raut and Srinivasan, 1994; Chakrabarti, 1999; Morand, 1999).

[4] Indeed, as pointed out by Nugent (1985, p. 78) "Such norms are typically maintained by assessing strong sanctions against those who deviate from them and by offering rewards for those who adhere to them… Individuals come to feel pride and satisfaction in being able to fulfill their obligations to their parent."

[5] The endogenisation of transfers are essentially based on backward altruism motives (Nishimura and Zhang, 1993; Zhang and Zhang, 1995).

the present study, we abstract from this issue but note that Azariadis and Drazen (1993) assume that the division of income between generations is determined as part of a bargaining process, while Cigno (1991, 1993), by stressing the fact that voluntary transfers can be explained by self-interest alone, suggests a contractual solution between parents and their descendants, which could also be implemented by a self-enforcing set of family rules.

Moreover, another feature that is worth to be mentioned is that in the papers that deal with models of old-age support, the endogenisation of fertility choices is based on the fact that children are valued exclusively as an investment good (e.g., Azariadis and Drazen, 1993; Nishimura and Zhang, 1993; Raut and Srinivasan, 1994), or, alternatively, by jointly considering both backward and forward altruism (e.g., Wigger, 1999).

In the past few decades a literature therefore emerged (e.g., Raut and Srinivasan 1994;[6] Chakrabarti, 1999; Morand, 1999) that analysed private systems of old-age support in OLG models with the aim of studying several aspects of real phenomena. However, the analysis of dynamic features (under rational expectations) in this class of models has not yet received in-depth attention. The present study attempts to fill this gap by building on a model with endogenous fertility and endogenous lifetime à la Chakraborty (2004) and assuming that the relationship between longevity and health spending is nonlinear with threshold effects (see the numerical experiments reported in Blackburn and Cipriani, 2002; de la Croix and Ponthière, 2010), as argued by some empirical studies (see, e.g., Fioroni, 2010). Moreover, we assume: (*i*) the fraction of wage income voluntarily transferred from children to parents is fixed and exogenous; (*ii*) the decision to have children is based not only on old-age-support motives but also on altruism towards children, i.e. each child represents both an investment good and consumption good, with individuals being assumed to draw utility directly from the number of children raised (forward altruism), instead of from old age consumption of parents (backward altruism).

As regards the health system, although the existence of a public financed health care services may cause, as in Chakraborty (2004), the appearance of multiple regimes of development as well as the opportunity to escape from poverty, we show that an increase in the health tax rate: (*i*) reduces the steady-state stock of capital in the high regime of development to a level even lower than the pre-existing (unique) low equilibrium; (*ii*) acts as an economic de-stabiliser and triggers complex cycles, when the health tax rate is included within a range of intermediate values, or determines a re-switch towards stability, when the health tax rate becomes larger. Moreover, other intriguing dynamic events are caused by the rise in intra-family transfers: indeed, the (local) dynamics of the economy is monotonic and convergent towards the unique steady state for either low or high levels of the gift rate, while becoming non-monotonic and divergent from the steady state for an intermediated-sized provision of old-age backing. However, a high enough level of intra-family gift can also have the potential to curb endogenous fluctuations[7] because of the existence of a subsequent re-switch towards stability. Interestingly, when threshold effects in the longevity function becomes stronger, raising the fraction of wage income voluntarily transferred by the young to the survival old-aged also causes the birth and death of multiple regimes of development.

Our findings are rather unusual in the business cycle literature with overlapping generations, especially because the economy is comprised of individuals with perfect

[6] In particular, Raut and Srinivasan (1994) is the study most closely related to ours from a dynamic point of view, as they analyse an endogenous growth model with old-age support from young to old people where growth paths divergent to a steady state or even chaotic can be observed.

[7] We note that in the present study we avoid to evaluate the normative implications of unpredictable fluctuations. However, even if the common sense seems to give a negative connotation to such fluctuations, it has been shown, by taking some utility comparisons into account, that chaotic behaviour can in some cases be beneficial because trajectories associated to a chaotic attractor may Pareto-dominate those associated to a fixed point (see Matsumoto, 2003).

foresight, lifetime utility is logarithmic and the production function is Cobb-Douglas. As is known, in fact, in addition to the modern equilibrium business cycle theory – which is essentially grounded on a stochastic origin of cycles (see, e.g., Kydland and Prescott, 1982; Long and Plosser, 1983) – according to which fluctuations in macroeconomic variables reflect transitional dynamics around balanced growth paths, a strand of theoretical literature exists that tries to explain economic cycles in an OLG context, and argues that exogenous shocks are not necessary for the emergence of fluctuations. Indeed, the emergence of periodic orbits and deterministic chaos when individuals have perfect foresight is due to distortions in the production side of the economy, such as imperfect competition, positive externalities and so on (see, e.g., Farmer, 1986; Reichlin, 1986; Grandmont et al., 1998; Cazzavillan, 2001),[8] and generally it requires at least a two-dimensional model which is notoriously more prone to instability, unless myopic expectations are assumed (Michel and de la Croix, 2000; de la Croix and Michel, 2002; Chen et al., 2008). In contrast, in this paper non-monotonic trajectories and deterministic chaos are observed in a simple economy with logarithmic preferences and Cobb-Douglas technology whose dynamics is completely characterised by a one-dimensional map, perfect competition between firms exists and individuals have perfect foresight. Moreover, we show the existence of bubbling phenomena,[9] so that a period doubling bifurcation process is triggered and subsequently curtailed and reversed, giving rise to period-halving bifurcations as either the health tax rate or the gift rate changes.[10] This twofold role of the policy and behavioural parameters is remarkable from an economic point of view: indeed, on the one hand, it may represent an explanation of the observed endogenous fluctuations (business cycles) – showing that an endogenous deterministic origin of cycles may complement the stochastic origin of it –, on the other hand these parameters may even be used to control, and potentially suppress, endogenous fluctuations.

To sum up, the equilibrium dynamics in our model economy may embody two undisputable stylised facts: the existence of both endogenous fluctuations (Grandmont, 1985) and the global stability (Galor and Ryder, 1989) of capitalist economies.

The rest of the paper is organised as follows. Section 2 presents the model. Section 3 deals with the study of the dynamics of the economy. Section 4 shows that chaotic motions under rational expectations are possible when either the health tax rate or gift rate increases. Section 5 concludes.

2 The model

Consider a general equilibrium OLG closed economy populated by rational and identical individuals, identical firms and a government that finances public investments in health at a balanced budget. Time is discrete and indexed by $t = 0,1,2\ldots$.

The lifetime of the typical agent of generation t is divided into childhood and adulthood, the latter period being, in turn, divided between youth, i.e. work and child-bearing time, and old age, i.e. retirement time. As a child, an individual does not make economic decisions and survives at the end of childhood with certainty, that is there is no child mortality.[11] When

[8] We note that similar production externalities can also amplify the propagation mechanisms able to clarify several business cycle puzzles that the standard real-business-cycles models fail to explain (see, e.g., Benhabib and Farmer, 1994; Benhabib and Wen, 2004).

[9] See Bier and Bountis (1984) and Stone (1993).

[10] A relevant mathematical reference on period-doubling and period-halving bifurcations is Nusse and Yorke (1988), while interesting economic applications are, e.g., Hommes (1994), Zhang (1996), Zhang (1999), Antoci and Sodini (2009), Brianzoni et al. (2009). Indeed, Gori and Sodini (2011) analyse the local and global dynamic properties of an OLG growth model with public health expenditure and young and old age labour supply.

[11] Note that, following Zhang et al. (2001, 2003) we abstract from considering child mortality and concentrate on both macroeconomic and stability effects of adult mortality decline.

adult, an individual draws utility from young-age consumption ($c_{1,t}$), old-age consumption ($c_{2,t+1}$) and the number of children raised (n_t), as in Eckstein and Wolpin (1985), Eckstein et al. (1988), and Galor and Weil (1996). Young individuals (N_t) are endowed with one unit of labour inelastically supplied to firms and receive wage income w_t. We assume that the probability of surviving from youth to old-age (i.e., adult mortality) is endogenous and determined by the individual health status, which is, in turn, augmented by public investments in health (see Chakraborty, 2004).[12] The survival probability at the end of youth of an individual started working at the beginning of time t, π_t, depends upon her health status h_t and it is described by the strictly increasing (though bounded) function $\pi_t = \pi(h_t)$. Following Blackburn and Cipriani (2002) and de la Croix and Ponthière (2010), we model this relationship as:[13]

$$\pi_t = \pi(h_t) = \frac{\pi_0 + \pi_1 \Delta h_t^{\delta}}{1 + \Delta h_t^{\delta}}, \tag{1}$$

where $\delta, \Delta > 0$, $0 \le \pi_0 < 1$, $\pi_1 > \pi_0 > 0$, $\pi(0) = \pi_0 \ge 0$, $\pi'_h(h) = \frac{\delta \Delta h^{\delta-1}(\pi_1 - \pi_0)}{(1 + \Delta h^{\delta})^2} > 0$,

$\lim_{h\to\infty} \pi(h) = \pi_1 \le 1$, $\pi''_{hh}(h) < 0$ if $\delta \le 1$ and $\pi''_{hh}(h) \gtrless 0$ for any $h \lessgtr h_T := \left[\frac{\delta - 1}{(1+\delta)\Delta}\right]^{\frac{1}{\delta}}$ if $\delta > 1$.

Eq. (1) allows us to capture several aspects of the length of life of the typical agent. Indeed, it encompasses the monotonic (concave) function used in numerical examples by Chakraborty (2004) when $\delta = \Delta = 1$ and $\pi_0 = 0$ as well as the S-shaped function when $\delta > 1$ (i.e., threshold effects exist), while also preserving (different from Chakraborty, 2004) a positive constant rate of longevity π_0 regardless of health investments, which can therefore be interpreted as an exogenous measure of the basic or natural rate of longevity (see, e.g., Ehrlich, 2000; Leung and Wang, 2010).[14]

We assume that at time t the individual health status is augmented by the public provision of health investments (h_t) financed with a constant proportional wage tax $0 < \tau < 1$ (see Chakraborty, 2004; Bhattacharya and Qiao, 2007) at a balanced budget, so that the government budget identity reads as follows:

$$h_t = \tau w_t. \tag{2}$$

Moreover, following Bental (1989), Ehrlich and Lui (1991), Raut and Srinivasan (1994), Chakrabarti (1999) and Morand (1999), we assume that every young voluntarily transfers an exogenous fraction $0 < d < 1$ of wage income to parents as a means of old-age insurance,[15] so

[12] We are therefore assuming an OLG context where an individual lives for three periods, while being a child (not economically active agent) during the first one, and she survives to childhood with certainty. In the second period (young adulthood), an individual inelastically offers labour to firms and gives birth and raise children. Moreover, an individual is alive at the end of young adulthood with uncertainty, i.e. there exists adult mortality, so that an agent may die before reaching old age. In the third period (old age), a surviving individual retires. To avoid confusion, however, we note that "since the agents do not make any economic decisions in their first period of life, this approach is equivalent to studying a model with two periods." (see Chakrabarti, 1999, p. 397).

[13] Although Blackburn and Cipriani (2002) assume human capital instead of public health spending as the independent variable in the longevity function, the line of reasoning to justify this formulation is the same.

[14] Empirical evidence shows a rather significant nonlinear relationship between longevity and either per capita income or per capita health spending. For instance, Fioroni (2010) provides evidence of this non-linearity at aggregate level (see Figures 1 and 2, p. 357, as drawn by World Development Indicators CD-ROM, World Bank, 2008).

[15] Some empirical evidence as regards old-age support exists (see Nugent and Gillapsy, 1983; Kağitçibaşi, 1982; Dharmalingam, 1994; Jensen, 1990) from which emerged, amongst other things, that historically the reasons for

that $\pi_{t-1} d w_t$ is the expected cost to each young[16] and $d w^e_{t+1} n_t$ is the benefit received when old, which is contingent on the number of children raised when young and the expected wage earned at time $t+1$. In period t, therefore, the budget constraint of the young reads as:

$$c_{1,t} + s_t + en_t + \pi_{t-1} d w_t = w_t (1-\tau), \tag{3.1}$$

i.e., the disposable income (net of wage taxation to finance health expenditure h_t) is divided amongst consumption, saving (s_t),[17] the (fixed) cost of raising n_t descendants (en_t), where $e > 0$ (the cost of children being represented by expenditures on commodities necessary for the upbringing of them, see, e.g., Raut and Srinivasan, 1994; van Groezen et al., 2003; van Groezen and Meijdam, 2008; Fanti and Gori, 2009, 2010), and the expected cost to transfer resources to sustain the old parents period by period. Indeed, in order to clarify the reason why the cost incurred by the young of generation t as a means of old-age insurance within the budget constraint Eq. (3.1) is weighted by the (individual) survival probability π_{t-1}, we note that as the cost of raising children depends upon the number of children raised, so the cost of transferring resources towards a parent depends on whether such a parent is alive at the end of her working period, namely at the end of time $t-1$, and this event occurs with probability π_{t-1}. In a framework with a representative agent, and knowing that every individual has a single parent, this translates into fact that the young started working at time t takes into account the fact that her parent is alive with probability π_{t-1}, while dying with probability $1-\pi_{t-1}$, so that only whether the former event occurs resources are inter-generationally transferred. In the present study, therefore, the fraction of wage income transferred by the young to the old is exogenous and based on social norms, so that its size varies as long as cultural and religion beliefs in societies change due to exogenous shocks.

If an individual survives to young adulthood, she retires and lives on: (*i*) the proceeds of savings plus the expected interest accrued the at rate r^e_{t+1}, and (*ii*) the amount of resources transferred by the young. Moreover, the existence of a perfect market for annuities (where savings are intermediated through mutual funds) implies that old survivors will benefit not only from their own past saving plus interest, but also from the saving plus interest of those who have deceased. Hence, the budget constraint at time $t+1$ of an old individual that belongs to generation t can be expressed as follows:

$$c_{2,t+1} = \frac{1 + r^e_{t+1}}{\pi_t} s_t + d w^e_{t+1} n_t . \tag{3.2}$$

By taking factor prices and the government budget as given, the individual representative of generation t chooses fertility and saving to maximise the expected lifetime utility function

$$U_t = \ln(c_{1,t}) + \pi_t \beta \ln(c_{2,t+1}) + \gamma \ln(n_t), \tag{4}$$

subject to Eqs. (3.1) and (3.2), where $0 < \beta < 1$ is the subjective discount factor and $\gamma > 0$ is the parents' taste for children. Expected utility Eq. (4) is usual in the OLG literature with exogenous fertility (see, e.g., Chakraborty, 2004; Chakraborty et al., 2010; Fanti and Gori,

the young to transfer resources towards their parents when old can essentially be explained by social and cultural norms (Willis, 1980; Nugent, 1985).

[16] The probability attached to the expected cost incurred by the young to transfer resources inter-generationally, is due to the fact every young knows that such an income will actually be transferred only whether her own parent does not die at the beginning of old-age (see Ehrlich and Lui, 1991).

[17] Note that as is usual in the OLG literature (see de la Croix and Michel, 2002), a young individual owns the stock of capital at time t and rents it to firms in the same period, while receiving the profits of the firm (that is, the return on capital) when she will be old because capital becomes productive one period forward, i.e., "capital K_t is productive at time t and is built from the savings of time $t-1$ (there is a one-period time-to-build), see de la Croix and Michel (2002, p. 4).

2012; de la Croix et al., forthcoming). In particular, following Abel (1985), in this class of models, the formulation for the uncertain lifetime utility function, which in our case depends on the consumption of material goods and services and the number of children (assumed to be normal goods), is based on the fact that the discounted utility index for a generic period depends on the individual probability of surviving in such a period, which indeed contributes to determine the degree of psychological subjective discount factor, i.e. the lower adult mortality, the higher the relative importance of old-age consumption with respect to young-age consumption. The utility index Eq. (4) is therefore obtained as the expected value of a utility function contingent on being alive at time t, plus a death-contingent utility index at the same time, which is, in turn, equal to zero (see also the discussion in Pestieau et al., 2008, as regards modelling expected lifetime welfare functions in the case of additive preferences and endogenous longevity). However, as clearly stated by Abel (1985, p. 779): "It is not necessary that the utility index is equal to zero in the case of death. All that is required is that utility in the state of death does not depend on the level of wealth."

The constrained maximisation of Eq. (4) gives the first order conditions for an interior solution, which are used together with the individual lifetime budget constraint to obtain, respectively, fertility and saving as follows:

$$n_t = \frac{\gamma w_t (1-\tau-\pi_{t-1} d)}{(1+\pi_t \beta+\gamma)\left(e-\pi_t d \frac{w^e{}_{t+1}}{1+r^e{}_{t+1}}\right)}, \tag{5.1}$$

$$s_t = \frac{w_t (1-\tau-\pi_{t-1} d)}{(1+\pi_t \beta+\gamma)\left(e-\pi_t d \frac{w^e{}_{t+1}}{1+r^e{}_{t+1}}\right)}\left[\pi_t \beta e-(\pi_t \beta+\gamma)\pi_t d \frac{w^e{}_{t+1}}{1+r^e{}_{t+1}}\right], \tag{5.2}$$

where $\tau+\pi_{t-1} d<1$.

A continuum of identical firms act competitively on the market. Aggregate production at time t (Y_t) takes place by combining capital (K_t) and labour ($L_t = N_t$ in equilibrium) through the constant returns to scale Cobb-Douglas technology $Y_t = A K_t^{\alpha} L_t^{1-\alpha}$, where $A>0$ is a scale parameter and $0<\alpha<1$ the output elasticity of capital. By assuming that output is sold at unit price and taking r_t and w_t as given, the representative firm maximises profits with respect to capital and labour, that is $\max_{\{K_t, L_t\}} A K_t^{\alpha} L_t^{1-\alpha}-(r_t+\rho)K_t-w_t L_t$, where $0\leq\rho\leq 1$ is the rate of depreciation of capital. Profit maximisation, therefore, implies that marginal products of capital and labour equal the gross return to capital and the wage, respectively (see de la Croix and Michel, 2002; Barro and Sala-i-Martin, 2004), that is:

$$\alpha A k_t^{\alpha-1} = \rho + r_t, \tag{6.1}$$

$$(1-\alpha) A k_t^{\alpha} = w_t, \tag{6.2}$$

where $k_t := K_t / N_t$ is the stock of capital per young.[18]

3 Dynamics

Under perfect foresight, individuals today expect that the future values of both the interest and wage rates depend on the future value of the stock of capital (see, e.g., Michel and de la Croix, 2000; de la Croix and Michel, 2002), that is

[18] Note that in what follows we assume that capital fully depreciates at the end of every period, that is $\rho = 1$. This hypothesis is not unrealistic as we assume that one period consists of 30 years (see de la Croix and Michel, p. 338).

$$\begin{cases} 1+r^{e}_{t+1} = \alpha A k_{t+1}^{\alpha-1} \\ w^{e}_{t+1} = (1-\alpha) A k_{t+1}^{\alpha} \end{cases}. \tag{7}$$

Given Eq. (2) and knowing that population evolves according to $N_{t+1} = n_t N_t$, the equilibrium condition in the capital market reads as

$$n_t k_{t+1} = s_t . \tag{8.1}$$

Combining now Eqs. (5.1), (5.2) and (8.1), the equilibrium condition can be written as follows:

$$k_{t+1} = \frac{\pi(h_t)}{\gamma}\left\{\beta e - [\pi(h_t)\beta + \gamma] d \frac{w^{e}_{t+1}}{1+r^{e}_{t+1}}\right\}. \tag{8.2}$$

Moreover, by using Eqs. (1), (2), (7) and (8.2), and knowing that $\pi(h_t)$ can be expressed as a function of the time t stock of capital k_t, because $h_t = \tau w_t$ and w_t depends on the capital stock through Eq. (6.2), so that $\pi(h_t) = \pi(\tau(1-\alpha)Ak_t^{\alpha}) = \Pi(k_t)$,[19] the dynamic path of capital accumulation is described by the following first order nonlinear difference equation:

$$k_{t+1} = \frac{\mathrm{P}\,\Pi(k_t)}{\alpha\gamma + d(1-\alpha)\Pi(k_t)[\Pi(k_t)\beta + \gamma]}, \tag{9}$$

where $\mathrm{P} := \alpha\beta e > 0$, which can alternatively be stated as:

$$k_{t+1} = \frac{\mathrm{P}\left(1+\mathrm{B}k_t^{\alpha\delta}\right)\left(\pi_0 + \pi_1 \mathrm{B}k_t^{\alpha\delta}\right)}{\alpha\gamma\left(1+\mathrm{B}k_t^{\alpha\delta}\right)^2 + d(1-\alpha)\left(\pi_0 + \pi_1 \mathrm{B}k_t^{\alpha\delta}\right)\left[\beta\left(\pi_0 + \pi_1 \mathrm{B}k_t^{\alpha\delta}\right) + \gamma\left(1+\mathrm{B}k_t^{\alpha\delta}\right)\right]}, \tag{10}$$

where $\mathrm{B} := \Delta[\tau(1-\alpha)A]^{\delta} > 0$.

From Eq. (10) we have the following results about: (*i*) existence either of a unique steady state or multiple steady states (see Result 1, which is shown through numerical experiments reported in Figures 1, 2 and 5), (*ii*) non-monotonic local dynamics and the occurrence of a flip bifurcation (see Proposition 1), and (*iii*) endogenous fluctuations when either the gift rate varies (see Result 2, which is shown through numerical experiments reported in Figures 1, 3, 5 and 6) or the health tax varies (see Result 3, which is shown through numerical experiments reported in Figures 2 and 4).

Result 1 *(Steady states). The dynamic system described by Eq. (10) admits either a unique positive steady state* $\{\bar{k}\}$*, which may be locally stable or unstable, or three positive steady states* $\{\bar{k}_1, \bar{k}_2, \bar{k}_3\}$*, where* $\bar{k}_1 < \bar{k}_2 < \bar{k}_3$*, the first is locally stable, the second is locally unstable, the third may be locally stable or unstable.*

We note that Propositions 1 and Results 2 and 3 that follow are stated for a generic steady state $\bar{k}$ but hold in either cases of existence of a unique steady state and multiple steady states. In particular, Proposition 1 characterises the non-monotonic local dynamics and the occurrence of a generic flip bifurcation, while Results 2 and 3 describe the stability properties,

[19] In order to clarify the reasons why demographic variables (i.e., fertility and longevity) are endogenous in the model, we note that while fertility is chosen by individuals by equating (private) marginal benefits and marginal costs or raising an additional child, through the constrained maximisation of lifetime utility function Eq. (4), longevity is endogenous because public health spending is assumed to be a fraction of wage income (see Chakraborty, 2004), which, in turn, depends on the current period stock of capital through Eq. (6.2), so that longevity varies as the capital stock changes. We note that a strand of literature exists where longevity is endogenous as it depends either on private health spending (Chakraborty and Das, 2005; Leung and Wang, 2010), chosen by equating the (individual) marginal benefits and marginal costs of spending in health, or both public and private health spending (Bhattacharya and Qiao, 2007).

for either the unique steady state $\bar{k}$, when a unique regime of development does exist, or high steady state $\bar{k}_3$, when multiple regimes of development do exist.

Proposition 1 *(Non-monotonic local dynamics). Let*

$$d > \tilde{d}(\bar{k}) \qquad (11)$$

hold, where

$$\tilde{d}(\bar{k}) := \frac{\alpha\gamma\left(1+\mathrm{B}\bar{k}^{\alpha\delta}\right)^2}{(1-\alpha)\beta\left(\pi_0+\pi_1\mathrm{B}\bar{k}^{\alpha\delta}\right)^2}. \qquad (12)$$

Then, the dynamics described by Eq. (10) is non-monotonic.

Moreover, if (11) holds, then a flip bifurcation generically occurs when $J'(\bar{k}) = -1$.

Proof Define the right-hand side of Eq. (10) as $J(k)$. Then differentiating $J(k)$ with respect to k and evaluating it at the generic steady state $\bar{k}$, gives:

$$J'(\bar{k}) = \frac{\mathrm{M}\,\bar{k}^{\alpha\delta-1}\left[\alpha\gamma\left(1+\mathrm{B}\bar{k}^{\alpha\delta}\right)^2 - d(1-\alpha)\beta\left(\pi_0+\pi_1\mathrm{B}\bar{k}^{\alpha\delta}\right)^2\right]}{\left\{\alpha\gamma\left(1+\mathrm{B}\bar{k}^{\alpha\delta}\right)^2 + d(1-\alpha)\left(\pi_0+\pi_1\mathrm{B}\bar{k}^{\alpha\delta}\right)\left[\beta\left(\pi_0+\pi_1\mathrm{B}\bar{k}^{\alpha\delta}\right)+\gamma\left(1+\mathrm{B}\bar{k}^{\alpha\delta}\right)\right]\right\}^2}, \qquad (13)$$

where $\mathrm{M} := \mathrm{PB}\alpha\delta(\pi_1-\pi_0) > 0$.

Since $\mathrm{sgn}\{J'(\bar{k})\} = \mathrm{sgn}\left\{\alpha\gamma\left(1+\mathrm{B}\bar{k}^{\alpha\delta}\right)^2 - d(1-\alpha)\beta\left(\pi_0+\pi_1\mathrm{B}\bar{k}^{\alpha\delta}\right)^2\right\}$, then (11) is necessary and sufficient to have $J'(\bar{k}) < 0$. Moreover, (11) is a necessary condition for the existence of a flip bifurcation which generically occurs when $J'(\bar{k}) = -1$ *Q.E.D.*

From extensive numerical simulations, examples of which are reported in Figures 1, 3, 5 and 6, we may conclude that when thresholds effects of health investments on longevity exists ($\delta > 1$), a value of the gift rate corresponding to which the unique steady state $\bar{k}$ (or the high equilibrium $\bar{k}_3$, when multiple regimes of development are in existence) looses stability through a flip bifurcation, i.e. $J'(\bar{k}) = -1$, can exist for a given value of the health tax rate. Moreover, Figures 2 and 4 refer to the case of changes in the health tax rate for a given value of gift rate, showing that the high equilibrium $\bar{k}_3$ can as well loose stability through a flip bifurcation when the health tax rate raises.

The following Result 2 (3) summarises the behaviour of the phase map (Eq. 10) when the gift rate d (the health tax rate τ) varies. In particular, several numerical experiments (not reported here for economy of space) show that two threshold values of the gift rate (health tax rate) can exist such that $J'(\bar{k}) = -1$. When d (τ) raises from zero, therefore, we can observe an increase in complexity of the dynamic events described by the nonlinear difference equation (10), when, in particular, d (τ) is included in an intermediate range of values, as well as a decrease in complexity and a re-switch towards stability, when d (τ) becomes larger. This phenomenon can also be ascertained by looking at both the shape of the capital accumulation locus shown in Figures 1 and 5, which depict the cases when the gift rate increases for a given value of the health tax rate, and the shape of the capital accumulation locus shown in Figure 3, which depicts the opposite case when the health tax rate increases for a given value of the gift rate. In particular, when threshold effects in the longevity function exist, a rise in d (τ) affects in a non-trivial way the shape of the map Eq. (10) by shifting it vertically and downwards (horizontally and leftwards), while also causing period-doubling phenomena followed by period-halving.

Result 2 *(Endogenous fluctuations when d varies). If $\delta > 1$ and:*
(1) if $0 < d < \underline{d}$, then the dynamics described by Eq. (10) is convergent to $\bar{k}$;
(2) if $d = \underline{d}$, then a flip bifurcation generically occurs;
(3) if $\underline{d} < d < \bar{d}$, then the dynamics described by Eq. (10) is non-monotonic and divergent from $\bar{k}$;
(4) if $d = \bar{d}$ a reverse flip bifurcation generically occurs;
(5) if $\bar{d} < d < 1$, then the dynamics described by Eq. (10) is convergent to $\bar{k}$.

Result 3 *(Endogenous fluctuations when τ varies). If $\delta > 1$ and:*
(1) if $0 < \tau < \underline{\tau}$, then the dynamics described by Eq. (10) is convergent to $\bar{k}$;
(2) if $\tau = \underline{\tau}$, then a flip bifurcation generically occurs;
(3) if $\underline{\tau} < \tau < \bar{\tau}$, then the dynamics described by Eq. (10) is non-monotonic and divergent from $\bar{k}$;
(4) if $\tau = \bar{\tau}$ a reverse flip bifurcation generically occurs;
(5) if $\bar{\tau} < \tau < 1$, then the dynamics described by Eq. (10) is convergent to $\bar{k}$.

The economic reason for the results established in Proposition 1 and Result 2 (3) is the following: an increase in the share of wage income as a means of old-age backing (health tax rate) reduces the disposable income of the young workers. As a consequence, saving and capital accumulation decrease as a general equilibrium effect. Interestingly, however, for intermediate values of the capital stock, threshold effects of health investments on longevity are triggered (see Eq. 1) and, hence, the depressing role exerted by the gift rate (health tax rate) becomes stronger because longevity raises. This, in turn, is due to both a higher expected cost to each young, as caused by the increased length of life when old, and a reduced intrinsic value of the expected benefit received when old, because the retirement time is now larger and thus the need to consume in such a period increases. For further increases in the capital stock and thus in health spending, the change in longevity becomes negligible and the negative effect of the gift rate on capital accumulation reduces, as the two depressive channels discussed above are sterilised by the trend of longevity to approach towards a constant ceiling.

In order to illustrate Result 1 as well as Proposition 1 and Results 2 and 3, we now perform numerical simulations to show the evolution of the phase map Eq. (10) together with the process of birth and death of the steady states when the gift rate d (Figures 1 and 3) and the health tax rate τ (Figure 2) are alternatively varied. The parameter set, chosen for illustrative purposes, is the following: $A = 10$, $\alpha = 0.33$ (Gollin, 2002), $\beta = 0.6$ (Žamac, 2007), $\gamma = 0.05$, $\pi_0 = 0.2$, $\pi_1 = 0.95$, $\Delta = 1$, $\delta = 60$, $e = 1$, $\tau = 0.12$ (Figure 1) and $d = 0.28$ (Figure 2). In particular, Figure 1 (Figure 2) illustrates, for any given value of the health tax rate τ (the gift rate d), (*i*) the evolution of the phase map and the unique positive steady state (multiple steady states), (*ii*) the loss of stability of the unique (high) fixed point, (*iii*) the appearance of endogenous fluctuations, provided that the relationship longevity-health spending is highly nonlinear with threshold effects, and (*iv*) the re-switch towards stability when the gift rate d (the health tax rate τ) becomes larger.[20]

[20] See Section 4 for a detailed discussion of Figures 1 and 2.

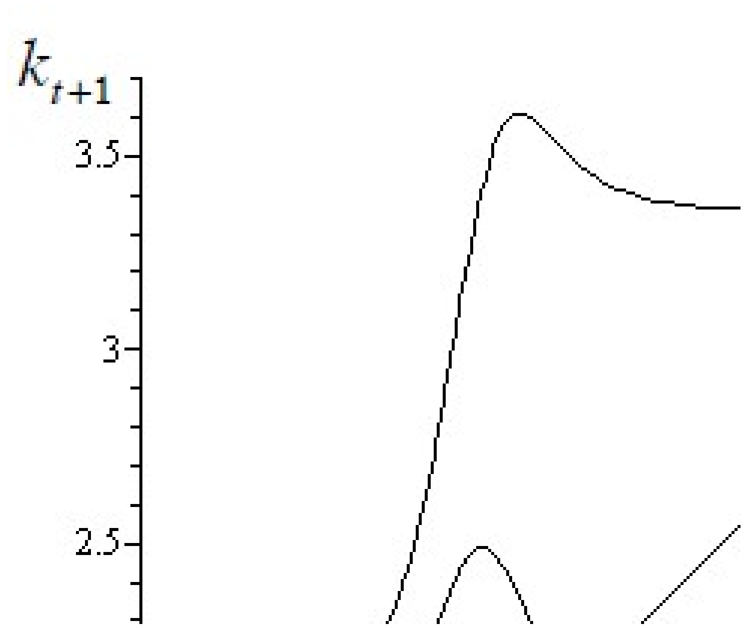

Fig. 1 Phase map and the unique steady state when d raises ($\tau = 0.12$)

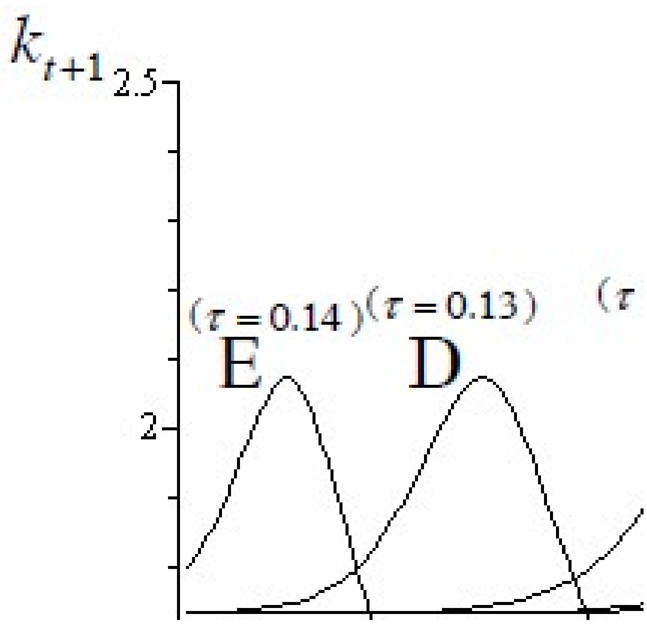

Fig. 2 Phase map and multiple steady states when τ raises ($d = 0.28$)

4 Chaotic dynamics under perfect foresight

We now investigate through numerical simulations[21] the local dynamic properties of the this simple economy by using the gift rate and the health tax rate as key bifurcation parameters. We show that deterministic chaos with bubbling phenomena in the case of both unique steady state and multiple steady states can be observed.

Changes in the gift rate (unique regime of development). In order to explain how a period-doubling cascade initiates and then reverses, it is useful to examine the picture of the phase

[21] We use the same parameter set as in Section 3.

map Eq. (10) shown in Figure 1 together with the corresponding bifurcation diagram (Figure 3), for different values of the parameter d. The capital accumulation locus has a flattened plateau region for small values of the capital stock as well as a plateau region that flattens out for large values of it. The effect of raising d is to translate vertically and downwards the curve as well as to reduce the plateau region in a stronger way for large values of the capital stock than for small values of it. Therefore, when the gift rate is continuously increased from zero, the fixed point is reduced and the slope of the curve at the intersection point with the 45° line reduces until the value -1 is reached (see curves A and B in Figure 1, where $d = 0.1$ and $d = 0.2$, respectively. The stable, with non-monotonic trajectories, steady state stock of capital is $\bar{k} = 3.361$ when $d = 0.1$ and the unstable, with non-monotonic trajectories, steady state is $\bar{k} = 2.113$ when $d = 0.2$).[22] A period-doubling bifurcation occurs at $d = \underline{d} = 0.195$. Beyond such a value, the fixed point becomes unstable and a stable two-period cycle, followed by a series of period-doubling cascades as d gradually raises, emerges (see curve C in Figure 1, where $d = 0.3$, and the corresponding bifurcation diagram depicted in Figure 3 in the range $0.195 < d < 0.315$. The steady state in this case is $\bar{k} = 1.903$). However, if we take the case of larger values of the gift rate into account, the equilibrium point turns out to become stable because it lies in the plateau region where the slope of the capital accumulation locus is approximately zero (see curve D in Figure 1, where $d = 0.4$. The steady state in this case is $\bar{k} = 1.584$, which is stable with monotonic trajectories). Indeed, as d is gradually reduced from 0.4, the slope of the map Eq. (10) continuously decreases until reaching -1, corresponding to which a period-doubling bifurcation emerges at $d = \bar{d} = 0.338$. Therefore, similar to the case discussed above, the fixed point becomes unstable, then a stable two-cycle emerges and for further reductions in d a series of period-doublings results. To sum up, as the gift rate increases from zero, the period-doubling route to chaos is eventually followed, at least for large enough values of d, by period-halving.

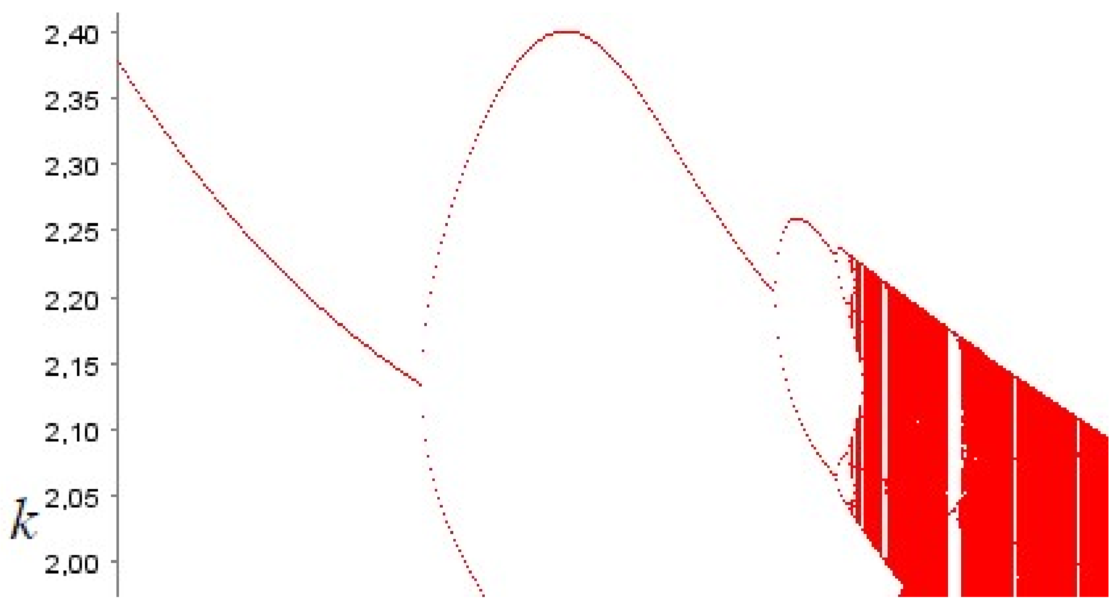

Fig. 3 Bifurcation diagram for d ($\tau = 0.12$). Enlarged view for $0.16 < d < 0.4$ and $1.6 < k < 2.43$ ($k_0 = 1$ is the initial value of the stock of capital)

[22] Note that Chakrabarti (1999) uses $d = 0.1$ as well as higher values of it, such as $d = 0.7$ and $d = 0.8$, in several numerical experiments.

Changes in the health tax rate (multiple regimes of development). We now turn to the analysis of the dynamic effects of the health tax rate τ, and examine the evolution of the phase map depicted in Figure 2, together with the corresponding bifurcation diagram (Figure 4), for increasing values of τ and for a given value of the gift rate ($d = 0.28$). As shown in Figure 2, the effect of a rise in τ is to translate the phase map horizontally and leftwards. Therefore, when the health tax rate is continuously increased from zero, both the unique steady state and the slope of the branch of the map at the intersection point with the 45° line are initially increased. Then, a rise in τ causes multiple (three) equilibria to appear (only both the low and high steady states are locally stable), also allowing for the possibility to permanently escape from poverty because only the high equilibrium eventually exists. Of course, the slope of the capital accumulation locus at the steady state reduces and a period-doubling bifurcation occurs: the high fixed point therefore becomes unstable and a stable two-period cycle, followed by a series of period-doubling cascades, occurs as τ is gradually raised. If we let the health tax rate increase further on, a horizontal shift in the phase map is observed, so that the slope of the map at the steady state is approximately zero, and thus a period-halving and a subsequent re-switch towards stability necessarily occur. Until when the health tax rate is fairly low (public health investments are relatively scarce), only the low equilibrium exists, where saving is discouraged by high mortality rates (see curve A in Figure 2, where $\tau = 0.113$ and the unique stable, with monotonic trajectories, steady state is $\bar{k}_1 = 1.742$). When τ slightly increases ($\tau = 0.115$, see curve B in Figure 2), multiple steady states appear and both the low $\bar{k}_1 = 1.775$ and high $\bar{k}_3 = 2.075$ equilibria are stable with monotonic trajectories. Indeed, the convergence towards each of them depends on initial conditions. Then, we observe the escape from poverty if we let τ increase further on (see curve C in Figure 2, where $\tau = 0.12$ and $\bar{k}_3 = 1.933$). The high equilibrium experiences further reductions (even beyond the value that existed when the tax rate was lower) as long as the health tax rate is raised by the government (see curves D and E in Figure 2).

Indeed, it is important to note that the rise in τ beyond a certain threshold allows to permanently escape from poverty (in the example depicted in Figure 2, such a threshold is $\tau = 0.1145$; see curves A and B in Figure 2 and the "jump" from the low equilibrium to the high equilibrium observed in the bifurcation diagram depicted in Figure 4), however, a rise in τ causes the loss in the low equilibrium, while also implying regular business cycles around the unique high steady state (a flip bifurcation occurs at $\tau = \underline{\tau} = 0.116$), which is however lower than when the health tax rate was lower (see curves C and D in Figure 2, where $\tau = 0.12$ and $\tau = 0.13$, respectively). Further increases in τ cause endogenous fluctuations to occur almost until $\tau = 0.123$; higher values of τ imply that the period-doubling bifurcation process curtails and then reverses, thus giving rise to period-halving bifurcations and eventually (beyond $\tau = \bar{\tau} = 0.1333$) a re-switch towards the stable equilibrium $\bar{k}_3$ which is, however, lower than before ($\bar{k}_3 = 1.57$, while the unique equilibrium $\bar{k}_1$ before the rise in τ had caused the appearance of the high equilibrium was $\bar{k}_1 = 1.78$).

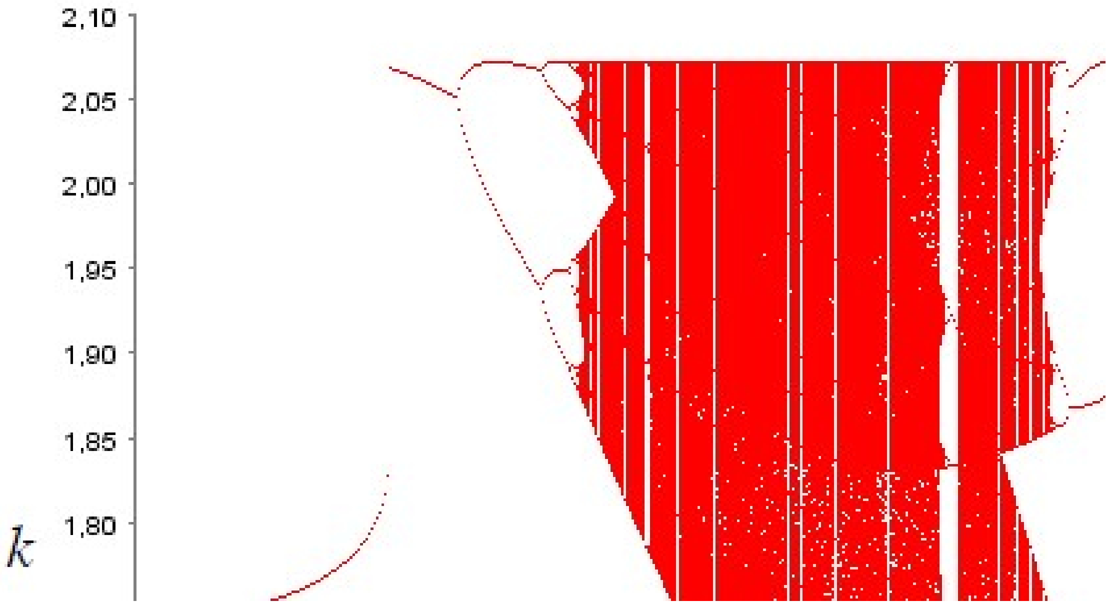

Fig. 4 Bifurcation diagram for τ ($d = 0.28$). Enlarged view for $0.1125 < \tau < 0.1375$ and $1.45 < k < 2.1$ ($k_0 = 1$ is the initial value of the stock of capital)

Changes in the gift rate (multiple regimes of development). It is important to note that when threshold effects in the longevity function becomes higher ($\delta = 80$), the change in the gift causes the birth and death of multiple steady states, as shown in Figures 5 and 6 which depict the evolution of the phase map Eq. (10) and the corresponding bifurcation diagram, respectively. The dynamic phenomena observed in this example are similar to those previously described.

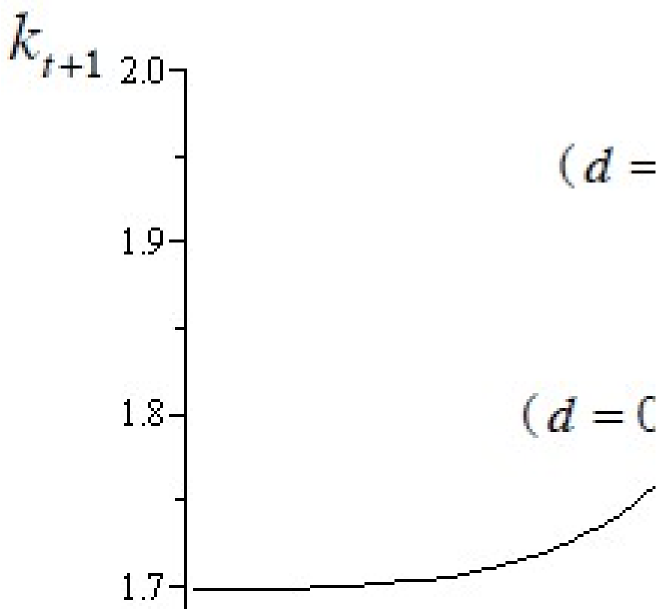

Fig. 5 Phase map and multiple steady states when d raises ($\tau = 0.12$)

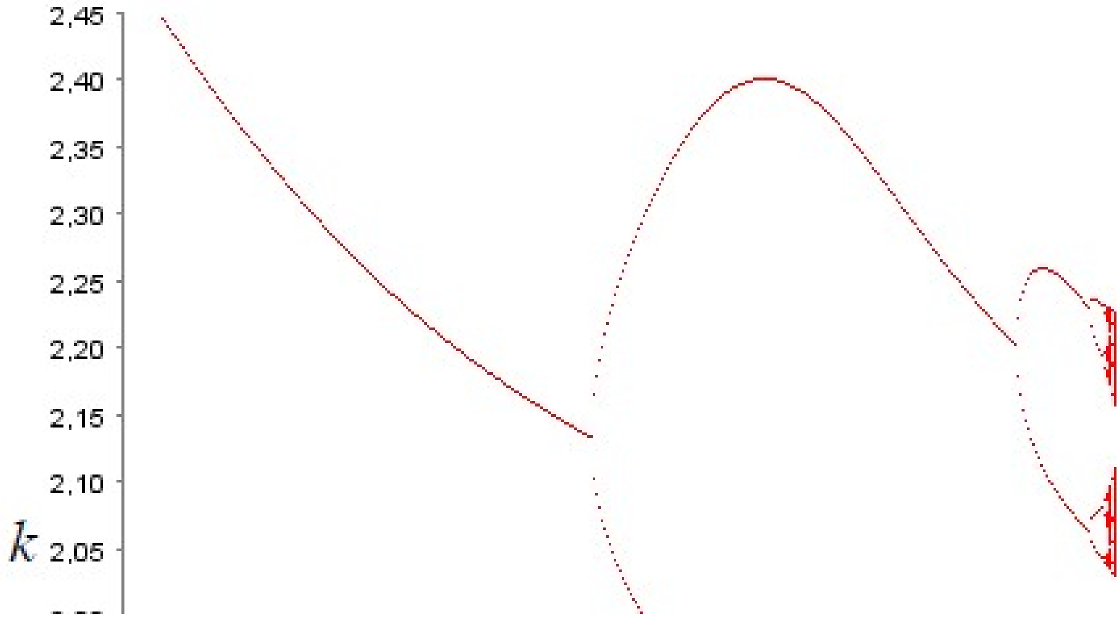

Fig. 6 Bifurcation diagram for d ($\tau = 0.12$). Enlarged view for $0.15 < d < 0.35$ and $1.6 < k < 2.45$ ($k_0 = 1$ is the initial value of the stock of capital)

The present analysis, therefore, has shown the remarkable complicated role of either a public health policy, through the effects on longevity, or transfers from children to parents, while also suggesting that the equilibrium dynamics may jointly explain, through the existence of the generic bubbling phenomenon, periodic and aperiodic dynamics, the existence of multiple regimes of development and the observed global stability of capitalist economies.

5 Conclusions

This paper examined how private intra-family transfers as a means of old-age backing (Raut and Srinivasan, 1994; Chakrabarti, 1999; Morand, 1999) and public investments in health (Chakraborty, 2004) affect the dynamics of a one-dimensional overlapping generations economy with endogenous demography (namely, fertility and longevity) and competitive markets under perfect foresight. Indeed, over the last decades the reduction in adult mortality as well as the rise in health spending represented two of the most important events that caused macroeconomic effects in several (underdeveloped, developing and developed) countries around the world. Therefore, the study of economic model that take fertility and longevity as endogenous variables into account may be highly valuable, especially for a policy perspective.

We showed that: (*i*) a unique regime or multiple regimes of development can occur as either the health tax rate (to finance public health expenditure) or gift rate (to voluntarily transfer resources from the young generation to the old generation) varies, and (*ii*) complex dynamics and bubbling phenomena can occur. Although, it is well known that overlapping generations models can indeed show unusual (i.e., non-monotonic and complex) behaviour in an endowment economy with or without money (see Kehoe and Levine, 1990), or in an economy with production externality or imperfect competition (see Benhabib and Farmer, 1999), no theoretical works explore, at the best of our knowledge, the dynamic effects of endogenous lifetime à la Chakraborty (2004) in a general equilibrium context à la Diamond (1965), where a private system of intra-family gifts from young to old people also exists. We found that the rise in public health investments, which is one the key determinants of adult mortality decline, may initially act as a destabilising device and cause deterministic chaos, while eventually acting as an economic stabiliser. Moreover, as regards the effects of private intra-family transfers, we found that the economy converges towards a stable equilibrium with monotonic trajectories when the gift rate is fairly low, while observing non-monotonic convergent dynamics and even deterministic chaos when the gift rate is included in an intermediate range of values. Interestingly, however, the equilibrium turns out to be stable with monotonic trajectories because of the existence of period-halving phenomena when the gift rate becomes larger.

Our findings suggest that the equilibrium dynamics of an overlapping generations growth model can endogenously reconcile the existence of periodic dynamics (cycles) or complex dynamics (chaos) and global stability of the long-run equilibrium of an overlapping generations economy when either a policy parameter or behavioural parameter varies.

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