

# Network Analysis and Calibration of the “Leveraged network-based financial accelerator”

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## Abstract

In this paper we analyze the network structure that endogenously emerges in the credit market of the agent-based model of Riccetti et al. (2011), where two kinds of financial accelerators are at work: the “leverage accelerator” and the “network-based accelerator”. We focus on the properties of network topology and its interplay with the overall economic performance. Moreover, we empirically calibrate the banking network in the model by using Japanese real data.

**Keywords:** network, bankruptcy cascades, calibration, leverage.

**JEL classification codes:** C63, D85, E32.

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# 1 Introduction

In this paper we analyze the credit network structure that endogenously emerges in the agent-based model of Riccetti et al. (2011), a model further developed in Riccetti et al. (2013). This model considers two kinds of financial accelerators: the “leverage accelerator” together with the “network-based financial accelerator” (Delli Gatti et al., 2010). Basically, the financial accelerator (Bernanke and Gertler, 1989; Bernanke et al. 1996) is a mechanism that can enlarge business fluctuations: negative shocks on firms’ output make banks less willing to loan funds, hence firms might reduce their investment and this leads again to lower output in a vicious circle. However, models of the financial accelerator available so far are generally limited, in our opinion, because of the Representative Agent assumption. The aggregate mainstream view of the financial accelerator abstracts from the complex nexus of credit relationships among heterogeneous borrowers and lenders that characterizes modern financially sophisticated economies. Instead, in Delli Gatti et al. (2010), the presence of a credit network may produce an avalanche of firms’ bankruptcies, so that even a small shock can generate a large crisis. Accordingly, an idiosyncratic shock on borrower (firm) deteriorates the lenders’ financial condition weakening the banking system; thus lenders increase the interest rates charged on borrowers (indirect interaction) worsening the non-financial sector conditions in a vicious circle that can make both firms and banks go bankrupt, possibly causing an avalanche of bankruptcies. Riccetti et al. (2011) merges the two mechanisms in a unified framework. The aim of present work is twofold:

1. to analyze the properties of the credit network and its influence on the overall economic performance;
2. to calibrate the banking network topology which emerges from the agent-based model by using Japanese real data.

It is natural to conceive credit markets as networks in which nodes represent agents and links represent credit claims and liabilities. Most works in this field focus specifically on the interbank market, since the latter is relevant for financial stability and, at the same time, well suited for a representation with basic network theory. The interconnectedness of credit institutions is a source of counterparty risk on interbank credit markets, which has been addressed recently by a number of theoretical models tackling the problem of contagious defaults (Gai and Kapadia, 2010; Amini et al., 2010, 2011; Battiston et al., 2012). These models, which go beyond previous simulation-based works (Nier et al., 2007; Elsinger et al., 2006), rely on complex network theory, which has become a prominent tool in this field. While earlier contributions (Allen and Gale, 2000) stressed the benefits of increasing diversification, suggesting that the more connections the better for financial stability, these later works have challenged this view, showing that diversification is not always beneficial for stability, and

underlining instead the systemic risk provided by default cascades and other contagion effects. Indeed, a large recent literature strand analyzes various sources of systemic risk focusing in particular on two channels, as explained by Gai and Kapadia (2010): first, there is the already mentioned direct contagion risk due to the network of exposures; second, there is the indirect contagion risk caused by the fire sale mechanism (also called “market liquidity risk”, as in Alessandri et al., 2009, or Cifuentes et al., 2005. The importance of the liquidity issue is debated also in the agent based model framework, see for instance Giansante et al., 2012), explained in many papers such as Choi and Cook (2012), Krishnamurthy (2009), Shleifer and Vishny (2011). Even in the fire sale context, Wagner (2011) shows that diversification is not always beneficial: on one hand a well diversified portfolio allocation reduces the bankruptcy probability, but on the other hand if many investors pursue the same strategy (that is a large diversification), there is a strong risk of facing higher liquidation costs due to a joint liquidation event. Focusing again on direct contagion effect, for instance, Battiston et al. (2012) show that, if market-related effects are considered along with credit-related effects by introducing a financial accelerator mechanism, then a potential trade-off between individual risk and systemic risk may exist for increasing connectivity of the network. Similar results are provided by Gai and Kapadia (2010), who show that financial systems exhibit a robust-yet-fragile tendency: while the probability of contagion may be low, once a default cascade is started its spread may be quite large. This effect is non-monotonic in connectivity: for a given range of values, connectivity increases the chances that institutions surviving the effects of the initial default will be exposed to more than one defaulting counterpart after the first round of contagion, thus making them more vulnerable to a second-round default.

In general terms, the dynamics of any contagion process depends crucially on network topology. This fact agrees with the following simple intuition: whenever a shock affects a node of a financial network, this will be transmitted to its neighbors with a probability that is proportional to the strength of their linkage to the shocked node. In this context, heterogeneity becomes of paramount importance: some nodes may be too big or too connected to fail, since their failure could hardly hit a large neighborhood. Empirical analyses find unequivocal evidence of heterogeneity in credit networks, such as De Masi et al. (2011) or Cont et al. (2012), thus providing a strong argument for a deeper analysis of network effects in financial markets. Moreover, empirical results show that networks cannot be easily estimated from partial data. For instance, it is well known that networks estimated with maximum entropy (ME) techniques, when shocked, behave quite differently from their real counterparts (Mastromatteo et al., 2012). In particular, ME networks are usually found to underestimate the extent of contagion, although non-linear effects also appear (van Lelyveld and Liedorp, 2006; Mistrulli, 2011). In this sense, a specific network heterogeneity needs to be addressed besides nodes’ heterogeneity to get a deeper understanding of credit markets.

On the other hand, the empirical support for the relevance of contagious defaults in the interbank market is mixed. This is not surprising at all since empirical works in this field

rely on a variety of simulation-based approaches and diverse behavioral assumptions<sup>1</sup>. For instance, those works which examine the effects of idiosyncratic shocks affecting a single bank, come to the conclusion that the scope of contagion is limited (Elsinger et al., 2006; Upper and Worms, 2004; Mistrulli, 2011). By adopting a more realistic setting, e.g. taking into account correlated market shocks and short-term 100% losses for creditors, quite different results have been obtained (Cont et al., 2012). Notwithstanding this uncertainty, central banks are becoming more and more interested in network analysis, supporting network-related research and dissemination, although most empirical work in this direction still looks merely descriptive (Castren and Kavonius, 2009; ECB, 2010). In order to develop more realistic models, the modelling framework should be grounded in the empirical evidence both qualitatively and quantitatively.

In recent years there has been a growing literature on the validation and calibration of agent-based models with real data (just to give some examples: Bianchi et al., 2007; Brenner and Werker, 2007; Fagiolo et al., 2007). Validation represents a set of techniques meant to verify if the model is able to reproduce the actual phenomena for which it has been designed within a satisfactory range of accuracy. Based on the matching between simulated and real data, calibration should improve the precision of the parameters' values. In this paper we focus on the calibration of the model parameter that governs the shape of credit network distributions. In particular, we aim at searching for the parameter's value that produces simulation results as close as possible to the real Japanese data, that we will describe below.

In order to attain the calibration task, we slightly modify the model by adding a parameter that influences the propensity of changing the lender according to the best interest rate. To the best of our knowledge this is the first work that tries to calibrate a parameter that shapes a network in an economic model. Calibration leads to a higher value of the parameter with respect to the baseline model, which implies a more concentrated banking system and a higher number of bank defaults.

The paper is organized as follows. In Section 2 we present the agent-based model of Riccetti et al. (2011). Section 3 contains the network analysis and discusses the interplay between network properties and aggregate economic features. The results of the calibration exercise are presented in Section 4. Section 5 concludes.

## 2 The model

Our economy is populated by firms and banks<sup>2</sup>. Firms – indexed by  $i = 1, 2, \dots, I$  – produce consumption goods. Banks – indexed by  $z = 1, 2, \dots, Z$  – extend credit to firms. In a sense, this is a “partial (dis)equilibrium model” focused on the credit market. Moreover, the model

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<sup>1</sup>For a survey see Upper (2011).

<sup>2</sup>In a sense, households exist to consume goods (final consumers) and to produce goods (labor suppliers), but they are not explicitly modelled.

analyzes the business cycle fluctuations and it does not assess the long-run growth topics.

## 2.1 Firms' capital structure

In this model, following Delli Gatti et al. (2010), the scale of activity of the  $i^{th}$  firm at time  $t$  - i.e. the level of production  $Y_{it}$  - is an increasing concave function of its net worth  $A_{it}$ . Indeed, we hypothesize that the production function (called “financially constrained output function”) is<sup>3</sup>:

$$Y_{i,t} = \phi K_{i,t}^{-\beta} \quad (1)$$

where  $\phi > 1$  and  $0 < \beta < 1$  are parameters uniform across firms and  $K_{i,t}$  is the total capital of the  $i^{th}$  firm at time  $t$ , composed by net worth and debt (see eq.5)<sup>4</sup>. However,  $Y_{i,t}$  is a function of  $A_{i,t}$  because we follow the dynamic trade-off theory for the capital structure of firms. Within the theoretical framework of the dynamic trade-off theory (see Flannery and Rangan, 2006), we hypothesize that all firms have a leverage<sup>5</sup> target  $L_{i,t}^*$  and fix a debt target  $B_{i,t}^*$  in the following way:

$$B_{i,t}^* = A_{i,t} L_{i,t}^* \quad (2)$$

The leverage level is set by firms by following an adaptive behavioral rule according to which the current leverage target is equal to the previous level modified by a random percentage increase (decrease) when the expected price is larger (smaller) than the interest rate on bank loans. The specific form of the leverage setting adaptive rule is the following:

$$L_{i,t}^* = \begin{cases} L_{i,t-1} \cdot (1 + \delta \cdot U(0, 1)), & \text{if } \bar{p}_{i,t} \geq r_{i,t-1} \\ L_{i,t-1} \cdot (1 - \delta \cdot U(0, 1)), & \text{if } \bar{p}_{i,t} < r_{i,t-1} \end{cases} \quad (3)$$

where  $r_{i,t}$  is the interest rate paid by firm  $i$  for the loan received at time  $t$  (see eq.6),  $\bar{p}_{i,t}$  is the expected price that is a modified exponential smoothing of recently observed firm-specific prices<sup>6</sup>,  $\delta$  is a parameter that sets the maximum leverage change between the two periods, and  $U(0, 1)$  is a random number drawn from a uniform distribution between 0 and 1.

Thus, the target leverage is a positive function of the expected mark-up on sales, and a negative function of the interest rate. This level changes among firms and over time given the evolution of  $\bar{p}_{i,t}$  and  $r_{i,t}$ .

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<sup>3</sup>The concavity of the financially constrained output function captures the idea that there are “decreasing returns” to financial robustness. Moreover, following Greenwald and Stiglitz (1993), this function can be considered as the solution of an optimization problem of firms' expected profits net of expected bankruptcy costs. For a detailed discussion see Delli Gatti et al. (2010, pp. 1630-1631).

<sup>4</sup>The relation between production and nominal capital in the production function implies that the “price of capital” is constant. This should be consistent with the assumption that the price of goods is exogenous and stationary, given that we only perform a short-run analysis.

<sup>5</sup>We are defining the leverage as debt/net worth.

<sup>6</sup>For a detailed description see Riccetti et al. (2011).

Following Equation 2, a firm that has some reinvested profits, increasing  $A_{i,t}$ , will also ask banks for new debt funds to reach the desired level of leverage: the debt is built during the growth periods.

To make the model more realistic, we hypothesize that debt lasts for two periods. To do this, in every period each firm asks for an amount of debt equal to the difference between the “debt target” and the residual amount of debt made in the previous period (and that will expire in the following one):

$$B_{i,t} = \max(B_{i,t}^* - B_{i,t-1}, 0) \quad (4)$$

Thus:

$$K_{i,t} = A_{i,t} + B_{i,t} + B_{i,t-1} \quad (5)$$

If a firm suffers high losses that, reducing the net worth, make the debt target smaller than the previous debt, the firm does not ask for new debt. This implies that firms’ network degree is one or two, that is a firm can borrow from one or two banks.

## 2.2 Interest rate setting

Each bank sets a different interest rate on loans and these differences imply that firms sometimes change banks to obtain a lower interest rate, following the mechanism that we will explain in Section 2.3.

We hypothesize that the  $z^{th}$  bank adopts the following rule in setting the interest rate on loans to the  $i^{th}$  borrower:

$$r_{i,t} = r_t^{CB} + f_1(A_{z,t}) + f_2(L_{i,t}, A_{i,t}) \quad (6)$$

Thus the interest rate is composed by three parts: the policy rate set by the central bank,  $r_t^{CB}$ ; a term  $f_1$  that decreases with the financial soundness of the bank (proxied by the  $z^{th}$  bank’s net worth  $A_{z,t}$ ); a term  $f_2$  that incorporates a risk premium increasing with borrower’s leverage.<sup>7</sup>

The presence of this endogenous premium in the interest rate is a channel of the network-based financial accelerator.

The hypothesis that the interest rate decreases with the financial soundness of the bank follows Delli Gatti et al. (2010). They supported this choice with many references, such as Kashyap and Stein (1995, 2000), Stein (1998), Kishan and Opiela (2000), and especially quoting Gambacorta (2008): “small, low liquid, and low-capitalized banks pay a higher premium because the market perceives them to be more risky. Since these banks are more exposed to asymmetric information problems they have less capacity to shield credit relationships in the case of a monetary tightening, and they should cut their supplied loans and raise the interest rate by a larger amount”.

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<sup>7</sup>See Riccetti et al. (2011) for details.

More in general, there are many theories on bank-firm relationship that can support this hypothesis. For instance, ample literature focuses on differences between large and small banks. Following Diamond (1984), banks should be very large and well diversified, then why do small banks exist (apart from the fact that small banks were supported by regulation before the deregulation process)? The most important answer is that small and large banks serve different kinds of borrowers. Large banks focus on large and well-established borrowers using “hard information” to evaluate them, while small banks concentrate on small and young firms using “soft information” (for a definition of “hard” and “soft” information” see for example Petersen, 2004). However, as reported for instance by Berger et al. (2007), producing “soft information” is difficult and costly, while bigger banks can obtain economy of scale with automated technologies (such as “credit scoring” methods as explained in Akhavein et al., 2005, or “internet banking” as reported by Furst et al., 2002) and this could cause higher spreads for small banks. Moreover small and young firms could be caught up with these small banks that could extract rents from them (Rajan 1992 and Sharpe 1990). Indeed, there is vast literature (for a survey see for instance Thakor and Boot, 2008) illustrating that the presence of switching costs is one of the main reason of banks’ rents. Degryse et al. (2009) explain that: “Asymmetric information, a phenomenon intrinsic to credit markets (...) makes bank credit unique. Repeated exchanges of information tie bank and borrower into a relationship and generate informational capital that is bank specific”. Haubrich (1989) supposes that the cost for a bank of maintaining long-term relationship is lower than repeated direct monitoring, and this is one of the reasons for which financial intermediaries exist. Degryse et al. (2009), quoting Sharpe (1990) and other papers, illustrate the theory on long-term bank relationships: “incumbent bank gains (...) informational advantage over competitors. A high-quality firm that tries to switch to a competing uninformed bank gets pooled with low-quality firms and is offered an even worse, breakeven interest rate. (...) Banks can extract higher profits from captured and locked-in (opaque) borrowers than from borrowers that have ready access to other financing alternatives”. This adverse selection effect should reinforce the mechanism of consolidation that we try to represent in the model, and it supports the idea that, if smaller banks operate with more opaque firms, they can set higher interest rates thanks to higher switching costs. Indeed, Berger and Udell (1996) and Carter et al. (2004) find that loans by larger banks tend to have lower interest rates (and better conditions: for instance less collateral)<sup>8</sup>.

In another research strand, since Petersen and Rajan (1995), many papers find that a growing competition reduces the interest rate spreads (even if young firms face increasing problems of credit availability); however, if on one hand this could support the idea that banking growth (and then market power and concentration growth) should increase interest rates, on the

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<sup>8</sup>However, both these studies use database without information about the borrower, then there is the possibility, consistent with the previous theories too, that large banks lend to better firms, with a smaller risk premium.

other hand, large banks usually operate in highly competitive markets, while small banks concentrate in small regional markets (see Brickley et al., 2003) exploiting that firms are often geographically limited in their funding choices (Ongena et al., 2007); indeed, Berger et al. (2007b) find that loan rates to small and medium firms are lower in markets in which there is a large bank. Moreover, studies as Berlin and Mester (1999) find that banks with more market power in deposits can help borrowers from adverse credit shocks with an interest rate smoothing. This is in line with the already reported reference quoted in Delli Gatti et al. (2010). The ability of well-capitalized banks to insulate their firms from adverse credit shocks (with an interest rate smoothing), is confirmed also during the last financial crisis, for instance see Gambacorta and Mistrulli (2011).

### 2.3 Firm-bank interaction

Initially, the credit network, i.e. the links among firms and banks, is random. Afterwards, in every period each borrower observes the interest rates of a number  $\xi$  of randomly selected banks. We assume, as done in Delli Gatti et al. (2010), that firm  $i$  changes bank with a propensity  $\theta_{i,n}$  of switching to the new lender  $n$ , that is decreasing (in a non-linear way) with the difference between the previous bank's interest rate and the interest rate set by the observed potential new bank, only if it finds another bank that charges an interest rate lower than the actual one. In symbols:

$$\theta_{i,n} = 1 - e^{(f_1^n - f_1^o)/f_1^n} \text{ if } f_1^n < f_1^o \quad (7)$$

where  $f_1^o$  is the bank specific term of the interest rate applied by the previous bank, and  $f_1^n$  is the term of the interest rate that could be set by the observed potential new bank.  $\theta$  is a switching probability: the firm extracts a random number from a uniform distribution  $U(0,1)$  and changes the bank if the extracted number is below  $\theta$ . This procedure to choose the partner is activated in every period, but the partner is changed less frequently. In this way, we model the sticky connection between a borrower and its banks, due to the (asymmetric) information on the firm owned by the bank.

However, the topology of the network is in a process of continuous evolution due to the changing interest rate charged by the banks. Indeed, banks characterized by more robust financial conditions can charge lower prices and therefore attract more new partners. As a consequence, their profits go up and their financial conditions improve, making room for even lower interest rates in the future and attracting more new partners. This self-reinforcing mechanism gives rise to an endogenous evolution of the credit network, that will be characterized by a right-skew distribution for node degree: there will be nodes characterized by a relatively high number of links (“hubs”) and nodes with a small number of connections.

The partner's selection mechanism could have interesting effects on the whole system: when a negative shock hits a node - for instance a firm goes bankrupt - the lenders of the bankrupt



firm react by raising the interest rate charged to all the other borrowers. This interest rate hike may induce the borrowers to switch to lenders who offer more favorable conditions, with two possible effects: on one hand mitigating the spreading of the shock to other firms, i.e. slowing down the financial accelerator (that is, the network effect mitigates financial instability); on the other hand, further weakening the bank that suffers from the bankruptcy (that is, the network effect amplifies financial instability).

## 2.4 Profits

Firm's profit ( $\Pi_{i,t}$ ) is a key component of the model for two reasons:

- it determines firm's net worth  $A_{i,t}$  in the following way:

$$A_{i,t+1} = A_{i,t} + \Pi_{i,t} \quad (8)$$

- it influences the target leverage, as already seen in Section 2.1.

Firm's profit is computed with the following formula:

$$\Pi_{i,t} = p_{i,t}Y_{i,t} - r_{i,t}B_{i,t} - r_{i,t-1}B_{i,t-1} \quad (9)$$

where  $Y_{i,t}$  is the output,  $r_{i,t}$  is the interest rate paid on the last loan ( $B_{i,t}$ ),  $r_{i,t-1}$  is the interest rate paid on the loan received the previous period ( $B_{i,t-1}$ ), and  $p_{i,t}$  is the stochastic gain on a unit of output (that contains the stochastic price net of the expenses for producing the output itself, except for financial costs). The price  $p_{i,t}$  is composed of two parts:

$$p_{i,t} = \alpha + \epsilon_{i,t} \quad (10)$$

where  $\alpha$  is the expected gross profit (that is net of financial costs), and  $\epsilon_{i,t}$  is the random component for each firm in each period. We assume that the random part is a variable distributed as a Normal with zero mean and finite variance. The rationale is the same as the one explained in Delli Gatti et al. (2010): given the predetermined supply, the relative price is an increasing function of the demand disturbance. A high realization of  $p_{i,t}$  can be thought of as a regime of "high demand" which drives up the relative price of the commodity in question. On the other hand in a regime of "low demand", the realization of  $p_{i,t}$  turns out to be low and may push the firm to bankruptcy. The demand side is sketched for the sake of simplicity, but endogenous demand should be taken into account<sup>9</sup>, and we will try to incorporate it in future developments of the model.

At the end of each period, the net worth of the  $i^{th}$  firm is defined, as already seen, by

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<sup>9</sup>Many agent based model assess the issue of an endogenous demand side. Just to make a few examples, see Deissenberg et al. (2008), Dosi et al. (2013), Fagiolo et al. (2004), Gaffeo et al. (2008), Raberto et al. (2012), Riccetti et al. (2012), Russo et al. (2007), Seppecher (2012).

$A_{i,t+1} = A_{i,t} + \Pi_{i,t}$ . The firm goes bankrupt if  $A_{i,t+1} < 0$ , i.e. if it incurs a loss (negative profit) and the loss is big enough to deplete net worth:  $\Pi_{i,t} < -A_{i,t}$ . When a firm goes bankrupt, we hypothesize that a new firm enters the market with a very small random net worth and chooses the lending bank at random.

Banks' net worth  $A_{z,t}$  evolves in the same way as firms:  $A_{z,t+1} = A_{z,t} + \Pi_{z,t}$ . However, the  $z^{th}$  bank's profit at time  $t$ ,  $\Pi_{z,t}$ , is given by:

$$\Pi_{z,t} = \sum r_{i,t} B_{i,t} + \sum r_{i,t-1} B_{i,t-1} - r_t^{CB} D_{z,t} - c \cdot A_{z,t} - \psi_{z,t}; \quad (11)$$

where  $r_{i,t}$  is the interest rate paid on  $B_{i,t}$  (if firm  $i$  has not gone bankrupt),  $r_t^{CB}$  is the Central Bank official interest rate,  $D_{z,t}$  is the amount of the  $z^{th}$  bank's deposits,  $c$  is a cost proportional to bank's size and  $\psi_{z,t}$  is the  $z^{th}$  bank's bad debt. In particular:

- deposits  $D_{z,t}$  are computed as the sum of all the lent credit, less the amount of the net worth; in this way, we do not constrain the money multiplier that determines the money supply;
- bad debt  $\psi_{z,t}$  is computed as the sum of all the credit lent to firms gone in default in period  $t$ , multiplied by the loss given default rate, that is 1 less the recovery rate; the recovery rate is computed as the ratio between the asset and the debt of the bankrupted firm and decreased by a fixed amount for the legal expenditure<sup>10</sup>.

The bank goes bankrupt if  $A_{z,t+1} < 0$ . In this case, we hypothesize that a new bank enters the market with a very small random net worth.

## 2.5 Business cycles

The simulation outputs reported in Riccetti et al. (2011) show the emergence of business cycle fluctuations due to two positive feedback mechanisms: the leverage cycle and the network accelerator. The leverage cycle works as follows: when the economy is growing, that is firms' net worth increases and the bad debt reduces, firms increase their leverage (because of growing expected profits) and banks are willing to lend the required credit; the pro-cyclicality of the leverage (i.e., the debt grows more than proportionally than the net worth) boosts firms' growth until the system reaches a critical point of financial fragility and the cycle is reversed through deleveraging. Indeed, when firms are very fragile, even a small shock is highly dangerous because it couples with the high interest set by banks, reducing firms' profit. Hence, firms reduce their investments (and banks are unavailable to loan funds to firms with low or negative expected profits), leading again to a lower output.

The reduced gains increase firms' bankruptcies, with an effect on the credit network: when a firm defaults, its banks record a non performing loan that reduces their net worth and

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<sup>10</sup>For further details see Riccetti et al.(2011).

they could go bankrupt too. If the bad debt is relatively small compared to the banks' net worth, they survive the loss, but, even in this case, banks increase the interest rates to other borrowers to cover the loss; the increased interest rates reduce the firms' profits, reinforcing the leverage cycle and, if other firms go bankrupt, enlarging the network-based mechanism in a vicious circle. In both cases, with or without bank defaults, the initial shock spreads across the financial network, with the possibility to create an avalanche of bankruptcies, which amplifies business fluctuations.

Ricchetti et al. (2011) report two cross-correlation functions between leverage and net worth or bad debt ratio, to show the cyclical behavior of leverage. Indeed, a positive correlation between net worth and the subsequent leverage and a negative correlation between leverage and the following net worth are detected. Moreover, a strong evidence of negative correlation between bad debt ratio and the subsequent leverage is observed, while a positive correlation between leverage and the following bad debt emerges.

Now, we focus on the other positive feedback mechanism, that is the network-based financial accelerator.

### 3 Network analysis

In this Section we apply the tools of network theory in order to investigate the topological properties of the model and to show the relevance of credit interlinkages for the overall economic dynamics. First of all, we study the evolution of bankruptcies focusing on contagion dynamics. Then, we analyze the relationship between network measures and macroeconomic variables. Finally, we investigate some features of the network topology that we will use for calibration in the next Section.

#### 3.1 Bankruptcies

By construction, in our model bank defaults are only due to contagion. Indeed, banks cannot go bankrupt without a bad debt caused by the failure of one or more borrowers. Instead, regarding firms, we can distinguish “fundamental defaults” from “contagious defaults”. We define “fundamental defaults” as those for which negative firms' profits lead to default independently of the risk premium spread charged by lenders. In other terms, in this case firms go bankrupt because of the combined effect of exogenous variables, namely the price shock and the policy rate set by the Central Bank.<sup>11</sup> In the opposite case we define the default as “contagious”, since it is determined by the spread, which is to a large extent an endogenous variable determined by credit connections. In the first place, it depends on the financial condition of the firm in question, where the latter is a function not only of the price shock but also

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<sup>11</sup>Hence, firms' net worth becomes negative even if the profit is calculated as follows:  $\Pi_{i,t} = p_{i,t}Y_{i,t} - r_t^{CB}(B_{i,t} + B_{i,t-1})$ . To make a parallel see Equation 9.

of the spread charged in the past by lenders. In the second place, it is also a function of the financial condition of the lender, which on its part is again a function of the bankruptcy rate among its borrowers. Thus, these defaults are endogenously determined by the past failure of other borrowers of the same lender, i.e. by credit connections.

Our simulation runs over 1000 periods, but we consider only the last 800 time steps, while the first 200 are left to initialize the system. The number of firms in our economy is 500 (while banks are 50). Over the time span considered we observe that the average percentage of firm defaults in each period is 15.6% (77.8 bankrupted firms), with a maximum of 21% (105 defaults) and a minimum of 11% (55 defaults). The mean of “contagious defaults” in each period is 5.1, therefore the share of “contagious defaults”, that is the number of “contagious defaults” divided by the total number of firm defaults, is 6.5%. It is worth noting that the share of “contagious defaults” in a single period reaches a maximum of 15.3%, with 11 “contagious defaults” in a time span in which 72 firms go bankrupt.

Given the specificity of our model, our definitions are different from the ones that are found in the literature, although they follow the same line of thought. Regarding for instance the interbank market, fundamental defaults are commonly defined as those determined by an exogenous market shock hitting the asset value of some banks and triggering their failure (see for instance Cont et al. (2012)), while contagious defaults are determined endogenously by the missed payments of the already bankrupted banks (Eisenberg and Noe, 2001). In the commercial credit network model of Fujiwara (2008), instead, contagious defaults are detected indirectly by weighting the probability of a simultaneous default of a supplier and a client firm, here defined as “domino effect”. In this case, again, the exogenous effect of a shock hitting the real economy is combined with an endogenous network effect determined either by the missed payment of the failed client or by the disruption of the production chain caused by the failure of the supplier.

### 3.2 Network and macroeconomic dynamics

Generally speaking, it is natural to conceive credit markets as networks in which nodes represent agents and links represent credit claims. A link between nodes  $i$  and  $j$  of a network  $G$  is denoted as  $a_{i,j}$ . Since a strength  $w_{ij}$ , representing the credit extended from  $i$  to  $j$ , is associated with  $a_{i,j}$ , credit markets are instances of a weighted network<sup>12</sup>. Furthermore, since it is not true that  $w_{i,j} \equiv w_{j,i}$ , they are also directed networks. Finally, credit relationships between banks and firms are represented by a bipartite (weighted and directed) network, i.e. a network whose nodes can be divided into two disjoint sets  $F$  (firms) and  $B$  (banks) so that every link connects an element of  $F$  with an element of  $B$ . One can extract two projected networks from a bipartite network, each one composed by just one kind of nodes. To compute

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<sup>12</sup>If otherwise links can take on only binary values  $a_{ij} \in \{0, 1\}$ , we obtain a binary network. We can always derive a binary network  $A$  from a weighted one by setting  $a_{ij} = 1$  if  $w_{ij} > 0$ .

these projections, we can employ the matrix representation of the binary network  $G$ , according to which the  $a_{ij}$  are arranged in an adjacency matrix  $A$ . Then the two projected networks are weighted networks represented by the strength matrices  $W_F \equiv A^T A$  and  $W_B \equiv AA^T$ . It's easy to see that, in the case of bank-firm credit networks, the elements of these matrices are respectively the number of lenders shared by two firms and the number of borrowers shared by two banks. Thus, two firms (banks) are connected in a projected credit network if they share at least one lender (borrower). The corresponding binary projected network is obtained as described in footnote 12. We remark that in our case we derive only the bank projected network since firms have a maximum of two lenders.

Networks are characterized by a variety of observable quantities. In the following, we introduce those measures which we employ in our analysis. Among the latter, the most well known are the degree  $d_i$  and strength  $w_i$  of a node  $i$ . The former is defined as the sum  $d_i = \sum_{j \neq i} a_{ij}$ , while the latter is  $w_i = \sum_{j \neq i} w_{ij}$ . If  $G$  is binary and directed, we need to distinguish between the out-degree and in-degree of the node  $i$ , since  $d_i^{out} \equiv \sum_{j \neq i} a_{ij} \neq \sum_{j \neq i} a_{ji} \equiv d_i^{in}$ . If  $G$  is weighted and directed, we also need to distinguish between the out-strength and in-strength of the node  $i$ , since  $w_i^{out} \equiv \sum_{j \neq i} w_{ij} \neq \sum_{j \neq i} w_{ji} \equiv w_i^{in}$ . Given that in our case we have a bipartite network, out- and in-degrees represent respectively the number of borrowers of a given bank and the number of lenders of a given firm, while out- and in-strengths represent respectively the total amount of loans extended by a given bank, and the total borrowing of a given firm. An important measure is *connectivity*, defined as  $l = \sum_i d_i^{out} \equiv \sum_i d_i^{in}$ . In other words, connectivity is the number of links in the network. By weighting connectivity over the maximum possible number of links we obtain the *density*  $\rho$  of  $G$ , which in the bipartite case reads:

$$\rho = \frac{l}{IZ} \quad (12)$$

where  $I = |B|$  and  $Z = |F|$ . In general, real networks are found to display low density, i.e. to be sparse. This property cohabits with another very common one, i.e. the fact that most nodes in a network are connected by a path, that is a sequence of nodes where each of them is linked with the next node in the sequence. In this case we also say that  $G$  has a giant component, meaning that most of the nodes lie on a single connected component, defined as a maximal connected subgraph of  $G$ <sup>13</sup>. Then we can compute the share  $\zeta$  of nodes included in this subgraph. The *distance*  $\varphi_{ij}$  between two nodes  $i$  and  $j$  may be defined as the shortest (geodesic) path between  $i$  and  $j$ . If  $i$  and  $j$  are not connected, we set  $\varphi_{ij} = +\infty$ . Then the average distance  $\bar{\varphi}$  may be computed only for the connected components of  $G$ .

For the bipartite network we use a measure  $\Sigma^2$ , taken from Bargigli and Gallegati (2011), which is proportional to the level of concentration in the network.<sup>14</sup> Bargigli and Gallegati

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<sup>13</sup>Two nodes  $i$  and  $j$  are said to be connected if  $G$  contains a path from  $i$  to  $j$ . A subgraph of  $G$  is said to be connected if it contains only connected nodes.

<sup>14</sup>In order to obtain this measure, we have to compute a normalized matrix  $M$ . In the case of a symmetric

(2011) show that, for a perfectly diversified network  $M^*$  obtained with the help of maximum entropy techniques,  $\Sigma^2(M^*) = 0$ , and further that  $\Sigma^2$  is increasing in the distance between  $M$  and  $M^*$ , defined with respect to the standard Frobenius matrix norm.

For the binary projected network of banks, we can define as an additional observable the *clustering coefficient* of a bank  $i$ , which in this case reads:

$$c_i = \frac{\sum_{h \neq j} a_{ij} a_{ih} a_{jh}}{d_i(d_i - 1)} \quad (13)$$

Clustering is often associated with heterogeneous local structures and neighborhoods in complex networks. In many social networks we find that if node  $i$  is linked to node  $j$  and node  $j$  to node  $h$ , then there is a fairly high probability that  $i$  and  $h$  are also linked. This property, therefore, measures the number of triangles in the network. In the jargon of social science, clustering answers the question whether a friend of a friend is also a friend.

Network variables are related with the main economic variables of the model (see Tables 1 and 2) in a plausible way, consistent with our theoretical expectations. An increasing economic activity (industrial production and firms' net worth) implies a denser network (positive correlations with connectivity and density both in the bipartite and in the bank projected network); this is obvious: a better economic environment reduces firm defaults, thus the number of links increases and the network is more connected. If the network becomes denser, the number of disjoint subgraphs reduces and these subgraphs increase their size. This has two opposite effects on distances among nodes: on one hand the subgraphs connect more and distant nodes, on the other hand the growing number of links creates shorter geodesic path among nodes. Thus, when the economic activity grows the average distance among nodes increases (positive correlation in Table 1), but the average distance among banks in the projected network decreases (negative correlation in Table 2): economic activity brings banks closer to each other, i.e. they share more borrowers. This last effect is confirmed by the increased average clustering among banks in the projected network. On the contrary, when the economy is in a downward phase, the number of firm defaults increases and the network becomes less dense, as shown by the correlations between firm defaults and the previously analyzed network variables.

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weighted network, normalization is defined by  $M = D^{-\frac{1}{2}} W D^{-\frac{1}{2}}$  where  $D$  is a diagonal matrix with elements  $\{w_1, w_2, \dots, w_n\}$ . In case of directed networks, normalization becomes  $M = D_{out}^{-\frac{1}{2}} W D_{in}^{-\frac{1}{2}}$  with obvious adaptation of the diagonal matrices. In general, from linear algebra we know that  $W = \sum_{i=1}^r \sigma_i u_i v_i^T$  for  $W \in \mathbb{R}^{n \times m}$ , where  $r = \min(n, m)$  and the  $u_i$  and  $v_i$  are its left and right singular vectors. In particular, thanks to normalization we have that  $\sigma_1(M) = 1$  for all  $M$ , so that by normalizing we obtain a measure that is consistent for matrices of different volumes  $v = \sum_i \sum_j w_{ij}$  and dimensions  $n, m$ . Finally, the measure  $\Sigma^2(M)$  is defined as follows

$$\Sigma^2(M) \equiv \sum_{i=2}^r \sigma_i^2$$

where  $\sigma_i$  are the (decreasing) singular values (SV) of  $M$ , starting from the second largest one.

Moreover, due to a growing industrial production and firms' net worth, the denser network implies a lower concentration on the credit market, shown by the negative correlation with  $\Sigma^2$ . This correlation could also be seen with the inverse causal link: a less concentrated network helps in creating a more robust economic environment.

Bank defaults, instead, do not impact on the overall density, given that firms search new banks when their banks go bankrupt. However, bank defaults are highly negatively related with the giant component size (both in the bipartite and in the projected network), positively correlated with the average distance among nodes and with the concentration ( $\Sigma^2$ ). Thus, bank defaults are very relevant given that they cause a more fractionated network, with smaller groups and with an increased distance also inside these smaller groups. Moreover, the relationship between increasing bank defaults and network concentration could have both causal implications: on one hand when banks go bankrupt firms tend to concentrate on the bigger remaining ones; on the other hand a more concentrated network could be riskier for banks and could induce a higher number of bank defaults. It is worth noting that all these effects depend on the absence of any mechanisms of bank's bailout, which has instead been used during the recent crisis, so leading in some cases to even bigger banks, for instance through mergers and acquisitions. Indeed, in the cases of a bailout, the rescued bank still remains in the credit sector without modifying the topology of the network, and with a smaller impact on its characteristics as the giant component, concentration, etc.

The correlations show that firms' leverage is positively correlated with network concentration, that is bigger banks increase their exposure as a consequence of the growing credit demand. Moreover, leverage is negatively correlated with banks projected network connectivity and density, and positively correlated with banks average clustering. This means that the growing leverage is associated with a concentration of the banking system: bigger banks increase their clients and are more connected among themselves (clustering), while smaller ones reduce their influence in the system (connectivity and density). The increase of banking concentration could couple the growing leverage as trigger of the subsequent crises.

Table 1: Pairwise correlation between economic and bipartite network variables (critical value at 5% = 0.0693). Network variables are: connectivity  $l$ , density  $\rho$ , giant component  $\hat{G}$ , average distance  $\bar{\varphi}$  and the concentration measure  $\Sigma^2$ .

	$l$	$\rho$	$\hat{G}$	$\bar{\varphi}$	$\Sigma^2$
Industrial production	0.2697	0.2704	0.0076	0.2475	-0.2261
Firms' net worth	0.2492	0.2474	0.0183	0.2213	-0.2577
Banks' net worth	0.0107	0.0099	-0.0301	0.0246	-0.0372
Firms' leverage	-0.0675	-0.0674	-0.0096	-0.0577	0.1116
Firm defaults	-0.1442	-0.1449	-0.0442	-0.114	0.0373
Bank defaults	-0.0006	0.0098	-0.3676	0.1805	0.1419

Table 2: Pairwise correlation between economic and projected network variables (critical value at 5% = 0.0693). Network variables are: connectivity  $l$ , density  $\rho$ , giant component  $\hat{G}$ , average distance  $\bar{\varphi}$  and average clustering  $\bar{c}$ .

	$l$	$\rho$	$\hat{G}$	$\bar{\varphi}$	$\bar{c}$
Industrial production	0.1954	0.1951	-0.0091	-0.1384	0.2166
Firms' net worth	0.1902	0.18	0.0016	-0.1173	0.199
Banks' net worth	-0.0448	-0.0444	-0.0481	-0.0038	0.1374
Firms' leverage	-0.0908	-0.0867	-0.0069	0.0566	0.1433
Firm defaults	-0.1181	-0.1202	-0.0396	0.0521	-0.0202
Bank defaults	-0.0757	-0.0317	-0.3808	0.0122	0.1335

### 3.3 Network topology

The topology of the network is in continuous evolution due to the changing interest rate charged by the banks, according to Equation 7. We analyze the emerging network structure with respect to three features: banks' degree distribution, banks' extended loans distribution, firms' received credit distribution. Figure 1 shows that the network structure is quite stable over time. However, an evident difference emerges among the three panels in Figure 1: the banks' degree distribution (Figure 1(a)) is much less skewed than the banks' credit distribution (Figure 1(b)), which on its part is less skewed than the firms' debt distribution (Figure 1(c)).

In Table 3, we compare the exponent  $\alpha$  of the power law  $p(x) \propto x^{-\alpha}$  fitted on the right tail of the simulated distribution, with the exponent fitted on Japanese real credit network data in 2000. We see that real data are much more skewed (lower  $\alpha$ ) for banks' degrees and banks' credits.

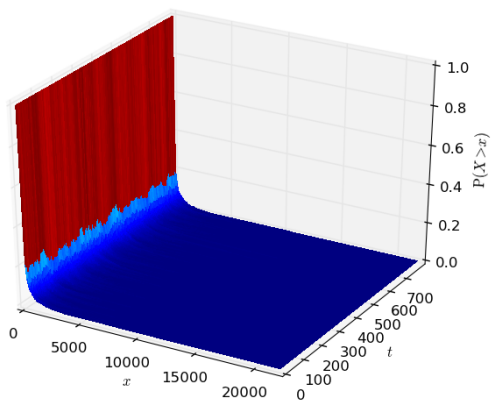
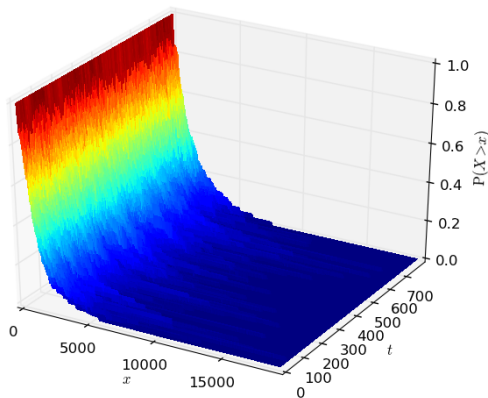
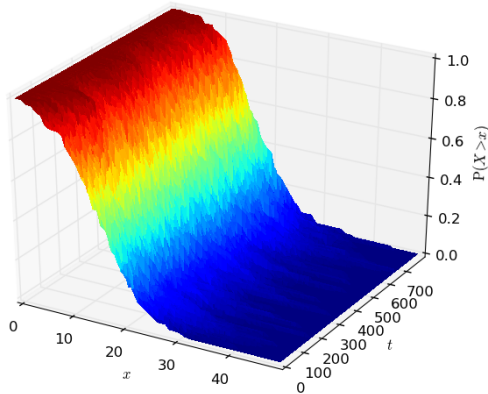
Table 3: Mean, minimum and maximum  $\alpha$  in the 800 periods of the simulated model VS the  $\alpha$  fitted into Japanese real data (year 2000).

$\alpha$	Mean	Maximum	Minimum	Japan
Banks' degrees	3.40	3.44	2.51	1.81
Banks' credits	2.08	2.49	1.40	1.49
Firms' credits	1.98	2.65	1.31	1.85

As an example, we can graphically show this difference in Figure 2 and 3 where we plot (panel a) the credit network emerged from the model at the end of the simulation (time 1000) and (panel b) the real Japanese credit network observed at year 2000. Indeed, the skewness of banking network real data is higher with respect to corresponding simulated data. Instead, in Figure 4, we do not observe a very different distribution between the model and Japanese firms' credit. Given these results, in the next Section we modify the model by adding a parameter which allows us to calibrate the banking network structure of the model with



Figure 1: *Time evolution of banks' degrees (panel a), banks' credit (panel b) and firms' credit (panel c) inverse cumulative distribution function (cdf).*



respect to banks' degree and banks' credit in Japanese data.

Figure 2: *Cdf of banks' degrees in the last period of the model simulation (panel a) compared to the real Japanese cdf in year 2000 (panel b).*

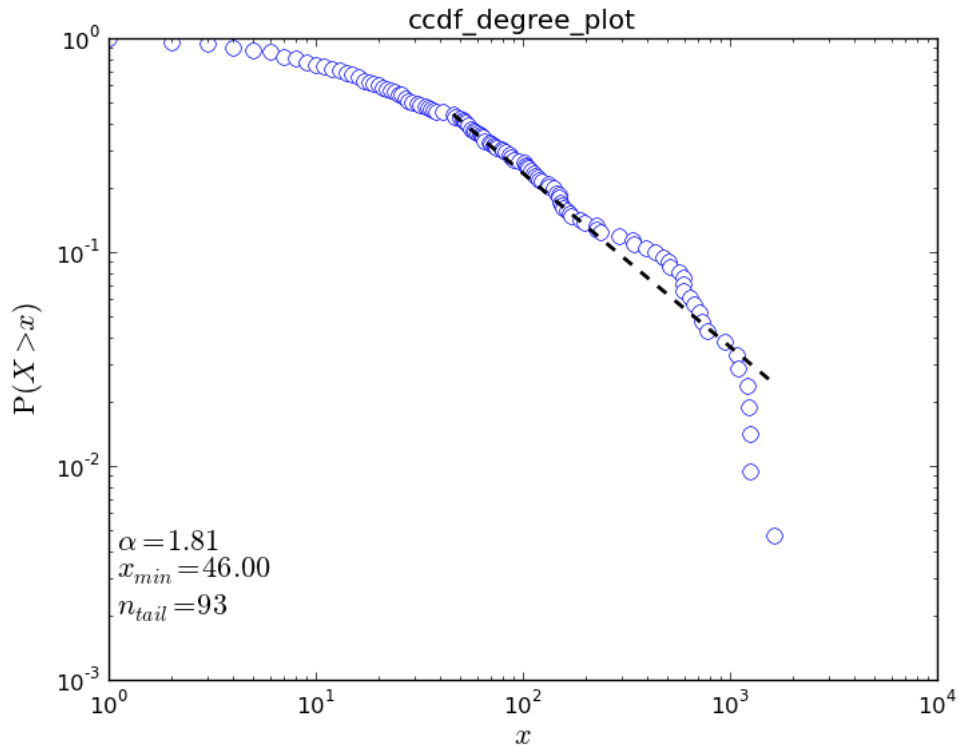
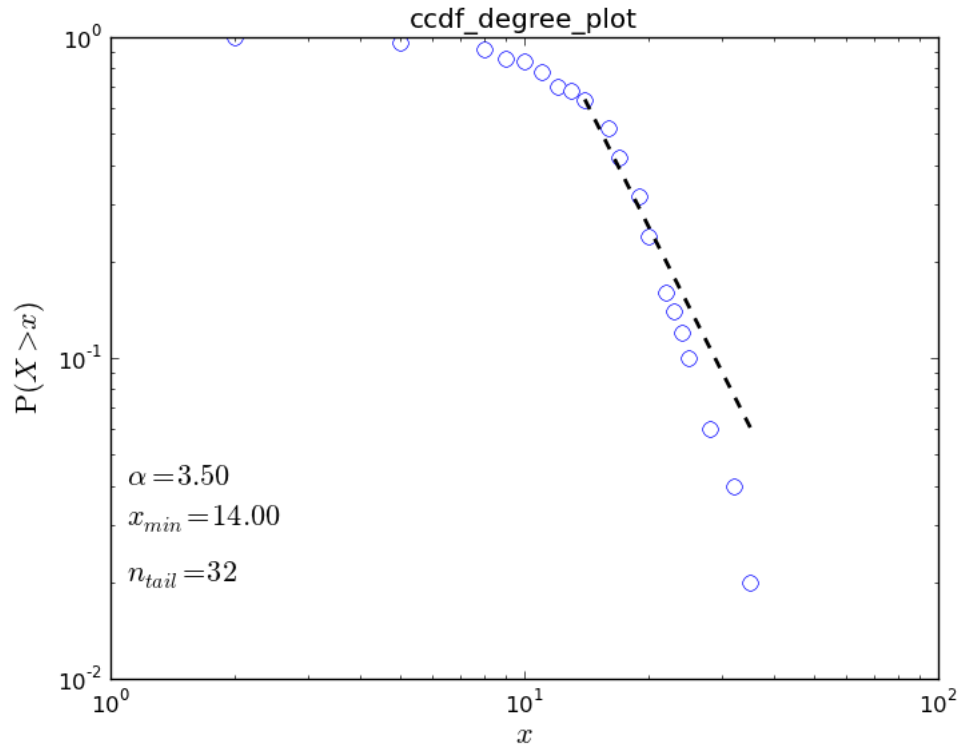


Figure 3: *Cdf of banks' credit in the last period of the model simulation (panel a) compared to the real Japanese cdf in year 2000 (panel b).*

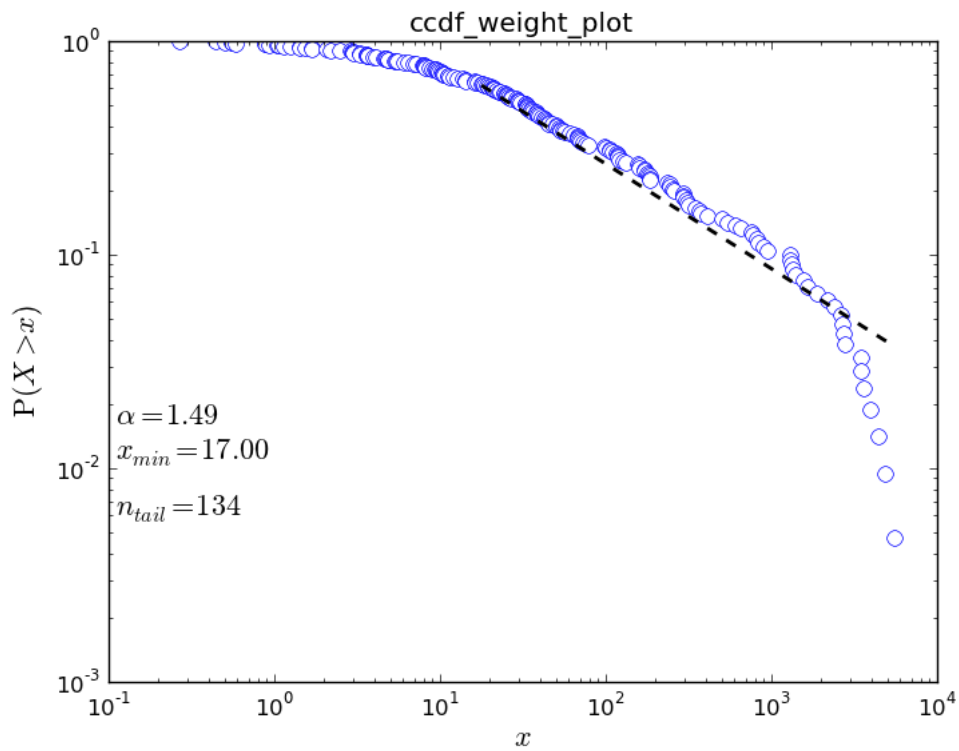
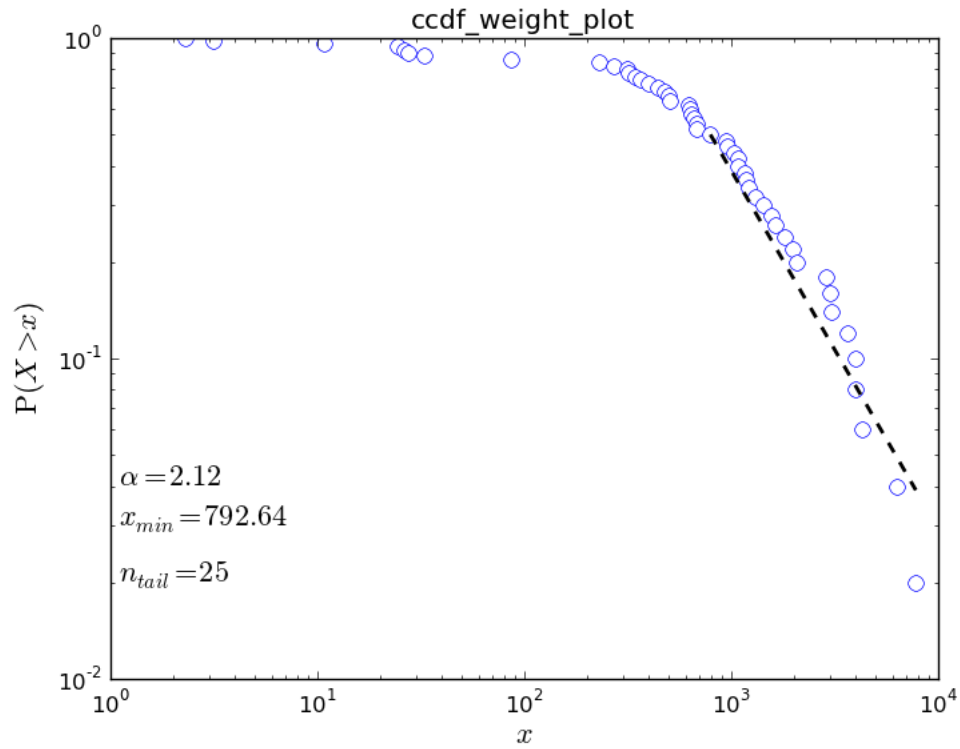
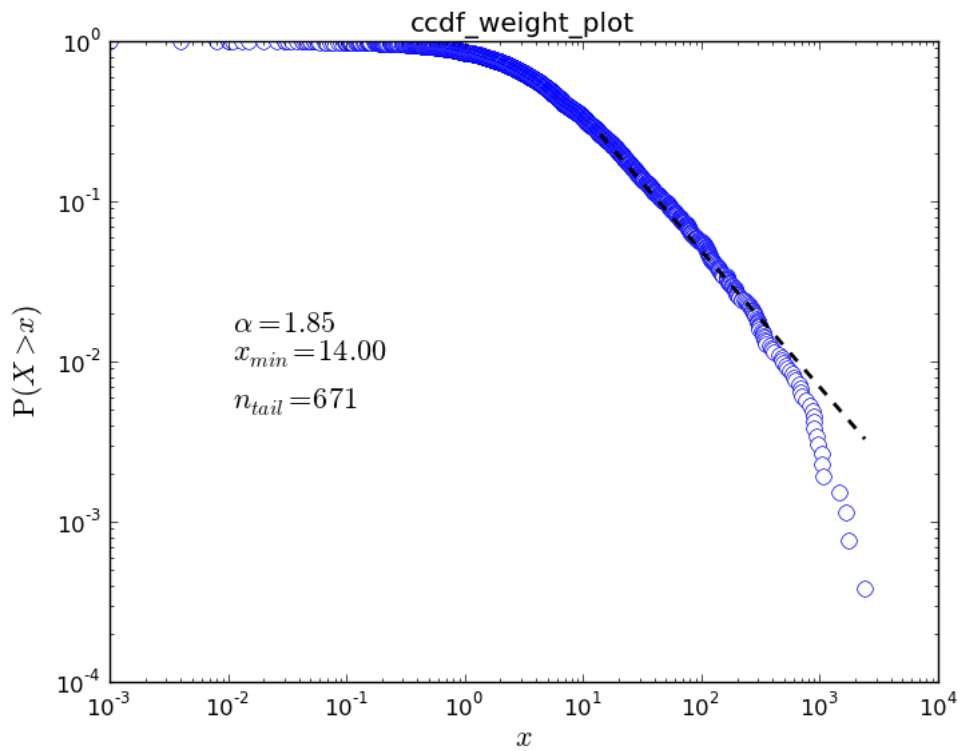
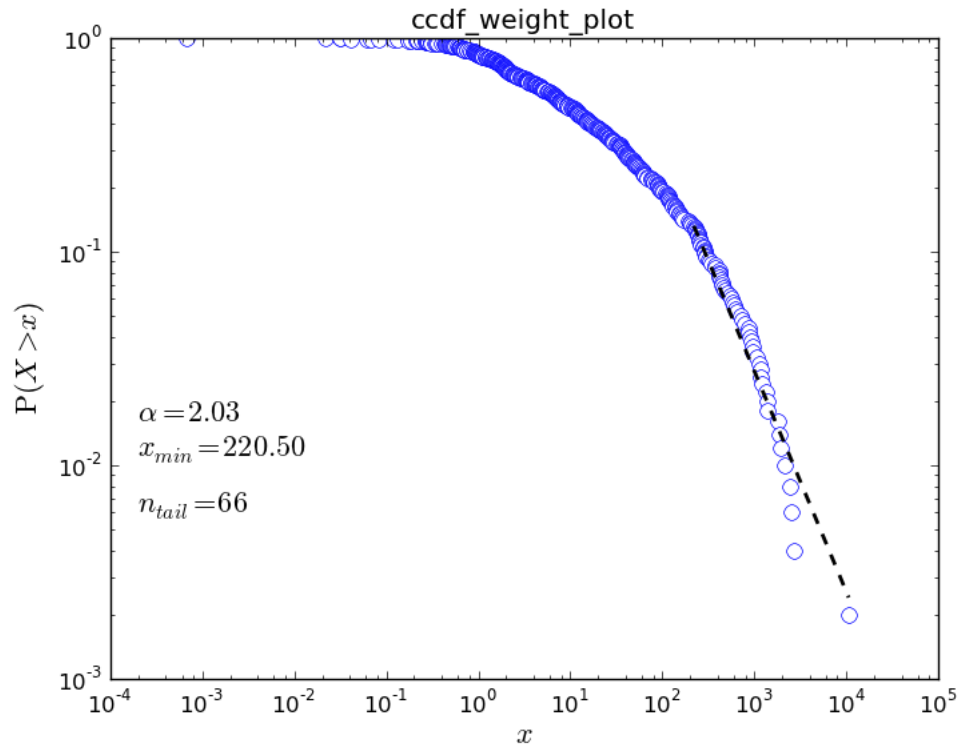


Figure 4: *Cdf of firms' credit in the last period of the model simulation (panel a) compared to the real Japanese cdf in year 2000 (panel b).*



## 4 Calibration

In this section we propose an empirical calibration of the simulation model, based on a dataset which refers to credit relationships between Japanese firms and commercial banks. This dataset, which has been analyzed under different perspectives in previous works (De Masi et al., 2011; Fujiwara, 2009; Bargigli and Gallegati, 2011), includes firms listed in the Japanese stock-exchange markets. Data are compiled from firms' financial statements, integrated by a survey of *Nikkei Media Marketing, Inc.* in Tokyo. They include the indication of the amount of borrowing obtained from each financial institution, both for short-term and long-term debt. Financial institutions, which for sake of simplicity are referred to as "banks", consist of long-term credit banks, city banks, regional banks (primary and secondary), trust banks and insurance companies, all of which represent the universe of financial institutions in Japan.<sup>15</sup> As we see from Table 4, the Japanese banking system underwent a progressive concentration in the period under study, which is a consequence of the protracted crisis of this sector, dating back to the burst of the real estate bubble in the early Nineties<sup>16</sup>.

Table 4: Descriptive statistics of the Dataset

Year	Firms	Banks	Links
2000	2,629	211	27,389
2001	2,714	204	26,597
2002	2,739	202	24,555
2003	2,700	192	22,585
2004	2,700	190	21,919
2005	2,674	182	21,811

In order to modify the skewness of banks' degrees and credits distributions, we insert the parameter  $\lambda > 0$  in Equation 7, following Delli Gatti et al. (2010):

$$\theta_{i,n} = 1 - e^{\lambda(f_1^n - f_1^o)/f_1^n} \text{ if } f_1^n < f_1^o \quad (14)$$

A higher  $\lambda$  represents a higher propensity of switching from one bank to another, according to the best interest rate. Modifying this parameter, indeed, the switching rate changes. For

<sup>15</sup>For a more extensive description of this network the reader can refer to De Masi et al. (2011).

<sup>16</sup>Since 1992, the Japanese banking system has experienced a sizable deterioration in its financial conditions. Commercial banks have recorded cumulative loan losses of about 83 trillion yen, leading to the failure of three large banks (and other small banks). The very poor financial conditions of major banks affected the whole credit system. In order to increase the financial stability of the system, in 1997 the Japanese regulators liquidated a large city bank and nationalized 2 of the 3 largest long-term credit banks. For more details see Brewer et al. (2003).

instance in the baseline case, that is with  $\lambda = 1$ , the percentage of non-defaulted firms that change bank is 2.57% in each period; considering that every firm has two loans, we compute the switching rate as half of the previous value, that is 1.29%. Instead, if  $\lambda = 2$  the switching rate passes to 3.38%, increasing as expected. We run the simulation for different values of  $\lambda$  and we report the switching rates in Table 5. In the table we also report the real switching rate computed on Japanese data between 2000 and 2005. Differently from our simplified model in which each firm has only two loans and changes one link every period, with real networks there is also a net creation or deletion of links. Indeed, if firms always maintained the same degree, it was easy to measure this probability  $\pi$  as follows:

$$\pi = 1 - f/l$$

where  $f = \sum_{ij} a_{ij}(t) \cdot a_{ij}(t+1)$  is the number of “conserved” links from  $t$  to  $t+1$  and  $l$  is the number of links in the network. Instead, in order to take into account the fact that in real networks there is a net creation or deletion of links, so that  $l$  is not constant over time, we define the switching probability as follows

$$\pi = 1 - f/\hat{l}$$

where  $\hat{l} = \sum_i \min(k_i(t), k_i(t+1))$  and  $k_i(t)$  is the degree of firm  $i$  at  $t$ , and  $\hat{l}$  represents the maximum number of links which can be conserved, net of link creation / deletion, so that  $\pi$  is properly normalized. With this method, we find the switching rates reported in the second panel of Table 5. Averaging the reported switching rates, we obtain a mean rate equal to 4.93%. Empirical data shows that relationships are very sticky. However, our model appears to be able to reproduce this transition rate for values of  $\lambda$  around 3 and, more in general, relationships are very sticky in the model too.

Table 5: Switching rate from one bank to another for different values of  $\lambda$  in simulations and mean switching rate in Japanese real data (year 2000-2005).

$\lambda$	0.5	1	2	3	4	6	8	16
Switching rate (%)	0.55	1.29	3.38	5.21	6.56	8.38	9.42	11.77
Japan	2000-01	2001-02	2002-03	2003-04	2004-05	mean		
Switching rate (%)	3.38	7.40	7.18	3.58	3.13	4.93		

We check that all the business cycle features found in the baseline simulation reported in Riccetti et al. (2011) are confirmed when we modify the value of  $\lambda$ . Moreover,  $\lambda$  does not affect the firms’ credit distribution, since credit demand is determined by the economic mechanisms independently of network formation. Instead, the banking network topology radically changes when  $\lambda$  grows from 1 to 4: the already explained self-reinforcing mechanism - that gives rise to the right-skew distribution for banks’ node degree - becomes stronger

and stronger; thus, the scaling exponent of simulated distribution decreases. Consequently, the banks' extended credit also shows an increasing positive skewness and a decreasing  $\alpha$  exponent. In other words, a growing propensity of switching from one bank to another tends to concentrate the credit market in the hands of the biggest banks (in terms of net worth). Regarding the banks' extended credit,  $\lambda = 3$  takes to a mean  $\alpha$  exponent similar to the real one estimated on Japanese data, as shown in Table 6. Instead, calibrating  $\lambda$  to reproduce Japanese banks' degree distribution, best fitting is  $\lambda = 4$ . When  $\lambda \leq 1$  or  $\lambda \geq 4$ , the network topology tends to stabilize.

Table 6: Mean simulated  $\alpha$  VS  $\alpha$  fitted into Japanese real data.

$\lambda$	0.5	1	2	3	4	6	8	16
$\alpha$ Banks' degrees	3.43	3.40	2.78	2.15	1.85	1.77	1.78	1.79
$\alpha$ Banks' credits	2.19	2.08	1.75	1.48	1.32	1.32	1.34	1.37
$\alpha$ Firms' credits	1.96	1.98	1.98	1.96	1.98	1.97	1.99	1.98
<i>Japan</i>	2000	2001	2002	2003	2004	2005	mean	
$\alpha$ Banks' degrees	1.81	1.82	1.91	1.96	1.97	1.98	1.91	
$\alpha$ Banks' credits	1.49	1.47	1.44	1.53	1.56	1.53	1.50	
$\alpha$ Firms' credits	1.85	1.84	1.83	1.99	1.92	1.94	1.90	

The sensitivity analysis shows that  $\lambda$  has an economic impact because it is positively related to the number of bank defaults. Indeed, this number largely increases from  $\lambda=1$  to  $\lambda=8$ , as shown in Figure 5. A bank with a larger bad debt increases its interest rate and therefore its borrowers will probably switch to other more financially robust banks that charge lower interest rates. Then, the bank that receives the initial negative shock ends up with a reduced number of borrowers, and consequently with a less diversified credit portfolio, in addition to the reduced net worth, leading to a higher default probability. In this context, bank defaults are both a cause and an effect of the increased market concentration.

Choosing  $\lambda=3$  (or more), following the previous calibration results, bank defaults become too large compared to real data although, as already explained in Section 3.2, in our model there are no mechanisms like government bailouts or mergers and acquisitions which usually reduce the number of bank defaults, thus preventing a real calibration on this feature. However, to better calibrate the overall output of the model, we can modify other parameters. For example, by lowering the parameter  $c$ , that proxies banks' operational cost, from 0.1 to 0.01 we largely reduce bank defaults, as shown in Table 7. Moreover, this parameter change does not largely affect the value of the  $\alpha$  exponent of the bank degree, bank credit and firm credit distributions, but for a modest decrease of bank degree concentration ( $\alpha$  of bank degree distribution slightly increases), correlated with a lower bank default rate.

To sum up, the value of the parameter  $\lambda$  which enables us to better reproduce the Japanese data on switching from a bank to another, banks' degree and banks' credit distributions is

Figure 5: *Mean percentage of banks' defaults.*

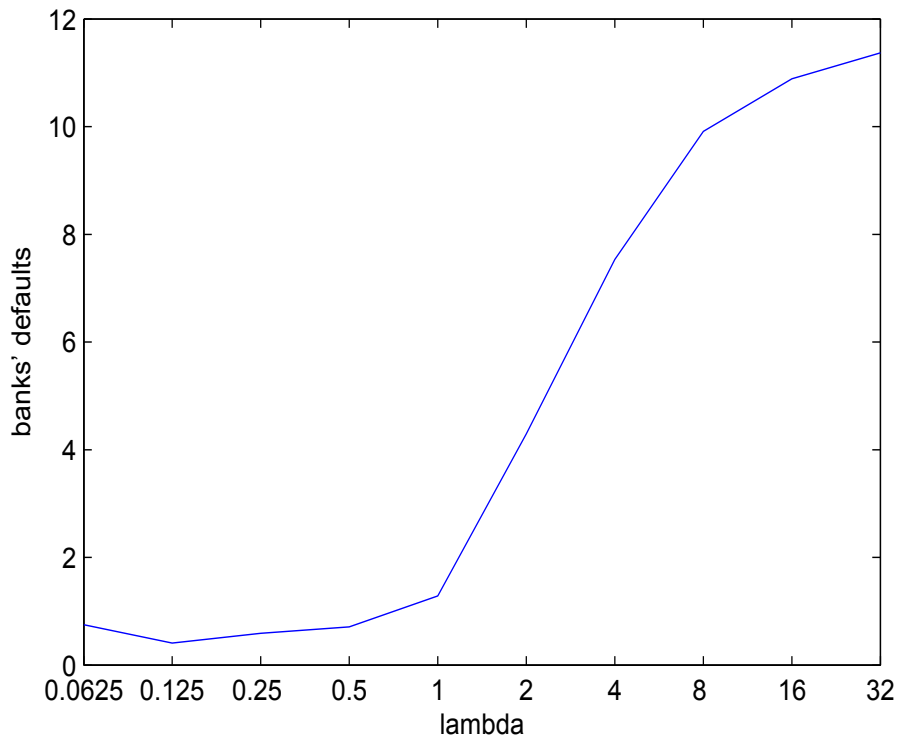


Table 7: Percentage of banks' defaults for different  $\lambda$  and  $c$ .

$\lambda$	0.5	1	2	3	4	6	8	16
$c = 0.1$	1.42	2.56	8.58	12.28	15.06	18.11	19.82	21.77
$c = 0.01$	0	0	0.50	1.92	6.50	12.18	15.36	17.50

around 3.

## 5 Conclusions

The aim of this paper is to analyze the credit network structure that endogenously emerges in the agent-based model of Riccetti et al. (2011) in which two kinds of financial accelerators are at work: the “leverage accelerator” and the “network-based accelerator”. We apply the tools of network theory in order to investigate the topological properties of the model and to show the relevance of credit interlinkages for the overall economic dynamics.

Generally speaking, it is natural to conceive credit markets as networks in which nodes represent agents and links represent credit claims. In this perspective, we analyze the relationship between network measures and macroeconomic variables. We also study the evolution of bankruptcies distinguishing “fundamental defaults” from “contagious defaults” and investigate some aspects of the network topology that we use to calibrate the model. Indeed, in



recent years there has been a growing literature on the validation and calibration of agent-based models. In this paper, we aim at searching for the parameter's value that produces simulation results as close as possible to the real Japanese data. To the best of our knowledge this is the first work that tries to calibrate a parameter that shapes a network in an economic model.

Network variables are related with the main economic variables of the model in a plausible way, consistent with our theoretical expectations. An increasing economic activity implies a denser network because a better economic environment reduces firm defaults, thus the number of link increases and the network is more connected. This brings banks closer to each other, i.e. they share more borrowers.

However, the leverage increases when the industrial production grows. Leverage increase is associated with a concentration of the banking system: bigger banks increase their clients, while smaller banks reduce their influence in the system and become isolated. The growing banking concentration could couple the increased leverage as trigger of the subsequent crises. Bank defaults cause a more fractionated network. Moreover, the relationship between bank defaults and network concentration could have both causal implications: on one hand, when banks go bankrupt firms tend to concentrate on the bigger remaining ones; on the other hand, a more concentrated network could induce a higher number of bank defaults.

The topology of the network is in continuous evolution due to the changing interest rate charged by the banks. We analyze the emerging network structure with respect to three features: bank degree distribution, bank extended loans distribution, firm received credit distribution. We find that real data are much more skewed for bank degrees and bank credits. To modify the skewness of bank degree and credit simulated distributions, we insert a parameter which controls the propensity of switching from one bank to another, according to the best interest rate. We run the simulation for different values of this parameter finding that all the business cycle features found in the baseline simulation are confirmed. However, we observe that the banking network topology radically changes when the parameter value grows: a growing propensity of switching from one bank to another tends to concentrate the credit market in the hands of the bigger banks. Indeed, a bank with a larger bad debt increases its interest rate and therefore its borrowers will probably switch to other more financially robust banks that charge lower interest rates. Thus, the bank that receives the initial negative shock ends up with a reduced number of borrowers, and consequently with a less diversified credit portfolio, in addition to the reduced net worth, leading to a higher default probability. In this context, bank defaults are both a cause and an effect of the increased market concentration. Accordingly, the sensitivity analysis shows that the additional parameter has an economic impact because it is positively related to the number of bank defaults. This number largely increases as the parameter value grows in a certain range.

This paper is a first step in the direction of validating simulation models with real data on credit networks. Other parameter changes can be tested in this framework to analyze their

(individual or joint) effects on network dynamics and macroeconomic evolution.

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