Distributed Energy-Efficient Power and Subcarrier Allocation for OFDMA-Based Small Cells

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Abstract—In this work, we derive a distributed resource allocation scheme for the uplink of an OFDMA-based small-cell network. The mobile terminals are modeled as utility-driven rational agents that aim at maximizing the number of bits correctly delivered at destination per unit of energy consumed, under minimum-rate constraints. The theoretical analysis of the underlying game equilibrium is exploited to derive an iterative and distributed algorithm that allows each terminal to select its optimal power allocation over subcarriers. Extensive simulations show that the proposed technique is able to properly allocate the resources across the network in a scalable and adaptive manner, while improving the performance of each user in terms of energy efficiency compared to an iterative waterfilling criterion.

I. Introduction

The data traffic in wireless networks has experienced a tremendous growth in the past couple of decades. At the same time, energy consumption and energy pollution of the Information and Communication Technology (ICT) industry are becoming main societal and economical concerns [1]. Although the ICT contribution to the global emissions still is and will probably remain a rather small percentage of the global figures (approximately 2%), the general trend of a 10% yearly increase in ICT-related carbon emission is alarming. Reducing the energy consumption is not only a matter of being environment-aware, it is also very much an economically important issue, especially from the standpoint of operators. Hence, more network capacity on the one hand, and less energy consumption on the other, are the seemingly contradictory future requirements on ICT. This has stimulated an intense research activity in both academia and industry in an innovative research area (recently spurred out by the SMART 2020 report [2] and the GreenTouch consortium [3]) called *green cellular networks* [4], whose ultimate goal is to design the architectures and technologies needed to meet the explosive growth in cellular data demand while reducing energy consumption.

A promising solution in this direction is the so-called *small-cell network* technology [5], based on the idea of a very dense deployment of operator-installed low-cost and low-power base stations endowed with multiple antennas and equipped with advanced auto-configuration and self-organization capabilities.

Operationally, small-cell networks could automatically be integrated into existing macro-cellular networks used to ensure wide-area coverage with small cells carrying most of data traffic. Despite its promise, the deployment of small-cell networks poses several technical challenges, mainly because small cells are likely to be connected via an unreliable backhaul infrastructure whose features may strongly vary from case to case, with variable characteristics of error rate, delay, and capacity. A possible solution in this direction is to find good signal processing techniques that approach the ideal cooperative gains while relaying on mostly local channel state information and local user data. This problem is referred to as *distributed cooperation* and is as challenging as important. One of the key frameworks to induce distributed cooperation among rational entities such as the small-cell terminals is provided by *game theory* [6], both in the cooperative and in the noncooperative formulation (see for example [7], [8] and references therein).

In this work, we exploit a noncooperative game-theoretic formulation to identify an efficient and scalable resource allocation scheme that can be used by the small-cell users to share the common available resources. In particular, we focus on the uplink of an OFDMA-based small-cell network, in which it is of paramount importance to properly allocate the resources so as to exploit the frequency diversity [9], in order to achieve some desired quality of service (QoS) requirements while keeping the interference at a tolerable level. Dealing with battery-powered mobile terminals, we also include in the problem formulation the concept of *link capacity per unit cost*, widely adopted in many different contexts [10], [11]. This is achieved by modeling the small-cell terminals as rational agents that engage in a noncooperative game, using their own local information to select their optimal subcarriers and powers so as to maximize their achievable rates per unit of power (including nonradiative ones) consumed, while satisfying some minimum-rate constraints.

II. System and Signal Model

We consider the uplink of a network composed by S small cells operating in an OFDMA-based open-access licensed spectrum.[1] The sth small cell uses a set of orthogonal subcar-

[1]The following notation is used throughout the paper. Matrices and vectors are denoted by boldface letters, $\mathbf{I}_L$, $\mathbf{0}_L$, and $\mathbf{1}_L$ are the $L \times L$ identity matrix, the $L \times 1$ all-zero column vector, and the $L \times 1$ all-one column vector, resp., and $\| \cdot \|^2$, $(\cdot)^T$ and $(\cdot)^H$ denote Euclidean norm of the enclosed vector, transposition and Hermitian transposition, resp. The notation $(x)^+$ stands for $\max(0, x)$, $W(\cdot)$ denotes the Lambert function [12], and $\mathbb{R}_+$ denotes the set of positive real numbers (including 0).

riers to serve the K_s user equipments (UEs) falling within its coverage radius ρ_s. For simplicity, we assume that the same set of subcarriers $\mathcal{N} = \{1, \ldots, N\}$ is used by all small cells. The latter is assigned by the macro-cell network and thus does not represent a parameter of our optimization problem. To exploit the frequency diversity, we assume the subcarrier spacing to be larger than the coherence bandwidth B_c experienced by each user. Each small-cell access point (SAP) is equipped with M receiving antennas, whereas a single antenna is employed at the UEs to keep the complexity of the front-end limited.

Let $\mathbf{h}_{kj,n} \in \mathbb{C}^{M \times 1}$ denote the uplink channel vector whose entries $[\mathbf{h}_{kj,n}]_m$ represent the (frequency) channel gains over subcarrier n from the jth UE to the mth receive antenna of user k's serving SAP, i.e., the sth SAP whose distance from user k is smaller than ρ_s with $k, j \in \mathcal{K} = \{1, \ldots, K\}$ and $K = \sum_{s=1}^{S} K_s$. The vector $\mathbf{x}_{k,n} \in \mathbb{C}^{M \times 1}$ collecting the samples received at the UE k's serving SAP over the nth subcarrier can be written as

$$\mathbf{x}_{k,n} = \sum_{j=1}^{K} \mathbf{h}_{kj,n} \sqrt{p_{j,n}} z_{j,n} + \mathbf{w}_{k,n} \qquad (1)$$

where $\mathbf{w}_{k,n} \in \mathbb{C}^{M \times 1}$ is a Gaussian vector with zero mean and covariance matrix $\sigma^2 \mathbf{I}_M$ accounting for background noise, whereas $p_{j,n}$ and $z_{j,n}$ denote UE j's transmit power and data symbol over subcarrier n, respectively. To keep the complexity of the SAP at a tolerable level, a simple linear detection scheme is employed for data detection. This means that the entries of $\mathbf{x}_{k,n}$ are linearly combined to form $y_{k,n} = \mathbf{g}_{k,n}^H \mathbf{x}_{k,n}$, where $\mathbf{g}_{k,n}$ is the vector employed for recovering the data transmitted by user k over subcarrier n. The signal-to-interference-plus-noise ratio (SINR) achieved by user k at its serving SAP over subcarrier n takes the form

$$\gamma_{k,n} = \mu_{k,n}(\mathbf{p}_{\backslash k,n}) p_{k,n} \qquad (2)$$

with

$$\mu_{k,n}(\mathbf{p}_{\backslash k,n}) = \frac{|\mathbf{g}_{k,n}^H \mathbf{h}_{kk,n}|^2}{\|\mathbf{g}_{k,n}\|^2 \sigma^2 + \sum_{j \neq k} |\mathbf{g}_{k,n}^H \mathbf{h}_{kj,n}|^2 p_{j,n}} \qquad (3)$$

where we have explicitly reported the dependence on $\mathbf{p}_{\backslash k,n} = [p_{1,n}, \ldots, p_{k-1,n}, p_{k+1,n}, \ldots, p_{K,n}]^T$, which is the vector collecting all powers transmitted over subcarrier n except user k's one. Using (2), the achievable rate (normalized to the subcarrier bandwidth, and thus measured in b/s/Hz) of the kth user is given by

$$r_k(\mathbf{P}) = \sum_{n=1}^{N} \log_2 (1 + \gamma_{k,n}/\Gamma) \qquad (4)$$

where Γ is the SINR gap with respect to the Shannon capacity [13], and $\mathbf{P} = [\mathbf{p}_1^T, \ldots, \mathbf{p}_K^T]^T \in \mathbb{R}_+^{K \times N}$ collects the transmit powers by all users over all subcarriers, where the (row) vector $\mathbf{p}_k = [p_{k,1}, \ldots, p_{k,N}]$ denotes user k's powers over all subcarriers, with $p_{k,n} \geq 0$ (if $p_{k,n} = 0$, user k is not transmitting over subcarrier n). Note that user k's multiple access interference (MAI), measured by the summation at the denominator of (3), comes from both intra-cell interference (generated by other UEs being served by the same SAP) and inter-cell interference (from UEs served by all other $S - 1$ SAPs), whereas macro-cell users are assumed to be orthogonal thanks to a proper frequency resource planning operated by the macro-cell network (if needed, macro-cell interference can be included into σ^2). For simplicity of notation, the dependence of $\mu_{k,n}$ and r_k on others' powers is neglected from now on.

III. GAME-THEORETIC RESOURCE ALLOCATION

As mentioned in Sect. I, an *energy-efficient* design of the network, that is of primary importance when dealing with mobile, battery-power UEs, must properly take into account the energy consumption incurred by each UE. To this aim, it is worth noting that, beside the radiative powers $\mathbf{p}_k$ at the output of the radio-frequency front-end, each terminal k also incurs circuit power consumption during transmission, mostly due to the power dissipated in the power amplifier [10]. The overall power consumption $P_{T,k}$ of the kth UE is thus given by $P_{T,k} = p_c + P_k$, where $P_k = \sum_{n=1}^{N} p_{k,n} = \mathbf{p}_k \mathbf{1}_N$ is the radiative power consumed by user k over the whole spectrum, and p_c represents the average current power consumed by the device electronics, which is assumed to be independent of the transmission state and equal for all UEs. Following [10], [14], the energy efficiency of the link can be measured (in b/J/Hz) by the utility function

$$u_k(\mathbf{P}) = \frac{r_k}{P_{T,k}} = \frac{\sum_{n=1}^{N} \log_2 (1 + \mu_{k,n} p_{k,n}/\Gamma)}{p_c + \sum_{n=1}^{N} p_{k,n}} \qquad (5)$$

where the dependence of all others' transmit powers over all subcarriers is collected by the gains $\{\mu_{k,n}\}_{n=1}^{N}$. Observe that, in data-oriented wireless networks, users are usually required to satisfy QoS requirements in terms of minimum achieved rates $\theta_k > 0$, i.e., $r_k \geq \theta_k$.

To summarize, the design of an energy-efficient resource allocation scheme, that encompasses both subcarrier allocation and power control (by setting, for each UE k, $p_{k,n} = 0$ on unused subcarriers, and $p_{k,n} > 0$ on used subcarriers), requires to solve, for each UE k, the following optimization problem:

$$\mathbf{p}_k^\star = \arg \max_{\mathbf{p}_k \in \mathbb{R}_+^N} \frac{\sum_{n=1}^{N} \log_2 (1 + \mu_{k,n} p_{k,n}/\Gamma)}{p_c + \sum_{n=1}^{N} p_{k,n}} \qquad (6)$$

$$\text{subject to} \quad p_{k,n} \geq 0 \quad \forall n = 1, \ldots, N \qquad (7)$$

$$\sum_{n=1}^{N} \log_2 (1 + \mu_{k,n} p_{k,n}/\Gamma) \geq \theta_k \qquad (8)$$

where the constraint (7) ensures each transmit power to be positive, whereas (8) forces each user to fulfill a requirement on the minimum normalized rate θ_k. Note that, unlike other formulations in the field of OFDMA resource allocation (e.g., [15], [16]), here the problem is tackled in a *joint* manner, by simultaneously addressing the subcarrier selection and the power loading among the K users (or the power allocation over the available subcarriers problem). Furthermore, the interplay among the UEs in $\mathcal{K}$ makes (6) a multidimensional optimization problem in which each UE $k \in \mathcal{K}$ aims at unilaterally choosing its own transmit power allocations $\mathbf{p}_k$ so as to optimize its own link energy efficiency $u_k(\mathbf{P})$. In doing this, each UE affects the choice of all other UEs as well.

The natural framework to study this kind of interactions is offered by the framework of non-cooperative game theory [6]. In particular, the underlying game $\mathcal{G}$ played by the UEs is defined as the tuple $\mathcal{G} = [\mathcal{K}, \{\mathcal{P}_k\}, \{u_k(\mathbf{P})\}]$, in which $\mathcal{K}$ is the player set; $\mathcal{P}_k \subseteq \mathbb{R}_+^N$ denotes the strategy set for which the constraints (7)-(8) are satisfied; and $u_k(\mathbf{P})$ is player k's payoff function defined in (5). Note that user k's action set depends on the actions of the other players, i.e., $\mathcal{P}_k = \mathcal{P}_k(\mathbf{P}_{\backslash k})$, where $\mathbf{P}_{\backslash k} = \mathbf{P} \backslash \mathbf{p}_k = [\mathbf{p}_{\backslash k,1}, \ldots, \mathbf{p}_{\backslash k,N}]$ is the power matrix $\mathbf{P}$ excluding the kth row $\mathbf{p}_k$, because of the rate constraint

$r_k \geq \theta_k$. In this case, the solution concept to be used is the generalized Nash equilibrium (GNE) [17], defined as follows.

Definition 1: Formally, a transmit power allocation $\mathbf{P}^\star = [(\mathbf{p}_1^\star)^T, \ldots, (\mathbf{p}_K^\star)^T]^T$ is a GNE of the game $\mathcal{G}$ if, for all players $k \in \mathcal{K}$, we have that

$$u_k(\mathbf{P}^\star) \geq u_k(\check{\mathbf{P}}_k) \tag{9}$$

for all powers $\mathbf{p}_k \in \mathcal{P}_k$ that meet the constraints (7)-(8), where $\check{\mathbf{P}}_k = [(\mathbf{p}_1^\star)^T, \ldots, (\mathbf{p}_{k-1}^\star)^T, (\mathbf{p}_k)^T, (\mathbf{p}_{k+1}^\star)^T, \ldots, (\mathbf{p}_K^\star)^T]^T$ (i.e., it differs from the matrix $\mathbf{P}^\star$ only for the kth row). ∎

The GNE is of particular interest in the context of distributed algorithms since it offers a predictable outcome of the game in which multiple agents (in this case, the small-cell UEs) with conflicting interests compete through self-optimization and reach a stable equilibrium point.

Proposition 1: If the problem (6) is feasible, i.e., if there exists a power allocation $\mathbf{P}^{\text{req}}$, with elements $0 \leq p_{k,n}^{\text{req}} < \infty$ for all $k \in \mathcal{K}$ and all $n \in \mathcal{N}$, such that the QoS constraint (8) is met with equality for all $k \in \mathcal{K}$, then there exists at least one power allocation $\mathbf{P}^\star$ that is a GNE of the game $\mathcal{G}$. The elements $p_{k,n}^\star$ of the matrix $\mathbf{P}^\star$ are the solutions to the following fixed-point system of equations:

$$p_{k,n}^\star = \begin{cases} \mathsf{br}(\mathbf{p}_{k,\backslash n}^\star, \mathbf{P}_{\backslash k}^\star) & \text{if } r_k(\tilde{\mathbf{P}}_{k,n}) \geq \theta_k \\ \mathsf{wf}(\mathbf{P}_{\backslash k}^\star) & \text{if } r_k(\tilde{\mathbf{P}}_{k,n}) < \theta_k \end{cases} \tag{10}$$

where $\mathbf{p}_{k,\backslash n}^\star = [p_{k,1}^\star, \ldots, p_{k,n-1}^\star, p_{k,n+1}^\star, p_{k,N}^\star]$, and $\tilde{\mathbf{P}}_{k,n} = [(\mathbf{p}_1^\star)^T, \ldots, (\mathbf{p}_{k-1}^\star)^T, (\tilde{\mathbf{p}}_{k,n})^T, (\mathbf{p}_{k+1}^\star)^T, \ldots, (\mathbf{p}_K^\star)^T]^T$, with $\tilde{\mathbf{p}}_{k,n}$ denoting the tentative power vector with nth component $\tilde{p}_{k,n} = \mathsf{br}(\mathbf{p}_{k,\backslash n}^\star, \mathbf{P}_{\backslash k}^\star)$, and $\tilde{p}_{k,\ell} = p_{k,\ell}^\star$ for any $\ell \neq n$; the *best-response* operator $\mathsf{br}(\cdot)$ is defined as

$$\mathsf{br}(\mathbf{p}_{k,\backslash n}, \mathbf{P}_{\backslash k}) = \begin{cases} 0 & \alpha_{k,n} < \beta_{k,n} \\ \frac{\Gamma}{\mu_{k,n}}[f(\alpha_{k,n}, \beta_{k,n}) - 1] & \alpha_{k,n} \geq \beta_{k,n} \end{cases} \tag{11}$$

where $f(\alpha_{k,n}, \beta_{k,n}) = e^{W((\alpha_{k,n}-1)\cdot e^{(\beta_{k,n}-1)}) - (\beta_{k,n}-1)}$ with

$$\alpha_{k,n} = \frac{\mu_{k,n}}{\Gamma}\left(p_c + \sum_{\ell \neq n} p_{k,\ell}\right) \tag{12}$$

$$\beta_{k,n} = \sum_{\ell \neq n} \ln(1 + \mu_{k,\ell} p_{k,\ell}/\Gamma) \tag{13}$$

and the *waterfilling* (WF) operator $\mathsf{wf}(\cdot)$ is defined as

$$\mathsf{wf}(\mathbf{P}_{\backslash k}) = \left(\nu_k - \frac{\Gamma}{\mu_{k,n}}\right)^+ \tag{14}$$

with the water level ν_k given by

$$\nu_k = \Gamma \cdot \sqrt[N]{e^{\theta_k \ln 2} \Big/ \prod_{\ell=1}^N \mu_{k,\ell}}. \tag{15}$$

Proof: The proof is briefly outlined in the Appendix. ∎

Remark 1: Similarly to [11] in which the authors deal with the same problem in (6) with no minimum rate constraints (i.e., $\theta_k = 0$ for all $k \in \mathcal{K}$), the GNE of the proposed game might *not* be unique. In particular, the fixed-point system of equations (10) might lead to more than one solution when the channel

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Algorithm 1 Iterative algorithm to solve problem (6).

set $t = 0$.
initialize $\mathbf{p}_k[t] = \mathbf{0}_N$ for all users $k \in \mathcal{K}$
repeat
  update $t = t + 1$.
  for $k = 1$ to K **do**
    {loop over the users}
    for $n = 1$ to N **do**
      {loop over the carriers}
      update $p_{k,n}[t]$ according to (11) {meeting (7)}
    end for
    if $\sum_{n=1}^N \log_2(1 + \gamma_{k,n}[t]/\Gamma) < \theta_k$ **then**
      apply Algorithm 2 {meeting (8)}
    end if
  end for
until $\mathbf{p}_k[t] = \mathbf{p}_k[t-1]$ for all $k \in \mathcal{K}$

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realizations among the users are particularly unbalanced. Further work is needed to formalize a condition under which the GNE of the game $\mathcal{G}$ is unique.

Remark 2: Unlike what we did for the single-carrier case in [18], a necessary and sufficient condition under which the optimization (6) is feasible (in the sense of Prop. 1) is hard to obtain. Further work is needed to fulfil this lack. Using the properties of non-negative matrices [19], only sufficient conditions (very loose in the practice) can be provided. A possible route to follow might be that of extending the Perron-Frobenius theorem illustrated in [19] to the problem at hand taking into account that an efficient subcarrier allocation scheme turn off those subcarriers experiencing deeply faded channels. In this work, the feasibility of the optimization problem is only assessed a-posteriori simply by letting each player achieve the minimum-rate constraint (8) with equality.

IV. DISTRIBUTED IMPLEMENTATION

To derive a practical criterion to let each small-cell UE $k \in \mathcal{K}$ reach the GNE of $\mathcal{G}$ in a distributed fashion, we start by assuming that the UEs with indices $j \neq k$ have already chosen their optimal transmit powers (i.e., in an asynchronous resource allocation scenario). This amounts to assuming $\mathbf{P}_{\backslash k} = \mathbf{P}_{\backslash k}^\star$. Hence, from (3), we have that the gains $\mu_{k,n}(\mathbf{p}_{\backslash k,n}^\star)$ needed to implement (10) can be obtained by

$$\mu_{k,n}(\mathbf{p}_{\backslash k,n}^\star) = \frac{\gamma_{k,n}}{p_{k,n}} \tag{16}$$

for all $n \in \mathcal{N}$. Otherwise stated, the only local information that is not available at the kth UE to compute the optimal powers $\{p_{k,n}^\star\}$ is the set of SINRs $\{\gamma_{k,n}\}$ measured at UE k's serving SAP. This can be fed back by the SAP with a modest feedback rate requirement on the return channel (a discussion on the impact of a limited feedback can be adapted to this specific scenario from [20]).

Based on the above considerations, we can derive an iterative and fully decentralized algorithm to be adopted by each UE k at each time step t to solve (10) with a low-complexity, scalable and adaptive procedure. The pseudocode for the whole network is summarized in Algorithm 1. Note that, in practice, each UE k needs only to implement the

Algorithm 2 Iterative waterfilling for user k.

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compute $\mathbf{q}_k$ using (17) {*sorting the power gains*}
set $n_a = N$
for $n = 1$ to N **do**
$\quad \tilde{\nu}_k = \Gamma \cdot \left\{ e^{\theta_k \ln 2} / \prod_{\ell=1}^{n} \mu_{k,q_{k,\ell}} \right\}^{1/n}$
$\quad$ **if** $\left(\tilde{\nu}_k < \Gamma / \mu_{k,q_{k,n}} \right)$ **then**
$\quad\quad n_a = n - 1$ {*computing the number of active carriers*}
$\quad\quad$ **break**
$\quad$ **else**
$\quad\quad \nu_k = \tilde{\nu}_k$ {*updating the water level*}
$\quad$ **end if**
end for
for $n = 1$ to n_a **do**
$\quad p_{k,q_{k,n}}[t] = \nu_k - \Gamma / \mu_{k,q_{k,n}}$
end for
for $n = n_a + 1$ to N **do**
$\quad p_{k,q_{k,n}}[t] = 0$
end for

Fig. 1. Random realization of a network with $S = 7$ small cells and $K = 28$ UEs, sharing $N = 12$ subcarriers in an OFDMA-based scenario.

steps enclosed in the inner cycle. For the sake of clarity, the WF algorithm is reported in Algorithm 2, in which n_a is the number of active carriers (a carrier n is active for user k if $p_{k,n} > 0$), and the vector $\mathbf{q}_k = [q_{k,1}, \ldots, q_{k,N}]$, with $1 \leq q_{k,n} \leq N$, is defined such that

$$\mu_{k,q_{k,1}} \geq \mu_{k,q_{k,2}} \geq \cdots \geq \mu_{k,q_{k,N}} \tag{17}$$

i.e., the power gains $\{\mu_{k,n}\}$ are sorted in a *descending* order (this can easily be done using standard sorting algorithms).

It is worth noting that the proposed iterative algorithm can be easily modified to accommodate further constraints on a per-subcarrier maximum power $\overline{p}_{k,n}$ (such that $p_{k,n} \leq \overline{p}_{k,n}$) and/or on a per-user maximum sum-power $\overline{P}_k$ (such that $\sum_{n=1}^{N} p_{k,n} \leq \overline{P}_k$). These additional constraints (usually required to ensure that power masks dictated by the standard are not violated) can be applied by introducing a further case on (11) (to account for $\overline{p}_{k,n}$) and a power-based WF after Algorithm 2 (that, although not reported for brevity, can be derived in a dual manner, to account for $\overline{P}_k$). Note that, similarly to the single-carrier case investigated in [18], introducing upper bounds on the transmit powers might impact on the existence of the GNE. This situation typically occurs when there is a UE that is too close to its serving SAP, and cannot thus reach its local optimum when the powers are limited. However, this can be effectively avoided by properly setting sufficiently high power limits and a reasonable forbidden drop radius within each small cell, as considered in Sect. V. Finally, note that this algorithm is suitable for a dynamic network configuration, in which UEs come and go: each UE just needs to stick to the steps listed in the inner cycle of Algorithm 1, that only require the SINRs fed back by the serving SAP, without any further information on the network and/or small-cell status.

V. SIMULATION RESULTS

In this section, we investigate the performance of Algorithm 1 by means of an extensive simulation campaign. Throughout the simulations, the following parameters are adopted. We consider $S = 7$ small cells with a radius $\rho_s = \rho = 20\,\mathrm{m}$ and randomly distributed over a $200\,\mathrm{m} \times 200\,\mathrm{m}$ area. Each SAP s, equipped with $M = 2$ receive antennas, serves $K_s = 4$
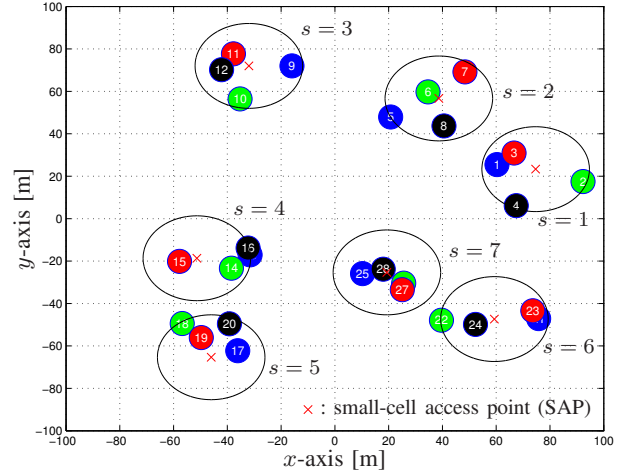
UEs, for a total of $K = 28$ UEs deployed in the network, assuming a forbidden disk radius equal to $0.2\,\mathrm{m}$. The set of available subcarriers is composed by $N = 12$ subcarriers, each having a bandwidth $B \approx 11\,\mathrm{kHz}$ and spaced by $100\,\mathrm{kHz}$, whereas each UE's coherence bandwidth is assumed to be $B_c \approx 90\,\mathrm{kHz}$, using a 24-tap channel model to reproduce multipath effects. To simulate the effects of fading and shadowing, we use a path-loss exponent equal to $\varsigma = 4$. For simplicity, perfect channel estimation is assumed at the receive side, and the maximum ratio combining (MRC) technique is considered, which amounts to setting $\mathbf{g}_{k,n} = \mathbf{h}_{kk,n}$ for all k and n. We also assume an SINR gap equal to $\Gamma = 1 = 0\,\mathrm{dB}$ and normalize all powers with respect to σ^2. In particular, we set $p_c/\sigma^2 = +20\,\mathrm{dB}$, $\overline{P}_k/\sigma^2 = +35\,\mathrm{dB}$ for all $k \in \mathcal{K}$, and $\overline{p}_{k,n}/\sigma^2 = +30\,\mathrm{dB}$ for all $k \in \mathcal{K}$ and for all $n \in \mathcal{N}$.

To evaluate the proposed algorithm in a practical setting, Fig. 1 reports a random realization of the network with the parameters described above. Using the distributed algorithm described in Sect. IV, after 9 iterations we get the solution of the system (10), representing the optimal power allocation at the GNE of the game $\mathcal{G}$, and reported in Fig. 2. As seen, this method tends to allocate the subcarriers in an exclusive manner whenever the MAI across UEs within the same small cell is too large (e.g., see the 2nd small cell, in which only subcarrier 6 is shared by 3 users), and to share the same subcarrier when the MAI across users is at a tolerable level (which also includes the interference generated by UEs from neighboring cells), as occurs in the 3rd small cell. On the right hand side, we report the minimum rate constraints versus the achieved rates at the GNE (the unit b/s/Hz is not reported due to space constraints). As can be verified, only in 3 cases out of 28, the UEs transmit at their minimum required rates θ_k, while it is convenient for the others to increase their transmit power so as to obtain better performance in terms of energy efficiency. It is interesting to observe that most WF users are in the 7th cell, in which there is one UE much closer to the SAP than the others (see Fig. 1): in this case, it gets most of the resources (black bars are much smaller than the other colors, due to a much better channel realization), which translates into (*i*) obtaining a rate much higher than the minimum one, and (*ii*) forcing the others to
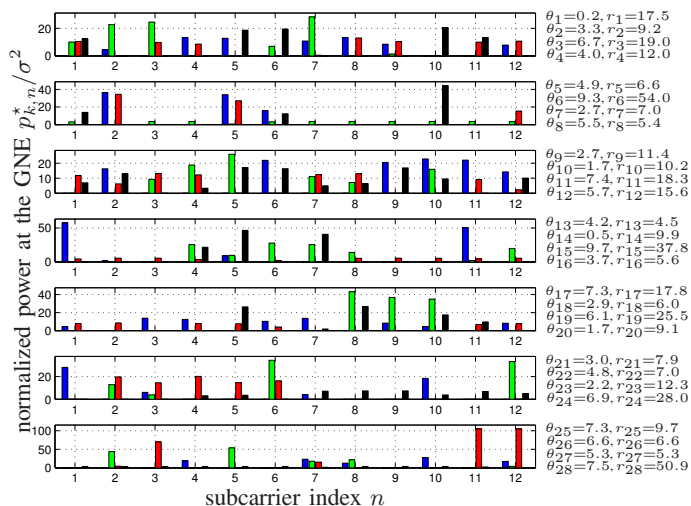
Fig. 2. Outcome of the resource allocation for the scenario of Fig. 1.

Fig. 3. Average normalized rate at the GNE as a function of θ_1.

Fig. 4. Average normalized utility at the GNE as a function of θ_1.

operate a WF so as to meet their constraints (8). Finally, note that maximum powers are not selected by any of the K users.

To evaluate the improvement in terms of energy efficiency of the proposed algorithm, we compare its performance with that achieved by a WF-based solution, in which all users aim at meeting θ_k with equality (in other words, each UE solves problem (6) when the equality in (8) holds, by using the WF criterion (14)). Fig. 3 reports the average rates obtained by averaging over all possible positions of a particular UE (say user 1) within a small cell, using $100,000$ independent realizations of a feasible network scenario,[2] whereas Fig. 4 depicts the average utility (normalized by the AWGN power σ^2) achieved at the GNE. Both results are plotted as a function of a specific minimum rate θ_1, while all others randomize their constraints θ_k for $k \neq 1$ in $[0, 20]$ [b/s/Hz].

As expected, WF users get exactly their demanded rate (Fig. 3), whereas energy-efficient users always achieve normalized rates from roughly 10 b/s/Hz on, due to a different optimization criterion. Interestingly, the energy efficiency achieved by users using (10) for larger θ_1 is lower than the one obtained by all users adopting the WF criterion. This result does not contradict the proposed formulation, as it is mainly due to a weaker MAI caused by the WF users, that transmit at lower powers than energy-efficient ones (not reported for the sake of brevity). To better illustrate this phenomenon, we also report the performance when only the observed user adopts the energy-efficient criterion (10), while all others use (14) (green curve). As can be seen, the energy efficiency achieved at the GNE is always higher than the WF-based one. To further support this conclusion, note that, averaging over all network realizations, the proposed algorithm achieves an average normalized utility of approximately 0.047, whereas the WF one obtains 0.042, also introducing fairness among the users, as its performance is weakly dependent on the QoS requirement θ_k. The drawback of the proposed technique is the larger convergence time as it requires about 14 iterations for its convergence, compared to roughly 7 steps employed
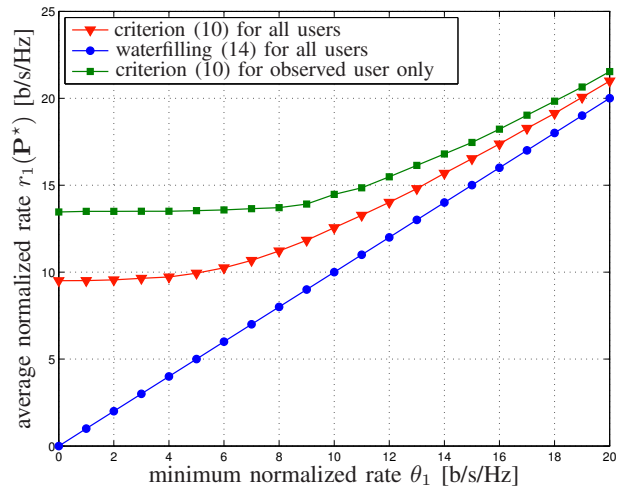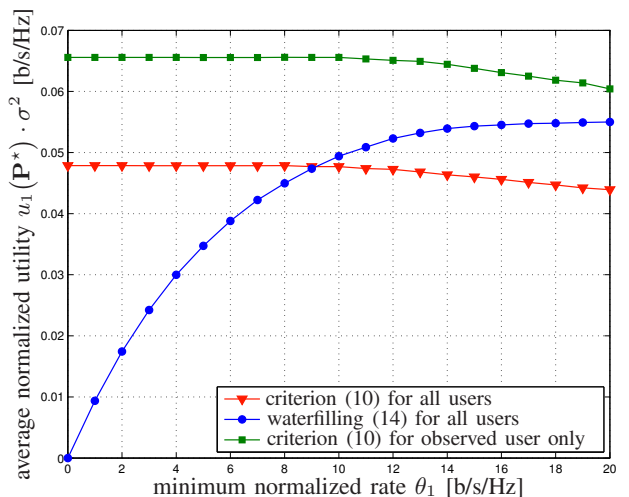
by the WF algorithm, while bearing a similar computational complexity at each step of the algorithm.

VI. CONCLUSIONS AND PERSPECTIVES

This work has investigated the problem of energy-efficient resource allocation for the uplink of an single-input multiple-output small-cell network, in which the users select their optimal subcarrier and power loading in a joint manner. The theoretical tools of noncooperative game theory are employed to solve the rate-constrained resource allocation problem, and to derive an iterative, decentralized and scalable algorithm. Numerical results are provided to show the effectiveness of the proposed solution and to evaluate the performance improvement with respect to an iterative waterfilling algorithm in terms of energy efficiency and fairness. Given the general formulation of the problem, this method can be applied to different contexts: macro-cell systems, relay-assisted communications, and cognitive networks, just to mention a few. Further work is needed to assess the feasibility of the problem given a particular network realization, and to measure the complexity of the algorithm as a function of the system parameters.

Appendix

Due to space limitations, only the main guidelines of the proof are given in the sequel. The GNE problem (6) is the multicarrier extension of the one investigated in [18], in which the case $N = 1$ is investigated for a relay-based cellular network. Similarly to [18], the existence of a GNE follows from the topology of the action set $\mathcal{P}_k(\cdot)$ [17], interpreted as a point-to-set mapping [21], jointly with the continuity and quasi-concavity properties of the payoff function $u_k(\cdot)$ as a function of $\mathbf{p}_k$ for all $k \in \mathcal{K}$.

To compute the optimal power allocation $\mathbf{P}^\star$ at the GNE, we must satisfy the Karush-Kuhn-Tucker (KKT) conditions [22]. To this aim, we can define the Lagrangian function

$$\mathcal{L}\left(\mathbf{p}_k, \boldsymbol{\xi}_k, \nu_k\right) = \frac{\sum_{n=1}^{N} \log_2\left(1 + \mu_{k,n} p_{k,n}/\Gamma\right)}{p_c + \sum_{n=1}^{N} p_{k,n}} + \sum_{n=1}^{N} \xi_{k,n} p_{k,n}$$
$$+ \nu_k \left(\sum_{n=1}^{N} \log_2\left(1 + \mu_{k,n} p_{k,n}/\Gamma\right) - \theta_k\right) \quad (18)$$

where $\boldsymbol{\xi}_k = \{\xi_{k,n}\}_{n=1}^{N}$ and ν_k are the KKT multipliers to include constraints (7)-(8).

Let us suppose that the constraints are not active (i.e., $\boldsymbol{\xi}_k = \mathbf{0}_N$, and $\nu_k = 0$). Hence, using (18) yields

$$\frac{\partial \mathcal{L}\left(\mathbf{p}_k, \boldsymbol{\xi}_k, \nu_k\right)}{\partial p_{k,n}} =$$
$$\frac{\frac{\mu_{k,n}/\Gamma}{1+\gamma_{k,n}/\Gamma}\left(p_c + \sum_{\ell=1}^{N} p_{k,\ell}\right) - \sum_{\ell=1}^{N} \ln\left(1 + \gamma_{k,\ell}/\Gamma\right)}{\ln 2 \cdot \left(p_c + \sum_{\ell=1}^{N} p_{k,\ell}\right)^2}. \quad (19)$$

Hence, when seeking the solution(s) of the equation $\partial \mathcal{L}\left(\mathbf{p}_k, \boldsymbol{\xi}_k, \nu_k\right)/\partial p_{k,n} = 0$, using (12), (13) and (19) we can write

$$\frac{\alpha_{k,n} + \gamma_{k,n}/\Gamma}{1 + \gamma_{k,n}/\Gamma} = \beta_{k,n} + \ln\left(1 + \gamma_{k,n}/\Gamma\right). \quad (20)$$

By manipulating on (20), we can finally get

$$p_{k,n} = \frac{\Gamma\left[f(\alpha_{k,n}, \beta_{k,n}) - 1\right]}{\mu_{k,n}}. \quad (21)$$

Let us now introduce back the constraint (7), and suppose that, for some subcarrier n, the constraint $p_{k,n} = 0$ is active. This means that $\xi_{k,n} > 0$. If we compute $\partial \mathcal{L}\left(\mathbf{p}_k, \boldsymbol{\xi}_k, \nu_k\right)/\partial p_{k,n}$, we get

$$\frac{\frac{\alpha_{k,n}+\gamma_{k,n}/\Gamma}{1+\gamma_{k,n}/\Gamma} - \left[\beta_{k,n} + \ln\left(1 + \gamma_{k,n}/\Gamma\right)\right]}{\ln 2 \cdot \left(\alpha_{k,n} + \gamma_{k,n}\right)^2 / \left(\mu_{k,n}\right)^2} = -\xi_{k,n} < 0. \quad (22)$$

By adopting similar steps as above, and using the properties of $W(\cdot)$ [12], we can see that (22) implies $\alpha_{k,n} < \beta_{k,n}$. This makes sense, because, if $\alpha_{k,n} < \beta_{k,n}$, $\partial u_k / \partial p_{k,n} < 0$. This implies that u_k is a decreasing function of $p_{k,n}$ for any $p_{k,n} \geq 0$, and hence the optimal choice is $p_{k,n} = 0$ (in other words, a rational user will *not* use subcarrier n, as the channel quality is too bad). Thus, in general, user k's best response can be summarized as in (11).

When instead only (8) is active, (6) is equivalent to minimize the sum-power P_k under the assumption $\sum_{n=1}^{N} \log_2\left(1 + \gamma_{k,n}/\Gamma\right) = \theta_k$. Using the KKT conditions, we get $1 - \nu_k \cdot \frac{\mu_{k,n}/\Gamma}{1+\mu_{k,n}p_{k,n}/\Gamma} = 0$ from which using (7) we get (14), where ν_k can be obtained as in (15), by using (8).

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