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Forecasting and pricing powers of option-implied tree models:

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Abstract: The aims of this paper are twofold. First, to investigate the accuracy of different option-implied trees in pricing European options in order to assess the power of implied trees in replicating the market information. Second, to compare deterministic volatility implied trees and stochastic implied volatility models (Bakshi et al. (2003)) in assessing the forecasting power of implied moments on subsequently realised moments, and ascertaining the existence, magnitude and sign of variance, skewness, and kurtosis risk-premia. The analysis is carried out using the Italian daily market data covering the period 2005-2014. This enables us to contrast the pricing performance of implied trees and to assess the magnitude and sign of risk premia in both a tranquil and a turmoil period.

The findings are as follows. First, the pricing performance of the Enhanced Derman and Kani (EDK, Moriggia et al. 2009) model is superior to that of the Rubinstein (1994) model. This superiority is stronger especially in the high volatility period due to a better estimation of the left tail of the distribution describing bad market conditions. Second, the Bakshi et al. (2003) formula is the most accurate for forecasting skewness and kurtosis, while for variance it yields upwardly biased forecasts. All models agree on the signs of the risk premia: negative for variance and kurtosis, and positive for skewness, but differ in magnitude. Overall, the results suggest that selling (buying) variance and kurtosis (skewness) is profitable in both high and low volatility periods.

*Keywords**:** risk management; pricing; forecasting; option-implied trees; risk-neutral moments.*

JEL classification: G13, G14.

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1. Introduction

Inverse and ill-posed problems are studied in various domains of science ranging from astronomy, physics, geophysics, medicine, and ecology to economics. Inverse and ill-posed problems are usually unstable and non-linear and, as a result, they are among the most difficult problems in operations research. A direct problem models a phenomenon by means of an equation and finds the solution to the equation. An inverse problem requires the identification of the equation coefficients, given that the solution to the equation is known. Direct problems are usually well-posed, i.e. they allow for a solution, which is unique and stable in the sense that it is not too sensitive to perturbation of the coefficients. On the other hand, inverse problems can be ill-posed in the sense that they may allow for multiple solutions and may be unstable with respect to perturbation in the data. This instability is undesirable for investors, since hedging ratios and portfolio weights may dramatically change with a small perturbation in the data.

A number of papers address the issue of inverse and ill-posed problems in finance. The majority of these papers deal with the inverse problem of estimating the local volatility in a Black–Scholes partial differential equation. One of the first contributions in this area is Dupire (1994), who shows that option prices given for any possible strike price and maturity completely determine the local volatility function. However, Dupire's formula cannot be used in practice because it is ill-posed. Cases of pure time-dependent and state-dependent volatility are addressed in Bouchouev and Isakov (1999) and Hein and Hoffman (2003), respectively. Egger and Engl (2005) investigate stable identifications of the Black-Scholes/Dupire (1994) equation, thanks to Tikhonov's regularization of the least squares formulation of the inverse problem. Hanke and Rosler (2005) focus mainly on the computational aspects, whereas Deng et al. (2008) adopt an optimal control framework to solve the inverse problem of deriving the implied volatility smile from a set of option prices with different times to maturity. Their approach is purely theoretical and difficult to implement with real option prices. The difficulties arise from the assumption of a limited number of strike prices and maturities, relative to the options traded in the market, in order to have local uniqueness. In order to price a claim with an arbitrary payoff (a volatility swap and a call on variance), Fritz and Gatheral (2005) show that the estimation of the weights used for a replicating portfolio of options is an inverse and ill-posed problem: the weights do not depend on the payoff function in an effective manner and may not be well-defined. Interestingly, the literature focuses on the theoretical aspects of the problem, paying little attention to empirical investigation. As a result, many proposed methods (e.g. Deng et al. (2008)) remain difficult to apply in practice.

The problem addressed in this paper is similar in nature to the literature on inverse problems. In our case the direct problem is how to price a set of options given the known risk-neutral distribution of the underlying asset. The inverse problem can be stated as follows: given the prices of a set of options written on the same underlying asset, what is the risk-neutral distribution of the underlying asset that would be consistent with the given set of option prices? To answer this question, we calibrate option-implied trees to the set of option prices and then extract the risk-neutral variance, skewness and kurtosis from the risk-neutral distribution implied by the trees. The inverse problem is addressed both in cases of a deterministic volatility setting (as in implied trees models of Derman and Kani (1994) and Rubinstein (1994)), and a stochastic volatility setting (Bakshi et al. (2003) formula). Risk-neutral moments are closely linked to the shape of the implied volatility smile (Neumann and Skiadopoulos (2013), Zhao et al. (2013)). For example, a steep implied volatility smile results in a highly negatively skewed model free risk-neutral distribution, while a high volatility attached to the extremes of the smile is associated with a high value of the kurtosis of the risk-neutral distribution. While for the purpose of option pricing we will focus on deterministic volatility models, given their simplicity with respect to stochastic volatility models, for the purpose of deriving the moments of the risk-neutral distribution, we investigate both the deterministic volatility model embedded in the implied trees and the stochastic volatility model of Bakshi et al. (2003).

The aim of the paper is twofold. First, to investigate the power of option-implied trees (constructed either by forward or backward induction) in pricing European options, in order to assess the proper representation of the smile. Second, to compare implied moments obtained with the use of option-implied trees with model-free moments obtained using the Bakshi et al. (2003) formula, in order to determine which method better forecasts the realized moments. Moreover, implied moments are used to assess the existence of variance, skewness and kurtosis risk premia that signal the potential profitability of variance, skewness and kurtosis trades. Variance, skewness and kurtosis trades may yield up to 20% per 100 Euro notional.

The comparison is pursued in the Italian market by analysing a data set consisting of daily data covering the 2005-2014 period, spanning both relatively tranquil periods (2005-2007 & 2013-2014) and the turmoil period of 2008-2012 (see also Albano et al. 2013). The Italian derivatives market (IDEM) is of great importance in Europe, trading around 200,000 contracts daily, corresponding to 3.5bn Euros of value traded. This market consists of more than 80 members, directly connected to the markets in Italy, the United Kingdom, France, The Netherlands, Germany, Luxembourg, Ireland, Belgium, the Czech Republic and Spain. The FTSE MIB index (FTSE Milano Indice Borsa) is the benchmark index for the Italian equity market, capturing approximately 80% of domestic market capitalization. The Italian market is an ideal setting to investigate the pricing performance of the different option-implied trees due to the high market stress conditions occurring within our dataset. The FTSE MIB index suffered a collapse of about 70% after the subprime bubble burst and a further decline of 50% during the European debt crisis. These features make the Italian market attractive for the study of both the pricing performance of implied trees, and tail risk measures such as skewness and kurtosis.

To the best of our knowledge, this is the first attempt to compare the implied moments and the risk premia obtained in a deterministic volatility model (implied trees) with the Bakshi et al. (2003) formula, which is consistent with a stochastic volatility setting and is considered by investors as the market standard. Moreover, given that the level (i.e. volatility) and the slope (i.e. skewness) of the implied volatility curve is closely linked (see e.g. Kozhan et al. (2013)), another aim of the present paper is to cast light on the pricing performance of option implied trees, the estimation of implied volatility and higher moments and the assessment of variance, skewness and kurtosis risk premia in both high and low volatility periods. Given the paucity of empirical applications of methods of solving the inverse problem of recovering the smile function and the risk-neutral distribution of the underlying asset (see e.g. Kabanikhin (2008)), the results of the present paper are of paramount importance for both investors and decision-makers.

The paper proceeds as follows. In section 2 we provide an updated literature review on option-implied trees (subsection 2.1) and implied moments (subsection 2.2). In section 3 we briefly describe the models used (Enhanced Derman and Kani in subsection 3.1 and Rubinstein in subsection 3.2). Section 4 outlines the computation of the risk-neutral moments and Section 5 describes the data (subsection 5.1) and the methodology (subsection 5.2). Section 6 contains the pricing performance results, investigated both for the entire sample period (subsection 6.1) and during high and low volatility periods (subsection 6.2). Section 7 discusses the forecasting power of implied moments, along with the assessment of variance, skewness and kurtosis risk premia, for the entire sample period (subsection 7.1) as well as in high and low volatility periods (subsection 7.2). The last section concludes.

2. Option implied trees and implied moments: Literature review

In this section we provide a detailed and updated literature review covering option-implied trees and the estimation of implied moments. We divide the review into two parts. The first part covers the forecasting performance of option-implied trees where many of the contributions present in the literature are reviewed. The second part covers the use of option-implied trees to derive the implied moments of the underlying asset distribution. This is a topic of current interest: our assessment shows that the existing work in the area of implied moments is limited and the scant available evidence on the subject is mixed in terms of the existence and sign of the risk premia. The limited empirical evidence on pricing and forecasting powers on future moments of option-implied trees provides the motivation for our study.

2.1 Option-implied trees

Implied trees are simple non-parametric discretizations of one- or two-dimensional diffusions, aimed at introducing non-constant volatility into option pricing models. To elaborate, let us assume that the stock price evolution is described by the following stochastic differential equation:

$$\frac{dS_t}{S_t} = \mu dt + \sigma(S,t) dZ_t \tag{1}$$

where S_t is the stock price, μ the expected return, $\sigma(S,t)$ is the local volatility function that is dependent on stock price and time, and dZ is a Wiener process with mean zero and variance dt. Instead of deriving $\sigma(S,t)$ by assuming a specific parametric form, the option-implied trees derive it numerically from the smile by requiring the option prices calculated from the model to fit the smile. The models are thus called smile-consistent models and their purpose is both to look at the distribution of the underlying asset return in the future for risk management purposes, and to price US and other exotic options consistently with traded European options.

There are two basic approaches for the construction of a deterministic volatility model: backward and forward induction as described earlier (an excellent literature review on option pricing is provided by Broadie and Detemple (2004)). Among the studies based on forward induction implied trees, Derman and Kani (1994) are the first to construct an option-implied tree consistent with the observed smile. One drawback of the Derman and Kani (DK) model is the possible occurrence of negative risk-neutral probabilities that denote arbitrage violations with their presence adversely affecting the performance of the implied tree in reproducing the smile.

Several authors have modified and extended the Derman and Kani (1994) model in an attempt to address this shortcoming, but they do not appear to have been successful. For example, Barle and Cakici (1998) introduce some modifications to the Derman and Kani implied tree, but do not resolve the negative probability problem at the edge of the tree, in particular in cases of high interest rates and pronounced smile functions. Derman et al. (1996) propose a trinomial tree offering a better adaptation to the real data than a binomial model, but presenting problems in the choice of the state space and not free from arbitrage violations in cases of high volatility. Charalambous et al. (2007) propose a non-recombining tree that is more flexible than a recombining tree, but complex from the computational point of view. More specifically, recombining trees have a substantially lower number of nodes, since after an up (down) and a down (up) movement the same node is reached. Finally, Moriggia et al. (2009) propose a check for the presence of arbitrage violations and a procedure to substitute the nodes which imply arbitrage violations throughout the entire tree. This last attempt is successful in erasing arbitrage opportunities, even if ad-hoc substitutions may limit the exact smile replication.

Among backward induction models, Rubinstein (1994) is the first to propose a two-step procedure for the derivation of the tree. In the first step, he estimates the ending nodes and probabilities. In the second step, he derives the tree by adopting a backward approach based on the hypothesis that the paths leading to the same ending node are equally probable. To elaborate, in the first step, a constrained optimization problem is solved in order to minimize the distance between prior density and the real density. The constraint requires market prices to match model prices under the no-arbitrage condition. This model suffers from at least two drawbacks: it cannot price options expiring before the maturity of the tree and it cannot price options that are path-dependent. In order to overcome these limitations, Hilliard et al. (1995) develop backward iteration methods for pricing certain types of path dependent securities. Along the same lines, Jackwerth (1997) proposes weighting the different paths in the tree in order to price path-dependent options. For path-dependent options, we also refer to the direct approach in Wong and Lo (2009), who adopt a bivariate trinomial lattice in order to price both barrier and American options in a stochastic volatility framework. Brown and Toft (1999) and Herwig (2005) outline different methodologies in order to calibrate Rubinstein's tree to intermediate maturity options. Tian (2015) also describes a methodology to diminish the number of unknowns in Rubinstein's optimization problem by grouping the ending nodal probabilities and using cubic spline smoothing. In addition, Costabile et al. (2014) show how to include parameters following a regime-switching model in a standard binomial setting. We argue that the Costabile et al. (2014) method is not an implied tree model, since it uses the standard Cox, Ross and Rubinstein model. The optimization process introduces a degree of freedom in the choice of the prior distribution, which could represent an issue for the practical use of the model. Bandi and Bertsimas (2014) propose a model to price a variety of options based on the combination of a robust optimization scheme and the idea of ϵ-arbitrage outlined by Bertsimas et al. (2001). Finally, given that it seems that an additional degree of freedom is needed in option-implied trees to include uncertainty in volatility, mention should be made of the attempts by Muzzioli and Torricelli (2000, 2002, 2005), who were the first to include uncertainty in forward-induction option-implied trees.

From the empirical point of view, the evidence about the pricing performance of different implied trees is limited. Dumas et al. (1998) show that option-pricing models based on deterministic volatility functions perform no better than ad-hoc procedures of smoothing Black–Scholes implied volatilities across strike prices and maturities in out-of-sample pricing and hedging. In practice, however, deterministic volatility functions are used more often for pricing options in-sample than out-of-sample: typically, an implied binomial tree is fitted to the cross-section of all liquid options to price other American or exotic options. Forward-induction implied trees are analysed in Brandt and Wu (2002) and Linaras and Skiadopoulos (2005). They compare the performance of the Derman and Kani (1994) and Barle and Cakici (1998) implied trees with different smile estimations in the UK and US markets (S&P100 options). They report mixed results that vary according to the moneyness of the option and the option type. With respect to time to maturity, they find that pricing errors decrease as time-to-maturity increases. Moreover, among the different interpolation schemes they find smaller errors using linear interpolation compared to the cubic spline one.

Lim and Zhi (2002) and Kim and Park (2006) analyze the pricing performance of the backward-induction implied trees of Rubinstein (1994) and Jackwerth (1997) by examining the UK and the Korean markets, respectively. Lim and Zhi (2002) use both American and European types of option. They find that both forward- and backward-induction implied trees fit European calls relatively well, but produce much larger errors in pricing otherwise identical European puts. While Derman and Kani (1994) is the best model in terms of smallest errors for near-term and intermediate maturities, Jackwerth (1997) is the best for longer-term maturities. The Derman and Kani (1994) model produces smaller deltas[1] for both European and American calls, while Jackwerth's (1997) deltas are much larger. Kim and Park (2006) compare the pricing performance of backward- and forward-induction trees for KOSPI 200 index options and find that short-term options are better priced in forward induction implied trees than in the CRR binomial tree. They also indicate the existence of a negative market volatility risk premium in the Korean market consistent with the findings by Bakshi and Madan (2006) in S&P 500 index options market.

Finally, Moriggia et al. (2009) test the performance of the Enhanced Derman and Kani (EDK) model comparing it to the Barle and Cakici (1998) implied tree in the Italian index options market. The results show that the Enhanced Derman and Kani model performs better in terms of pricing, and that the improvement in pricing performance can be attributed to an improved pricing of far-from-the-money options and a lower number of no-arbitrage violations. As a result, the EDK model is the simplest (based on Derman and Kani (1994)) and the most accurate model among forward-induction implied trees. For this reason the EDK model was taken as the representative of forward-induction implied trees and implemented in the empirical part of this study. For backward-induction implied trees, the Rubinstein (1994) model was adopted, since it is the simplest and fastest among the backward-induction methods. Moreover, given that the options under investigation are European in origin, more elaborate methods for pricing path-dependent options are unnecessary.

To sum up, the findings of these papers draw dissimilar pictures of the competition between

[1] Measurement error

backward- and forward-induction implied trees, making it difficult to assess the superiority of one over the other in absolute terms. For medium and long maturity options, the majority of the papers prefer forward-induction implied trees to backward-induction ones. For short maturity options, there is no clear pattern, since the performance of backward- and forward-implied tree models is dependent on moneyness and type of the options being priced. It must be stressed that in this paper short maturity options are used, in order to supplement the literature with our results. All the papers reviewed here agree on the close relationship between the characteristics of the smile function and the shape of the risk-neutral distribution of the underlying asset in terms of variance, skewness and kurtosis, and this is investigated in the next subsection.

2.2 Option-implied moments

Estimation of the risk-neutral distribution of asset returns at a future date is of paramount importance for investors since they can use the information for portfolio optimization and risk management. The risk-neutral distribution, or implied distribution, is the distribution that the market attributes to the underlying stock price in the future through quoted options written on the asset (Giamouridis and Skiadopoulos, 2012). Having a reliable estimate of future asset return distribution is important for investors because it helps them understand what the market sentiment or the market view is about the underlying asset, the business sector of the company, or the country. Indeed, using options' market prices to estimate the risk-neutral distribution means using the most recent information available, yielding a more accurate forecast of the relevant moments.

Several studies investigate the relationship between the volatility surface and the risk-neutral distribution of asset returns. Jackwerth and Rubinstein (1996) argue that if we observe a sufficient number of option prices, all estimation methods tend to produce similar estimates of the central part of the distribution, though they differ in the modelling of the distribution tails. Campa et al. (1998) prefer the implied-tree approach for estimating the risk-neutral distribution, to other non-parametric methods, due to its flexibility in allowing different smile representations and good representation of the data. However, they highlight the presence of outliers in the tails of the Rubinstein (1994) tree that could distort the pricing performance (based on the risk-neutral distribution options to be priced).

Zhang and Xiang (2008) theoretically present a relationship between the risk-neutral standard deviation, skewness and kurtosis and the implied volatility smirk based on a model-free formula. In particular they claim that changes in risk-neutral volatility, skewness and kurtosis are associated with the level, slope and curvature of the implied volatility surface. Neumann and Skiadopoulos (2013) study the relationship between higher moments and the volatility surface and find that the dynamics of the volatility surface are predictable on the basis of the implied higher moments.

A number of recent studies show that higher moments are indeed useful in the assessment of risk and for devising trading strategies. Grubbstrom and Tang (2006) provide general closed formulae for computing higher-order moments of a compound distribution, with the only limitation that the random variable needs to be non-negative. More specifically, skewness is used for measuring the probability of returns two or more standard deviations below the mean, and kurtosis is employed to assess the weight of the tails. In addition, some studies investigate the dynamics of risk-neutral moments and highlight the possibility of adopting profitable trading strategies based on forecasted moments. For instance, investors could use the implied moments to predict future returns of assets (Han (2008), Bali and Hovakimian (2009)), or to design profitable trading strategies, based on forecasted moments (Neumann and Skiadopoulos (2013)). Rehman and Vilkov (2012) show that the currently observed option implied ex-ante skewness is positively related to future stock returns. Moreover, by examining the dynamics of the implied moments in an autoregressive model, they show that these moments are linked to one another over time.

Others, study the estimation and predictability of implied moments for individual stocks, and find that they can be explained by a number of firm characteristics. Theodossiou and Savva (2015) find empirical evidence that contradict the theoretical positive relation between risk and return as result of negative skewness in the distribution of portfolio excess return. Along similar lines, Zhang and Xiang (2008) show that changes in risk-neutral volatility, skewness and kurtosis are related to the level, slope and curvature of the implied volatility surface and these relationships can be used to better calibrate option pricing models. Finally, Kang et al. (2010), by incorporating the risk preferences of investors, propose a new volatility forecast embedding variance and higher order risk-neutral moments. They show that this forecast is superior to other estimates in terms of prediction power over subsequently realized volatility. Finally, some studies find a relationship between risk-neutral moments and the monetary policy (see Bekaert et al. (2013), Bruno and Shin (2015)).

Although the existence of a variance risk premium is investigated in a number of studies (e.g. Carr and Wu (2009) and Bakshi and Madan (2006)), the evidence about skewness and in particular kurtosis risk premiums remains limited and mixed. The skewness risk premium is computed in different ways (the difference between risk-neutral and physical moments (Zhao et al. (2013), a generalization of the variance swap to a skewness swap (Kozhan et al. (2013), the difference between upside and downside variance risk premiums (Fenou et al. (2015)). It should be noted that Bali and Murray (2013) use trading strategies to investigate the pricing of risk-neutral skewness,[2] highlighting the existence of a negative skewness risk premium, and indicating a preference for assets with positively skewed return distributions by investors. All the different approaches produce results consistent with a risk-neutral distribution more negatively skewed than the physical one, pointing to a negative skewness risk premium. Moreover, Kozhan et al. (2013) show that the skewness risk premium is closely related to the variance risk premium: they both vary over time and are driven by a common factor (strategies to capture one and hedge out exposure to the other generate insignificant trading profits). They only investigate variance and skewness, not kurtosis and they find that writing variance (skewness) swap results on average in a profit (loss). Chang et al. (2013) investigate the pricing of volatility, skewness and kurtosis risks and find a negative skewness risk premium, but a less clear-cut result on market volatility and kurtosis. Zhao et al. (2013) find a significantly negative kurtosis risk premium. These findings suggest the presence of profitable variance, skewness and kurtosis trades, even if the sign of the risk premia is debatable.

3. Description of the implied tree models.

In the following, we briefly describe the Enhanced Derman and Kani method and the Rubinstein method. These methods will then be used in the empirical section to estimate the underlying asset risk-neutral

[2] Risk-neutral variance, skewness and kurtosis are the second, third and fourth moments of the risk-neutral distribution of the underlying asset return which is implied from option prices, while physical variance, skewness and kurtosis are the same moments obtained from the empirical time-series of the underlying asset return.

distribution and price options with regard to the Rubinstein method, to compute implied moments and to assess the existence of variance, skewness and kurtosis risk premia.

3.1. The Enhanced Derman and Kani (EDK) implied tree

Derman and Kani (1994) construct an implied tree using forward induction for the purpose of pricing other options consistently with the options used for the derivation of the implied tree. They build a recombining binomial tree which uses as inputs the market prices of European-style index options across all strikes and expirations. Their model has uniformly spaced levels Δt apart. Let $j=0,...,n$ be the number of levels of the tree, that are spaced by Δt. As the tree recombines, $i=1,...,j+1$ is the number of nodes at level j. Let us assume that the tree has already been constructed up to the current level $j-1$. We wish to derive the stock prices for the next period j. The known stock price $S_{i,j-1}$ prevailing over this time period can evolve in two states in the next period j: the up one, $S_{i+1,j}$, and the down one, $S_{i,j}$. The risk-neutral probability of an up-jump is $p_{i,j}$. This probability is defined as: $p_{i,j} = \frac{F_{i,j}-S_{i,j}}{S_{i+1,j}-S_{i,j}}$, where $F_{i,j}$ is the forward price. Arrow-Debreu prices $\lambda_{i,j}$, defined as the prices of a security that pays one euro in state (i,j) and zero in all the other states, and can be used to discount the payoff of the option in each state, are derived as the sum over all paths leading to node (i,j) of the product of the risk-neutral probabilities discounted at the risk-free rate at each node. If the level is odd, the centering condition is given by equation (2), whereas if the level is even, the two central nodes will have to satisfy equation (3):

$$S_{\frac{j}{2}+1,j} = S_{0,0} \tag{2}$$

$$S_{j/2,j} = \frac{S_{0,0}}{S_{\frac{j}{2}+1,j}} \tag{3}$$

The centering condition allows the tree to grow around the current spot price S_0. It ensures that the final nodes of the tree are sufficiently dispersed around the current spot price. Let $C_{i,j-1}$ and $P_{i,j-1}$ be the price of a call and a put with strike $S_{i,j-1}$ and maturity j, respectively. These prices are computed using Black-Scholes formulae with constant volatility obtained from the smile function. In the upper part of the tree, the recursive formula used to compute $S_{i+1,j}$, given $S_{i,j}$, can be described as (4). Similarly, in the lower part of the tree the recursive formula to compute $S_{i,j}$, given $S_{i+1,j}$, can be described as (5):

$$S_{i+1,j} = \frac{S_{i,j}[e^{r\Delta t}C_{i,j-1}-\Sigma_C]-\lambda_{i,j-1}S_{i,j-1}(F_{i,j-1}-S_{i,j})}{[e^{r\Delta t}C_{i,j-1}-\Sigma_C]-\lambda_{i,j-1}(F_{i,j-1}-S_{i,j})} \tag{4}$$

$$S_{i,j} = \frac{S_{i+1,j}[e^{r\Delta t}P_{i,j-1}-\Sigma_P]-\lambda_{i,j-1}S_{i,j-1}(F_{i,j-1}-S_{i,j})}{[e^{r\Delta t}P_{i,j-1}-\Sigma_P]-\lambda_{i,j-1}(F_{i,j-1}-S_{i,j})} \tag{5}$$

where r is the risk-free rate, $\Sigma_C = \sum_{k=i+1}^{i}\lambda_{k,j-1}(F_{k,j-1} - S_{i,j})$, $\Sigma_P = \sum_{k=1}^{i-1}\lambda_{k,j-1}(F_{k,j-1} - S_{i,j})$ and $F_{i,j-1}$ is the forward value of $S_{i,j-1}$.

These equations can be used only if the level j is odd. If level j is even, we combine equations (3) and (4) to obtain (6):

$$S_{i+1,j} = \frac{S_{0,0}[e^{r\Delta t}C_{i,j-1}+\lambda_{i,j-1}S_{0,0}-\Sigma_C]}{\lambda_{i,j-1}F_{i,j-1}-e^{r\Delta t}C_{i,j-1}+\Sigma_C} \tag{6}$$

The transition probability $q_{i,j-1}$ of an up move is computed as equation (7):

$$q_{i,j-1} = \frac{F_{i,j-1} - S_{i,j}}{S_{i+1,j} - S_{i,j}} \quad (7)$$

The main problem in the derivation of the implied tree is the presence of risk-free arbitrage opportunities, represented by a risk-neutral probability falling outside the (0, 1) interval. The Derman and Kani (1994) implied tree, even with the Barle and Cakici (1998) modifications, is not free from arbitrage, in particular at the boundary of the tree. Moreover, this tree may become numerically unstable when the number of steps becomes large. As a result, we adopt the Enhanced Derman and Kani (EDK) method to ensure the absence of no-arbitrage violations in the Derman and Kani implied tree. The EDK method provides no-arbitrage checks and proposes no-arbitrage replacements for all the nodes in the tree (for further details see Moriggia et al. (2009)). This method enhances the Derman and Kani model to determine option prices and implied moments free of arbitrage.

3.2. Rubinstein's implied binomial tree

Rubinstein (1994) proposes an implied binomial tree using backward induction in order to incorporate the smile into an option pricing model. Rubinstein's procedure can be divided into two steps. In the first step the risk-neutral probability distribution of the underlying asset at the end of the tree is estimated. In the second step, the tree is derived using a backward technique with a simple three-step algorithm. Rubinstein's method consists of minimizing the square difference between prior and posterior risk-neutral probabilities, under certain constraints. Let us define $Q_{i,n}$ and $Q_{i,n}^{'}$, respectively, as the posterior and the prior probability of arriving at node (i,n) at expiry date n. The posterior ($Q_{i,n}$) can be derived as the solution of the following optimization problem where we minimize the sum of squared deviations between prior and posterior probabilities:

$$\min \sum_{i=0}^{n} (Q_{i,n} - Q_{i,n}^{'})^2 \quad (8)$$

subject to: $\sum_{i=0}^{n} Q_{i,n} = 1 \;\; and \;\; Q_{i,n} > 0 \quad for\ i = 0, \dots, n, C_k^b \leq C_k \leq C_k^a,\ S^b \leq S \leq S^a$ (9)

where C^b and C^a are, respectively, the option bid and ask price quotes observed for the European call with strike K_k with $k = 1, \dots, m < n$, expiring at n, and S^b and S^a are the bid and ask prices of the underlying asset, C_k is the price of a call with maturity n and strike price K_k:

$$C_k = e^{-rn} \sum_{i=0}^{n} Q_{i,n} \left(S_{i,n} - K_k\right)^{+} \quad (10)$$

and $S_{0,0}$ is the value of underlying asset at time 0:

$$S_{0,0} = e^{-rn} \sum_{i=0}^{n} Q_{i,n} S_{i,n} \quad (11)$$

The posterior implied risk-neutral probabilities are called nodal probabilities since $Q_{i,n}$ is the probability to reach node i at expiry n regardless of the path to reach that node. The rather arbitrary and restrictive assumption of equal path probabilities makes it possible to build the tree in a simple way with a three-step procedure. First, calculate the nodal probabilities at the preceding nodes as described by equation (12). Second, compute the probability of an upward move over the next time interval as described by equation (13). This process rules out negative probabilities. Finally, compute the stock price at the preceding level with the risk-neutral valuation formula as described by equation (14). The implied tree is derived by repeating this simple algorithm up to the first node, $S_{0,0}$.

$$Q_{i-1,n-1} = (1 - w_{i,j}) * Q_{i-1,n} + w_{i,j} * Q_{i,n} \quad (12)$$

$$q_{i-1,n-1} = w_{i,j} * \frac{Q_{i,n}}{Q_{i-1,n-1}} \quad (13)$$

$$S_{i-1,n-1} = e^{-r\Delta t}[(1 - q_{i-1,n-1}) * S_{i-1,n} + q_{i-1,n-1} * S_{i,n}] \quad (14)$$

Despite its simplicity and the absence (by construction) of arbitrage opportunities, the Rubinstein model presents some drawbacks: it requires a costly optimization routine, with different possible outcomes depending on the choice of the prior distribution and it is suitable only for European-type options. However, given the characteristics of our data-set (consisting of European-type options), and the aim of our exercise (pricing plain-vanilla options) the model is consistent with the purpose.

4. Derivation of implied moments of the underlying assets

In order to obtain implied moments for the distribution of the underlying asset return, we follow two alternative methods. First, we derive them by using the risk-neutral density estimated with the implied trees described in Section 3. Second, we compute directly the implied moments by using the model-free (Bakshi et al. (2003)) formula. In the first method, implied moments are computed as integrals of the risk-neutral density estimated with the implied trees as follows:

$$m_\alpha = \int_{-\infty}^{\infty} x^\alpha f(x)dx \quad (15)$$

where $\alpha = 1,2,3,4$, $x = ln\frac{S_n}{S_t}$ and $f(x)$ is the risk-neutral density of the variable x. As the implied tree yields a discrete cumulative distribution, a discrete summation over all nodes approximates the continuous integral in formula (15). With these moments, variance, skewness and kurtosis are easily obtained as follows:

$$VAR(t,n) = m_2 - m_1^2 \quad (16)$$

$$SKEW(t,n) = \frac{m_3 - 3m_1m_2 + 2m_1^3}{(m_2 - m_1^2)^{3/2}} \quad (17)$$

$$KURT(t,n) = \frac{m_4 - 4m_1m_3 + 6m_1^2m_2 - 3m_1^4}{(m_2 - m_1^2)^2} \quad (18)$$

In the second method, the Bakshi et al. (2003) formula is used. Bakshi et al. (2003) developed a model-free method in order to extract volatility, skewness and kurtosis of the risk-neutral distribution on the expiry date from a cross-section of call and put option prices. Their method is called model-free because it is consistent with many underlying asset dynamics. Model-free variance, skewness and kurtosis are obtained from the following equations:

$$VAR(t,n) = e^{rn}V(t,n) - \mu(t,n)^2 \quad (19)$$

$$SKEW(t,n) = \frac{e^{rn}W(t,n) - 3e^{rn}\mu(t,n)V(t,n) + 2\mu(t,n)^3}{[e^{rn}V(t,n) - \mu(t,n)^2]^{3/2}} \quad (20)$$

$$KURT(t,n) = \frac{e^{rn}X(t,n) - 4e^{rn}\mu(t,n)W(t,n) + 6e^{rn}\mu(t,n)^2V(t,n) - 3\mu(t,n)^4}{[e^{rn}V(t,n) - \mu(t,n)^2]^2} \quad (21)$$

with

$$\mu(t,n) = e^{rn} - 1 - \frac{e^{rn}}{2}V(t,n) - \frac{e^{rn}}{6}W(t,n) - \frac{e^{rn}}{24}X(t,n) \quad (22)$$

$$V(t,n)=\int_{S_t}^{\infty}\frac{2\left(1-ln\left[\frac{K}{S_t}\right]\right)}{K^2}C_{t,n,K}dK+\int_{0}^{S_t}\frac{2\left(1+ln\left[\frac{S_t}{K}\right]\right)}{K^2}P_{t,n,K}dK \tag{23}$$

$$W(t,n)=\int_{S_t}^{\infty}\frac{6\ln\left[\frac{K}{S_t}\right]-3\left(\ln\left[\frac{K}{S_t}\right]\right)^2}{K^2}C_{t,n,K}dK-\int_{o}^{S_t}\frac{6\ln\left[\frac{S_t}{K}\right]+3\left(\ln\left[\frac{S_t}{K}\right]\right)^2}{K^2}P_{t,n,K}dK \tag{24}$$

$$X(t,n)=\int_{S_t}^{\infty}\frac{12\left(\ln\left[\frac{K}{S_t}\right]\right)^2-4\left(\ln\left[\frac{K}{S_t}\right]\right)^3}{K^2}C_{t,n,K}dK-\int_{o}^{S_t}\frac{12\left(\ln\left[\frac{S_t}{K}\right]\right)^2+4\left(\ln\left[\frac{S_t}{K}\right]\right)^3}{K^2}P_{t,n,K}dK \tag{25}$$

where $C_{t,n,K}$ and $P_{t,n,K}$ are, respectively, the current price of a call and a put option with maturity n and strike K. We stress that formula (19) is the same as Britten-Jones and Neuberger (2000), who derive the model-free implied volatility formula, with the aim of characterizing the underlying asset processes consistent with observed option prices, extending the Derman and Kani (1994), Dupire (1994, 1997) and Rubinstein (1994) models that only consider deterministic volatility to a stochastic volatility setting.

Theoretically, the two methods (implied trees and the Bakshi et al. formula) are based on different assumptions about volatility: a deterministic volatility assumption for implied trees and a stochastic volatility assumption for the Bakshi et al. formula. As a result, we can expect different estimates for implied moments in the two methods. Moreover, there may be others that could explain the deviations among the different estimation methods. For example, if it is true that the risk-neutral density determines a unique option price across all strikes, this price could be different from the real market price, because the implied tree fits some strikes better than others. On the other hand, a continuum of option prices in strikes determines a unique risk-neutral density. However, given that only a limited number of strikes (around 15 in the Italian market) are listed on the financial markets, interpolation and extrapolation errors could be present, yielding a risk-neutral distribution that does not well represent the entire set of option prices. Moreover, with the interpolation-extrapolation method, we could generate option prices that insert arbitrage opportunities into the model-free formula (e.g. kinks in the smile function). One advantage of the EDK model is that it rules out these option prices in the construction of option-implied trees. This aim is achieved by introducing no-arbitrage checks. The aim of this exercise is to compare the moments obtained with various implied trees with those obtained using the model-free method.

5 Data and methodology

In this section we describe the data set used (subsection 5.1) and the methodology (subsection 5.2) in order to assess the pricing performance of implied tree models and the forecasting power of risk-neutral moments on future realized moments. Variance, skewness and kurtosis risk premia are also discussed.

5.1 Data

The data set consists of closing prices on FTSE MIB-index options (MIBO), recorded from 1 January 2005 to 31 November 2014 and kindly made available by Borsa Italiana. MIBO are European options on the

FTSE MIB index, which is a capital weighted index composed of 40 major stocks quoted on the Italian market. As for the underlying asset, closing prices of the FTSE MIB-index recorded in the same t period, adjusted for dividends, are used. As a proxy for the risk-free rate, the Euro Interbank Offered Rates (EURIBOR) with maturities of one week and one, two and three months are used. Appropriate yields to maturity are computed by linear interpolation. The data-set for the FTSE MIB index and the MIBO was kindly made available by Borsa Italiana S.p.A, Euribor rates and dividend yields were obtained from Datastream.

Several filters are applied to the option data set. First, we eliminate options near to expiry which may suffer from pricing anomalies that might occur close to expiration (Hentschel, 2003). To be consistent with the computation methodology of quoted volatility indexes, we use the most conservative filter that eliminates options with time to maturity of less than eight days. Second, following Ait-Sahalia and Lo (1998), only at-the-money and out-of-the-money options are retained; in-the-money options are discarded because they are less often traded. Finally, option prices violating the standard no-arbitrage constraints are eliminated because they would introduce arbitrage violations into the models.

5.2 Methodology

In order to derive the implied moments based on implied trees, we follow the method described below and we reiterate the process for both near and next-term options in each date of the sample. The benchmark is the Cox, Ross and Rubinstein (1979) tree, constructed using a constant volatility equal to an average of at-the-money implied volatilities of a call and a put. The benchmark tree is also used as initial input for the Rubinstein tree in order to have the prior estimate of the risk-neutral distribution which is used for the optimization process (equations (8-9)).

In order to derive the Enhanced Derman and Kani tree, and to implement the Bakshi et al. (2003) formula, we first obtain the smile function by using an interpolation-extrapolation scheme. We recover Black-Scholes implied volatilities from traded option prices and interpolate between strikes by using cubic splines in order to generate the missing points (pairs of strike and implied volatility) to use in the generation of the implied tree. We then extrapolate volatilities outside the listed strike price range using a constant extrapolation scheme where the implied volatility is supposed to be equal to the volatility of the minimum (maximum) strike price traded, for strikes below (above) the minimum (maximum) strike price traded. We extrapolate outside the existing domain of strike prices by using a factor u=10 such that: $S_0/(1+ u) \leq K \leq S_0(1+ u)$, where S is the current spot price and K refers to the generated strike prices. In order to achieve sufficient discretization of the integration domain, we compute strikes spaced by an interval $\Delta K = 10$. Following Muzzioli (2010, 2013), parameters u and ΔK have been chosen, in order to have insignificant truncation and discretization errors. For the Rubinstein model, we generated the missing prices, to obtain at least 15 prices for each day, used in the optimization routine, by adopting the cubic spline smoothing of the implied volatilities. Unlike Tian (2015) who proposes selecting a smaller number of ending probabilities to be estimated and interpolated by cubic-spline smoothing to derive all the ending probabilities, we adopted the cubic spline interpolation for the implied volatility smile function (similarly to the steps taken in the EDK model) to generate the missing prices. All implied trees are derived with 50 steps.

In order to compare the pricing performance of the EDK and Rubinstein implied trees, we adopt the following metrics widely used in the literature for assessing the accuracy in pricing (see e.g. Moriggia et al. (2009)). In particular, we use the mean absolute error (MAE), mean absolute percentage error (MAPE) between model prices and market prices, and the mispricing index (MISP), defined below, to obtain a measure of absolute errors (both in terms of absolute and percentage error) and to assess the amount of over-pricing or under-pricing of each model:

$$MAE = \frac{1}{m}\sum_{k=1}^{m}\left|P_k^T - P_k^M\right| \tag{26}$$

$$MAPE = \frac{1}{m}\sum_{k=1}^{m}\frac{\left|P_k^T - P_k^M\right|}{P_k^M} \tag{27}$$

$$MISP = \frac{\sum_{k=1}^{m}\left(\frac{P_k^T - P_k^M}{P_k^M}\right)}{\sum_{k=1}^{m}\left|\frac{P_k^T - P_k^M}{P_k^M}\right|} \tag{28}$$

In these formulae, P^T and P^M indicate, respectively, the theoretical (obtained from different models) and market price of the options, and *m* is the number of options in the class. The mean absolute error (MAE) measures the average mispricing of each model relative to the actual price in terms of index points (the multiplier for each index point is 2.5 Euros). Given that, depending on moneyness, the option prices differ in absolute terms, the mean absolute percentage error (MAPE) is intended to measure the average absolute percentage error (regardless of the price magnitude) and it is particularly useful to compare across different moneyness levels. On the other hand, the mispricing index (MISP) ranges from -1 to 1 and provides a tangible measure of under-pricing (MISP < 0, if all prices are under-evaluated by the model: MISP = -1) or over pricing (MISP > 0, if all the prices are over-evaluated by the model: MISP = 1).

In order to obtain a constant 30-day measure for the implied moments, we use linear interpolation as in the computation of the VIX index:

$$\sigma_{30} = \sqrt{\left\{\frac{T_1}{365}\sigma_{T_1}^2\left[\frac{T_2 - 30}{T_2 - T_1}\right] + \frac{T_2}{365}\sigma_{T_2}^2\left[\frac{30 - T_1}{T_2 - T_1}\right]\right\} * 365/30} \tag{29}$$

where T_i is the number of calendar days to expiry of the *i*-th maturity index option, $i = 1{,}2$, *i=1* for near term and *i=2* for next term. Near and next term moments are derived by using near and next term options, respectively, with maturity closest to 30-days. Risk-neutral moments represent the forecast of future realised moments embedded in option prices. Given the option price at time *t*, for an option with maturity *T*, the moments extracted are for the implied distribution of the underlying asset return from *t* to maturity *T*. As a result, they reflect the investors' opinion on the future characteristics of the return of the underlying asset in terms of variance, skewness and kurtosis. Hence, it will be interesting to compare the investors' forecasts embedded in option prices with the realized physical moments. The physical moments are computed at time *T*, by looking backward (from time *t* to time *T*) at the actual realizations of the underlying asset return and computing on the underlying asset return, the realised variance, skewness and kurtosis. Physical moments are derived from daily log-returns of the underlying index by using a rolling window of 30 calendar days and then annualised. In order to gauge the ability of the implied trees to forecast physical moments, we compute the Root Mean Squared Error (RMSE) metric:

$$RMSE = \sqrt{\frac{1}{m}\sum_{k=1}^{m}(M_k^P - M_k^{RN})^2} \tag{30}$$

where m is the number of observations in the sample, M^P is the physical moment and M^{RN} is the risk-neutral moment (variance, skewness or kurtosis).

In order to explain the difference between risk-neutral and physical moments, it should be noted that risk-neutral and physical moments can be considered as the two legs of a variance, skewness and kurtosis swap. In a variance swap, at maturity, the long side pays a fixed rate (the variance swap rate) and receives a floating rate (the realised variance). A notional Euro amount is multiplied by the difference between the two rates. The payoff at maturity is:

$$N(\sigma_R^2 - VSR) \tag{31}$$

where N is a notional Euro amount, σ_R^2 is the realised variance (the physical moment), and VSR is the fix variance swap rate which by no arbitrage is equal to risk-neutral variance (see e.g. Carr and Wu (2009)).

Variance swaps, traded over the counter, are used by investors to have a pure exposure to the future level of variance. Zhao et al. (2013) propose two new types of contracts similar to variance swaps: skewness swap and kurtosis swap. The payoffs of these contracts are similar to the payoff of the well-known variance swaps, i.e. the difference between physical (M^P) and risk-neutral (M^{RN}) moments. The payoff of a long position in a variance (skewness or kurtosis) swap with the notional amount of N=100 Euro, held up to expiry, is called the variance (skewness or kurtosis) risk premium:

$$RP = 100 * (M^P - M^{RN}) \tag{32}$$

Where RP is the risk premium, M^P is the physical moment and M^{RN} is the risk-neutral moment, i.e. the amount that investors are willing to pay in order to be hedged against peaks of variance, kurtosis, or against drops in skewness. The risk premium is computed daily and then averaged across the sample. For example, in a variance swap, the long part is willing to pay a fixed swap rate (risk-neutral variance), decided at time t, which is usually higher than the realized variance, which is received at time T. This is explained by the fact that the long part fears increases in future realized variance and is willing to accept a negative return (the difference between realized and risk-neutral variance) to be hedged against peaks in variance. Increases in variance are considered critical for the investors' portfolio. As a result, the difference between realized and risk-neutral variance is referred to as the variance risk premium (the amount investors are willing to pay to be hedged against increases in variance).

Following the same reasoning, we examine the skewness and kurtosis swaps. In the literature, there is almost a consensus in modelling investor preferences as kurtosis averse, preferring stocks with a low probability in the tails and high accumulation around the mean (see Dittmar, 2002). On the other hand, it would be natural to assume that investor preferences are positive-skewness inclined, namely that investors prefer stocks with upsides that are more frequent than downsides. The empirical evidence on the relationship between skewness and future returns is mixed (see e.g. Xing, Zhang and Zhao, 2010 for a positive relationship and Conrad, Dittmar and Ghysels, 2013 for a negative relationship). Unlike Zhao et al. (2013), we stick to equation (23) also for the skewness risk premium and we use skewness (kurtosis) instead of the third (fourth) cumulant in the computation of skewness (kurtosis) risk premia. This is intended to help make the interpretation of the results simpler.

6. Empirical results

In this section we describe the pricing performance of implied trees, both over the entire sample period (subsection 6.1) and during the high and low volatility periods (subsection 6.2). The analysis of the pricing performance during the high and low volatility periods is especially motivated by the importance of having a correct pricing model when market conditions are adverse (high volatility).

6.1 The pricing performance of the implied binomial tree models: The entire sample

We first assess the performance of implied binomial trees in reproducing the smile, i.e. in pricing the options in our sample. The aim of this exercise is to verify the ability of different trees to reproduce the underlying asset distribution. Based on standard practice in the implied tree literature (e.g. Lim and Zhi (2002), Kim and Park (2006)), we take the standard Cox, Ross and Rubinstein model (1979) (CRR) as the benchmark. This model is derived on the assumption of a constant volatility equal to an average of at-the-money call and put implied volatilities.

The results for the entire sample are shown in Table 1. According to the mean absolute percentage error (MAPE), the best model in reproducing underlying asset distribution is the Enhanced Derman and Kani (EDK), followed by the Rubinstein (RUB) model. CRR demonstrates a slightly inferior performance in this regard. The mispricing index, MISP (Table 1, line 7), is negative in all the models, indicating that all the binomial trees substantially underprice the options considered compared to the actual price observed in the market. Among these models, the EDK model shows the greatest underpricing, while the CRR shows the least. While CRR shows similar errors for call and put options, both EDK and RUB tend to model the price call options better than pricing the puts. The Euro amount of loss in terms of mispricing is subsumed in the MAE index (Table 1, line 4), confirming the better pricing performance of the EDK model, mainly due to a better pricing of put options.

In order to determine which option class is the best priced in each model, we divide the options into three moneyness categories depending on the ratio of the strike price (K) to the underlying market price (S) (e.g. for calls: in-the-money, if $K/S \leq 0.97$, at-the-money, if $0.97 < K/S < 1.03$, out-of-the-money, if $K/S \geq 1.03$). Unlike Moriggia et al. (2009), we use a coarser partition of the option moneyness categories in order to obtain a homogeneous number of options in each class on each day (around five per class). The results for the three categories are shown in Table 2. According to the MAPE, among the three models considered, the best performance is attained for in-the-money options and it gradually deteriorates with the decrease in moneyness level. Overall, call options are far better priced than put options, since they are less subject to hedging pressure (see e.g. Bollen and Whaley (2004), Chan et al. (2004), Garleanu et al. (2009)). The EDK model performs better than both the CRR and the RUB model for all moneyness classes. The only exceptions are the cases of at-the money calls (Panel B of Table 2) for which CRR is the preferred model and out-of-the-money calls (Panel C of Table 2), for which RUB is the preferred model.

In order to determine whether the differences between the model price and the market price of options are statistically significant, we apply the Diebold and Mariano test of equal predictive accuracy (Diebold and Mariano (1995)). The Diebold and Mariano test is intended to assess the superiority of a model in predictive accuracy from a statistical point of view. In order to identify the winner, it is not sufficient for a model to display lower errors than another model, but the difference should also be significant from a statistical point of view (the fact that model A is better than model B in a particular sample realization does not mean that it is necessarily better with regard to the entire population). Our criterion for comparison of the model price and the actual price is the MAPE metric, allowing comparisons across different degrees of moneyness characterized by different levels of prices. Results are shown in Table 3. A positive (negative) t-statistic in this table indicates that the model specified by the corresponding line produced a larger (smaller) average loss than the model specified by the corresponding column. The Diebold and Mariano test statistic under the null of equal predictive accuracy is distributed as a *N (0, 1)* distribution. The results confirm the best pricing performance of the EDK model for the whole sample (Panel A of Table 3) and for put options (Panel B of Table 3) at the 1% level. However, the EDK model fails to achieve the best pricing performance for in-the-money and at-the-money call options, which are found to be better priced in the other models. The Rubinstein model performs slightly better than the CRR model for out-of-the-money calls (Panel C of Table 3). In the whole sample (Panel A of Table 3) the difference between CRR and RUB is not statistically significant.

In terms of mispricing, with constant volatility the CRR model overprices call options and underprices put options, as expected. The implied tree models underprice all option categories, with the only exception of the RUB model for in-the-money calls (Panel C of Table 3). Overall, the better performance of option-implied trees relative to the CRR model is determined mainly by the better pricing of out-of-the-money options, in particular call options in the RUB model and put options in the EDK. The highest underpricing in the EDK model is attained for out-of-the-money options and in the RUB model for at-the-money options. Both the EDK and RUB models underprice most out-of-the-money puts and calls, though the underpricing on out-of-the-money calls is less severe. As a result, implied tree models attach a lower probability to the left tail of the risk-neutral distribution more than they do for the right tail. The results shown here for the whole sample are confirmed by the MAE index (Table 1, lines 5-6) that provides a measure of mispricing in terms of Euro values. The highest errors are attained for put options, compared to the errors for call options across all models. The highest errors are found in the RUB model (except for call options, where they are the highest for the CRR model).

6.2 The pricing performance in high and low volatility periods.

During the sample period (2005-2014), the world financial markets witnessed a high level of turbulence and major losses. In particular, during the period 2008-2012, the financial markets in Italy suffered a loss of about 70%. Furthermore, model-free implied volatility, which remained low for the entire period 2005-2007, has increased significantly since 2008 and remained high on average until 2012. Figure 1 depicts the FTSE MIB index during the sample period and the volatility index for the Italian market computed by

Bloomberg (which uses traded put option prices). Different patterns are to be seen in the sample period. While the period 2008-2012 is characterized by high volatility and a bearish market, the first and the last years of the sample are characterized by lower volatility and a positive market trend. Accordingly, we divide the sample into two sub-periods of low volatility (2005-2007 & 2013-2014) and high volatility (2008-2012) and we contrast the pricing performance of the three models in these two periods. The results are shown in Table 4. Surprisingly, we do not observe dramatic changes in terms of performance across the two time-periods in any of the models. However, an examination of the findings across different moneyness classes, shown in Tables 5 and 6, does point to some notable results. First, the best-priced option class in both sub-periods, for all the models, remains the in-the-money category and the worst is the out-of-the-money category. By examining the Mean Absolute Percentage Error (MAPE) criterion, all the models show a better performance in the high volatility period for all moneyness classes. According to the MISP criterion, all models underprice all moneyness classes in both sub-periods. While in the EDK model the underpricing is quite similar in the two sub-periods, for both CRR and RUB models, the underpricing is more severe in the low volatility period. The MAE index confirms the better pricing performance in terms of Euro values of the EDK model for all moneyness classes both in the low (except for out-of-the-money options) and in the high volatility periods. Each option class is priced with a similar Euro error in the two periods, since the smaller error in the high volatility period is offset by the higher option price in the high volatility period.

Second, across models, the EDK model performs better than other models in both low and high volatility sub-periods, in terms of having the smallest MAPE and MAE values, with the only exception for the out-of-the-money options being in the low volatility period, for which RUB obtains a slightly smaller error (Table 5, Panel C). The performance of the EDK implied tree in particular for at-the-money and out-of-the-money options in the high volatility period (Table 6, Panels B and C) is crucial, since these types of options are the most liquid in the market and, thus, the most important for the estimation of the smile and the computation of model-free moments. On the other hand, the RUB model outperforms a standard CRR model only in the low volatility period for out-of-the-money options.

The Diebold and Mariano (1995) test results, shown in Table 7, confirm the superiority of the EDK model in both sub-periods (Panel A). At the 1% level, in the low volatility period the EDK model is better than the RUB model, except for out-of-the-money options (significant at the 5% level). In the high volatility period, the EDK model is the best, followed by the standard CRR model.

As a result, we conclude that the better performance of option-implied trees (particularly EDK) with respect to the standard CRR binomial model is mainly due to the better pricing performance of out-of-the-money options. This result is particularly important for traders who usually concentrate their operations on out-of-the-money puts which are used for downside risk hedging. Implied trees underprice all option categories. In connection with the two implied trees, the Enhanced Derman and Kani (EDK) model and Rubinstein (RUB) model, the results point to a better performance of the EDK model in both sub-periods, mainly determined by better pricing in the high volatility period (Panel B of Table 4). These findings are relevant for practitioners because, according to the evidence, they can safely rely on the Enhanced Derman and Kani model, which turns out to be a valid pricing instrument even in periods of turmoil. In addition, the model can be used for the pricing of exotic options which could be an issue mainly in high volatility periods.

The RUB model performs better than the CRR model only in the pricing of out-of-the-money options in the low volatility period (Panel C of Table 5). Hence, we can conclude that, among implied trees, the EDK is the best model, since it obtains better results (smaller errors), it is flexible in the pricing of plain vanilla and more exotic options, and it performs better in turmoil periods where correct pricing is problematic and accuracy acquires more importance than in tranquil periods.

7 Risk-neutral moments

In this section we describe the estimation of risk-neutral moments in implied binomial trees and in the model-free method both for the entire sample period (subsection 7.1) and during the high and low volatility periods (subsection 7.2). The comparison of the high and low volatility periods is motivated both by the need to disentangle the higher moments' estimation from the level of volatility and the importance of having reliable forecasts when market conditions are adverse (high volatility).

7.1 Forecasts of risk-neutral moments in the whole sample

A reliable estimation of variance, skewness and kurtosis has implications both at the micro and the macro levels. At the micro-level, investors can use different estimation methods to forecast implied moments more accurately. At the macro-level, regulators can use the forecast of market implied moments as an early warning on future market conditions. In order to assess whether the implied moments computed by means of implied binomial trees convey better information on future realised moments than the risk-neutral moments computed using the Bakshi et al. (2003) model free formula (MF), we report the descriptive statistics of both physical and risk-neutral moments in Table 8. The variance estimated with the implied binomial tree models is lower than model-free variance, as shown in Table 8, panel A. As a result, we can conclude that the EDK and the RUB models tend to overestimate the realised variance less than the MF procedure does. This is significant since a drawback of the model-free formula is its dependence on the (possibly infinite) number of options to be used: the more the options used, the more the level of implied volatility increases. Moreover, supposing volatility as an independent source of risk (as in the model-free formula), yields an estimate of implied volatility higher than that obtained by supposing volatility as a deterministic function of asset price and time (as in implied trees). This result is also consistent with the evidence that the implied binomial tree models tend to under-price options on average. Among implied trees, the variance computed with the RUB model is the smallest one.

Risk-neutral skewness is negative for all models and higher in absolute terms than physical skewness. This is consistent with the results in Conrad et al. (2013) and Zhao et al. (2013): extreme price decreases are more likely than extreme price rises, but they are more often expected (under the risk-neutral distribution) than subsequently realised. EDK displays the most negative risk-neutral skewness value: therefore it attributes to the left tail the highest probability if compared to the other models.

Risk-neutral kurtosis is higher than physical kurtosis for all the estimation methods, pointing to the existence of a negative kurtosis risk premium. This means that in all the models a high probability is attached both to the left and the right tail, the highest one being the one in the RUB model. Overall, the findings are consistent with previous empirical evidence (e.g. Bakshi et al., 2003) on a negatively skewed risk-neutral distribution with heavy tails.

The forecasting accuracies of the three risk-neutral moments in terms of the root-mean-square error (RMSE) are reported in Table 9. The forecasting performance of future realised moments of the two implied tree models is therefore inferior to the model-free method in terms of magnitude of errors. Specifically, MF obtains smaller RMSE for risk-neutral skewness and kurtosis than the implied trees. The best forecasting method for realised variance is the RUB model, whereas the EDK model ranks as the second best for all the moments of the distribution.

In order to understand the economic implications of the estimation of moments with different methods, we report the variance, skewness and kurtosis risk premia in Table 10. It should be noted that the risk-neutral moment corresponds to the fixed leg of the variance (skewness or kurtosis) swap, i.e. to the fixed payment that an investor who is long on variance (skewness or kurtosis) is willing to pay in exchange for the floating realised variance (skewness or kurtosis). A 100 Euro notional is multiplied to the long swap payoff.

In line with the literature (e.g. Carr and Wu (2009)), the variance risk premium is negative and significant for all the models. This means that investors do price variance risk, implying that variance buyers can accept a negative premium in order to hedge against an unexpected increase of stock market volatility. As a result investors are not only averse to variance of returns, but also to changes in variance. In terms of magnitude, for 100 Euros of notional value, the average loss ranges from €4.33, if we use model-free moments methodology to price variance, to €0.91 (€0.31) for the EDK (RUB) model. As expected the highest risk premium is attained in the model-free method. Thus, on average, selling variance could be profitable.

The skewness risk premium is positive and significant in all models: it is the smallest in the RUB model (€9.86) per 100 Euro notional, and the highest in the EDK model (€16.09). This means that investors attach a more negative risk-neutral skewness to the underlying distribution than the realized one, meaning that they expect a distribution of the underlying asset to be more left-skewed than the real one. As a result, in line with Kozhan et al. 2013, we find that buying skewness could be profitable, on average. The kurtosis risk premium is negative and significant in all models and, in absolute terms, it is the smallest in the EDK model (€14.22) and highest in the RUB (€23.23). Thus, investors tend to price kurtosis risk and possible profits stem from being short on a kurtosis swap. The results in terms of the sign of the risk premia, in both the EDK and the RUB model, are coherent with the standard results in the literature. This confirms that the binomial tree models work well and could be used by traders to assess not only the consistency but also the sign of the risk premia.

7.2 Forecasts of risk-neutral moments in the two sub-periods

Bearish markets are usually characterized by a high level of volatility, and high volatility may have an effect on the other two moments, namely skewness and kurtosis. Moreover, it is more important to have reliable estimates on implied moments in bearish markets (in order to contain losses), than in bullish markets when gains are expected. Thus, in this section we try to disentangle the effect of volatility on the other two moments by assessing how the estimation of the moments vary in bullish or bearish periods. The estimates of physical and risk-neutral variances, skewness and kurtosis in the two sub-periods are reported in panels B and

C of Table 8, respectively. According to the figures, the difference in physical variance of the FTSE MIB index between the two sub-periods is high (almost three times higher in the high volatility period). Risk-neutral variance is also much greater in the high volatility period, meaning that investors expect a higher variance during turmoil periods.

In line with Dennis and Mayhew (2002), we find that risk-neutral skewness tends to be more negative during the turmoil period (i.e. investors attach a higher probability to extreme negative returns during turmoil periods), with the sole exception being the MF estimation method. Therefore, according to implied binomial trees, extreme price falls are more frequently expected than extreme price rises in the turmoil period. The opposite holds for the model-free estimation method, though the difference here is not large in the two sub-periods. On the other hand, the results for realised skewness show that extreme price falls took place more than extreme price rises in the low volatility period, if compared to the high volatility period.

Unlike physical skewness, physical kurtosis varies little across the two periods, with a slightly higher value in the low volatility period. However, the risk-neutral measures show mixed results: kurtosis is higher in the high volatility period for the RUB model and the opposite is true for the EDK and MF measures. This means that while the realised distribution showed a higher volatility level, but with less frequent extreme price moves (and more symmetric moves) in the turmoil period, the investors' expected distribution displayed higher volatility and more frequent and more asymmetric extreme price moves in the high volatility period.

The forecasting performance of risk-neutral moments in the two sub-periods is reported in Panels B and C of Table 9, respectively. Overall, all the models demonstrate a better performance in the low volatility period, which is not surprising due to the powerful effect of the negative macroeconomic conditions in the high volatility period. As for the variance and the skewness estimation, all the methods demonstrate a worse performance in the high volatility period, with the sole exception of MF methodology. For kurtosis, the performance improves in the high volatility period for the EDK and MF models, while it deteriorates for the RUB model.

In order to understand the economic implications of the estimation of moments with different methods, we report the variance, skewness and kurtosis risk premia in high and low volatility periods in Panels B and C of Table 10. Regardless of the estimation method, we find significant risk-neutral density deviations from the corresponding physical counterparty for all moments as the risk premia are statistically different from zero (the t-statistics are adjusted for serial dependence, according to Newey-West) with the sole exception being the variance risk premia in high volatility period according to the RUB model. Notably, model-free variance, which is derived consistently with a stochastic volatility assumption, is much higher than that obtained with implied binomial trees, that assume deterministic volatility, in any volatility regime. According to the model-free methodology, the variance risk premia are higher in the high volatility period where trading strategies are more profitable. For implied binomial trees, on the other hand, selling variance turns out to be more profitable in the low volatility period. The result of implied binomial trees is closer to what might be expected: usually selling variance is profitable as long as the market does not move too much: therefore in high volatility periods lower returns are expected on selling variance.

As for skewness, we find significant positive risk premia in both periods: this is explained by the more negative risk-neutral skewness than subsequently realised. As a result, buying skewness would have been profitable, especially in the low (high) volatility period according to MF and EDK (RUB) method. Kurtosis risk premia are significant at the 1% level in both subsamples for all the models, therefore selling kurtosis is profitable, especially in the low (high) volatility period according to EDK and MF (RUB) model. The reasoning about kurtosis selling follows the reasoning about volatility selling: we expect profits in the low volatility period more than in the high volatility period. For skewness it is not easy to have a prior: we can say that small profits (or losses) are expected in low volatility periods and large profits (or losses) are expected in high volatility periods. Therefore the implied trees forecasts are superior to the model-free ones since they capture the expected magnitude of variance skewness and kurtosis risk premia in high and low volatility periods. Overall, these findings signal that profitable volatility, skewness and kurtosis trades do exist in both low and high volatility periods and could be exploited by settling appropriate strategies in options. Since investors are willing to accept a strongly negative return, being long (short) in variance and kurtosis (skewness) swaps, in order to be hedged against positive (negative) peaks of variance and kurtosis (skewness), the results suggest the need to hedge exposure to any moment of the distribution.

8. Conclusions

In this paper we compared two types of implied trees, based on backward and forward induction, for the dual purpose of option pricing and extracting the risk-neutral distribution of the underlying asset in the Italian index market. We analyzed the pricing performance of the implied trees both in tranquil and turbulent market conditions, using the Cox, Ross and Rubinstein (1979) tree as a benchmark. The tranquil period covered the years 2005-2007 and 2012-2014, while the turmoil period covered the years 2008-2012. Moreover, we used the implied trees to extract the risk-neutral moments of the underlying asset's distribution, and compare them with the model-free moments (Bakshi et al. (2003)). Implied trees were based on the assumption that volatility is a deterministic function of the underlying asset and time, while the Bakshi et al. formula was derived in line with a stochastic volatility model. As a result, the comparison of the two methodologies was also used to assess the need to include stochastic volatility in the picture. Finally, we compared risk-neutral moments with physical moments in order to assess the existence of variance, skewness and kurtosis risk premia.

From the theoretical point of view, backward induction implied trees are simple to construct and rely on the minimization of the distance between (a few) market prices and model prices to estimate the risk-neutral distribution. These models are generally overdetermined and, hence, face serious challenges in gaining stability in the parameters, which are also sensitive to the choice of the function to be minimized. As a result, while on a specific date the fit might be good, on a different date, the fit might dramatically worsen, limiting the applicability and usefulness of these models. Approaches based on forward induction, on the other hand, may suffer from the necessary interpolation and extrapolation used to generate the missing options prices (or volatilities) and the resulting solution may violate standard arbitrage bounds if not ad-hoc corrected (as it is in the Enhanced Derman and Kani model). Thus, from the theoretical point of view, we may also address the greater accuracy, the better fit to the smile, and the better pricing performance of forward induction implied trees with respect to backward induction ones.

From the empirical point of view, the results favour the Enhanced Derman and Kani (EDK) model relative to the Rubinstein (1994) model in both sub-periods. The better performance of EDK is mainly due to the smaller errors in the high volatility period and an improved pricing of put options for all moneyness classes. In other words, the better performance of EDK model is due to its better estimation of the left tail of the distribution which describes the unfavourable market conditions.

These results are in line with other empirical studies (e.g. Linaras and Skiadopoulos (2005)) which claim that a "golden choice" for option pricing consistent with the smile does not exist. The EDK model seems to be the best with regard to many criteria. It obtains better results (smaller errors) on the left tail of the distribution (out of the money put options), it is flexible in the pricing of plain vanilla and more exotic options, and it performs better in turmoil periods where correct pricing is problematic and accuracy deserves more importance than in tranquil periods. These findings are relevant for practitioners because, according to the evidence, they can safely rely on the Enhanced Derman and Kani model, which turns out to be a valid pricing instrument even in periods of turmoil and for the pricing of exotic options which could be an issue, mainly in high volatility periods. Given the good pricing performance of the Enhanced Derman and Kani in both high and low volatility periods, a promising line for future research is to combine the use of implied trees with a regime-switching model. In particular, in line with Costabile et al. (2014) who implement a standard Cox-Ross-Rubinstein binomial tree for any regime, we may think of option pricing in different volatility regimes characterized by a volatility smile function shifted up (high volatility) or down (low volatility).

Despite the good pricing performance within the sample, the EDK model is not the best model for forecasting physical moments: the model-free formula is the most accurate in this regard for both skewness and kurtosis. Notably, model-free variance, which is derived in line with a stochastic volatility assumption, is much greater in magnitude than the variance obtained with implied binomial trees, that assume deterministic volatility. Among implied trees, the Rubinstein model is the best in forecasting variance. All models agree on the sign of the risk premia: negative for variance and kurtosis, positive for skewness. Regardless of the estimation method, we find significant risk-neutral density deviations from the corresponding physical counterpart in all three models as variance, skewness and kurtosis risk premia are statistically different from zero (with the only exception for variance in the Rubinstein model in the high volatility period).

Overall, the results suggest that selling (buying) variance and kurtosis (skewness) is profitable both in low and in high volatility periods. Therefore, our results indicate the need to hedge exposure to any moment of the distribution. According to the model-free measure, we detect a higher profitability of variance (skewness and kurtosis) trades in the high (low) volatility period. On the other hand, consistently to what is expected, implied binomial tree models point to a higher profitability of variance (skewness) trades in low (high) volatility periods. As a result, we can consider implied trees' moments as superior to model-free ones in capturing the expected sign of the variance, skewness and kurtosis risk premia in high and low volatility periods. The evidence about the higher profitability of kurtosis trades in high or low volatility periods is mixed depending on the estimation method used. The results point to the evidence that investors are willing to pay a high fixed rate, in order to be hedged against positive (negative) peaks of variance and kurtosis (skewness). The results for variance and skewness swap are consistent with Kozhan et al. (2013).

Our results are of practical importance for traders who may rely on implied trees in order to price less liquid exotic options consistently with the European ones and for determining profitable trades on risk-neutral moments. This study should, however, be extended with the analysis of the determinants of variance, skewness and kurtosis risk premia. Moreover, the dynamics and the relationship among implied moments deserve careful investigation along with the importance of risk-neutral moments for forecasting future returns. Finally, since it is noted in Conrad et al. (2013) that skewness estimates based on sample averages are prone to measurement errors more than other moments, we leave the use of other asymmetry measures less sensitive to outliers for future research.

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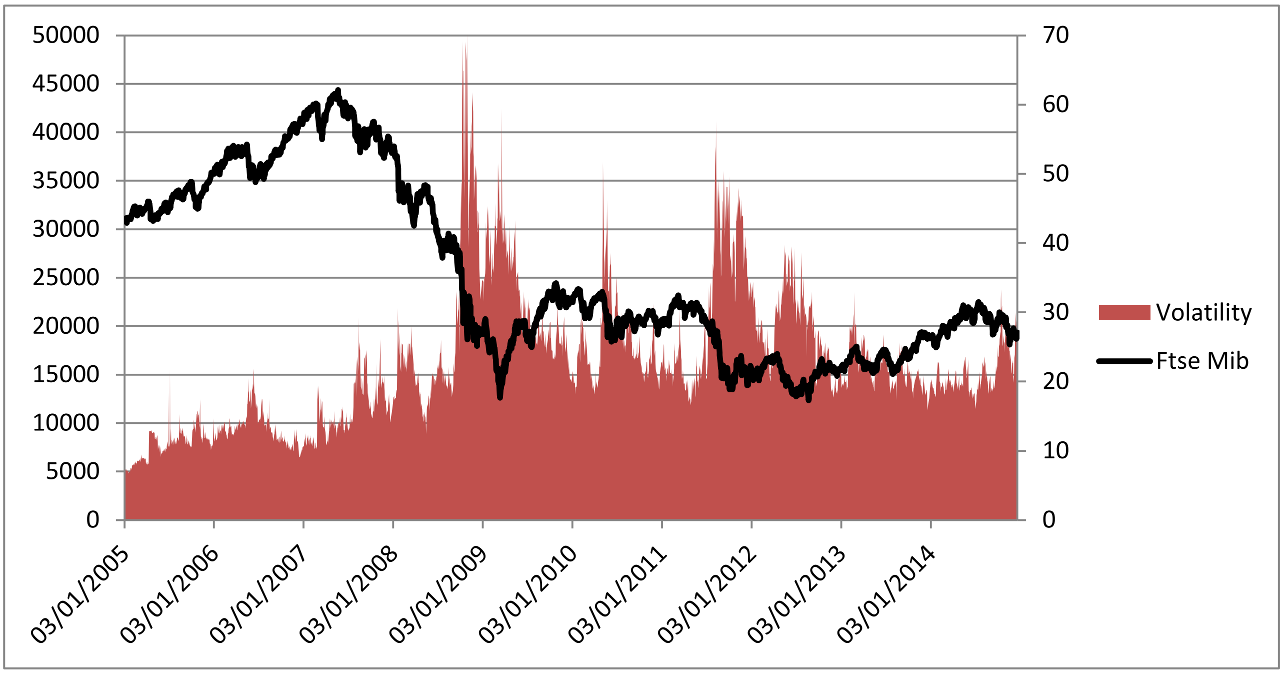
Figure 1. The FTSE MIB index (i) and the Volatility index (ii): sample period (2005-2014)



(i) left-hand scale, (ii) right-hand scale

Table 1. Pricing performance of the three models considered (Entire sample:2005-2014)

Models	*CRR*	*EDK*	*RUB*
MAPE	0.0473	0.0289	0.0444
MAPE Call	0.0464	0.0182	0.0128
MAPE Put	0.0481	0.0397	0.0761
MAE	9.38 (€23.45)	6.20 (€15.51)	15.04 (€37.60)
MAE Call	4.30 (€10.75)	3.41 (€8.53)	3.96 (€9.91)
MAE Put	13.95 (€34.87)	9.23 (€23.08)	26.35 (65.88)
MISP	-0.1371	-0.5649	-0.3988
MISP Call	0.5424	-0.5731	0.0373
MISP Put	-0.8166	-0.5566	-0.8350

Note: The Table shows the pricing performance for the Cox, Ross and Rubinstein (CRR), Enhanced Derman and Kani (EDK), and Rubinstein (RUB) models. MAPE = Mean Absolute Percentage Error, MAE = Mean Absolute Error, MISP = Mispricing Index. $MAPE = \frac{1}{m}\sum_{k=1}^{m}|P_k^T - P_k^M|/P_k^M$, *with* P_k^T *theoretical price and* P_k^M *market price;* $MAE = \frac{1}{m}\sum_{k=1}^{m}|P_k^T - P_k^M|$, *MAE is expressed in terms of index points, in brackets the corresponding value in € (each Index point is equal to €2.5).* $MISP = \sum_{k=1}^{m}\left(\frac{P_k^T - P_k^M}{P_k^M}\right) / \sum_{k=1}^{m}\left|\frac{P_k^T - P_k^M}{P_k^M}\right|$, *measures overpricing (MISP>0) or underpricing(MISP<0), it ranges from -1 to 1.*

Table 2. Pricing performance of the three models by moneyness (entire sample: 2005-2014).

Models	*CRR*	*EDK*	*RUB*
		Panel A: In-the-money	
MAPE	0.0033	0.0020	0.0061
MAPE Call	0.0013	0.0013	0.0020
MAPE Put	0.0054	0.0028	0.0104
MAE	7.94 (€19.86)	4.87 (€12.18)	14.57 (€36.42)
MAE Call	3.45 (€8.62)	3.51 (€8.78)	5.33 (€13.33)
MAE Put	10.99 (€27.47)	5.70 (€14.24)	21.03 (€52.56)
MISP	0.0256	-0.4996	-0.0636
MISP Call	0.7235	-0.5698	0.8339
MISP Put	-0.6723	-0.4294	-0.9612
		Panel B: At-the-money	
MAPE	0.0244	0.0180	0.0490
MAPE Call	0.0069	0.0094	0.0108
MAPE Put	0.0419	0.0266	0.0873
MAE	12.65 (€31.62)	9.32 (€23.30)	25.38 (€63.45)
MAE Call	3.47 (€8.67)	4.72 (€11.81)	5.39 (€13.47)
MAE Put	22.44 (€56.10)	14.22 (€35.54)	46.71 (€116.77)
MISP	-0.3689	-0.4816	-0.7559
MISP Call	0.1417	-0.5474	-0.5834
MISP Put	-0.8796	-0.4158	-0.9284
		Panel C: Out-of-the-money	
MAPE	0.1141	0.0667	0.0781
MAPE Call	0.1311	0.0437	0.0257
MAPE Put	0.0970	0.0897	0.1305
MAE	7.55 (€18.88)	4.42 (€11.04)	5.17 (€12.92)
MAE Call	5.99 (€14.96)	2.00 (€4.99)	1.17 (€2.93)
MAE Put	8.42 (€21.05)	7.78 (€19.46)	11.32 (€28.30)
MISP	-0.0680	-0.7134	-0.3770
MISP Call	0.7619	-0.6022	-0.1385
MISP Put	-0.8980	-0.8246	-0.6154

Note: The Table shows the pricing performance for in-the-money options (Panel A), at-the-money options (Panel B) and out-of-the-money options (Panel C), for the Cox, Ross and Rubinstein (CRR), Enhanced Derman and Kani (EDK), and Rubinstein (RUB) models. See Table 1 for the definition of MAPE, MAE and MISP.

Table 3. Diebold and Mariano (1995) tests of significant difference in pricing performance (entire sample: 2005-2014).

Panel A: Entire sample

	EDK	*RUB*
CRR	*2.93*	*0.44*
	0.00	*0.66*
EDK		*-6.39*
		0.00

Panel B: by type (Call vs Put)

	Call			***Put***	
	EDK	*RUB*		*EDK*	*RUB*
CRR	*2.34*	*2.55*	*CRR*	*6.04*	*-9.57*
	0.02	*0.01*		*0.00*	*0.00*
EDK		*2.64*	*EDK*		*-10.54*
		0.01			*0.00*

Panel C: Call options by moneyness

	in-the-money			***at-the-money***			***out-of-the-money***	
	EDK	*RUB*		*EDK*	*RUB*		*EDK*	*RUB*
CRR	*-0.05*	*-8.48*	*CRR*	*-2.81*	*-3.23*	*CRR*	*2.42*	*2.67*
	0.96	*0.00*		*0.01*	*0.00*		*0.02*	*0.01*
EDK		*-1.97*	*EDK*		*-0.72*	*EDK*		*3.28*
		0.05			*0.47*			*0.00*

Panel D: Put options by moneyness

	in-the-money			***at-the-money***			***out-of-the-money***	
	EDK	*RUB*		*EDK*	*RUB*		*EDK*	*RUB*
CRR	*2.84*	*-9.40*	*CRR*	*5.69*	*-8.95*	*CRR*	*3.56*	*-5.18*
	0.01	*0.00*		*0.00*	*0.00*		*0.00*	*0.00*
EDK		*-5.85*	*EDK*		*-9.22*	*EDK*		*-5.74*
		0.00			*0.00*			*0.00*

Note: The Table shows the Diebold and Mariano tests(t-stats and p-values) using the MAPE measure, in the whole sample (Panel A) by type of option (Panel B) and by moneyness (Calls in Panel C and Puts in Panel D), for the Cox, Ross and Rubinstein (CRR), Enhanced Derman and Kani (EDK), and Rubinstein (RUB) models.

Table 4. Pricing performance of the three models in the high and low volatility periods.

Models	*CRR*	*EDK*	*RUB*
	Panel A: Low volatility period (2005-2007 & 2013-14)		
MAPE	0.0653	0.0369	0.0456
MAE	10.22 (€25.56)	6.69 (€16.72)	15.09 (€37.73)
MISP	-0.1972	-0.5740	-0.4832
	Panel B: High volatility period (2008-2012)		
MAPE	0.0296	0.0211	0.0433
MAE	8.39 (€20.98)	5.72 (€14.29)	15.44 (€38.61)
MISP	-0.0780	-0.5503	-0.3159

Note. The Table shows the pricing performance in the low volatility period (Panel A: 2005-2007 & 2013-14) and in the high volatility period (Panel B: 2008-2012) for the Cox, Ross and Rubinstein (CRR), Enhanced Derman and Kani (EDK), and Rubinstein (RUB) models. See Table 1 for the definition of MAPE, MAE and MISP.

Table 5. Pricing performance of the three models by moneyness (low volatility period: *2005-2007 & 2013-14*).

Models	*CRR*	*EDK*	*RUB*
	Panel A: In-the-money		
MAPE	0.0042	0.0025	0.0074
MAE	9.41 (€23.53)	5.59 (€13.98)	16.61 (€41.53)
MISP	-0.0242	-0.4861	-0.0920
	Panel B: At-the-money		
MAPE	0.0254	0.0201	0.0504
MAE	12.30 (€30.75)	9.73 (€24.31)	24.42 (€61.06)
MISP	-0.4746	-0.5249	-0.8200
	Panel C: Out-of-the-money		
MAPE	0.1664	0.0882	0.0789
MAE	8.96 (€22.39)	4.74 (€11.86)	4.25 (€10.62)
MISP	-0.0928	-0.7111	-0.5375

Note. The Table shows the pricing performance in the low volatility period (2005-2007 & 2013-14) for in-the-money options (Panel A), at-the-money options (Panel B) and out-of-the-money options (Panel C) for the Cox, Ross and Rubinstein (CRR), Enhanced Derman and Kani (EDK), and Rubinstein (RUB) models. See Table 1 for the definition of MAPE, MAE and MISP.

Table 6. Pricing performance of the models by moneyness (high volatility period: *2008-2012*).

Models	***CRR***	***EDK***	***RUB***
	Panel A:In-the-money		
MAPE	0.0025	0.0016	0.0049
MAE	5.2 (€13.03)	4.16 (€10.40)	12.58 (€31.45)
MISP	-0.1984	-0.5210	-0.0357
	Panel B: At-the-money		
MAPE	0.0238	0.0160	0.0477
MAE	13.1 (€32.68)	9.11 (€22.77)	27.18 (€67.94)
MISP	-0.3925	-0.4129	-0.6929
	Panel C: Out-of-the-money		
MAPE	0.0624	0.0456	0.0773
MAE	5.6 (€13.93)	3.88 (€9.70)	6.57 (€16.42)
MISP	-0.0310	-0.7171	-0.2190

Note: The Table shows the pricing performance in the high volatility period (2008-2012) for in-the-money (Panel A), at-the-money (Panel B), out-of-the-money (Panel C) options for the Cox, Ross and Rubinstein (CRR), Enhanced Derman and Kani (EDK), and Rubinstein (RUB) models. See Table 1 for the definition of MAPE, MAE and MISP.

Table 7. Diebold and Mariano test of significant difference in pricing performance (high and low volatility periods)

Panel A: Low versus high volatility period

low volatility			**high volatility**		
	EDK	RUB		EDK	RUB
CRR	*2.67*	*1.99*	CRR	*5.04*	*-4.55*
	0.01	*0.05*		*0.00*	*0.00*
EDK		*-3.37*	EDK		*-6.95*
		0.00			*0.00*

Panel B: Low volatility period: ***2005-2007 & 2013-14***

in-the-money			**at-the-money**			**out-of-the-money**		
	EDK	RUB		EDK	RUB		EDK	RUB
CRR	*1.62*	*-8.60*	CRR	*2.73*	*-7.28*	CRR	*2.58*	*2.71*
	0.11	*0.00*		*0.01*	*0.00*		*0.01*	*0.01*
EDK		*-3.70*	EDK		*-5.79*	EDK		*2.31*
		0.00			*0.00*			*0.02*

Panel C: High volatility period: ***2008-2012***

in-the-money			**at-the-money**			**out-of-the-money**		
	EDK	RUB		EDK	RUB		EDK	RUB
CRR	*3.36*	*-8.67*	CRR	*2.96*	*-5.30*	CRR	*4.60*	*-2.73*
	0.00	*0.00*		*0.00*	*0.00*		*0.00*	*0.01*
EDK		*-8.43*	EDK		*-6.60*	EDK		*-5.05*
		0.00			*0.00*			*0.00*

Note: The Table shows the Diebold and Mariano tests(t-stats and p-values) using the MAPE measure, in the low and high volatility period (Panel A) and by moneyness in the low volatility period (2005-2007 & 2013-14 - Panel B) and in the high volatility period (2008-2012 - Panel C) for in-the-money options, at-the-money options and out-of-the-money options, for the Cox, Ross and Rubinstein (CRR), Enhanced Derman and Kani (EDK), and Rubinstein (RUB) models.

Table 8. Moments of the physical and the risk-neutral distribution (Variance, Skewness and Kurtosis)

Models	***EDK***	***RUB***	***MF***	***PHYSICAL***
		Panel A: Entire sample		
Variance	0.0701	0.0641	0.1043	0.0610
Skewness	-0.1671	-0.1048	-0.1077	-0.0062
Kurtosis	3.1426	3.2327	3.1636	3.0005
		Panel B: Low volatility Period: *2005-2007 & 2013-14*		
Variance	0.0374	0.0327	0.0519	0.0282
Skewness	-0.1590	-0.0607	-0.1151	-0.0103
Kurtosis	3.1482	3.2144	3.2065	3.0026
		Panel C: High volatility period: *2008-2012*		
Variance	0.1024	0.0952	0.1561	0.0935
Skewness	-0.1752	-0.1484	-0.1004	-0.0023
Kurtosis	3.1371	3.2509	3.1212	2.9984

Note. Risk-Neutral Moments are all calculated over a 30-day period by interpolating near and next term moments for Enhanced Derman and Kani (EDK), Rubinstein (RUB), and model-free (MF). Physical moments (PHYSICAL) are obtained with a rolling window over a 22-working days period (equivalent to 30 days), as described in Section 6. Panel A: whole sample, Panel B: low volatility period (2005-2007 & 2013-14), Panel C: high volatility period (2008-2012).

Table 9. Forecasting performance of higher moments.

Models	*EDK*	*RUB*	*MF*
	Panel A: Entire sample		
RMSE Var	0.0627	0.0585	0.0856
RMSE Skew	0.2450	0.2970	0.1592
RMSE Kurt	0.3757	0.5053	0.3103
	Panel B: Low volatility period: *2005-2007 & 2013-14*		
RMSE Var	0.0222	0.0186	0.0337
RMSE Skew	0.2311	0.2709	0.1662
RMSE Kurt	0.3872	0.4838	0.3786
	Panel C: High volatility period: *2008-2012*		
RMSE Var	0.0857	0.0804	0.1160
RMSE Skew	0.2580	0.3206	0.1520
RMSE Kurt	0.3641	0.5256	0.2232

Note: The Table shows the forecasting performance of subsequently realised variance, skewness and kurtosis, for the Enhanced Derman and Kani (EDK), Rubinstein (RUB) and model-free (MF) models, in terms of root mean squared error defined as: $RMSE = \sqrt{\frac{1}{m}\sum_{k=1}^{m}(M_k^P - M_k^{RN})^2}$, *where* M^P *is the physical moment and* M^{RN} *is the risk-neutral moment for variance, skewness and kurtosis. Panel A: whole sample, Panel B: low volatility period (2005-2007 & 2013-14), Panel C: high volatility period (2008-2014).*

Table 10. Variance, skewness and kurtosis risk premia.

Models	*EDK*			*RUB*			*MF*		
	Variance RP	**Skewness RP**	**Kurtosis RP**	**Variance RP**	**Skewness RP**	**Kurtosis RP**	**Variance RP**	**Skewness RP**	**Kurtosis RP**
Panel A: all sample									
mean	-0.91	16.09	-14.22	-0.31	9.86	-23.23	-4.33	10.15	-16.32
t-stat	-7.15	42.68	-20.03	-2.56	17.25	-25.38	-28.70	40.53	-30.30
p-value	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
Panel B: Low volatility period: *2005-2007 & 2013-14*									
mean	-0.93	14.87	-14.57	-0.45	5.04	-21.18	-2.38	10.48	-20.39
t-stat	-15.91	29.05	-14.03	-8.64	6.55	-16.82	-34.36	28.09	-22.09
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Panel C: High volatility period: *2008-2012*									
mean	-0.89	17.29	-13.87	-0.16	14.61	-25.25	-6.25	9.81	-12.28
t-stat	-3.61	31.39	-14.32	-0.70	17.80	-19.04	-22.24	29.37	-22.90
p-value	0.00	0.00	0.00	0.49	0.00	0.00	0.00	0.00	0.00

Note: The Table shows the mean variance, skewness and kurtosis risk premia (in Euro) for Enhanced Derman and Kani (EDK), Rubinstein (RUB) and model-free (MF) models. Risk premia (RP) are defined as: RP = $N * (M^P - M^{RN})$ *where* M^P *is the physical moment and* M^{RN} *is the risk-neutral moment, N=100 is a notional Euro amount. Panel A: whole sample, Panel B: low volatility period (2005-2007 & 2013-14), Panel C: high volatility period (2008-2012).*