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Fear or greed? What does a skewness index measure?

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Abstract

The *Chicago Board Options Exchange (CBOE) SKEW* index is designed to capture investors' fear in the US stock market. In this paper we pursue two objectives. First, we investigate the properties of the *CBOE SKEW* index in order to assess whether it captures fear or greed in the market. Second, we introduce and compare three measures of asymmetry of the Italian index options return distribution. These measures include: (i) the *CBOE SKEW* index adapted to the Italian market (we call it *ITSKEW*) and (ii) two model-free measures of skewness based on comparison of a bear and a bull index. Finally, we explore the existence and sign of the skewness risk premium. Several results are obtained. First, the Italian skewness index *(ITSKEW)* presents many advantages compared to the two model-free measures: it has a significant contemporaneous relation with market index returns and with model-free implied volatility. Both the *ITSKEW* and the *CBOE SKEW* indices act as measures of market greed (the opposite of market fear), since returns react positively to an increase in the skewness indices. Trading strategies show that the Italian market is characterized by a significant positive skewness risk premium.

Keywords: skewness index, risk-neutral moments, implied volatility, skewness risk premium.

JEL classification: G13, G14

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1. Introduction

The Chicago Board Options Exchange (CBOE) Volatility Index (VIX) has been called the "investors' fear gauge" (Whaley, 2000) since it measures the investors' consensus view about expected future stock market volatility (market sentiment). The higher the value of VIX, the greater the fear is considered to be. The fact that the VIX spikes during periods of market turmoil (bad news) is because when expected volatility increases, investors demand higher rates of return and stock prices fall. However, the correlation is not exact and sometimes it is even in the opposite direction (VIX spikes when stock prices rise). In this case, it can be said that the VIX index captures the investors' excitement. As a result, we can consider the "investors' greed gauge" (or expectation of earnings) to be the opposite of market fear. In other words, in this case, VIX becomes a measure of the investors' consensus view of expected future stock market volatility (market sentiment) characterized by positive returns (good news) (Whaley, 2000 defines greed as the investors' excitement in a market rally).

The $CBOE\ SKEW$ index has been listed on the CBOE since February 2011 to measure the tail risk not fully captured by VIX. While VIX measures the overall risk in the 30-day S&P500 log-returns, without disentangling the probabilities attached to positive and negative returns; the skewness index $(CBOE\ SKEW)$ is intended to measure the perceived tail risk, i.e. the probability that investors attach to extreme negative returns. The $CBOE\ SKEW$ index varies around a value of 100. Values above the threshold level 100 tend to point to a negative risk-neutral skewness and a distribution skewed to the left. These values indicate that negative returns are more often expected than positive ones. The opposite is true for values below 100: they indicate that positive returns are more often expected than negative ones. Moreover, a high value of the $CBOE\ SKEW$ index indicates that buying protection against downturn (put options) is more expensive. In practice, however, the $CBOE\ SKEW$ index has been found to spike both in periods of market downturn and in periods of market excitement, and its relation with returns is less clear-cut than that of the VIX index (which has been called the "investors' fear gauge") as highlighted in the descriptive analysis of the $CBOE\ SKEW$ index in Section 3 and Section 7 below.

Our contributions include the following. First, we provide a descriptive analysis of the properties of the $CBOE\ SKEW$ index, in order to highlight its role as a measure of market fear or greed (or expectation of earnings).

Second, in the lines of the *CBOE SKEW* index formula, we delineate a skewness index for an important European market: the Italian stock market, that we call *ITSKEW*. While the implied volatility of the aggregate Italian market is currently measured by the implied volatility index (the *IVI* index),[1] a measure of the asymmetry in the return distribution and tail risk has yet to be adopted for this market. Third, this paper complements the existing studies by casting light on the properties of both the *CBOE SKEW* and the *ITSKEW* indices as barometers of the investors' fear of the downside, or of the investors' greed (excitement) of the possible upside. Fourth, we contribute to the literature by casting additional light on both the relation between risk-neutral skewness and future returns and the existence and the sign of a skewness risk premium.

The predictive power of risk-neutral skewness on future realized returns and, in turn, the sign of the skewness risk premium, are debated in the literature. Bali and Murray (2013) and Conrad et al. (2013) find a negative relation between risk-neutral skewness and future stock returns. This indicates that, consistent with investors' preference for positive skewness, stocks with a left skewed risk-neutral distribution earn higher future returns to compensate for their higher left-tail risk. However, several other studies find the opposite (positive) relation between risk-neutral skewness (or similar proxies) and future stock returns (Xing et al. (2010), Yan (2011), Cremers and Weinbaum (2010), Rehman and Vilkov (2012), Faff and Liu (2016), Stilger et al. (2016)). This finding is consistent with the theory that informed investors trade in options rather than in the underlying asset. According to this theory, good news is immediately reflected in higher call prices, while bad news is manifested in higher put prices. Good and bad news are subsequently incorporated into the underlying asset. Higher call prices make the risk-neutral distribution more skewed to the right (risk-neutral skewness increases) and when the positive information is incorporated into the underlying asset, returns become positive. On the other hand, higher put prices make the risk neutral distribution more skewed to the left (risk-neutral distribution decreases) and when the negative information is incorporated into the underlying asset, returns become negative.

The paper proceeds as follows. First, we provide a descriptive analysis of the properties of the *CBOE SKEW* index, in order to highlight its role as a measure of fear or greed. Second, we introduce an index similar to the

[1] The *IVI* index is computed by FTSE Russell, by using the *VIX* index formula adapted to the Italian market.

CBOE SKEW index for the Italian stock market and we call it *ITSKEW*. We highlight that this is the first attempt of computing a skewness index for a European market. Third, we investigate on the Italian market, two model-free asymmetry indices based on the Faff and Liu (2016) intuition of measuring skewness as the ratio between the right and left part of the distribution of the asset return, as explained in detail in Section 3. Fourth, we investigate the properties of the skewness indices. In particular, we consider whether these indices measure fear, as they are intended to, or greed in the market. Fifth, we investigate the relationship between skewness indices and future market returns. Sixth, we investigate the existence and sign of the skewness risk premium by analyzing portfolio strategies based on skewness and disentangle the contribution of the left and the right part of the risk-neutral distribution to the profitability of these strategies.

We obtain several findings. First, if the *ITSKEW* index falls by 100 basis points, the FTSEMIB index decreases by 0.349%. On the other hand, if the Italian *ITSKEW* index increases by 100 basis points, the FTSEMIB index goes up by 0.14%, manifesting an asymmetric behaviour. However, unlike the sign of the effect of volatility on returns (the FTSEMIB increases when expected volatility decreases and vice versa) a decrease (increase) in the *ITSKEW* index is associated with a decrease (increase) in the FTSEMIB index. We find a similar correlation between the *CBOE SKEW* index and S&P500 returns in the US market, though changes in the *CBOE SKEW* do not show any asymmetric effect. As a result, in contrast with the *VIX*, both the *ITSKEW* and the *CBOE SKEW* indices can be considered more as a measure of market greed than market fear, due to their positive relation with market returns. Both in the US market and the Italian market, high returns are in general associated with low levels of the volatility index and with high levels of the skewness index. The higher the volatility, the greater the fear; the higher the skewness index, the greater the greed (expectation of earnings). The model-free asymmetry indices based on the comparison of a bear and a bull index are not useful as indicators of current risk or greed because they are unrelated to market returns. Therefore, we find that the *ITSKEW* index presents many advantages relative to asymmetry indices: it has a significant contemporaneous relation with both returns, it has model-free implied volatility and it is still significant in the explanation of returns, even after controlling for model-free implied volatility.

Furthermore, our results point to an investor preference for positive skewness and a positive skewness risk premium (unlike the results in Bali and Murray (2013)). The positivity of the risk premium is supported also by portfolio trading strategies based on risk-neutral skewness. We find that selling out-of-the-money puts and buying out-of-the-money calls (a long position in risk-neutral skewness) is on average profitable. Moreover, the better performance of the portfolio composed of only put options, compared to the portfolio consisting of only call options, indicates that the mispricing of options is mainly focused on the left part of the distribution, meaning that put options are overpriced. As a result, buying risk-neutral skewness yields a positive return, consistently with the findings in Kozhan et al. (2013) and unlike the results in Bali and Murray (2013), who find that buying risk-neutral skewness yields a negative return. To make a comparison with variance, selling risk-neutral variance is found to be profitable in the literature (Carr and Wu, 2009). Selling risk-neutral variance is an insurance selling strategy, buying risk-neutral variance is an insurance buying strategy. Taking a long position on a variance swap (selling physical variance i.e. receiving floating and buying the risk-neutral one, i.e. paying fixed) yields a negative profit, and as a result the variance risk premium (defined as the difference between physical and risk-neutral variance) is negative. The negative sign on the variance risk premium means that investors regard as a negative shock to their portfolios any increase in market volatility and are willing to pay a premium in order to hedge against increases in market volatility. In our study buying risk-neutral skewness was found to be profitable, representing an insurance selling strategy, whereas selling risk-neutral skewness represents an insurance buying strategy. Taking a long position on a skewness swap yields a positive profit (selling physical skewness i.e. receiving floating and buying the risk-neutral one, i.e. paying fixed). As a result, the skewness risk premium (defined as the difference between physical and risk-neutral skewness) is positive. The positive sign on the skewness risk premium means that investors consider an increase in skewness to be a positive shock (the risk-neutral distribution shifts to the right). Investors are willing to pay a premium to be hedged against decreases in market skewness.

2. Literature review

After the October 1987 stock market crash, many authors recognized that the implied volatility of index options varies according to a pre-specified pattern: out-of-the-money put options are more expensive than out-of-the-money call options (the so-called skew or smirk). This phenomenon has been called "crash-o-phobia" (Rubinstein, 1994), due to the fact that put options are deemed to be more expensive than call options as they provide protection against stock market crashes. Jackwerth and Rubinstein (1996) investigate S&P 500 index option prices over an eight-year period from April 1986 to December 1993. They find that risk-neutral skewness and kurtosis show a discontinuity across the 1987 market crash, indicating that there was a significant change in the investors' downward risk perception. Moreover, the risk-neutral probability of another significant decline in the S&P 500 index increased after the 1987 crash. Aït-Sahalia and Lo (1998) propose a non-parametric technique for estimating the state price density implicit in option prices which can account for the skewness and the kurtosis of the risk-neutral density. Even though the skew pattern of implied volatilities has been widely documented in the literature (see Rubinstein, 1994 and Jackwerth and Rubinstein, 1996), only recently has it attracted the attention of researchers from the modelling perspective. The skew is reflected in a (negatively) skewed risk-neutral distribution, pointing to the presence of sizeable risk premia and the need for hedging against negative realizations of the underlying asset (tail risk).

Bakshi et al. (2003) propose a formula (BKM formula hereafter) to extract implied moments from a cross-section of option prices. The formula is model-free because it is not based on an option-pricing model and it is consistent with many different asset price dynamics. Many authors find that risk-neutral skewness computed using the Bakshi et al. (2003) formula to be a good predictor of the realized skewness. Lin et al. (2008) investigate the structure of the implied volatility smile in the LIFFE[2] option market by using prices from 79 individual stock options and the FTSE 100 index options, recorded from March 1992 to December 2002. They find a significantly positive relation between the physical and the risk-neutral moments. More recently, Neuberger (2012) finds that implied skewness predicts future realized skewness (computed from high-frequency returns) in the S&P500 index options market from December 1997 to September 2009.

[2] London International Financial Futures and Options Exchange

Several methods are proposed in literature to investigate the skewness risk premium. First, the skewness risk premium can be computed as the simple difference between physical and risk-neutral skewness. Foresi and Wu (2005) are the first to point to the existence of a premium charged by the market on downside index movements. They analyze twelve major equity indices using ten years of data (May 1995-May 2005). The discrepancy between these two skewness measures suggests that the market charges a high risk premium on downside index movements. Second, Zhao et al. (2013) propose computing the skewness risk premium as the negative difference between the physical and risk-neutral third moments, in order to ensure that the swap rate of the contract will be positive. They find the skewness risk premium and the risk-neutral skewness in the S&P500 index option market between January 1996 and December 2005 to be significantly negative and time-varying for all the considered sub periods, and for the 30, 60, and 90 day time to maturity. This evidence is consistent with a risk-neutral distribution more negatively skewed than the physical one. Third, an alternative measure of the skewness risk premium is proposed by Kozhan et al. (2013), who generalize the notion of variance swap (Carr and Wu, 2009) to higher order moments where the fixed leg is the option-implied moment and the floating leg is the realized moment. The average profit from the third moment swap can be interpreted as the premium for being exposed to the moment's risk. In the S&P500 equity index option market, in the period between January 1996 and January 2012, they find the average realized skewness to be negative and substantially smaller, in absolute terms, than the average implied skewness, indicating a more symmetrical realized distribution compared to the implied one. In addition, they show that the skewness risk premium is closely related to the variance risk premium: they both vary over time and are driven by a common factor (strategies to capture one and hedge out exposure to the other earn insignificant trading profits). Similarly, Neumann and Skiadopoulos (2013) find a positive relation between risk-neutral skewness and the variance risk premium. Fourth, along the lines of Muzzioli (2013a), Feunou et al. (2015) break down the variance risk-premium into upside and downside premia, supposing that investors like good uncertainty (as it increases the probability of gains) and dislike bad uncertainty (since it increases the likelihood of losses). In this setting, the skewness risk premium is computed as the difference between upside and downside variance risk premia. They find it to be negative in the S&P500 market in the period from September 1996 to December 2010, consistent with a risk-neutral distribution more negatively skewed than the physical one. Fifth, another strand of literature investigates the skewness risk premium by using portfolio strategies consisting of positions in options and in the underlying asset. Javaheri (2005) looks for profit opportunities arising from the mispricing of options. Based on the assumption that the option-implied distribution is in general more negatively skewed than the historical one, the author suggests a strategy of buying out-of-the-money calls and selling out-of-the-money put options. This portfolio can be interpreted as an insurance selling strategy. In fact the trade generates consistent profits if no crash occurs. However, it results in a significant loss in the case of a sudden large downward movement. By using S&P500 options from January 2002 to January 2003, the author finds mixed evidence on the profitability of skewness trades. Liu (2007) implements vega and delta neutral strategies by using FTSE 100 index options data from January 1996 to April 2000. Portfolios with long positions in put options and short positions in call options result in significant negative returns. The evidence suggests that out-of-the-money put options are overpriced compared to out-of-the-money call options. However, the profitability of the opposite strategy is unlikely to materialize because arbitrage profits are eroded by bid-ask spreads. Chang et al. (2013), measuring the skewness risk premium by means of portfolio sorting techniques in the American market in the period from January 1996 to January 2012, find a negative and economically significant skewness risk premium, not explained by other common risk factors. Bali and Murray (2013) investigate the pricing of risk-neutral skewness by using options on individual stocks in the American market from January 1996 to October 2010. The portfolios are delta and vega neutral, isolating a position in skewness (hence the portfolios are called skewness assets). They find a strong and robust relationship between risk-neutral skewness (measured with BKM methodology) and the skewness asset returns which represent a long skewness position. They argue that these results are consistent with a negative skewness risk premium and an investor preference for positive skewness. Similar results are obtained by Conrad et al. (2013) on a sample of individual stock options in the American market from January 1996 to December 2005. They find a strong and negative relationship between the third-order moment and the subsequent returns: firms with less negative or positive skewness are associated with lower returns over the following month. This means that investors seem to prefer assets with positive skewness. The relationship between skewness and returns is both economically and statistically significant and persists even after various controls. Moreover, they find that risk-neutral skewness can be considered as a market-based forwardlooking prediction of physical skewness. An investor preference for positively skewed assets is documented also in Amaya et al. (2015), where a physical measure of skewness is used in place of the option implied one.

Overall we can say that the majority of the papers focus on individual stocks, pointing to the existence of a significant skewness risk premium. The evidence is mixed regarding the relationship between skewness and subsequently realized returns. More specifically, although Bali and Murray (2013) and Conrad et al. (2013) find a negative relation, indicating that stocks characterized by a positive or less negative skewed distribution earn lower future returns and vice versa, many other papers find a positive relation. Xing et al. (2010) investigate the relationship between the shape of the volatility smirk and the cross-section of future equity returns, by using options on individual stocks in the period from January 1996 to December 2005. They find that stocks with the steepest smirks in the option market underperform stocks with a less pronounced smirk. Yan (2011) in the Option Metrics database from January 1996 to January 2005 finds that low slope portfolios earn higher returns than high slope portfolios, where the average stock jump size is proxied by the slope of option implied volatility smile. Cremers and Weinbaum (2010), using the Option Metrics database from January 1996 to January 2005 find that stocks with relatively expensive calls outperform stocks with relatively expensive puts. Rehman and Vilkov (2012) in the Option Metrics database from January 1996 to June 2007 find that option-implied ex-ante skewness is positively related to future stock returns. Finally, Faff and Liu (2016) in the S&P index options market on the period from January 1996 to August 2013 find that the more negatively skewed the risk-neutral distribution, the lower the future returns in the SPX market, suggesting that a skewness index is useful in forecasting the future index returns. Stilger et al. (2016) investigate the relationship between risk-neutral skewness of individual stocks and future realized stock returns over the period January 1996 to December 2012. By using a strategy that is long, the quintile portfolio with the highest risk-neutral skewness stocks and short the quintile portfolio with the lowest risk-neutral skewness stocks, they find that the relationship is significant and positive. They also argue that the underperformance of the portfolios with the lowest risk-neutral skewness is driven by those stocks that are perceived as overpriced by investors but hard to sell short.

To sum up, the majority of the papers find a positive relation between risk-neutral and physical skewness, indicating that the implied option measure is a good predictor of the realized measure (as is the case for volatility, see e.g. Muzzioli (2010) and Muzzioli (2015)). The skewness risk premium is found to be significant but the evidence on the sign is mixed (most of the studies find that risk-neutral skewness is generally greater in absolute value than physical skewness). The relationship between skewness and subsequently realized returns is debated: some papers find a positive relation, others a negative one. The majority of the papers have investigated the US market and individual stocks. The evidence on European markets and market indices remains scant.

3. Skewness measures

Skewness is a statistical measure of the asymmetry of the distribution. It is defined as:

$$Skewness = \frac{E[(R(\mathrm{t}) - \mu)^3]}{\sigma^3} = \frac{E[(X - \mu)^3]}{(\sigma^2)^{3/2}} \quad (1)$$

where $(R(\mathrm{t}) - \mu)^3$ is the third central moment, μ is the mean of returns, σ is its standard deviation, and $E()$ is the expectation operator under the physical measure. Skewness of a normal distribution is zero, indicating symmetry in the returns around the mean; if skewness is negative, it means that the mass of the distribution is concentrated in the left tail, meaning that the left tail is longer or thicker, or both, compared to the right tail. Conversely, if skewness is positive it means that the right tail is longer or thicker, or both, relative to the left tail (the mass of the distribution is concentrated in the right tail). It should be noted that in the literature, other statistical measures of asymmetry have been proposed (see e.g. Bowley's (1901), Pearson's (1895) and Groeneveld & Meeden's (1984))[3] which are not discussed here.

[3] Pearson's (1895) and Groeneveld & Meeden's (1984) are based on the relationship between mean, median and standard deviation of a distribution, while Bowley's (1901) exploits the quantiles of the distribution.

3.1 The model-free formula and the *CBOE-SKEW* index

Bakshi et al. (2003) develop a model-free method to extract volatility, skewness and kurtosis of the risk-neutral distributions from a cross-section of option prices. Their methodology is called model-free, since it does not rely on any option pricing model, being consistent with many underlying asset price dynamics. According to Bakshi et al. (2003) model-free skewness is obtained from the following equation, which represents the risk-neutral version of equation (1):

$$SK(\mathrm{t},\tau) \equiv \frac{E_t^q\{(R(\mathrm{t},\tau) - E_t^q[R(\mathrm{t},\tau)])^3\}}{\{E_t^q(R(\mathrm{t},\tau) - E_t^q[R(\mathrm{t},\tau)])^2\}^{3/2}} \tag{2}$$

In this specification, $R(\mathrm{t},\ \tau)$, $[R(\mathrm{t},\ \tau)])^2$ and $[R(\mathrm{t},\ \tau)])^3$ are the payoffs of the contracts, at time t with maturity τ, based on first, second and third moment of the distribution, respectively, and the prices of these contracts are obtained under the risk-neutral expectation (E_t^q) (for a more detailed discussion of the contracts see the Appendix). Equation (2) is used in the computation of the skewness index called *CBOE SKEW*, which measures the investors' perceived skewness of the Chicago Board Option Exchange (CBOE). Since the risk-neutral skewness attains typically negative values for equity indices because the implied option distribution is in general asymmetric to the left, in order to enhance the interpretation, CBOE defines *SKEW* as:

$$SKEW = 100 - 10 \times SK \tag{3}$$

where *SK* is the 30-day measure of risk-neutral skewness, computed by using linear interpolation between two values of risk-neutral skewness which refer to option series with different time to maturity (in general a first option series with a maturity of less than 30 days and a second one with time to maturity greater than 30 days are used). As a result, for a symmetric distribution, risk-neutral skewness is equal to zero and the *CBOE SKEW* index is equal to 100. This value can be considered as a threshold level for the *CBOE SKEW* index (zero risk-neutral skewness) since values greater (lower) than 100 mean that the risk-neutral distribution displays asymmetry to the left (right). Furthermore, $CBOE\ SKEW$ measures the slope of the implied volatility curve:[4] the steeper the curve, the higher the $CBOE\ SKEW$ index will be in value. Therefore, $CBOE\ SKEW$ can also be considered as a measure of the perceived tail risk of S&P500 log-returns in a 30-day horizon. Tail risk is the risk associated with the probability of extreme negative returns, in other words, returns two or more standard deviations below the mean (market crash, black swan). The probability of this type of event may be small for a normal distribution, but it could be significantly greater for a skewed distribution with fat tails. This is the case of the distribution of S&P500 log-returns which has a sizeable left tail and it is, therefore, riskier than a normal distribution with the same mean and variance: $CBOE\ SKEW$ quantifies this additional risk.

Historically, $CBOE\ SKEW$ has varied over a range of about 50 points and with an average value of 115. Its maximum value is 153.66 reached on 28 June 2016, a few days after the UK Brexit referendum. $CBOE\ SKEW$ reached its all-time low of 101.09 on 21 March 1991 at the end of the recession that started in July 1990. This means that the implied distribution of S&P500 log-returns has historically always been left-skewed (Neumann and Skiadopoulos (2013)). It is worth noting that in order to apply formula (2) to the financial market, where a continuum of option prices in strikes is not traded, both truncation and discretization errors occur. In the computation of the $CBOE\ SKEW$ index, only at-the-money and out-of-the money options with maturity of at least one week are considered. Furthermore, the interval of strike prices used is cut when two consecutive options with zero bid prices are found. As a result, if a change in volatility occurs, then the number of options considered in the computation may change. Other critical issues in the CBOE construction method concern the linear interpolation between near- and next-term maturities, which may induce a bias if model-free skewness is not a linear function of maturity. Furthermore, the use of the average between the lowest ask price and the highest bid price as a proxy of the option price may lead to errors when the bid-ask spreads are wide. Nonetheless, the $CBOE\ SKEW$ index formula is considered the market standard for the computation of skewness indices.

The data set for the US market consists of closing prices recorded from 3 January 2011 to 28 November 2014 (the same time period used for the Italian data set). The data for the S&P500 index is obtained from

[4] The volatility curve is obtained by plotting implied volatilities against strike prices for a given expiry. Since volatility decreases as the strike price increases in equity options, the implied volatility curve is negatively sloped, and the slope can be considered as a proxy for risk-neutral skewness.

Datastream, the data for the $CBOE\ VIX$ and the $CBOE\ SKEW$ indices are obtained from the CBOE website. The descriptive statistics for the $CBOE\ SKEW$ index, the $CBOE\ VIX$ index and the S&P500 returns are shown in Table 1. In this sample, the CBOE SKEW index varied between 111.31 and 146.08, with an average value of 123.68, pointing to a pronounced left-skewed risk-neutral distribution. The VIX index varied between 10.32 and 48.00, with an average value of 17.63, pointing to a heavy tailed risk-neutral distribution. All the series in levels are far from normality, as highlighted by the Jarque Bera test which strongly rejects the hypothesis of a normal distribution. For this reason, daily changes are defined in logarithmic terms as follows: $\Delta x_{t+1} = \ln(x_{t+1}/x_t)$, where x is the series under investigation.

The correlation coefficients between the $CBOE\ SKEW$ index, the VIX index and the S&P500 daily returns are shown in Table 2, both in terms of levels and the daily changes. We can see that the $CBOE\ SKEW$ index displays a negative significant correlation (-0.331) with the VIX index and a marginally significant correlation (positive) with daily market returns. On the other hand, the relation between the VIX index and S&P500 daily returns is negative and statistically significant. We find that high values of the VIX index are associated with negative returns and low values of the $CBOE\ SKEW$ index. The results are similar in terms of daily changes. Daily changes in the $CBOE\ SKEW$ index display a negative significant relation with daily changes in the VIX index and a positive significant relation with market returns. On the other hand the daily changes in the VIX index show a strong negative relation with market returns (-0.814). Therefore, while we expect the VIX index to act as a measure of market fear (Whaley (2000)), the role of the $CBOE\ SKEW$ index, due to its positive relation with market returns, is not clear and will be investigated in Section 7.

3.2 The SIX asymmetry index proposed by Faff and Liu (2016).

In order to overcome some of the limits of the $CBOE\ SKEW$ methodology when only a few strike prices are traded, Faff and Liu (2016) propose a model-based methodology to compute skewness in a Black-Scholes (1973) framework, by using a state-preference pricing approach. They use Black-Scholes implied volatilities instead of the model-free formula extracted from a few options: only two at-the-money call and put options with maturity closest to a 30-day period. They define the skewness index SIX, which is computed as the ratio of the lower partial moment volatility to the upper partial moment volatility of market returns as follows:

$$SIX = \frac{BEX}{BUX} \tag{4}$$

where BEX (the bear index) and BUX (the bull index) are the lower and upper partial moment volatility indices of market returns. Liu and O'Neill (2016) define BEX^2 as a financial asset that pays a USD amount of $ln(S_T/S_t)^2$ at some future date T, for every future index level S_T and spot price level S_t if $S_T \leq S_t$, or \$0 otherwise. BEX^2 can be obtained as:

$$BEX^2 = \sum_{s=1}^{S} \Phi_s \left[ln\left(\frac{S_T}{S_t}\right) - h \right]^2 I_{\ln(S_T/S_t) \leq h} \tag{5}$$

with S_t that is the current stock market index, S_T is the index value at time T, Φ_s is the risk-neutral probability of reaching S_T (or equivalently the state price density) and h is the threshold level that can be set to any arbitrary value in order to distinguish between positive and negative returns (e.g. 0, the risk-free rate, or the expected return $E(R)$). Symmetrically BUX^2 is computed as follows:

$$BUX^2 = \sum_{s=1}^{S} \Phi_s \left[ln\left(\frac{S_T}{S_t}\right) - h \right]^2 I_{\ln(S_T/S_t) > h} \tag{6}$$

i.e. it pays a USD amount of $ln(S_T/S_t)^2$ if $S_T > S_t$, or \$0 otherwise. The argument underlying the skewness index (SIX) formula is the following: if the risk-neutral distribution is symmetric, SIX in equation (4) is equal to 1; if the risk-neutral distribution is left (right) skewed, SIX is greater (lower) than 1. Liu and O'Neill (2016) estimated the state prices $\Phi(K_i, K_{i+1})$ as:

$$\Phi(K_i, K_{i+1}) = e^{-rT}\{N[d_2(K_i)]\} - \{N[d_2(K_{i+1})]\} \tag{7}$$

where $K_i < K_{i+1}$ denotes the *i-th* strike price,

$$d_2(K) = \frac{\ln(S_t/K) + (r - \delta - \sigma^2/2)T}{\sigma\sqrt{T}} \tag{8}$$

where δ is the dividend yield and σ is estimated as the average of four Black-Scholes implied volatilities from two at-the-money calls and two at-the-money puts with maturities the closest to a 30-day period. In order to discretize the state price density, Liu and O'Neill (2016) choose a grid of states spanning from 0.1 to 9999, with 0.10 increments.

It is worth noting that in order to capture the asymmetry of the distribution, many authors in the literature use the difference between the implied volatilities of two options with different moneyness as a proxy for risk-neutral skewness. In a skewed risk-neutral distribution, the smile or smirk is downward sloping; therefore the difference between the implied volatility of two options with opposite delta can be considered as a measure of the smile slope. Bali et al. (2016) use the difference in implied volatilities between an out-of-the-money call option and an out-of-the-money put option with delta equal to -0.25 and 0.25, respectively. Yan (2011) uses the difference in implied volatilities between a near-the-money put option and a near-the-money call option with delta equal to -0.5 and 0.5, respectively. Xing, Zhang and Zhao (2010) measures the slope of the volatility smile as the difference between the implied volatility of an out-of-the-money put option whose moneyness is between 0.80 and 0.95 and the implied volatility of an at-the-money call option with moneyness between 0.95 and 1.05. Similar proxies are used in the present paper to define portfolio trading strategies.

4. Data and methodology

The data set consists of closing prices on FTSE MIB[5]-index options (MIBO), recorded from 3 January 2011 to 28 November 2014. MIBO are European options on the FTSE MIB, a capital-weighted index composed of 40 major stocks listed on the Italian market. As for the underlying asset, closing prices of the FTSE MIB-index recorded in the same time-period are used. The FTSE MIB is adjusted for dividends as follows:

$$\hat{S}_t = S_t e^{-\delta_t \Delta t} \tag{9}$$

where S_t is the FTSE MIB index value at time t, δ_t is the dividend yield at time t and Δt is the time to maturity of the option. As a proxy for the risk-free rate, Euribor rates with maturities of one week, one month, two months, and three months are used: the appropriate yield to maturity is computed by linear interpolation. The data set for the MIBO is kindly provided by Borsa Italiana S.p.A; the time series of the FTSE MIB index, the dividend yield and the Euribor rates are obtained from Datastream.

[5] **F**inancial **T**imes **S**tock **E**xchange **M**ilano **I**ndice di **B**orsa

Several filters are applied to the option data set in order to eliminate arbitrage opportunities and other irregularities in the prices. First, consistently with the computational methodology of other indices (such as the $CBOE\ SKEW$), we eliminate options near to expiry (options with time to maturity of less than eight days) because they may suffer from pricing anomalies that occur close to expiration. Second, following Ait-Sahalia and Lo (1998) only at-the-money option and out-of-the-money options are retained. These include put options with moneyness (X/S, where X is the strike price and S the index value) lower than 1.03 and call options with moneyness higher than 0.97. Finally, option prices violating the standard no-arbitrage constraints are eliminated.

In order to compute risk-neutral skewness, we follow two different model-free methods: the procedure adopted by the CBOE that relies on the Bakshi et al. (2003) formula and the ratio between the volatility of the left part and the volatility of the right part of the distribution. The Faff and Liu (2016) formula provides an appealing insight into the possibility to measure skewness as the ratio between the right and left part of the distribution of the asset return. However, this formula suffers from the following drawbacks. First, it is a model-based approach, since it relies on the Black-Scholes formula. Many studies have highlighted the inconsistency of the assumption of a constant volatility, as supposed in the Black-Scholes model, with the empirical evidence in the financial markets (see e.g. Rubinstein (1985), Rubinstein (1994), Jackwerth and Rubinstein (1996)). Second, it considers only four around-the-money options in the estimation of the implied volatility to plug in the Black-Scholes formula, other options traded are discarded. This limitation results in a considerable loss of information. In order to overcome these limits, we propose to compute the asymmetry index, which we indicate as $\overline{SIX}_{mf}$, in a model-free setting.

We compute the ratio between the volatility of the left part and the volatility of the right part of the distribution. However, in order to obtain a model-free measure of the upside and downside volatility, we use the enhanced Derman and Kani method (Moriggia et al. (2009)) to derive the risk-neutral distribution of the underlying asset at the maturity date T. The implied tree has uniformly spaced levels Δt apart. Let $j=0,\dots,n$ be the number of levels of the tree that are spaced by $\Delta t = T/n$. As the tree recombines, the number of nodes at level j *will be:* $i=1,\dots,j+1$. The use of the Enhanced Derman and Kani method ensures the absence of no-arbitrage violations in the implied tree and it is motivated by its simplicity and the good replication of the smile pattern, as documented e.g. in Muzzioli (2013a, 2013b). The volatilities in the upper and lower part of the tree are computed as in Faff and Liu (2016), with the only difference that we adopt a model-free approach in order to estimate the risk-neutral distribution:

$$VOL_{UP}(t,T) = \sqrt{\sum_{i=1}^{j+1} \Phi_i [ln(S_i/S_t) - h]^2 \, I_{ln(S_i/S_t)>h}} \tag{10}$$

$$VOL_{DW}(t,T) = \sqrt{\sum_{i=1}^{j+1} \Phi_i [ln(S_i/S_t) - h]^2 \, I_{ln(S_i/S_t)\leq h}} \tag{11}$$

In this specification, Φ_i is the state price density and corresponds to the risk-neutral probability of reaching the ending node i at time T, with $i = 1, \ldots, j+1$; $ln(S_i/S_t)$ is the log-return of the underlying asset at node i; S_i is the underlying asset price at the ending node i; S_t is the underlying spot price at time 0 and h is threshold level. In particular, following Faff and Liu (2016) we use two values for h:

- $h = 0$ to compute SIX_{mf0} ;
- $h = E(R)$ to calculate SIX_{mfR} .

The new skewness index $\overline{SIX}_{mf}$ is computed as follows:

$$\overline{SIX}_{mf} = \frac{VOL_{UP}}{VOL_{DW}} \tag{12}$$

In order to have a constant 30-day measure for implied skewness, we derive the skewness indices by using a linear interpolation with the same formula, which is adopted for the computation of the $CBOE\ SKEW$ index (CBOE, 2010):

$$SK = w\, SK_{near} + (1-w) SK_{next} \tag{13}$$

with $w = \frac{T_{next}-30}{T_{next}-T_{near}}$, and T_{near} (T_{next}) is the time to expiration of the near (next) term options, SK_{near} (SK_{next}) is the skewness measure which refers to the near (next) term options, respectively.

Physical moments are obtained from daily FTSE MIB log-returns by using a rolling window of 30 calendar days. In this way the physical measures refer to the same time-period covered by the risk-neutral counterparts. Following the methodology adopted by the CBOE, to facilitate the interpretation and the comparison with *CBOE SKEW* index, we compute the Italian skewness indices (*ITSKEW)* as in equation (3). For the SIX_{mf} index SK = $(1-\overline{SIX}_{mf0})$ or $(1-\overline{SIX}_{mfR})$ in equation (3) in order to have the same interpretation as *CBOE SKEW:* values above the threshold level 100 suggest that the distribution displays negative skewness and vice versa. Physical skewness is computed as in equation (3) for ease of comparison.

5. Results for the Italian skewness indices

Skewness indices for the Italian market are shown in Figure 1. From this figure, we can observe the two skewness indices computed as the ratio between the volatility of the left and the right tail of the distribution which use a different barrier level h in order to separate positive and negative returns. SIX_{mf0} and SIX_{mfR}, show the same pattern, but SIX_{mf0} is shifted upward and its range of variation is slightly narrower. This differential pattern is due to the different barrier levels h used in the two measures ($h=0$ for SIX_{mf0} and $h=E(R)$ for SIX_{mfR}): in the first case a return is defined as positive if it is bigger than zero: in the second, if it is bigger than the expected return. Compared to these measures, *ITSKEW* displays a higher standard deviation. Moreover, SIX_{mf} indices present fewer peaks than *ITSKEW* does.

5.1 Descriptive analysis

Table 3 provides the summary statistics for the FTSE-MIB index returns, model-free implied volatility, physical and risk-neutral skewness indices, daily changes in the model-free implied volatility, and daily changes in the risk-neutral skewness indices. For each variable the last rows provide the Jarque-Bera test for normality and the p-value of the test. A high Jarque-Bera statistic value indicates that the null hypothesis of a normal distribution for the variable is rejected. A few observations are in order in this connection. First, the physical returns display slightly negative skewness and a pronounced excess kurtosis; the hypothesis of a normal distribution is rejected, according to the Jarque-Bera test. Also for model-free implied volatility, the hypothesis of a normal distribution is strongly rejected, indicating the presence of extreme movements in volatility in the form of fat tails. Second, all the skewness indices are on average higher than the threshold level of 100 (103.78, 103.11 and 101.44 for $ITSKEW$, SIX_{mf0} and SIX_{mfR} respectively), suggesting that both physical and risk-neutral skewness are in general negative in the sample period, with the physical distribution (100.13 for $SKEW_{PH}$) less negatively skewed than the risk-neutral distribution (as measured by the $ITSKEW$ index). Third, we find that extreme price decreases are more likely to occur than extreme price rises. Moreover, extreme price decreases are more frequently expected during the sample period (under the risk-neutral distribution) than subsequently realized (similar findings are in Conrad et al. (2013)). Fourth, all the risk-neutral skewness measures ($ITSKEW$, SIX_{mf0} and SIX_{mfR}) display positive skewness and excess kurtosis and the hypothesis of a normal distribution is strongly rejected based on the Jarque-Bera test. This indicates the presence of extreme movements also in the skewness measures. Fifth, physical skewness is found to be the most symmetric among all skewness measures, followed by the $ITSKEW$ index. The SIX_{mf} indices are the furthest from the normal distribution. The SIX_{mf} indices are not directly comparable in the levels to the other indices due to their different construction methods. They are less volatile than $ITSKEW$ and on average SIX_{mf0} points to a more negative skewed distribution than SIX_{mfR}, being the distribution sliced slightly left of zero. This suggests that the expected return implied in an option price is slightly less than zero.

The correlation coefficients between the skewness measures and the other moments of the return distribution are shown in Table 4, both in terms of levels and the daily changes. According to this table, the $ITSKEW$ index displays the highest significant correlation (0.156) with realized skewness index ($SKEW_{PH}$), while the SIX_{mf} indices are almost unrelated with physical skewness. Therefore, only the $ITSKEW$ index can be used to forecast realized skewness. The $ITSKEW$ measure also presents a positive and significant correlation (0.21) with daily returns, the highest in absolute value, while the SIX_{mf} indices are almost unrelated to daily returns. Interestingly, while the $ITSKEW$ index has a negative and significant correlation with the model-free implied volatility, the SIX_{mf} indices show a positive and significant correlation. Therefore, according to the $ITSKEW$ (SIX_{mf}) index, the risk-neutral distribution of the FTSE-MIB index returns is less (more) negatively skewed when model-free implied volatility is high. The value of the correlation between $ITSKEW$ and model-free implied volatility (-0.284) is similar to the value of the correlation between the $CBOE\ SKEW$ index and the $CBOE\ VIX$ index computed over the same period (-0.331). The $ITSKEW$ index also shows an average value of about 103.78 over the period 2011-2014, far lower than the corresponding average value of $CBOE\ SKEW$ index (123.68). This means that in general the risk-neutral distribution of log-returns of the FTSE MIB index is less asymmetric in the sample period than that of the S&P 500 index. One possible explanation for this evidence is that S&P500 index is broadly representative of the stock market (compared to the Italian market index) and, as a results, of the investment opportunity set. This makes the S&P500 put options an ideal candidate in order to hedge portfolios against downside market risk and this may increase the volatility of the put and in turn the asymmetry of the distribution. On the other hand the average implied volatility of the Italian market is about twice that of the US market (measured by the VIX index). This is due to the higher sectorial diversification of the S&P500 index (the FTSE MIB index is composed by a limited number of stocks and it is highly representative of the financial sector) and to the European debt crisis that affected the Italian market during the 2011-2012 time-period.

The correlation between the daily changes of the $ITSKEW$ index and the daily changes in model-free implied volatility is negative, suggesting that a greater model-free implied volatility is associated with more symmetry in the distribution. Daily changes in the SIX_{mf} indices are almost unrelated to volatility changes. The correlation between the daily changes of the $ITSKEW$ index and the returns is positive, suggesting that a positive return is associated with a positive change in the $ITSKEW$ index. The SIX_{mf} indices are almost unrelated to returns. Figure 2 shows the graphs of the $ITSKEW$ index and of the SIX_{mf0} index along with the FTSE MIB index (the graphical comparison with SIX_{mfR} is not shown, since it shares the same pattern as SIX_{mf0}).

5.2 Properties of the skewness indices

In order to investigate the relation between changes in the skewness measures and changes in model-free implied volatility, we estimate the model described by equation (14). To this end, we compute the changes in model-free implied volatility using the model-free methodology as in Muzzioli (2013b) with an extrapolation outside the existing domain of strike prices with a constant volatility function.

$$\Delta skewness_t = \alpha + \beta \Delta IV_t + \varepsilon_t \tag{14}$$

In this model $\Delta skewness_t$ is proxied by daily changes in $ITSKEW$ index ($\Delta ITSKEW_t$), SIX_{mf0} index (ΔSIX_{mf0_t}) and SIX_{mfR} index (ΔSIX_{mfR_t}); ΔIV_t is the daily change in model-free implied volatility. The estimation results, presented in Table 5, point to a negative relation between volatility changes and changes in the $ITSKEW$ index as β is negative and statistically different from zero. This indicates that an increase in model-free implied volatility is associated with a decrease in the $ITSKEW$ index or a less negative risk-neutral distribution. We do not find any significant relation between model-free implied volatility and the SIX_{mf} indices. The results for the $ITSKEW$ index are consistent with the findings in Chang et al. (2013) in the S&P500 index options market. Moreover, Neuberger (2012) also finds a positive correlation coefficient between model-free variance and skewness, implying that the higher the variance, the less skewed the distribution will be (the magnitude of the correlation is 0.297 in the period 1997-2009, on the S&P500 index options market). Recall that the correlation between volatility and returns is negative (leverage effect). A possible explanation is that when volatility is high, returns are low (for example, in a stressed market or after a market crash) and a repeat crash (as indicated by the $ITSKEW$ index) may not be viewed as that likely. On the other hand, when volatility is low, returns are high (tranquil market) and a crash is deemed to be more probable.

In order to investigate whether the skewness indices act as a measure of market fear or market greed, we analyze the relation between changes in the skewness measures and the returns of the FTSE-MIB index. To this end we estimate the following regression:

$$R_t = \alpha + \beta \Delta skewness_t + \varepsilon_t \tag{15}$$

In this model $\Delta skewness_t$ is proxied by $\Delta ITSKEW_t$, ΔSIX_{mf0_t} , ΔSIX_{mfR_t}. Results are presented in Table 6. We intend to assess whether the skewness measures can be considered as indicators of market fear or market greed, i.e. whether they measure more investors' excitement than investors' fear. The slope coefficient of changes in the $ITSKEW$ is positive and significant, suggesting that an increase in the $ITSKEW$ index (i.e. the risk-neutral distribution becomes more negatively skewed), is associated with positive returns. Therefore, positive peaks in $ITSKEW$ can be considered as indicators of investor greed, negative peaks in $ITSKEW$ can be considered as indicators of investor fear (market stress). When skewness is proxied by the SIX_{mf} indices, the slope coefficients are insignificant, pointing to the uselessness of the SIX_{mf} indices as indicators of current risk.

In order to disentangle the effect of positive and negative changes in the $ITSKEW$ index on FTSE-MIB index returns, we divide the changes in the $ITSKEW$ index into positive ones:

$$\Delta ITSKEW_t^+ = \Delta ITSKEW_t \text{ if } \Delta ITSKEW_t > 0\text{, otherwise } \Delta ITSKEW_t^+ = 0 \quad (16)$$

and negative ones:

$$\Delta ITSKEW_t^- = \Delta ITSKEW_t \text{ if } \Delta ITSKEW_t < 0\text{, otherwise } \Delta ITSKEW_t^- = 0 \quad (17)$$

and estimate the following regression:

$$R_t = \alpha + \beta_1 \Delta ITSKEW_t^+ + \beta_2 \Delta ITSKEW_t^- + \varepsilon_t \quad (18)$$

Table 7 shows the regression results on asymmetry of effects from positive and negative changes in $ITSKEW$ on returns. Both positive and negative changes in the $ITSKEW$ index are highly significant as both slope coefficients are positive. In terms of magnitude, the coefficient of negative changes in the $ITSKEW$ index is more than twice as large as that of the positive changes, indicating an asymmetric effect (the Wald test for H0: β_1 is statistically equal to β_2 is strongly rejected at the 1% level). Specifically, a decrease in the $ITSKEW$ index (risk-neutral skewness increases) is associated with a strong decrease in returns (0.349), while an increase in the $ITSKEW$ index (risk-neutral skewness become more negative) is associated with a less pronounced increase in returns (0.140). The market reacts more negatively to decreases in the $ITSKEW$ index than it reacts positively to increases in the $ITSKEW$ index. Therefore, in this setting the $ITSKEW$ index acts as a measure of market greed and the opposite of the $ITSKEW$ index (risk-neutral skewness) acts as a measure of market fear.

A comparison with volatility can be useful for a better understanding of this relation. An increase in market volatility is reflected in a deterioration of the investment opportunity set (Chang et al., 2013), due to its negative correlation with market returns, an increase in the $ITSKEW$ index (decrease in risk-neutral skewness) is viewed as an improvement of the investment opportunity set, due to its positive correlation with market returns. Moreover, the negative relation between market returns and the VIX index allows us to define it as an indicator of investor fear (Whaley, 2000). Symmetrically, the positive relation between market returns and the $ITSKEW$ index allows us to define it a barometer of investor excitement (greed) rather than investor fear. While the investors' aversion to changes in model-free implied volatility is well documented in the literature (see Carr and Wu, 2009), a few papers investigate investor preference for skewness uncertainty (Chang et al. 2013, Chabi-Yo, 2012). It is assumed that risk-averse investors prefer right-skewed assets and as a consequence are willing to obtain a lower return on these assets (Chang et al. 2013). Therefore, a positive change in the $ITSKEW$ index, i.e. a decrease in risk-neutral skewness, is related to an increase in the possibility of negative jumps representing bad news for investors. In this context, stocks which react positively to a decrease in risk-neutral skewness, which is perceived as bad news, provide a desirable hedge against downside jump risk and as a consequence require lower future returns. As the FTSE MIB index reacts positively to a decrease in risk-neutral skewness, it provides a hedge against skewness risk since it increases in value when the riskiness of the distribution (in term of skewness) increases. The market index is perceived as a well-diversified portfolio, and therefore it is less vulnerable to jump risk compared to individual stocks.

As a final step, in order to assess the correlation between returns, changes in model-free implied volatility and changes in the skewness measures, we estimate the following regression:

$$R_t = \alpha + \beta_1 \Delta skewness_t + \beta_2 \Delta IV_t + \varepsilon_t \tag{19}$$

In this specification the change in skewness ($\Delta skewness_t$) is proxied alternatively by the three measures $\Delta ITSKEW_t$, ΔSIX_{mf0_t} , ΔSIX_{mfR_t}, and ΔIV_t is the change in model-free implied volatility. Estimation results for the model are shown in Table 8. We can see that the coefficient of changes in model-free implied volatility (ΔIV_t) is negative and highly significant in every regression. Among skewness measures, changes in the $ITSKEW$ index are the only significant ones, demonstrating the power of this skewness measure in the explanation of market returns. Therefore, the $ITSKEW$ index presents several advantages, compared to the SIX_{mf} indices: it has a significant contemporaneous relation with both index returns and model-free implied volatility, and it augments the volatility explanation content with valuable information about returns (in fact the adjusted R-squared increases when we add the $ITSKEW$ index into the regression).

As a second goal of our study, we aim to assess whether the *ITSKEW* index can be used to forecast future market returns. A previous study, Muzzioli (2013b) found that changes in implied volatility (as measured by both Black-Scholes implied volatility and model-free implied volatility) can serve as an early-warning indicator of market stress, and that the returns have explanatory power in forecasting future implied volatility. Pan and Poteshman (2006) find that publicly observable option signals are able to predict stock returns only for the next one or two trading days, before stock prices subsequently reverse. Other papers (Xing et al. 2010) find that the predictability from volatility smirks persists for a much longer period (six months).

To investigate the power of skewness indices in predicting future market returns, we estimate a vector autoregression (VAR) model as follows:

$$R_t = c + \sum_{l=1}^{K} a_l \Delta\, ITSKEW_{t-l} + \sum_{l=1}^{K} b_l R_{t-l} + u_t \tag{20}$$

$$\Delta\, ITSKEW_t = c + \sum_{l=1}^{K} a_l R_{t-l} + \sum_{l=1}^{K} b_l \Delta\, ITSKEW_{t-l} + u_t \tag{21}$$

The lag length $k = 2$, is chosen according to both the Schwarz and Hannan-Quinn information criteria. The estimation output of the model is shown in Table 9. The VAR coefficient estimates show that changes in the *ITSKEW* index can be explained by past returns (one lag) and past changes in the *ITSKEW* index (lags one and two). On the other hand, returns cannot be explained by past returns, but can be explained by past changes in the *ITSKEW* index (one lag). We perform a Granger causality test for the null hypothesis of zero effect (H: $a_l = 0$, $l = 1{,}2$) from skewness on returns in equation (20) and from market returns on skewness in equation (21) in order to understand whether $\Delta ITSKEW$ Granger causes returns R in the first regression, as opposed to R Granger causing $\Delta ITSKEW$ in the second regression. The results are shown in Table 10. The Granger causality test shows that both the null hypothesis that returns do not Granger cause changes in the *ITSKEW* index and that changes in the *ITSKEW* index do not Granger cause returns are rejected at the 5% level. Therefore, there is weak evidence that positive changes in the *ITSKEW* index are reflected in a negative return the following day, and that, on the other hand, a positive return is reflected in an increase of the *ITSKEW* index in the following day. This is in line with Harvey and Siddique (2000), who find that when past returns have been high, the investors' forecast of skewness becomes more negative. This is consistent with the so-called "bubble theory" purporting that if past returns have been high, this indicates that the bubble has been inflating and, therefore, a significant drop can be expected when the bubble bursts.

6. Sub-period analysis

In order to investigate if high or low levels of implied volatility have an effect on skewness, we propose a comparison between the FTSE MIB index, model-free implied volatility and the $ITSKEW$ index, as shown in Figure 3. From the graph we can observe two different medium-term trends: a negative trend (bearish market) characterized by a higher volatility between February 2011 and the end of July 2012, and a positive trend (bullish market) in the second part of the sample period, which is characterized by lower volatility. We may attribute the reversal in trend to the positive effect of the "whatever it takes" London Speech of the ECB President Mario Draghi (26 July 2012) that put an end to the acute phase of the European sovereign debt crisis. Proclaiming that the ECB would do "whatever it takes" to save the Euro was the incipit to the Outright Monetary Transactions[6] policy, putting an end to speculation on government bonds of the peripheral countries. The speech was followed by an immediate increase in the market indices of the European stock markets. Therefore, in order to assess the behaviour of the skewness indices in high and low volatility periods, we split the data set according to these volatility periods and report the descriptive statistics of the skewness indices in the two sub-periods in Table 11.

Physical skewness is negative in the first sub-period characterized by a bearish market, and slightly positive in the second sub-period characterized by a bullish market. Risk-neutral skewness indices attain a value higher than 100 in both sub-periods, pointing to an overall negative skewness. The $SKEW$ index is high in the bullish period and low in the bearish period, pointing to a more negatively skewed distribution in the period characterized by a stable market and low volatility. These findings are consistent with the findings in Han (2008), Faff and Liu (2016) in the S&P500 options market. Also Neuberger (2012) finds in the S&P500 options market

[6] Outright Monetary Transactions, announced on 2 August, 2012, is a programme which allows the ECB to purchase, under certain conditions, sovereign bonds issued by Eurozone member-states. http://www.ft.com/cms/s/0/448a6f28-f822-11e1-828f-00144feabdc0.html

that in the period when index volatility was low (2003-2007) negative skewness was relatively high, whereas it was rather low in the volatility spike of 2008. When the market returns are positive (bullish market) risk-neutral skewness tends to be more negative. On the other hand, in periods of bearish market, risk-neutral skewness tends to be more positive, since investors are expecting an inversion of the tendency. In fact, when the market is bearish, investors may purchase out-of-the-money calls instead of buying the underlying asset, shifting the risk-neutral distribution to the right. The information we obtain from the SIX_{mf} is the opposite. The behavioral pattern of the SIX_{mf} index is found to be more consistent with physical skewness which is slightly less negative (more positive) in the low volatility period. The SIX_{mf} index is consistent with Dennis and Mayhew (2002), who find that risk-neutral density tends to be more negatively skewed for stocks in periods of higher market volatility. The difference between physical and risk-neutral skewness ($SKEW$ index) is higher in the bullish market period. This indicates the presence of a skewness risk premium in the Italian market, that is more consistent in the low volatility period. This means that in bullish market periods investors expect a more negatively skewed risk-neutral distribution than subsequently realized. This is investigated further in the following section by means of portfolio trading strategies.

7. A comparison with the CBOE SKEW index

In order to assess whether the properties of the $ITSKEW$ index observed in the Italian market also apply to the US market, we replicate the analysis based on equations (14)-(21) for the US market on the same time-period. We estimate the regression in equation (14) by using changes in the $CBOE\ SKEW$ index as a proxy for $\Delta skewness_t$ and changes in the VIX index as a proxy for ΔIV_t. The results, presented in Table 12, show a negative statistically significant relation between changes in the $CBOE\ SKEW$ index and changes in the VIX index, in line with the results found in the Italian market. This indicates that an increase in the VIX index is associated with a decrease in the $CBOE\ SKEW$ index (less negative risk-neutral distribution). However, the adjusted R-squared is lower than the one for the $ITSKEW$ index in Table 5.

In order to investigate the relation between changes in the *CBOE SKEW* index and S&P500 daily returns, we estimate equation (15) where $\Delta skewness_t$ is proxied by changes in the *CBOE SKEW* index and R_t by S&P500 daily returns. The results, shown in Table 13, display a positive statistically significant relationship between changes in the *CBOE SKEW* index and S&P500 returns: an increase in the *CBOE SKEW* index is associated with an increase in market returns. Again, the relation, though weaker than the one in the Italian market (lower adjusted R-squared) is of the same sign.

In order to disentangle the effect of positive and negative changes in the *CBOE SKEW* index on S&P500 index returns, we divide the changes in the *CBOE SKEW* index into positive and negative ones (equations 16-17) and estimate the regression in equation (18). The results, reported in Table 14, show positive coefficients for both positive and negative changes in the *CBOE SKEW* index. However, while the coefficient for positive changes in the skewness index is statistically significant at the 1% level, the one for negative changes in the *CBOE SKEW* index is only marginally significant at the 5% level. This result suggests that positive changes in *CBOE SKEW* are associated with positive returns; and negative changes in the *CBOE SKEW* are associated with negative returns. Unlike the results for the *ITSKEW,* we do not detect any asymmetric effect (the Wald test for H0: β_1 is statistically equal to β_2 is not rejected). This is the main dissimilarity between the US and the Italian stock market. In fact, in the Italian case the coefficient for negative changes in the *ITSKEW* index (β_2) is more than twice the coefficient for positive changes in the Italian skewness index (β_1), as reported in Table 7.

We also investigate whether the relationship between changes in the *CBOE SKEW* index and market returns persists if we add changes in the *VIX* index as an explanatory variable (equation 19). The results, reported in Table 15, show that the coefficient for changes in the *VIX* index (β_2) is negative and statistically different from zero, while the coefficient for changes in the *CBOE SKEW* index is positive but only marginally significant. This result suggests that changes in the *CBOE SKEW* index lose explanatory power when we consider changes in the volatility index in the model. In contrast, in the Italian market, when we add changes in implied volatility as an explanatory variable (equation 19), the *ITSKEW* coefficient is still statistically different from zero. To sum up, we find that *CBOE SKEW* index has similar properties to the *ITSKEW* index. In particular changes in the *CBOE SKEW* index present a positive relation with market returns and a negative relation with the *VIX* index. Therefore we can state that also the *CBOE SKEW* index acts more as a measure of market greed than as a measure of market fear.

Another interesting and debated issue is the relation between skewness and future market returns. In order to assess whether we can use the *CBOE SKEW* index in order to forecast future market returns, or vice versa, market returns to forecast future changes in the *CBOE SKEW* index, we estimate the following VAR models:

$$R_t = c + \sum_{l=1}^{K} a_l \Delta\, CBOE\ SKEW_{t-l} + \sum_{l=1}^{K} b_l R_{t-l} + u_t \tag{22}$$

$$\Delta\, CBOE\ SKEW_t = c + \sum_{l=1}^{K} a_l R_{t-l} + \sum_{l=1}^{K} b_l \Delta\, CBOE\ SKEW_{t-l} + u_t \tag{23}$$

where R_t is proxied by S&P500 daily returns. We chose the lag length equal to 3, according to both the Schwarz and Hannan-Quinn information criteria. The estimation output of the model is reported in Table 16. We can see that the S&P500 returns can be explained only by lagged returns (one, two and three lags). On the other hand, changes in the *CBOE SKEW* index can be explained mainly by lagged changes in the *CBOE SKEW* index (one, two and three lags) and by market returns (at lag two). We perform a Granger causality test for the null hypothesis of zero effect (H: $a_l = 0,\ l = 1,2,3$) from change in the *CBOE SKEW* index on returns in equation (22) and from market returns on changes in the skewness index in equation (23) in order to understand whether $\Delta CBOE\ SKEW$ Granger causes returns R in equation (22), as opposed to R Granger causing $\Delta CBOE\ SKEW$ in equation (23). The results, reported in Table 17, show that the null hypothesis that returns do not Granger cause changes in the *CBOE SKEW* index is rejected at the 10% level. Therefore, there is weak evidence that positive returns are reflected in positive changes in the *CBOE SKEW* index the following day; this result, although weaker, is in line with the one found in the Italian market. On the other hand, we cannot reject the null hypothesis that changes in the *CBOE SKEW* index do not Granger cause future market returns. Therefore, unlike the *ITSKEW* index, changes in the *CBOE SKEW* index are not useful to forecast future market returns.

8. Trading strategies

The difference between risk-neutral and physical skewness may be exploited by skewness trades which allow investors to profit from an overvalued or undervalued third moment. When the implied third moment is undervalued with respect to the physical skewness, Javaheri (2005) suggests a strategy consisting of buying out-of-the-money calls and selling out-of-the-money puts. This strategy is exploited also in Bali and Murray (2013) where three different skewness assets (they are named skewness assets since their value depends solely on the skewness of the underlying asset) are used to test mispricing in different portions of the risk-neutral density of returns. Therefore, in order to assess whether it is possible to exploit the difference between risk-neutral and physical skewness, in line with Bali and Murray (2013), we create three different portfolios: a $PUTCALL\ asset$ (a short position in out-of-the-money (OTM) puts and a long position in out-of-the-money (OTM) calls) a $PUT\ asset$ (a short position in out-of-the-money (OTM) puts and a long position in at-the-money (ATM) puts) and a $CALL\ asset$ (a long position in out-of-the-money (OTM) calls and a short position in at-the money (ATM) calls). In order to isolate the effect of skewness, the exposure to changes in the underlying asset (delta-neutral) and volatility (vega-neutral) is removed. As a result, each asset represents a long skewness position. A comparison with volatility trading strategies can be useful for a better understanding of these portfolios. Indeed, as a long straddle position is considered as a long volatility position since it increases (decreases) in value when the volatility of the underlying security increases (decreases), skewness assets increase (decrease) in value when the skewness of the underlying security increases (decreases). Portfolio strategies are investigated also in Kozhan et al. (2013): they find that buying low-strike puts and selling high-strike calls generates on average a negative return, and as a result buying risk-neutral skewness is profitable.

The $PUTCALL\ asset$, described by equation (24), is designed to change value if there is a change in the skewness of the risk-neutral return density coming either from a change in the left tail, or from a change in the right tail, or from both:

$$PUTCALL\ asset = C_{OTM} - \frac{V_{C_{OTM}}}{V_{P_{OTM}}}\ P_{OTM} - \left(\Delta_{C_{OTM}} - \frac{V_{C_{OTM}}}{V_{P_{OTM}}} \Delta_{P_{OTM}}\right) S \tag{24}$$

where C_{OTM} and P_{OTM} indicate the price of out-of-the-money call and put, respectively, Δ is the delta of the option, V is the vega of the option, and S is the position of the investor in the underlying asset. The return of the $PUTCALL\ asset$ is expected to be positive if OTM calls are undervalued relative to OTM puts. This condition is consistent with an implied distribution more negatively-skewed than the physical one.

The $PUT\ asset$, described by equation (25), is designed to change in value if there is a change in the skewness of the underlying asset coming from a change in the left tail of the risk-neutral density:

$$PUT\ asset = -P_{OTM} + \frac{V_{P_{OTM}}}{V_{P_{ATM}}} P_{ATM} - \left(-\Delta_{P_{OTM}} + \frac{V_{P_{OTM}}}{V_{P_{ATM}}} \Delta_{P_{ATM}}\right) S \tag{25}$$

where P_{OTM} and P_{ATM} indicate the price of out-of-the-money put and at-the-money put, respectively, Δ is the delta of the option, V is the vega of the option and S is the position in the underlying asset. The return of the $PUT\ asset$ is expected to be positive if OTM puts are overvalued relative to ATM puts.

The $CALL\ asset$, described by equation (26), is designed to change value if there is a change in the skewness of the underlying asset arising from a change in the right tail of the risk-neutral density.

$$CALL\ asset = C_{OTM} - \frac{V_{C_{OTM}}}{V_{C_{ATM}}} C_{ATM} - \left(\Delta_{C_{OTM}} - \frac{V_{C_{OTM}}}{V_{C_{ATM}}} \Delta_{C_{ATM}}\right) S \tag{26}$$

where C_{OTM} and C_{ATM} indicate the price of out-of-the-money put and at-the-money put, respectively, Δ is the delta of the option, V is the vega of the option and S is the position in the underlying asset. The return of the $CALL\ asset$ is expected to be positive if OTM calls are undervalued relative to ATM calls.

We create the skewness assets in equations (24)-(26), by using next-term option prices that usually have a maturity between 30 and 70 days. The options with the closest strike price to the underlying asset value are taken to be the at-the-money options. Out-of-the-money options are taken to be the ones whose strike price to underlying asset price ratio is the closest to 0.90 for puts and 1.10 for call options, respectively. In order to have delta and vega neutral portfolios, trades are set at day t and are closed at day $t+1$. Daily profits and losses are computed as the difference between the value of the portfolios in $t+1$ and in t and represent the daily risk premium for being exposed to skewness. Daily return is computed as:

$$r = \frac{P_{t+1} - P_t}{|P_t|} \tag{27}$$

where P_{t+1} and P_t are the prices of the skewness asset at day $t + 1$ and t, respectively. In line with Bali and Murray (2013), we use the absolute value of the skewness asset price at time t because skewness asset prices are not guaranteed to be positive. Transaction costs are not considered.

The cumulative return of the three skewness assets is reported in Figure 4. We can observe that the cumulative return of all skewness assets is positive during the sample. The descriptive statistics of the skewness assets' returns are reported in Table 18. Average daily returns are ascertained to be statistically different from zero, by using the Newey West adjusted errors. The $PUTCALL\, asset$ achieves the best performance with a cumulative return of 56.43%, the average daily return is statistically different from zero, pointing to a heavy overvaluation of out-of-the-money put options and symmetrically, an undervaluation of out-of-the-money call options. The $PUT\, asset$ achieves a cumulative return of 49.16% and the average daily returns are statistically different from zero. This result suggests that out-of-the-money put options are highly overvalued with respect to at-the-money options. The $CALL\, asset$ realizes a cumulative return of 10.64%. However, the average daily return is not statistically different from zero. Therefore, the undervaluation of out-of-the-money call options with respect to at-the-money call options is not statistically significant. We can conclude that the mispricing of options is concentrated in the left tail of the distribution. This means that the implied distribution is in general more negatively-skewed than the physical one and that buying skewness is on average profitable. Although Bali and Murray (2013) find the opposite result, this evidence is consistent with Kozhan et al. (2013) and the literature that documents the overvaluation of out-of-the-money put options with respect to out-of-the-money call options (see e.g. Javaheri (2005) and Liu (2007)). The dissimilarity might be related to the different rebalancing in the skewness assets implementation: in fact, Bali and Murray (2013) do not rebalance the position daily for delta and vega, retaining the same portfolios until option expiration.

9. Conclusions

In this paper we analyzed the role of the *CBOE SKEW* index as a as a measure of market fear or greed and proposed different skewness measures for the Italian index option market. In order to delineate for the first time a skewness index for the Italian stock market we exploited both the CBOE method to compute a skewness index (we call it *ITSKEW*) and a model-free measure based on the ratio between the volatilities on the left and on the right part of the risk-neutral distribution (SIX_{mf}). The CBOE method yields a skewness index which is negatively related to the Italian volatility index, both in terms of levels and daily changes, indicating that higher volatility is associated with less negative skewness. This is consistent with the results obtained for the S&P500 option market, in line with Han (2008) and Faff and Liu (2016). Unlike the *ITSKEW*, the SIX_{mf} index has a weak positive correlation with model-free implied volatility. We also find a negative relation between volatility changes and changes in the *ITSKEW* index: an increase in model-free implied volatility is associated with a decrease in the *ITSKEW* index (less negative risk-neutral distribution). We do not find a significant relation between model-free implied volatility and the other asymmetry indices (SIX_{mf}) indicating that the SIX_{mf} is not related to model-free implied volatility. Unlike the SIX_{mf} indices, the *ITSKEW* index can also be considered as a predictor of future realized skewness, thanks to its good correlation with the latter. Moreover, the *ITSKEW* index behaves similarly to the *CBOE SKEW* index: positive changes in the *CBOE SKEW* index are associated with negative changes in the *VIX* index.

By investigating the relation between the skewness indices and market returns, we find that an increase in the *ITSKEW* index (i.e. the risk-neutral distribution becoming more negatively skewed), is associated with an increase in returns. Therefore, in this setting the *ITSKEW* index acts as a measure of market greed. We also find that the effect of positive and negative changes in *ITSKEW* is asymmetric: a decrease in the *ITSKEW* index, indicating that the distribution becomes more skewed to the right, is associated with a strong decrease in the returns, while an increase in the *ITSKEW* index is associated with a less pronounced increase in returns. The market reacts more negatively to decreases in the *ITSKEW* index than it reacts positively to increases in the *ITSKEW* index. When skewness is proxied by the SIX_{mf} indices, the slope coefficients in equation (15) are insignificant, pointing to the uselessness of the SIX_{mf} indices as indicators of current risk. Therefore, we find that the $ITSKEW$ index presents some advantages compared to the SIX_{mf} indices: it is a better predictor of future realize skewness, it has a significant contemporaneous relation with returns and model-free implied volatility, and it is significant in the explanation of returns, even after having controlled for volatility. The $CBOE\ SKEW$ index behaves, similarly to the Italian skew index, as a measure of market excitement, even if it does not show any asymmetric effect. Moreover, changes in the $CBOE\ SKEW$ index lose explanatory power on returns when the VIX index is considered in the model. This suggests that part of the information content of the $CBOE\ SKEW$ index is already embedded in the VIX index. This highlights the importance of investigating other skewness measures that may better complement the implied volatility information in explaining market returns, which is left for future research, since also the skewness measure based on the ratio between the volatilities on the left and on the right part of the risk-neutral distribution (SIX_{mf}) do not have any explanatory power.

We also find weak evidence that positive changes in the $ITSKEW$ index are reflected in a negative return the following day, and that a positive return is reflected in an increase of the $ITSKEW$ index. This is in line with Harvey and Siddique (2000), who find that when past returns have been high, the investors' forecast of skewness becomes more negative, consistent with the so-called "bubble theory": if past returns have been high, this means that the bubble has been inflating and, therefore, a large drop can be expected when the bubble bursts. In particular, the combination of a high skewness index and a low implied volatility may indicate an overly complacent market, and signal the creation of speculative bubbles.

Finally, our findings point to the existence of a positive skewness risk premium in the Italian market, consistently with an investors' preference for positive skewness. This emerges both from the fact that implied skewness is more negative than physical skewness in the sample period and from the profitability of skewness trading strategies. The positive returns of the three portfolios (a short position in out-of-the-money puts and a long position in out-of-the-money calls; a short position in out-of-the-money puts and a long position in at-the-money puts; a long position in out-of-the-money calls and a short position in at-the money calls) confirm that the implied distribution of log-returns is more negatively skewed than the physical one. In addition, the better performance of the portfolios composed by only put options indicates that the mispricing of options is mainly focused on the left part of the distribution. Unlike Bali and Murray (2013), but consistently with Kozhan et al. (2013), portfolio strategies show that buying skewness (long out-of-the-money call options and short out-of-the-money put options) is on average profitable.

As investors are averse to volatility, the VIX has been called the investors' fear gauge, since it has been found to spike mainly during high levels of market turmoil. From the findings of the paper, both in the Italian and in the US market the $SKEW$ index has been found to be an investors' greed gauge, given its positive relation with market returns. The higher the volatility, the greater the fear, the higher the $SKEW$ measure, the greater the greed.

Given the possibility to use the Italian $SKEW$ index for settling portfolio strategies and for forecasting future returns, and the properties of the $SKEW$ index as an indicator of market greed, we believe that the results of the paper can be of importance for both investors and regulators. Investors could take advantage of the discrepancy between physical and risk-neutral skewness by creating skewness assets and use skewness in order to forecast future returns. Regulators could monitor the information embedded both in volatility and skewness indices. In particular, a large negative change in skewness indices, combined with an increasing implied volatility, may be regarded as an early warning of a strong fall in the stock market.

This analysis may be extended in many directions. Further research is needed in order to assess the relationship among implied moments and the study of other asymmetry measures which, similarly to the portfolio strategies, are able to capture changes in the implied distribution coming from the different tails. Moreover, as the skewness coefficient is a normalized measure which is divided by variance, the study of non-normalized measures which react only to asymmetry, and not to both asymmetry and variance, will be useful to better understand the properties of the skewness indices.

Appendix A.

We provide in this section further details about the model-free formula proposed in Bakshi et al. (2003) in order to compute higher moments of the option implied return distribution. According to Bakshi et al. (2003) model-free skewness is obtained from the following equation as:

$$SK(\mathrm{t},\tau) \equiv \frac{E_t^q\{(R(\mathrm{t},\tau) - E_t^q[R(\mathrm{t},\tau)])^3\}}{\{E_t^q(R(\mathrm{t},\tau) - E_t^q[R(\mathrm{t},\tau)])^2\}^{3/2}} = \frac{e^{r\tau}W(\mathrm{t},\tau) - 3e^{r\tau}\mu(\mathrm{t},\tau)V(\mathrm{t},\tau) + 2\mu(\mathrm{t},\tau)^3}{[e^{r\tau}V(\mathrm{t},\tau) - \mu(\mathrm{t},\tau)^2]^{3/2}} \tag{A1}$$

where $\mu(t,\tau)$, $V(t,\tau)$, $W(t,\tau)$ and $X(t,\tau)$ are the prices of the contracts, at time t with maturity τ, based on first, second, third and fourth moment of the distribution, respectively; their value are computed as:

$$\mu(t,\tau) \equiv E^q\, ln[S(t+\tau)/S(t)] = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V(t,\tau) - \frac{e^{r\tau}}{6}W(t,\tau) - \frac{e^{r\tau}}{24}X(t,\tau) \tag{A2}$$

$$V(t,\tau) = \int_{S(t)}^{\infty} \frac{2(1 - ln[K/S(t)])}{K^2}\, C(t,\tau;\, K)\, dK + \int_0^{S(t)} \frac{2(1 + ln[S(t)/K]}{K^2} P(t,\tau;\, K) dK \tag{A3}$$

$$W(t,\tau) = \int_{S(t)}^{\infty} \frac{6\, ln[K/S(t)] - 3\, ln[K/S(t)]^2}{K^2} C(t,\tau;\, K) dK - \int_0^{S(t)} \frac{6\, ln\,[S(t)/K] + 3\, ln\,[S(t)/K]^2}{K^2} P(t,\tau;\, K) dK \tag{A4}$$

$$X(t,\tau) = \int_{S(t)}^{\infty} \frac{12\, ln[K/S(t)]^2 - 4\, ln[K/S(t)]^3}{K^2} C(t,\tau;\, K) dK + \int_0^{S(t)} \frac{12\, ln\,[S(t)/K]^2 + 4\, ln\,[S(t)/K]^3}{K^2} P(t,\tau;\, K) dK \tag{A5}$$

where $C(t,\tau;\ K)$ and $P(t,\tau;\ K)$ are the prices of a call and a put option at time t with maturity τ and strike K, respectively, $S(t)$, is the underlying asset price at time t.

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Table 1 – Descriptive statistics for the S&P500 market.

	CBOE SKEW	*VIX*	*R*	*ΔVIX*	*ΔCBOE SKEW*	*ΔCBOE SKEW+*	*ΔCBOE SKEW-*
Mean	123.68	17.63	0.00	-0.00	0.00	0.01	-0.01
Median	122.88	15.95	0.00	-0.00	-0.00	0.00	-0.00
Maximum	146.08	48.00	0.05	0.41	0.14	0.14	0.00
Minimum	111.31	10.32	-0.07	-0.31	-0.13	0.00	-0.14
Std. Dev.	6.19	6.16	0.01	0.07	0.03	0.02	0.02
Skewness	0.67	2.05	-0.60	0.75	0.13	3.24	-3.18
Kurtosis	3.06	7.32	8.79	6.43	7.95	18.49	17.95
Jarque-Bera	74.39	1455.18	1431.07	573.87	1003.55	11544.41	10809.20
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note: The table reports the descriptive statistics for the *CBOE SKEW* index, the *VIX* index, S&P500 returns and daily changes in both the *VIX* and the *CBOE SKEW* indices. R is the S&P500 daily return (continuously compounded); $\Delta\ CBOE\ SKEW^{+}$ and $\Delta\ CBOE\ SKEW^{-}$ are the positive and negative changes in the $CBOE\ SKEW$ index, respectively. Daily changes are defined in logarithmic terms as follows: $\Delta x_{t+1} = \ln(x_{t+1}/x_t)$, where x is the series under investigation. The p-value refers to the Jarque-Bera test for normality.

Table 2 – Correlation table for the S&P500 market.

	CBOE SKEW	*VIX*	*R*	*ΔVIX*	*ΔCBOE SKEW*	*ΔCBOE SKEW+*	*ΔCBOE SKEW-*
CBOE SKEW	1.000						
VIX	-0.331***	1.000					
R	0.066**	-0.156***	1.000				
ΔVIX	-0.026	0.122***	-0.814***	1.000			
ΔCBOE SKEW	0.257***	-0.029	0.187***	-0.173***	1.000		
ΔCBOE SKEW+	0.322***	-0.053	0.161***	-0.143***	0.819***	1.000	
ΔCBOE SKEW-	0.090***	0.008	0.142***	-0.139***	0.803***	0.315***	1.000

Note: The table reports the correlation coefficients between the measures used in the study of the S&P500 market. For the definition of the measures see Table 1. Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *.

Table 3 – Descriptive statistics for the Italian market.

	$SKEW_{PH}$	IT $SKEW$	SIX_{mf0}	SIX_{mfR}	IV	R	ΔIV	ΔIT $SKEW$	ΔIT $SKEW^{+}$	ΔIT $SKEW^{-}$	Δ SIX_{mf0}	Δ SIX_{mfR}
Mean	100.13	103.78	103.11	101.44	33.83	0.00	0.00	0.00	0.01	-0.01	0.00	0.00
Median	100.08	103.84	103.27	101.62	31.20	0.00	0.00	0.00	0.00	-0.00	-0.00	0.00
Maximum	103.52	126.36	115.99	112.24	75.43	0.06	0.30	0.15	0.15	0.00	0.13	0.11
Minimum	95.97	89.11	96.31	95.27	14.95	-0.07	-0.52	-0.20	0.00	-0.20	-0.10	-0.09
Std. Dev.	1.11	4.56	2.03	1.77	9.73	0.02	0.08	0.03	0.02	0.02	0.02	0.02
Skewness	-0.00	0.39	0.65	0.49	1.28	-0.24	-0.81	0.31	3.13	-3.10	0.22	0.21
Kurtosis	4.37	4.92	5.97	5.61	4.51	4.44	6.82	7.70	15.39	20.13	4.74	4.44
Jarque-Bera	76.58	174.53	428.85	319.10	360.88	92.98	700.75	913.73	7833.07	13493.71	130.34	91.87
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note: The table reports the descriptive statistics of physical and risk-neutral skewness indices, the model-free implied volatility, FTSE MIB returns and daily changes in volatility and skewness measures. We indicate as $SKEW_{PH}$ the index of subsequently realized skewness in the next 30 days, $ITSKEW$ is the index we compute using the CBOE method, SIX_{mf0} and SIX_{mfR} refer to the SIX_{mf} indices computed as the ratio between upside and downside corridor implied volatilities with barriers equal to *0* and R, respectively, where R is the expected return, IV is the model-free implied volatility multiplied by 100 (VIX methodology), R is the FTSE MIB daily return (continuously compounded); $\Delta\ ITSKEW^{+}$ and $\Delta\ ITSKEW^{-}$ are the positive and negative changes in the $ITSKEW$ index, respectively. The p-value refers to the Jarque-Bera test for normality.

Table 4 – Correlation table for the Italian market.

	$SKEW_{PH}$	$ITSKEW$	SIX_{mf0}	SIX_{mfR}	IV	R	ΔIV	ΔIT $SKEW$	ΔIT $SKEW^{+}$	ΔIT $SKEW^{-}$	Δ SIX_{mf0}	Δ SIX_{mfR}
$SKEW_{PH}$	1.000											
$ITSKEW$	0.156***	1.000										
SIX_{mf0}	0.030	0.063**	1.000									
SIX_{mfR}	0.047	0.090***	0.991***	1.000								
IV	0.008	-0.284***	0.209***	0.142***	1.000							
R	-0.017	0.208***	-0.038	-0.032	-0.117***	1.000						
ΔIV	0.057*	-0.145***	0.021	0.019	0.134***	-0.573***	1.000					
ΔIT $SKEW$	0.003	0.354***	-0.083**	-0.073**	-0.059*	0.439***	-0.435***	1.000				
ΔIT $SKEW^{+}$	-0.026	0.352***	-0.063**	-0.063*	-0.060*	0.286***	-0.432***	0.830***	1.000			
ΔIT $SKEW^{-}$	0.034	0.214***	-0.072**	-0.054*	-0.034	0.431***	-0.264***	0.788***	0.310***	1.000		
Δ SIX_{mf0}	0.004	0.009	0.565***	0.565***	0.006	-0.026	0.031	-0.061*	-0.038	-0.062*	1.000	
Δ SIX_{mfR}	0.005	0.020	0.561***	0.564***	0.004	-0.015	0.023	-0.036	-0.0206	-0.039	0.997***	1.000

Note: The table reports the correlation coefficients between the measures used in the study of the Italian market. For the definition of the measures see Table 3. Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *.

Table 5- Regression output for the changes in the skewness measures and changes in model-free implied volatility in the Italian market (equation (14)).

	α	β	R^2
$\Delta ITSKEW_t$	-0.000 (0.766)	-0.177 (0.000)	0.189
$\Delta\ SIX_{mf0_t}$	-0.000 (0.976)	0.009 (0.351)	0.003
$\Delta\ SIX_{mfR_t}$	-0.000 (0.975)	0.006 (0.487)	0.001

Note: The table presents the estimated output of the regression: $\Delta skewness_t = \alpha + \beta \Delta IV_t + \varepsilon_t$, where for $\Delta skewness_t$ we use daily changes in *ITSKEW* index ($\Delta ITSKEW_t$), SIX_{mf0} index ($\Delta\ SIX_{mf0_t}$) and SIX_{mfR} index ($\Delta\ SIX_{mfR_t}$); ΔIV_t are the daily changes in model-free implied volatility, p-values in parentheses.

Table 6 - Regression output for the changes in the skewness measures and daily returns on the FTSE-MIB (equation (15)).

	α	β	R^2
$\Delta ITSKEW_t$	-0.000 (0.922)	0.237 (0.000)	0.192
$\Delta\ SIX_{mf0_t}$	-0.000 (0.986)	-0.019 (0.414)	0.001
$\Delta\ SIX_{mfR_t}$	-0.000 (0.987)	-0.013 (0.618)	0.000

Note: The table presents the estimated output of the regression: $R_t = \alpha + \beta \Delta skewness_t + \varepsilon_t$, where for $\Delta skewness_t$ we use daily changes in *ITSKEW* index ($\Delta ITSKEW_t$), SIX_{mf0} index ($\Delta\ SIX_{mf0_t}$) and SIX_{mfR} index ($\Delta\ SIX_{mfR_t}$); p-values in parentheses.

Table 7 - Regression output for positive and negative changes in the *ITSKEW* index and daily returns on the FTSE-MIB (equation (18)).

	α	β_1	β_2	R^2
	-0.002 (0.000)	0.140 (0.000)	0.349 (0.000)	0.212

Note: The table presents the estimated output of the regression: $R_t = \alpha + \beta_1 \Delta ITSKEW_t^+ + \beta_2 \Delta ITSKEW_t^- + \varepsilon_t$; p-values in parentheses.

Table 8 - Regression output for the changes in the skewness measures, changes in model-free implied volatility and daily returns on the FTSE-MIB (equation (19)).

	α	β_1	β_2	R^2
$\Delta\, ITSKEW_t$	-0.000 (0.944)	0.126 (0.000)	-0.103 (0.000)	0.373
$\Delta\, SIX_{mf0_t}$	-0.000 (0.886)	-0.006 (0.760)	-0.126 (0.000)	0.328
$\Delta\, SIX_{mfR_t}$	-0.000 (0.885)	-0.002 (0.925)	-0.126 (0.000)	0.328

Note: The table presents the estimated output of the regression: $R_t = \alpha + \beta_1 \Delta skewness_t + \beta_2 \Delta IV_t + \varepsilon_t$, where for $\Delta skewness_t$ we use daily changes $ITSKEW$ index ($\Delta ITSKEW_t$), SIX_{mf0} index ($\Delta\, SIX_{mf0_t}$) and SIX_{mfR} index ($\Delta\, SIX_{mfR_t}$); p-values in parentheses.

Table 9 - VAR Estimation output for the Italian market.

	R_t	$\Delta\, ITSKEW_t$
R_{t-1}	0.009831 (0.27549)	0.182693*** (2.86033)
R_{t-2}	-0.019276 (-0.53848)	0.066370 (1.03584)
$\Delta\, ITSKEW_{t-1}$	-0.042144** (-2.11985)	-0.306903*** (-8.62454)
$\Delta\, ITSKEW_{t-2}$	0.001620 (0.08129)	-0.085880** (-2.40804)
c	-2.93E-06 (-0.00544)	-0.000221 (-0.22926)

Note: The table reports the estimation output (t-stat in parentheses) of the VAR model:

$$R_t = c + \sum_{l=1}^{K} a_l \Delta\, ITSKEW_{t-l} + \sum_{l=1}^{K} b_l R_{t-l} + u_t$$

$$\Delta\, ITSKEW_t = c + \sum_{l=1}^{K} a_l R_{t-l} + \sum_{l=1}^{K} b_l \Delta\, ITSKEW_{t-l} + u_t$$

Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *.

Table 10 - Granger causality test between daily returns on the FTSE-MIB and daily changes in $ITSKEW$ index.

Null Hp.	X^2	*p-value*
$\Delta ITSKEW$ does not Granger cause R	7.12	0.029
R does not Granger cause $\Delta ITSKEW$	8.68	0.013

Note: The table reports the Granger causality test for the VAR model as defined in note to Table 9.

Table 11 – Descriptive statistics of skewness measures in the bearish and bullish sub-periods.

03/01/2011 – 25/07/2012: *Bearish market*				
	$SKEW_{PH}$	$ITSKEW$	SIX_{mf0}	SIX_{mfR}
Maximum	103.09	114.77	115.99	112.24
Minimum	97.95	93.68	96.31	95.27
Mean	100.43	103.35	103.47	101.68
Median	100.43	103.65	103.50	101.77
Std. Dev.	0.93	3.70	2.41	2.08
Skewness	0.25	-0.09	0.65	0.57
Kurtosis	0.17	0.00	2.82	2.83
26/07/2012 – 28/11/2014: *Bullish market*				
	$SKEW_{PH}$	$ITSKEW$	SIX_{mf0}	SIX_{mfR}
Maximum	103.52	126.36	109.22	107.05
Minimum	95.97	89.11	99.46	98.15
Mean	99.93	104.16	102.86	101.28
Median	99.82	104.10	103.12	101.57
Std. Dev.	1.18	5.03	1.67	1.51
Skewness	0.13	0.41	0.15	0.03
Kurtosis	1.59	1.77	0.17	-0.01

Note: for the definition of the measures see Table 3.

Table 12 - Regression output for the changes in the *CBOE SKEW* index and changes in the *VIX* index.

	α	β	R^2
$\Delta\ CBOE\ SKEW_t$	0.000 (0.909)	-0.061 (0.000)	0.029

Note: The table presents the estimated output of the regression: $\Delta CBOE\ SKEW_t = \alpha + \beta \Delta VIX_t + \varepsilon_t$; p-values in parentheses.

Table 13 - Regression output for the changes in the *CBOE SKEW* and daily returns on the S&P500.

	α	β	R^2
$\Delta\ CBOE\ SKEW_t$	0.000 (0.074)	0.073 (0.000)	0.034

Note: The table presents the estimated output of the regression: $R_t = \alpha + \beta \Delta CBOE\ SKEW_t + \varepsilon_t$; p-values in parentheses.

Table 14 - Regression output for positive and negative changes in the *CBOE SKEW* index and daily returns on the S&P500.

	α	β_1	β_2	R^2
	0.000 (0.301)	0.080 (0.000)	0.065 (0.018)	0.033

Note: The table presents the estimated output of the regression: $R_t = \alpha + \beta_1 \Delta CBOE\ SKEW_t^+ + \beta_2 \Delta CBOE\ SKEW_t^- + \varepsilon_t$; p-values in parentheses.

Table 15 - Regression output for the changes in the *CBOE SKEW* index, changes in *VIX* index, and daily returns on the S&P500.

	α	β_1	β_2	R^2
	-0.000 (0.007)	0.019 (0.025)	-0.110 (0.000)	0.664

Note: The table presents the estimated output of the regression: $R_t = \alpha + \beta_1 \Delta CBOE\ SKEW_t + \beta_2 \Delta VIX_t + \varepsilon_t$; p-values in parentheses.

Table 16 - VAR Estimation output for the US market.

	R_t	$\Delta\ CBOE\ SKEW_t$
R_{t-1}	-0.064059** (-1.97125)	-0.058126 (-0.72967)
R_{t-2}	0.054833* (1.68783)	0.166732** (2.09363)
R_{t-3}	-0.092471*** (-2.84256)	0.128214 (1.60781)
$\Delta\ CBOE\ SKEW_{t-1}$	-0.001543 (-0.11679)	-0.314535*** (-9.71138)
$\Delta\ CBOE\ SKEW_{t-2}$	0.004750 (0.34820)	-0.185152*** (-5.53668)
$\Delta\ CBOE\ SKEW_{t-3}$	-0.001265 (-0.09461)	-0.138401*** (-4.22250)
c	0.000545 (1.74901)	0.0000413 (0.05415)

Note: The table reports the estimation output (t-stat in parentheses) of the VAR model:

$$R_t = c + \sum_{l=1}^{K} a_l \Delta\ CBOE\ SKEW_{t-l} + \sum_{l=1}^{K} b_l R_{t-l} + u_t$$

$$\Delta\ CBOE\ SKEW_t = c + \sum_{l=1}^{K} a_l R_{t-l} + \sum_{l=1}^{K} b_l \Delta\ CBOE\ SKEW_{t-l} + u_t$$

Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *.

Table 17 - Granger causality test between daily returns on the S&P500 and daily changes in $CBOE\ SKEW$ index.

Null Hp.	X^2	*p-value*
$\Delta CBOE\ SKEW_t$ does not Granger cause R	0.29	0.963
R does not Granger cause $\Delta CBOE\ SKEW_t$	7.76	0.051

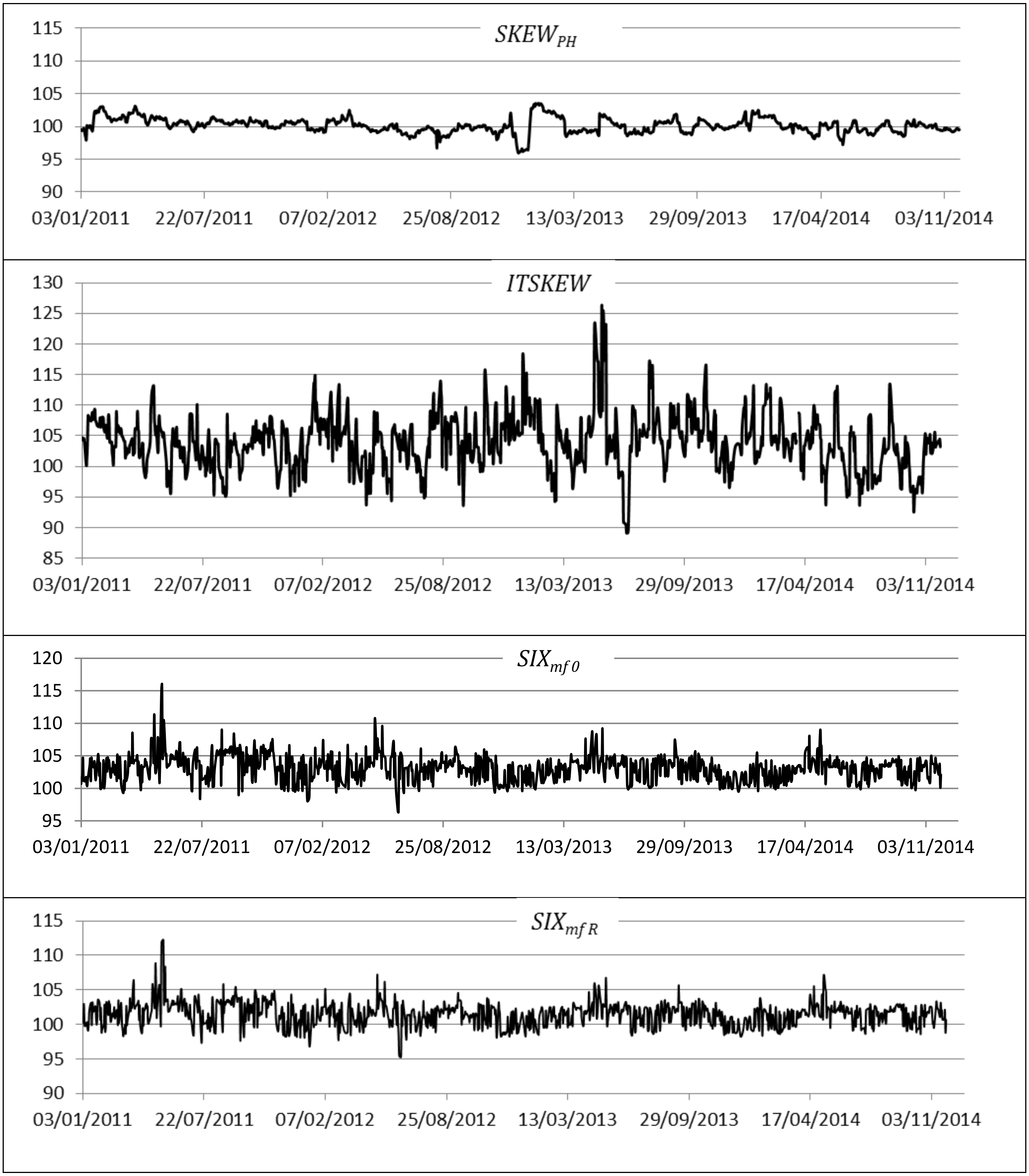
Note: The table reports the Granger causality test for the VAR model as defined in note to Table 16.

Table 18 - Skewness assets returns for the entire sample period.

	$PUTCALL\ asset$	$PUT\ asset$	$CALL\ asset$
Cumulative return	56.43%	49.16%	10.64%
Average daily return	0.05%	0.04%	0.01%
t-statistic	5.49	4.05	0.95
p-value	0.000	0.000	0.343
Average ann. return	11.44%	10.28%	2.73%
Annualized volatility	4.14%	5.03%	5.70%

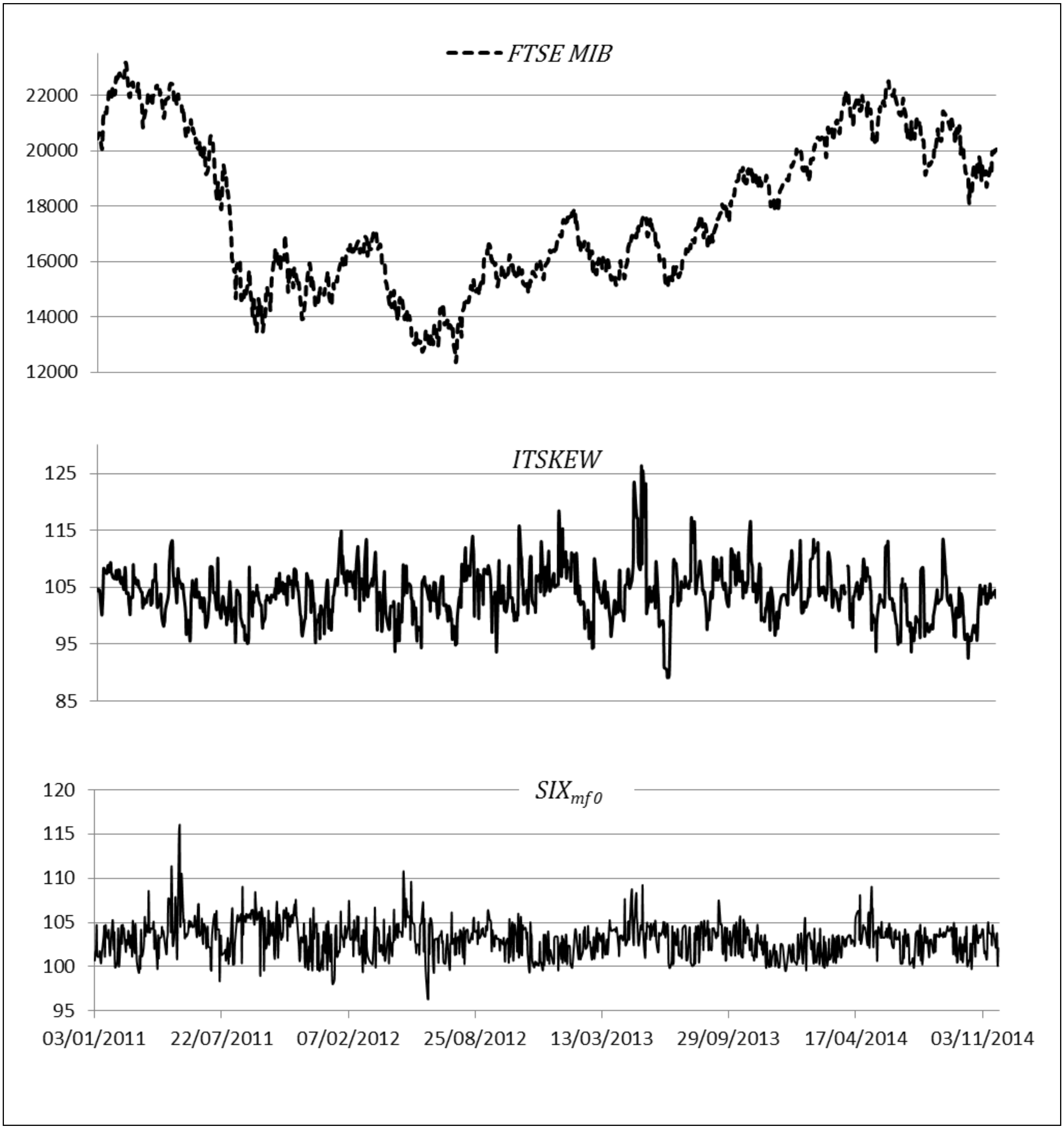
Note: The table reports the descriptive statistics for the Skewness assets returns used in the study in order to disentangle the contribution to the profitability of differences between the physical and the risk-neutral distribution in the left ($PUT\ asset$) or in the right ($CALL\ asset$) parts of the distribution or in both ($PUTCALL\ asset$) .

Figure 1 - Graphical comparison of skewness indices for the Italian market.



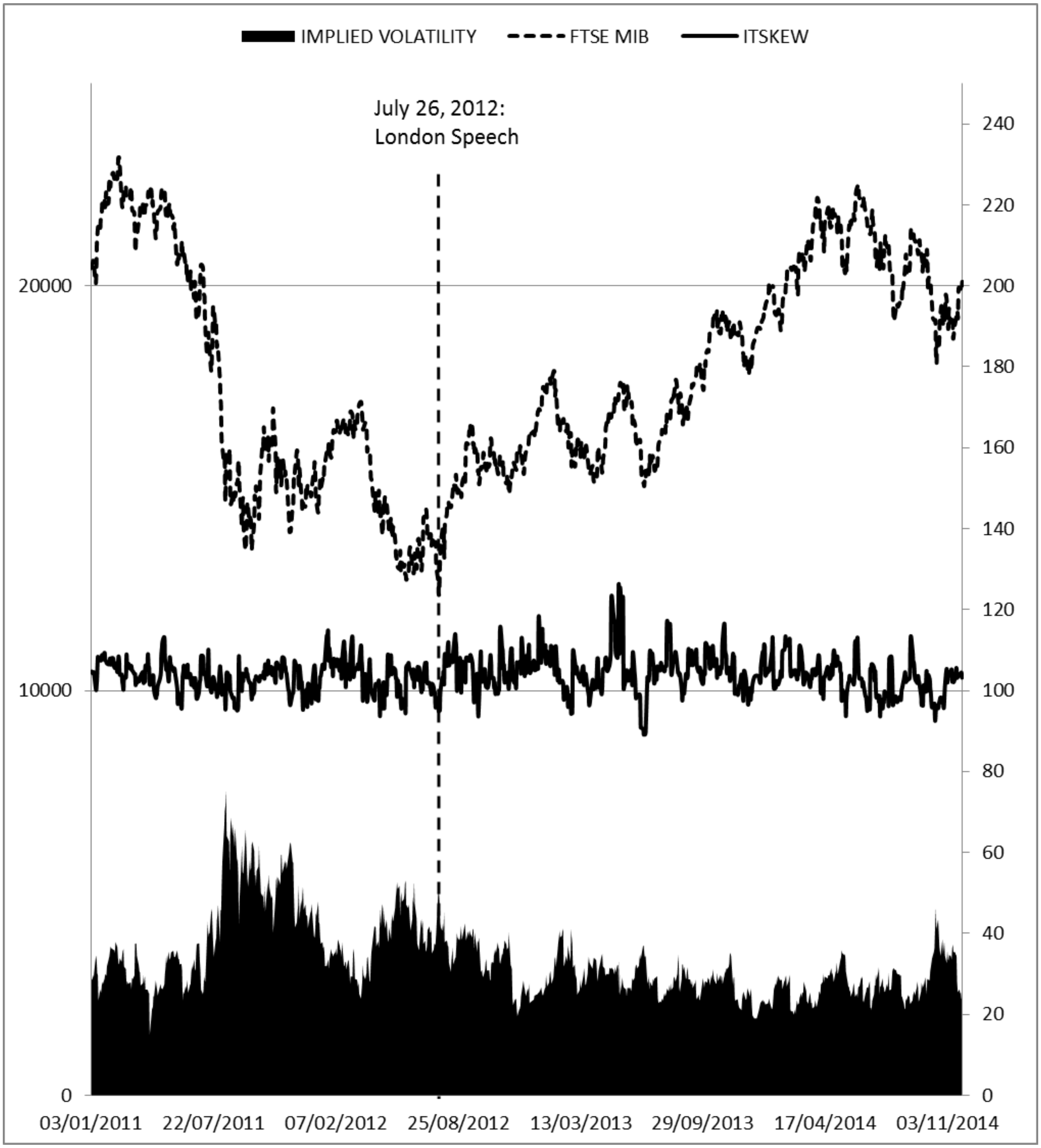
We indicate as $SKEW_{PH}$ the index of subsequently realized skewness in the next 30 days, $ITSKEW$ is the index we compute using the CBOE method, SIX_{mf0} and SIX_{mfR} refer to the SIX_{mf} indices computed as the ratio between upside and downside corridor implied volatilities with barriers equal to *0* and *R*, respectively, where *R* is the expected return.

Figure 2 – Comparison between the FTSE MIB index, the $ITSKEW$ index and the SIX_{mf0}.



Note: The Figure reports the closing values of the Italian market index FTSE MIB and the skewness indices ($ITSKEW$ and SIX_{mf0})

Figure 3 – Comparison between FTSE MIB index, model-free implied volatility and *ITSKEW* index



Note: FTSE MIB index refers to the values on the left, while implied volatility and *ITSKEW* index refer to the values on the right. Implied volatility values are obtained as the model-free implied volatility multiplied by 100 (*VIX* methodology).

Figure 4 – Skewness assets returns in the Italian market (notional 1 m. Euro investment).