A Task-priority Based Control Approach to Distributed Data-Driven Ocean Sampling

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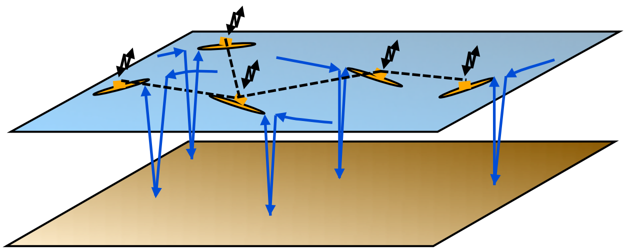
Abstract – **The paper illustrates the basic ideas and relevant algorithmic developments underlying the proposal for a task-priority based control approach to distributed data-driven ocean sampling applications. This approach is deemed allowing a better formalization of the overall motion problem of the involved team of agents; that apart the ultimate mission objective, also result characterized by other different control objectives directly related with both operability and safety aspects of the entire sampling system.**

Also, the proposed approach, other than leading to a unifying algorithmic structure, also seems allowing to foresee good possibilities for different types of downgrading toward efficient decentralized implementations.

I Introduction

The paper explicitly deals with the problem of adaptively governing the motions of a team of AUV's, which is employed for performing a distributed sampling mission within an assigned marine area; in the way sketched in the figure that follows



More precisely, in this framework, a team composed by a number n of AUV's that, when surfaced at a so-called k-th stage, are connected by a wireless network with given topology, are asked to perform (possibly in a synchronous way) a vertical dive and relevant re-emersion; during which the associated k-th stage sample measurements of an oceanic field variable $\theta(x)$, at the reached depth, are collected by each vehicle composing the team.

Then, after surfacing, the acquired geo-referenced data are made converging toward a central processing station (in our diagram supposed coinciding with one of the AUV's always let on surface) termed as "the center"; where such data are then processed for producing an upgrade of the previous geo-referenced field estimate $\hat{\theta}(I_{k-1},x)$ (resulting from the whole distribution of previous sampling points; where I_{k-1} just represents the set of all geo-referenced data acquired till the previous stage) to the one $\hat{\theta}(I_k,x)$, based on the current stage acquisitions.

Thus, upon completion of the k-th stage updating, the problem becomes that of adaptively driving the agents toward a new sampling configuration $x_{i(k+1)}\ ; i=1,...,n$ for the next stage $k+1$; and so on for all successive stages, till the complete coverage the assigned area A.

Qualitatively speaking, each new set of sampling points should obviously be located within a not yet explored zone; that in order to avoid useless oversampling should stay at a suitable distance from all previous sampled points; and for the same reason they should also exhibit a suitable sparse configuration among them; but nevertheless by always maintaining an a-priori established "measure of quality" for the progressively obtained field estimates, within pre-assigned suitable bounds.

Moreover, during the motions toward the next sampling configuration, the vehicles are required to maintain their connectivity; while also maintaining all distances among them above a minimal threshold, for avoiding any risk of collisions within the team.

Further, during the process, the sequential positioning of the sample points should also obey as much as possible to the general rule of incrementally covering the assigned area; that is by possibly avoiding the arising of disjoint unexplored zones; in such a way to also avoid as much as possible the need of non negligible "transfer jumps" of the team between such zones.

An adaptive distributed solution to the above problem has been recently provided in [1], based on the translation of the various requirements into suitable potential (or penalty) functions; and on the adoption of a specific measure of quality of the field estimates (see next section).

Previous works to [1] on the subject were for instance the ones chronologically reported in the list [2-7] of the present work references section. We just refer to [1] for comments on such and others previous works, and on the advances that [1] was presenting with respect to them.

In the present work, by then having [1] as the basic reference work, we present the fundamental ideas underlying a new formulation of the overall problem, this time based on a task priority control approach, together with some basic indications on how to translate such an approach into a corresponding algorithmic framework; that will result it also suitable to be efficiently implemented in a distributed form.

The task priority based approach, originally developed for concurrently achieving different control objective within the motion and positioning of robotic kinematic chains (see for instance [8]) can in fact be easily extended to any system exhibiting similar kinematic characteristics; as just it is for our multi-vehicle (planar) case; which can be assimilated to a kinematic chain without any rigid linking constraint; that is to a completely free highly redundant kinematic system.

As a matter of fact, like what happens for robotic kinematic chains, the task priority based approach generally allows for well-defined and clearer formalizations of multi-objective motion control problems, while also resulting into a unifying algorithmic framework (i.e. supported by a basic algorithmic

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structure largely independent from the specific nature of the different control objectives); thus motivating the present attempt to extend its use also to the adaptive sampling problem in hand.
To this aim, the paper is organized as follows. Section II will provide a brief recall, from [1], about the quality measure of the field estimate be used; that will greatly influence the definition of the ultimate mission goal, as well as all other control requirements, together with their prioritization.
Then, section III, IV, V will progressively provide, till its final organization in section V, the algorithmic structure needed for managing the resulting list of prioritized tasks. Section VI will instead comment about the existing possibilities of implementing the resulting adaptive control policy in a suitably distributed form.
Finally section VII will conclude the paper with some final consideration and indications about future work to be done.

II THE FIELD ESTIMATE QUALITY MEASURE AND THE LIST OF PRIORITIZED CONTROL REQUIREMENTS

Let us assume that, as it is commonly done, the current estimation update $\hat{\theta}(I_k,x)$ is centrally evaluated by the center via weighted interpolation; that is via the form

$$\hat{\theta}(I_k,x)=\sum_{ij}\rho_{ij}v_{ij}\phi(x-x_{ij}) \quad (2.1)$$

With $\phi(x)$ a suitable radial base function translated to the ij -th sample coordinate; v_{ij} the corresponding sample value and ρ_{ij} an associated suitable weight.
Then, by letting the field estimate true error be

$$\varepsilon(I_k,x)=\left\|\theta(x)-\hat{\theta}(I_k,x)\right\| \quad (2.2)$$

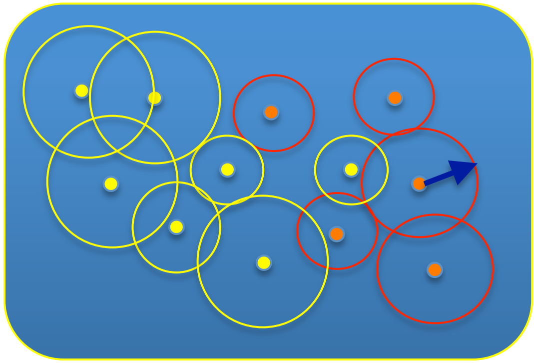
The detailed studies developed in [1] show that an upper-bounding error field $\hat{\varepsilon}(I_k,x)$; hat is a field such that

$$\varepsilon(I_k,x)\le\hat{\varepsilon}(I_k,x) \quad (2.3)$$

is always provided (under reasonable assumption about the continuity of the true field $\theta(x)$) by the interpolation procedure itself, exhibiting the following properties: a) the upper bounding error field is zero in correspondence of the coordinates $x_{ij}\ ;\ i=1,\dots,n\ ;\ j=1,\dots,k$ where samples have been collected; b) it is radially increasing starting from each sample coordinate; c) starting from each sample coordinate, it therefore assumes local maximum values along directions intersecting those coming from samples coordinates proximal to the considered one; d) it instead indefinitely increases along directions oriented toward not yet explored zones.
Then, accordingly with the above properties, it follows that the estimate-upgrading procedure can always provide, upon completion of the k -th sampling stage, the set of positive values $d_{ij}\ ;\ i=1,\dots,n;\ j=1,\dots,k$, each one corresponding to the radius of the circle c_{ij} centered on the corresponding sample coordinates x_{ij} , inside which the upper-bounding error field remains below a pre-assigned precision threshold D ; that is such that

$$\hat{\varepsilon}(I_k,x)\le D\ ;\ \forall x\ s.t\ \left\|x-x_{ij}\right\|\le d_{ij}\ ;\begin{cases} i=1,\dots,n \\ j=1,\dots,k\end{cases} \quad (2.4)$$

Where the maximum value D is obviously achieved only on the arcs of the generic circumference c_{ij} not entering inside any other neighboring circle. Such situation corresponds to what is roughly sketched in the figure that follows; where the yellow circles are assumed relevant to sampling points preceding the ones within the current k -th stage, that instead appear surrounded by red circles.



Thus the problem becomes that of surface transferring the vehicles toward a new set $x_{i(k+1)}\ ;\ i=1,\dots,n$ of sampling coordinates, each one located on the non-intersecting arcs of the corresponding red circumference; that will therefore represent the positions for next $(k+1)$ -th sampling stage.
However, since the points located on the non-intersecting arcs are, from the point of view of the estimate improvement at the next stage, all equivalent each other; an efficient policy for their choice must necessarily be based on the concurrent fulfillment of some other requirements; aiming to preserve the team operability during the transfer motions. That is the already mentioned ones of 1) maintaining the agent connectivity; 2) while also guaranteeing the vehicles safety (i.e. all agents staying each other above a given safety distance; and 3) by contemporarily directing each agent toward wider unexplored zones. And then so forth for all needed future sampling stages.
In particular we must now explicitly notice how the assigned ordering 1), 2), 3) to the above reminded requirements, actually also reflects the decreasing importance (or in equivalent terms, the decreasing priorities) that must be attributed to each one of them, from the point of view the team operability; simply meaning that connectivity keeping is therefore considered to be more important than safety distance maintenance; which is in turn more important than the sampling itself, notwithstanding the fact that this last qualifies the ultimate mission objective.
As already announced in the introduction, the forthcoming sections III and IV will sequentially provide the algorithmic answer to the stated prioritized control needs

III CONNECTIVITY AND SAFETY DISTANCES KEEPING

By then considering the connectivity-keeping objective to be the highest priority one, formerly note how a connection topology between a couple of stages $k,k+1$ is nothing more than a connected graph G^k embracing all n vehicles plus the center; and to this regard it not necessarily needs to be a tree (i.e. without loops), even if a the tree structure assignment corresponds to the simplest and most common case. Moreover a connection topology not necessarily need to be the same for all adjacent couples of stages; even if we require to be so within a couple of them (i.e. not changing in between of the vehicle transfers from one configuration stage to the next).

Moreover, when a connection topology is assigned, it is obviously understood that the module σ_{ij} of vector s_{ij}, which directly connects vehicles i and $j \neq i$ within G^k, is initially lower than the coverage distance d_M ; that is

$$\left|s_{ij}\right| \doteq \sigma_{ij} < d_M \quad ; \quad \forall s_{ij} \in G^k \tag{3.1}$$

And consequently we will require such condition maintained during the entire vehicles transfers between stages $k,k+1$.

To this end, formerly consider the time derivatives

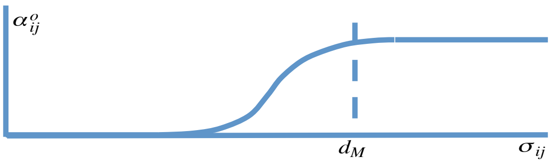
$$\dot{\sigma}_{ij} = h_{ij}^T(\dot{x}_i - \dot{x}_j) \doteq H_{ij}\dot{\chi} \quad ; \quad \forall s_{ij} \in G^k \tag{3.2}$$

With $h_{ij} \doteq s_{ij}/\sigma_{ij}$; and where $\dot{\sigma}_{ij}$ itself has been formally expressed also in terms of vector $\dot{\chi} \doteq col\,(x_l\ ; l = 0,1,...,n)$ of all the vehicles velocities (including the center, numbered as the 0-th one); with H_{ij} the resulting row vector (simply constituted by the row stacking of null 1x2 blocks, apart those $h_{ij}^T, -h_{ij}^T$ located at positions i,j, respectively).

Then consider the association of each one of them with a *feedback reference rate* of the type

$$\dot{\bar{\sigma}}_{ij}^o = \gamma_o(d_o - \sigma_{ij}) \quad ; \quad \gamma_o > 0 \quad ; \quad d_o < d_M \tag{3.3}$$

To be in turn associated with a *feedback activation function* $\alpha_{ij}^o(\sigma_{ij})$ of sigmoidal type as in the following figure (right activation function)



With this in mind, now consider at each time instant the problem of evaluating the set S_o of the vehicles velocities $\dot{\chi}$ minimizing the sum of quadratic forms which appears in the following expression

$$S_o \doteq \left\{ \dot{\chi} = arg\min_{\dot{\chi}} \sum_{ij \in G^k} \left\| \alpha_{ij}^o(\dot{\bar{\sigma}}_{ij}^o - H_{ij}\dot{\chi}) \right\|^2 \right\} \tag{3.4}$$

Then, by keeping into account the sigmoidal form assigned to each activation function and to each reference feedback laws (3.3), it should be consequently clear how (3.4) actually translates the general need of pushing toward the inside of bounds (3.1), only those σ_{ij} that at current instant are, for some reason, greater of the guard value represented by the "foot" of the corresponding activation function.

By the way, the fact of employing a sigmoidal activation functions (thus continuous in their arguments), instead of a simpler Boolean one, clearly also results as a *necessary* condition for having the vehicles velocities exhibiting continuous behaviors in any condition.

At this point, by formerly conveniently re-expressing (3.4) in the more compact form

$$S_o \doteq \left\{ \dot{\chi} = arg\min_{\dot{\chi}} \left\| A_o(\dot{\bar{\sigma}}_o - H_o\dot{\chi}) \right\|^2 \right\} \tag{3.5a}$$

With

$$\begin{cases} \dot{\bar{\sigma}}_o \doteq col(\dot{\bar{\sigma}}_{ij}^o) \\ A_o \doteq diag(\alpha_{ij}^o) \ ; i,j \in G^k \\ H_o \doteq col(H_{ij}) \end{cases} \tag{3.5b}$$

Then the solution set S_o immediately follows in terms of the following linear manifold

$$S_o \doteq \left\{ \dot{\chi} = \dot{\chi}_o + Q_o\dot{z}_o \,; \forall \dot{z}_o \right\} ; \begin{cases} F_o \doteq A_o H_o \\ \dot{\chi}_o \doteq F_o^{\#}(A_o\dot{\bar{\sigma}}_o) \\ Q_o \doteq (I - F_o^{\#}F_o) \end{cases} \tag{3.6}$$

With $\dot{\chi}_o$ corresponding to so-called minimal norm solution; to which any vector of the form $Q_o\dot{z}_o\,; \forall \dot{z}_o$ can be added, without altering the assumed minimum value of the quadratic form. That is, as imposed by the form assigned to the various activation functions, without altering the best achievable approximations to the sole $\dot{\bar{\sigma}}_{ij}^o$ corresponding to the distances σ_{ij} currently located at the right of their guard value; that can be achieved by acting on the global velocity vector $\dot{\chi}$ [1].

Obviously, the arbitrary vector $Q_o\dot{z}_o\,; \forall \dot{z}_o$ represents the one to be used for also possibly forcing, at the second priority level, all vehicle staying each other above a given safety distance d_m (naturally with $d_m < d_M$, typically much smaller, due to obvious feasibility reasons).

To this end, by then considering the control objective

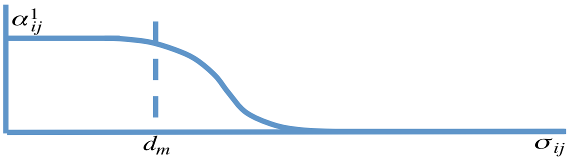
$$\left|s_{ij}\right| \doteq \sigma_{ij} > d_m \quad ; \quad \forall s_{ij} \tag{3.7}$$

We can proceed in a way strictly similar as before; that is, by introducing the feedback reference rates

$$\dot{\bar{\sigma}}_{ij}^1 = \gamma_1(d_1 - \sigma_{ij}) \quad ; \quad \gamma_1 > 0 \quad ; \quad d_1 > d_m \tag{3.8}$$

[1] What stated is actually rigorously true only in correspondence of unitary and/or zero assumed by the activation functions. When one or more activation functions instead fall within their transition zone $0 < \alpha_{ij}^o < 1$ the necessity of resorting to *regularized pseudo-inversions* (and in particular to special ones specifically tailored for task priority control, as in [9]) to the end of managing with continuity the algorithmic singularities resulting from decreasing α_{ij}^o toward zero, this forces the above statement to assume an approximated character. Thus, though regularization represents the *necessary* condition for the continuity of the vehicle velocities; unfortunately it also imposes a price to be paid for guaranteeing such behavior.

Together with the associated sigmoidal feedback activation function $\alpha_{ij}^{1}(\sigma_{ij})$, now of left type as in the following figure



Then, by similarly proceeding as before, we preliminary get the following definition for the associated conditioned (by the membership constraint (3.6)) set S_1 of solutions

$$S_1 \doteq \left\{ \dot{\chi} = \arg \min_{\dot{\chi} \in S_o} \left\| A_1 (\dot{\bar{\sigma}}_1 - H_1 \dot{\chi}) \right\|^2 \right\} \tag{3.9a}$$

Now with

$$\begin{cases} \dot{\bar{\sigma}}_1 \doteq col(\dot{\bar{\sigma}}_{ij}^{1}) \\ A_1 \doteq diag(\alpha_{ij}^{1}) \ ; \forall i,j \\ H_1 \doteq col(H_{ij}) \end{cases} \tag{3.9b}$$

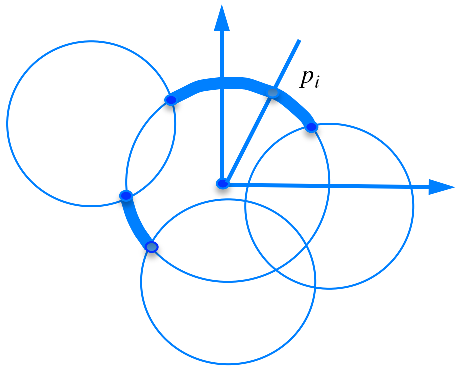
At this point, by explicitly solving the minimization in (3.9a), after some algebra (i.e. by formerly representing $\dot{\chi}$ as imposed by (3.6); then solving the minimization with respect to $\dot{z}_o$; and then returning back to $\dot{\chi}$) the conditioned solution set S_1 eventually follows, as the linear manifold

$$S_1 \doteq \left\{ \dot{\chi} = \dot{\chi}_1 + Q_1 \dot{z}_1 ; \forall \dot{z}_1 \right\} ; \begin{cases} F_1 \doteq A_1 H_1 \\ \hat{F}_1 \doteq F_1 Q_o \\ \dot{\chi}_1 = (I - Q_o \hat{F}_1^{\#} F_1) \dot{\chi}_o + \\ \quad + Q_o \hat{F}_1^{\#} (A_1 \dot{\bar{\sigma}}_1) \\ Q_1 = Q_o (I - \hat{F}_1^{\#} \hat{F}_1) \end{cases} \tag{3.10}$$

Obviously, as confirmation to what it was understood to do, since by construction $S_1 \subseteq S_o$, motion laws (3.10) certainly guarantee the achievement of the priority-one objective (connectivity keeping); meanwhile doing their best for also achieving the secondary one (safety distance keeping).
Here also note how the residual additional arbitrary vector $Q_1 \dot{z}_1 ; \forall \dot{z}_1$ can be now used for finally possibly forcing, at the lowest priority level (even if qualifying the entire mission) the vehicles transfers toward the maximum error arcs, as it will be illustrated in the next section.
Finally, as completion of the present section, the following remarks seem worth to be made.
First of all note how the two till now considered control objectives are de-facto independent from the specific distributed sampling mission; but are instead common to any mission requiring the cooperation among team of agents (e.g. patrolling, search & rescue, etc.) where connectivity and safety distance reveals important to be maintained, for both safety and operability reasons. Thus with the non negligible possibility to be used within many others different contexts.
Second of all we must not forget that for the time being the devised two laws are still of centralized nature (i.e. still to be implemented by the center vehicle); and that a preliminary discussion about their possible decentralization is however reported in section VI.

IV Transfers toward the maximum errror arcs

By passing now considering the third lowest-priority task, however qualifying the entire mission, we can start by assuming the availability (at center level, where all the geo-referenced data are made converging) of a simple algorithm capable of preliminary finding, for each surfaced vehicle (with the exclusion of the center) and on the corresponding circle $c_i \; ; i = 1,...,n$ centered on it at the end of k-th stage, the set of arcs non intersecting any one the other circles centered on the other agents (red circles in second figure); nor centered on any one relevant to previous sampling points (yellow circles in the same figure), as in the following reported other figure



Moreover we also assume the availability at central level of an additional algorithm, capable of selecting, still for each agent except the center, the arc and a point p_i on it, toward which the i-th vehicle should direct, by considering it as its next sampling location.
Such more complex additional algorithm will be developed in the next section V and, as we shall see, it will reflect as much as possible the need of sampling along not yet explored directions, which are deemed to be of "maximally preferable", accordingly with what will be suggested within the same section.
Thus, by assuming the existence of such point for each vehicle, apart the center (i.e. $i = 1,2,...,n$), still similarly to what has been done before, it will be now sufficient to consider formerly the set of reference feedback laws

$$\dot{\bar{x}}_i = \gamma_2 (\bar{p}_i - x_i) \quad ; \quad \gamma_2 > 0 \quad ; \quad i = 1,2,...,n \tag{4.1}$$

Each one corresponding to the one that would by itself drive the i-th vehicle toward its associated point p_i (note: now corresponding to a set of equality control objectives, instead then to inequality ones as before); and then proceeding by consequently establishing the set of all admissible vehicles velocity vectors (now also including the center) as the following one

$$S_2 \doteq \left\{ \dot{\chi} = \arg \min_{\dot{\chi} \in S_2} \left\| \dot{\bar{\chi}} - H_2 \dot{\chi} \right\|^2 \right\} \tag{4.2a}$$

With

$$\begin{cases} \dot{\bar{\chi}}^- \doteq col\,(\dot{\bar{x}}_i \;;\; i=1,...,n) \\ H_2 \doteq [\; 0_{2nx2} \,\vdots\, I_{2nx2n} \;] \end{cases} \tag{4.2b}$$

That via the same algebra as before now leads to the following other linear manifold

$$S_2 \doteq \left\{ \dot{\chi} = \dot{\chi}_2 + Q_2 \dot{z}_2 \,;\forall \dot{z}_2 \right\} ; \begin{cases} \hat{H}_2 \doteq H_2 Q_1 \\ \dot{\chi}_2 = (I - Q_1 \hat{H}_2^{\#} H_2)\dot{\chi}_1 \\ \quad + Q_1 \hat{H}_2^{\#} \dot{\bar{\chi}}^- \\ Q_2 = Q_1 (I - \hat{H}_2^{\#} \hat{H}_2) \end{cases} \tag{4.3}$$

Where, being the above the one corresponding to the least priority task, we can use the simplest choice $\dot{z}_2 = 0$.

Here again we can note how (4.3) results it also independent from the way the new sampling points are assigned; thus meaning that it also could be employed within different contexts.

V THE UNIFIED MOTION CONTROL STRUCTURE

By looking at the sequential construction of the devised manifolds $S_o \subseteq S_1 \subseteq S_2$, to be at each sampling time implemented via (3.6), (3.10) and (4.3), respectively; with the final choice for $\dot{\chi}$ to be performed within (4.3), we can easily see that accordingly with the introduction of the following associations, actually introduced for further unifying the used notations

$$\begin{aligned} &\dot{\bar{y}}_o \doteq \dot{\bar{\sigma}}_o \\ &\dot{\bar{y}}_1 \doteq \dot{\bar{\sigma}}_1 \\ &\dot{\bar{y}}_2 \doteq \dot{\bar{\chi}}^- \;\;;\;\; A_2 = I \end{aligned} \tag{5.1}$$

A unifying algorithmic structure, for the resulting whole motion control law, can actually be easily devised, in the terms of the following algorithm, to be applied at each sampling time.

$$\dot{\chi}_{-1} \doteq 0 \quad ; \quad H_{-1} = 0 \quad ; \quad Q_{-1} = I$$

$$for\ j = 0,1,2$$

$$F_j \doteq A_j H_j$$

$$\hat{F}_j \doteq F_j Q_{j-1}$$

$$\dot{\chi}_j = (I - Q_{j-1} \hat{F}_j^{\#} Fj)\dot{\chi}_{j-1} + Q_{j-1} \hat{F}_j^{\#} (A_j \dot{\bar{y}}_j)$$

$$Q_j = Q_{j-1}(I - \hat{F}_j^{\#} \hat{F}_j)$$

$$\dot{\chi} \doteq \dot{\chi}_3$$

As a matter of fact this does not happen incidentally; because it is instead the natural consequence of having applied task-priority based motion control procedure to a system which can be assimilated to a robotic kinematic chain (even if of very particular nature); just always leading to a uniform basic algorithmic structure as above, independently from the number and the ordering of the prioritized task (see again [8]. Thus by the way also implying that even more motion tasks, other than the considered three could actually be added, whenever necessary

VI PREFERENTIAL EXPLORATION DIRECTIONS

At the completion of the k-th field estimate upgrading, the n vehicles will appear each one centered on the associated circle $c_i \;;\; i = 1,...,n$; as on the other hand it will appear for the points corresponding to all previous sampling locations.

The surface domain contained within the envelop of all such circles will therefore constitute an estimate (in extension and shape) of the part of the assigned area which has been covered till the k-th sampling stage included.

On the other hand, on the border of each circle c_i on which each vehicle is centered, there are one or more arcs, with each one of them that can also be intended as a "door" open toward not yet explored zones; to the edge of which the vehicles must be transferred.

Then, by assuming (as on the other hand it is expected it has to typically occur) that there exists a portion of the envelope occupied by the n vehicles *adjacently* one each other; and that such portion of envelope is *convex toward the external*; then a seemingly reasonable criterion to be adopted, for selecting the arcs opened toward unexplored zones, together with their associated direction points p_i, simply consists in exploring the perimeter L of the assigned overall sampling area A, by progressively considering on it only the points x_L such that each vector $(x_L - \hat{x}_i) \;;\; i = 1,...,n$ (each one connecting the center $\hat{x}_i$ of the circle c_i relevant to the i-th vehicle) do not intersect any of the circles among those centered on the previous sampling points. Then, on such reduced perimeter $\hat{L}_n$, we shall in turn fix $\hat{x}$ as the point such thatsuch that

$$\hat{x} = arg \max_{x \in \hat{L}_n} \left(\min_i \left\| x - \hat{x}_i \right\|^2 ; i = 1,...n \right) \tag{6.1}$$

By translating into more simple terms, what has been above proposed, substantially means that we must try to find ,on the perimeter L, the "point of view" exhibiting the maximum distance from the vehicles, from which point the departing "rays" $(x_L - \hat{x}_i) \;;\; i = 1,...,n$ can "light" each vehicle; that is without having any one of them interrupted by any other circle (i.e. among those relevant to the other vehicles, as well as to previous sampling locations).

Such sort of "optical analogy" actually represents the principle that inspired what has been above proposed.

Then, from the knowledge point $\hat{x}$ the various points $p_i \;;\; i = 1,...,n$ located on the consequently so selected n arcs, immediately follow.

Naturally enough, in case it were instead resulting $\hat{L}_n = \varnothing$ It would means that during the perimeter exploration at least a vehicle (not necessarily always the same) which results "shadowed" whatever is the point of view on the perimeter

But should this sometime occur, the perimeter exploration could however be repeated, for alternatively now searching for different sub-perimeters, each one clustering different vehicles, not necessarily in a disjoint form, for each of which a point of type (5.1) can consequently be found.

After that, a set of disjoint clusters (whose union must however however include all vehicle) can always be extracted from the previous, possibly superimposed, ones; for then separately proceeding for each one of them.

In such cases the different clusters will be consequently addressed toward different preferred exploration directions.

Nevertheless this will never give rise to any risk about loss of connectivity or violation of the safety distances, just because these objectives are always maintained at their higher priorities for *all* vehicles.

Anyway, it might even occur that no clusters can actually be found. This may for instance happen when the team, after different sampling stages, eventually get very close to the perimeter, possibly with very many of the previous circles located "behind its back"; thus in a way that it can no longer way out, via the proposed policy.

In this occurrence it is then clear that the use of proposed policy has to be stopped; for letting space to the forced block-transfer of the entire team toward zones located beyond the already sampled one (by passing on them without anymore sampling), for then restarting again with the same policy.

Moreover it is here also al deemed, in case of some previous sampling stages for some reason executed without employing the here suggested policy, that they also might leave some vehicle (if not all) without any possibility of exiting out; thus similarly requiring a forced transfer of these lasts

The development of such additional forced transfer policies, whenever required, is for the moment however let outside the scopes of the present work. Meanwhile the foreseen possibility of their occasional employment is here interpreted as a confirmation of the fact that, also for the considered application, the popularly known "no free lunch theorem" is however always lurking.

Moreover, among the possible causes rising additional problems, we could even formally include also the pathological case (however largely unrealistic, from a practical point of view) of so large radii of the circles $c_i\ ; i = 1,...,n$, also joined with a so small connectivity distance bound d_M, that the associated preferred next sampling points $p_i\ ; i = 1,...,n$ could actually not resukt connectible by the graph G^k (the one to be maintained during the transition to the next stage) without violating the connectivity bounds. Thus, in case of such very pathological situation, it is quite clear how the highest priority task of connectivity keeping will consequently prevent the team from reaching all established next sampling points. If this were however the so unlikely case (however straightforwardly detectable by the center) different heuristic policies could even be proposed, for exiting from the resulting impasse: for instance by trying other graphs; while also possibly temporarily imposing the suitable reduction of some of radii, etc. Obviously the managing of such so rare occurrences, even for sake of completeness, is it also out of the scopes of the present work

At this point of the discussion it seems finally worthwhile noticing how the computational burden required for implementing the above suggested general policy, is de-facto only apparently high; because the optical analogy used for its explanation clearly also shows how it can instead result very simple, at least whenever compared with the most standard accelerate graphical applications now available many day-to-day used common devices (tablets, smart-phones, etc.).

VII Possibilities of decentralization

The entire set of developed control algorithms has been proposed by still adopting a centralized point of view; where at center level (at the end of each stage, after the acquisition od all collected geo-referenced data) the preferred exploration directions for each vehicle (and related preferred next sampling point) are preliminary evaluated; after that all subsequent vehicles motions are they also real-time managed by the center itself, within the established priority list.

Obviously enough, in this centralized assumption, the motion management might risk to require an unsustainable full duplex communication effort between the center and the set of vehicles; or among the vehicles in case centralized motion control were instead simply replicated on board of each vehicle.

An alternative for reducing such communication effort could for instance be that of pre-simulating in an accelerated way the overall motion control at central level; and then transmitting to each agent only some key way-points extracted from the resulting simulated trajectories; to be then independently tracked by each vehicle.

Such an implementation, rather simple and very much communication saving, would however result into a rather traditional strategy composed by a preliminary *planning phase*, then followed by an *open-loop execution phase*, with respect to possible perturbation changes incurring in the current vehicles configuration (for instance, local marine current might actually perturb the connectivity and/or safety distances among the agents); to which the team should react on the basis of the current and measured global state of such distances; and not instead indirectly, with each vehicle reacting independently from the others, only to changes with respect to its own pre-planned trajectory.

Though this solution should however guarantee good performances, at least within the most common cases of moderate currents, the real alternative for tackling the possibilities of decentralization, by still remaining within a full closed loop context, should consist in fully exploiting the great sparsity exhibited by the various matrices involved in the motion control part; that should consequently allow a downgrading toward a decentralized structure where, even at the cost of some slight loss of performances, each vehicle only has to communicate with its closet neighbors, instead than with all others.

This will consequently constitute the next methodological step to be executed within the subject.

VIII CONCLUSIONS

The paper has presented the basic ideas underlying the algorithmic development of a task priority based approach to the control of distributed data-driven ocean sampling system applications.

Such an approach is deemed to be noticeable for different reasons; mainly including the following: a) it provides a clearer formalization of the overall problem; b) it also provide a unified algorithmic structure for the entire motion control of the involved team of vehicle; c) the high sparsity of the involved system matrices should allow for efficient decentralizations of the overall motion control policy.

Preliminary validating simulations will therefore be the subject of future works, together with future studies devoted to the development of suitable decentralized control architectures, downgraded from the centralized one; to be they also validated by relevant simulation activities, before performing any field trial at sea.

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