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*Original*

The new INRIM rotating encoder angle comparator (REAC) / Pisani, Marco; Astrua, Milena. - In: MEASUREMENT SCIENCE & TECHNOLOGY. - ISSN 0957-0233. - 28:4(2017), p. 045008. [10.1088/1361-6501/aa5af6]

*Availability:*

This version is available at: 11696/54927 since: 2021-01-28T14:56:54Z

*Publisher:*

IOP Publishing [Society Publisher]

*Published*

DOI:10.1088/1361-6501/aa5af6

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## The new INRIM Rotating Encoder Angle Comparator (REAC)

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### Abstract

A novel angle comparator has been built and tested at INRIM. The device is based on a double air bearing structure embedding a continuously rotating encoder, which is read by two heads: one fixed to the base of the comparator and a second fixed to the upper moving part of the comparator. The phase measurement between the two heads' signals is proportional to the rotation of the moving table. The advantage of this solution is to reduce the encoder graduation errors and to cancel the cyclic errors due to the interpolation of the encoder lines. By using only two pairs of reading heads, we have achieved an intrinsic accuracy of  $\pm 0.04''$  (rectangular distribution) that can be reduced through self-calibration. The residual cyclic errors have shown to be less than  $0.01''$  peak-to-peak. The random fluctuations are less than  $0.01''$  rms on a 100 s time interval. A further advantage of the rotating encoder is the intrinsic knowledge of the absolute position without the need of a zeroing procedure. Construction details of the rotating encoder angle comparator (REAC), characterization tests, and examples of practical use are given.

**Keywords:** Angle metrology, angle encoder, absolute encoder, nanoradians, autocollimators

### 1 Introduction

High accuracy angle measurements are needed in many fields of research and industrial production, from precision engineering and robotics to astronomical telescope or sub-nanometre topography. Nowadays there are many commercial instruments such as angular encoders (AEs) and autocollimators (ACs) with an extremely high resolution, as low as  $0.001''$ , that could answer the need of the industry or of the scientific research. ACs are generally calibrated by comparison with AEs so ultimately the latter is the preferred device to implement an angle standard. Nevertheless, the accuracy of AEs, i.e. the ability of providing the "true value", is limited by two main effects. The first is caused by non-uniform spacing of the grating lines or non-perfect centering of the encoder in the rotating structure. Both have the same effect of causing an error which has different values for different position of the table, although has zero mean over a full circle rotation. These errors are related to the physical construction of the device and can be characterized by comparison with a more accurate reference encoder. The second effect is the so-called "interpolation error". Since usually the resolution of an encoder is much smaller than the spacing of the grating lines, the reading is obtained by subdividing the interval between lines into thousands of parts by means of a mix of opto-electronics and computing tricks. In fact, the division is never exact and, also in this case, we have an error which is proportional to the position between the lines and has zero average. This error is also referred to as "cyclic error" because it is more or less identical each grating period. This error can be evaluated and partially cancelled by some self-calibration procedure, e.g. by rotating at constant speed the encoder while recording the output [1].

Typical values of these errors in high end encoders are at best several tenths of an arcsecond, thus, for applications where higher accuracy angle measurements are needed, any angular encoder must be calibrated with reference to a better quality angular standard. The traceability chain eventually ends up with the state of the art angle reference standards maintained in the National Metrological Institutes (NMIs).

Today, only few NMIs in the world have designed and realized their own special facilities to provide angle calibrations at the uncertainty level of thousandths of arcseconds [1-3].

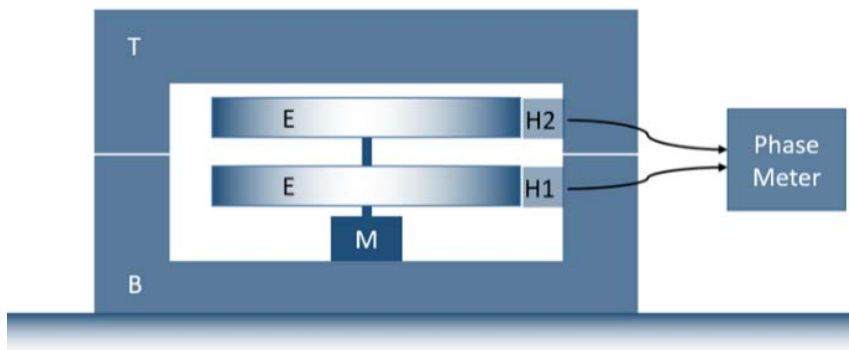
Up to now, the INRIM angle standard was based on two index tables Moore 1440 with an accuracy limited to 0.1" and with the disadvantage of needing a long fully manual procedure. Recently, in the framework of the European project "Angles" [4], we have designed and built a new high accuracy automated angle standard.

The device described in this paper, called Rotating Encoder Angle Comparator (REAC), is based on a double air bearing structure where a continuously rotating encoder is embedded. This allows the elimination of the interpolation errors and the reduction of encoder errors.

## 2 Working principle and realization

### 2.1 Hardware and software

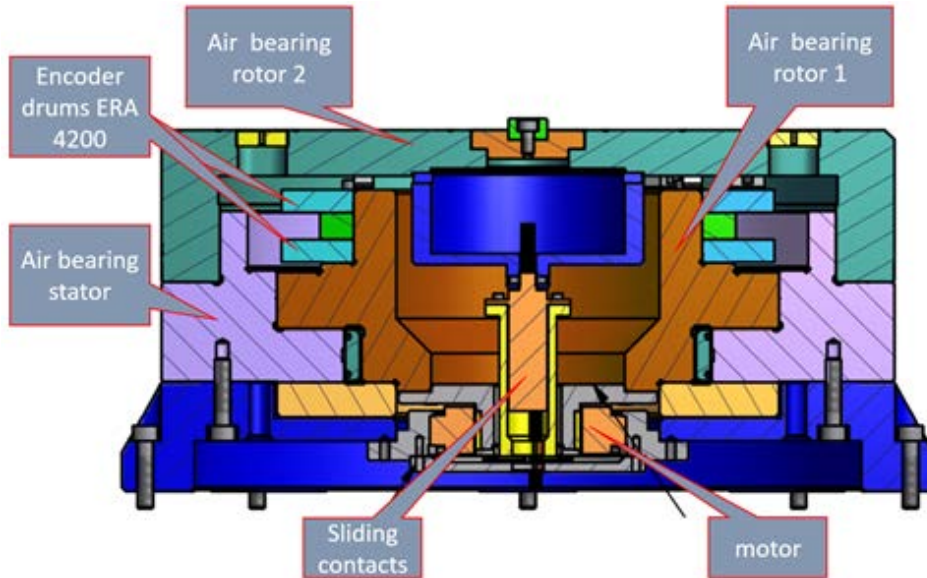
The core of the system is an angular encoder which is maintained in rotation at constant speed and is read by two heads, the first one fixed to the base and the second one rotating with the measurement table, according to the principle demonstrated by E.W. Palmer [5]. If the signal coming from the fixed head is used as a reference signal, the angle of rotation of the measurement table can be obtained as the phase difference between the reference signal and the signal coming from the rotating head. This configuration has two main significant advantages: first of all, the encoder errors due to circle division errors are cancelled by averaging the data of each complete revolution of the encoder. Secondly, since a dynamic phase measurement is performed, the cyclic errors due to the signal interpolation between encoder's graduations are not present. Figure 1 shows the working principle scheme of the REAC.



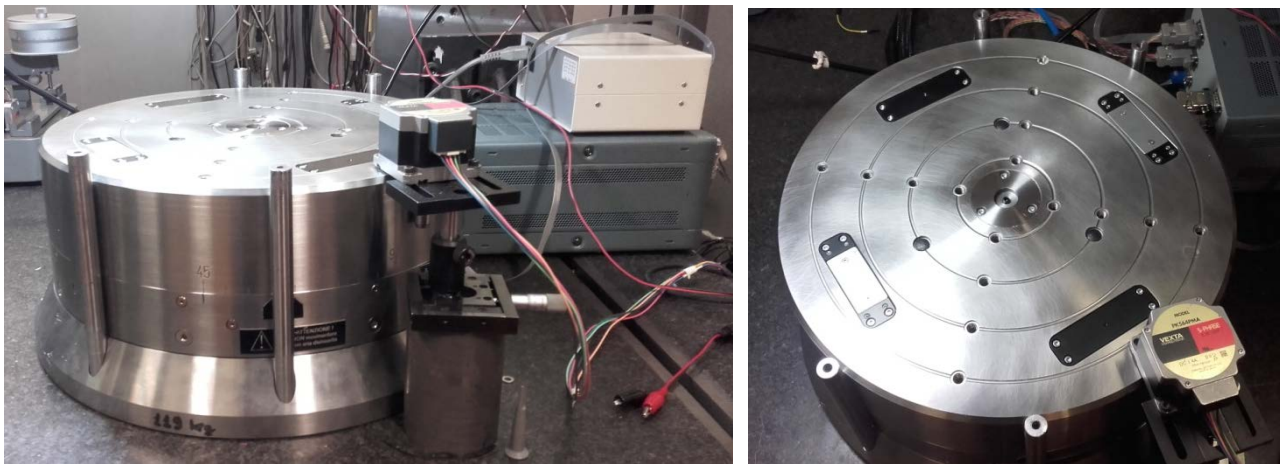
**Figure 1.** Working principle of the rotating encoder angle comparator: B is the base of the instrument, T is the upper table which can rotate with respect to the base and is used for the calibrations, E are two identical encoders placed in rotation at constant speed by means of motor M, H1 and H2 are the encoder reading heads fixed to the base and to the rotating table respectively. The signals coming from the two heads are sent to the phase meter.

The table has been implemented by Mager S.r.l. [6] with the realization of a steel structure composed of three parts: a base, an inner rotating part holding the encoders and an upper measuring table available for the user. The three parts can mutually rotate thanks to a double air bearing. In figure 2 a vertical section of the REAC is shown, where the different parts are highlighted. The base (lower grey part) supports the air bearing stator (green), while the motor (red) drives the air bearing rotor (blue) which is fixed to the encoder and rotates continuously at a speed that can be selected up to 30 rpm. For practical purposes the encoder is a couple of equal stacked encoders (white) manufactured by Heidenhain, model ERA 4200 with  $4 \cdot 10^4$  lines and 20  $\mu$ m pitch (equivalent to 32.4"). The encoders are read by a total of four heads, instead of the minimal number of two: two of them are fixed to the base while the other two are fixed to the upper measuring table (grey). The signals from the moving heads leave the REAC through sliding contacts (yellow). Each head generates two quadrature sinusoidal signals at a frequency which depends on the rotation speed of the encoder (i.e. 20 kHz for 30 rpm). The signals are amplified by a low noise circuit and sent to a 16 bit ADC

board (NI-USB-6259 BNC). A software implemented in Labview® elaborates the signals and calculates the phase difference through a Matlab® algorithm. The phase is calculated in real time for any cycle of the reference signal and is accumulated in a register. After a complete revolution of the drum (when 40000 cycles have been counted) the average is calculated and is displayed and recorded. In the results presented here we have chosen to rotate the encoder at 30 rpm, so that the output of the REAC is updated every two seconds.

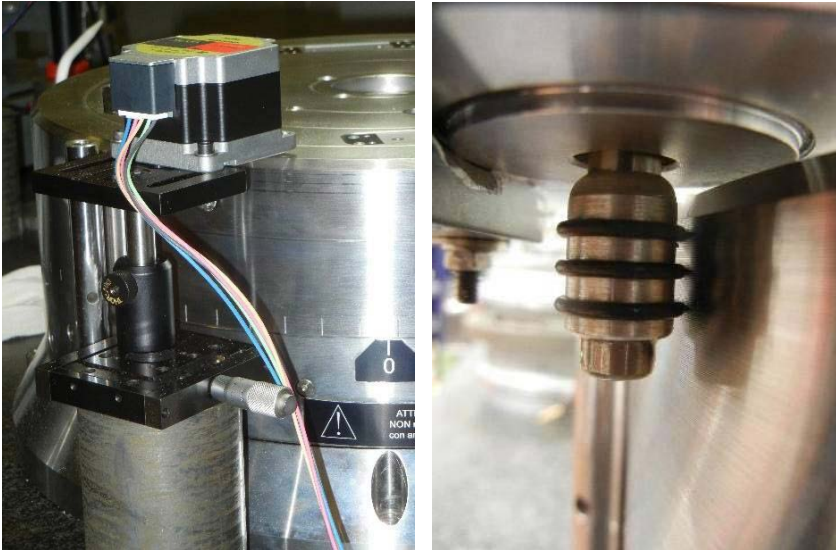


**Figure 2.** Vertical section of the Rotating Encoder Angle Comparator (REAC)



**Figure 3.** Picture of the REAC: side view on the left; top view on the right. The four windows visible on the top are the access to the four reading heads of the two encoders.

The driving system of the upper table is based on a microstep motor (Orientalmotor, 5-phase stepping, CRK Series) that allows a fast positioning with a step resolution of 0.1". The motor shaft is inserted into a cylinder provided with 3 horizontal grooves to host some o-rings which drag the REAC upper table, as shown in figure 4. A software implemented in LabView® controls a digital output of the data acquisition device to send an electrical signal to run the motor. Therefore, a complete automatically procedure to generate angular steps of whatever amplitude can be set up. Further realization details can be found in [7, 8].



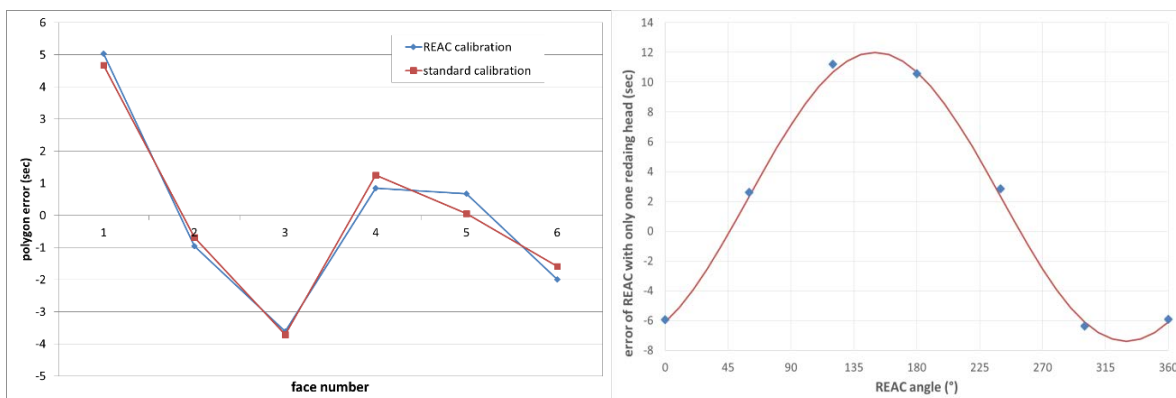
**Figure 4.** Driving system and detail of the mechanical coupling between the motor shaft and the REAC upper table. The micrometric screw visible in the left picture allows to engage the pulley, trim the contact force and fine positioning when the motor is off.

## 2.2 Preliminary test

In order to check for the mounting accuracy of the device and the error cancellation principle we have performed a preliminary test by using as a reference an optical polygon previously calibrated at INRIM.

The polygon is placed at the centre of the REAC upper table, while an autocollimator is positioned in front of the polygon aiming at one of its faces and perpendicular to its rotation axis. The calibration is performed by rotating the REAC upper table by the nominal angle of the polygon and measuring the deviation from the nominal value with the autocollimator. In order to minimize the measurement errors, the “cross calibration” or “multi-step” technique was used, which means repeating the measurement of the  $N$  angles between adjacent faces of the polygon in  $N$  different relative orientations between the artefact and the REAC, obtaining a set of  $N^2$  data. This cross calibration procedure allows obtaining the errors of the polygon and the errors of the rotary table.

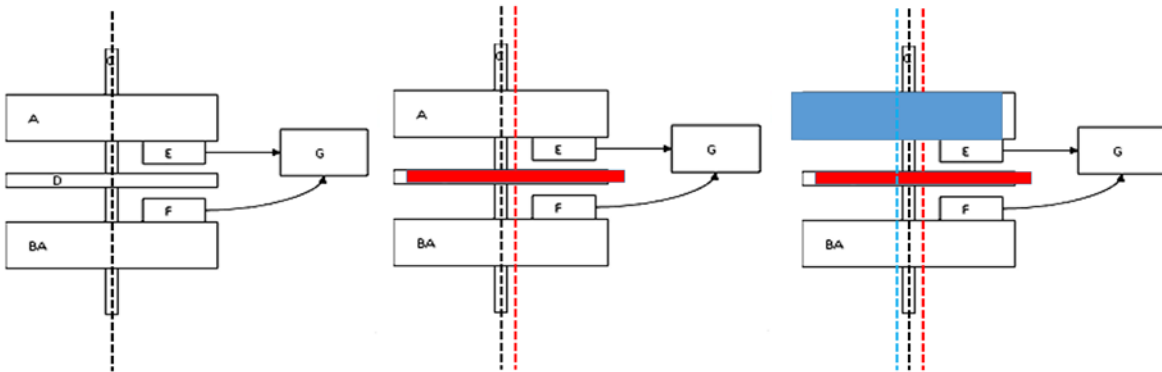
The agreement between the two calibrations reported in figure 5a) is not excellent (within 0.6”), due to the fact that only one of the two reading heads of the REAC was used for this measurement. As said, the cross calibration allows us to estimate of the errors of REAC, shown in figure 5b). This error has an almost perfect sinusoidal behaviour with an amplitude of 18” peak-to-peak. This is explained as follows.



**Fig 5:** a) Polygon calibration results by REAC using only one head (in blue line) and by Moore indexing tables (in red lines); b) REAC errors (in blue) fitted with a sine function (red). The error is caused by a non ideally coaxial construction and can be cancelled by using two opposite heads.



As shown in figure 6, two different kinds of errors of the encoder are caused by a non-perfect mechanical construction. The first is the centering error of the encoder (i.e. encoder axis not coincident with the motor axis) that causes a cyclic error. The second is the centering error of the upper table (i.e. upper table axis not coincident with the motor axis), also in this case an eccentricity periodical error will occur. While the first error is cancelled by averaging over one full revolution of the encoder, the second, which cannot be cancelled, is the error plotted in figure 5b). The 18'' error is explained with a distance between the two axes (black and blue in picture 6) of 14  $\mu\text{m}$ . The value (although not negligible) is within the construction specifications. By using two opposite heads is it possible to strongly reduce this error. In the device presented here, two opposite heads are used in the upper table and the signals are independently analyzed by two identical softwares running in parallel. The average of the two outputs is used as the output of the REAC.



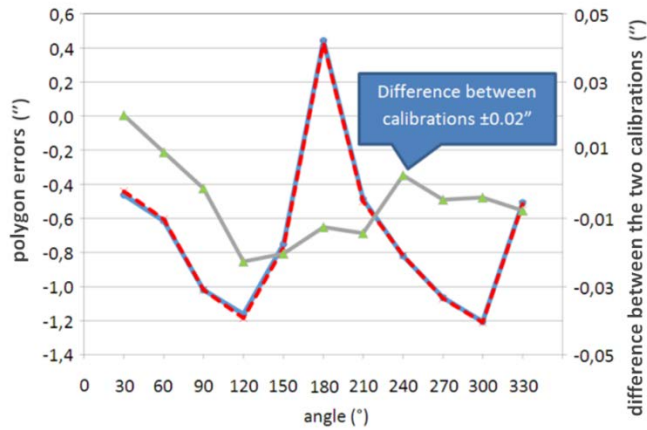
**Fig 6:** Errors due to non-perfect mechanical construction. a) the system is represented schematically with BA being the basis of the instrument, D the rotating encoder, A the upper table, and E and F the two heads fixed to the table and to the base, respectively; the dashed black line is the encoder rotation axis taken as a reference. b) if the encoder is mounted with its axis (red) not coincident with the motor axis, an instantaneous error occurs which is cancelled by averaging over one full revolution. c) if the upper table carrying the head E rotates on an axis (blue) not coincident with the motor axis a sinusoidal error is generated. This error can be cancelled by using two opposite heads and averaging the results.

In order to characterize the new instrument and evaluate its performances, different tests have been performed, such as comparisons with angular artefacts (optical polygons and index tables), test on linearity, and noise analysis. In the following chapters, we will discuss in detail all the measurements performed.

### 3 Validation by comparison with angle standard

#### 3.1 Polygon calibration

The first test aimed at the validation of the REAC by means of a transfer standard. To this purpose, the REAC was used to calibrate an optical polygon previously calibrated with the INRIM angle standard. The artefact is a 12-sided polygon made of steel manufactured by Möller Wedel Optical. The procedure to calibrate this polygon is the same used for the calibration of the 6-sided polygon described in section 2, but now both the reading heads of the REAC are used. In this case, a set of  $12^2$  data was obtained from the multi-step technique. In figure 7 the deviation from nominal angular values of the 12-sided polygons is shown, measured with the REAC and with the Moore index tables.



**Figure 7.** Deviation from nominal angular values of the 12-sided polygons, measured with REAC and with Moore index tables (red and blue curves, left scale). In green the difference between the two calibrations (right scale).

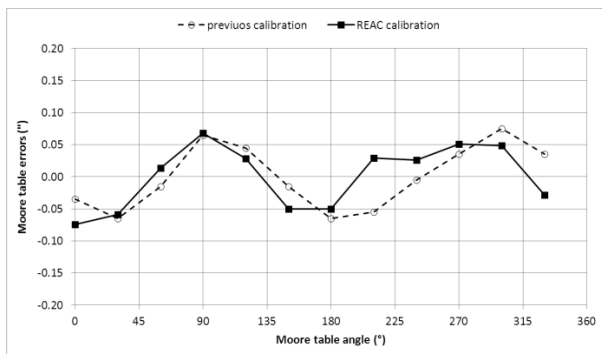
The difference between the two calibrations is about  $\pm 0.02''$ , hence well within the INRIM calibration and measurement capabilities (CMC) declared in the BIPM database [11].

### 3.2 Direct comparison with a Moore index table

The second test was a direct comparison between the REAC and the previous INRIM national angle standard, made by two Moore index tables provided with 1440 uniformly spaced teeth. Each tooth corresponds to an angle of  $15'$ , so the angular displacement is related to the number of teeth by which the table has been rotated.

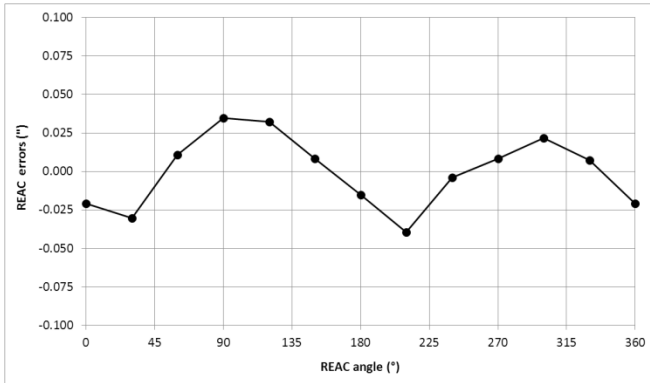
For that measurement, the two Moore index tables are superposed, one above the other, on top of the REAC upper table so that the rotation axis of the tables is coaxial with the REAC axis within few tens of micrometres. A mirror is placed above the higher Moore table and an autocollimator aims at the mirror. The calibration is performed by rotating the higher Moore table by an angle  $\alpha$  and the REAC by an angle  $-\alpha$ , so that the mirror is always in front of the autocollimator which measures the deviation from the initial position. The value of  $\alpha$  is chosen to be  $30^\circ$ , so the measurement is repeated 12 times in order to complete an entire revolution of the tables. Also in this case the 'multi-step' technique has been used: hence the set of 12 measurements is repeated in 12 different relative positions obtained by rotating the lower Moore table by angles multiple of  $\alpha$ .

At the end of the calibration procedure, two important outputs are supplied by the multistep technique. First, the errors of the Moore table (i.e. the deviation from the nominal angles which are multiples of  $30^\circ$ ) are obtained and can be compared with the same errors obtained in a previous calibration. The results, reported in figure 8, are in good agreement and are compliant with the uncertainty of the previous calibration which was  $0.05''$ .



**Figure 8.** Deviation from  $30^\circ$  nominal angular interval of a Moore index table, measured with REAC (dashed line) compared with a previous calibration (solid line)

The second and most important result is the estimation of the errors of REAC by the multistep technique. These errors, reported in figure 9, are within  $\pm 0.04''$  and show a double periodicity. They could be explained by a non-perfect cancellation of the eccentricity error due to an error in the rotation of the table as explained in paragraph 2.3. Anyway, it is possible to characterize this error in detail and correct the output of the REAC for that.



**Figure 9.** REAC systematic errors estimated by multi-step cross-calibration procedure.

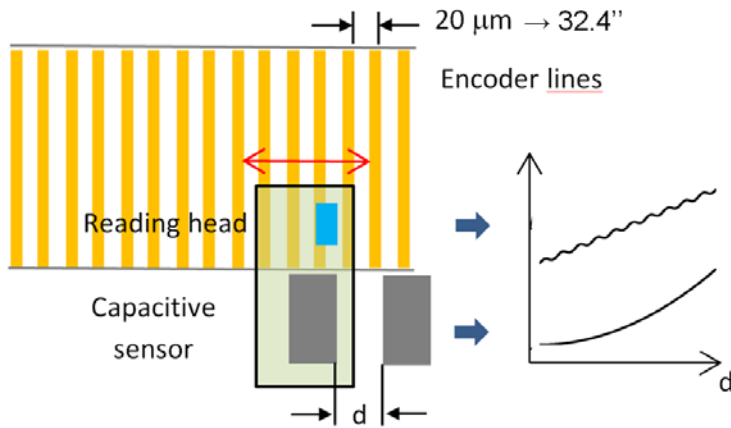
#### 4 Cancellation of interpolation errors

Another important issue is the interpolation or cyclic error characterization. Indeed, all the encoders in general are affected by errors due to non-ideal signal interpolation between the graduation lines, which generally represents a big limit to the encoder's accuracy. In order to solve the problem, high accuracy encoders are often provided by an electronic circuit or a software, which after a calibration procedure, estimates the shape of the interpolation errors and corrects the output signal for that effect.

Thanks to its working principle, the system developed at INRIM is free from non-linearity due to signal interpolation between graduation lines since the phase measurement is performed dynamically rather than statically. In order to verify this feature, we have performed the experiment described here.

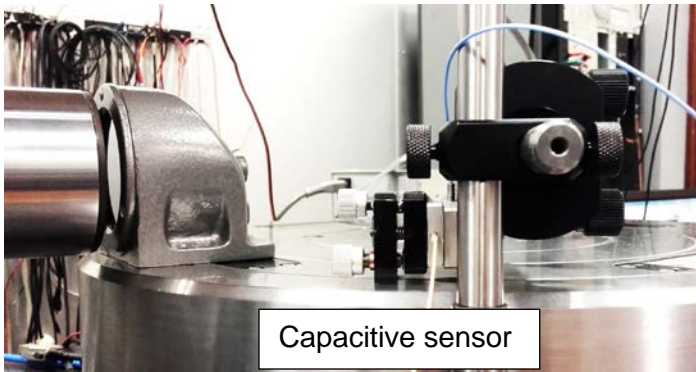
The idea is to compare the reading of the encoder, of which we want to analyse non-linearity, with the reading of a capacitive sensor (Fig 10). The capacitive sensor is able to measure displacements with very high resolution and good linearity. The unavoidable residual non-linearity of the capacitive sensor is a polynomial function of the distance (mainly second order) and we can assume that no cyclic errors are present. Thus, if we subtract the reading of the two instruments and look for nonlinearities, we can easily distinguish between residual capacitive polynomial errors and residual encoder cyclic errors. This method has indeed been successfully adopted by the authors to characterize classical encoders residual cyclic errors [13].





**Figure 10.** Scheme of the principle. While rotating the table, the output of the REAC with possible cyclic nonlinearities is recorded together with the output of the capacitive sensor.

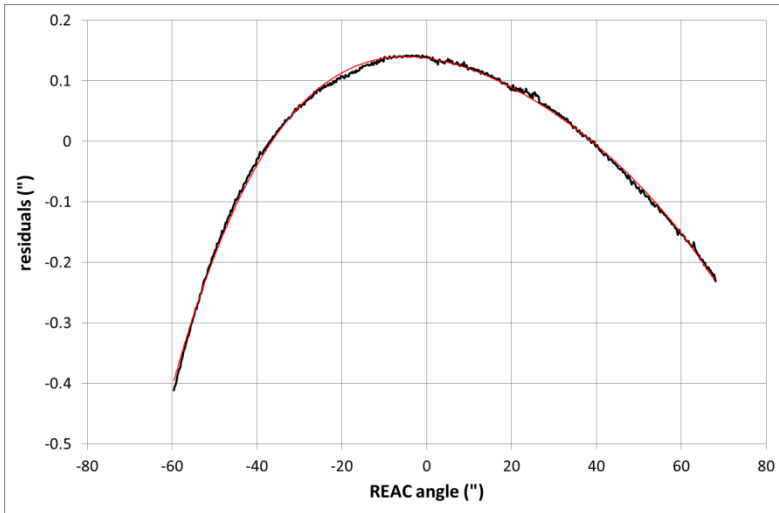
The capacitive transducer is manufactured by Physik Instrumente (model PZ 106E D-100) with a measuring range of 300  $\mu\text{m}$  and nanometre level resolution. Its capacitance value depends on the distance  $d$  between the plates and is compared with an internal reference capacitor (10 pF).



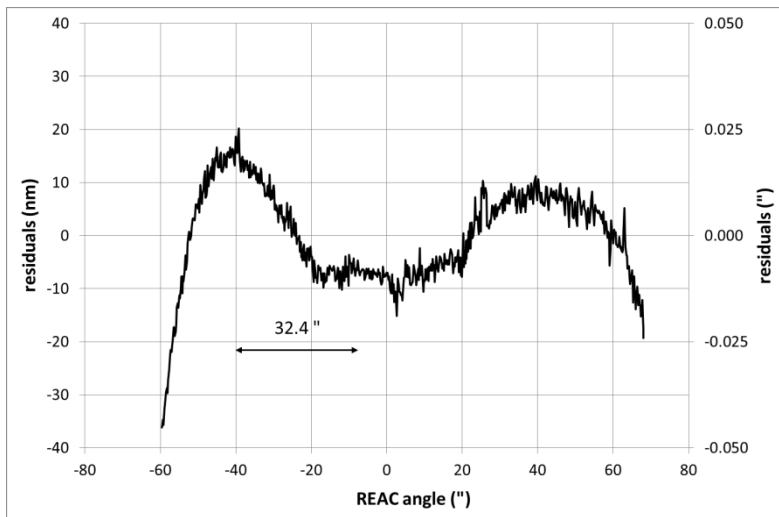
**Figure 11.** picture of the experimental set-up to investigate the system's linearity.

As shown in figure 11, the left plate of the capacitor is fixed to an optical mounting glued on the rotating table of the REAC, while the right plate is fixed to a mounting which stands directly on the base of the REAC and faces the first plate to form an air capacitor. Vertical and angular adjustments are provided to set the plates parallel and at a suitable distance. The distance from the capacitive sensor centre to the centre of the table (i.e. of the encoder) is 160 mm, so that the output of the capacitive sensor can be converted in angle knowing that 1  $\mu\text{m}$  displacement corresponds to 6.25  $\mu\text{rad}$  or about 1,3  $''$ .

An interval of about 130 $''$  (corresponding to about 100  $\mu\text{m}$  distance change on the capacitive sensor) is slowly explored while the readings of the two instruments are recorded. The difference is plotted in figure 12. As expected, the residuals have a sort of parabolic shape, which is the expected non-linear behaviour of a capacitive sensor. Finally, if we remove the second and third order terms, the residual errors are shown in figure 13. We can still observe higher order non-linearities due to the capacitive sensor, while no cyclic errors with 32.4 $''$  period are visible, at least at the resolution level of the measurement, limited by electronic noise and mechanical stability, that can be estimated to be well below 0.01 arcsec.

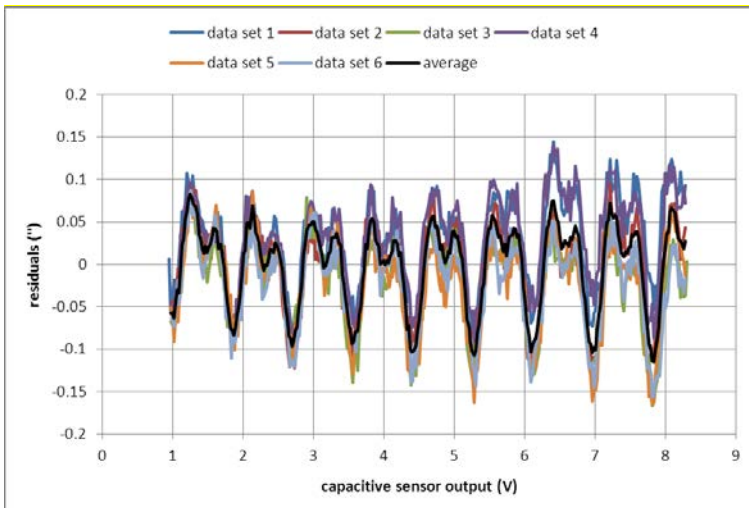


**Figure 12.** Residual errors of capacitive sensor versus REAC converted to arcseconds. The typical capacitive sensor's non-linearity is evident.



**Figure 13.** The same data as in figure 12 after removal of a 2<sup>nd</sup> and 3<sup>rd</sup> order polynomial. The residual higher order nonlinearity is attributed to the capacitive sensor. It is evident that no significant errors with a 32.4" period (horizontal arrow in the graph) are visible within the measurement resolution.

As a countercheck of the effectiveness of the application of the capacitive sensor to the measurement of cyclic errors, we made the following simple test. We stopped the motor which rotates the REAC's encoder so that the encoder stays fixed with respect to the base. Then we have acquired the quadrature signals coming from the head fixed to the rotating table. This way the system behaves like a standard table equipped with an angular encoder. In fact, in a normal case, the head is fixed to the base and the encoder is fixed to the rotating part, nevertheless the concept is identical. The quadrature signals coming from the head are sampled together with the signal coming from the capacitive sensor. A software implemented in LabView® calculates the phase angle of the encoder without any correction. By comparing the outputs of the capacitive sensor and of the encoder, the combination of the non-linearities of both instruments is obtained. We expect to find the typical interpolation error of classical encoders. The residuals from six data sets are shown in figure 14 together with the average residuals (black line): the measurement range is about 8 encoder's periods (i.e.  $\approx 260''$ ) and the encoder's non-linearity due to interpolation errors is evident with a magnitude about 0.15" peak-to-peak.



**Figure 14.** With the encoder not rotating, the system behaves like a standard encoder-table assembly. The difference between the angle measured with the encoder and the same measured with the capacitive sensor is shown for six data sets (in black, the average). The cyclic error without any correction is about 0.15" peak-to-peak.

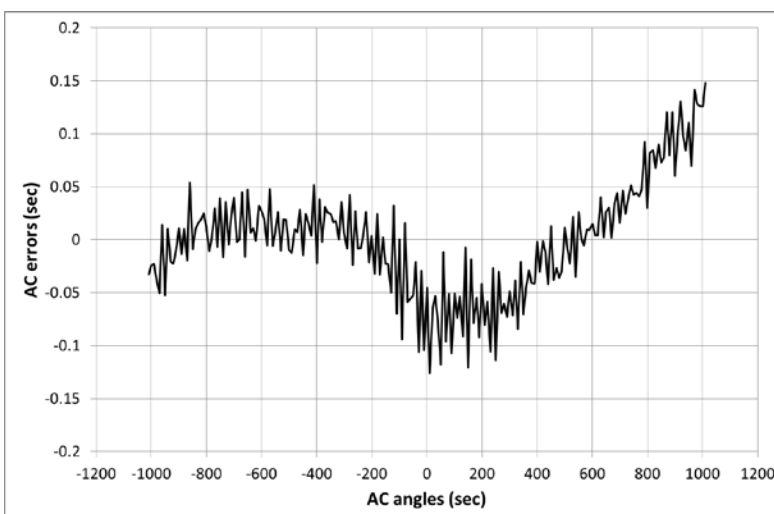
## 5 Calibration application and error analysis

### 5.1 Autocollimator calibration

Up to now, the autocollimators calibration at INRIM was performed by means of a sine-bar small angle generator (SAG) or with the nanoangle generator (NAG) [9]. The 0.2" accuracy of the SAG is limited by the resolution of the micrometric screw which generates the angle and the calibration procedure is completely manual. On the other side, the NAG is more accurate and automatic, but it can be used only for angular intervals down to 25".

Therefore, we are planning to substitute the old facilities for autocollimator calibration by the REAC. In order to test this possibility, the REAC was used to calibrate an autocollimator previously calibrated with the SAG. The autocollimator is a Moeller-Wedel Elcomat 3000, with a large angular range (about  $\pm 1000''$ ) and a resolution of 0.01".

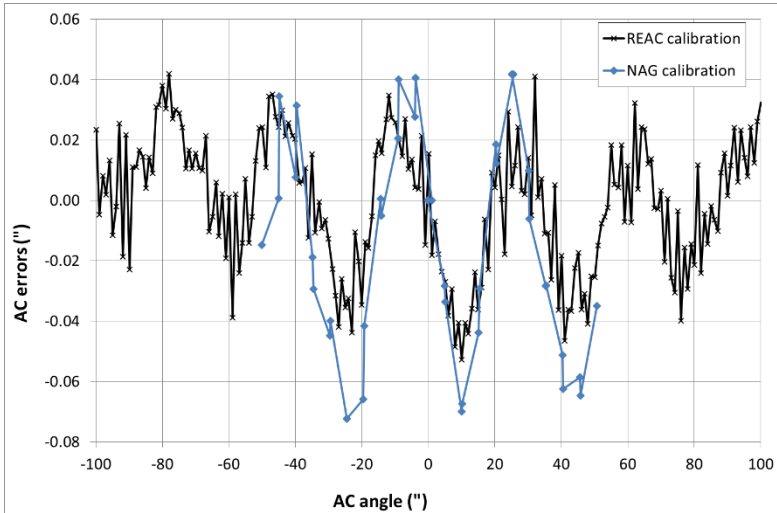
In figure 15, the AC errors measured with the REAC are shown. The results are compatible with the previous calibration performed with SAG with an uncertainty of 0.2".



**Figure 15.** AC Elcomat 3000 errors over the entire angular range measured by REAC. The fluctuations which appear to be random are instead mainly due to the undersampling of the oscillating AC behaviour visible in the short range calibration (fig.16).

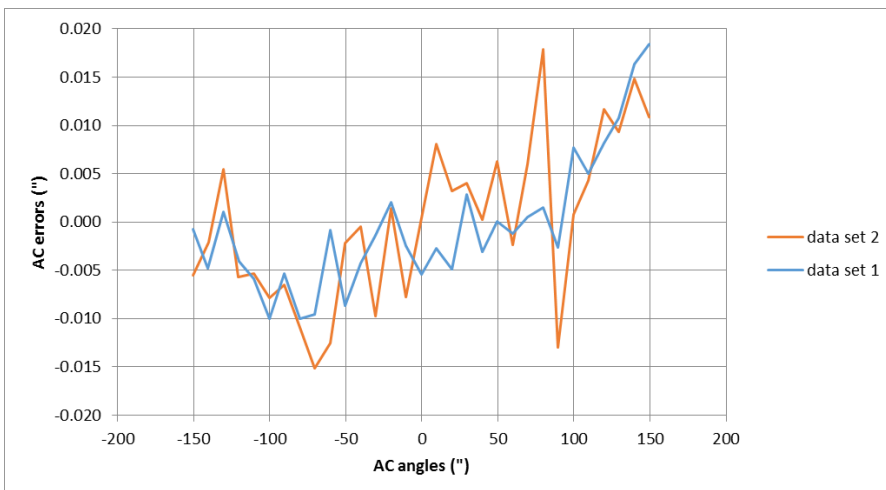
Then we have compared the calibration performed with the REAC with the one formerly made with the NAG in a small interval around zero, where typical AC nonlinearities (due to quantization of the pixel of the CCD sensor) must be investigated for high accuracy applications. The NAG measurement is in fact the stitching of four calibrations because of the limited range of the NAG (four runs  $\times 25'' = 100''$ ).

Also in this case, the agreement is within the uncertainty associated with the old measurements. The fluctuations visible in the short-range calibration (fig 16) are mainly due to the above quoted “pixel error” plus some additional random noise of the AC.



**Figure 16.** AC Elcomat 3000 errors in a limited angular range measured with the REAC (black) and with the NAG (blue).

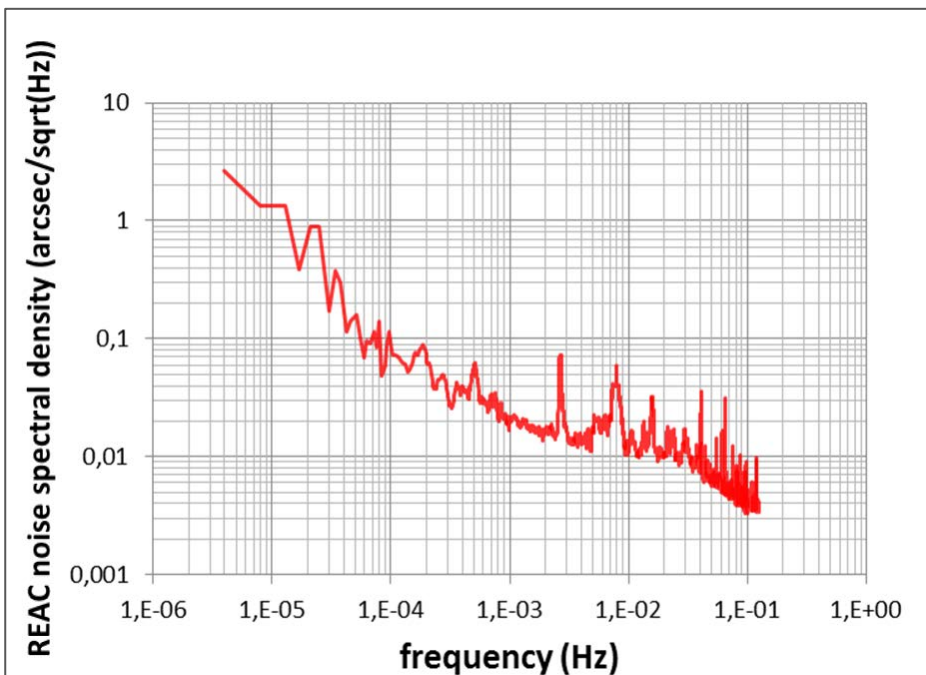
Finally, the REAC was used to calibrate a high-performance autocollimator, model Elcomat HR, with a resolution of  $0.001''$  in an angular range of  $\pm 150''$ . The measurement has been taken in two different positions of the REAC. The AC errors measured with the REAC are shown in figure 17. The results are compliant with the accuracy of  $\pm 0.02''$  stated by the manufacturer.



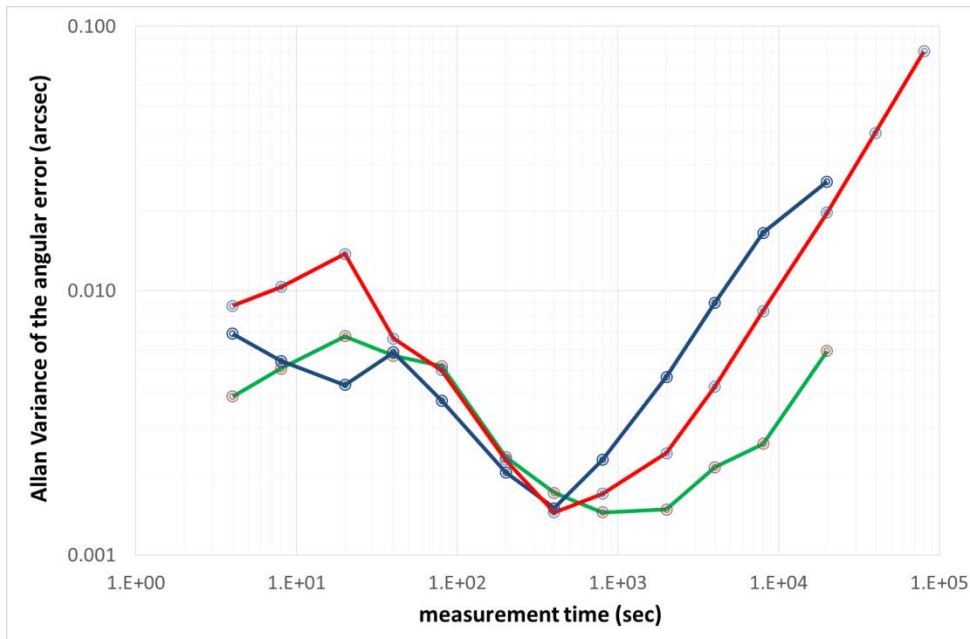
**Figure 17.** AC Elcomat HR errors measured by REAC. The two sets of measurement have been taken in two different positions of the REAC.

## 5.2 Noise analysis

In order to find the short and medium term stability of the REAC, we have performed a series of measurements with the upper table held fixed and recording the time series for several hours. Finally, the noise spectral density of the time series is calculated and plotted in figure 18. The plot gives the measurement limit of the REAC as a function of the measurement time. If the measurement time is of the order of few tens of seconds, the stability is of the order of 0.01 arc sec, while for measurements of the order of one hour or more, thermo-mechanical drifts cause instabilities of several hundredths of a second. In one day or more, the drift reaches the level of one second. This limit can be attributed mainly to the mechanical stability of the structure that is made of steel and has a relatively high thermal expansion coefficient, in combination with the temperature variations of the laboratory and of the pressurized air. Periodical fluctuations are visible at about  $3 \times 10^{-3}$  Hz (about 300 seconds) and at higher frequencies. These can be attributed to conditioning cycle as well as periodic pressure changes due to the compressor cycle. Both effect will be investigated. In order to evaluate the effect of the fluctuations on the measurement error, the best estimator is the Allan variance [12] since it gives the statistical error integrated in a time interval. In figure 18 is shown the Allan variance for different time series taken in different conditions. What we learn from the Allan variance plot is that for short time intervals ( $< 1$  min) the above described fluctuations have an important contribution. After 1 min integration time these effects are reduced to few thousandths of an arcsecond, for time intervals longer than about 1000 s (about 20-30 min) the error increases because of thermomechanical drifts. We can conclude that in the present working conditions a measurement carried on in the time interval 100 to 1000 s has a statistical error less than 0.01''.



**Figure 18.** REAC noise spectral density of the REAC with the table held still.



**Figure 19.** Allan variance of the REAC output with the table held still in three different runs. The red curve comes from the same time series treated in figure 18. The vertical axis is the angular error in arcseconds, the horizontal axis is the measurement time in seconds.

### 5.3 Uncertainty budget

The validation of the REAC through the comparison with the pre-existing INRIM angle facilities has shown full compatibility between the old and the new instruments, thus demonstrating the possibility of using the REAC for angle calibrations, both for small angles (ACs calibration) and for large angles (polygons and tables), from now on. Nevertheless, the purpose of the work was not only to update the metrological facilities, rather to reduce the uncertainty. Here we try to give an evaluation of the REAC uncertainty on the basis of the data we have collected up to now.

The main uncertainty sources we have considered are the large angle systematic errors of the encoder reading due to a non-perfect cancellation of the eccentricity error, the cyclic errors and the stability/noise errors. As a base to evaluate the systematic errors we have the two comparison made with the Moore tables and with the 12 sided polygon (described in paragraphs 3.1 and 3.2). Although the first one gave the worst result, we consider it as a reliable figure being it a complete cross-calibration. Therefore, referring to the results plotted in figure 9, the error of the table without any correction is  $\pm 0.04''$  (rectangular distribution). Obviously, once proven that this error is a constant of the instrument, it can be corrected thus greatly reducing the uncertainty, but in absence of further information we conservatively take this value for good.

As for the cyclic errors, we can state that, both from theoretical considerations and from the result shown in figure 13, the effect is negligible. We can conservatively attribute a maximum value of this effect equal to  $0.01''$  peak-to-peak. On the other hand, the stability is an important contribution to the uncertainty budget. The fluctuations measured in paragraph 5.2 will affect the result of a single measurement, leading to a spread in repeated measurements. The effect depends on the measurement time, i.e., when measuring an angular interval, the time occurring between the two measurements. As learned by the Allan variance plot we will perform angle measurements in the time interval between 1 min to half an hour (that means averaging the results taken in said time interval) and we can conservatively attribute to the measurement a normal distribution error with  $\sigma = 0.01''$ .



If the measurement must be performed in shorter time intervals the uncertainty will increase. In case of long measurements (e.g. polygon calibrations), as said instrumental drifts will become important, nevertheless, thanks to the self-zeroing properties of angular measurements, the drifts can be evaluated and corrected. In these cases, the uncertainty will be evaluated case by case.

## 6 Absolute encoder

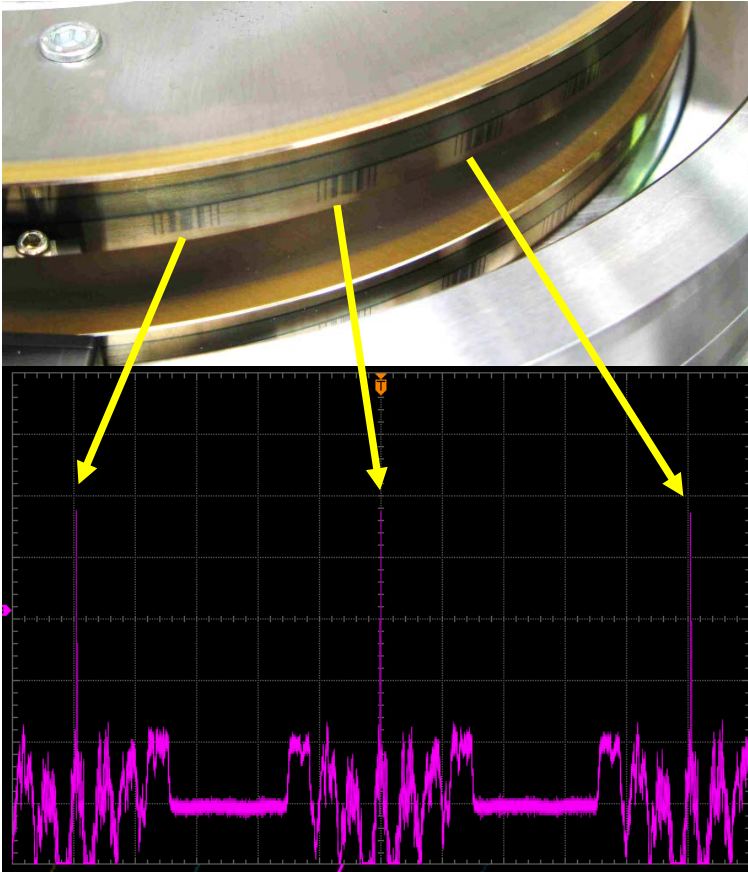
The rotating encoder principle has a further advantage with respect to the classical encoder configuration: the knowledge of the absolute angle is straightforward, so a simple and accurate absolute encoder can be realized.

A classical incremental encoder only provides the measurement of an angular interval, that is when is switched on the encoder “ignores” its absolute position and starts counting from its present position taken as the reference. Therefore, the knowledge of the absolute position of a table (i.e. the angular position with respect to a physical reference or zero), is usually implemented through a zeroing procedure in which the encoder is rotated until a reference mark (always present on precision encoders) is found, and starting the incremental measurement from said reference. In the worst case, the zeroing procedure requires a full 360° rotation. More refined encoders, as the one embedded in the REAC, has several reference marks, placed at unequal distance, so that the zeroing procedure requires a rotation of only a fraction of a revolution (9 degrees for the ERA 4200 40000).

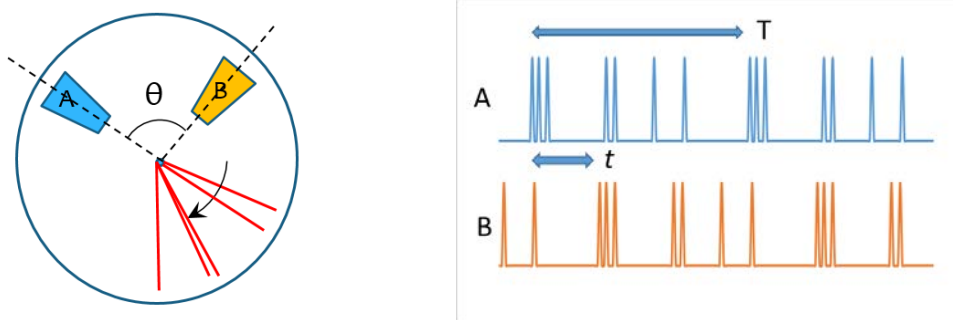
In application where a zeroing procedure is not desirable, “absolute encoders” are used. In contrast with an incremental encoder that measure angular intervals, an absolute encoder gives a unique value for any drum position. Absolute encoders have widespread applications in industrial plants (e.g. robots arms, elevators, etc.). The disadvantage is a relatively large uncertainty (at best few tenths of arc second) and a relatively high complexity of the drum and the reading heads.

The REAC has the advantage of being based on a high-resolution incremental encoder, and, thanks to the continuously rotating part, the reference marks are always read and the absolute angle of the table is known. Therefore, intrinsically the REAC can be referred to as an “absolute encoder” with the resolution and accuracy of an incremental encoder.

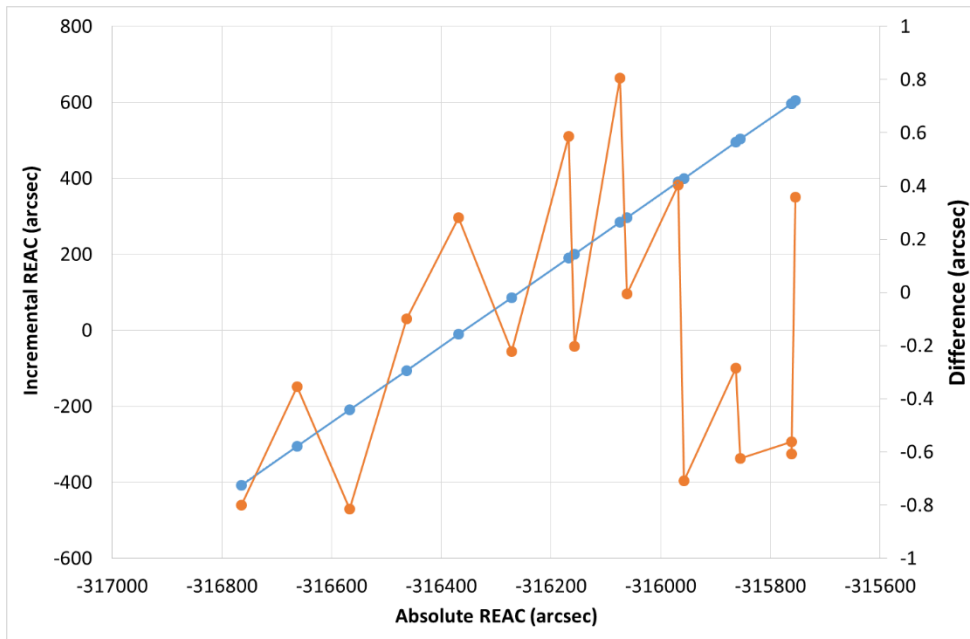
Another possible feature of the rotating encoder principle, which we want to underline, is that it is possible to realize an absolute encoder without the need of a regularly patterned rotating drum. Indeed, we envisage the realization of a simple encoder (a simplified version of the REAC) with a rotating drum, which could have any patterned structure on it read by two heads as in the REAC. The idea is that if the drum rotates at constant speed with period  $T$ , the two heads read the same structure with a time delay  $t$ , which is proportional to the angle  $\alpha$  with the simple relation  $\alpha = 2\pi \cdot t/T$ . Therefore, by applying some time dependent function, like autocorrelation, between the two signals it is possible to find the angle with an accuracy that depends on the quality of the electronic signals and of the uniformity of the rotation speed. In order to demonstrate the principle we performed an experiment by using the REAC. We have used the reference signals, quoted above, coming from the reading heads. These spike-like signals have non uniform angular distances with respect to each other and can be used as an example of a “random” structure to apply the autocorrelation principle. In figure 20, a record of three out of the 40 reference signals is shown. In figure 21 the simplified schematic of the experimental set-up is depicted. The angle is measured with the autocorrelation method for 11 angular positions over a 100° span and is compared with the angle measured with the REAC reading used here as a reference. The results are shown in figure 22: although the reference signals of the ERA 4200 were not intended to measure angles, it was possible to measure absolute angles with an accuracy better than 1 arcsec. The major limit of the experiment is the timing resolution of the acquisition system (500 kHz) which was not optimized for the purpose. To our opinion, this experiment paves the way to the realization of a new generation of simple though accurate absolute encoders.



**Figure 20:** The reference marks present on the ERA4200 drum are used as a pseudo-random pattern to test the absolute angle measurement principle.



**Figure 21:** The absolute angle measurement principle. The readings of the two heads have a repetition period  $T$ , and are shifted by a time  $t$  proportional to the absolute angle between the heads.  $t$  is found by autocorrelation function.



**Fig 22.** Comparison between the absolute reading (horizontal axis) and the REAC incremental reading (left vertical axis). The difference (in orange, right vertical axis) shows an agreement better than  $\pm 1''$ .

## Conclusions and discussion

A novel rotary table, called REAC, to be used as the angle standard in INRIM, has been built and tested. The table is based on the rotating grating principle. In principle the instrument is not affected by encoder errors and has virtually no interpolation errors. The structure is rather simple and compact and does not require complex electronics and computing tools. The REAC has been compared with the former INRIM's angle facilities and all tests have shown full compatibility so that the instrument can be used from now on to substitute the old national standards (except for the calibration of levels requiring a vertical angle). Furthermore both theoretical considerations and tests have been carried on to estimate the uncertainty of the device. The error of the table without any correction is within  $\pm 0.04''$  (rectangular distribution). Errors due to interpolation errors have been estimated to be less than  $0.01''$  peak-to-peak, while errors due to noise and stability are less than  $0.01''$  rms for measurement time intervals between about 1 minute to 1 hour. A software based on LabView and MatLab allows (almost) fully automated calibration of encoders, polygons and autocollimators. The REAC is now the new INRIM angular standard.

Future activity will be devoted to the comparison with other NMIs facilities by the circulation of accurate transfer standards such as ACs, polygons and angular encoders in order to give a definitive estimation of the accuracy of the REAC. A further activity will be the repetition of the measurement described in paragraph 5.1 with an autocollimator with better performances (Elcomat HR) also by adopting the shearing method described in [10]. Finally we plan to substitute the driving system with a piezo motor with the aim of increasing the positioning resolution and stability.

## Aknowlegments

The present work has been mainly funded by the EMRP Joint Research Project SIB 58 "Angles". The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union.

We wish to thank our colleague Massimo Zucco for his help in the realization of the REAC software, Tanfer Yandayan, Ralph Geckeler, José Salgado and Tsukasa Watanabe for their precious suggestions.

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